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FROM FLUX MODELS**

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ESTIMATES OF CURRENT DEBRIS FROM FLUX MODELS

Gregory H. Canavan

Flux models that balance accuracy and simplicity are used to predict the growth of space debris to the present. Known and projected launch rates, decay models, and numerical integrations are used to predict distributions that closely resemble the current catalog—particularly in the regions containing most of the debris.

Estimates of the growth and decay of space debris from flux models can balance accuracy and simplicity to permit fidelity in modeling production and decay processes while retaining transparency of results. Production and decay are of comparable importance in determining the rate of growth, level of saturation, and time for decay of space debris. These estimates use a combination of historical data and current launch rates projections to estimate debris production and models of the decay of eccentric orbits to treat removal by orbital decay. These launch rates are used to predict spacecraft, rockets, and mission related debris. Fragments are estimated from laboratory, field, and on-orbit experiments. Known explosions account for most of the fragment end of the catalog; the satellites on now on orbit account for most of the intact objects. Other sources are assumed to have placed objects at low altitudes, where they would not be treated accurately by these calculations. The estimates include collisional cascading, although it is a small for the parameters and times studied.

This note focuses on understanding the current debris environment and how its structure constrains the assumptions that can be made in various models. Integrating the model forward to the present from an initial condition of zero debris 35 years ago gives a distribution that closely resembles the current catalog—particularly in the regions containing most of the debris and having some potential for cascade growth. Discrepancies with the current catalog, which are modest, are discussed and used as the basis for comments on possible extensions of this work. The principal discrepancy is that between predicted and observed densities below 600 km, which cannot be satisfactorily addressed by the fragment growth rate and areal density, the two main parameters in the estimates. The note also explores a model that averages out the effects of fragment eccentricity to produce a simpler set of equations. While there are significant errors in the initial formulation of the model, it can be modified to 10% accuracy, which is about that in the catalog and better than that in the source models.

It appears possible to predict the current catalog with some accuracy, particularly at the altitudes that contain most of the debris, and it is possible to do so with simple treatments of sources, losses, and eccentricity. However, the results depend on only a few parameters, which cannot be adjusted to address the difference between the predicted and observed debris densities and fluxes at 600 km and below. The consistency of these results and their transparent dependence

on the initial conditions and model indicate that these models and sources are calibrated about as well as possible for other, related predictions. The sections below discuss launch rates, debris production, orbital decay, model calculations, results, interpretations, and conclusions in order.

Launch rates are reviewed in a companion note.¹ Its principal result relevant to this study is shown in Figure 1, in which the solid symbols represent the global launch rates per decade to all orbits and the open symbols represent the launch rates to low Earth orbit (LEO). Launch rates to LEO decline primarily due to the 2-fold decrease of the CIS launch rate in the last decade and the continuing shift of defense and commercial payloads from LEO to geosynchronous Earth orbit (GEO). This shift has been roughly linear in time, is occurring about equally rapidly in the U.S. and CIS, and has now reached a fraction of almost 50% of successful launches directed to GEO.

There would be many fewer launches to LEO in the next century were it not for planned communication constellations. The bottom curve represents a 100 satellite constellation replenished by 4 satellites per launcher every 10 years, which requires ≈ 25 launches per decade. The upper open curve represents a 1000 satellite constellation replenished by 2 satellites per launcher every 10 years, which requires 50 launches per year. For the 100 satellite constellation, the launch rate falls to about 30/year by the end of the century, which is $\approx 1/4$ of the peak rate of the last decade. For the 1000 satellite constellation, the launch rate falls to about 80/yr, which is $\approx 2/3$ rd of the peak. Planned U.S. communication constellations are used for specificity, but they are typical of those in preliminary discussion elsewhere. History suggests it is unlikely that more than one will be economically viable and that the preferred system is likely to be the one deployed first.

To resume the historical peak launch rate of the previous decade would require that the CIS resume cold-war launch rates or for the current shift of payloads from LEO to GEO to be reversed. There is little basis for projecting growth in defense or civil LEO activities. Thus, achieving launch rates half as large as the historical peak would require a ≈ 1000 satellite LEO communication constellation, and achieving a rate one quarter that large would require ≈ 100 satellites. For smaller constellations, the launch rates to LEO would fall to about 10% of the historical peak.

