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**CHARACTERIZATION OF MULTIPHASE FLUID FLOW DURING AIR-SPARGED
HYDROCYCLONE FLOTATION BY X-RAY CT**

Final Report for the Period August 14, 1990--August 13, 1994

**By
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ABSTRACT

Air sparged hydrocyclone (ASH) flotation is a new particle separation technology that has been developed at the University of Utah. This technology combines froth flotation principles with the flow characteristics of a hydrocyclone such that the ASH system can perform flotation separations in less than a second. This feature provides the ASH with a high specific capacity, 100 to 600 times greater than the specific capacity of conventional flotation machines. In an effort to develop a more detailed understanding of ASH flotation, multiphase flow characteristics of the air sparged hydrocyclone were studied and the relationship of these characteristics with flotation performance was investigated.

This investigation was divided into four phases. In the first phase, the time-averaged multiphase flow characteristics of the ASH during its steady state operation were studied using x-ray computed tomography (x-ray CT). In this regard, a model system, mono-sized quartz flotation with dodecyl amine as collector, using a 2" diameter ASH unit (ASH-2C), was selected for study. Various flow regimes, namely, the air core, the froth phase, and the swirl layer, were identified and their spatial extent established for different experimental conditions by x-ray CT analysis.

In the second phase, a detailed parametric study of flotation response of the ASH for the same system was carried out in order to establish the effect of various operating variables on flotation response. The findings of this phase of investigation were then correlated with the multiphase flow characteristics as revealed by x-ray CT in the first phase. Thus, the impact of various operating variables on the flow regimes, and hence, on flotation response was established.

PREFACE

Research for the project sponsored by the Department of Energy through the Grant, No. DE-FG22-90PC90311, has been successfully completed in August 1994. The project titled "Characterization of Multiphase Fluid Flow During Air-Sparged Hydrocyclone Flotation by X-Ray CT", was sponsored by the DOE for the period beginning August 14, 1990, and ending August 13, 1994.

A graduate student, Mr. Avimanyu Das, of the Department of Metallurgical Engineering, University of Utah, was primarily responsible for carrying out the research work on this project. His Ph. D. dissertation reports the findings of the DOE-sponsored project in some detail and constitute the basis of this final report.

In the third phase, the axial flow reversal characteristics of the ASH under steady state operation, were studied using a tracer injection technique. In this regard, the surface of zero axial velocity, which essentially determines the fraction of the froth being carried to the overflow, was located for various operating conditions. The results were reviewed and the importance of the surface of zero axial velocity with respect to flotation recovery was established.

In the final phase of this research, the flow characteristics and flotation response were examined from first principles and valuable information regarding bubble trajectory, flotation rate and bubble attachment mechanism is discussed. The findings from these fundamental studies support earlier speculations about the residence time of air bubbles in the swirl layer, air split, and flotation rate. Finally, based on all of the results mentioned above, a phenomenological description of the ASH flotation process was offered.

All the studies in this research were carried out for the first time and have improved our understanding of the ASH flotation technology from a fundamental standpoint. Based on this understanding, certain operational considerations were suggested. It is expected that the results of this research will be of great use in the future design and operation of the air-sparged hydrocyclone and help achieve better separation efficiency.

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CHAPTER 1

INTRODUCTION

Flotation is undoubtedly one of the most important and versatile separation processes used in mineral processing technology. The use of flotation is expanding to treat a greater tonnage of raw material with new applications being developed in many additional areas. The theory of froth flotation is very complex and is not yet completely understood. Froth flotation utilizes differences in physico-chemical surface properties of various particles. In a particular system, such differences in surface properties between the particles in suspension are generally achieved after treatment with specific flotation reagents. An air-bubble must be able to attach itself to a particle in suspension and transport it to the froth phase for flotation to take place. This process is limited to relatively fine particles, because if the particles are too large detachment will occur due to inertial forces. As with all processes, flotation also has its advantages and disadvantages. One of the drawbacks of the flotation process is its inability to float very fine particles efficiently. With the inception of the air sparged hydrocyclone (ASH) flotation a new technology for fine particle flotation has been developed (1-6).

One of the most important features of the ASH is that flotation is achieved in a strong centrifugal force field in a matter of a second or less in some cases. Therefore, the ASH provides a high specific capacity that is 100 to 600 times greater than the specific capacity of the conventional flotation equipment. This feature and the fact

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that there are no moving parts in the ASH, make the air sparged hydrocyclone a very useful piece of equipment from an industrial perspective.

1.1 ASH Technology Status

One of the most recent advances in flotation equipment design has been the development of the high specific capacity air sparged hydrocyclone (ASH) for fine particle flotation in a centrifugal field. The concept of air sparged hydrocyclone for fine particle flotation is based on the proposition that the energy of the inertial collision between fine particle and air bubble in a strong centrifugal force field will be increased sufficiently to achieve film rupture, bubble attachment and flotation. The development of the ASH flotation Technology began at the University of Utah in the early 1980s and has continued since that time. By way of introduction, various aspects of ASH technology are discussed in this chapter.

The air-sparged hydrocyclone is distinguished by its high specific capacity for fine particle flotation in a centrifugal field. For example, it is now evident that the ASH has a specific capacity of at least 100 times that of conventional flotation equipment. During the 1980s, small ASH units, 2" diameter (ASH-2C), were tested for a large number of mineral commodities, and design modifications were made as necessary (7). The air-sparged hydrocyclone has shown promising potential for the flotation of copper porphyry ore, low grade placer gold and auriferous pyrite ores, iron ore, phosphate rock, potash, various other industrial minerals, and of course, fine coal cleaning. Development efforts have been extended to other countries, including Australia, Brazil, Canada, Chile, China, Finland, India, Israel, Mexico, Netherlands, Poland and South Africa.

One of the promising applications has been found to be the use of the ASH for fine coal flotation (8). As a result of this pioneering research, the effectiveness of the ASH for fine coal flotation has been demonstrated for a number of U.S., Canadian,

Mexican and South African coals, and plant-site evaluation of large-diameter ASH units is in progress. Testing of the first 15" air-sparged hydrocyclone (ASH-15C) was accomplished during 1992/93 at the Homer City Coal Preparation Plant, and the test results indicate that effective separations will be possible for fine coal cleaning (9).

Also during the last few years, attention has been given to application of ASH flotation technology outside the mineral industry, including the pulp and paper industry and the food industry, as well as its possible utilization to solve environmental problems (waste water, oil spills, soil remediation, hazardous waste, etc.). For example, the ASH technology has now been licensed to Advanced Processing Technologies, Inc. in the environmental area and implementation of the ASH technology is expected in 1994. Preliminary field trials for the removal of volatile organic compounds (VOCs) from contaminated water by air stripping with the ASH are now in progress. In one case, initial results show that more than 99% of the VOCs can be removed using the ASH technology (10).

The most significant development, however, was the announcement by Kamyr in 1991 to construct a \$20-million wastepaper recycling plant which uses air-sparged hydrocyclone technology for deinking flotation in the production of recycled paper. The plant, which has been in operation since June 1992, employs a three-stage flotation system using twenty 6" ASH units (ASH-6C) to handle a flow of about 4000 gpm (11,12). As a result of this success, a second plant is now scheduled for construction by Ahlstrom-Kamyr. This new \$250-million plant, to be located near Sanford, West Virginia, will be almost 10 times the capacity of the first plant. Ahlstrom-Kamyr has an exclusive license from the University of Utah for use of the ASH technology in the pulp and paper industry. It is expected that as many as 250, 6" diameter air-sparged hydrocyclones will be used in this modern state-of-the-art plant, which should be in operation by the end of 1995.

As the ASH flotation technology finds more and more applications in various areas, inside and outside the mineral industries, it becomes essential to achieve a more in-depth phenomenological understanding of the fundamentals of the ASH flotation technology, both from a design and process control point of view. For example, it is well known from the froth flotation literature that the characteristics of bubble formation and the bubble size significantly influence the flotation performance (13-18). In this regard, fundamental research has been taken-up to examine the nature of bubble formation in the swirl flow of the ASH and determine the corresponding bubble size distributions using high-speed photography and high-speed video techniques (19,20).

Also, fundamental research has been taken-up to study the time-averaged multiphase flow characteristics of the ASH flotation process using x-ray computed tomography (x-ray CT). As a result of these x-ray CT studies, a better appreciation of the influence of system variables on the nature of the segregated flow has been developed (21-23). In addition, efforts have been made to study the axial flow reversal characteristics during ASH flotation using a tracer analysis technique and thus offer a phenomenological explanation of the observed flotation response of the ASH under various experimental conditions (24). These fundamental studies have helped to establish design features and operating conditions for even more efficient separations using ASH technology and are the subject of this thesis.

1.2 ASH Geometry and Principles of Operation

Although there are some similarities between the conventional hydrocyclone and the air-sparged hydrocyclone with regard to basic flow type and construction of the inlet section, the innumerable differences with regard to geometry, working principles, nature of multiphase flow, underflow configuration, etc., makes the ASH distinctly different from the conventional hydrocyclone.

1.2.1 Conventional Hydrocyclone and the ASH

There are few similarities between the two types of hydrocyclones. Both hydrocyclones have the same inlet section configuration, an inlet header with a tangential entry and a vortex finder. The primary flow is a swirl flow in both cases where the fluid flow creates a centrifugal force in addition to the ever-present gravitational force. On the other hand, the differences between the hydrocyclones are many. For example, the conventional classification hydrocyclone has a cylindrico-conical body geometry whereas the air-sparged hydrocyclone has a cylindrical geometry.

Separations are based on size and density in the case of a conventional hydrocyclone whereas the ASH separates particles primarily on the basis of particle hydrophobicity. The air phase has no interaction with the slurry phase in a conventional hydrocyclone. There is no flotation in a conventional hydrocyclone. On the contrary, flotation is the primary means of separation in the ASH. In the ASH, air is injected through a porous tube wall and the air phase is dispersed into the swirl flow of the suspension.

The underflow outlet is circular and located at the apex of the conical section in a conventional hydrocyclone, whereas the underflow opening is annular and located at the periphery of the cylindrical ASH.

Thus, the multiphase flow characteristics and working principles are entirely different in the two cases. The flow behavior and the working principles of the conventional hydrocyclone are fairly well understood (25-29). However, the multiphase flow characteristics of the ASH are more complex and not yet well understood.

1.2.2 ASH Configuration

The air-sparged hydrocyclone design has a cylindrical geometry. Figure 1.1 shows a perspective view of the ASH. It consists of two concentric right-vertical tubes. The inner tube is a porous tube, typically with an average pore diameter of 50 microns (pore size range : 40 - 60 microns), through which air is sparged. The outer nonporous tube serves to provide an air jacket to establish uniform distribution of air. Thus there is an air chamber between the two tubes. The top, or inlet section of the ASH consists of a conventional hydrocyclone header with a vortex finder and an involute or tangential feed entry. The incorporation of a tapered froth pedestal at the bottom of the ASH facilitates easy and continual variation of the underflow annular opening, even during operation of the unit, by simply rotating the threaded froth pedestal support. In the case of the 2" diameter ASH, the ASH-2C which was used for this study, each rotation of the pedestal support corresponds to an underflow annular opening increase or decrease of 0.95% of the porous tube radius.

1.2.3 ASH Principles

The concept of air-sparged hydrocyclone for fine particle flotation is based on the proposition that the energy for the inertial collision between a fine particle and an air bubble will be increased sufficiently in a strong centrifugal force field to achieve film rupture, bubble attachment and flotation. In ASH flotation the centrifugal force field is generated by conversion of pressure head into the rotational motion of swirl flow.

The feed slurry, the particulate suspension, enters through the tangential inlet at the top of the ASH and follows a helical path before it exits in swirl flow through the underflow opening. See Figure 1.1. In the process, it shears the air flow coming into the ASH through the porous tube into air bubbles. Thus, during passage, contact between centrifuged particles and air bubbles takes place, bubble attachment to

AIR-SPARGED HYDROCYCLONE

Figure 1.1. Perspective of the air-sparged hydrocyclone.
(Adapted from reference 8)

hydrophobic particles occurs, and the bubble-hydrophobic particle aggregate is transported along with the froth towards the vortex finder into the overflow stream. The high-speed swirl flow exerts a considerable shear force at the inner surface of the porous tube. This, coupled with the fact that the air is introduced through fine pores, results in the generation of a large number of small air bubbles which facilitates the flotation of fine hydrophobic particles. The general principles of ASH flotation have been discussed in considerable detail in the literature (30-34).

1.3 Related Work

Studies have been carried out by various researchers to understand the fundamentals involving swirl flow and ASH fluid dynamics. For example, in the past, efforts have been made to explain some of the fluid dynamic aspects of swirl flow in general in terms of vorticity and swirl decay (35). Early ASH research to characterize the swirl flow was accomplished using a transparent plexiglass tube having no froth pedestal with water being forced through an involute entrance at the top of the tube (31). It was found that a wave pattern forms on the surface of the swirl layer with streamlines approximately 45 degrees from the normal when the water is fed at a sufficiently high pressure. This led to the expectation that, under such conditions, the axial and tangential velocities are almost equal in magnitude. Also in this study (31), tangential and axial velocities of the swirl layer were measured using Laser-Doppler anemometry. It was found that both velocities drop rapidly in the region immediately below the inlet section and then rise again. From this point downwards, the tangential velocity decays significantly with axial distance, whereas the axial velocity decays only slightly. From first principles, the radial component of velocity has to be zero at the tube wall and at the air/water interface. Therefore, in view of the small thickness of the swirl layer (10% of the tube radius), the radial component of velocity was assumed to be zero at all axial and radial positions in this study. Efforts have been

made to compare these experimentally measured tangential velocity profiles with theoretical calculations (36). In this study (36), the vorticity-stream function form of the Navier-Stokes equation was solved assuming the flow to be of high Reynolds number but still in the laminar regime. No turbulent closure equations were used. The calculated velocity profiles compared reasonably well with the limited amount of experimental data reported elsewhere (31).

Efforts have also been made to understand multiphase flow in the ASH by studying bubble characteristics generated from a single orifice. Consideration of the size and slip velocity of air bubbles from a single orifice (31) had led to speculations about multiphase flow in the ASH system. A simple theoretical expression for the size of a bubble was derived. Using a high-speed video system for motion analysis, it was found that effect of surface tension on bubble size was negligible. Bubble coalescence does not occur at the point of formation. Bubble size increases with air flow rate and fluid viscosity. It was confirmed that movement of individual bubbles follows Stoke's law. The capacity of the ASH is limited by the short residence time of the bubble phase. However, no efforts were made to study flow characteristics in the ASH during actual flotation (31).

In another study (20) bubble size distribution in the ASH was studied using high speed photography for the two phase air-water system. In contrast to the previous research on single bubble formation in swirl flow, it was found that an increase in frother (MIBC) concentration (a decrease in surface tension) significantly decreased the average bubble size and narrowed the size distribution for water flow rates from 30 lpm to 75 lpm. The surfactant minimized the effect of coalescence at the point of bubble formation. The average bubble size decreased from 400 microns to 200 microns when the water flow rate increased from 30 lpm to 75 lpm at a frother concentration of 10 ppm. Larger bubbles tended to be produced at higher air flow rates. However, this effect was not very substantial.

The effect of some operating and design variables was studied for a number of mineral commodities by Gopalakrishnan (37). These variables included vortex finder diameter, underflow opening, slurry flow rate, air flow rate, reagent addition and pulp density, which were studied for fine coal flotation, placer gold flotation and sulfur flotation from low grade volcanic sulfur deposits. Efforts were made to optimize these variables for each specific system so as to obtain maximum separation efficiency. However, the optimum levels of these variables were established by trial and error. Fundamental studies were not carried out and the findings were mainly based on empiricism.

Notwithstanding these previous ASH studies a thorough understanding of fluid flow behavior and its relationship to flotation performance remains incomplete. In this regard, initial efforts to characterize the multiphase flow behavior using x-ray computed tomography (x-ray CT) were found to be very encouraging (38,39). In these studies qualitative observations of the time-averaged multiphase flow were made using x-ray CT. A suspension of hydrophilic limestone in water was fed to the ASH unit and x-ray CT scans were taken at different locations along the axis of the unit. The time-averaged imaging of the swirl flow showed the presence of a radial particle concentration gradient of the slurry in the ASH unit from a high density sedimented limestone particles at the porous wall to a zero density air core at the center. These qualitative studies indicated that detailed quantitative studies of time-averaged multiphase flow in the ASH can effectively be made using the x-ray CT technique. Therefore, further study was undertaken in 1991 to provide a more detailed understanding of the ASH flow behavior and flotation performance by direct experimental observation using x-ray CT (40). Fairly accurate quantification in terms of density profiles along the axis and along the radius of the ASH was found to be possible by x-ray computed tomography. In addition, different flow regimes, e.g., air

core, froth phase and swirl layer were identified and their spatial extent established (21).

1.4 Research Objectives

This research is oriented towards improving the present understanding of the multiphase flow behavior of the ASH and its impact on flotation performance. It is fundamental in nature and the studies were carried out under near-ideal conditions so as to eliminate extraneous variations due to uncontrolled variables.

1.4.1 Inadequacy of Existing Phenomenological Explanation

Since the time of its inception a lot of work has been done to examine the applicability of ASH flotation technology in different practical situations. Although the applicability of the ASH in various areas has been established, a phenomenological explanation encompassing all the observed behavior of the ASH has not been available. This is primarily because the swirl flow in the ASH is extremely complex in nature and involves all three phases, solid, liquid and gas. In addition, the surface chemistry and kinetics of froth flotation coupled with bubble coalescence and breakage, froth stability, and its transport makes the understanding of the process a formidable task. Measurements have been difficult because of the very short residence time of both the air phase and slurry phase in the ASH. In addition to that, the presence of the opaque, inner porous tube makes the use of conventional measurement techniques such as high-speed photography and laser-doppler anemometry very difficult. Thus, the highly complex multiphase swirl flow makes a detailed theoretical analysis difficult and at the same time, the experimental difficulties associated with the design features of the ASH makes a detailed experimental analysis difficult. Therefore, a thorough understanding of ASH flotation has not been possible. As of now, most of the phenomenological understanding of the ASH flotation

technology is based on empiricism, measurements in a transparent plexiglass tube, and studies with a single orifice. Theoretical analysis of the system from first principles has been limited. Study of the swirl flow characteristics during actual ASH flotation has not been made. Thus, a thorough understanding of the multiphase flow characteristics of the ASH and its relationship to flotation performance still remains incomplete.

1.4.2 Scope of Research

Preliminary research to use x-ray CT for the study of multiphase flow behavior in the ASH has been found to be encouraging (38,39). Therefore, this thesis research was taken up with a view to improve our present understanding of the ASH flow behavior and its impact on the performance of the ASH. The objectives of this research are manifold :

- Experimental procedure and data analysis to establish flow characteristics
- Parametric study of operating variables
- Correlation between flow characteristics and flotation response
- Phenomenological explanation of ASH flotation from multiphase fluid flow characteristics.

The results of this research will provide valuable information for scale up and control of the ASH flotation process. A phenomenological description of the ASH flotation process has been sought in this research in order to describe, for example,

- The spatial extent and average density of different flow regimes
- Froth stability criteria and its transport

- Axial flow reversal and its influence on flotation recovery
- Bubble motion in the ASH
- The maximum particle size flotation limit
- The mechanism of bubble attachment.

CHAPTER 2

X-RAY COMPUTED TOMOGRAPHY

The area of medical imaging has been revolutionized with the introduction of x-ray based scanning machines involving the concept of computed tomography (CT). The first CT scanner was developed by Hounsfield in 1973 and CT machines, which have been in the market for the past 21 years, have proved to be excellent tools in the diagnosis of cancerous tissues at an early stage. The power of x-ray CT was quickly recognized by researchers in the area of nondestructive testing (NDT) and several machines have been developed specifically devoted to NDT work. The engineering and physics of CT are discussed in great detail in the work of Herman (41) while the mathematics has been elaborately described by Natterer (42), Kak (43) and many others (44-50).

2.1 Historical Perspective

It was proved by Radon (51) in 1917 that any arbitrary function could be recovered from its set of line integrals taken along various chords and various directions. However, the inherent mathematics was so complex that his method could not be implemented at that time. Bracewell (52) reported an application of CT in radio astronomy in 1956 and Cormack (53) derived an inversion formula in 1963, both of which were closer to being implemented than Radon's solution. Bracewell and Riddle (54) and Ramachandran and Lakshminarayan (55) showed how computations involved in the CT methodology could be accelerated by adopting the use of

convolutions. However, it was almost 55 years after the publication of Radon's paper that a CT machine was pioneered by Hounsfield (56).

2.2 Background Theory

The working principle of x-ray CT is based on the absorption of an x-ray beam as it passes through an object. A simplified approach to understand this principle, as discussed elsewhere (39), is presented here. The relationship between the initial intensity of an x-ray beam as it leaves the source and the final intensity of the same beam after it has passed through a section of the object (Figure 2.1), can be described by the following equation.

$$I_f = I_i \exp\left[- \int_{Ray} \mu(x,y) ds\right] \quad (2.1)$$

where

I_i = initial intensity of the x-ray beam

I_f = final intensity of the x-ray beam

μ = linear attenuation coefficient

Equation (2.1) assumes that the x-ray beam is monochromatic in nature. Rearranging Equation (2.1) in the following manner,

$$\ln\left[\frac{I_i}{I_f}\right] = \int_{Ray} \mu(x,y) ds \quad (2.2)$$

it can be seen that the left-hand side of this equation is the measured quantity, the value of which can be obtained from the CT machine (39). The right-hand side of

Figure 2.1. A diagram showing an x-ray beam as it emanates from the source, passes through the sample, and finally is collected at the detector. (Adapted from reference 39)

Equation (2.2) represents the ray integral of the x-ray beam that passed through the object. From Figure 2.1 it can be seen that when the angle θ is measured from the x-axis, the ray can be defined as

$$x \cos \theta - y \sin \theta = \text{Ray} \quad (2.3)$$

and the line integral of the corresponding ray can be written as

$$P_{\theta}(\text{Ray}) = \int_{\text{Ray}} f(x, y) ds \quad (2.4)$$

where $P_{\theta}(\text{Ray})$ = the projection of the x-ray beam passing through the object.

The right-hand side of Equation (2.4) is essentially the same as the right-hand side of Equation (2.2) with the unknown function $f(x, y)$ simply taking the place of the linear attenuation coefficient $\mu(x, y)$. By using the definition of Dirac delta function, Equation (2.4) can be written as (39)

$$P_{\theta}(\text{Ray}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta - y \sin \theta - \text{Ray}) dx dy \quad (2.5)$$

Equation (2.5) is known as the Radon transform of the function $f(x, y)$. If a series of line integrals are taken through the unknown function $f(x, y)$ at a fixed angle θ , a projection, $P(\theta)$, is obtained. This is illustrated in Figure 2.2.

Figure 2.2. An illustration showing several rays traversing through the sample at a fixed angle θ . The data from the series of rays are called a projection $P(\theta)$. (Adapted from reference 39)

$P(\theta)$ can be obtained by using an assembly of sources and detectors and translating them at predefined intervals to cover the entire object being scanned. In order to correlate the projection $P(\theta)$ to a reconstructed CT image, the properties of Fourier transform is used (39).

