EVALUATION OF THE GAUSSIAN BEAM MODEL FOR PREDICTION OF LDV FRINGE FIELDS

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ABSTRACT

A simple model is developed to estimate the fringe field geometry at the intersection of two gaussian laser beams. Comparison of the model results to experimentally measured fringe spacing demonstrates that while the model predicts the fringe geometry well when the beam waists are far from the intersection volume, it performs poorly under nominally ideal conditions—when the beam waists are located at the intersection. Data obtained with two different laser sources indicate that the discrepancies between the theory and experiment are likely due to deviations of the laser beam from an ideal gaussian beam. With a high quality laser, the details of the fringe field geometry are still not well duplicated by the gaussian beam model, although the magnitude of the variation in fringe spacing and the effect of the controlling system parameters are correctly predicted.

1. INTRODUCTION

Over the past thirty years, Laser Doppler velocimetry (LDV) has evolved into the technique of choice for obtaining accurate, non-intrusive measurements of fluid velocity. In the dual-beam, real-fringe configuration (Fig. 1), the frequency of the Doppler signal can be viewed as being due to passage of a scattering particle through a set of interference fringes created by two coherent laser beams which are brought to a focus (waist) and crossed by a single transmitting lens. Due to continuous improvements in optics, electronics, and signal processing techniques, the spatial uniformity of these interference fringes may now be the limiting factor in determining the accuracy with which flow velocities can be measured.

In recent experimental investigations, large variations in the spatial uniformity of the fringe field are observed along the coordinate directions defined by both the major and minor probe volume axes (Miles and Witze, 1994). These non-uniformities occur even under conditions of nominally ideal optical alignment—especially in regions off the probe volume axes. In attempting to understand the source of these variations we found that existing analytical models of the fringe spacing variation provided limited guidance. These models are valid for gaussian beams with equal beam waist sizes and equal (or equal but opposite) distances of the beam waists from the measuring volume; the effects of differing waist sizes and positions on the fringe field uniformity cannot be ascertained. Additionally, the predicted fringe spacings are strictly valid only on the probe volume axes. Computed results from a more rigorous, general model are similarly available only for a restricted subset of conditions and only along the probe volume major axis.

Previous experimental measurements also provided few points for comparison—data obtained under conditions of near ideal alignment are sparse, and are generally limited to measurements along the longitudinal probe volume axis. In addition, these data were obtained with techniques that averaged over the transverse extent of the probe volume, a procedure that masks the details of the fringe field.

In view of the preceding discussion, the goals of this paper are outlined as follows:
(1) To provide an alternative gaussian beam model which permits simple evaluation of the fringe field throughout the probe volume, for arbitrary beam waist sizes and positions.
(2) To compare the model results with experimental data in order to a) determine the applicability of the models to the prediction of fringe fields in a dual-beam velocimeter and b) to understand the origin of the large variations in the fringe field which have been experimentally observed.
(3) To provide guidance to the user or designer of LDV systems on how to minimize the effects of the fringe field non-uniformities on the accuracy of the data obtained.

Fig. 1 Dual-beam, real-fringe LDV configuration

2. **PREVIOUS LDV FRINGE FIELD STUDIES**

Fringe field uniformity was first investigated by Hanson (1973), who demonstrated that misaligned LDV systems are characterized by a gradient in fringe spacing along the optical (longitudinal) axis. An expression for this gradient was obtained which is valid at the intersection of the two beam centerlines when the waists of the two beams have equal longitudinal positions. Comparison of the predicted gradient to experimental measurements demonstrated reasonable agreement, although data were obtained under conditions of rather severe misalignment.

Later, Hanson (1976) demonstrated the existence of a fringe spacing gradient along the transverse probe volume axis, which occurs when the two beam waists are located at equal distances from the intersection volume, but on opposite sides. His resulting expression is valid on the probe fringe spacing gradient along the transverse probe volume axis, equal distances from the intersection volume, but on reasonable agreement, although data were obtained under conditions of near-ideal alignment.