Fragment production. Most debris comes from spacecraft, rockets, mission-related objects released during deployment, and fragmentation or explosions. Spacecraft and rockets are observed and cataloged. Mission-related objects are fewer, smaller, and shorter lived. Fragments are more difficult to treat because they are produced on orbit through processes that are poorly understood and quantified, so their number, size, and mass is known imperfectly. Over the last 35 years, there have been about 120 explosions out of 3500 launches, which have produced about 7000 LEO fragments, i.e., about 200 fragments per year. That gives 60 fragments per explosion, or 2 fragments per launch, which is comparable to the number of satellites, rockets, and mission-related objects per launch.

However, the density of fragments on orbit is not directly related to these launch or

explosion rates. Figure 2 shows that the number of fragments and mission related objects saturated at 3,000 and 1,000 objects, respectively, about two decades ago. They have not increased since, although the rate of fragmentations has remained significant and about constant. It would appear that fragments and mission-related objects have come into a quasi-equilibrium, in which they are removed by orbital decay as fast as they are produced by launches and explosions. If so, their steady-state level should be roughly the ratio of these sources and losses. If launch rates fall by a factor of two to four and the number of explosions per launch did not change, the number of fragments should fall by factors of two to four, at which their numbers would be small relative to the number of large, intact objects discussed below.

All on-orbit fragmentations are thought to have been observed, and an average of ≈ 60 fragments per explosion have been placed in the Air Force Space command (AFSPC) catalog and tracked to decay or to their present location. To provide a source of fragments for this study, the ephemerides of the fragmentation sources² were combined with the measured ≈ 0.2 km/s isotropic fragment distributions to provide the fragment source, which is shown together with the AFSPC catalog, in Fig. 3.³ The two curves have peaks at similar altitudes, and similar minima in between, but they also have significant differences. Below about 700 km, the fragment source lies far above the catalog, because the source represents the number of fragments produced, before their reduction by orbital decay, while the catalog includes the full 35 years of decay, which is strong at those altitudes. The source is larger than the catalog up to about 900 km. The source is slightly below the peak of the catalog at 950 km and far below the peak at 1,450 km, which indicates that objects larger than fragments must be invoked to explain those peaks. The source lies above the catalog above about 1,550 km, although there are relatively few particles at those altitudes.

The rate of fragmentation has been relatively constant over the last 35 years, so dividing the total historical production by 35 years gives the average number of fragments produced per year. Integrating that result over 35 years, using the decay rates derived below, roughly recovers the fragment portion of the current debris catalog, which is a check on the consistency of the estimate. Mission-related debris objects are generally smaller, are less well known, and decay faster, so they are modeled below by a constant multiplier to the altitude-resolved average fragment source rate.

Intact object production. The large, intact spacecraft and rockets in orbit represent a small fraction of the total number of debris objects, but a significant fraction of the area in the catalog and the mass in estimates derived from it. Because of their large mass to area ratios, large objects are less affected by drag, so they remain in orbits for decades, centuries, or millennia, depending on their orbits.⁴ Figure 2 indicates that of the roughly 3500 rockets and 4500 spacecraft launched to date, about 2,000 spacecraft and 1,000 rockets—about 38% of the total launched—remain in orbit, which gives an average growth rate of about $3000/35 \approx 85$ intact objects per year during the peak launch rates of the previous decade.⁵ The total number of debris objects has not

grown in the last decade, because the increase in the number of intact objects has been offset by the decrease in the number of fragments and mission-related objects. Spacecraft have higher densities than empty rockets, which gives them about a factor of three to five higher mass per unit area, and hence about a factor of three to five longer lifetime;⁶ thus, spacecraft are more of a concern than is indicated by their current 2:1 numerical weighting relative to rockets.

Figure 4 shows the number of spacecraft in LEO, which currently totals to about 1,240.⁷ The peaks correspond to those in the catalog, although the spacecraft are generally a small fraction of the catalog objects at each altitude. Comparing Figs. 3 and 4 shows that the spacecraft distribution does not exceed the catalog at any altitude, because these distributions are for the current, decayed distributions of both spacecraft and the full catalog. The spacecraft peak at 950 km in fig. 4 is more than large enough to make up for the fragment gap in Fig. 3. There are relatively few spacecraft at 1,00 to 1,400 km, so they do not degrade the agreement between the fragments and catalog there shown in Fig. 3. The spacecraft peak at 1,450 km in Fig. 4 contributes much of the peak in the catalog by itself. Spacecraft contribute little at higher altitudes.