In the case of a parallel beam geometry, if a 2-D Fourier transform is performed on a function $f(x,y)$, the following equation will result.

$$F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx dy \quad (2.6)$$

The Fourier transform of a 2-D function is illustrated in Figure 2.3. The shaded circle in Figure 2.3 is the 2-D Fourier transform of the function $f(x,y)$ where the transformed coordinates are designated as u and v . (39)

Similarly, if a 1-D Fourier transform is performed on the projection $P(\theta)$, Equation (2.7) will result.

$$S_{\theta} = \int_{-\infty}^{\infty} P(\theta,t) e^{-j2\pi\omega t} dt \quad (2.7)$$

To relate the 1-D Fourier transform of the projection of an unknown function $f(x,y)$ to the 2-D Fourier transform of the same function, Fourier Slice Theorem (42) is used. For example, for the angle of measurement $\theta = 0$, Equation (2.6) reduces to

$$F(u,v)|_{\theta=0} = \int_{-\infty}^{\infty} P(0,x) e^{-2\pi(ux)} dx \quad (2.8)$$

Figure 2.3. An illustration showing the function $f(x,y)$ after it undergoes a 2-D Fourier transform operation. The transformed coordinate system is designated by variables u and v . (Adapted from reference 39)

The right-hand side of Equation (2.8) is just Equation (2.7) measured at $\theta = 0$. Figure 2.4 helps in visualizing the relationship of Equation (2.7) with Equation (2.8). Figure 2.4 shows that if a 1-D Fourier transform is performed on projection $P(\theta)$, the result would correspond to the data points in the line at an angle θ in the transformed coordinate axes u and v (39). The u and v coordinate system represents the 2-D Fourier transform of the unknown function $f(x,y)$. This is the same coordinate system shown in Figure 2.3.

Extending this logic, it can be concluded that, to construct a 2-D Fourier transform of an unknown function, the projection data at different angles have to be collected, a 1-D Fourier transform on the projection data has to be performed, and finally the data have to be plotted on the u and v coordinate system. Once the 2-D Fourier map is completed, it is a simple matter of performing a 2-D inverse Fourier transform to reconstruct the image (39).

However, the above procedure is not followed exactly because of algorithm instability and the inefficiency of computation. What is done in actual practice is, as a first step, projection data from a CT machine are collected. The next step is to perform a convolution between the projection data and a suitable filter. The filtered projection is then back-projected to obtain the reconstructed image. Back-projection is a discrete operation that mimics the operation of the CT machine but is done in the reverse manner. The projection and back-projection operations are illustrated in a simplified fashion in Figure 2.5 and Figure 2.6 respectively.

In these figures, the sample has been conveniently subdivided into nine different parts or pixels. The nine pixels have a density value attached to each one of them as shown in Figure 2.5. If the ray is passed through this sample at two different angles, 0 degrees and 90 degrees, the projection of the sample will be simply addition of the pixel values in these directions. The two different projection values are shown in Figure 2.5.

Figure 2.4. A drawing showing how the 1-D Fourier transform of the projection $P(\theta)$ can be mapped into the 2-D domain of the transformed coordinate system. (Adapted from reference 39)

PROJECTION DATA AT 90 DEGREES

Figure 2.5. Example of projection

0	0	0
0	0	0
0	0	0

1) Initialize Pixels

0+1 =1	0+1 =1	0+1 =1	← 1
0+4 =4	0+4 =4	0+4 =4	← 4
0+2 =2	0+2 =2	0+2 =2	← 2

2) Back-project the first angle

3	0	4
↓	↓	↓
1+3 =4	1+0 =1	1+4 =5
4+3 =7	4+0 =4	4+4 =8
2+3 =5	2+0 =2	2+4 =6

3) Back-project the second angle

4	1	5
7	4	8
5	2	6

4) Final image

Figure 2.6. Example of back-projection.

Back-projection is shown in Figure 2.6. First, the nine pixels are initialized to zero value. The projection data for the different angles are then smeared back, one angle at a time, as shown in Figure 2.6. Then superposition (summation) of the back-projections on the pixels is done. When projection data for a large number of angles are back-projected, the image is reconstructed.

It appears that the results of the reproduction operation are different from the values of the original sample. However, if the projection and back-projection operations are performed using a large number of angles and if the final back-projected image is normalized by the number of angles used, the reconstructed image approximates the original sample very closely (39).

In order to obtain more accurate reconstruction of tomographic images, a new algorithm is being developed (57), which uses filtered back-projection method but is iterative in nature.

2.3 Relevance to this Research

The essential feature of x-ray tomographic imaging is the determination of material density (more accurately attenuation coefficient) of a small region of 3-D space called a voxel (volume element, the thickness of which is the width of the beam). Tomography can determine the density of all voxels in the three-dimensional region of the scan section. Of course, the position of each voxel is known precisely. Thus, it is possible to determine the geometric characteristics of any region of space with respect to density variation. Subject to resolution limitations, it is possible to determine precisely the shape, and therefore the volume, as well as the mass, of each individual phase within the target volume. Since a 2-D target volume of several thousand cubic centimeters (beam thickness of 1 mm) can be examined rapidly, samples containing a very large number of particles having different densities can be studied.

X-ray CT analysis is very well suited for the study of multiphase fluid flow, so common to mineral processing operations. The nature of the flow regimes in grinding, classification, concentration, flotation, thickening or filtration can be revealed by the observed density profiles. In the case of slow processes such as thickening and filtration, this can be done on a real time basis, while for fast processes such as the multiphase flow in conventional hydrocyclones and air-sparged hydrocyclones, time-averaged characteristics can be observed (38).

In the case of air-sparged hydrocyclone flotation, the time-averaged imaging of the swirl flow will reveal the concentration gradient in the swirl layer at the porous tube wall to the air core at the center at any section of the ASH. This will help establish the spatial extent and average density of different flow regimes, including the air core diameter and other spatial features distinguished by density variation. Of particular interest, information about froth stability can be obtained. With this detailed understanding of time-averaged multiphase fluid flow in the air-sparged hydrocyclone, a better phenomenological understanding will be achieved and theoretical treatment of the phenomena can be considered with greater confidence. By way of example, the spatial extent of the froth phase and its relationship to flotation recovery under various conditions will provide guidelines for improving design and operation criteria which should lead to improved separation efficiency.

CHAPTER 3

X-RAY CT ANALYSIS OF MULTIPHASE FLOW

In view of the fundamental nature of this research, a well-defined system was chosen for study. ASH flotation of mono-sized quartz particles with MIBC as frother and dodecyl amine as collector was selected as a model system. The x-ray CT technique, the experimental results, and data analysis are discussed in this chapter. It should be noted that since the feed suspension contained only hydrophobic quartz particles, there was no separation from other particles.

3.1 Experimental Set Up and Data Acquisition

Quartz particles were used as a model system because they do not disintegrate and are of sufficiently high density to create a significant variation in suspension density thus allowing for x-ray CT utilization. In this research the effect of the following operating variables on flow behavior and froth phase stability were studied :

- A^* - the ratio of overflow opening area to underflow opening area
- Q^* - the ratio of air flow rate to slurry flow rate
- Φ - percent solids in the feed suspension
- P - slurry inlet pressure.

3.1.1 Materials

For these experiments quartz particles of size 100 x 200 mesh were prepared from SIL-CO-SIL 250 by wet screening with a 24" SWECO screen. Since the quartz

particles are intrinsically hydrophilic a collector (dodecyl amine) was added to create a hydrophobic state at the quartz surface. MIBC was used as frother. Table 3.1 gives a summary of the materials used in the x-ray CT experiments.

3.1.2 Equipment

An ASH-2C system, designed and manufactured by Advanced Processing Technologies Inc. of Salt Lake City, Utah, was used in this study. The ASH unit consists of a KREBS inlet header with a rectangular (0.80" x 0.33") inlet opening and a 0.875" ID vortex finder, a 1.9" ID porous tube (12" in length), and an underflow tapered froth pedestal.

The x-ray CT scanning was carried out by renting time on a modified medical scanner from a local research company, M/S TerraTek. This second generation CT scanner, DeltaScan 100, has a PDP 11-04 host computer. It incorporates both rotational and translational motion of the x-ray source-detector assembly to scan and reconstruct a cross sectional image at any desired position.

Table 3.1. Materials used in the x-ray CT experiments

Material	Description	Supplier	Purity
Quartz	SIL-CO-SIL 250 100 x 200 mesh	US Silica Co.	99.8 %
MIBC	Methyl Isobutyl Carbinol as frother	Aldrich Chemicals	99.0 %
Dodecyl Amine	12-Carbon Amine as Collector	Rhone Poulenc Inc.	98.8 %

In addition to the above mentioned equipment, other accessories used in these experiments were, a 300 liter sump for the feed suspension, a small 50 liter sump for the underflow, a 1.5 VACSEAL pump to feed the ASH and a SALA pump to return the underflow back into the feed sump, two CS type 1/2 HP LIGHTNIN mixers, several two-way Apollo ball valves, a compressor to supply air, a pressure gauge to monitor slurry inlet pressure, and a rotameter to monitor air flow rate. Table 3.2 provides a summary of the equipment used for the x-ray CT experiments.

3.1.3 Procedure

The quartz particles were wet-screened and a 100 x 200 mesh size fraction was taken. The ASH-2C unit was placed horizontally inside a large plexiglass tube (11.5" ID and 96" long) around which the x-ray source/detector assembly rotates on a frame. The system is mounted on a table which can be adjusted to set the ASH unit at the desired axial location. The movement of the table is controlled by a computer interface to precisely locate the section to be scanned. A photograph of the ASH unit mounted on the CT scanner is presented in Figure 3.1.

In common practice, the ASH is operated in a vertical position. However, because of the practical difficulties the ASH was operated in a horizontal position during x-ray CT experiments. In order to establish the effect of the orientation of the ASH, flotation tests were carried out for horizontal and vertical ASH orientations under different experimental conditions. The results of these experiments are presented in the next chapter. Examination of these data revealed that the orientation of the ASH has negligible effect on flotation performance of the ASH.

A suspension of quartz particles (100 x 200 mesh) in water at the desired solids concentration was prepared in the feed sump. The reagents used for this study were MIBC as frother and dodecyl amine as collector. The frother concentration was

Table 3.2. Equipment used for the x-ray CT experiments

Equipment	Model / Description	Supplier
X-Ray CT Scanner	DeltaScan 100, Model : 1102	Ohio Nuclear Inc.
Air-Sparged Hydrocyclone	Model : ASH-2C	Advanced Processing Technologies Inc.
1.5 VACSEAL Feed Pump	Model: D-1.5-VR-A101	The Galigher Co.
PACEMAKER Motor	3 HP, Model: 19284R-0	The Louis Allis Co.
Underflow Pump	SPV 180-1.5	SALA Machine Works
Motor	5 HP, Model : B0054DGF2A3	Toshiba Corporation
LIGHTNIN Mixers	Model : ND-2, 1725 RPM	Mixing Equipment Co. Inc.
Compressor	Model : 44643	DeVilbiss (Can.) Ltd., Canada
Rotameter	Model : 10A1755A	Fisher & Porter Co.
Pressure gauge	Ashcroft	Cole-Parmer Instrument Co.
Valves	Apollo, two-way ball valves	Conbraco Industries Inc.

Figure 3.1. A photograph of the ASH unit mounted inside a plexiglass tube in the x-ray CT scanner.

maintained at 40 ppm (water basis) for all experiments. The collector concentration was kept at 800 g per ton of dry solids. Two LIGHTNIN mixers kept the solids in a suspended state and homogenized the slurry such that the system was well mixed. The slurry flow rate was controlled by regulating the ASH inlet pressure with a two-way Apollo ball valve. The inlet pressure was monitored by a pressure gauge which was calibrated by measuring the volume flow rate at different pressure levels. A compressor was used to supply air to the ASH. The air was sparged through the inner porous tube which had an average pore diameter of about 50 microns (pore size range 40-60 microns). The air flow rate was monitored and controlled with a rotameter. A schematic diagram of the experimental set up is given in Figure 3.2.

In the case of x-ray CT experiments, the ASH was operated as a closed system. The overflow (froth product) was discharged back to the feed sump. See Figure 3.2. The underflow rejects were collected in another sump and pumped back into the feed sump continuously. Thus, the net flow into and out of the feed sump was zero. The slurry in the feed sump was conditioned for 10 minutes and then the system was started. Steady state was reached quickly and scans were taken at different locations as preprogrammed with the CT scanner.

For most cases, 11 sections of the ASH unit were scanned. However, in some cases only three sections, at the top, middle and bottom, were scanned. The locations of the scanned sections are shown in Figure 3.3. The CT scanner scans the cross section from 60 different angles over a span of 180 degrees and measures the intensity values. Thus the projection data at every 3 degrees were collected. From these projection data a 2-D image of the cross section was generated using the standard convolution back-projection reconstruction algorithm. Generation of the projection data at each angle takes about 2 seconds. Thus, generation of each cross sectional image took about 2 minutes and as such the information obtained provides the

Figure 3.2. A schematic diagram of the x-ray CT experimental set up.

Figure 3.3. The porous tube section of the ASH unit showing the location of the sections scanned during x-ray CT experiments. The distances were measured from the line AA.

characteristic features of the time-averaged multiphase flow since the retention time of the suspension is on the order of a second.

It should be noted that because of the practical difficulties of sampling the product streams at the x-ray CT site, flotation experiments were carried out at the university laboratory with the same experimental set up as was used for the x-ray CT experiments. The flotation response was measured at the same experimental conditions for which x-ray CT measurements were taken in order to relate flotation response to the flow characteristics, particularly the froth features revealed from image reconstruction.

Objects of known density were attached to the outer surface of the ASH unit in order to provide for density calibration. These objects were also scanned while scanning the ASH section. The reconstruction algorithm assigns a CT number to every pixel (picture element) depending upon its density or attenuation coefficient. The average CT number of the pixels representing the object were thus related to the actual density and the calibration curve was established.

3.2 Data Analysis

To analyze the images, a special computer algorithm was designed which involved a C++ computer code. The code was compiled using the DJGPP compiler which supports a large number of graphic tools and was run on an IBM PC-486. A suitable graphics-user interface was provided for easy operation of the computer program (58). The software is able to extract quantitative data from the images such as the air core dimension, average density of the swirl layer, density value at any pixel and the radial density profile (averaged) of the cross section. The software is available with Professor J. D. Miller of Metallurgical Engineering, University of Utah. The computer code and the related aspects are described in this section.

3.2.1 The Algorithm

The computer code designed to analyze the images is explained with the box-diagram shown in Figure 3.4. The first step was to read the image and display it on the screen in a 256 pixels X 256 pixels square region. Since the dimensions of the porous tube were known a priori, the spatial extent of one pixel in millimeters could be determined. The code reads the intensity of the pixels, the CT numbers associated with each pixel, and normalizes them into a 0-255 intensity scale, zero being the lowest density and 255 being the highest density. These normalized CT numbers were then plotted in the 256 x 256 pixels square area to display the image.

In the second step, the center of the image, the geometric center of the circular cross section, was located. The pixels in the annular region representing the outer shell, which has a very high density, had intensity values of 160 or more. All other pixel values in the 256 x 256 square matrix were 100 or less because of the lower density of the material in this region. This information was used to identify the center of the image. The x and y-coordinates of all pixels in the peripheral annular region, having an intensity value of 160 or more were recorded by the code. The x and y-coordinates of all such pixels were averaged separately. The average x-coordinate was assigned to be the abscissa and the average y-coordinate was assigned to be the ordinate of the center of the image.

Then, the region of interest, the region surrounded by the porous tube, was isolated from the image and considered for analysis. This step was accomplished by bitwise AND operation (59). When two values are ANDed, the binary representations of the two values are "compared" bit by bit. If A and B represent corresponding bits of the two operands, then the following table, Table 3.3, shows the results of A ANDed with B for all possible values of A and B.

Figure 3.4. A box diagram showing the steps involved in the computer code for image analysis.

Table 3.3. The results of AND operation for various values of the operands

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

In this case the operand A was the whole image where the bits corresponding to the region of interest as well the bits outside this region were assigned the value of 1. The operand B also was the whole image where the bits corresponding to the region of interest were assigned a value of 1 and the bits outside this region were assigned a value of 0. Now, when A and B were ANDed, it can be seen from Table 3.3 that only the bits representing the region of interest retained the value of 1 whereas all other bits contained a value of 0. Thus, the region of interest was retained on the screen while the rest of the image was cleared.

In the fourth step, the air core diameter was calculated by counting the number of pixels with a value zero. Thus, the area covered by these pixels was determined and the radius of a circle having this area was calculated to establish the average radius of the air core.

The average density value was calculated at every pixel on a radial line in the next step. In this step, the air core radius was incremented by one pixel value and at this radial position 360 pixel values, each at an angular distance of one degree apart, were read. The density at this radial position was the average of these 360 values. The radial position then was incremented by one pixel and the process was repeated until the surface of the porous tube was reached.

In the final step, the radial density profile was generated using these average density values at various radial positions. Using the previously determined air core radius and these average density values, the image was reconstructed.

It should be noted that the air core is not exactly circular and its center does not always coincide with the geometric center of the ASH unit. However, these two centers were found to be very close. This program assigns a circular region of the calculated radius to the air core, and places it at the geometric center of the ASH, and reconstructs the image with the average density values at each pixel radially. This results in some source of error and, in this regard, analysis was also done to measure density profiles from the center of the air core instead of from the center of the ASH unit. Density values were measured along eight different radial directions, 45 degrees apart, from the center of the air core. Thus, eight different radial density profiles at eight different angular positions at the same cross section were generated. These data are presented in Figure 3.5 for the conditions mentioned in the figure caption and consist of eight different radial density profiles at different angular positions in the middle of the ASH. The averaged density profile is also shown in the figure. As can be seen from this figure, the variation in the density profiles in various directions is very small. Thus, the averaged density profile can be considered to be a good representation of the actual density distribution inside the ASH. A statistical analysis of the confidence level of this data is presented in the next subsection.

Other features, such as selecting a particular region and finding out its average density and histogram, the density value at any pixel, selecting any circular region and finding out the radial distribution of density in this circular area, etc., are also available with this data analysis code.

Figure 3.5. Radial density profile in eight different directions at the same section (section 6) in the middle of the ASH for 15% solids (100 x 200 mesh quartz particles with 800 g/ton amine), $A^* = 0.74$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

3.2.2 Confidence Level of the Data Set

As mentioned in the previous section, a statistical analysis for the data presented in Figure 3.5 was carried out to establish the confidence level (60,61). In this analysis, the averaged density values were denoted as $\bar{d}$ and the directional densities (as measured) were denoted as d . The magnitude of the error, $(d-\bar{d})$, was calculated for all the radial positions in each direction. These values were then plotted against radial position and the plot is shown in Figure 3.6. It can be seen that these values are randomly scattered around zero and lie inside a band except for the two narrow regions enclosed between the dotted vertical lines. In these two regions the absolute value of $(d-\bar{d})$ is high because of the fact that the center of the air core is not exactly at the geometric center of the ASH, but slightly off from it. Therefore, while measuring the density values in different directions from the air core center, the distance to the porous tube wall is not equal in all the directions. Thus, by approximating the air core as circular and placing it at the geometric center some amount of error is introduced in the region around the air core surface and in the region close to the porous tube surface. This error is reflected in these two regions.

Examination of Figure 3.6 reveals the fact that there is a random scatter in the variable $(d-\bar{d})$ around a value close to zero. Hence it can be said that $(d-\bar{d})$ follows a normal distribution with a mean μ and a variance of σ^2 at each radial position. Since the data lie within a band, equal variance at all radial positions can be assumed and a pooled analysis can be made. The variance of the population σ^2 can be approximated by the pooled variance of the sample s^2 . Since the true values of the parameters are not known, the normal distribution should be approximated by the t -distribution. However, for a large value of n such as in this case (over 400), the t -distribution becomes the same as normal distribution. Therefore the confidence interval for the mean value of $(d-\bar{d})$, μ , can be approximated as

Figure 3.6. Scatter plot of the error in density, $(d-\bar{d})$, for the data corresponding to Figure 3.5, in order to check for randomness and establish the confidence interval.

$$\bar{x} \pm z(\alpha/2)(s/\sqrt{n})$$

where $\bar{x}$ is the sample mean, $z(\alpha/2)$ is the 100(1- $\alpha/2$) percentile of the N(0,1) distribution and n is the sample size. These parameters were calculated for $n = 416$ and it was found that for these observations $\bar{x}$ was 1.028×10^{-2} , s was 4.584×10^{-2} .

For the 95% confidence level, $z(\alpha/2) = z(0.05/2) = 1.96$. Therefore the confidence interval was calculated to be $1.028 \times 10^{-2} \pm 4.405 \times 10^{-3}$. That is, 95% of the time the mean value of the observations ($d\text{-dbar}$) is likely to lie in the range 5.875×10^{-3} to 1.468×10^{-2} . In other words, in this research if the density at a point was found to be 1.00 g/cc after averaging, using the same set of data (same image), 95% of the time another observer will find the average density value at this point to lie between 1.0058 to 1.0146. Thus by averaging the density, an error of 0.6% to 1.4% is being introduced to the data. Since this error is relatively small, it can be said that the average density profile is a good representation of the actual density distribution in the ASH.

However, excluding the two regions of high error from the analysis, $\bar{x}$ was calculated to be 9.984×10^{-3} , s to be 2.621×10^{-2} and n to be 388. Therefore the interval for 95% confidence level of the error, ($d\text{-dbar}$), was determined to be $9.984 \times 10^{-3} \pm 2.608 \times 10^{-3}$.

3.2.3 Identification of the Flow Regimes

Figure 3.7 shows a reconstructed CT image and the corresponding radial density profile as generated by the data analysis software in Figure 3.8. The regions identified from the density profile in Figure 3.8 are the following : A - Air Core, B - Froth Phase, C - Swirl Layer, D - Porous Tube, E - Air Chamber, F - Outer Shell. The air core on the axis of the ASH is defined by the region of zero density.

Figure 3.7. A reconstructed CT image showing different flow regimes in the middle of the ASH for 15% solids in feed, $A^* = 0.74$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

A - Air Core	D - Porous Tube
B - Froth Phase	E - Air Chamber
C - Swirl Layer	F - Outer Shell

Figure 3.8. The radial density profile corresponding to the CT image shown in Figure 3.7, showing different flow regimes in the middle of the ASH for 15% solids in feed, $A^* = 0.74$, $Q^* = 4.55$ and inlet pressure = 10.5 psi.

To identify the froth phase and the swirl layer the slope of the radial density curve at every point was calculated and changes in the gradient of the radial density profile curve, as shown in Figure 3.9, were examined. It can be seen from Figure 3.9 that as one goes from air core/froth phase interface towards the porous tube wall, the slope gradually decreases, goes through a minimum (not necessarily zero) and rises again, finally becoming zero at the highest density point. These two points of minimum (just before the slope starts rising) and zero slope are identified in Figure 3.9 as Point A and Point B respectively. The midpoint between this minimum slope point and the zero slope point at maximum density was chosen to be the point of distinction between the froth phase and the swirl layer.

The spatial extent of region D, region E and region F (Figure 3.8) were known a priori from the dimensions of the porous tube and the outer shell. Note that the porous tube density has been measured independently to be about 0.6 g/cc and its thickness is about 6 mm. As can be seen in Figure 3.8, the width of region D is 6 mm but only the middle portion (3 mm in the middle) has a density of 0.6 g/cc. In fact, a sharp drop from the peak swirl layer density to the density of the porous tube is expected as one goes to region D from region C, i.e., a step response. In fact, as is evident from Figure 3.8, the transition is not that sharp. This error is introduced by the limited resolution due to beam size and by the limitations of the reconstruction algorithm.

3.2.4 Density Calibration

In the CT scanner, the reconstruction algorithm assigns a CT number to every pixel based on its density, more correctly its attenuation coefficient. In the image analysis program, these CT numbers were normalized into a 0-255 scale and then were correlated with density through a calibration curve. For this purpose, objects of known density were also scanned along with the ASH cross sections. Table 3.4 shows

Figure 3.9. Variation in the gradient of the radial density profile with radial position for 15% solids, $A^* = 0.74$, $Q^* = 4.55$, and 10.5 psi inlet pressure.

