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R. Kumar, D. P. Edwards

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KAPL ATOMIC POWER LABORATORY

SCHENECTADY, NEW YORK 12301

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Interfacial Shear Modeling in Two-Phase Annular Flow

Ranganathan Kumar and David P. Edwards

Lockheed Martin Corporation

Schenectady, NY 12301

ABSTRACT

A new interfacial shear stress model called the law of the interface model, based on the law of the wall approach in turbulent flows, has been developed and locally applied in a fully developed, adiabatic, two-phase annular flow in a duct. Numerical results have been obtained using this model in conjunction with other models available in the literature that are required for the closure of the continuity and momentum equations. These results have been compared with droplet velocity data (using laser Doppler velocimetry and hot film anemometry), void fraction data (using gamma densitometry) and pressure drop data obtained in a R-134A refrigerant test facility. Droplet velocity results match the experimental data well, however, the prediction of the void fraction is less accurate. The poor prediction of void fraction, especially for the low void fraction cases, appears to be due to the lack of a good mechanistic model for entrainment.

Nomenclature

A'''	Interfacial area density
B,C,E,K	Modeling constants
d	Diameter
f_i	Friction factor
i	Interfacial node
I	Intermittency
k	Kinetic energy
k_s	Wall roughness
p	Neighboring node
P	Pressure, Pa
Re_t	Reynolds number based on duct thickness
t	Duct thickness, m
u	Axial velocity, m/sec
u^*	Friction velocity, m/sec
V_R	Relative velocity, m/sec
w	Mass rate of flow, kg/hr
y	Thickness dimension, m

Subscripts and superscripts

avg	Average
cv	Continuous vapor field
cl	Continuous liquid field
dl	Dispersed liquid field
dv	Dispersed vapor field
i	Interface
rel	Relative
sp	Single phase
h	Hydraulic

Symbols

α	Void fraction
δ	Liquid film thickness
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Density
τ	Shear stress

Introduction

In annular flow, the interface between the liquid and the vapor core is characterized by interfacial waves which play a dominant role in governing the system parameters. As described by Hall-Taylor *et al.* (1963), there are two types of waves: disturbance or roll waves, and small surface ripples, which act to both increase the interfacial shear stress and introduce droplets into the gas core.

By considering that the waves on the interface act as roughness elements, a two-phase friction factor similar to the pipe friction factor for single-phase flow can be developed. The liquid film is treated as a rough surface and can be expressed in terms of an equivalent sand roughness. Gill (1963) calculated the equivalent sand roughness from the pressure drop and gas Reynolds number in air-water flows using a single-phase friction factor chart. The roughness height appears to vary quadratically with the film thickness and is approximately independent of the film flow rate. Although this deduction was made with limited data, the data showed at least qualitatively that the interfacial shear increased with increasing film thickness.

The most widely used model for interfacial shear using a friction factor was proposed by Wallis (1969) who defined an interfacial friction factor, f_i , as

$$f_i = f_{sp} \left(1 + 300 \frac{\delta}{d} \right) \quad (1)$$

where δ is the film thickness and f_{sp} is the single-phase friction factor taken to be 0.005. Despite its popularity, Wallis' correlation does not predict interfacial shear in ripple-wave dominated air-water flows [Sekoguchi *et al.* (1978)] or in steam-water flows [Whalley and Hewitt (1978)].

Andreussi (1983) and Asali *et al.* (1985) used friction length as the characteristic length scale instead of the tube diameter. A law of the wall model has been attempted by Tomida (1974),

Hewitt (1991), Dobran (1983) and others. However, either they attempted to predict the friction factor from a logarithmic profile or they used it to evaluate the momentum diffusivity as a function of film thickness. None of the authors applied the law of the wall directly to calculate interfacial shear. Further, no attempt was made by any of these authors to apply the interfacial shear model in a multidimensional prediction.

From a review of the existing models, it is apparent that there is a need to develop a physically based model for interfacial shear that can be applied to a multidimensional prediction and that has the capability to include the wave characteristics. In the current paper, a law of the interface has been introduced to determine the interfacial shear and has been applied in a fully developed annular flow using a two-dimensional, two-fluid pressure-based finite volume code. The flow is divided into four fields (continuous liquid (cl), dispersed vapor (dv), dispersed liquid (dl), and continuous vapor (cv)). This four field prediction capability was previously described by Siebert *et al.* (1995) and has been found to be superior to the traditional two fluid model in capturing the true behavior of the flow. Void fraction and droplet velocity profiles have been obtained using the new interfacial model and have been compared with experimental data.