The spacecraft launch rate has been relatively constant over the last 35 years, and decay has been modest, so dividing the number in LEO by 35 gives the average source of LEO spacecraft of about $1240 / 35 \text{ years} \approx 35/\text{year}$. That is lower than the aggregate number given above, but that value is for peak rates and includes spacecraft at all altitudes, as well as rocket bodies, which are included here with the fragments because of their lower areal densities and shorter lifetimes. Since large objects experience less drag, they largely accumulate on orbit. As they do not appear to have saturated yet, a two to four-fold decrease in launch rate would be expected to decrease their source rate by similar factors to about 20 and 10 objects/year, respectively.

Decay rates for circular orbits are known; the decay rates for eccentric orbits can be deduced from them.⁸ Circular orbit decay lifetimes, $T_0(z)$, as functions of altitude, z , are based on atmosphere and drag models due to Jacchia,⁹ They can be used to estimate eccentric orbit decay times through analysis by King-Hele.¹⁰ Those lifetimes can be used with an approximate correction for fragment eccentricity and velocity developed by the NASA JSC to derive average lifetimes for the fragments. JSC starts with the observation that for an isotropic distribution of fragments, half are scattered up, and half are scattered down, in velocity, energy, and altitude. Averaging these results over their assumed isotropic angular distributions and measured velocity distributions provides effective lifetimes for all fragments.¹¹

A companion note solves the results of Jacchia and King-Hele as functions of the displacement of fragments from their altitude of production, infers the distribution of fragments from their velocity distribution, and averages the result over fragment velocities to produce average lifetimes $\langle T_+ \rangle$ and $\langle T_- \rangle$, and decay rates $\langle 1/T_+ \rangle$ and $\langle 1/T_- \rangle$, for fragments scattered upward and downward, respectively,¹² in good agreement with those stated by JSC.¹³

These results must be averaged over the measured fragment distribution to provide inputs for flux models, which only keep track of the number of particles, not their distribution over size.¹⁴ This averaging produces average decay rates and times that correspond to fragments about a factor of two larger than the smallest objects in the catalog, because it is dominated by the numerous, small fragments.¹⁵ Their areal density, i.e., mass per unit area or ballistic coefficient $\beta \approx 3 \text{ kg/m}^2$, properly weights the effect of drag on the small particles that experience it most, but is too large for the intact objects, which have values of $\beta \approx 100\text{-}200 \text{ kg/m}^2$, so that the effects of drag on them are proportionately less. This sensitivity to object size is studied below by examining large and small β separately. Calculations with small β are used to study the evolution of small objects, which contain most of the fragments, while calculations with large β are used to study the evolution of large objects, which contain most of the mass.

This averaging provides the average decay times $\langle T_+ \rangle$ and $\langle T_- \rangle$ shown in Fig. 5 for particles with $\beta \approx 100 \text{ kg/m}^2$, along with the average lifetime $\langle T \rangle = (\langle T_+ \rangle + \langle T_- \rangle)/2$ used in JSC stability calculations. These lifetimes are exponential in altitude, with scale heights of $\approx 60 \text{ km}$ below 900 km and $\approx 240 \text{ km}$ above it. Below 900 km , $\langle T_+(z) \rangle \approx 2T_0(z)$ and $\langle T_- \rangle \approx T_0/2$, so $\langle T \rangle \approx (2T_0 + T_0/2)/2 = 1.25T_0$, which means JSC's average decay times for all orbits are essentially those for circular orbits.