Table 3.4. Objects of known density and the corresponding normalized CT numbers

Object	Density (g/cc)	Normalized CT No.
Air	0.000	0
Porous tube material	0.590	28
Acetone	0.785	37
Water	1.000	49
Teflon	1.470	73

the objects used for density calibration, their densities and the corresponding normalized CT numbers based on 20 observations for each object.

The normalized CT numbers presented in Table 3.4 are the average values of 20 observations. The normalized CT numbers of these objects are plotted against their actual density in Figure 3.10. and a straight line was fitted to these data through linear regression analysis. It can be seen from Figure 3.10 that the data fit a linear relationship with a correlation coefficient of 0.999. In this manner, then, the density calibration curve was generated. It was found that the normalized CT numbers were related to actual density (g/cc) in the following manner :

$$\text{Density} = 0.02 \times (\text{Normalized CT number}) + 0.017 \quad (3.1)$$

Figure 3.10. Density calibration with normalized CT numbers. The center line is the estimated regression line and the outer lines represent 95% prediction interval (± 0.011 g/cc).

A statistical analysis was carried out in order to determine the 95% prediction interval for density based on 20 observations for each object (60,61). The estimated regression line is given by

$$\hat{y} = \alpha + \beta x \quad (3.2)$$

where $\hat{y}$ is the estimated density, x denotes normalized CT number and α and β are the regression coefficients which were found to be 0.017 and 0.02 respectively in this case. The residual, e_i , was then calculated for each observation

$$e_i = y_i - \hat{y} \quad (3.3)$$

where y_i is the observed and $\hat{y}$ is the estimated density.

The sum of squares due to error, SSE, and the mean square due to error, MSE, were then evaluated from the following equations.

$$SSE = \sum_{i=1}^n e_i^2 \quad (3.4)$$

$$MSE = \frac{SSE}{n-2} \quad (3.5)$$

The 95% prediction interval for density, Y_k , at any level of normalized CT number, x_k , is given by

$$Y_k = \alpha + \beta x_k \pm [t(0.025; n-2) \sqrt{MSE(1 + \frac{1}{n} + \frac{(x_k - \bar{x})^2}{\sum (x_i - \bar{x})^2})}] \quad (3.6)$$

where, t denotes the Student's t -distribution, and $\bar{x}$ denotes the average normalized CT number. The term included in the square bracket in Equation (3.6) was evaluated for several values of x_k and it was observed that this value is almost constant at 0.011. Therefore, 95% of the time, the actual density will lie within ± 0.011 of the estimated density. This is shown as a band in Figure 3.10.

3.3 Froth Density

In order to understand the radial density profiles and identify the froth phase regime during ASH flotation, it is useful to have a point of reference. In this regard, flotation tests were carried out in a gravitational field using a conventional flotation cell to float quartz and in these experiments the froth density of the concentrate was measured as a function of time using the following procedure.

In a 2-liter cell, a suspension of quartz in water was prepared with 15% solids. Then 40 ppm frother (MIBC) and 800 g/ton collector (dodecyl amine) were added and the suspension was conditioned for 10 minutes. Air flow was initiated and the concentrate was collected at different time intervals. Three concentrates (500 cc sample) were collected, weighed, and the average densities of these froth samples were determined from their weights and volumes. In this way the average density of the froth sample which consists of water, hydrophobic quartz particles and air bubbles was determined. The three concentrates, collected at the 1st, 4th and 7th minute, were found to have average froth densities of 0.38, 0.27 and 0.14 g/cc respectively. It should be noted that in this case the suspension was continuously being depleted of solids which accounts for the decrease in froth phase density with time. These density values are given to provide a reference point in the analysis of the radial density profiles during ASH flotation as determined by x-ray CT. For example the froth phase for the data presented in Figure 3.8 (Region B), is distinguished by a range of densities from 0.8 g/cc to 0.2 g/cc. As can be seen from this data and subsequent data

to be examined, the froth phase densities in ASH flotation generally have been found to be somewhat greater than the froth phase densities found for conventional flotation in a gravitational field.

3.4 Results and Discussion

In the x-ray CT experiments, scans at different locations of the ASH were taken for various levels of operating variables. The data analysis algorithm was used to generate the radial density profiles from the images. The images are available with Professor J. D. Miller. Variation in density with radial position at the top, middle and bottom of the ASH is tabulated in Appendix A for various experimental conditions. These radial density profiles were studied for each slice and the flow regimes were identified. The experimental results are presented and discussed in this section.

3.4.1 Reproducibility of Data

The experimental results have been found to be fairly reproducible and the reliability of the data sets has been established. For example, Figure 3.11 shows the variation in data on the same day. In this figure the radial density profiles at the top, middle and bottom of the ASH are shown for replicate experiments but at two different times on the same day. It can be seen from this figure that there is very little variation in data on the same day.

Figure 3.12 shows the reproducibility of the results after a certain period of time. The radial density profiles for identical conditions are presented in which the second set (Set-II) of experiments was carried out 6 months after the first set (Set-I). It can be seen from this figure that the profiles are very similar in the two cases. The density values compare favorably. The small differences can be attributed to the difficulty of maintaining exactly the same conditions when high flow rates are involved such as these.

Figure 3.11. Radial density profiles at the top, middle and the bottom of the ASH for 15% solids (100 x 200 mesh quartz particles with 800 g/ton amine), $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure for two experiments under the same conditions at two different times in order to establish the variation in data on the same day (8/24/1992).

Figure 3.12. Radial density profiles at the top, middle and the bottom of the ASH for 5% solids (100 x 200 mesh quartz particles with 800 g/ton amine), $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure for two experiments under the same conditions conducted 6 months apart in order to establish the long term variation in experimental data.

Examining Figures 3.11 and 3.12, it can be said that the results are fairly reproducible and establish an indication of the experimental error that can be expected.

3.4.2 Density Profiles - No Solids

Figure 3.13 shows the density profiles at the top, middle and bottom of the ASH when there are no quartz particles in the feed (water and reagents only). It should be noted that in most instances, including this case, 11 scans were taken along the length of the ASH but only 3 are shown in Figure 3.13 to facilitate presentation and understanding of the results. A very narrow froth phase (hardly defined) can be detected at the top as revealed by the gradually increasing density region between the air core and the swirl layer. The gradual increase in density is indicative of decreasing air bubble concentration in the liquid phase which becomes insignificant in the swirl layer. The froth phase is absent elsewhere. An almost cylindrical air core and a thick swirl layer which is fairly uniform along the axis of the ASH (except at the very top) are notable in this case which, in the absence of quartz particles, serves as a reference for comparison with other cases.

3.4.3 Characteristic Features of the Flow Regimes

3.4.3.1 Axial View

The internal flow in the ASH is characterized by three distinct flow regimes, namely, the central air core, the froth phase and the swirl layer. It has been observed that irrespective of the experimental conditions, there is always an air core at the center when the slurry is fed at a sufficiently high pressure. X-ray CT measurements reveal these features clearly. Examination of the radial density profiles at various axial positions provides information regarding the dimensions of the above mentioned flow regimes and their average densities. Thus, by studying the radial density profiles at

Figure 3.13. Radial density profiles at the top, middle and bottom of the ASH for no solids in the feed, $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

various axial positions of the ASH the axial view of the flow regimes along the entire length of the ASH can be constructed. Such a representation is shown in Figure 3.14. The experimental conditions are given in the figure caption.

For this purpose, the radial density profiles are stacked according to their axial positions as shown in Figure 3.14(a). From this arrangement it is easy to see the spatial extent and average density of the flow regimes. The axial view is presented in Figure 3.14(b). It can be seen from this figure that under these experimental conditions, the swirl layer thickness increases down the axis of the ASH and the average density of the swirl layer decreases down the axis and eventually becomes close to 1.00 g/cc at the very bottom. The froth phase thickness decreases down the axis and the froth phase disappears at the very bottom of the ASH. However, the air core diameter does not change very much.

3.4.3.2 Dimensionless Area Ratio (A^*) - 5% Solids

The variable A^* is defined as the ratio of overflow opening area to underflow opening area. Figure 3.15 shows the effect of A^* on the density profiles for 5% solids in the feed slurry (feed density = 1.032 g/cc). The experimental conditions are given in the figure caption. In this figure, the density profiles at the top, middle and bottom of the ASH are presented for three different A^* values. It can be seen from this figure that the air core is larger at the top and smaller at the bottom for all cases.

For $A^* = 1.00$, the swirl layer thickness is small at the top with an average density of about 1.03 g/cc and the froth phase could be observed at the top as indicated by the gradual increase in density from the air core to the swirl layer (Figure 3.15). The froth phase is almost absent in the middle, in this case, and the swirl layer has expanded. At the bottom, there is no froth phase and an even thicker swirl layer can be observed. Note that the swirl layer density for both the middle and bottom sections is very close to 1.0 g/cc signifying the absence of solids in the swirl layer at

CONSTRUCTION OF AXIAL VIEW FROM RADIAL DENSITY PROFILES

Figure 3.14. Construction of the axial view of the flow regimes
 (a) Radial density profiles stacked according to their axial positions for 15% solids, $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure
 (b) Corresponding axial view of the flow regimes.

Figure 3.15. Radial density profiles at the top, middle and bottom of the ASH for different A^* values with 5% solids in the feed, $Q^* = 4.55$ and 10.5 psi inlet pressure.

these locations. The overwhelming similarity between the density profiles at the very bottom in this case (column 1, Fig. 3.15) and the no solids case (Fig. 3.13) corroborates the analysis. This finding coupled with the observation that the froth phase is absent from the middle of the ASH downward, in this case, represent two important points that are revealed from the x-ray CT measurements :

- (1) The froth phase is stabilized in the presence of hydrophobic solids.
- (2) All the solids are removed in the top region of the ASH for 5% solids in the feed, inlet pressure = 10.5 psi and $A^* = 1.00$. In fact, it can be seen from Table 3.5, more than 90% recovery has been achieved in this case.

For lower A^* values ($A^* = 0.85$ and 0.74), a thin froth phase could be identified by the inflection point on the density profiles at all axial locations. Also, the swirl layer density in these two cases ranges from 1.005-1.025 g/cc at all axial locations as can be seen from Figure 3.15. These results suggest the presence of hydrophobic solids in the swirl layer at all axial locations for $A^* = 0.85$ and 0.74 . The flotation recoveries in the overflow are reduced, as can be seen from Table 3.5, to 85% for $A^* = 0.85$ and to 68% for $A^* = 0.74$. These flotation results support the x-ray CT finding that hydrophobic solids are present in the swirl layer even at the bottom of the ASH in these cases and, consequently, the stabilized froth phase extends over the full length of the ASH. In these two cases, froth and hydrophobic particles are lost to the underflow which accounts for the lower recovery.

The axial views of the flow regimes for $A^* = 0.74$ and $A^* = 1.00$ are compared in Figure 3.16. Expansion of the air core diameter with a decrease in A^* for 5% solids in feed (feed density = 1.032 g/cc) is clearly visible in this figure. Also, a stable froth phase can be observed even at the bottom of the ASH for $A^* = 0.74$. This observation along with the observation that the average swirl layer density at the bottom is about 1.02 g/cc, confirms the presence of hydrophobic solids at this location.

Figure 3.16. Comparison between the axial views of the flow regimes for two different A^* values constructed from 11 density profiles, 3 of which are shown in Figure 3.15 for 5% solids (100 x 200 mesh quartz particles with 800 g/ton amine), $Q^* = 4.55$, inlet pressure = 10.5 psi.

Table 3.5. Flotation response of 100 x 200 mesh quartz particles for specified experimental conditions with a 12" long porous tube.

Collector : 800 g/ton ; pH = 7.4.

Solids in Feed (wt%)	A*	Q*	Inlet Pressure (psi)	Slurry Flow Rate (lpm)	Recovery (%)
5	1.00	4.55	10.5	63.5	94.2
5	0.85	4.55	10.5	63.5	85.6
5	0.74	4.55	10.5	63.5	68.4

For $A^* = 1.00$, the froth phase is absent in the bottom half of the ASH and an average swirl layer density of 1.00 g/cc indicates that flotation is complete. There are no solids present in this region. In fact, more than 90% recovery for $A^* = 1.00$ and less than 70% for $A^* = 0.74$ were obtained as can be seen from Table 3.5. Generally, the swirl layer thickness can be observed to increase substantially from the top to the middle of the ASH and thereafter remains uniform for $A^* = 1.00$. On the other hand, the thickness of the swirl layer for $A^* = 0.74$ does not change significantly with respect to axial position.

3.4.3.3 Dimensionless Area Ratio (A^*) - 15% Solids

Figure 3.17 shows the effect of A^* on density profiles with 15% solids in the feed slurry (feed density 1.103 g/cc). The density profiles at the top, middle and bottom of the ASH are shown in this figure for three different A^* values. Comparison between Figure 3.15 and Figure 3.17 reveals that, at any A^* value, if the percent solids in the feed is increased from 5 to 15, the air core shrinks considerably. It can be seen from Figure 3.17 that, for $A^* = 1.00$, the froth phase is present at the top and middle sections as indicated by the inflection in the density profiles. Also, the swirl

Figure 3.17. Radial density profiles at the top, middle and bottom of the ASH for different A^* values with 15% solids in the feed, $Q^* = 4.55$ and 10.5 psi inlet pressure.

layer density (1.05-1.125 g/cc) is substantially higher at these locations. Therefore, it can be concluded that hydrophobic solids are present in the swirl layer at these locations. However, at the bottom, the absence of a froth phase and a thick swirl layer of average density 1.00 g/cc indicates the absence of solids at this axial position. Examination of the other density profiles for $A^* = 1.00$ reveals that the froth phase extends all the way down to the bottom of the ASH, gradually diminishing in thickness and finally disappearing at the very bottom. Indeed, 97% recovery of the solids in the overflow was achieved in this case as can be seen from Table 3.6.

For lower A^* values ($A^* = 0.85$ and 0.74) the froth phase is stable at all axial locations, even at the bottom, as can be seen by the inflection in the density profiles (Figure 3.17). Also, the swirl layer density is in the range 1.05-1.10 g/cc thus indicating the presence of hydrophobic solids in the swirl layer at all axial locations for these two cases. In fact, poor recovery was obtained in these two cases (51% and 15% respectively) as can be seen from Table 3.6.

Table 3.6. Flotation response of 100 x 200 mesh quartz particles for specified experimental conditions with a 12" long porous tube.

Collector : 800 g/ton ; pH = 7.4.

Solids in Feed (wt%)	A^*	Q^*	Inlet Pressure (psi)	Slurry Flow Rate (lpm)	Recovery (%)
15	1.00	4.55	10.5	63.5	97.3
15	0.85	4.55	10.5	63.5	50.9
15	0.74	4.55	10.5	63.5	15.5

The axial views of the flow regimes with 15% solids in the feed for $A^* = 0.74$ and $A^* = 1.00$ are compared in Figure 3.18. It can be seen from this figure that the air core expands as A^* is decreased from 1.00 to 0.74. A stable froth phase and a high density of the swirl layer (~ 1.1 g/cc) at the bottom of the ASH for $A^* = 0.74$, confirms the presence of hydrophobic solids at this location.

For $A^* = 1.00$, the froth phase decreases in thickness down the axis of the ASH and eventually disappears at the bottom. The absence of the froth phase at the bottom of the ASH and an average swirl layer density of 1.00 g/cc at this location indicates that flotation is complete. More than 95% recovery for $A^* = 1.00$ and less than 20% for $A^* = 0.74$ were obtained as can be seen from Table 3.6.

3.4.3.4 Percent Solids

The effect of solids concentration in the feed on the flow regimes is depicted in Figure 3.19. The experimental conditions are given in the figure caption. It can be seen from this figure that as the percent solids in the feed slurry is increased from 5 to 15, the air core shrinks and the swirl layer thickness at the top of the ASH is reduced. For 5% solids in the feed, the froth phase is relatively narrow and does not extend down to the bottom of the ASH. In this case, the flow region in the bottom half of the ASH, essentially consists of an air core and a thick swirl layer of average density 1.0 g/cc, indicating the total absence of solids in this region.

On the other hand the froth phase is much more extensive for the 15% solids case and extends all the way down to the bottom of the ASH, gradually diminishing in thickness. In the previous section, it was established that the froth phase is stabilized by the presence of hydrophobic particles. Therefore, from the dependence of froth stability on the presence of hydrophobic particles (62), it can be inferred that, in this case - 15% solids, the hydrophobic solids in the froth phase extend the full length of the ASH decreasing in amount from the top to the underflow discharge at the bottom.

Figure 3.18. Comparison between the axial views of the flow regimes for two different A^* values constructed from 11 density profiles, 3 of which are shown in Figure 3.17 for 15% solids (100 x 200 mesh quartz particles with 800 g/ton amine), $Q^* = 4.55$, inlet pressure = 10.5 psi.

Figure 3.19. Effect of percent solids in the feed on the flow regimes constructed from 11 density profiles, 3 of which are shown in Figures 3.15 and 3.17 for 5% solids and 15% solids (100 x 200 mesh quartz particles with 800 g/ton amine) respectively, with $A^* = 1.00$, $Q^* = 4.55$, inlet pressure = 10.5 psi.

Also, the disappearance of the froth phase at the very bottom and an average swirl layer density of 1.0 g/cc at this location confirms the fact that, in this case - 15% solids, hardly any solids are present at the very bottom of the ASH. It is evident then that the solids concentration gradually decreases along the ASH length (from top to bottom) and eventually becomes almost zero at the very bottom.

It was found that more than 90% of the solids are recovered to the overflow in both cases, 5% and 15% solids. See Tables 3.5 and 3.6. Comparison between the 5% and 15% solids cases with $A^* = 1.00$, brings out one very important design aspect. The stability and the extent of the froth phase is dependent on the amount of hydrophobic solids in the system. Therefore, at $A^* = 1.00$, but almost all the solids have been removed in the top portion of the ASH for 5% solids, the complete length of the ASH is required to float the solids in the 15% solids case. A different set of flotation experiments with a shorter ASH unit was designed to verify this point and will be discussed in the next chapter.

3.4.3.5 Inlet Pressure

Of course, the inlet pressure at which the slurry is being fed to the ASH is expected to change the flow characteristics of the ASH as it changes the inlet velocity of the slurry and hence the resulting centrifugal force field. In this regard, the effect of slurry pressure for 15% solids in the feed was studied. Figure 3.20 shows the effect of inlet pressure on the radial density profiles. The experimental conditions are given in the figure caption. At a low inlet pressure (6.5 psi) the recovery of solids in the overflow was found to be 70% as can be seen from Table 3.7. This case is characterized by the presence of the froth phase at all axial locations and a high swirl layer density (1.02-1.08 g/cc) as can be seen from Figure 3.20. This may be explained by the fact that the upward axial velocities are not great as the inlet velocity itself is lower in this case. As a result of a lower inlet velocity the shear force and thus the

Figure 3.20. Comparison between the radial density profiles at the top, middle and bottom of the ASH at two different inlet pressures with $A^* = 1.00$, $Q^* = 4.55$ and 15% solids in the feed.

Table 3.7. Flotation response of 100 x 200 mesh quartz particles for specified experimental conditions with a 12" long porous tube.
Collector : 800 g/ton ; pH = 7.4.

Solids in Feed (wt%)	A*	Q*	Inlet Pressure (psi)	Slurry Flow Rate (lpm)	Recovery (%)
15	1.00	4.55	10.5	63.5	97.3
15	1.00	4.55	6.5	46.6	69.9

bubble generation/particle collision may be expected to be reduced. Consequently, froth transport to the overflow becomes sluggish resulting in a poor flotation recovery.

On the other hand, the higher shear rate at 10.5 psi should facilitate a higher particle/bubble collision frequency. The magnitudes of the velocities are higher in the 10.5 psi case which facilitates transport of froth to the overflow giving higher recovery. The froth phase gradually diminishes in thickness down the axis and disappears at the bottom, and the average swirl layer density decreases and eventually becomes 1.00 g/cc at the bottom indicating the absence of hydrophobic solids at this position. Indeed, a high recovery (97%) was obtained for this case. However, it should be kept in mind that beyond a certain inlet pressure, the resulting velocities become too great and may actually destabilize the hydrophobic particle-air bubble aggregates and thus result in lower recovery of the solids.

The axial views for these two inlet pressures are presented in Figure 3.21. X-ray CT measurements revealed that a thick, stable froth phase is present all along the axis of the ASH for a lower inlet pressure (6.5 psi). From the dependence of froth stability on the presence of hydrophobic solids, it can be said that solids are present even at the bottom of the ASH in this case. This is supported by the high density of

Figure 3.21. Effect of inlet pressure on the flow regimes constructed from 11 density profiles, 3 of which are shown in Figure 3.20 for 15% solids (100 x 200 mesh quartz particles with 800 g/ton amine), with $A^* = 1.00$ and $Q^* = 4.55$.

the swirl layer as discussed earlier. Thus, at this low inlet pressure (6.5 psi), some froth along with hydrophobic particles are being carried to the underflow. Quartz particles are thus being lost to the underflow thereby accounting for the observed lower recovery. Visual observations regarding the swirl flow characteristics in a plexiglass tube having the same inlet configuration, which will be discussed in the next chapter, revealed that vortex formation at 6.5 psi is barely complete. The upward axial velocities are not great in view of the low inlet velocity. The shear force and thus the bubble generation/particle collision may be reduced. Consequently, froth transport to the overflow is diminished.

On the other hand, at 10.5 psi the swirl layer becomes depleted of hydrophobic quartz particles down the axis of the ASH, the froth phase diminishes in thickness, and the swirl layer density approaches 1.00 g/cc. Thus, loss of froth phase and hence quartz particles to the underflow is negligible. This is revealed by the high recovery as has been discussed earlier. It was found that the air core shrinks as the slurry pressure increases. This feature has also been observed visually in the case of swirl flow through plexiglass tube.

3.4.3.6 Dimensionless Flow Rate - Q^*

The variable Q^* is defined as the dimensionless ratio of the air flow rate to the slurry flow rate. Figure 3.22 shows the effect of Q^* on the density profiles for 15% solids in the feed (feed density = 1.103 g/cc). Experimental conditions are given in the figure caption. In this figure, the radial density profiles at the top, middle and bottom of the ASH are presented for three different Q^* values.

For $Q^* = 2.28$, it can be observed that a froth phase (not very well defined) is present only at the top and the froth phase is absent elsewhere in the ASH. Also, the average swirl layer density at all axial positions is around 1.1 g/cc indicating the presence of large amount of solids in the swirl layer. At a fixed inlet pressure, a lower

Figure 3.22. Radial density profiles at the top, middle and bottom of the ASH for different Q^* values with 15% solids in the feed, $A^* = 1.00$ and 10.5 psi inlet pressure.

value of Q^* implies a lower air flow rate. Thus, in this case, there is not enough air available to capture the hydrophobic particles and form a stable froth. Consequently, most of the solids remained suspended in the swirl layer. Only 52% recovery of solids to the overflow was obtained in this case as can be seen from Table 3.8. It appears that there is insufficient air flow at $Q^* = 2.28$ and the corresponding froth phase instability results in a lower recovery.

At higher air flow rates ($Q^* = 4.55$ and 6.83), good recoveries have been achieved (97% and 96% respectively). See Table 3.8. The striking similarity between the density profiles at all axial positions between these two cases is notable. In both these cases the froth phase thickness gradually decreases down the axis and eventually becomes zero at the very bottom as can be seen from Figure 3.22. Also, in these two cases, the swirl layer density decreases down the axis and eventually becomes 1.00 g/cc at the bottom indicating good flotation. Examination of Figure 3.22 also reveals that as Q^* is increased the air core expands because of the higher proportion of air.

Table 3.8. Flotation response of 100 x 200 mesh quartz particles for specified experimental conditions with a 12" long porous tube.
Collector : 800 g/ton ; pH = 7.4.