Durst and Stevenson (1979) extended Hanson's longitudinal analysis by referring the results to system parameters: the transmitting lens focal length and the location of the waists of the input laser beams. Implicit in the interpretation of their result is the assumption that the longitudinal gradient in fringe spacing is constant over the length of the probe volume. Extensive measurements of the longitudinal gradient were also obtained, and showed good agreement with the analytical result. As will be shown below, the assumption of a constant longitudinal gradient is appropriate under the experimental conditions employed. For these conditions, however, little structure in the fringe field is to be expected under near-ideal alignment conditions, and the large variations seen by Miles and Witze (1994) were not observed.

In addition, Durst and Stevenson obtained an alternate expression for the fringe spacing variation along the transverse axis; like Hanson's result, the analysis was restricted to equal but opposite distances of the beam waists from the intersection volume, and is valid only along the transverse axis. Qualitative confirmation of this effect is reported, but no quantitative verification of the result was possible.

A more recent, frequency-based analysis is given by Durst, et al. (1990), which allows for different beam waist sizes and positions and provides a framework for examination of the effects of non-gaussian beams. The expressions obtained are complex, and the effect of various system parameters on the longitudinal variation in fringe spacing is therefore computed and presented as a series of graphs. Experimental verification of the computed results demonstrates fair agreement between theory and experiment under misaligned conditions; under near-ideal alignment conditions, however, the experimental variation in fringe spacing exceeds the theoretical variation by a factor of approximately three.

Collectively, the above cited studies have verified the applicability of gaussian beam models in predicting the longitudinal variation in fringe spacing under conditions in which the longitudinal gradient is nearly constant—this typically occurs when the beam waists are far from the probe volume. Although the variation in fringe spacing under near-ideal alignment conditions is also of interest, the existing data obtained under these conditions are few and tend to show a large discrepancy with theoretical predictions. Furthermore, until recently, experimental determination of the transverse variation in fringe spacing had not been achieved, and no comparison to gaussian beam model predictions has been made.

3. **THE GAUSSIAN BEAM MODEL**

There are at least two approaches by which a model for the fringe spacing in the intersection volume of two gaussian beams can be developed. The first approach, based on the difference in phase of the two beams expressed in beam coordinate systems, permits a rigorous analysis without approximation (Miles, 1996). The second approach, presented in this paper, has much in common with the earlier fringe-based models and is both intuitive and mathematically simple, although some implicit approximations must be made. The effect of these approximations, however, can be shown to be negligible simply by comparison with results obtained using the first approach. Although the following analysis is specific to gaussian beams, we believe that the indicated effects of various system parameters carry over, at least qualitatively, to the beams delivered by single-mode optical fibers as well.

An excellent discussion of the properties of Gaussian beams and their transformation by optical elements can be found in the text by Siegman (1986); here we review only those aspects relevant to the present work. Fig. (2) depicts the divergence of a Gaussian beam propagating away from its waist. At the waist, the surfaces of constant phase (wave fronts or phase fronts) are locally planar, and gradually acquire curvature and diverge as distance from the waist is increased. At any location z along the beam axis, the local radius of curvature and the local beam spot size (the radius defined by the $1/e^2$ intensity contour) are known solely in terms of the spot size at the beam waist $w_0$. With the Rayleigh range $z_R$ defined by

$$z_R = \frac{2w_0^2}{\lambda},$$

the local radius of curvature, $R(z)$, and spot size, $w(z)$, can be found from
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\[ R(z) = z + \frac{z^2}{z} \quad (2) \]
and
\[ w^2(z) = w_0^2 \left( 1 + \frac{z}{z_0} \right)^3 \quad (3) \]

Note that the center of curvature lies on the beam axis. The usual sign convention is that positive \( R(z) \) indicates a diverging beam.

In the following derivation, we consider a 'fringe' to be defined by the locus of all points for which the phase difference between the two beams is fixed. As a practical matter, this may correspond to a surface of maximum light intensity, minimum light intensity, or neither. The geometry to be considered is shown schematically in Fig. 3. The locations \((x_{cl}, z_{cl})\) and \((x_{cz}, z_{cz})\) refer to the locations of the centers of curvature of the wavefronts of each beam at an arbitrary location \((x, z)\) within the measurement volume. The wavefront centers of curvature are known to lie along the lines \(x_{cl} = -\frac{x}{\tan \alpha}\) and \(x_{cz} = \frac{z_{cz}}{\tan \alpha}\) where \(\alpha\) is the usual half-angle of the beam crossing and the origin of the coordinate system is at the crossing of the beam centerlines.