These calculations are intended for time scales of decades to centuries; thus, objects that decay on much shorter time scales are of lesser concern. For the $\beta \approx 100 \text{ kg/m}^2$ of Fig. 5, decay times are measured in decades below about 700 km . Conversely, for objects above $1,000 \text{ km}$, drag is of little importance even over a century, so objects at those altitudes accumulate. Between about 600 and $1,000 \text{ km}$, fragments are subject to both production and decay on relevant time scales. However, the altitude regions of interest vary with the mass per area of the fragment. For objects with $\beta \approx 10 \text{ kg/m}^2$, drag is significant as high as $1,200 \text{ km}$, and fragments are taken out quickly as low as $\approx 800 \text{ km}$. The decay times are all reduced by a factor of 10 for a 10-fold reduction in β , which at lower altitudes corresponds to a reduction in altitude of $\approx 60 \text{ km}$ $\ln(10) \approx 140 \text{ km}$, in accord with the values above and JSC's estimate that there is appreciable drag even on circular orbits at those altitudes.¹⁶

Model. The orbital decay model used is an adaptation of one developed by JSC for stability calculations,¹⁷ which retains some measure of the eccentricity of the debris distribution but remains simple enough for understanding and parameter variations. It starts with the observation that for an assumed isotropic distribution of fragments from an explosion or collision on a circular orbit, half the particles are scattered up and the other half down—in energy, semi-major axis, and altitude. In fact, orbits are not circular, distributions are not isotropic, and neither is known with precision, but more general assumptions quickly lead to purely numerical solutions, which tend to imbed similar assumptions in obscure places. For typical debris velocities, a

fragment's apogee is raised—or its perigee lowered—by about 100 km, respectively. Thus, in a region of the atmosphere with a scale height of about 100 km, the former would have a lifetime about a factor of 3 longer than that of particles at the production altitude, while the latter would have lifetimes about a factor of three shorter, are described by the averaged lifetimes calculated and discussed in the section on Decay Rates.

The model divides the atmosphere below 2,000 km into 100 km bins, and divides the number of particles in each altitude bin into two groups: those that have been scattered up, N_+ , and those scattered down, N_- , which evolve under

$$dN_+/dt = S/2 - N_+/\tau_+ + F_+ + QN^2/2, \quad (1)$$

$$dN_-/dt = S/2 - N_-/\tau_- + F_- + QN^2/2. \quad (2)$$

The first term on the right hand side of each equation is the rate of production of fragments from launches and explosions. Since they are assumed equal, the sources for up- and down-scattered objects are each half the totals shown in Figs. 3 and 4. The second term represents decay, at the rates for the up- and down-scattered particles, respectively. The third is the flux of particles decaying from higher orbits, which is discussed further below. The last is the collisional cascading rate for a total of $N = N_+ + N_-$ particles. While the first and last terms could be combined into a single equation for $d(N_+ + N_-)/dt = dN/dt$, it is not possible to combine the second terms into an effective decay rate for N , so in general it is necessary to retain two separate equations to retain the first order effects of eccentricity.

Vertical flux. The calculations below use a simplified model for the vertical flux of particles, without which the debris density would be zero wherever the source was zero, in contrast to observations. The flux model assumes that in the bin at altitude z , the density of either the + and - eccentricity components vary separately as

$$dN_z/dt = S/2 - N_z/\tau + N_{z+1}/\tau_{z+1}, \quad (3)$$

where the cascade rate, $QN^2/2$, is included in S for simplicity, and the downward flux at each altitude is approximated by the ratio of the density to lifetime, $F_z = N_z/\tau_z$. As the flux into the bin at z is from $z + 1$, it is from the bin above and is $F_{z+1} = N_{z+1}/\tau_{z+1}$, as indicated. Given N_{z+1} , for S constant, the solution to Eq. (3) over the time interval dt is

$$N_z(t+dt) = N_z(t)\exp(-dt/\tau_z) + (S/2 + N_{z+1}/\tau_{z+1})\tau_z[1 - \exp(-dt/\tau_z)], \quad (4)$$

which is solved from z large, where the flux is zero, downward in z . In addition to this dynamic result, there is a conservation law that the number of particles in the bin at time t plus the source and the flux into it must equal the particles in the bin at $t + dt$ plus the flux out of it, so that

$$N_z(t) + S/2 dt + F_{z+1}dt = N_z(t+dt) + F_zdt, \quad (5)$$

which can be solved for

$$F_z = F_{z+1} + S/2 + [N_z(t) - N_z(t+dt)]/dt, \quad (6)$$

as the flux into the bin below. Using Eq. (6) to compute the fluxes conserves particles identically

even if Eq. (4) becomes inaccurate at low altitudes. For large dt the solution to Eq. (4) is