Governing Equations

The ensemble averaged phasic governing equations for the four fields are given in the following Cartesian tensor form.

Conservation of phasic mass:

$$\frac{\partial}{\partial t}(\alpha_i \rho_i) + \frac{\partial}{\partial x_k}(\alpha_i \rho_i u_i^k) = \sum_{j \neq i} (\Gamma_{ji} - \Gamma_{ij}) \quad (2)$$

Conservation of phasic momentum:

$$\begin{aligned} \frac{\partial}{\partial t} \left(\alpha_i \rho_i u_i^l \right) + \frac{\partial}{\partial x_k} \left(\alpha_i \rho_i u_i^k u_i^l \right) = & -\alpha_i \frac{\partial p}{\partial x_l} + \frac{\partial}{\partial x_k} \left(\alpha_i \mu_i \left[\frac{\partial u_i^l}{\partial x_k} + \frac{\partial u_i^k}{\partial x_l} \right] \right) \\ & + \underline{M_i^l + \sum_{j \neq i} \left[D_{ij} (u_j^l - u_i^l) + \Gamma_{ji} u_{ji}^{l-int} - \Gamma_{ij} u_{ij}^{l-int} \right]} \end{aligned} \quad (3)$$

Volume fraction constraints:

$$\sum_i \alpha_i = 1; \quad 0 \leq \alpha_i \leq 1 \quad (4)$$

Here, subscript i denotes the field under consideration ($i = \text{cl, dv, dl, cv}$). A double subscript denotes a quantity transferred from the field indicated by the first index (donor phase) to the field indicated by the second index (receptor phase). Superscript l denotes a vector component of velocity. Superscript int indicates an interface value. The double subscript on the interfacial value denotes the side of the interface that is under consideration. Γ represents the interfacial mass transfer rate between fields due to entrainment and deposition. $D_{ij}(u_j^l - u_i^l)$ represents the interfacial drag force between fields, and M_i^l represents non-drag interfacial forces (including Reynolds stresses) between fields.

The phases are coupled via interfacial transfer terms (underlined in Eqs. 2 and 3), and jump conditions are used to relate the transfer of mass, momentum, and energy across the interface. The continuity jump condition requires zero mass accumulation on the interface. The momentum jump condition dictates that the net force acting on the interface is balanced by surface tension effects. In drag-like terms, the velocity difference across the interface is equal to the difference between the phasic velocities, but in mass transfer terms, the mass is assumed to depart the donor phase at the donor phase velocity and arrive in the receptor phase with that same velocity. The

platform used for code development is FLOW3D which has been significantly modified to include the momentum transfer model (wall lateral force), the interfield model (entrainment/deposition), and the new interfacial shear model described below.

Wall Lateral Force

For bubbles flowing near a wall, a lateral force acts to move the bubbles toward the center of the duct. This force is balanced by the remaining bubbly flow forces resulting in a volume fraction peak slightly displaced from the wall. The model developed by Antal *et al.* (1991) has been shown to be effective for bubbly flows. In annular flows, a similar force is required to maintain the thin continuous liquid film on the walls, or conversely, to maintain the continuous vapor flow in the core. For this analysis, it was assumed that the annular flow force and this bubbly flow force take the same form.

The final form of the wall lateral force is written as

$$M^w = \frac{2\alpha_{dv}\rho_l V_{rel}^2}{d_b} \left(\left(C_{w2} \frac{D_h}{2y_{wall1}} \right)^a + (-C_{w1})^a \right)^{1/a} - \left(\left(C_{w2} \frac{D_h}{2y_{wall2}} \right)^a + (-C_{w1})^a \right)^{1/a} \quad (5)$$

where a is a smoothing parameter, y_{wall1} is the distance to the nearer wall, and y_{wall2} is the distance to the farther wall. The smoothing function was used to remove a discontinuity in the wall force. The hydraulic diameter was used as the characteristic vapor length. For this analysis, following an optimization study on the coefficients, values of -0.0000167 and 0.00002 were used for C_{w1} and C_{w2} .