Solids in Feed (wt%)	A^*	Q^*	Inlet Pressure (psi)	Slurry Flow Rate (lpm)	Recovery (%)
15	1.00	2.28	10.5	63.5	51.9
15	1.00	4.55	10.5	63.5	97.3
15	1.00	6.83	10.5	63.5	95.6

The axial views of the flow regimes for $Q^* = 2.28$ and $Q^* = 6.83$ are presented in Figure 3.23. With a low air flow rate ($Q^* = 2.28$), as can be seen from Table 3.8, only 52% of the hydrophobic solids were recovered in the overflow. In this case, a very small froth region, could be observed at the top of the ASH. The absence of a stable froth elsewhere and the very high density (1.05-1.08 g/cc) of the swirl layer indicate the presence of a large amount of solids in the swirl layer at all axial locations which would be expected in view of the poor flotation recovery in this case. Hydrophobic particles remain in the swirl layer and are lost to the underflow, thus accounting for a low recovery. At a higher air flow rate ($Q^* = 6.83$), the froth phase stabilizes and extends all the way down to the bottom of the ASH, again gradually diminishing in thickness. It was found that, in this case, the froth phase disappears at the very bottom and that the swirl layer is less dense at the bottom (about 1.0 g/cc) indicating the absence of any hydrophobic quartz particles at this location. Thus, loss of quartz particles in the underflow stream is very low. Indeed, about 96% recovery was obtained for this case as can be seen from Table 3.8. Expansion of the air core with an increase in Q^* is another observation that can be made from the x-ray CT data as is evident from Figure 3.23.

3.4 Summary

The experimental results clearly show the utility of x-ray CT for the analysis of the complex multiphase flow during air-sparged hydrocyclone flotation. Even though the x-ray CT measurements describe only the time-averaged steady state flow, the calculated radial density profiles provide sufficient information to distinguish between the froth phase and swirl layer during actual flotation experiments. On this basis, the variation in the flotation response can be described from the variation in the spatial extent of the flow regimes with changes in design and operating variables (A^* , Q^* ,

Figure 3.23. Effect of Q^* on the flow regimes constructed from 11 density profiles, 3 of which are shown in Figure 3.22 for 15% solids (100 x 200 mesh quartz particles with 800 g/ton amine), with $A^* = 1.00$, and 10.5 psi inlet pressure.

percent solids, inlet pressure). The importance of froth stability and its transport were shown to be of particular significance in the analysis of quartz flotation by the air-sparged hydrocyclone.

The air core radius was found to vary from 4.5 mm to 11 mm during quartz flotation. Specifically, the air core expands as the value of A^* is reduced and as Q^* is increased, but shrinks when the percent solids in the feed is increased. In general, the air core/froth phase interface has surface instabilities which are manifested in minor deviations from a cylindrical shape.

The froth phase dimension changed considerably with changes in design and operating variables. The results from x-ray CT measurements clearly establish the dependence of froth stability on the presence of hydrophobic particles. The features of the froth phase and the swirl layer (e.g., spatial extent, average density, etc.), as revealed by x-ray CT measurements, has been correlated with flotation performance. Since the froth phase thickness depends on the amount of hydrophobic particles present in the system, the spatial extent of the froth phase is substantially greater with 15% solids in the feed than with 5% solids in the feed. On this basis, it was found that a longer residence time, and thus a longer ASH, is required for efficient flotation at a higher solids concentration in the feed.

Froth phase features are favorable for flotation at a high A^* value ($A^* = 1.00$). Froth transport and its removal through the vortex finder appears to be better at this A^* value which improves flotation recovery.

Froth transport is efficient at higher inlet pressures apparently because of the greater axial velocities. At low inlet pressure, froth transport is sluggish and consequently hydrophobic particles remain in the swirl layer and stabilize the froth at the bottom of the ASH thus resulting in its discharge to the underflow and poor recovery. A higher air flow rate (higher Q^*) up to a certain level leads to increased froth stability and thereby an improved flotation response.

The swirl layer experiences a stronger force at the top when compared to that at the bottom because of the decay in the force field as the suspension travels downward (30,31,35). The swirl layer thickness has always been found to be smaller at the top and larger at the bottom because of this axial variation in the centrifugal force. The extent of flotation was clearly evident from the change in density of the swirl layer and complete flotation was realized when the swirl layer was depleted of solids and its density was reduced to 1.0 g/cc.

CHAPTER 4

QUARTZ FLOTATION EXPERIMENTS

Flotation experiments were carried out in order to quantify the flotation response and relate these data to the results from x-ray CT experiments. Flotation response is characterized by the recovery of solids in the overflow. Detailed parametric study of ASH flotation revealed the influence of various operating variables on flotation performance. In this research, effects of the following operating variables on flotation response were studied :

- A^* - the ratio of overflow opening area to underflow opening area
- Q^* - the ratio of air flow rate to slurry flow rate
- Φ - percent solids in the feed suspension
- P - slurry inlet pressure
- d_p - particle size
- C_c - collector concentration
- pH - suspension pH.

The results from these flotation experiments along with the experimental procedure are described in this chapter.

4.1 Experimental Technique

For flotation experiments the ASH system was operated as an open system. The two product streams were not pumped back to the feed sump in order to facilitate sampling of the overflow and underflow streams. The system was stopped each time

after samples were collected from the two product streams. Details of these experiments are given in this section.

4.1.1 Materials

The materials used in these experiments are the same as those used in x-ray CT experiments. However, two different sizes of quartz particles were used in this study in order to establish the effect of particle size on flotation response. Also, in one set of experiments a solution of sodium hydroxide in water was used to change the pH of the pulp in order to examine the effect of flotation chemistry on flotation response. Table 4.1 gives a summary of the materials used in the flotation experiments.

Table 4.1. Materials used in the flotation experiments

Material	Description	Supplier	Purity
Quartz	SIL-CO-SIL 250 100 x 200 mesh 200 x 325 mesh	US Silica Co.	99.8 %
MIBC	Methyl Isobutyl Carbinol as frother	Aldrich Chemicals	99.0 %
Dodecyl Amine	Collector	Surfactant Division Rhone Poulenc Inc.	98.8 %
NaOH	Granular, to prepare alkaline solution	Fisher Scientific Co.	98.3 %

4.1.2 Equipment

An ASH-2C system, designed and manufactured by Advanced Processing Technologies Inc. of Salt Lake City, Utah, was used, the same unit as was used in the x-ray CT experiments. The ASH unit consists of a KREBS inlet header with a rectangular (0.80" x 0.33") inlet opening and a 0.875" ID vortex finder, a 1.9" ID porous tube (12" in length), and an underflow tapered froth pedestal assembly.

In addition to that, other accessories used in these flotation experiments were, a 300-liter sump for the feed suspension, a 1.5 VACSEAL pump to feed the ASH, two CS type 1/2 HP LIGHTNIN mixers, several two-way Apollo ball valves, a compressor to supply air, a pressure gauge to monitor slurry inlet pressure, and a rotameter to monitor air flow rate. To separate the solids from the sampled product streams, a pressure filter was used. Table 4.2 provides a summary of the equipment used for the flotation experiments.

4.1.3 Procedure

Quartz particles were wet-screened and the 100 x 200 mesh and 200 x 325 mesh size fractions were taken. In these studies the system was run as an open system. The product streams, overflow and underflow, were open in order to facilitate sample collection. The feed pump was shut off after collection of each sample.

The quartz suspension at the desired solids concentration and the desired levels of frother and collector was prepared in the feed sump. The same reagents, MIBC as frother and dodecyl amine as collector, were used in these experiments. The frother concentration was maintained at 40 ppm (water basis) for all experiments. Unless otherwise mentioned, the collector concentration was kept at 800 g per ton of dry solids, the particle size was 100 x 200 mesh and the pH of the suspension was 7.4. Two LIGHTNIN mixers kept the solids in a suspended state and homogenized the slurry such that the system was well mixed. The slurry flow rate was controlled by

Table 4.2. Equipment used for the flotation experiments

Equipment	Model / Description	Supplier
Air-Sparged Hydrocyclone	Model : ASH-2C	Advanced Processing Technologies Inc.
1.5 VACSEAL Feed Pump PACEMAKER Motor	Model: D-1.5-VR-A101 3 HP, Model: 19284R-0	The Galigher Co. The Louis Allis Co.
Pressure Filter	Model : F-1	The Booth Co.
LIGHTNIN Mixers	Model : ND-2, 1725 RPM	Mixing Equipment Co. Inc.
Compressor	Model : 44643	DeVilbiss (Can.) Ltd., Canada
Rotameter	Model : 10A1755A	Fisher & Porter Co.
Pressure gauge	Ashcroft	Cole-Parmer Instrument Co.
Valves	Apollo, two-way ball valves	Conbraco Industries Inc.

regulating the ASH inlet pressure with a 2-way Apollo ball valve. The inlet pressure was monitored with a pressure gauge which was calibrated by measuring the volume flow rate at different pressure levels. A compressor was used to supply air to the ASH. The air was sparged through the inner porous tube which had an average pore diameter of about 50 microns (pore size range 40-60 microns). The air flow rate was monitored and controlled with a rotameter.

In the case of flotation experiments, the slurry in the feed suspension was conditioned for 10 minutes before the experiments were initiated. The operating variables were set at desired levels and the system was started. In the ASH, steady state is reached quickly and the product streams were sampled simultaneously for equal time intervals (typically 5-10 seconds). These samples were then weighed, filtered using a pressure filter and dried. From the weights of the solids in the overflow and underflow samples, recovery of the solids to the overflow was determined for each experiment.

4.2 Results and Discussion

A large number of flotation experiments (over 200) were carried out in order to establish the influence of different operating variables on flotation response. These data along with the water split to the overflow are shown in Appendix B in tabular form. Replicate experiments were carried out to test the reproducibility of the data. These data are furnished and discussed in this section.

4.2.1 Reproducibility of Data

The results have been found to be satisfactorily reproducible within experimental error. Figure 4.1 shows the variation in data for replicate experiments. It can be seen from this figure that there is very little variation in data and the measurements are fairly accurate and consistent. Thus, reliability of the data sets collected at the indicated dates was established.

A statistical analysis was carried out to determine the 95% confidence interval of this data (60,61). In this analysis the average recovery at any inlet pressure was denoted as $\bar{r}$, and the individual measurements were denoted as r . The error, $(r - \bar{r})$, was calculated for each observation and was plotted against the inlet pressure. Such a plot is shown in Figure 4.2. It can be seen from this figure that the scatter in

Figure 4.1. Replicate experiments to show reproducibility of flotation recovery data for $A^* = 1.00$, $Q^* = 4.55$, 5% solids and 10.5 psi inlet pressure.

Figure 4.2. Scatter plot of the error in recovery, $(r-rbar)$, with inlet pressure corresponding to the data presented in Figure 4.1, in order to check for randomness in the scatter and establish the confidence level.

the error is fairly random and does not follow any pattern. Thus a normal distribution may be assumed and the confidence interval for the mean value of ($\bar{r}$) can be approximated as

$$\bar{x} \pm z(\alpha/2)(s/\sqrt{n})$$

where $\bar{x}$ is the mean value of the observations ($\bar{r}$), $z(\alpha/2)$ is the 100(1- $\alpha/2$) percentile of the N(0,1) distribution, s is the pooled variance of the sample (assuming equal variance in the observations at various pressure levels), and n is the sample size. These parameters were calculated for $n = 15$ and it was found that for these observations $\bar{x}$ was 2.543×10^{-6} , s was 0.683.

For the 95% confidence level, $z(\alpha/2) = z(0.05/2) = 1.96$. Therefore the confidence interval was calculated to be $2.543 \times 10^{-6} \pm 0.346$. Since the above interval is fairly close to zero, it can be said that the data sets are fairly reliable and hence the reproducibility is established.

From the above analysis, it can be said that if the recovery has been reported to be 80% on a certain day in this research, then 95% of the time another observer will observe a recovery between 79.65% to 80.35% under the same conditions on any other day.

4.2.2 Flotation Response

4.2.2.1 Horizontal and Vertical ASH

It was mentioned in Chapter 3 that the ASH was operated in a horizontal position during x-ray CT experiments. To establish the effect of the orientation of the ASH, flotation tests were carried out for horizontal and vertical ASH under various experimental conditions. The results of these experiments are presented in Table 4.3.

Table 4.3. Flotation response of 100 x 200 mesh quartz particles for specified experimental conditions with two different ASH orientations.

Collector level : 800 g/ton, pH = 7.4.

A*	Q*	Inlet Pressure (psi)	Solids in the feed (wt%)	Recovery with horizontal ASH (%)	Recovery with vertical ASH (%)
0.74	4.55	10.5	5	66.8	68.4
1.00	4.55	10.5	5	92.4	94.2
0.74	4.55	10.5	15	14.2	15.5
1.00	4.55	10.5	15	94.9	97.3
1.00	6.83	6.5	5	76.2	80.1
1.00	2.28	6.5	5	38.0	41.5
1.00	4.55	13.5	5	58.5	58.4
1.00	4.55	8.5	15	81.9	83.1
1.00	6.83	13.5	15	68.0	68.9

Examination of the results presented in Table 4.3 reveal that the effect of the orientation of the ASH on flotation response is not significant. Therefore, the results from the x-ray CT experiments may be correlated with the results from flotation experiments.

4.2.2.2 Dimensionless Area Ratio (A*) - 5% & 15% Solids

The variable A*, defined as the dimensionless ratio of overflow opening area to the underflow opening area, appears to be one of the most important operating variables in ASH flotation. Tests were carried out with different solids concentration in the feed, 5% and 15% solids by weight, and the flotation response in terms of recovery of solids in the overflow was observed for three different A* values. These

results are presented in Figure 4.3. It can be seen from this figure that recovery of hydrophobic quartz particles increased with an increase in A^* for both cases.

Examination of Figure 4.3 reveals that the influence of A^* on flotation response is more prominent when the percent solids in the feed suspension is higher (15%). The recovery rises slowly and tends to become almost constant with an increase in A^* for 5% solids case. On the other hand, a steep, almost linear, rise in recovery can be observed for 15% solids when the level of A^* is raised from 0.74 to 0.85 and finally to 1.00.

From other test results, it was found that the conditions in these tests (e.g. levels of Q^* , inlet pressure etc.) are fairly conducive for flotation. However, the conditions at the underflow exit change as the annular underflow opening area changes with a change in A^* . It should be noted that the vortex finder diameter and hence the overflow opening area was not altered in these experiments. A change in A^* was brought about by changing the underflow opening area. As A^* is increased (or decreased), the annular underflow opening area decreased (or increased) thus allowing less slurry to exit through the underflow opening. Consequently, a greater amount of slurry reports at the overflow indicating a change in the axial flow reversal characteristics with a change in A^* is evident. Thus, the amount of hydrophobic particles being carried to the overflow and hence recovery changes with A^* . Evidently, particle transport plays a major role in this regard (63).

In Chapter 3, a discussion on the significant changes in froth phase characteristics with a change in A^* was given. The froth phase characteristics as revealed by x-ray CT measurements, were described for two different A^* values with two different solids concentrations in the feed in Figures 3.15 through 3.18. It was found that the froth phase was stable and relatively thick even at the bottom of the ASH for both 5% and 15% solids when A^* was low ($A^* = 0.74$). See Figures 3.16 and 3.18. However, when A^* was increased to 1.00, the froth phase gradually

Figure 4.3. Variation in flotation response with A^* for two different solids concentrations. $Q^* = 4.55$, inlet pressure = 10.5 psi.

decreased in thickness down the axis of the ASH. The froth phase disappeared in the middle of the ASH for 5% solids whereas for 15% solids the froth phase disappeared at the very bottom. These observations are explained better in the following discussion which is given as a supplement to the discussion in Chapter 3.

The froth pedestal offers an obstruction to the downward swirl flow and prevents all of the slurry from exiting through the underflow opening. Therefore, an upward flow is generated. It is expected that at any elevation of the ASH the axial velocity changes its direction at a certain radial position depending upon the experimental conditions. Therefore, a surface of zero axial velocity exists inside the ASH and the axial flow reversal characteristics change as A^* changes. The surface of zero axial velocity shifts with regard to its radial position as the boundary conditions at the underflow exit changes with a change in A^* . As a consequence the amount of froth carried to the overflow will change and flotation recovery will increase or decrease accordingly. In this regard, the surface of zero axial velocity was located for different experimental conditions using a tracer analysis technique (24) and will be discussed in the next chapter.

At higher A^* more froth is carried to the overflow because of the favorable axial flow reversal characteristics. This results in the depletion of hydrophobic solids in the suspension as it travels downwards. Thus, depending on the solids concentration of the feed, at certain axial position in the ASH there are not enough hydrophobic particles to stabilize the froth and consequently, the froth phase disappears. However, at low A^* values, the froth removal is not very efficient and the recovery of hydrophobic particles in the overflow is not great. The hydrophobic particles get trapped inside, stabilize the froth all along the axis of the ASH, and as a result some are discharged to the underflow.

4.2.2.3 Porous Tube Length - 5% and 15% Solids

As discussed in the previous section, the solids concentration becomes important when the value of A^* is low. However, at a high A^* ($A^* = 1.00$) value comparable recoveries have been achieved with different solids concentrations in the feed suspension.

In the previous chapter, it was pointed out that a longer ASH unit and hence a longer residence time is required to obtain higher recovery when solids concentration in the feed slurry is high. To verify this point, flotation experiments were carried out at a shorter porous tube length, 6" instead of 12", for different levels of operating variables and the flotation recoveries were compared. The results with a shorter ASH (6" porous tube length) are compared with the results with a longer ASH (12" porous tube length) in Table 4.4 for 5% solids in the feed.

It can be seen from Table 4.4 that when the conditions (e.g. A^* , Q^* and inlet pressure) are favorable to achieve good flotation, comparable recoveries have been obtained with the 6" and 12" porous tubes. Therefore, it is evident that under these conditions the length of the porous tube has no noticeable impact on flotation performance when the feed suspension contains 5% solids.

Table 4.4. Flotation response of 100 x 200 mesh quartz particles for specified experimental conditions with two different porous tube lengths.

Solids in feed : 5%, collector level : 800 g/ton, pH = 7.4.

A^*	Q^*	Inlet Pressure (psi)	Slurry Flow Rate (lpm)	Recovery with 12" length ASH (%)	Recovery with 6" length ASH (%)
1.00	4.55	10.5	63.5	91.2	88.4
1.00	6.83	6.5	46.6	80.1	80.8

Table 4.5 compares the recovery values for 15% solids in the feed with two different porous tube lengths, 6" and 12". It can be seen from Table 4.5 that, under favorable conditions as discussed above, the recovery drops drastically from 92% to 56% and from 80% to 59% for the two conditions shown, when the porous tube length is reduced to 6" from 12". Evidently, a longer porous tube is required to increase the recovery of hydrophobic quartz particles in the overflow when the feed suspension contains 15% solids.

From the preceding discussion it can be said that for a feed which contains 5% solids, when the level of the operating variables are favorable for good flotation (more than 80% recovery), the recovery obtained with the shorter ASH is comparable to that with the longer ASH. However, in the case of 15% solids, reducing the length of the porous tube from 12" to 6", resulted in a large drop in recovery. Thus, it appears that a longer ASH is required for good flotation with higher solids percent in the feed slurry. However, with a lower percent solids in the feed, a shorter ASH is sufficient to produce high recovery depending upon the operating variables.

Table 4.5. Flotation response of 100 x 200 mesh quartz particles for specified experimental conditions with two different porous tube length.
Solids in feed : 15%, collector level : 800 g/ton, pH = 7.4.

A*	Q*	Inlet Pressure (psi)	Slurry Flow Rate (lpm)	Recovery with 12" length ASH (%)	Recovery with 6" length ASH (%)
1.00	4.55	10.5	63.5	92.4	56.0
1.00	6.83	6.5	46.6	80.6	59.0

4.2.2.4 Dimensionless Flow Rate Ratio (Q^*) and Inlet Pressure (P)

The influence of inlet pressure and Q^* on flotation response has been summarized graphically in Figures 4.4 and 4.5. In these figures, recovery is plotted against inlet pressure for different Q^* values with 5% and 15% solids in the feed. Three very interesting aspects of ASH flotation can be observed from these figures. First, comparison between Figure 4.4 (for 5% solids) and Figure 4.5 (for 15% solids) reveals that when A^* is maintained at a high level ($A^* = 1.00$), the solids concentration in the feed (within the range considered) has no noticeable impact on the flotation response. Second, at any Q^* value, the recovery passes through a maximum as inlet pressure is increased from 6.5 psi to 16.5 psi. The recovery increases first and then starts to drop beyond about 10 psi. Third, with respect to Q^* there appears to be an optimum level for this dimensionless variable. At any inlet pressure, the recovery increases with Q^* upto a certain point ($Q^* = 6.83$) and beyond this point it drops; however this decrease in recovery is very small.

These observations lead to the following phenomenological explanations. First, at a given Q^* , increase in the inlet pressure enhances the shear rate for improved bubble generation and the higher velocity and pressure appear to facilitate froth transport to the overflow. In view of the high centrifugal force and hence a high tangential velocity of the fluid in the ASH, bubble generation is expected to be governed by the shear at the wall (64). At higher inlet pressures, because of the high shear at the wall, smaller bubbles are expected to be ripped off from the porous tube wall. The smaller size of the bubbles enhances flotation and consequently the recovery improves. However, beyond a certain inlet pressure, no further decrease in bubble size is possible. Also, the resulting tangential velocity and hence the centrifugal force for detachment may be too high for the bubble/particle aggregate to remain stabilized. Under such conditions detachment occurs resulting in a drop in recovery. Second, at a given inlet pressure, as Q^* is increased the concentration of bubbles increases because of the increased air flow rate. This results in enhanced recovery. However, as found in earlier research (20), bubble size increases with

Figure 4.4. Variation in flotation response with inlet pressure at different Q^* values. $A^* = 1.00$, 5% solids in the feed.

Figure 4.5. Variation in flotation response with inlet pressure at different Q^* values. $A^* = 1.00$, 15% solids in the feed.

an increase in air flow rate. Thus, at higher relative air flow rates (higher Q^*), the bubble concentration may reach a constant value with the increase in air simply being accommodated by the generation of bubbles of larger size. A larger bubble size may be detrimental to flotation and cause the recovery to drop (65-68). However, since the bubble size does not increase substantially with Q^* , the drop in recovery is also small.

It is expected that the number of pores in the porous tube actively participating in bubble formation plays an important role in determining flotation recovery. At a low Q^* (low air jacket pressure), when the air flow rate is low, the number of these active pores is also low. Thus, the number of air bubbles formed is relatively smaller. With an increase in air flow rate (air jacket pressure) more pores of the porous tube get activated and take part in bubble formation. Therefore, at higher Q^* the number of bubbles are also relatively larger. This results in a higher collision rate and hence a higher recovery. However, beyond a certain Q^* , there is no further increase in the number of active pores as most of the available pores have already been activated at this Q^* . Thus, further increase in Q^* does not result in an increase in the number of bubbles and consequently the recovery does not increase at higher Q^* values.

From Figures 4.4 and 4.5, it can be seen that there is a trade off involved both with respect to inlet pressure and Q^* . These two variables need to be maintained at optimum levels to maximize recovery. It appears that, for quartz (100 x 200 mesh) flotation, the system reaches an optimum at an inlet pressure of about 10 psi and a Q^* value of about 7.00.