$$N_Z = (S/2 + N_{Z+1}/\tau_{Z+1})\tau_Z, \quad (7)$$

which is influenced equally by the source and decay. For S large, $N_Z = \tau_Z S/2$, which is the idealized saturation case discussed above. For $S = 0$, the solution reduces to

$$N_Z = (N_{Z+1}/\tau_{Z+1})\tau_Z, \quad (8)$$

from which $N_Z/N_{Z+1} = T_Z/T_{Z+1}$. Since the right hand side falls exponentially with z , this result leads to a decrease in N_Z/N_{Z+1} by a factor of about $\exp(-100/60) \approx 1/5$ at each bin in an exponential atmosphere of scale height of ≈ 60 km, as is the case where the orbital decay flux is significant. The flux is not dominant at altitudes where cascade growth is possible.

Fragment results. Figure 6 shows the number of fragments predicted by calculations that integrate Eqs. (1)–(6) forward from zero fragments at any altitude 35 years ago using the fragment sources shown in Fig. 3 and a fraction $3/100 \text{ kg/m}^2$ of the large object decay times shown in Fig. 5. The bottom curve is N_- , the next curve is N_+ , the next is their sum $N_+ + N_-$, and the top curve is the catalog. Comparing Fig. 6 with Fig. 3 shows that N_- is strongly reduced from the fragment source, even at an altitude of 1,050 km, in accord with Fig. 5 for 3 kg/m^2 fragments. N_+ is significantly reduced from the source by 850 km. Their sum $N_+ + N_-$ has peaks at 950 km and 1,550 km. The former is within about 70% of the catalog value there; the latter is only about 30% of it. What this agreement means must await the evaluation of the contribution from the intact objects, which is performed below.

The sum $N_+ + N_-$ falls far below the catalog at altitudes below about 850 km. Indeed, N_- falls to about zero below 600 km, because fragments scattered down from 600 km reach 500 km densities, where the lifetimes from Fig. 5 are measured in months. However, even N_+ is small there, because its lifetime is only a few years. This result is not altered by the contribution from the intact objects below. The discrepancy could be due to additional sources such as rocket bodies that are not treated explicitly, or it could be due to inaccuracy in the flux model, which was designed less for accuracy than for speed and stability—the primary requirement in a model that must work over a range of 6 orders of magnitude in time scales.

Ballistic coefficient. Since the fragment sources are known and the decay rates are fairly conventional, the two primary parameters through which this discrepancy could be addressed are the fragment β , and the overall source rate S . As noted above, the average β is dominated by the smallest fragments, which are produced in space, where only their radar cross sections (RCS) can be measured. There are three sources of information on β : laboratory tests, field tests, and on-orbit observations. Laboratory railgun tests are designed to determine how the number of fragments scales on target size, rather than to infer the β of large targets.¹⁸ However, reasonable interpretation of those results implies a value of $\beta \approx 10 \text{ kg/m}^2$ for 1 kg objects. The Atlas field test gives a value of $\approx 3 \text{ kg/m}^2$.¹⁹ On-orbit observations attempting to infer an object's β from radar

measurements of the rate at which its orbit changes, which would give an independent and directly relevant measurement of β . Probability histograms for most of the fragments from the 1981 Landsat 3 explosion at 900 km have been reviewed in detail, producing values of β from 0.2 to 20 kg/m^2 , with a few out to 300 kg/m^2 , which correspond to large objects. About 20% of the β 's were negative, which indicates the trajectories were too noisy for the inference of precise ballistic coefficients—and Landsat 3 produced one of the better behaved fragment distributions.²⁰ The initial study indicated that most fragments had β 's in the range of 1 to 10 kg/m^2 , which span the range above. They are somewhat lower than those inferred from laboratory studies,²¹ but compatible with those measured from the Atlas test.

Figure 7 shows the result of increasing β to 30 kg/m^2 , which is an extreme excursion that takes it outside the bounds established above by a factor of two to three. Comparing it to Fig. 6 shows little change above 1,100 km, where the fragments simply accumulate. At lower altitudes, N_+ changes little above 850 km, but N_- increases to almost equal N_+ . The distinction between them is largely eliminated in this excursion, where neither strongly experiences drag at those altitudes. Their sum still falls below the catalog; by 600 km it is about a factor of two low. For this β , at intermediate altitudes $N_+ + N_-$ is very close to the whole catalog, leaving little opportunity for a significant contribution from the intact objects. However, the peak at 1,450 km still has little contribution from fragments. Intermediate values of β produce distributions below 1,000 km intermediate between those shown in Figs. 6 and 7.