Entrainment/Deposition Models

The entrainment and deposition models of Kataoka and Ishii (1982) were adapted to

multidimensional predictions by extending the original one-dimensional models onto a local basis. This was done by using local parameters in the definitions of the film Reynolds number, Re_f , liquid Reynolds number, Re_{ff} , and Weber number, We , as

$$Re_f = \frac{\rho_f (\alpha_{dl} u_{dl} + \alpha_{cl} u_{cl}) D_h}{\mu_f} \quad (6)$$

$$Re_{ff} = \frac{\rho_f \alpha_{cl} u_{cl} D_h}{\mu_f} \quad (7)$$

$$We = \frac{\rho_g (\alpha_{cv} u_{cv})^2 D_h}{\sigma} \left(\frac{\rho_f - \rho_g}{\rho_g} \right)^{1/3} \quad (8)$$

The local entrainment and deposition rates were calculated using these parameters in the original correlations.

Interfacial Area Density

In order to model the cv-cl interfacial area density, some assumptions about the interface must be made. For this analysis, the continuous liquid is assumed to flow as a film along the wall. The wall is assumed to be completely wetted ($\alpha_{cl} = 1.0$ at the wall), and the cl volume fraction decreases rapidly in the first few computational nodes away from the wall. In the real flow, the steepness of the transition from the wall ($\alpha_{cl} = 1$ to the core $\alpha_{cl} \sim 0$) is determined by the film thickness and wave characteristics of the interface. This waviness is determined by the flow rates, heat fluxes, and momentum forces acting on the interface. In this modeling, the smooth transition in α_{cl} may be interpreted as the effect of a wavy interface on the mean void distribution, although the details of the wavy structure of the interface are not captured in the modeling.

For smooth films flowing parallel to the walls, the interfacial area density can be calculated

from the gradient normal to the wall in the cl void fraction, or

$$A_{cl-cv}''' = \nabla \alpha \cdot \hat{n} \quad (9)$$

$\hat{n}$ is a vector normal to the wall. In the computational cells adjacent to the walls, the gradient was calculated assuming that the cl void fraction is 1.0 at the wall.

Development of Law of the cl-cv Interface

The two-phase mean velocity profiles for given flow conditions plotted as u_m vs. $\log(y/R_0)$ have been found to be linear as in single-phase velocity profiles [Gill (1964)]. The mean velocity profile slopes, i.e., the gradient of u_m with respect to $\log(y/R_0)$ change with changing flow conditions. In single-phase flow, the slope of the u_m curve remains constant for rough walls with the profiles shifting downward a constant amount depending upon the roughness factor [Clauser (1956)]. A 'rough' wall may be considered similar to a 'wavy' surface, and guidance may be sought from single-phase flow analysis in developing a two-phase law of the interface model. Wallis (1969) showed that at atmospheric pressure for air-water flows, a wavy annular film is equivalent to a sand roughness of four times the film thickness over the range $0.001 < k_s/D < 0.03$, where k_s is the grain size of the equivalent sand roughness.

In single-phase flow the turbulent boundary layer consists of a viscous sublayer and the defect layer. When the outer and the inner layers are matched, the velocity profile in the overlap region can be expressed in terms of u^+ and y^+ based on friction velocity. The law of the wall may be written as

$$u^+ = \frac{1}{\kappa} \ln y^+ + B - \Delta B \left(k_s^+ \right) \quad (10)$$

where B is the smooth wall single-phase flow constant which is usually 5.5. The Von Karman constant, κ , is 0.4 for single-phase flow and the function, ΔB captures the effects of the rough wall. ΔB increases with roughness, k_s^+ . Based on Clauser's (1956) data for large k_s^+ , ΔB can be written as

$$\Delta B = \frac{1}{\kappa} \ln \left(1 + C k_s^+ \right) \quad (11)$$

where C is 0.3 in single-phase flows, and is as yet undetermined in two-phase flows.