4.2.2.5 Particle Size

Figure 4.6 shows the effect of particle size on ASH flotation of quartz. Flotation recoveries of two different particle sizes at various Q^* values are plotted against inlet pressure. It can be seen from this figure that fairly high recovery can be achieved even at a low Q^* (recovery is about 70% at $Q^* = 2.28$) in the case of smaller particles (200 x 325

Figure 4.6. Effect of particle size on flotation response for $A^* = 1.00$ and 5% solids in the feed.

mesh). The recovery of 200 x 325 mesh quartz particles is fairly insensitive to inlet pressure over a wide range (10-35 psi) at any Q^* as can be seen from Figure 4.6. Also, for these smaller particles, the recovery starts to drop beyond about 36 psi whereas for larger particles (100 x 200 mesh) the recovery starts to drop beyond about 10 psi. This observation can be explained by the fact that the centrifugal acceleration required for detachment is greater in the case of smaller particles because of their lower mass. Therefore, the stability of the bubble/particle aggregates is maintained even at high inlet pressures (~36 psi) and the hydrophobic quartz particles are transported into the froth phase. Evidently, smaller particles form a more stable aggregate with the bubbles (69).

On the other hand, for the larger particles the centrifugal acceleration required for detachment is smaller, and the bubble/particle aggregates become unstable at a relatively low inlet pressure (~10 psi). Bubble detachment begins to take place at an inlet pressure exceeding 10 psi resulting in a drop in recovery. In an earlier publication (34), it was established from theoretical considerations that the maximum diameter of a particle that can be floated in a centrifugal field, should be inversely proportional to the square root of centrifugal acceleration. From a balance of the surface tension and the centrifugal force acting on the particle/bubble aggregate, the following relationship was found and the maximum size particle to be floated should vary inversely with the tangential velocity :

$$d_{\max} = \frac{C}{(V^2 / R)^{1/2}} \quad (4.1)$$

where,

$d_{\max}$ = maximum particle size to be floated

V = tangential velocity

R = radius of the ASH

C = constant which includes surface tension, particle density, bubble size, contact angle, etc.

Thus, the ratio of maximum size particles to be floated at two different inlet pressures should be as follows :

$$d_{1,\max} / d_{2,\max} = V_2 / V_1 \quad (4.2)$$

In this study, the inlet pressure, P (psi), and the slurry flow rate, Q (lpm), were found to be related in the following manner :

$$Q = 23.66P^{1/2} - 13.47 \quad (4.3)$$

The flow rate Q (lpm) is related to the inlet area, A_{inlet} (cm²), and inlet velocity, V_{inlet} (cm/sec), as follows

$$Q = (A_{inlet} V_{inlet}) \frac{60}{1000} \quad (4.4)$$

For these experiments the inlet area was constant.

$$A_{inlet} = 1.716 \text{ cm}^2 \quad (4.5)$$

Therefore, from Equation (4.3)

$$V_{inlet} = 229.8P^{1/2} - 130.8 \quad (4.6)$$

where, the inlet velocity is in cm/sec and inlet pressure is in psi.

It is expected that the ratio of the $d_{\max}$ values should be equal to the inverse ratio of the tangential velocities as determined by the corresponding pressures. That is, approximating the velocities by the inlet velocity in (4.2),

$$\frac{d_{\max, p=p_1}}{d_{\max, p=p_2}} = \frac{V_{\text{inlet}, p=p_2}}{V_{\text{inlet}, p=p_1}} = \frac{(229.8P_2^{1/2} - 130.8)}{(229.8P_1^{1/2} - 130.8)} \quad (4.7)$$

The above equation (Equation 4.7) describes the relationship between the feed inlet pressure and the maximum particle size that can be floated in the resulting centrifugal field. Thus, from the data presented in Figure 4.6, $d_{\max, 10 \text{ psi}} = 105$ microns and $d_{\max, 36 \text{ psi}} = 47.5$ microns, the validity of Equation 4.7 can be evaluated and is found to be substantiated by the experimental results. For example, using the $d_{\max}$ values and a pressure of 10 psi, the other pressure can be calculated to be 39.7 psi which agrees rather well with the experimental value of 36 psi.

The observations from Figure 4.6 that the recovery drops at about 10 psi for the larger (100 x 200 mesh) particle size and at about 36 psi for the smaller (200 x 325 mesh) particle size corroborate the earlier results and analysis about the maximum size of particles that can be recovered by ASH flotation (34). It should be noted that in the above calculations the tangential velocity has been approximated by the maximum tangential velocity (inlet velocity).

4.2.2.6 Collector Level

Flotation experiments were also carried out to establish the influence of collector concentration on flotation response. These tests were done at three different Q^* levels and three different collector concentrations with 5% solids in the feed, $A^* = 1.00$, inlet pressure = 10.5 psi, and with 100 x 200 mesh quartz particles. The recovery values for

different experimental conditions are tabulated in Table 4.6. The pH of the feed suspension was maintained at a constant level of pH 7.4, the natural pH of the system, for all these tests. A graphical representation of these results is given in Figure 4.7 for $Q^* = 6.83$ and $Q^* = 2.28$. It can be seen from this figure that, in general, an increase in the amount of collector increases the ASH flotation recovery. At all collector levels, the recovery rises initially with inlet pressure and beyond a critical inlet pressure it starts to drop. Highest recoveries are obtained in all cases at the highest level of collector concentration (800 g/ton) considered at pH 7.4. The increase in recovery when the collector concentration is increased from 533 g/ton to 800 g/ton is quite substantial in both cases as can be seen from the data presented in Figure 4.7.

Examination of Figure 4.7 also reveals that at a low Q^* value ($Q^* = 2.28$), the air flux is insufficient and hence even with a high collector concentration (800 g/ton) only about 50% recovery can be achieved.

Table 4.6. Recovery values at pH 7.4 for different Q^* and collector concentrations.

Inlet Pressure (psi)	Collector : 800 g/ton			Collector : 533 g/ton			Collector : 267 g/ton		
	Q^*			Q^*			Q^*		
	6.83	4.55	2.28	6.83	4.55	2.28	6.83	4.55	2.28
6.5	80.6	69.9	42.2	26.1	22.2	10.2	14.8	12.9	2.4
8.5	91.1	83.1	47.9	33.3	32.2	14.0	21.8	22.7	4.1
10.5	95.6	92.4	51.9	39.8	37.6	16.2	25.2	25.7	8.2
13.5	68.9	58.7	46.0	33.2	33.2	14.6	19.1	17.9	8.6
16.5	41.1	41.7	38.8	31.5	27.8	12.9	14.3	12.1	7.2

Figure 4.7. Variation in the recovery of hydrophobic quartz particles (100 x 200 mesh) with inlet pressure for various collector concentrations at a pH of 7.4, an A^* of 1.00 for $Q^* = 6.83$ and $Q^* = 2.28$.

When the air flux was increased ($Q^* = 6.83$), the recovery also increased. However, with lower collector concentrations, 267 g/ton and 533 g/ton, about 20% and 40% solids were recovered in the overflow even at a high Q^* ($Q^* = 6.83$). It appears that, at this pH, the quartz surface coverage by dodecyl amine is inadequate with 267 g/ton. With a higher collector concentration, 533 g/ton, the surface becomes slightly more hydrophobic but still the recovery is limited to 40% as shown in Figure 4.7. Further increase in collector concentration (to 800 g/ton) at this pH apparently increases surface coverage by the collector making the particles sufficiently hydrophobic with an excellent recovery of more than 90% at 10.5 psi. Again, at a higher inlet pressure, detachment starts to occur and causes a drop in the recovery.

4.2.2.7 Suspension pH

To study the effect of pH on flotation response, ASH experiments were carried out at a different pH (pH = 10.1). Sodium hydroxide was added to the pulp and the pH was raised to 10.1. After homogenizing the pulp for 10 minutes, flotation tests were done at three different Q^* values and three different collector levels with 5% solids (100 x 200 mesh) in the feed, $A^* = 1.00$, and inlet pressure = 10.5 psi. Table 4.7 summarizes the results of these tests which are also presented graphically in Figure 4.8 for $Q^* = 2.28$ and $Q^* = 6.83$.

It can be seen that, at a high value of Q^* ($Q^* = 6.83$), the air flow rate is adequate to stabilize the froth phase. At this Q^* , when the collector concentration was increased from 267 g/ton to 533 g/ton, a marked increase in recovery was observed as can be seen from Figure 4.8. However, further increase in collector concentration (to 800 g/ton) did not result in any further increase in recovery.

At a low value of Q^* ($Q^* = 2.28$), insufficient air flow limits the recovery to low values because of froth instability. However, when the collector concentration increased from 267 g/ton to 533 g/ton, recovery also increased although not substantially. See

Figure 4.8. Variation in the recovery of hydrophobic quartz particles (100 x 200 mesh) with inlet pressure for various collector concentrations at a pH of 10.1, an A^* of 1.00 for $Q^* = 6.83$ and $Q^* = 2.28$.

Table 4.7. Recovery values at pH 10.1 with different Q^* and collector concentrations.

Inlet Pressure (psi)	Collector : 800 g/ton Q^*			Collector : 533 g/ton Q^*			Collector : 267 g/ton Q^*		
	6.83	4.55	2.28	6.83	4.55	2.28	6.83	4.55	2.28
6.5	75.8	71.5	42.2	79.4	70.0	43.7	25.3	23.3	14.9
8.5	90.1	88.0	48.6	88.4	85.9	47.3	30.0	29.1	19.7
10.5	92.3	90.2	52.0	92.1	93.4	53.9	34.9	38.5	28.3
13.5	74.3	67.7	48.3	71.3	72.4	52.2	32.6	38.6	29.7
16.5	48.8	47.2	42.8	50.2	53.9	47.4	28.4	35.3	28.1

Figure 4.8. Further increase in collector concentration resulted in a small drop in recovery.

Comparison of Figure 4.8 (suspension pH = 10.1) with Figure 4.7 (suspension pH = 7.4) reveals that the flotability of quartz increases at lower collector levels as the pulp becomes more alkaline (70). Under this condition, a smaller amount of collector is sufficient to achieve good flotation. Thus, it can be concluded that a more alkaline pulp reduces the consumption of collector. However, at a high pH (pH = 10), beyond a collector level of about 700 g/ton ($\sim 10^{-4}$ M) a drop in recovery may be expected from surface chemistry considerations (71). At a collector level of 800 g/ton and a pH of 10, a very small drop in recovery has been observed. See Figure 4.8. However, this drop was not very significant. A detailed discussion of the surface chemistry aspects of the amine/quartz system is beyond the scope of this research.

In this research, most of the experiments were carried out at a high collector concentration of 800 g/ton of dry solids and a suspension pH of 7.4. It should be noted that the objective of this research was not to optimize the level of reagent addition but

rather to examine the effect of multiphase flow phenomena on ASH flotation. It is evident from the data presented in Figures 4.7 and 4.8 that the collector consumption can be reduced substantially by appropriate control of suspension pH.

4.3 Summary

Flotation performance of the ASH is strongly dependent on the characteristics of the multiphase fluid flow which, in turn, depend on the levels of the operating variables. The effects are manifested in the shape of the air core and its dimension, froth phase and swirl layer dimensions and their densities, froth stability as depicted in the flow regimes and of course the recovery of solids in the overflow as described in Chapter 3. In this Chapter (Chapter 4), the effects of these variables on flotation performance are discussed. The major findings regarding how the operating variables influence flotation response are summarized in this section.

Recovery of hydrophobic solids to the overflow appears to be determined by froth stabilization, axial flow reversal characteristics, magnitude of the centrifugal acceleration which limit the maximum size particle to be floated, the applied shear (inlet pressure) which determines bubble size and froth transport. Flotation performance was found to improve with increasing A^* . With an increase in A^* , the transport of froth appears to be improved as the axial flow reversal characteristics become more favorable.

The centrifugal acceleration required for detachment, for a given ASH diameter, depends on the mass of the particle and the tangential velocity (inlet pressure) of the slurry. A high tangential velocity appears to enhance froth transport. Too high a tangential velocity, however, may result in the disengagement of the bubble-particle aggregate due to increased centrifugal force and thus under such conditions poor recovery occurs. Also, a higher inlet pressure and higher velocity increases shear rate. The size of the air bubble, being a function of the resulting tangential velocity or shear at the wall, is expected to be reduced with an increase in inlet pressure. Thus the number density of the bubbles would

be higher while having a smaller size at high slurry inlet pressures which results in higher recovery. At inlet pressures beyond a certain limit, the bubble size becomes independent of tangential velocity and the high centrifugal force for detachment causes a decrease in recovery.

High air flow rates ensure sufficient air availability to stabilize the froth phase and thus better flotation recovery. However, too high an air flow rate results in an increase in average bubble size which may be detrimental to flotation. The increase in bubble size is not substantial and therefore a small drop in recovery was observed at very high Q^* values. It appears that both the tangential velocity and the air flow rate are required to be kept at optimum levels in order to preserve good flotation conditions in the ASH.

It was confirmed from this study that the maximum size particle that can be floated in a centrifugal field is indeed inversely proportional to the square root of the centrifugal acceleration. Flotation of smaller particles is relatively insensitive to the operating variables over a wide range. Because of the higher centrifugal acceleration required for detachment in the case of smaller particles, the loaded bubble is much more stable. Therefore, transport of the mineralized bubble to the overflow stream occurs without any significant detachment, even under severe flow conditions, resulting in good recovery.

In general flotation recovery was found to increase with an increase in the collector concentration. Although quartz floats well at low pH (pH = 7.4), the amount of collector needed to achieve good flotation is substantial. It was shown that the consumption of collector can be reduced by increasing the pH of the feed suspension. The results indicate that about 800 g of collector is required per ton of dry solids for good flotation when the pH is low (pH = 7.4). However, only about 500 g of collector per ton of dry solids was enough to achieve good flotation at higher pH (pH = 10.1). Evidently, collector consumption can be reduced by appropriate control of the suspension pH.

CHAPTER 5

FLOW CHARACTERISTICS IN ASH FLOTATION

The performance of the ASH depends strongly on the nature of the multiphase fluid flow. Formation and stability of the vortex, the axial flow reversal and the conditions at the system boundaries are of paramount importance in this regard. Both theoretical and experimental approaches were adopted in this portion of the research in order to characterize some of the phenomena which occur inside the ASH during its steady state operation. Specifically, axial flow reversal is examined based on experimental measurements and bubble trajectories are predicted from first principles. Finally, swirl flow in a plexiglass tube is characterized by photography. The results from these studies are presented and discussed in this chapter.

5.1 Axial Flow Reversal

The suspension enters the ASH through a tangential inlet at the top and exits through the annular underflow opening at the bottom in swirl flow. In the absence of the froth pedestal at the bottom, all of the swirl flow would exit through the bottom of the ASH. However, the tapered froth pedestal prevents all of the fluid from exiting through the underflow opening at the bottom and causes the fluid to have an upward motion. Therefore, some of the flow (ideally froth and hydrophobic particles) exits through the overflow opening (the vortex finder pipe) at the top. Thus, as in a conventional hydrocyclone, it is expected that at any axial location there will be a point at some radial position where the axial velocity is zero (72,73). In the region

between this point and the air core interface, fluid will be flowing upward towards the vortex finder whereas in the region between this point and the porous tube surface, fluid will be flowing downward towards the underflow opening. Therefore, there is a surface of zero axial velocity (ZAV) inside the ASH. A change in the operating conditions is expected to change the spatial coordinates (location) of this surface at any axial position of the ASH and thus cause a change in the transport characteristics of the ASH leading to a change in the recovery of the overflow product. Therefore, it was intended to determine the location of this surface of ZAV for various operating conditions in order to characterize the performance of the ASH with regard to its transport properties. Experiments were carried out to locate this surface using a Tracer Analysis Technique. Details of this study are described in the following sections.

5.1.1 Materials

A concentrated solution of lithium chloride (LiCl) in water was used as the tracer in this study. MIBC was added to the tracer solution achieve have the same surface tension as that of the feed pulp. Table 5.1 gives a summary of the materials used in this study.

5.1.2 Equipment

The same set up that was used for flotation experiments was used in this study. The same ASH unit, pump, mixers, pressure gauge, valves and rotameters were used. In order to inject the tracer into the ASH during steady state operation, 5 cc syringes and 0.8 mm diameter needles were used. After filtration of the product samples the aqueous phase were taken to Directly Coupled Plasma (DCP) analyzer to determine the concentration of tracer solution in the product streams. Table 5.2 gives a summary of the equipment used in this study.

Table 5.1. Materials used for the axial flow reversal study

Material	Description	Supplier	Purity
Quartz	SIL-CO-SIL 250 100 x 200 mesh	US Silica Co.	99.8 %
MIBC	Methyl Isobutyl Carbinol as frother	Aldrich Chemicals	99.0 %
Dodecyl Amine	Collector	Surfactant Division Rhone Poulenc Inc.	98.8 %
LiCl	Granular, to prepare tracer solution	Fisher Scientific Co.	98.9 %

5.1.3 Procedure

Quartz particles were wet-screened and the 100 x 200 mesh fraction was taken. In these studies the ASH system was run as an open system. The product streams were open such that they could be sampled after tracer injection. The feed pump was shut off after collection of each sample.

The quartz suspension in water was prepared in the feed sump at the desired solids concentration and the necessary levels of frother and collector were added. The same reagents, MIBC as frother and dodecyl amine as collector, were used in these experiments. The frother concentration was maintained at 40 ppm (water basis) for all experiments. The collector concentration was kept at 800 g per ton of dry solids, the particle size was 100 x 200 mesh and the pH of the suspension was 7.4. Two LIGHTNIN mixers kept the solids in a suspended state and homogenized the slurry such that the system was well mixed. The slurry flow rate was controlled by regulating the ASH inlet pressure with a two-way Apollo ball valve. The inlet pressure was monitored by a pressure gauge which was calibrated by measuring the volume flow rate at different pressure levels. A compressor was used to supply air to the

Table 5.2. Equipment used for the axial flow reversal study

Equipment	Model / Description	Supplier
Air-Sparged Hydrocyclone	Model : ASH-2C	Advanced Processing Technologies Inc.
1.5 VACSEAL Feed Pump	Model: D-1.5-VR-A101	The Galigher Co.
PACEMAKER Motor	3 HP, Model: 19284R-0	The Louis Allis Co.
LIGHTNIN Mixers	Model : ND-2, 1725 RPM	Mixing Equipment Co. Inc.
Compressor	Model : 44643	DeVilbiss (Can.) Ltd., Canada
Rotameter	Model : 10A1755A	Fisher & Porter Co.
Pressure gauge	Ashcroft	Cole-Parmer Instrument Co.
Valves	Apollo, two-way ball valves	Conbraco Industries Inc.
Syringes	5 cc	Becton Dickinson & Co.
Needles	0.8 mm dia, 1.5" long	Becton Dickinson & Co.
DCP analyzer	Model : SpectraSpan V No. 53040	Spectrametrics Inc.

ASH. The air was sparged through the inner porous tube and the air flow rate was monitored and controlled with a rotameter.

Seven small holes were drilled in the porous tube section of the ASH at different axial positions. These holes were located at 30, 75, 110, 155, 200, 240 and 280 mm from the bottom end of the porous tube. A line diagram of the porous tube section showing the location of the sampling ports is given in Figure 5.1. These sampling ports were then sealed from inside and outside with silicone glue. Needles were then inserted through these holes. The silicone provided for easy movement of the needles while preventing any leakage at the same time. Thus, through any hole, it was possible to inject the tracer solution at any radial position inside the ASH by moving the needle a prescribed distance in the radial direction.

A tracer solution of high concentration was prepared by dissolving lithium chloride in water. To prepare the tracer solution, 488.65 grams of LiCl was dissolved in 1 liter of water such that the concentration of lithium in the tracer solution was 80 g/l.

From x-ray CT data the air core diameters for various experimental conditions were known a priori as well as the combined spatial extent of the froth phase and swirl layer. Depending upon these dimensions, tracer solution was injected at different radial positions in the region between the air core interface and the porous tube surface, through the port-holes. The tracer solution (5 cc) was injected for each radial position at a given axial position and the product streams were sampled simultaneously. It was anticipated that at any axial location, if the tracer solution was injected at the porous tube side of the surface of ZAV, most of the lithium chloride solution would report to the underflow stream. On the other hand, if the tracer solution was injected at the air core side of the surface of ZAV, the overflow stream would have a high concentration of lithium chloride solution.

Figure 5.1. A line diagram of the porous tube section of the ASH showing the axial positions of the sampling ports used for tracer injection. The distances were measured from the line AA.

The slurry was prepared in the feed sump with desired levels of solids, frother and collector. It was conditioned for 10 minutes, the operating variables were set at desired values and then the experiments were initiated. Because of the small residence time of the slurry in the ASH, tracer injection and sample collection were done at the same time. Through each sampling port, tracer solution was injected at various radial positions, each position being 2 mm apart. While tracer was injected through one needle, all other needles were blocked with rubber padding so that there was no leakage. The sampled product streams were vacuum filtered to separate out the solids. The aqueous phase of the product streams were then taken for DCP analysis to determine the concentration of lithium in each of these solutions. From these data then the concentration of lithium in the two product streams were known for specific radial and axial positions.

5.1.4 Data Analysis

In order to determine the location of the surface of ZAV, the following steps were followed. As a first step, the lithium concentrations in the overflow stream were plotted against radial position (distance from the porous tube wall where the tracer was injected) for all axial locations. Such a plot is shown in Figure 5.2. It can be seen from this figure that there is indeed a substantial rise in tracer concentration in the overflow as one goes from the porous tube wall to the air core surface. This observation is true for almost all axial positions except for the bottom most port, 30 mm from the bottom. Thus, the existence of the surface of ZAV is evident. Examination of Figure 5.2 reveals that the slope of the concentration curve has a maximum point.

As the final step, to locate the surface of ZAV more precisely, the gradient of tracer concentration vs. radial position was calculated at every point. These gradients were then plotted against radial position for all axial positions. Such a plot is shown in

Figure 5.2. Lithium concentration in the overflow stream as a function of the position of tracer injection through each sampling port for 5% solids, $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure. The corresponding lithium concentrations in the underflow are given in Appendix C.

Figure 5.3. It is evident from this figure that the gradient of the concentration goes through a maximum for all axial positions. At any axial position, this point of maximum slope was taken to be the point of ZAV. In such a way, then, the points of ZAV were found at seven axial positions and the surface of ZAV was located for different experimental conditions. It should be noted that the tracer solution was injected at radial positions 2 mm apart. Therefore, the position of the surface for ZAV could have a maximum error of ± 1 mm. Lithium concentrations in the product streams for various axial and radial position of tracer injection are tabulated in Appendix C.

5.1.5 Results and Discussion

Using the technique described above, the influence of the following operating variables on the position of the surface of ZAV was studied :

- A^* - the ratio of overflow opening area to underflow opening area
- Q^* - the ratio of air flow rate to slurry flow rate
- Φ - percent solids in the feed suspension
- P - slurry inlet pressure.

These results are presented and discussed in the following sections.

5.1.5.1 Effect of Dimensionless Area (A^*)

Figure 5.4 compares the surface of ZAV for two different A^* values with $Q^* = 4.55$, 5% solids in feed, and inlet pressure = 10.5 psi. The dotted lines on the stacked radial density profiles are the surfaces of ZAV for the two different conditions as determined by the above-mentioned technique. It can be seen from this figure that as A^* is increased from 0.74 to 1.00, the surface of ZAV shifted to the porous tube wall side. Although not substantially large, this shift corresponds to a larger amount of froth being carried to the overflow for higher A^* value ($A^* = 1.00$) and thus accounting for the higher recovery found in the flotation experiments.

Figure 5.3. Gradient of lithium concentration profile in the overflow stream as a function of the position of tracer injection through each sampling port for 5% solids, $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

Figure 5.4. The surface of zero axial velocity (ZAV) at two different A^* values with $Q^* = 4.55$ and 10.5 psi inlet pressure for 5% solids in the feed.

Interestingly, the surface of ZAV intersects the radial density profiles at different points in the two cases. It can be seen clearly from this figure that, because of the location of the surface of ZAV, almost all the froth is being carried to the overflow for $A^* = 1.00$. In this case the surface of ZAV does not intersect the froth phase at any axial location. Thus, there is no loss of froth to the underflow. This agrees very well with the observation (Chapter 4) that the recovery in this case is more than 90%.