If the phase difference between the two beams at their centers of curvature is taken to be fixed (for convenience we take this difference to be the phase difference which defines a fringe), then the condition for the existence of a fringe at \((x, z)\) is simply that the optical path length, \(R_{cl}\), between the two centers of curvature to \((x, z)\) differs by an integral number of wavelengths. This optical path length can be identified as the radius of curvature of the phase front at \((x, z)\). The condition for the existence of a fringe can therefore be simply expressed as:

\[ R_{cl} - R_{cz} = \sqrt{\left(x - x_{cl}\right)^2 + \left(z - z_{cl}\right)^2} - \sqrt{\left(x - x_{cz}\right)^2 + \left(z - z_{cz}\right)^2} = n\lambda \quad (4) \]

The spacing \(L\) between adjacent fringes in the \(x\)-direction is determined by differentiating Eq. (4) with respect to \(x\) and rearranging to obtain

\[ L = \left( \frac{\lambda R_{cl} R_{cz}}{R_{cl} (x - x_{cl}) - R_{cz} (x - x_{cz})} \right) \quad (5) \]

Note that the fringe spacing given by Eq. (5) has a \(z\)-direction dependency which is implicit in the definitions of the \(R\). It is also noteworthy that as \(x\) varies, the local phase of each beam at \((x, z)\) varies, and the corresponding radius of curvature (and center of curvature) of the local phase front also varies. In deriving these equations, we have neglected this \(x\)-dependency of \(x_{cl}\) and \(z_{cl}\). It can be shown (Miles, 1996) that neglecting the \(x\)-dependency of the wavefront center of curvature introduces an error of order \((\lambda/2\pi\mu_0)^3\), which is negligible for practical LDV systems.

It is interesting to examine some limiting cases of Eq. (5) to establish consistency with earlier studies. Along the optical axis \((x = 0)\), when the beam waists are of equal sizes and are located at equal distances from the beam crossing (but on the same side of the crossing), \(R_{cl} = R_{cz} = R\) and \(x_{cl} = -x_{cz}\), so that Eq. (5) reduces to

\[ L = \frac{\lambda R}{2x_{cl}} \quad (6) \]

Eq. (6) is identical to the result obtained by Hanson (1973).

A second limiting case of interest occurs when the beam waists are equal distances from the crossing but on opposite sides. In the \(z = 0\) plane, \(R_{cl} = -R_{cz} = R\) and \(x_{cl} = x_{cz}\), so that

\[ L = \frac{-\lambda R}{2(x - x_{cl})} \quad (7) \]

This is identical to the expression obtained by Hanson (1976). Differentiating Eq. (7) with respect to \(x\), the transverse gradients in fringe spacing are found to be

\[ \frac{dL}{dx} = \frac{\lambda}{2(x - x_{cl})} \left( \frac{R}{R_{cl} (x - x_{cl}) - R_{cz} (x - x_{cz})} \right) \quad (8) \]

Accounting for the different coordinate systems used, Eq. (8) is the expression obtained by Durst and Stevenson (1979). All previously obtained fringe model results can thus be shown to be special cases of Eq. (5).

The fringe spacing at any location within the probe volume can be computed from Eq. (5) with \(R\) obtained from Eq. (2). The longitudinal beam coordinate \(z_i\), for use in Eq. (2), can be determined from (see Fig. 3)

\[ z_i = (z - z_w) \cos \alpha - (x - x_w) \sin \alpha \quad (9) \]
and
\[ z = (z - z_w) \cos \alpha + (x - x_w) \sin \alpha \quad (10) \]

where \((x_w, z_w)\) and \((x_w', z_w')\) represent the coordinates of the beam waists. The \(x\)-coordinates of the wavefront centers of curvature, for use in Eq. (5), can be obtained from

\[ x_{cl} = x_w + (R_{cl} - z_i) \sin \alpha \quad (11) \]
and
\[ x_{cz} = x_w - (R_{cz} - z_i) \sin \alpha \quad (12) \]