Justification of the Non-Dimensional Parameters

The non-dimensional quantities are defined relative to the wall conditions in single-phase flows. In the vapor core in annular two-phase flow, it is more appropriate to non-dimensionalize the velocity using interfacial conditions. Thus,

$$u^+ = \frac{(u - u_i)}{u^*} \quad (12)$$

where $u^* = \sqrt{\tau_i / \rho_{cv}}$ and u_i is the average interface velocity. The non-dimensional distance, y^+ , from the interface is defined in terms of $(y - \delta)$ where δ is the distance from the wall to the interface. The definition of y^+ and u^+ require that the interfacial velocity and the interface distance be known *a priori*. Due to distortion of the interface by waves of small and large amplitudes, it is difficult to define δ . The air-water experiments of Chien and Ibele (1964) and Ueda and Nose (1974) indicate that the liquid film can be divided into two layers: the continuous liquid layer that extends from the wall to the wave troughs, and the disturbed wavy layer of thickness from the crests to the troughs of the waves. In the continuous layer of the liquid film, the turbulent momentum diffusion is governed by the conditions near the wall, and the effective momentum

diffusivity should have a structure similar to that for single-phase flow. In the wavy region of the film, the diffusion distance is proportional to the wave height [Dobran(1983)]. The use of a nondimensional parameter involving δ may not be appropriate. Furthermore, the difficulty in identifying δ makes implementation difficult.

Because the interface is covered by small ripples and disturbance waves, it can be characterized by patches of rough and smooth surfaces of liquid of varying thickness. Based on these observations and previous discussions, in a discretized domain for numerical simulation, it is justified to assume that the interface exists at every node. Following Figure 1, the interface exists at node i . On this node i , the interfacial shear can be calculated using quantities at node p , one node away from the interface node i . Thus, $(y-\delta)$ is proportional to Δy , and u_i becomes u_{cl} . The non-dimensional parameters are rewritten as

$$u^+ = \frac{(u - u_{cl})}{u_*} \quad (13)$$

and

$$y^+ = \frac{K \Delta y u_* \rho_{cv}}{\mu_{cv}} \quad (14)$$

where $K\Delta y$ is the distance between the interface node and the next node perpendicular to the wall where continuous vapor quantities will be used for computation. To allow for the patches of rough and smooth film surfaces, a factor of intermittency, K , based on the wave frequency and the wave height may be used. In this paper, K is taken to be 1.

Interfacial Shear Model

Near the interface, the interfacial stress is related to the turbulence kinetic energy by

$$\tau_i = C_\mu^{0.5} \rho_{cv} k_{cv} \quad (15)$$

This relationship, with the friction velocity, can be used to redefine the scaled variable, y^+ , as

$$y^+ = \frac{\rho_{cv} (y_p - y_i) C_\mu^{0.25} k_{p,cv}^{0.5}}{\mu_{cv}} \quad (16)$$

The wall function type approximation requires that the velocity normal to the interface is zero in the neighborhood of the wall, and the velocity parallel to the interface is related linearly to the interfacial shear stress. Therefore, the interfacial shear stress can be written as

$$\tau_i = T_m (u_{p,cv} - u_{i,cl}) \quad (17)$$

where

$$T_m = \frac{\rho_{cv} C_\mu^{0.25} k_{p,cv}^{0.5}}{u_p^+} \quad (18)$$

The non-dimensional velocity, u_p^+ , can be obtained from Eq. (10) and recast as

$$u_p^+ = \frac{1}{\kappa} \ln(E y^+) \quad (19)$$

where E and κ are new constants to be determined for two-phase flows. Equations (17), (18), and (19) provide the interfacial shear stress at node i in terms of the neighboring node p away from the wall. It is calculated at every nodal point in the flow field including the nodes in the vapor core where there is no cl-cv interface. The local implementation of this is described in the next section.

C_μ in Eq. (18) takes a constant value of 0.09. The constants E and κ in Eq. (19) are 9.0 and 0.4, respectively, in single-phase flows, but may vary in two-phase flows.

Wall Roughness

The current model differs from the Wallis model by considering the turbulent interface rather than calculating an overall friction factor. To obtain the modeling constants and compare them with previous models, the interfacial shear stress was converted to a friction factor. Although Wallis developed his correlation based on air-water data for selected flow conditions, such a comparison provides a useful estimate of the modeling coefficient to a first approximation.