Intact object results. Figure 8 shows the number of intact objects predicted by calculations that integrate Eqs. (1)—(6) forward from zero fragments 35 years ago using the fragment source of Fig. 4 and the decay rates of Fig. 5. As before, the bottom curve is N_- and the rest are N_+ , $N_+ + N_-$, and the catalog. Comparing it with Fig. 4 shows little change below 750 km. Indeed, N_+ and N_- are very similar for those altitudes. They only differ by a factor of two at 550 km, which shows the relative insensitivity of large objects to drag. The locations and magnitudes of the peaks at 950 and 1,450 km are insensitive to drag. There is some sensitivity to drag below 500 km, but the number of intact objects there is so small compared to the catalog that this insensitivity is unlikely to be the explanation of the discrepancy between them. Since drag is ineffective, the only relevant parameter in varying the number of intact objects is increasing or decreasing their overall source rates, which increases or decreases the number of objects at a given time proportionately.

Combined fragment and intact object distributions from these calculations are shown in Fig. 9, in which the bottom curve is the intact object distribution from Fig. 8, the next curve is N_+ and N_- from Fig. 6, and the top curve is the catalog. The debris distribution agrees well with the catalog at and between the two peaks, i.e., in the region that contains most of the objects. The contribution from large objects complements that from the fragments to produce $\approx 5\%$

agreement at and around the peak at 950 km. Roughly 10% agreement persists from 950 km to 1,300 km, after which it improves to better than 5% by the second peak at 1,450 km. Beyond 1,700 km the predicted distribution is well above the catalog. As decay is inoperative there, the catalog represents an accumulation of the fragment source over time; thus, this discrepancy is actually a measure of the error in the approximate treatment of the fragmentation source used there. The number of fragments is not large.

At 850 km, the predicted distribution is about 5% below the catalog. That discrepancy increases to about 25% by 750 km, and 50% by 650 km. Although the number of particles is small—about 5% of the total—the discrepancy is significant because it lies in the region of manned spacecraft. From the discussion above, three parameters could address it: the intact object growth rate, the fragment growth rate, or the fragment β . In practice, it would be difficult to vary any of them. The preferred choice would be the intact object growth rate, since it would provide additional objects that could survive longer at low altitudes. However, it would have to be increased significantly to have any effect, and increasing it would disrupt the agreement on both peak values. Increasing the fragment growth rate would increase the peak at 1,450 km more than it increased the peak at 950 km, which would degrade agreement there. It would increase the fragment distribution at every altitude, but would lead to few additional objects at 500 km. Increasing the fragment β is studied in Fig. 9. It does increase the number of fragments at low altitudes, e.g., it reduces the discrepancy to a factor of two at 550 km. However, $\beta = 30 \text{ kg/m}^2$ leaves little for intact objects to contribute and degrades the agreement at both peaks, unless offset by reductions in intact objects. Thus, large values of β essentially blur the distinction between fragments and intact objects, and smaller values do not have the required impact. It appears the discrepancy at low altitudes is due instead to the ignored sources or approximate fluxes discussed above.

Average decay time calculations. Figures 6 shows that the difference between the decay times of objects scattered up and down can make a factor of two or more difference between N_+ and N_- for altitudes of interest, although Fig. 7 indicates that those differences are reduced at higher ballistic coefficients. Carrying two sets of densities and lifetimes for fragments and intact objects is not difficult, but it is useful to examine the extent to which it is necessary. JSC states that averaging the fragment distribution over size, direction, and altitude produces a distribution of fragments from a point source S_0 that decays as²²

$$S = S_0 [\exp(-t/\tau_+) + \exp(-t/\tau_-)]/2, \quad (9)$$

where S_0 is independent of t and τ_+ and τ_- are the average lifetimes computed above of up- and down-scattered fragments, respectively. JSC notes that for a steady source of fragments, dB/dt , it is possible to integrate Eq. (9) over times $t \gg \tau_+$ and τ_- to produce an "equilibrium spatial density"

$$\langle S \rangle = \int_0^t dt' S(t - t') dB/dt' = dB/dt S_0 \tau, \quad (10)$$

where dB/dt is assumed constant and

$$\tau = (\tau_+ + \tau_-)/2, \quad (11)$$

is an average decay time, which by the estimates above is roughly equal to the circular orbit decay time. Using this approximate decay time in time-dependent calculations would reduce the equations for the evolution of the number of fragments and intact objects to the form

$$dN/dt = S - N/\tau, \quad (12)$$

which would be numerically simpler, and analytically integrable in some cases of interest.