On the other hand, since the surface of ZAV intersects the froth phase at almost all axial locations for $A^* = 0.74$, a lot of froth is expected to be carried to the underflow. This has indeed been observed as the recovery to the overflow is less than 70% in this case.

Figure 5.5 shows the axial view of the flow regimes along with the surface of ZAV for these two cases. It can clearly be seen from this figure that the surface of ZAV does not intersect the froth phase for $A^* = 1.00$ resulting in a high recovery. On the other hand, for $A^* = 0.74$ the surface of ZAV intersects the froth phase and thus carries some froth to the underflow. This results in a loss of hydrophobic particles to the underflow and hence a lower recovery.

Figure 5.6 compares the surfaces of ZAV for two different A^* values with 15% solids in the feed, $Q^* = 4.55$ and 10.5 psi inlet pressure. In this case too, a shift in the surface of ZAV towards the porous wall was observed as A^* was increased from 0.74 to 1.00 which accounts for the higher recovery with $A^* = 1.00$ than with $A^* = 0.74$.

Again, as was the case for 5% solids, it can be seen from Figure 5.6 that the surface of ZAV intersects the froth phase at almost all axial positions for $A^* = 0.74$ and therefore a large amount of froth is lost to the underflow. Thus, it accounts for the very low recovery (about 20%) as has been reported in Chapter 4.

Figure 5.5. Axial view of the flow regimes with the surface of zero axial velocity (ZAV) at two different A^* values with $Q^* = 4.55$ and 10.5 psi inlet pressure for 5% solids in the feed.

Figure 5.6. The surface of zero axial velocity (ZAV) superimposed on the radial density profiles at two different A^* values with $Q^* = 4.55$ and 10.5 psi inlet pressure for 15% solids in the feed.

On the other hand, at $A^* = 1.00$, the surface of ZAV does not intersect the froth phase at any axial position. In this case all the froth is carried to the overflow. Indeed a high recovery (more than 90%) was achieved in this case.

These features can be seen more clearly from Figure 5.7 which shows the axial view of the flow regimes and the location of the surface of ZAV for the above two cases. The surface of ZAV does not intersect the froth phase for $A^* = 1.00$, carrying almost all the froth to the overflow. On the other hand, the froth phase is intersected by the surface of ZAV at all axial locations for $A^* = 0.74$ which results in a substantial loss of froth to the underflow and hence a low recovery.

5.1.5.2 Effect of Dimensionless Flow Rate (Q^*)

The influence of Q^* on the surface of ZAV is shown in Figure 5.8. The surface of ZAV is plotted for $Q^* = 2.28$ and $Q^* = 6.83$ with 15% solids in feed, $A^* = 1.00$ and 10.5 psi inlet pressure. It can be seen from this figure that Q^* has no noticeable impact on the surface of ZAV. The surface was found to have the same location in both these cases. Thus, the axial flow reversal characteristics that determine froth transport appear to be similar in these two cases. Therefore, froth transport must not be the factor that limits the recovery of solids to the overflow in these cases. However, a lower recovery was observed with a lower Q^* ($Q^* = 2.28$) than with a higher Q^* ($Q^* = 6.83$). As discussed in the previous chapter, insufficient air flow for lower Q^* prevents the formation of a stable froth and causes the recovery to drop, the axial flow reversal characteristics being comparable in the two cases. In other words, had there been enough froth with $Q^* = 2.28$, it would have been carried to the overflow, resulting in good recovery.

The axial view of the flow regimes along with the surface of ZAV for the above two cases is shown in Figure 5.9. It can be seen from this figure that in both the cases the surface of ZAV does not intersect the froth phase. At a low Q^* ($Q^* = 2.28$)

Figure 5.7. Axial view of the flow regimes with the surface of zero axial velocity (ZAV) at two different A^* values with $Q^* = 4.55$ and 10.5 psi inlet pressure for 15% solids in the feed.

Figure 5.8. Effect of Q^* on the location of the surface of zero axial velocity (ZAV) for 15% solids in the feed, $A^* = 1.00$ and 10.5 psi inlet pressure.

Figure 5.9. Axial view of the flow regimes with the surface of zero axial velocity (ZAV) at two different Q^* values with $A^* = 1.00$ and 10.5 psi inlet pressure for 15% solids in the feed.

the froth phase is not stabilized due to insufficient air flow. The hydrophobic particles remained centrifuged at the wall and were eventually carried to the underflow resulting in a poor recovery. On the other hand, the froth phase is stabilized at a high Q^* ($Q^* = 6.83$), and is readily carried to the overflow giving good recovery.

5.1.5.3 Effect of Percent Solids

Figure 5.10 shows the surfaces of ZAV for two different solids concentration with $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure. Examination of this figure reveals that the surface of ZAV does not change position with an increase in solids concentration at this value of A^* . In this regard, it should be noted that the recovery was not affected by a change in solids concentration when A^* was kept at 1.00 and was found to be high under these conditions (Chapter 4). This implies that froth transport in both these cases was favorable.

The axial views of the flow regimes and the surface of ZAV for these two cases are shown in Figure 5.11. It can be seen from this figure that in both the cases all the froth is being carried to the overflow since the surface of ZAV does not intersect the froth phase in either case. In fact, more than 90% recovery has been observed in both cases (Chapter 4).

5.1.5.4 Effect of Inlet Pressure

The inlet pressure at which the slurry is being fed to the ASH is expected to change the flow characteristics of the ASH as the entire velocity distribution is dependent on the inlet velocity and hence inlet pressure. The influence of inlet pressure on axial flow reversal is shown in Figure 5.12. In this figure, the surface of ZAV is shown for two different inlet pressures. The experimental conditions are given in the figure caption. Examination of this figure indicates that the surface of ZAV shifts a little towards the porous tube wall as the inlet pressure is increased from 6.5

Figure 5.10. Effect of percent solids on the location of the surface of zero axial velocity (ZAV) for $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

Figure 5.11. Axial view of the flow regimes with the surface of zero axial velocity (ZAV) at two different solids concentrations with $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

Figure 5.12. Effect of inlet pressure on the location of the surface of zero axial velocity (ZAV) for 15% solids in the feed, $A^* = 1.00$ and $Q^* = 4.55$.

psi to 10.5 psi. Consequently, the amount of froth being carried to the overflow is more in the case of 10.5 psi. This explains the observation that recovery is greater at 10.5 psi than at 6.5 psi.

Figure 5.13 compares the axial view of the flow regimes along with the surface of ZAV for these two cases. It can be seen from this figure that because of the location of the surface of ZAV, some froth is being carried to the underflow at a low inlet pressure (6.5 psi) giving lower recovery while all the froth is carried to the overflow at a high inlet pressure (10.5 psi) resulting in a high recovery. Therefore, it appears that froth transport to the overflow is one of the limiting factors in these two cases with regard to flotation response.

5.2 Theoretical Analysis of Bubble Trajectories

Modeling efforts were made to characterize the two-phase flow in the ASH. In this regard, the Navier-Stokes equation was appropriately modified for the movement of air bubbles through the swirl layer taking into consideration the interaction between bubbles (similar to hindered settling). The slip velocities of the bubbles were calculated in both radial and axial directions. To calculate the absolute velocity of the bubbles, the fluid velocity profile was required. However, reliable data on fluid velocity are not available at this point. Velocity data were taken from a few measurements by Kinneberg (74) and unpublished work by Yin et al. (75). It should be noted here that the data from Kinneberg's measurements are available only for a narrow region near the tube wall in the absence of air and the velocity profiles calculated by Yin et al. (75) have not yet been substantiated by experimental measurements. However, in the absence of any other source, these velocity data were used in this calculation to determine the absolute velocity of the bubbles. Finally, the trajectory of an air bubble, when introduced at a certain axial position, was calculated using these absolute velocities.

Figure 5.13. Axial view of the flow regimes with the surface of zero axial velocity (ZAV) at two different inlet pressures with $A^* = 1.00$, $Q^* = 4.55$ and 15% solids in the feed.

Slip velocity calculations were also done for the case when swarms of bubbles were introduced at the porous tube wall at regular time intervals, assuming axial symmetry in the ASH. From the results of these calculations the steady state bubble concentration profile was calculated. From this profile, then, the radial density profiles at various axial positions were determined. The calculated radial density profile and the radial density profile observed experimentally using x-ray CT measurements were compared qualitatively.

The calculations for the single bubble case and for the case in which there are swarms of bubbles are similar and based on a general formulation. However, the specifics were different. Details of this procedure and the calculated results are presented in the following sections.

5.2.1 General Formulation of the Problem

The two phases, air and water, are considered separately. In most text books on fluid mechanics, the continuity and momentum equations for the two phases are dealt with in great detail (76-81). The Stoke's equation for a swarm of particles settling in a fluid can be arrived at using mass and momentum balance. In the ASH, the creeping flow conditions, under which Stoke's equation is valid, are not satisfied. However, in this analysis we are interested in the relative velocity between the two phases, which is expected to be fairly small. Thus, Stoke's equation may still be assumed to be useful if a coordinate system moving with the fluid is chosen. In fact, such assumptions have been made by researchers to describe fluidized bed reactors in which high absolute velocities are encountered but the relative velocities between the two phases are small (82,83).

In this problem the equations governing a swarm of bubbles rising through water while settling against a centrifugal force were modified as necessary. In order to account for the interaction between the bubbles, the hindered settling approach was

adopted. In this regard, the development of the governing equations as given by Wallis (79) is followed and a brief description is presented here.

5.2.1.1 Continuity Equation

The equations of conservation of mass, or continuity equations, can be written quite generally in differential form for one-dimensional separated flow in which the phases are considered separately. In the absence of external source terms, they can be written as :

$$\frac{\partial}{\partial t}[\rho_1(1-\alpha)] + \nabla \cdot [\rho_1(1-\alpha)V_1] = S_{12} \quad (5.1.a)$$

$$\frac{\partial}{\partial t}[\rho_2\alpha] + \nabla \cdot [\rho_2\alpha V_2] = -S_{12} \quad (5.1.b)$$

S_{12} is a source term that represents the mass rate of phase change per unit volume. The subscripts 1 and 2 denote phase 1 and phase 2 respectively. ρ, α and V stand for density, volume fraction of phase 2 and velocity vector respectively.

In 1-D form, after integration across the area (A) of the tube Equations (5.1) become (79),

$$\frac{\partial}{\partial t}[\rho_1(1-\alpha)A] + \frac{\partial}{\partial z}[\rho_1(1-\alpha)v_1A] = \int S_{12}dA \quad (5.2.a)$$

$$\frac{\partial}{\partial t}[\rho_2\alpha A] + \frac{\partial}{\partial z}[\rho_2\alpha v_2A] = \int -S_{12}dA \quad (5.2.b)$$

where v is the component of the velocity vector in the direction under consideration.

5.2.1.2 Momentum Equations

The momentum equations, equations of motion, or Newton's law for the two phases can be written generally in the 3-D vector form :

$$\rho_1 \left(\frac{\partial V_1}{\partial t} + V_1 \cdot \nabla V_1 \right) = b_1 + f_1 - \nabla p \quad (5.3.a)$$

$$\rho_2 \left(\frac{\partial V_2}{\partial t} + V_2 \cdot \nabla V_2 \right) = b_2 + f_2 - \nabla p \quad (5.3.b)$$

where, b_1 and b_2 are the body forces, per unit volume of that component, which act on each component. ∇p is some average pressure gradient. Evaluation of f values depends on the particular flow regime and conditions of the problem. The f values contain components due to hydrodynamic drag, apparent mass effects during relative acceleration, particle-particle forces, forces due to momentum changes, and so on.

F_1 and F_2 are defined as the equivalent of the f values per unit volume of the whole flow field. Thus

$$F_1 = f_1(1 - \alpha) \quad (5.4.a)$$

$$F_2 = f_2\alpha \quad (5.4.b)$$

If these forces are due to mutual hydrodynamic drag, action and reaction are equal and we have

$$F_1 = -F_2 = F_{12} \quad (5.5)$$

5.2.1.3 Application to this Research

In the case of 1-D flow, Equations (5.2) are resolved in the direction of motion to give (79)

$$\rho_1 \left(\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial z} \right) = b_1 + f_1 - \frac{\partial p}{\partial z} \quad (5.6)$$

$$\rho_2 \left(\frac{\partial v_2}{\partial t} + v_2 \frac{\partial v_2}{\partial z} \right) = b_2 + f_2 - \frac{\partial p}{\partial z} \quad (5.7)$$

At this point, a force balance on the particles/fluid system is useful. Consider an experiment in which an array of particles is maintained stationary in a horizontal fluid stream and the force on a typical particle is measured in terms of the fluid velocity, properties and concentration. See Figure 5.14. The total force necessary to restrain a particle with volume v_p will be (79)

$$F' = v_p \left(f_s - \frac{dp}{dz} \right) \quad (5.8)$$

F' can be correlated in terms of the fluid flux and properties by defining a drag coefficient, for a given α , as follows (79)

$$(C_D)_\alpha = \frac{F'}{\frac{1}{2} \rho_f j_{f0}^2 A_p} \quad (5.9)$$

Figure 5.14. Forces on particles in an array. The particles are held against the fluid drag by a force F' per particle.

where A_p is a characteristic cross section for the particle. The drag coefficient contains both the effect of the net pressure gradient and the effect of forces contained in f_s . The fluid flux relative to particles is denoted by j_p .

The entire volume between lines aa and bb in Figure 5.14 must be in equilibrium. The end effects can be made vanishingly small by choosing a sufficiently large volume. Therefore, since the external force on the particles balances the pressure gradient,

$$F'(1-\alpha) = -\left(\frac{dp}{dz}\right)v_p \quad (5.10)$$

The drag force on the particles must be equal and opposite to the drag force on the fluid. Therefore

$$F_{fs} = \alpha f_f = -(1-\alpha)f_s \quad (5.11)$$

A further useful parameter is

$$f_{fs} = f_f - f_s \quad (5.12)$$

Combining Equations (5.8) to (5.12) all of the various forces can be expressed in terms of each other as follows:

$$\begin{aligned}
 \frac{F'}{v_p} &= (C_D)_\alpha \frac{1}{2} \rho_f j_{f0}^2 \frac{A_p}{v_p} = -\frac{1}{(1-\alpha)} \frac{dp}{dz} = -\frac{f_f}{(1-\alpha)} \\
 &= \frac{f_s}{\alpha} = -f_{fs} = -\frac{F_{fs}}{\alpha(1-\alpha)}
 \end{aligned}
 \tag{5.13}$$

If the drag coefficient can be determined as a function of void fraction and fluid properties for the specific system, all of the forces can be evaluated.

For a stationary fluidized bed in which the particles are supported against gravity by the upward flow of fluid around them, Equations (5.6) and (5.7) reduce to (82,83)

$$\frac{dp}{dz} = -g\rho_f + f_f = -g\rho_s + f_s
 \tag{5.14}$$

where g is the acceleration due to gravity.

In this analysis, the fluid is moving. However, in the radial direction the fluid is assumed to have no velocity. In the axial direction the bubble is assumed to move with the fluid. Therefore, in a coordinate system moving with the fluid, the fluid velocity in the axial direction may be considered to be zero. In other words, from the standpoint of slip (relative) velocity, a stationary fluid assumption may be justified. Thus, in the case of a bubble rising up through the liquid, the momentum equation of the i th bubble species is given by (84)

$$0 = -\frac{dp}{dz} + \frac{\rho_i g}{g_c} + f_i
 \tag{5.15}$$

and the momentum equation for the liquid (water) is given by (84)

$$0 = -\frac{dp}{dz} + \frac{\rho_l g}{g_c} + f_l \quad (5.16)$$

where f_i is the drag force acting on the i th bubble species by the fluid and f_l is the drag force acting on the liquid by the bubbles and g_c is a conversion factor.

The number of bubble species in the suspension is N . Therefore, equilibrium considerations give (84),

$$\alpha_l f_l + \sum_{i=1}^N \alpha_i f_i = 0 \quad (5.17)$$

where α_l and α_i are volumetric fractions of the liquid and the i th bubble species respectively. The hydrodynamic interaction, drag force, is given by (79)

$$f_i = -\frac{18\mu_l(v_i - v_l)}{d_i^2 g_c F(\alpha_l)} \quad (5.18)$$

This can be assumed to be valid for a mono-dispersed and a multispecies system as well. Combining Equations (5.15)-(5.18) yields (84)

$$\frac{g(\rho_i - \rho_l)}{18\mu_l} \alpha_l F(\alpha_l) = \sum_{k=1}^N \frac{(v_k - v_l)}{d_k^2} \alpha_k + \frac{(v_i - v_l)\alpha_l}{d_i^2} \quad (5.19)$$

Equation (5.19) represents a set of N equations with N unknowns in the slip velocity, $(v_i - v_l)$. The solution for the i th species is (84)

$$(v_i - v_l) = \frac{g d_i^2 F(\alpha_l)}{18\mu_l} [(\rho_i - \rho_l)(1 - \alpha_i) - \sum_{\substack{k=1 \\ k \neq i}}^N (\rho_k - \rho_l) \alpha_k] \quad (5.20)$$

for $i=1,2,\dots,N$.

Defining the slurry density as

$$\rho_{sl} = \rho_l \alpha_l + \sum_{k=1}^N \rho_k \alpha_k \quad (5.21)$$

the quantity enclosed by the square bracket in equation (5.20) reduces to $(\rho_i - \rho_{sl})$.

The slip velocity for the i th bubble species is then given by (84)

$$v_i - v_l = \frac{g d_i^2}{18\mu_{sl}} F(\alpha_l) (\rho_i - \rho_{sl}) \quad (5.22)$$

The more general equation for non-Stokes flow (in the case of a bubble swarm) is given by (84)

$$v_i - v_l = \frac{g d_i^2}{18\mu_{sl}(1 + 0.15 \text{Re}_{bs}^{0.687})} F(\alpha_l) (\rho_i - \rho_{sl}) \quad (5.23)$$

Equation (5.23) is valid up to a Reynolds number of about 1000.

Defining slip velocity of bubble for a mono-dispersed system as U_{sb} (85),

$$U_{Sb} = \frac{g d_b^2}{18\mu_{sl}(1 + 0.15 \text{Re}_{bs}^{0.687})} F(1 - \alpha_b)(\rho_b - \rho_{sl}) \quad (5.24)$$

where,

$$\text{Reynolds number for bubble swarm, } \text{Re}_{bs} = \frac{d_b U_{Sb} \rho_{sl} (1 - \alpha_b)}{\mu_{sl}} \quad (5.25)$$

and

$$F(1 - \alpha_b) = (1 - \alpha_b)^{m-1} \quad (5.26)$$

and (86)

$$m = [4.45 + 18(d_b / d_c)] \text{Re}_b^{-0.1} \quad 1 < \text{Re}_b < 200 \quad (5.27)$$

$$m = 4.45 \text{Re}_b^{-0.1} \quad 200 < \text{Re}_b < 500 \quad (5.28)$$

and Reynolds number for a single bubble

$$\text{Re}_b = \frac{d_b U_b \rho_{sl}}{\mu_{sl}} \quad (5.29)$$

where U_b is the terminal rise velocity of a single bubble.

In the case of a two-phase air-water system Equation (5.24) reduces to (84)

$$U_{Sb} = -\frac{g d_b^2 \rho_l}{18\mu_l(1 + 0.15 \text{Re}_{bs}^{0.687})} (1 - \alpha_b)^{m-1} \quad (5.30)$$

If d_b is known, U_{sb} can be calculated for a given α_b . This requires an iterative solution because $Re_{bs} = f(U_{sb})$ and $Re_b = f(U_b)$. In this calculation a Newton-Raphson procedure was adopted to solve by iteration.

5.2.2 Computational Procedure (29, 87-89)

In this calculation, the bubble was tracked from the point where it was introduced until it reached one of the three boundaries, namely, the air core/water interface, the overflow opening, or the underflow opening, through which it can 'exit' from the calculation domain. The computations for a single bubble and a swarm of bubbles are similar, the difference being the void fraction. However, the bubble movement is a strong function of the void fraction, as can be seen from Equation 5.30. Therefore, the trajectory that an individual bubble follow in a swarm of bubbles is different from the trajectory it follows in the absence of other bubbles.

From Equation (5.30), the axial slip velocity of the bubble was estimated. To estimate the radial slip velocity of the bubble, g in Equation (5.30) was substituted by v_t^2/r , the centrifugal acceleration, and the resulting equation was solved iteratively using the Newton-Raphson method (29). In this calculation, the Newton-Raphson subroutine typically took about 15-25 iterations to converge.

To estimate the absolute velocity of the bubble, the axial and radial fluid velocity data were necessary. In this regard, the unpublished fluid velocity data, theoretically generated by Yin et al. (75) were used. Figure 5.15 shows these axial, radial and tangential velocity profiles in the ASH which were used in these calculations. These velocity profiles were calculated by Yin Di solving the Navier-Stokes equation in its stream function-vorticity form for single phase and using Prandtl's mixing length model to estimate the turbulent viscosity. These data were validated for the narrow region close to the tube wall with the experimental data reported by Kinneberg (74). However, it should be noted that the reliability of the

Figure 5.15. The velocity profiles that were used in the bubble trajectory and radial density profile calculations.

theoretically generated data in the other regions of interest was not proven. In view of this, the calculations described in this section may be considered as qualitative rather than quantitative.

The theoretical fluid velocity data were added to the corresponding slip velocities to evaluate the absolute velocity of the bubble in the radial and axial directions. These velocities were calculated at every grid point of the calculation domain for each bubble and the radial and axial distance traversed by the bubble in the current time step was determined. Thus the trajectory of the air bubble was tracked.

Depending upon the slip velocities, the bubble reported at the air core/water interface or at the overflow stream or at the underflow stream, when it was considered to be out of the computational domain. At any time step the number of air bubbles associated with each volume element of the domain was stored in memory by the computer code. From these data, at steady state, the void fraction at every grid point was determined, which led to the generation of radial density profiles at various axial positions. The radial density profiles generated theoretically were compared with the density profiles observed experimentally using x-ray CT for the two-phase system in a qualitative sense.

For this calculation, the air core was assumed to be cylindrical, the radius of which was estimated from x-ray CT data to be 8 mm under the conditions for which these two-phase flow calculations were done. The swirl layer in the porous tube section was the domain (16.13 mm x 304.8 mm) for these calculations. The calculation domain along with the grid structure is shown in Figure 5.16. The domain was divided into 300 axial grid spaces (301 grid points) and 30 radial grid spaces (31 grid points). The grid structure was uniform in both radial and axial directions. The grid spacing in the radial direction was denoted as dr and the grid spacing in the axial direction was denoted as dz . However, the grid structure used in the calculation of fluid velocity profiles was coarser than the grid structure used in these calculations.

Figure 5.16. The calculation domain (swirl layer) and the grid structure used for bubble trajectory and radial density profile calculations.

Therefore, interpolation was done using Lagrange Polynomial (90) to obtain velocity values at all the grid points. In this analysis, axial symmetry was assumed and thus these calculations were limited to a 2-D vertical plane.

As shown in Figure 5.16, the bubbles were introduced in swarms through the porous tube wall (BC), one bubble at every grid point at every time step. The time interval at which bubbles appeared at these grid points was calculated from the air flow rate. Depending on their slip velocities and the fluid velocities, the bubbles reported at different boundaries, e.g., the air core surface, the underflow opening or at the overflow opening, were then considered to be out of the calculation domain. The calculations were terminated when the number of bubbles coming into the domain was equal to the number of bubbles going out of the domain for quite a few consecutive time steps. In this regard, the number of bubbles going out of the domain was plotted with time and shown in Figure 5.17. It should be noted that the number of bubbles coming into the domain at any time step was equal to the number of vertical grid points (301). When the number of bubbles going out of the domain was about this value for a quite a few time steps, as can be seen from Figure 5.17, steady state was assumed to have been reached and the computations were terminated. It can be seen that steady state was probably reached at around 175th time step. However, the computations were continued until 300th time step to make sure that a steady profile for bubble concentration was obtained.