The modeling constant, E , in Eq. (19) depends on the value of C in Eq. (11). As a first approximation, these values can be estimated by comparing the friction factor from the current model with the Wallis model. The duct Reynolds number is written as

$$Re_t = \frac{\rho t u_{avg}}{\mu} = \int_0^{t^+} u^+ dy^+ = \frac{t^+}{\kappa} \left(\ln t^+ + B\kappa - \Delta B\kappa - 1 \right) \quad (20)$$

The interfacial friction factor, f_i , becomes

$$f_i = 2 \frac{u^2}{u_{avg}^2} = 2 \frac{t^{+2}}{Re_t^2} \quad (21)$$

Using the above equations, the following relationship for friction factor can be obtained (see Appendix A).

$$\sqrt{\frac{1}{4f_i}} = 2 \log_{10} \left[\frac{Re_t \sqrt{4f_i}}{1 + \frac{C}{\sqrt{8}} Re_t \sqrt{4f_i} \frac{k_s}{t}} \right] + 0.142 \quad (22)$$

The factor $(4f_i)$ is the Darcy friction factor, Λ . From the classic Prandtl-Schlichting sand-grain roughness curve for single-phase flow, $C \sim 0.3$. For this value of C , Eq. (22) reduces to that given for single-phase flow [White (1974)]. For large values of $Re_t \sqrt{4f_i}$, the interfacial friction factor is written as

$$f_i = \frac{1}{4} \left[0.868 \ln \left(\frac{\sqrt{8} t}{C k_s} \right) + 0.142 \right]^{-2} \quad (23)$$

Simplifying this to Wallis' form, C is found to be 12.2. The law of the interface becomes

$$u_p^+ = \frac{1}{\kappa} \ln \left[\frac{e^{B\kappa}}{1 + 12.2 k_s^+ y^+} \right] \quad (24)$$

The quantity within brackets is Ey^+ . In two-phase flow, the E -value is dependent on the wave roughness which in turn depends on the wave frequency and wave velocity. It is two orders of magnitude smaller than that for single-phase flow, reducing the value of u_p^+ by at least an order of magnitude, and increases the interfacial shear. For example, for $k_s^+ \sim 30$, and $y^+ \sim 100$, the shear calculated with wall roughness is about 7.58 times that for the smooth wall. A value of 7.0 was used in the present calculations which provided the same exit volume fraction as Wallis' model for the same inlet conditions.

Discussion of Results

The current model was implemented into the multi-field, multidimensional solver, and the results were compared with adiabatic annular flow data obtained in a refrigerant (R-134A) test facility. Averaged void fraction data and droplet velocity were obtained using a gamma densitometer and a laser Doppler velocimeter, respectively. Table 1 provides the flow conditions for which the computational results were obtained. A 16x16 grid was used for low void fraction

cases, and the results were grid independent. For grid independent results at higher void fractions, denser grids of 16x28 or 16x32 were required to resolve the thin films.

The axial development of the phasic volume fractions is shown in Figure 2. The annular flow structure develops very rapidly, and the annular film develops and reaches a fully developed condition within the first half of the axial flow length. However, the development of the droplet flow in the core takes a slightly longer development length.

Results have been obtained using both Wallis' model and the current model. When Wallis' interfacial shear is applied on a local basis, it must be modified to account for the effect of averaging on the model. In the formulation of the interfacial shear, the magnitude of the shear depends on V_R^2 . In the original one-dimensional model, this relative velocity is the difference between the average liquid film velocity and the average vapor core velocity. In the local formulation, the relative velocity is the difference between the local liquid and vapor velocities, the difference being much smaller. The relative velocity has been multiplied by a distribution parameter of 1.34 to account for the effect of this difference. A comparison of the local prediction and the equivalent one-dimensional interfacial shear rate using this distribution parameter is shown in Figure 3. From this figure, it appears that the distribution parameter should increase slightly with the mass flux to properly capture the effects of averaging, but for this analysis it has been maintained as a constant. It should be noted that such a distribution parameter was not used in the law-of-the-interface model to predict interfacial shear.

The friction factor for the current model was obtained and matched with the Wallis correlation. The non-dimensional roughness factor, k_s^+ , or alternatively, T_m , given in Eq. 18, was obtained by matching the inlet and the exit void fractions using both models. Although the same value of k_s^+ was used for the rest of the runs, at high void fractions and high flow rates, this value

may have to be changed to account for very thin films.