However, this approximation assumes that N_+ and N_- are equal, as their sources and decay rates are equal, which is violated by factors of two or more in the direct solutions shown in Fig. 6. The cause of the discrepancy is that for altitudes and times of interest, the approximation involved in evaluating $\langle S \rangle$ is not accurate.²³ Its actual integral is

$$\langle S \rangle = dB/dt S_0 \{ \tau_+ [1 - \exp(-t/\tau_+)] + \tau_- [1 - \exp(-t/\tau_-)] \} / 2. \quad (13)$$

At 850 km, where the number of fragments and the error are both significant, $\tau_+ \approx 50$ years and $\tau_- \approx 8$ years, so at $t = 35$ years the quantity in parenthesis for the up-scattered fragments is $\approx 50(1 - 0.5) \approx 25$ years, while that for the down-scattered fragments is ≈ 8 years. Thus, it is appropriate to use the limiting value of $\tau_- dB/dt S_0$ to evaluate the density of down-scattered fragments, but it is in error by a factor of two to use $\tau_+ dB/dt S_0$ to evaluate the density of up-scattered fragments. That approximation is only valid for $t \gg \tau_+ \approx 50$ years, which is outside the scope of these calculations. For intermediate times $8 < t < 50$ years, a more accurate approximation is

$$\langle S \rangle \approx dB/dt S_0 \{ \tau_+ [1 - (1 - t/\tau_+)] + \tau_- [1 - \exp(-t/\tau_-)] \} / 2 \approx dB/dt S_0 (t + \tau_-) / 2. \quad (14)$$

At any time the error in the fragment density due to this approximation is the difference between Eqs. (10) and (14), which is

$$\Delta \langle S \rangle \approx dB/dt S_0 [(\tau_+ + \tau_-) / 2 - (t + \tau_-) / 2] \approx dB/dt S_0 (\tau_+ - t) / 2, \quad (15)$$

which is initially about half as large as the final value. This expression holds without alteration for the higher altitudes that contain most of the fragments.

Despite these conceptual flaws, the reduction to a single decay time and distribution for fragments and intact objects would have some advantages, if it could be performed with adequate accuracy. Figure 10 compares the current up- and down-scattered fragment densities predicted above to the results of the integration of Eqs. (1)–(6) forward from zero fragments with the same sources, using a single decay time for the fragments equal to the average decay time

$$\langle T \rangle = (\langle T_+ \rangle + \langle T_- \rangle) / 2, \quad (16)$$

at each altitude. The bottom curve is N_- , the next curve is half the density from the average decay time $\langle T \rangle$ model, and the top curve is N_+ . As expected, the average density, which uses a decay time about a factor of three larger than $\langle T_- \rangle$, produces a larger number of particles at every altitude than N_- . At 950 km, the ratio of N_-/N_{avg} is about 75%; at 850 it is about 50%; at 750 km it is about 40%. Overall, the N_- distribution contains about 20% fewer particles than the circular orbit distribution N_{avg} .

Conversely, the N_+ distribution contains more particles than the circular orbit one. The excess is about 10% at 850 km and 25% at 750 km. The percentages are larger at lower altitudes, but there are fewer particles there. Overall, the N_+ distribution contains about 10% more particles than the circular orbit distribution, which partially makes up for the 20% shortfall in N_- . At the level of the disaggregated eccentric orbit fluxes, the circular orbit approximation appears to be accurate to within 10 to 20%.