By revisiting the simplified, limiting cases described
above, some insight into the geometry of the fringes can be gained. It is interesting to consider the changes in fringe spacing along the major axis of the probe volume \((x = 0)\), when the beam waists have equal sizes and \(z\)-coordinates \(z_w\) (as described by Eq. (6)). This corresponds to a typical, conventional LDA using a path compensated beam splitter (note that we do not assume that the input laser beam has been appropriately "collimated"). Under these conditions, Eqs. (2), (6) and (9)-(12) give

\[
L = \frac{\lambda}{2\sin\alpha} \left[ 1 + \frac{\cos\alpha}{z_w^2} \left( \frac{w_0}{d} \right) \right].
\]

From the form of Eq. (13), it is apparent that the fringe spacing gradients in the \(z\)-direction cannot be assumed constant unless \(z_w > > z_R\), that is, under fairly severe conditions of misalignment. Further note that for beams characterized by large \(z_R\), the variation in fringe spacing can be small even for misaligned systems. With perfect alignment of the beam waists with the crossing \((z_w = 0)\), Eq. (13) reduces to

\[
L = \frac{\lambda}{2\sin\alpha} \left[ 1 + \left( \frac{\cos\alpha}{z_w} \right)^2 \right].
\]

from which the minimum possible variation in fringe spacing for a given \(\alpha\) and waist diameter can be computed.

Eqs. (13) and (14) allow the longitudinal variation in fringe spacing to be determined in terms of properties of the focused beams near the intersection volume. While these properties can be readily measured, it is of greater value to understand how the fringe variation is affected by various system parameters, e.g., input beam diameter or beam crossing angle. It is shown elsewhere (Miles, 1996) that, to a good approximation, Eq. (13) can be expressed as

\[
L = \frac{\lambda}{2\sin\alpha} \left[ 1 - \frac{1}{2} \cos^2 \left( \frac{\alpha}{\alpha_e} \right) + \left( \frac{w_0}{d} \right)^2 \right].
\]

In Eq. (15), \(w_0\) and \(d\) are the incident beam waist radius and offset from the optical axis, respectively, as defined in Fig. 1. The parameters \(z_w\) and \(f\) in the "normalized mis-alignment" \((z_w - f)/z_w\) are also defined in Fig. 1, and the coordinate \(\tilde{z}\) is defined as the \(z\)-coordinate normalized by one-half of the probe volume length, \(w_0/\sin\alpha\). With this definition, \(\tilde{z}\) is constrained to the range \(-1 < \tilde{z} < 1\). Eq. (15) can be used with good accuracy for \(w_0/d < 0.10\) and \((z_w - f)/z_w < 2\), or alternatively, for arbitrary \(w_0/d\) when \((z_w - f)/z_w = 0\). Note that the variation in fringe spacing under all alignment conditions is determined by the ratio \(w_0/d\). In order for fringe spacing non-uniformities to be large, the beam offset \(d\) is typically small, which implies small \(\alpha\). It follows that for most applications in which the variation in fringe spacing is important, Eq. (15) can be further approximated by

\[
L = \frac{\lambda}{2\sin\alpha} \left[ 1 - \frac{1}{2} \left( \frac{w_0}{d} \right)^2 \right].
\]

Like Eq. (13), Eq. (16) can be reduced under ideal alignment conditions to

\[
L = \frac{\lambda}{2\sin\alpha} \left[ 1 + \frac{1}{2} \left( \frac{w_0}{d} \right)^2 \right],
\]

in which case the fringe spacing non-uniformity is determined solely by the ratio \(w_0/d\).