Table 5.3 provides a summary of the operating conditions for which these calculations were made. Coalescence and breakage of the bubbles were not considered and the bubbles were assumed to have a single size (300 microns). The trajectories of the bubbles were tracked and the steady state profile of the bubble concentration was generated. From the steady state bubble concentration profile, then, the radial density profiles at various axial locations were calculated. These profiles were compared on a qualitative basis with the experimental density profiles as measured by x-ray CT.

Figure 5.17. Plot of the number of bubbles going out of the calculation domain with time in order to establish steady state.

Table 5.3. Operating conditions for which the density profile calculations were done.

ASH diameter	48.26 mm	Inlet pressure	10.5 psi
Pedestal diameter at the underflow exit	43.64 mm	Slurry flow rate	60.0 lpm
Porous tube length	304.8 mm	Slurry viscosity	0.001 Pa.s
Air core diameter	16.0 mm	Slurry density	1.00 g/cc
Vortex finder ID	20.62 mm	Air flow rate	270 lpm

In order to track the trajectory of an individual bubble, only one bubble was introduced at a certain axial position at zero time. No other bubble entered the domain in the subsequent time steps. This bubble was tracked until it reached one of the boundaries and thus its trajectory was calculated.

5.2.3 Computer Code

The fluid velocity profiles, axial, radial and tangential, were given as an input to this program. Then the time step was calculated from the number of bubbles appearing at the grid points at each time step and the air flow rate. The Reynolds number for a single bubble was then estimated, which required an iterative Newton-Raphson technique. Then the program calculated the following, in the indicated order, for each bubble at every grid points, sweeping the domain first horizontally and then vertically :

- 1) Radial position of the current grid point
- 2) The void fraction of air from the number of bubbles associated with the current grid point. In this regard, the volume element having a rectangular cross section ($dr \times dz$) and a length of $2\pi r$, r being the current radial position, was considered.

From this volume and the total number of bubbles in this volume element, the void fraction was calculated, individual bubble volume being known

- 3) The value of m from Equation (5.27) or (5.28) depending upon the Reynolds number
- 4) The axial slip velocity of the bubble from Equation (5.30). To calculate the radial slip velocity, the acceleration due to gravity, g , in Equation (5.30) was substituted by the centrifugal acceleration, v_t^2 / r
- 5) The absolute velocities of the bubble in the radial and axial directions, from the slip velocities and corresponding fluid velocities
- 6) How much radial and axial distance the bubble has traversed in that time step
- 7) The grid point the bubble should be associated with in the next time step. If it reached any of the three boundaries, it was considered to be out of the calculation domain and was not considered in the next time step.

The program also kept track of the number of bubbles going out of the calculation domain at each time step which automatically became zero at the beginning of the next time step. When for quite a few consecutive time steps the number of bubbles going out of the domain was equal to the number of bubbles coming into the domain, it was assumed that steady state has been reached and the calculations were terminated. Then, from the number of bubbles associated with every grid points at steady state, the density at every grid point was calculated. Thus, theoretical radial density profiles were generated at each axial position.

5.2.4 Results and Discussion

5.2.4.1 Single Bubble Case

The trajectory of a single bubble, when introduced at a particular axial position, is presented in Figure 5.18. It can be seen from this figure that after being released from the porous tube wall the bubble travels towards the air core and

Figure 5.18. The trajectory of a single bubble when introduced at different axial positions of the porous tube during two phase air/water flow with $A^* = 1.00$, $Q^* = 4.55$, and 10.5 psi inlet pressure.

downward simultaneously for a while. The inward radial movement of the bubble can be thought of as the bubble 'settling' against the centrifugal force ('radial buoyancy'). Since the radial velocity of the fluid at all positions is negligible, this 'radial buoyancy' dominates, and the bubble moves towards the air core. In the axial direction too, the bubble is 'settling' against gravity (usual buoyancy). However, in the axial direction, the downward fluid velocity is too great near the wall and it dominates over buoyancy. Therefore, the bubble is dragged with the fluid downward, the slip velocity being very small. Depending upon the axial point of entry in the ASH, the bubbles reported at different boundaries and consequently the residence time of the bubble in the swirl layer was found to vary from 0.05 second to 0.15 second.

After a few time steps, by virtue of its steady radial velocity, the bubble crosses the point of zero axial velocity. At that point onward, the bubble starts rising upwards since the upward axial velocity of the fluid dominates in this region. However, the centrifugal force is very strong at all locations and causes the bubble to have a high slip velocity in the radial direction. This radial velocity of the bubble takes it to the air core surface. A few bubbles, when introduced at the very top or very bottom of the porous tube, find an outlet through the overflow exit or underflow exit respectively. In most cases, when the bubble reports at the air core surface, the time for which the bubble remain in the swirl layer is about 0.1-0.15 seconds. The bubbles which exit through the overflow and underflow opening, spend even less time in the swirl layer (0.05-0.08 seconds). Thus, it is evident from these calculations that the residence time of air bubbles in the swirl layer is very low.

5.2.4.2 Bubble Swarm Case

However, the bubble trajectories are different when a swarm of bubbles are introduced into the domain at each time step. This is because of the mutual interactions between the bubbles (as in hindered settling). As can be seen from

Equation (5.30), the bubble slip velocity is a strong function of volume fraction of air. Thus, in the presence of other bubbles, the bubbles travel relatively slower. Therefore, at steady state, each grid point will have certain number of bubbles associated with it. In view of the very high radial slip velocity and hence a short residence time of the air bubbles in the swirl layer, saturation of control volume with air bubbles was not encountered in this calculation.

From the number of air bubbles associated with the current grid point, the void fraction at that location was calculated using the procedure described in the previous section. Thus, from the steady state bubble concentration profile, the volume fraction of air and hence the radial density profile was generated at all axial positions. The calculated radial density profiles at the top, middle and bottom of the ASH for two-phase air-water flow and the corresponding radial density profiles experimentally observed through x-ray CT measurements are presented in Figure 5.19.

It can be seen from these figures that the experimental and calculated density profiles compare reasonably well from a qualitative standpoint. However, in view of the simplistic approach of these calculations and the fact that the reliability of the fluid velocity profiles used in the calculations was not proven, a quantitative comparison is not warranted. In these calculations it has been established that the trend of the radial density profiles can be predicted from theoretical considerations. However, more work is needed to be able to make accurate quantitative predictions and in this regard, these calculations can be modified and refined to a great extent.

Finally, an estimate of the air split to the overflow was made. In this regard, it was assumed that the bubble reports at the overflow if it exits through the overflow opening or if it reaches the air core interface. Figure 5.20 shows the total number of bubbles reporting at the overflow and the number of bubbles exiting through the underflow opening for the last 100 time steps, i.e., at steady state. It can be seen from this figure that these values are fairly constant with small fluctuations. The average

Figure 5.19. Comparison between calculated and experimental radial density profiles at the top, middle and bottom of the ASH for no solids in the feed, $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

Figure 5.20. Plot of the number of bubbles reporting at the overflow and underflow for the last 100 time steps of calculation in order to determine the air flow split at steady state.

value of the number of bubbles reporting at the overflow was estimated to be 261 and the average number of bubbles reporting to the underflow was estimated to be 32. Therefore the air split to the overflow was estimated as $(261)/(261+32) = 0.891$. Thus 89.1% of the air reported at the overflow at steady state. In this regard, it should be noted that the air split was measured by Kinneberg (74) and it was observed that about 82% of the air reports at the overflow at steady state. Thus the theoretical and measured values of the air split compare reasonably well.

5.3 Swirl Flow in a Nonporous Plexiglass Tube

To compliment the understanding of the nature of flow inside the ASH, visual observations of the swirl flow behavior of water through a nonporous plexiglass tube were made by photography. The effects of inlet pressure and A^* on the flow characteristics were observed and recorded in these experiments as described in the following sections.

5.3.1 Materials and Equipment

In these experiments only water was used as the feed material to the ASH. The porous tube section of the ASH-2C was replaced by a 2.4" ID transparent right vertical plexiglass tube. The geometry of the inlet was the same (involute). The same set up that was used for flotation experiments was used in these experiments. However, no air was sparged. Therefore, accessories such as compressor and rotameter were not necessary.

5.3.2 Procedure

Since there were no solids in the feed and no air was sparged, the only variables studied in these experiments were A^* and inlet pressure. Three different A^* values and six different inlet pressure levels were chosen. The nature of flow

characteristics at all possible combinations of these two variables was recorded through photography. The system was started after the variables were set at desired levels. The system was allowed to reach steady state and photographs of the transparent plexiglass tube section were taken. Examination of these photographs revealed valuable information about the shape of the air core, interfacial instabilities and streamline patterns at the tube wall. The observations are presented and discussed in the following section.

5.3.3 Results and Discussion

Visual observations of the flow patterns in a plexiglass tube are represented by the photographs shown in Figures 5.21 and 5.22. The three columns are for $A^* = 1.00$, 0.85 and 0.74 respectively from left to right. The rows are for inlet pressures 1.5, 3.5 and 5.5 psi in Figure 5.21 and 7.5, 10.5 and 13.5 psi in Figure 5.22 respectively from top to bottom. It can be seen from Figure 5.21 that for all A^* values, wetting of the tube surface is incomplete at 1.5 psi. There is no vortex formation at 3.5 psi and the tube is flooded at the bottom for all A^* values.

For $A^* = 0.74$, vortex formation is complete at 7.5 psi. See Figure 5.22. The effect of pressure-drop oscillation (91) is reflected in the air core surface. For such a low value of A^* ($A^* = 0.74$) the underflow opening area is large and hence the water split to the overflow is small. In this case the fluctuating volume flow rate in the overflow stream is indicative of a fluctuating axial pressure drop. Because of this unsteady nature of the pressure drop, surface instabilities result. A wave pattern, 3-D in character, forms at the interface. This wave pattern at the air-water interface can be seen from Figure 5.22 at all inlet pressures for $A^* = 0.74$.

For $A^* = 0.85$, although vortex formation is complete at 7.5 psi, the effect of pressure-drop oscillation can be observed at the air-water interface from Figure 5.22. The wave pattern forms at the interface owing to fluctuating pressure drop characteristics.

Figure 5.21. Variation in swirl flow behavior with A^* at low inlet pressures for water flow in a right vertical plexiglass tube.

Figure 5.22. Variation in swirl flow behavior with A^* at high inlet pressures for water flow in a right vertical plexiglass tube.

However, at very high inlet pressure (13.5 psi) the pressure drop and the volume flow rate to the overflow stream stabilize to a great extent. Therefore at this inlet pressure, with $A^* = 0.85$, the surface instabilities are reduced substantially resulting in a near-cylindrical air core. It can also be seen from this figure that the air core shrinks with inlet pressure.

For $A^* = 1.00$, vortex formation is incomplete at 5.5 psi. See Figure 5.21. At 7.5 psi vortex formation is complete and an air core extending to the bottom of the tube is clearly visible from Figure 5.22. At this high value of A^* ($A^* = 1.00$), the underflow opening is relatively small and the flow rate to the overflow stream was found to be steady and even at lower values of inlet pressure. The wave pattern, not so prominent, at the air-water interface can be seen only at 7.5 psi. Above this inlet pressure the pressure drop stabilizes completely and the surface instabilities disappear, resulting in a cylindrical air core. The air core shrinks a little with inlet pressure. However, no further change is noticeable above 10.5 psi.

Comparison at any inlet pressure between these three A^* values reveals that the air core expands as A^* is decreased. Vortex formation is complete at about 7 psi for all cases. At lower A^* values ($A^* = 0.85$ and 0.74), a wave pattern forms at the air-water interface even at high inlet pressures and the flow instabilities are very prominent. At a high A^* value ($A^* = 1.00$) the flow is more stable and the wave pattern disappears from the interface at lower inlet pressures. The pressure-drop oscillations are no longer visible at the high A^* value.

5.4 Summary

It was found that froth transport plays a very important role with regard to the performance of the ASH. Therefore, in spite of all other conditions being favorable the flotation recovery can be poor under circumstances when froth transport is sluggish. In this regard the location of the surface of zero axial velocity is critical, because this surface actually determines the amount of froth being transported to the

overflow. Ideally, if this surface coincides with the froth phase-swirl layer interface, best results will be obtained. However, depending upon the conditions, this surface may intersect with the froth phase and consequently a portion of the froth is lost to the underflow, causing the recovery to drop.

It was found that as the value of A^* is increased, the surface of ZAV shifts towards the porous tube wall and thereby carries more froth to the overflow. Consequently, the recovery increases. The surface of ZAV also shifts towards the porous wall with an increase in inlet pressure causing a larger amount of froth to be transported towards the vortex finder. Therefore, an increased recovery was observed at higher inlet pressures up to some limit depending on particle mass. It appears that froth transport is critical as far as these two parameters, A^* and inlet pressure, are concerned. However, Q^* and solids concentration of the feed slurry did not have any noticeable impact on the location of the surface of ZAV.

From theoretical considerations, bubble trajectories were calculated. The results match reasonably well with earlier speculation of a short residence time for the air bubbles (about 0.1 second). The results also show that the bubble motion is dominated by the fluid velocity in the axial direction whereas the 'radial buoyancy' dominates motion in the radial direction.

The void fraction of air is critical in determining bubble motion in the case of a bubble swarm. However, a short residence time of air bubbles keeps the void fraction low and allows the bubble swarm to travel in the case of two-phase flow. The steady state bubble concentration profile was calculated and, from this, the radial density profiles were generated. The results show that the air hold up in the swirl layer is indeed low when the system is operating under steady state. The experimental and theoretical radial density profiles compare favorably at least in a qualitative sense. Also, the air split to the overflow, predicted to be 89% from theoretical

considerations, compares reasonably well with experimental observations of 82% made by Kinneberg (74).

In swirl flow experiments with a transparent plexiglass tube, it was shown that vortex formation is complete at about 7 psi inlet pressure. There are instabilities at the surface of the air core under certain conditions. Because of these instabilities, wave patterns form at the air core surface. These instabilities disappear at high inlet pressures and high A^* values and the air core becomes more cylindrical.

CHAPTER 6

PHENOMENOLOGICAL DESCRIPTION OF ASH FLOTATION

Various researchers have carried out studies, both theoretical and experimental, to characterize the important features of ASH flotation. However, confirmation of a detailed phenomenological explanation of ASH flotation has not been possible. Most of the understandings of multiphase flow in the ASH as well as the scale up and design criteria are still based on inferences and empiricism. In the previous chapters of this thesis, additional information about the flow phenomena and characteristics of ASH flotation has been discussed based on new experimental results, in particular the results from x-ray CT analysis. Now, in this chapter a more detailed phenomenological description of ASH flotation is offered in view of our improved understanding. Also, speculations regarding the rate of flotation and the mechanism of bubble attachment are presented.

6.1 Flotation Rate Constant

Flotation is usually reported to be a first order rate process in the technical literature (92-95). Conventionally, the flotation rate constant is calculated assuming first order kinetics and this rate constant is an index of flotation performance. In this section, an expression for flotation rate is derived for steady state ASH flotation of quartz. The rate, derived from theoretical considerations, was found to follow first order kinetics and on this basis a rate constant is estimated for ASH flotation during steady state operation.

Flotation rate is defined as the number of solids particles of specified type captured and removed per unit time. The expression for the flotation rate can be written as :

$$\frac{\text{\# of particles captured}}{\text{time}} = \left[\frac{\text{\# of particle captured per bubble}}{\text{time}} \right] \times [\text{\# of bubbles}] \quad (6.1)$$

The terms in Equation (6.1) are considered and derived separately below.

During x-ray CT measurements it was observed that in the absence of hydrophobic solids in the swirl layer, the density of the swirl layer was always very close to 1.00 g/cc which indicates that the bubble concentration in the swirl layer is small. In view of the high air flow rate, this would be possible only if the bubble residence time in the swirl layer is very small. Therefore, it is difficult for the bubbles to capture particles during motion as they sweep through a small thickness of swirl layer during motion. In addition, because of the very high tangential velocity of the slurry in the ASH, the shear rate at the wall is very high. Therefore, a lot of slurry sweeps past the bubble while it is forming at the surface of the porous tube. In view of this, in this analysis it is assumed that a significant amount of bubble attachment takes place at the wall itself during the formation of the bubble. It is also assumed that bubble attachment is almost complete in the annular region close to the porous tube wall, which the solids particles would occupy if all of them were centrifuged to the wall with certain packing factor. Thus, this volume of the particle bed, V , is important for bubble attachment. Therefore, the term in the left-hand side can be written as :

$$\frac{\text{\# of particles captured}}{\text{time}} = V \frac{dc_p}{dt} \quad (6.2)$$

where c_p is the number of particles per unit volume.

The volume of the particle bed can be estimated easily from the solids concentration of the slurry and the slurry hold up volume. In this regard, the volume fraction of solids in the suspension can be estimated from the weight percent of solids,

$$\text{Volume fraction of solids} = \frac{\rho_s s}{\rho_l s + \rho_s (100 - s)} \quad (6.3)$$

where

s = wt% of solids in the suspension

ρ_l = density of the liquid phase

ρ_s = density of the solid phase

The slurry hold up volume in the ASH is given as

$$\text{Slurry hold up volume} = h\pi(R^2 - r^2) \quad (6.4)$$

where h is the length of the ASH, R is the ASH radius, and r is the radius of the air core.

Now, assuming a packing factor of 0.76, the volume of the particle bed under consideration, is given as

$$V = h\pi(R^2 - r^2) \times \left[\frac{\rho_l s}{\rho_l s + \rho_s (100 - s)} \right] \times \frac{1}{0.76} \quad (6.5)$$

The first term in the right-hand side of Equation 6.1 can be written as :

$$\frac{\text{\# of particles captured per bubble}}{\text{time}} = \left(\frac{\pi}{4} d_b^2 u_t c_p E_k \right) \quad (6.6)$$

where

u_t = average tangential velocity in the annular volume

E_k = collection efficiency which is defined as the fraction of particles swept out by a bubble that collides with, attaches to and remains attached to the bubble

The last term in the right-hand side of Equation 6.1 is derived from the bubble diameter, tangential velocity and air flow rate :

$$\text{\# of bubbles} = \text{\# of bubbles per unit time} \times \text{time}$$

Thus,

$$\text{\# of bubbles} = \left(\frac{Q_g}{\frac{\pi}{6} d_b^3} \right) \times \left(\frac{L}{u_t} \right) \quad (6.7)$$

where

Q_g = air flow rate

L = the total length of the helical path that the fluid travels in the porous tube section

Assuming that the fluid travels at an angle of 45 degrees with the vertical, the helical path length can be easily calculated for a 12 inch long porous tube,

$$L = 1.414h \quad (6.8)$$

Therefore, Equation (6.1) becomes :

$$V \frac{dc_p}{dt} = [u_t (\frac{\pi d_b^2}{4}) c_p E_k] \times [\frac{Q_g}{\pi d_b^3 / 6}] \times [\frac{1.414h}{u_t}] \quad (6.9)$$

Substituting the expression for V and simplifying, Equation (6.9) becomes :

$$\frac{dc_p}{dt} = [\frac{1.612 Q_g E_k (\rho_l s + \rho_s (100 - s))}{\rho_l s \pi d_b (R^2 - r^2)}] c_p \quad (6.10)$$

It can easily be seen that Equation (6.10) has the form of a first order rate equation. The term included inside the square bracket can be identified as the flotation rate constant, k. To evaluate the flotation rate constant, the collection efficiency factor needs to be evaluated for each specific system. The air flow rate, wt% solids in the feed, air core radius, ASH radius, and bubble diameter can be easily estimated, calculated or measured.

In this regard a little discussion about the flotation rate constant derived above is warranted. From Equation (6.10), the expression for the flotation rate constant is found to be :

$$k = [\frac{1.612 Q_g E_k (\rho_l s + \rho_s (100 - s))}{\rho_l s \pi d_b (R^2 - r^2)}] \quad (6.11)$$

It can be seen from Equation (6.11) that as the air flow rate increases the flotation rate increases. This has indeed been observed in Chapter 4. See Figures 4.3 and 4.4. However, bubble diameter also increases with air flow rate and beyond a certain air

flow rate this increase may be significant and reduce the flotation rate. Bubble size distribution has been studied by Lelinski (19). In this study (19) it was found that an increase in air flow rate resulted in an increase in average bubble size. However, the increase in bubble size with air flow rate was not great. Since the increase in bubble diameter with air flow rate is not great, the effect of increased bubble diameter is not too significant. This again has been observed in Chapter 4. The recovery dropped only slightly above a certain Q^* value. See Figures 4.3 and 4.4.

Equation (6.11) also suggests an increase in flotation rate with a decrease in bubble diameter. In the ASH, the size of the bubble is determined by the shear at the wall (96-99). As the tangential velocity (inlet pressure) increases the shear also increases. Therefore, smaller bubbles are ripped off from the wall at higher inlet pressures which enhance flotation. An increase in flotation recovery with inlet pressure upto a certain point has indeed been observed in Chapter 4. See Figures 4.3 and 4.4. However, beyond a certain value of inlet pressure, no further reduction in bubble size would be possible as the bubble has to grow upto a minimum size before it can be sheared off from the porous tube wall by the slurry. At the same time the centrifugal force increases with increasing inlet pressure (tangential velocity) and the hydrophobic particle will be detached from the bubble which carry them. These two effects coupled together explain the drop in recovery beyond a certain inlet pressure as was observed in chapter 4. See Figures 4.3 and 4.4. Thus, the form of the flotation rate constant agrees qualitatively with the experimental observations.

A rough estimation of the magnitude of the flotation rate constant can also be made. It should be noted that the collection efficiency depends on the particle size, particle density, bubble diameter, air flow rate and other characteristics of the specific system. For conventional flotation columns, the collection efficiency is typically in the range 0.2 - 4% (100). This was not evaluated for the ASH system and a value of 1%

is assumed for an order of magnitude estimate of the flotation rate constant. Other values were taken from experimental conditions. Thus taking,

$$Q_g = 250 \text{ lpm} = 4166 \text{ cc/sec}$$

$$R = 0.95 \text{ inch} = 2.413 \text{ cm}$$

$$d_b = 300 \text{ } \mu\text{m} = 0.03 \text{ cm}$$

$$\rho_l = 1.00 \text{ g/cc}$$

$$\rho_s = 2.65 \text{ g/cc}$$

$$s = 15$$

$$r = 0.5 \text{ cm}$$

the rate constant k , from Equation (6.11), was determined to be $2.048 \times 10^3 \text{ sec}^{-1}$. It should be noted that this value is very high compared to the flotation rate constant for conventional column flotation which is typically on the order of 0.02 sec^{-1} . Thus it appears that the flotation rate constant for the ASH is about five orders of magnitude greater than that for conventional flotation. This explains the high recovery achieved with an ASH in such a short time. In this regard, it should be noted that the residence time of the air bubble in the ASH is about 0.1 sec whereas in the conventional flotation column it is about 10 minutes, which is 6000 times greater. Also, in ASH flotation the bubble diameter is about 300 microns whereas in conventional flotation the bubble diameter is about 1.5 mm, which is five times greater than that in ASH flotation. Therefore, such a high value of flotation rate constant may not be unreasonable. However, it should be kept in mind that the estimation made here is rough and based on approximate values of the collection efficiency and bubble diameter.