In Table 1, the test conditions are provided along with the computed values of the interfacial shear stress, the average velocities in the cl, dl and cv fields and the test section pressure drop from the current model. For the four cases predicted here, the interfacial shear and pressure drop in the test section decrease with increasing void fraction. This behavior is consistent with the available one-dimensional models. The interfacial shear stress calculated from the current model differed from Wallis' model predictions by 6% to 8%.

A comparison of the measured and predicted pressure drop for these flows is shown in figure 4. The predicted pressure drop for the smooth wall is seen to be 15% to 20% lower than the measured values. This is due to the application of the standard, single-phase turbulent smooth wall function boundary conditions used in the momentum equations. It is apparent from this analysis that the wall shear is larger in the film than is predicted by the wall functions and requires some modification. The law of the wall has been modified to account for roughness ($60 < k_s^+ < 100$), and predicts pressure drop much better.

The droplet velocity and the void fraction profiles were available from the refrigerant test facility, and they have been compared with the current model and Wallis' model (Figures 5 through 8). In every case, the two models predict very similar void profiles, but tend to overpredict the void data, in general. As the average void fraction increases, the two models tend to predict the experimental data better, with the best agreement for $\alpha=92\%$. Since both models were run with the same entrainment/deposition models and the same droplet size of 0.5 mm, the conclusion one may draw from these overpredictions of α_{cv} is that the entrainment model used is not adequate. It should be noted that no entrainment data are currently available to validate the entrainment model that is adapted for the duct using circular tube data.

The striking difference in the predictions between the current model and Wallis' model appears in the droplet velocity prediction. In almost every case, the current model predicts much lower velocity than Wallis' model, and predicts the data much better. In the current model, the shear stress is calculated using the c_l and c_v velocities at neighboring nodes, i.e., across the interface. This appears to be a more appropriate way to calculate the interfacial shear in annular flow. In addition, the average relative velocity in the c_v - d_l fields is much higher in the current model compared to Wallis' model. For example, the average c_v - d_l velocity difference in the current model is nearly 40% for Case 1, whereas it was less than 10% using Wallis' model.

Summary

A law of the interface model has been developed in annular flow to determine the interfacial shear and implemented in a modified computer code, FLOW3D, along with models for wall lateral force, entrainment and deposition. Interfacial shear relates the relative velocity in the continuous liquid and vapor fields across the interface, and is applied locally. The model deviates from Wallis' correlation in that the current model calculates the interfacial shear directly and does not use a friction factor. The current model appears to predict the droplet velocity data better than Wallis' correlation. The entrainment model used here severely underpredicts the droplet volume fraction. Further modeling work in this area is warranted. As a consequence of the interfacial waves breaking at the tips, droplet deposition and droplet entrainment processes have a significant impact on the shear stress. The current models for entrainment/deposition need to be reevaluated and their impact on the current interfacial shear model tested.

Additional data on the interfacial wave frequency, amplitude and velocity will be needed to support further development of the proposed interfacial shear model. The current model allows wave characteristics which should be incorporated.

Table 1: Computational Results for the Test Cases

Case #*	α	grid	τ_i , Pa	$\overline{u_{cl}}$ m/sec	$\overline{u_{dl}}$ m/sec	$\overline{u_{cv}}$ m/sec	ΔP Pa
1	0.65	16x16	2.89	0.37	0.58	0.98	6100
2	0.75	16x16	2.58	0.41	0.66	1.05	4930
3	0.84	16x28	2.23	0.49	0.80	1.14	4330
4	0.92	16x32	1.76	0.59	0.84	1.29	3965

* All cases were run at $w=106$ kg/hr and $P=2440$ kPa.