In a similar manner, the situation in which the beam waists are equal distances from the crossing but on opposite sides can be revisited. Substituting Eq. (11) into Eq. (7) and expanding in a power series one obtains

\[
L = \frac{\lambda}{2\sin\alpha} \left[ 1 + \left( \frac{\sin\alpha}{R\tan\alpha} \right)^2 + \left( \frac{\sin\alpha}{R\tan\alpha} \right)^2 + ... \right].
\]

After evaluating Eq. (9) at \(z = 0\) and introducing the approximation \(\sin\alpha << \cos\alpha\), Eq. (18) can be expressed as

\[
L = \frac{\lambda}{2\sin\alpha} \left[ 1 - \frac{1}{2} \cos^2 \left( \frac{\alpha}{\alpha_e} \right) \right].
\]

Eq. (19) expresses the transverse variation in fringe spacing in the \(x = 0\) plane in terms of the properties of the focused beams. Like Eqs. (13) and (14), it is of greater interest to express this variation in terms of system variables, which results in (Miles, 1996)

\[
L = \frac{\lambda}{2\sin\alpha} \left[ 1 - \frac{1}{2} \cos^2 \left( \frac{\alpha}{\alpha_e} \right) \right].
\]

The transverse variation in fringe spacing is thus shown to also be fully determined by the dimensionless ratios \(w_0/d\) and \((z_w - f)/z_w\), and is approximately linear in the normalized transverse coordinate \(\tilde{x}\).

Although the above discussion has clarified the variation in fringe spacing observed along the longitudinal and transverse probe volume axes, we have not yet addressed the off-axis fringe spacing variations, nor the effects of differing beam waist sizes and arbitrary beam waist positions. These effects are considered by using Eq. (3), with Eqs. (9)-(12), to compute the fringe spacing at arbitrary \((x, z)\) locations throughout the probe volume.

In general, the computed results show that the on-axis variation in fringe spacing, as given by Eqs. (13) and (19), can be used as a reasonable predictor of the fringe spacing throughout the probe volume, and that different waist sizes and positions do not dramatically vary the nature of the variation in fringe spacing.

For path-compensated (and ideally aligned) systems, when \(z_w = z_{w'}\), the longitudinal \((z)\) variation in fringe spacing is found to have negligible dependence on the transverse coordinate \(x\). Likewise, the transverse variation is essentially independent of \(z\) and is negligible throughout the probe volume.

In a similar fashion, when the beam waists are equal but opposite distances from the beam crossing \((z_w = -z_{w'})\), we find that the longitudinal profile retains the parabolic form given by Eq. (14), but the total variation decreases with increasing \(|z_w|\). The longitudinal profile remains nearly identical off-axis, although it is offset by an amount dictated by the expected lateral variation. The lateral profile is found
to be closely linear (cf. Eq. (19)), with a slope which does not change significantly with $z$ (as required by the observed variation in the longitudinal profiles).

To a rough approximation, the computed results indicate that when the beam waists are at arbitrary locations, the longitudinal variation in fringe spacing can be estimated from Eq. (13) with $z_{w}=(z_{o}+z_{w})/2$, while the lateral variation can be estimated from Eq. (19), using $z_{n}=-z_{o}=(z_{o}-z_{w})/2$. Under no circumstances were longitudinal profiles observed which deviated from the approximately parabolic form dictated by Eq. (13). Similarly, the lateral profiles were observed to remain linear, with a slope which never exceeded the slope estimated from Eq. (19) as described above.

The effects of varying beam waist sizes are similar to the effects of arbitrary waist locations—both variables change the local evolution of the wavefront curvature and subsequently the spatial fringe spacing variation. For the present purposes, it suffices to note that for modest changes in waist size ($\pm 10\%$) the induced variation in fringe spacing was small compared to the longitudinal variation which exists under ideal alignment conditions, and negligible in comparison to the variation associated with non-ideal beam waist locations. Fringe spacing variation was computed for various combinations of different beam waist sizes and locations—none of these combinations resulted in departures in the longitudinal profiles from a nearly parabolic form or in a departure from linearity in the transverse profiles.

4. EXPERIMENTAL SET-UP AND PROCEDURE

Due to the desire to examine the variation in fringe spacing throughout the probe volume (including the transverse variation), a simple experimental technique in which the beat frequency of light scattered from a moving surface is measured cannot be employed. The signal frequency resulting from such a technique is an average over a cross-section of the probe volume, and additional difficulties are introduced due to random phase fluctuations associated with multiple scattering centers within the probe volume. To overcome these problems, a commercial beam profiling device (Photon Inc., Model 1180), consisting of a rotating drum surrounding a photodetector is employed. On the drum surface a small aperture is mounted which sweeps across the incident beam in front of the photodetector. The resulting signal is proportional to the intensity of the light falling on the aperture. By scanning the aperture through the probe volume, a signal is generated which is analogous to the Doppler burst that would be generated by a particle following the same trajectory of the aperture. This apparatus is similar in function to the scanning pinhole used by Durst, et al. (1990). Because the fringe spacing within the probe volume may be smaller than the available apertures, the measurement of the variation in fringe spacing is made in a magnified image of a cross-section of the probe volume. In earlier work, Miles and Witze (1994) demonstrated that the fringe field in the magnified image is related to the fringe