Figure 11 compares $N_+ + N_-$ to the circular orbit N_{avg} . The agreement is excellent above 1,000 km. The average model predicts about 5% more fragments at 950 km and 10% more at 850 km; the differences become smaller at lower altitudes. Thus, at the level of the overall distributions, the circular orbit model appears reasonably accurate. The fragments are only one component of the total density. When the intact objects are added, the discrepancy becomes even smaller. The distribution for the circular orbit could be adjusted to agree with the catalog as well as the eccentric orbit model by increasing the fragment ballistic coefficient. The adjustment would be so slight that it would not be able to distinguish on the basis of field or on-orbit observations. It would be necessary to go to more detailed catalog information to differentiate between these the models of the current distribution.

Summary and conclusions. This note uses flux models to predict the growth of space debris to the present. It balances accuracy and simplicity to achieve fidelity in modeling production and decay while retaining transparency of results. It uses historical data and current launch rates projections to estimate future debris production. Estimates launch rates to LEO fall due to the 2-fold decrease of CIS launch rates and the continuing shift of defense and commercial payloads from LEO to GEO—even with planned communication constellations. Those launch rates can be used to predict spacecraft, rockets, and mission related debris. Fragments are created on orbit by explosions by poorly-known processes. However, their density is not directly related to launch or explosion rates; they saturated several decades ago. Observed fragmentations provide an accurate source for most of the fragment portion of the catalog. Known explosions account for most of the fragment end of the catalog; the satellites on now on orbit account for most of the intact objects. It is assumed that other sources have placed objects at low altitudes, where they would not be treated accurately by these calculations. The calculations also include cascading due to collisions, which is small for the parameters and times studied.

Decay models due to Jacchia for circular orbits and their extension to eccentric orbits by King-Hele provide decay lifetimes, which when averaged over fragment distributions, agree well with JSC's approximate treatment. The estimates combine these source and loss mechanisms into a 20-level model of the atmosphere below 2,000 km, which tracks the up- and down-scattered fragments and intact objects separately. Integrating the model forward to the present from an initial condition of zero debris 35 years ago gives a distribution that closely resembles the current

catalog—particularly in the regions containing most of the debris. Below 850 km, the predicted distribution fall below the catalog; the agreement is only a factor of two by 650 km.

Of the parameters that could be used to address the discrepancy, changing the intact object growth rate would disrupt agreement on the peak values, increasing the fragment growth rate would degrade agreement on the main peak at 950 km, and increasing β would just blur the distinction between fragments and intact objects. It appears the discrepancy at low altitudes is due to ignored sources or the approximate treatment of vertical fluxes.

The differences between the decay times of objects scattered up and down can make factor of two or more difference between their densities for altitudes of interest, but it is useful to examine the extent to which it is necessary. JSC's reduction to a single decay time has unacceptable errors, but a modified version is accurate to about 20% at the level of individual fluxes and 10% at the level of overall distributions, which is comparable to the errors involved in the statistics in the catalog and less than those involved in the estimates of the debris sources.

The principal result of this note is the demonstration that it is possible to predict the current debris distribution with some accuracy, particularly at the altitudes that contain most of the debris and have some potential for cascade growth. Moreover, it is possible to significantly simplify the treatments of sources, losses, and eccentricity without losing that agreement. However, the results depend strongly on only a few parameters, which cannot be adjusted to address the differences between predicted and observed debris densities at 600 km and below. The only remaining parameters that can be varied profitably are the fragment source rate and areal density, neither of which can significantly reduce the discrepancy without degrading agreement in the regions of peak density. The consistency of these results and their transparent dependence on the initial conditions and model structure and parameters indicate that these models and sources are both credible and calibrated about as well as possible for other, related predictions.

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Fig. 1. Global launch rate for various communication constellations.

Fig. 2- On-orbit cataloged population (corrected for delayed cataloging).

Source: Orbital Debris (National Research Council)

Fig. 3. Fragment source and debris catalog vs altitude at present.

Fig. 4. Distribution of spacecraft and debris catalog vs altitude at present.

Fig. 5. Eccentric and circular decay times versus altitude for $\beta = 100$

Fig. 6. Components of fragment density versus altitude for $\beta = 3 \text{ kg/m}^2$.

Fig. 7. Components of fragment density versus altitude for beta = 30 kg/m².

Fig. 8. Components of large, intact object density versus altitude at present for S/C source and beta = 100 kg/m².

Fig. 9. Fragment and intact components of current catalog as a function of altitude.

Fig. 10. Eccentric and circular orbit densities versus altitude.

Fig. 11. Total eccentric and circular orbit densities versus altitude.

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