References

- Andreussi, P. (1983) "Droplet Transfer in Two-Phase Annular Flow," *Int. J. Multiphase Flow*, Vol. 9, No. 6, pp. 697-713.
- Antal, S.P, Lahey, R.T., Jr. and Flaherty, J.E. (1991) "Analysis of Phase Distribution in Fully Developed Laminar Bubbly Two-Phase Flow," *Int. J. Multiphase Flow*, Vol. 17, No.5, pp. 635.
- Asali, J.C., Hanratty, T.J. and Andreussi, P. (1985) "Interfacial Drag and Film Height for Vertical Annular Flow," *AIChE J.*, Vol. 31, pp. 895.
- Chien, S. and Ibele, W. (1964) "Pressure Drop and Liquid Film Thickness of Two-Phase Annular and Annular-Mist Flow," *J. Heat Transfer*, Vol. 86, pp. 80-96.
- Clauser, F.H. (1956) *Adv. Appl. Mech.*, Vol. 4, pp. 1-51.
- Dobran, F. (1983) "Hydrodynamic and Heat Transfer Analysis of Two-Phase Annular Flow with a New Liquid Film Model of Turbulence," *Int. J. Heat Mass Transfer*, Vol. 26, No. 8, pp. 1159-1171.
- Gill, L.E., Hewitt, G.F., Hitchon, J.W. and Lacey, P.M.C. (1963) "Sampling Probe Studies of the Gas Core in Annular Two-Phase Flow-I: The Effect of Length on Phase and Velocity Distribution," *Chem. Eng. Sci.*, Vol. 18, pp. 525-535.
- Hall Taylor, N.S., Hewitt, G.F. and Lacey, P.M.C. (1963) "The Motion and Frequency of large Disturbance Waves in Annular Flow in a Long Vertical Tube," *UKAEA Report AERE-R 6012*.
- Hewitt, G.F. (1991) "Annular Flow Phenomena," *Proc. Japan Soc. Multiphase Flow on Waves and Turbulence in Multiphase Flow, and its Application*, September, pp. 126Nig-154.
- Kang, H.C. and Kim, M.H. (1992) "Measurement of Three-Dimensional Wave Form and Interfacial Area in an Air-Water Stratified Flow," *Nuclear Engineering Design*, Vol. 136, pp. 347-360.
- Sekoguchi, K., Hori, K., Nakazatomi, M. and Nishikawa, K. (1978) "On Ripple of Annular Two-Phase Flow: Part 2: Characteristics of Wave and Interfacial Friction Factor," *Bull. JSME*, Vol. 21, No. 152, pp. 279-286.
- Tomida, T., Yamamoto, T., Takebayashi, T. and Okazaki, T. (1975) "Correlation of Data on the Apparent Friction Coefficient in Upward Two-Phase Flow of Air-Liquid Mixtures," *J. Chem. Eng. of Japan*, Vol. 8, No.2, pp. 113-118.

Ueda, T. and Nose, S. (1974) "Studies of Liquid Film Flow in Two-Phase Annular and Annular-Mist Flow Regions; Part 2, Upflow in a Vertical Tube," *Bull. Japan. Soc. Mech. Engrs*, Vol. 17, pp. 614-624.

Wallis, G.B. (1969), *One-dimensional Two-Phase Flow*, McGraw-Hill, New York.

Whalley, P.B. and Hewitt, G.F. (1978), The correlation of liquid entrainment fraction and entrainment rate in annular two-phase flow, *AERE-R9187*.

White, F.M. (1974), *Viscous Fluid Flow*, McGraw-Hill Book Company, New York.

Figure 1: Schematic of the computational grid across the interface

Figure 2: Volume fraction distribution for the exit plane and cross sectional average volume fraction development of an adiabatic flow prediction. Phase 1 is cl, phase 2 is dv, phase 3 is dl, phase 4 is cv.

Figure 3: Comparison of the average cl-cv interfacial shear stress for a two-dimensional prediction of flow in R-134A at $P=2440$ kPa. The results from the current models are compared to the original one-dimensional models for the same average flow conditions.

Figure 4: Comparison of experimental and predicted pressure drop for R-134A flow in a rectangular duct.

Figure 5: FLOW3D Predictions of Void Fraction and Droplet Velocity Using the Current Model and Wallis' Model and Comparison with Case 1 data ($\alpha=65\%$)

Figure 6: FLOW3D Predictions of Void Fraction and Droplet Velocity Using the Current Model and Wallis' Model and Comparison with Case 2 data ($\alpha=75\%$)

Figure 7: FLOW3D Predictions of Void Fraction and Droplet Velocity Using the Current Model and Wallis' Model and Comparison with Case 3 data ($\alpha=84\%$)

Figure 8: FLOW3D Predictions of Void Fraction and Droplet Velocity Using the Current Model and Wallis' Model and Comparison with Case 4 data ($\alpha=92\%$)