field in the object plane of the imaging system simply by the ray-optics magnification $M$. It was further found that this relationship can be achieved experimentally with good accuracy. Because the fringe spacing in the magnified image is considerably larger than in the probe volume (and the signal from the beam profiler of significantly lower frequency), the signal from the beam profiler can be sampled digitally and used to accurately determine the spacing between individual fringes. The need to use counter or FFT type signal processors to determine the signal frequency is thus eliminated, as is the bias due to an averaging over many fringes which is inherent in such schemes.

A full discussion of the experimental technique is given by Miles and Witze (1994), and only a summary is presented here. The beam profiler and imaging lens (20X microscope objective) are mounted on an optical rail with a fixed separation corresponding to the image distance which gives the desired magnification. Because the distance of the image plane from the imaging lens is fixed, translation of the entire optical rail simply changes the effective object plane at fixed magnification. The signal output by the beam profiler is digitally sampled at a rate of about 50 times the signal frequency. Digital filtering is used to remove the signal pedestal and high frequency noise without introducing skewing of the zero crossings due to non-linear filter phase characteristics. The fringe spacing is obtained from the time between zero-crossings, which are determined using linear interpolation between the two samples on either side of each zero crossing. A threshold signal modulation of approximately 10% of the maximum modulation is used to delimit the signal.

The experimental apparatus used to create the probe volume and position the waists of the input beams is shown in Fig. 4. The beam from the laser is split by an equal path length beam splitter and each beam passes through a Keplerian telescope. The telescope lenses have independent x-y-z adjustments, which enables placement of the waists of the laser beam at arbitrary locations and allows for precise beam positioning in the front focal plane of the transmitting lens. Various beam expansion ratios can be obtained by changing the telescope lenses and their spacing. Placement of the waists of the laser beams at the desired z-coordinate $z_{w}$ is accomplished by placing the beam profiler within the beam path and adjusting the beam expander until the beam has equal diameters at $z_{w} \pm \Delta z$. The distance $\Delta z$ is chosen such that the beam diameter has changed by a measurable amount, typically 5% of the waist diameter. Due to the rather

Fig. 4 Experimental configuration. The separate telescopes (collimators) permit adjustment of the location of each beam waist individually.
Fig. 5 Far-field intensity profile of the Ar+ laser beam. The symbols are the measured intensities, while the solid line is the gaussian function which best fits the data.

slow variation in beam size (or maximum intensity) near the waist this method appears to give the most repeatable results.

For most of the results presented below, a water-cooled Ar+ laser (Cooper Lasersonics Excel 5000) operating at 514.5 nm was employed. Even at low power, the near-field profile measured just at the beam exit was markedly non-gaussian, though characterized by a single central maximum in intensity. Diffraction rapidly smooths the profile to a near-gaussian shape in the far-field (approximately 20-30 cm from the laser exit), where the beam is round to better than 1%, and is very nearly gaussian, as shown in Fig. 5. Nevertheless, the near-field profile indicates that the beam must contain higher order hermite-gaussian modes. Although the theoretical model described above is for gaussian beams, it is appropriate to compare the model predictions to the measurements obtained with this laser due to the common use of such lasers for LDV and the prevalent assumption that their beams are closely approximated by a gaussian (TE\(lm_0\)) beam. To more closely correspond to the model predictions, we have also obtained measurements of the fringe field with a low power HeNe laser with a near-field beam profile that is very closely approximated by a gaussian function.