Appendix A

Determination of Modeling Constant

The duct Reynolds number is defined in terms of the law of the cl-cv interface as follows:

$$Re_t = \frac{\rho t u_{avg}}{\mu} = \int_0^{t^+} u^+ dy^+ = \frac{t^+}{\kappa} \left(\ln t^+ + B\kappa - \Delta B\kappa - 1 \right) \quad (A.1)$$

The friction factor is now written in terms of Reynolds number as

$$f_i = 2 \frac{u^{*2}}{u_{avg}^2} = 2 \left(\frac{\rho t u^*}{\mu} \right)^2 \left(\frac{\mu}{\rho t u_{avg}} \right)^2 = 2 \frac{t^{*2}}{Re_t^2} \quad (A.2)$$

from which

$$Re_t = \sqrt{\frac{2}{f_i}} t^+ \quad (A.3)$$

Substituting Eq. (A.3) into Eq. (A.1),

$$\sqrt{\frac{2}{f_i}} t^+ = \frac{t^+}{\kappa} \left(\ln t^+ + B\kappa - \Delta B\kappa - 1 \right) \quad (A.4)$$

where

$$\Delta B = \frac{1}{\kappa} \ln(1 + Ck_s^+) \quad (A.5)$$

where C is a constant to be evaluated. Eq. (A.4) becomes

$$\sqrt{\frac{2}{f_i}} = \frac{1}{\kappa} \left(\ln \left(\frac{t^+}{1 + Ck_s^+} \right) + B\kappa - 1 \right) \quad (A.6)$$

Using $B\kappa = 2.2$, and converting natural log to $\log_{10}$,

$$\sqrt{\frac{2}{f_i}} = \frac{1}{\kappa} \left[2.035 \log_{10} \left(\frac{Re_t \sqrt{\frac{f_i}{2}}}{1 + Ck_s^+} \right) \right] + 1.2 \quad (A.7)$$

Now, k_s^+ can be rewritten as

$$\begin{aligned} k_s^+ &= \frac{k_s u^*}{v} = \frac{t u^*}{v} \frac{k_s}{t} = \frac{t u_{avg}}{v} \frac{u^*}{u_{avg}} \frac{k_s}{t} \\ &= Re_t \sqrt{\frac{f_i}{2}} \frac{k_s}{t} \end{aligned} \quad (A.8)$$

Dividing both sides of Eq. (A.7) by $\sqrt{8}$ to obtain Darcy friction factor, $\Lambda = 4f_i$, and using Eq. (A.8),

$$\frac{1}{\sqrt{\Lambda}} = 2\log_{10}\left(\frac{Re_t\sqrt{\Lambda}}{1 + \frac{C}{\sqrt{8}}Re_t\sqrt{\Lambda}\frac{k_s}{t}}\right) + 0.142 \quad (\text{A.9})$$

For $C=0.3$, this equation reduces to that given for single-phase flow [White (1974)]. For large $Re_t\sqrt{\Lambda}$, the friction factor, f_i , is

$$f_i = \frac{1}{4}\left[2\log_{10}\left(\frac{\sqrt{8}t}{Ck_s}\right) + 0.142\right]^{-2} \quad (\text{A.10})$$

The above is then converted back to natural log so that series expansion on the logarithmic function can be performed. Retaining only the first term in the expansion, and assuming $k_s/t \ll 1$, the friction factor becomes

$$f_i = \frac{1}{4}\left[1.879 - 0.614C\frac{k_s}{t}\right]^{-2} \quad (\text{A.11})$$

This expression can then be compared to Wallis' friction factor given by

$$f_i = f_{sp}\left(1 + 300\frac{\delta}{d_h}\right) \quad (\text{A.12})$$

For $d_h=2t$, assuming $k_s \sim \delta$, and using algebraic manipulations, it can be shown that equations (A.11) and (A.12) are equivalent if $C=12.2$. Now, the law of the wall can be written in terms of the new wall roughness coefficient as

$$u_p^+ = \frac{1}{\kappa} \ln \left[\frac{e^{B\kappa}}{1 + 12.2k_s^+} y^+ \right] \quad (\text{A.13})$$

The quantity within brackets is Ey^+ . In single-phase flows, E is approximately 9.0. In two-phase flow, the above expression shows that E is at least two orders of magnitude smaller than that for single-phase flow, reducing the value of u_p^+ by at least an order of magnitude. This increases the interfacial shear which varies as the inverse of u_p^+ .