5. GAUSSIAN BEAM MODEL ASSESSMENT

In Fig. 6 the variation in fringe spacing along the probe volume longitudinal axis is shown for three different alignment conditions: \(z_0 = z_m = -5.0\) mm, \(z_0 = z_m = 0.0\) mm, and \(z_0 = z_m = 5.0\) mm. The model predictions agree reasonably well with the data for the misaligned conditions—especially at locations within the probe volume which are furthest from the beam waists. At probe volume locations approaching the beam waists, however, the deviations between the model predictions and the data grow larger. Despite this agreement for misaligned conditions, the model

† With \(w_0 = 47\) \(\mu m\), 5.0 mm corresponds to approximately 37% of the beam Rayleigh range.

Fig. 6 Longitudinal variation in fringe spacing at \(x = 0\) for beam waists before, at, and after the crossing. The solid lines indicate the model results, while the symbols indicate the experimental data. The \(z\)-coordinate has been normalized by the half-length of the intersection volume, \(w_0/\sin\alpha = 1.13\) mm.

Fig. 7 Lateral variation in fringe spacing at \(z = 0\) for both beam waists at the crossing and for waist positions on opposite sides of the crossing.

Fig. 8 Lateral variation in fringe spacing under ideal alignment conditions at different longitudinal locations within the probe volume. The model predicts negligible fringe spacing variation throughout the volume.
fails to predict both the magnitude and the trends in the data under conditions of near-ideal alignment. In particular, the presence of two local extrema in the longitudinal profile is a recurring feature in our experimental measurements which is not consistent with the single extremum near \( z = 0 \) given by the functional form of Eq. (13).

Fig. 7 presents the fringe spacing variation in the transverse coordinate direction along the probe volume minor axis \( (z=0) \), for \( z_m = z_{m'} = 0.0 \) mm, \( z_n = -z_{n'} = 5.0 \) mm, and \( z_a = -z_{a'} = -5.0 \) mm. In contrast to the longitudinal profiles, these experimental lateral profiles appear to be well approximated by the Gaussian beam model. For \( z \neq 0 \), however, the transverse experimental profiles obtained with \( z_n = z_{n'} = 0.0 \) mm do not agree well with the model predictions. These off-axis transverse profiles are shown in Fig. 8. Although the model predicts a negligible transverse fringe spacing variation throughout the probe volume, an overall variation of approximately 7% is observed. The characteristic shapes of these transverse profiles, concave up at negative \( z \) and concave down at positive \( z \), are also recurring features in our measurements.

It is, perhaps, not surprising that the transverse variations are small along the minor axis but large elsewhere. Only along the minor axis are the transverse beam coordinates equal \( (\text{i.e. } x_1 = x_2 \) see Fig. 3), and deviations in the phase of the two beams from the ideal gaussian beam phase profile may be self-canceling when \( z = 0 \).

It is important to recognize that in obtaining the data presented in Figs. 6–8, great care was taken to ensure that aberrations in the beam profile were not introduced by the optical system employed. The (presumably innocuous) beam splitter was found to introduce measurable deviations from a gaussian profile into the beams; replacement with an alternate design which did not introduce these deviations did not materially affect the results. Nowhere in the optical system were beam aperturing effects significant, and replacement of the beam expanders with a Galilean design (in which lens aberrations are expected to cancel to some extent) did not change the results. The transmitting lens used was a high quality achromat, characterized by a marginal ray f-number of \( f/12 \). The possibility remains, however, that aberrations of this lens, though not significantly affecting the intensity profiles in the probe volume, introduced wavefront aberrations which led to the observed fringe non-uniformity. This possibility was investigated with the apparatus shown in Fig. 9, where the transmitting lens was replaced with individual focusing lenses for each beam. Equivalent results were obtained.

In light of the above discussion, it is likely that the differences between the model predictions and the data are associated with the higher order hermite-gaussian modes in the \( \text{Ar}^+ \) laser beam. To investigate this hypothesis, fringe spacing measurements were obtained using the \( \text{HeNe} \) laser described above. Longitudinal profiles of the fringe spacing variation obtained with this laser are shown in Fig. 10, for three different values of the ratio \( w_0/d \), under ideal alignment conditions \( (z_n = z_{m'} = 0.0) \). Although the profiles retain the two extrema seen in Fig. 6, the gaussian beam model appears to correctly estimate the magnitude of the longitudinal fringe spacing variation, if not the details of the profile. Note in particular that the total variation seen for \( w_0/d = 0.079 \) is reduced to about 0.5%, as compared to the variation in Fig. 6 \( (w_0/d = 0.083) \) of over 2%. Transverse profiles are again observed to have significant departures from linearity, although of considerably lesser magnitude than seen in Fig. 8. By way of comparison, the overall variation in fringe spacing observed for \( w_0/d = 0.079 \) is about 2.5%, versus about 7% in Fig. 8. The observed lateral variation appears to scale with \( w_0/d \), although the overall observed variation for \( w_0/d = 0.079 \) and \( w_0/d = 0.064 \) are similar. In addition, the characteristic concave up curvature for \( z < 0 \) and concave down for \( z > 0 \) is again observed, as well as very little transverse variation when \( z = 0 \).
6. SUMMARY, CONCLUSIONS, AND IMPLICATIONS FOR LDV SYSTEM DESIGN

A simple, yet general, analysis of the fringe field in the intersection of two gaussian beams has been presented and shown to reduce to all previously obtained fringe model results. By comparison with the results of a more rigorous analysis, the approximations inherent in this simple approach can be shown to be negligible.

Expressions for the variation in fringe spacing along both the longitudinal and transverse probe volume axes are provided in terms of the beam waist size and locations near the intersection volume. Referring these expressions to system parameters demonstrates that the dimensionless ratio $w_0/d$ controls the overall fringe spacing variation for both properly aligned and mis-aligned systems. The model results are also used to compute the variation in fringe spacing for arbitrary (and different) beam waist locations and sizes, both on and off the probe volume axes. It is found that fringe spacing variation throughout the probe volume is generally well approximated by the on-axis results. Furthermore, the fringe spacing profiles calculated for arbitrary beam waist positions and differing waist sizes are found to be qualitatively similar to the profiles seen when the beam waists have equal or equal but opposite longitudinal locations.

Comparing the model results to measured spacing between the fringes formed by nominally gaussian Ar+ laser beams, it is found that the model adequately predicts the experimental data only when the beam waists are relatively far from the intersection volume. Under near ideal alignment conditions, large longitudinal and off-axis ($z \neq 0$) transverse variations in fringe spacing are observed. Neither the magnitude nor the characteristic shapes of the fringe spacing profiles can be reproduced from the gaussian beam model, even with mismatched beam waist sizes and arbitrary waist locations. Experimental fringe spacing profiles obtained with more nearly gaussian HeNe laser beams retained the same characteristic shapes observed using the Ar+ laser, but the magnitude of the variation was reduced to approximately the levels predicted by the gaussian beam model.

Both the theoretical and the experimental results obtained have implications for the design of LDV systems when highly accurate measurements of fluid velocity are required. Due to the large fringe spacing variation observed experimentally in the $z$-direction, we concur with the recommendation of Durst et al. (1990) that the collection optics should restrict the effective probe volume to the central half of the actual probe volume length. In particular, we observe the largest longitudinal ($z$) gradients in fringe spacing before the center of the probe volume ($z < 0$), a region which should be avoided. The large $x$-direction variations which exist off the probe volume axes, however, indicate that the effective probe volume should be limited to the central portion of the probe volume height as well, provided that the number of available fringes and signal processing considerations permit this course.

Perhaps the single most significant improvement in fringe field uniformity, however, can be achieved by using a laser with a more nearly gaussian beam profile. The peak variation in fringe spacing was found to decrease by a factor of approximately 4-5 by changing from an Ar+ laser to a HeNe, for similar values of the ratio $w_0/d$.

Provided that a high quality laser is used, the theoretical and experimental investigations indicate that the overall longitudinal variation in fringe spacing scales with the square of the non-dimensional ratio $w_0/d$. Reducing this parameter can reduce the overall variation in fringe spacing, though it must be recognized that this action can adversely affect other aspects of the measurement, such as signal-to-noise ratio and spatial resolution. Additionally, we observe experimentally that the off-axis lateral variation in fringe spacing is reduced as $w_0/d$ is reduced, though the gaussian beam model predicts this variation to be negligible at all values of $w_0/d$.

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REFERENCES