6.2 Mechanism of Bubble Attachment

In conventional flotation cells, the air bubble rises to the surface and on its way hydrophobic particles are attached. It is very well established that for these cells a

long residence time (on the order of 10 minutes) is required to achieve good flotation. Consequently, the columns are quite tall. In these columns, the critical factor in determining the recovery is the amount of slurry that sweeps past the bubble during its rise (101). In a conventional column flotation cell, the bubble travels typically about 10 meters before it becomes fully loaded. During this period a large amount of slurry sweeps past the bubble and thus the bubble collects hydrophobic particles during its upward motion.

However, during ASH flotation, the results from the x-ray CT study show that even though a large volume of air passes through the swirl layer, the air hold up in the swirl layer is very small (less than 5%). Therefore, it is evident that the air bubbles have a very short residence time in the swirl layer. The theoretical analysis in the previous chapter supports this point. In this regard, it should be noted that in an earlier study (74), it was speculated from theoretical considerations that the air phase has a residence time of about 0.1 second and the slurry phase has a residence time of about 0.8-1.0 seconds. Kinneberg's (74) speculation is supported by the results of this study.

When the bubble is ripped away from the porous tube by the fluid, it travels along with the slurry in the angular direction. Therefore, the relative velocity in this direction is negligibly low. Also the bubble travels in the axial direction with the fluid having a low relative velocity. Thus, in the angular and axial directions very little slurry sweeps past the bubble after its release from the wall owing to the negligible slip velocities in these directions. In the radial direction, although the slip velocity is high, the bubble traverses only a few millimeters (6-12 mm depending upon the conditions) in the case of an ASH-2C before it reaches the froth surface. This, then, becomes equivalent to having a conventional flotation column which is only a few millimeters tall. Therefore, if the mechanism of attachment in the ASH were to be the same as in a conventional flotation column, flotation with a high recovery would be

impossible with an ASH. However, high recoveries can be obtained in ASH flotation as evidenced by the very high flotation rate constant ($k = 2.265 \times 10^3$), much higher than that for conventional flotation (typically, $k = 0.03$). Thus experimental findings corroborate the possibility of a high flotation rate in the case of ASH flotation and it appears that the mechanism of attachment of hydrophobic particles to bubbles is significantly different from that in a conventional flotation column.

During ASH flotation, the slurry shears the air flow at the porous tube wall since the directions of flow of the two phases are mutually perpendicular at the porous tube surface. At sufficiently high inlet pressures, when vortex formation is complete, bubble generation is controlled by the shear at the wall. The bubble is ripped off from the wall as soon as it grows to a certain size by the fluid depending upon the tangential velocity. Since, after being sheared off from the wall very little slurry sweeps past the bubble, complete loading of the bubble during its passage would be difficult if the bubble were to pick up particles during its motion. Therefore, it appears that the attachment of particles takes place right at and near the porous tube wall during bubble formation as the bubble is heavily loaded by the time it is sheared from the porous tube surface. This appears to be reasonable in view of the fact that during its formation a large volume of slurry sweeps past the bubble as the slurry flows orthogonal to the air bubbles as they grow. This compensates for the negligible amount of slurry sweeping past the bubble after its release. This mechanism of attachment then explains the high recovery that can be achieved with ASH flotation.

6.3 Operational Considerations

It appears from the discussion in Chapters 3, 4 and 5 that there is a clear relationship between the flotation response and the spatial extent and average density of the flow regimes. The average density of different regimes and their spatial extents

are in turn dependent upon the operating conditions. Thus the operating conditions need to be adjusted in such a way that the characteristics of the flow regimes become favorable for flotation and maximum recovery is achieved at steady state.

In general, there are three different flow regimes under normal operating conditions. The central air core, the swirl layer near the porous tube wall and a froth phase in between those two regimes. The first criterion for good flotation is that the froth phase must be stabilized. In this regard, it was shown that the froth phase stabilizes in the presence of hydrophobic solids. The collector level should be high enough, depending on the pH of the pulp, so that a hydrophobic surface is created on the quartz particles. A high pH should be maintained to reduce the consumption of collector. The air flow rate should be adequate to stabilize the froth phase. It has been shown that insufficient air results in a weak froth phase.

The solids concentration of the pulp is important in the selection of the right design for the ASH. A higher solids concentration will, in general, require a longer residence time and hence a longer ASH unit for a given ASH diameter.

Axial flow reversal plays a major role with regard to transport of froth to the overflow. The transport of froth phase to the overflow should be efficient in order to achieve a high flotation recovery. It is desired to collect as much froth in the overflow as possible, keeping the loss of froth to the underflow at a minimum. The location of the surface of zero axial velocity is important in this regard. This surface can be manipulated by two operating variables, A^* and inlet pressure. Ideally, it would be best to have the surface of ZAV coincide with the interface between the froth phase and the swirl layer. Thus, the two above-mentioned variables must be adjusted such that this goal is achieved.

CHAPTER 7

SUMMARY AND CONCLUSIONS

Air sparged hydrocyclone flotation is strongly dependent on the characteristics of the multiphase fluid flow which, in turn, depends on the levels of many different operating variables. The effects are manifested in the shape of the air core and its dimension, froth phase and swirl layer dimensions, swirl layer density, froth stability as revealed from x-ray CT density profiles, and of course the recovery of hydrophobic particles in the overflow. The major findings of this thesis research on the flotation of mono-sized quartz particles are summarized and presented in this concluding chapter.

Three distinct flow regimes, the central air core, the froth phase and the swirl layer near the porous wall, characterize the internal flow regimes in the ASH. The feed slurry enters the ASH at the top through a tangential or involute inlet and follows a helical path before exiting through the annular underflow opening at the bottom. Because of the swirling flow, a strong centrifugal force field develops at the top of the ASH. As the suspension travels downwards this force field decays in strength because of the wall friction. As a result, the tangential velocity component also decays (31, 35). Thus, the swirl layer experiences a stronger force at the top when compared to that at the bottom. In fact, the swirl layer thickness at the top has always been found to be smaller than at the bottom because of this axial variation in the centrifugal force. It is expected that this variation influences the air distribution for single section jacketed porous tube and, in fact, experiments are in progress to confirm this expectation (102).

7.1 Utility of X-Ray CT

The experimental results clearly show the utility of x-ray CT for the analysis of the complex multiphase flow during air-sparged flotation. Even though the x-ray CT measurements describe only the time-averaged steady state flow, the calculated density profiles provide sufficient information to distinguish between the froth phase and swirl layer during actual flotation experiments. On this basis, the variation in the flotation response can be described from the variation in the spatial extent of the flow regimes with changes in the design and operating variables (percent solids, A^* , Q^* , inlet pressure, particle size, collector level, and suspension pH). The criteria for froth stability were established and the importance of froth stability and its transport were shown to be of particular significance in the analysis of quartz flotation by the air-sparged hydrocyclone.

It has been demonstrated in Chapter 3, how the spatial extent of the flow regimes and their average density, as determined by x-ray CT measurements, relate to the flotation performance of the ASH. The effects of operating variables on these flow regimes, particularly the froth phase features and flotation response, were established. Important operating variables were identified to be A^* - the nondimensional area ratio, Q^* - the nondimensional flow rate ratio, inlet pressure and percent solids in the feed

It was found that the air core expands with a decrease in A^* whereas an increase in Q^* causes the air core to expand. However, the air core diameter shrinks when the solids concentration of the feed slurry is increased. When conditions in the ASH-2C are favorable for quartz flotation, i.e., A^* , Q^* and inlet pressure are maintained at 1.00, 4.55 and 10.5 psi respectively, it was established from the froth stability features that the ASH length required for good recovery (80% or more) is smaller for lower percent solids (5%) than for higher percent solids (15%). The froth phase is stabilized by the presence of hydrophobic particles. In the absence of

hydrophobic particles it is difficult to detect a steady state froth phase in the high centrifugal force field of the ASH. Good flotation is achieved when the froth phase is stabilized and gradually decreases in thickness down the axis of the ASH eventually disappearing, under which circumstances the swirl layer density at the bottom is reduced to about 1.00 g/cc. If the operating and design conditions are such that a stable froth phase is present even at the bottom, then some froth is carried to the underflow stream. This causes a loss of hydrophobic quartz particles in the underflow and consequently the recovery drops.

It is clear from the discussion in Chapter 3 that fairly accurate estimation of flotation performance can be made by investigating the spatial extent and average density of the flow regimes, namely, air core, froth phase and swirl layer. The thickness of the froth phase appears to be proportional to the solids concentration when a stable froth is formed. The froth phase thickness was found to be greater with 15% solids in feed than with 5% solids in feed and in comparison to conventional flotation the froth phase density ($\rho_{froth} \sim 0.14 - 0.38$ g/cc) was found to be significantly greater in the case of ASH flotation ($\rho_{froth} \sim 0.2 - 0.8$ g/cc). From the x-ray CT data it was speculated that froth transport may be the critical factor in some cases for determining flotation performance. In this regard the slurry inlet pressure and A^* - the dimensionless area ratio were found to be particularly significant.

7.2 Flotation Recovery

Detailed parametric study of the ASH flotation response was presented in Chapter 4. The dependence of flotation response on a number of operating and design variables was established. It was confirmed that the presence of hydrophobic particles greatly enhances froth stability. Experiments with two different porous tube lengths established that the length of the ASH and thus the residence time required for good flotation indeed depends on the solids concentration of the feed suspension. There is a

clear relationship between the spatial extent and the average density of the flow regimes and the flotation performance.

It was also shown in Chapter 4 that flotation performance is improved with increasing A^* . The recovery of solids to the overflow increases faster with A^* at a high solids concentration (15%) than when the solids concentration in the feed is low (5%). Flotation performance is restricted by the magnitude of the centrifugal force for detachment and also by the bubble size. The centrifugal force required for detachment, for a given ASH diameter, depends on the mass of the particle and the tangential velocity (inlet pressure) of the slurry. A high tangential velocity would enhance froth transport. However, too high a tangential velocity may result in the detachment of the bubble-particle aggregate by raising the centrifugal force for detachment which results in a poor recovery. Also, a high tangential velocity may lead to the generation of smaller bubbles by increasing the shear rate, which facilitates flotation. However, beyond a certain value of inlet pressure, bubble size would become independent of inlet pressure and the resulting high force for detachment destabilizes the bubble-particle aggregate causing a drop in recovery. Loading of very small bubbles is also difficult when the particles are comparable in size to the size of the bubbles.

A high air flow rate (high Q^*) ensures the formation of a stable froth and thus results in a better flotation recovery. Also, the flotation rate constant is increased with an increase in the relative air flow rate (higher Q^*). However, too high an air flow rate may result in an increase in average bubble size and consequently in a reduced recovery. However, the increase in bubble size with Q^* has been found to be not so great (19). Therefore the adverse effect on flotation recovery by an increase in Q^* is not very significant. It was established in this study that a higher Q^* value beyond an optimum level certainly does not enhance the flotation recovery of quartz. Thus, it appears that both the tangential velocity and the air flow rate must be maintained at

optimum levels, depending upon the specific characteristics of the system, in order to maintain a good condition for ASH flotation.

It is also confirmed in this study that the maximum particle size that can be floated in a centrifugal field is indeed inversely proportional to the square root of the centrifugal acceleration. Thus, given a centrifugal force field, there is a ceiling on the maximum size of the particle that can be floated using the ASH. Flotation of smaller particles is relatively insensitive to the operating variables. Because of the higher centrifugal force required for detachment in the case of smaller particles, the loaded bubble is much more stable. Therefore, transport of the mineralized bubble to the overflow stream occurs without any significant detachment, even under severe flow conditions, resulting in good recovery.

At neutral pH (pH = 7.4), an increase in the collector level was found to improve flotation performance. At this pH better surface coverage by the collector is achieved with a higher collector concentration and the recovery increased at higher collector levels. However, quartz appears to float even better in a more alkaline medium at lower concentrations of collector. At a higher pH (pH = 10.1), good flotation recovery was achieved at a low collector concentration. It was established that collector consumption can be reduced by appropriate control of the suspension pH.

7.3 Axial Flow Reversal

In Chapter 5, the importance of froth transport characteristics was established. In this regard the location of the surface of zero axial velocity was studied a tracer analysis technique. It was found that this surface shifts towards the porous tube wall as A^* is increased and also as the inlet pressure is increased. Since this surface determines how much froth is being carried to the overflow and thereby flotation recovery, control of the ASH flotation can be achieved by adjusting these two

variables. It was demonstrated that under certain conditions, the surface of ZAV intersects the froth phase and carries some froth to the underflow. This results in a loss of hydrophobic particles in the underflow stream causing a drop in flotation recovery. In the cases where the surface of ZAV does not intersect the froth phase, most of the froth is carried to the overflow stream and the recovery is high. The variables such as percent solids in the feed and Q^* have no noticeable impact on the location of the surface of ZAV.

7.4 Bubble Trajectory

Theoretical considerations in Chapter 5 provides a description of bubble trajectory and the steady state bubble concentration profile. The trajectories of single bubbles were calculated and their residence time in the flow field were shown to be very low, in the range 0.05-0.15 second. This compares favorably with earlier speculation of a 0.1 second bubble residence time (74). It was also shown that for the air-water two phase flow it is possible to calculate the steady state radial density profiles in the ASH by tracking the bubbles. The theoretical calculations agreed reasonably well with the experimental results obtained by x-ray CT measurements for two phase flow. Finally, the air flow split to the overflow was theoretically estimated to be 89% which compared favorably with the experimental observation of 82% by Kinneberg (74).

7.5 Photographic Analysis of Single Phase Swirl Flow

Based on photographic analysis of single phase swirl flow in the absence of air, vortex formation was found to be complete at about 7 psi inlet pressure. Below this inlet pressure, the system remains flooded and the air core-water interface has instabilities, which are caused by a fluctuating pressure drop. These instabilities are manifested as almost a sinusoidal wave pattern at the air core-water interface. These

instabilities tend to disappear with increasing A^* and increasing inlet pressure rendering the air core more cylindrical in shape.

7.6 Phenomenological Description

Chapter 6 provides a phenomenological description of ASH flotation. An expression for the flotation rate constant was derived. ASH flotation was described from a first order kinetic expression. Very high recovery values with a very short residence time during ASH flotation suggests a much higher flotation rate constant as compared to the flotation rate constant for conventional column flotation. In fact, the flotation rate constant was estimated for ASH flotation and it was found to be about five orders of magnitude higher than that for conventional flotation column. This, coupled with the fact that the residence time of the air bubbles in the swirl layer is very low, indicates that the bubble attachment mechanism is different in the case of ASH flotation and led to the proposition of a new mechanism of bubble attachment. It was proposed that loading of the air bubbles occurs during bubble formation and in a thin annular region close to the porous tube wall. The slip velocities of the bubble in the axial and angular directions are very small and the bubble travels a very short distance in the radial direction. Thus, the amount of slurry sweeping past the bubble after its release is very small and during passage through the swirl layer to the froth phase, very little attachment is expected to take place. Therefore, in order to achieve such high recoveries observed in practice, bubble attachment must be almost complete at the porous tube wall during its formation and shortly after release.

In essence, this fundamental study of air sparged hydrocyclone flotation, increased our present understanding of multiphase flow during ASH flotation. Based on the findings of this research it is now known which conditions lead to an enhanced flotation recovery and why. On this basis improved control strategies can be developed and improved design and scale up criteria can be established to achieve

better separation efficiency. For the first time a quantitative study of time-averaged multiphase flow during ASH flotation by x-ray CT, a detailed parametric study of ASH flotation of quartz, and a study of axial flow reversal in the ASH during steady state operation have been reported. These findings constitute an important contribution to the science and technology of ASH flotation.

7.7 Future Work

Flotation in the ASH is a very complex process. Direct measurements of the flow characteristics are difficult because of the opaque porous tube. In this regard, x-ray CT has been shown to be a very useful tool for the analysis of ASH flotation. This work should be extended to study and verify the findings of this thesis research with a larger diameter ASH unit during steady state operation.

In this thesis research an ideal system was chosen in view of the fundamental nature of the study. The understandings from this research should be extended to study the practical situations where separation of one mineral component from another takes place with a low concentration of hydrophobic particles in the ASH.

Bubble characteristics during ASH flotation need to be studied in greater detail. Details of bubble generation, movement, stability, coalescence and breakage, etc., must be known in order to be able to characterize ASH flotation more completely. However, considering the very low residence time of both the phases, slurry and air, it is indeed a difficult task to characterize the bubbles. Moreover, the level of turbulence inside the ASH subjects the measurements by a probe to a lot of noise. These difficulties must be overcome.

Another important aspect is the fluid dynamics. Efforts were made to measure velocities of the swirl flow in a plexiglass tube (31). These measurements were limited to a narrow region close to the tube wall. Also, no air was sparged during these measurements. Since the tangential velocity is one of the most significant variables in

determining the magnitude of the centrifugal force field, it is essential to make measurements of the velocities in the entire domain during actual ASH flotation. Again, the presence of the opaque porous tube and air bubbles limits the use of conventional LDV techniques. Some ways must be found to be able to make these measurements in order to establish whether or not the air flow through the swirl layer affects the flow pattern significantly.

Finally, a detailed theoretical analysis needs to be performed to explain the three phase flow during ASH flotation. No theoretical model has yet been developed to describe the ASH flotation phenomena in its entirety. However, the complexity of this analysis must be kept in mind in this regard. The swirl flow in the ASH has all three phases, solid, liquid and gas, involved. In addition, flotation is also taking place in the ASH. This requires considerations of froth characteristics, surface chemistry and flotation kinetics along with the fluid flow characteristics of the ASH. These, taken together, make such an all-inclusive analysis a formidable task. However, to make the best use of a wonderful technological innovation, the air-sparged hydrocyclone, these problems must be solved and a more in-depth understanding of the phenomena taking place in the ASH must be realized.

APPENDIX A

RADIAL DENSITY PROFILES

Table A.1. Density with radial position at three different axial positions of the ASH. $A^* = 0.74$, $Q^* = 4.55$, inlet pressure = 10.5 psi (ref. Figures 3.15 and 3.17).

Radius (mm)	5% Solids			15% Solids		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
0.447	0	0	0	0	0	0
0.894	0	0	0	0	0	0
1.341	0	0	0	0	0	0
1.787	0	0	0	0	0	0
2.234	0	0	0	0	0	0
2.681	0	0	0	0	0	0
3.128	0	0	0	0	0	0
3.575	0	0	0	0	0	0
4.022	0	0	0	0	0	0
4.469	0	0	0	0	0	0
4.915	0	0	0	0	0	0
5.362	0	0	0	0	0	0
5.809	0	0	0	0	0	0
6.256	0	0	0	0	0	0
6.703	0	0	0	0	0	0
7.15	0	0	0	0	0	0
7.596	0	0	0	0	0	0.162
8.043	0	0	0	0	0.24	0.162
8.49	0	0	0.222	0	0.296	0.18
8.937	0	0	0.259	0	0.352	0.218
9.384	0	0.259	0.296	0	0.371	0.236
9.831	0	0.296	0.334	0	0.409	0.274
10.278	0	0.334	0.371	0	0.427	0.311
10.724	0	0.371	0.409	0	0.446	0.349
11.171	0	0.409	0.446	0.201	0.465	0.386
11.618	0	0.446	0.465	0.295	0.502	0.423
12.065	0	0.483	0.502	0.351	0.521	0.461
12.512	0	0.502	0.521	0.389	0.539	0.479
12.959	0	0.539	0.558	0.445	0.558	0.498
13.406	0	0.558	0.577	0.501	0.577	0.517
13.852	0.258	0.595	0.614	0.538	0.595	0.535
14.299	0.332	0.614	0.633	0.576	0.614	0.554
14.746	0.407	0.633	0.689	0.594	0.614	0.573

Table A.1. (continued)

Radius (mm)	5% Solids			15% Solids		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
15.193	0.482	0.652	0.726	0.613	0.633	0.592
15.64	0.538	0.67	0.783	0.613	0.652	0.629
16.087	0.594	0.708	0.82	0.632	0.652	0.666
16.534	0.632	0.726	0.876	0.669	0.67	0.723
16.98	0.706	0.764	0.913	0.706	0.689	0.779
17.427	0.762	0.783	0.951	0.762	0.708	0.853
17.874	0.819	0.82	0.97	0.819	0.745	0.91
18.321	0.875	0.857	0.988	0.875	0.801	0.966
18.768	0.912	0.895	1.007	0.931	0.857	1.003
19.215	0.949	0.932	1.026	0.968	0.913	1.022
19.661	0.987	0.97	1.026	1.006	0.97	1.022
20.108	1.006	0.988	1.044	1.043	1.007	1.04
20.555	1.024	1.007	1.044	1.062	1.026	1.04
21.002	1.043	1.026	1.044	1.08	1.044	1.04
21.449	1.043	1.026	1.044	1.08	1.063	1.04
21.896	1.062	1.026	1.044	1.099	1.063	1.04
22.343	1.062	1.026	1.044	1.099	1.063	1.04
22.789	1.043	1.007	1.026	1.099	1.044	1.022
23.236	1.043	0.97	0.988	1.08	1.026	0.984
23.683	1.006	0.932	0.951	1.043	0.97	0.947
24.13	0.931	0.857	0.876	0.968	0.895	0.853

Table A.2. Density with radial position at three different axial positions of the ASH. $A^* = 0.85$, $Q^* = 4.55$, inlet pressure = 10.5 psi (ref. Figures 3.15 and 3.17).

Radius (mm)	5% Solids			15% Solids		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
0.447	0	0	0	0	0	0
0.894	0	0	0	0	0	0
1.341	0	0	0	0	0	0
1.787	0	0	0	0	0	0
2.234	0	0	0	0	0	0
2.681	0	0	0	0	0	0
3.128	0	0	0	0	0	0
3.575	0	0	0	0	0	0
4.022	0	0	0	0	0	0.201
4.469	0	0	0	0	0	0.276
4.915	0	0	0	0	0	0.295
5.362	0	0	0	0	0	0.314
5.809	0	0	0	0	0	0.332
6.256	0	0	0	0	0.224	0.332
6.703	0	0	0	0	0.318	0.351
7.15	0	0	0	0.162	0.355	0.37
7.596	0	0	0	0.218	0.392	0.389
8.043	0	0	0.183	0.274	0.43	0.407
8.49	0	0.25	0.22	0.311	0.448	0.445
8.937	0	0.306	0.295	0.367	0.486	0.463
9.384	0	0.362	0.351	0.423	0.523	0.501
9.831	0.143	0.4	0.37	0.461	0.561	0.538
10.278	0.199	0.437	0.37	0.517	0.579	0.557
10.724	0.274	0.475	0.389	0.554	0.617	0.594
11.171	0.33	0.512	0.426	0.61	0.636	0.613
11.618	0.386	0.549	0.482	0.648	0.654	0.65
12.065	0.423	0.568	0.519	0.666	0.673	0.669
12.512	0.479	0.605	0.538	0.685	0.692	0.688
12.959	0.517	0.624	0.538	0.704	0.71	0.706
13.406	0.554	0.643	0.576	0.704	0.729	0.725
13.852	0.592	0.662	0.613	0.704	0.748	0.744
14.299	0.629	0.68	0.65	0.723	0.748	0.762
14.746	0.648	0.699	0.706	0.723	0.766	0.781

Table A.2. (continued)

Radius (mm)	5% Solids			15% Solids		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
15.193	0.685	0.718	0.762	0.741	0.785	0.8
15.64	0.723	0.736	0.8	0.76	0.785	0.837
16.087	0.76	0.755	0.837	0.779	0.804	0.875
16.534	0.797	0.774	0.893	0.816	0.822	0.912
16.98	0.835	0.811	0.949	0.853	0.841	0.931
17.427	0.872	0.849	0.987	0.891	0.879	0.968
17.874	0.891	0.867	0.987	0.928	0.897	0.987
18.321	0.928	0.905	1.006	0.966	0.935	1.024
18.768	0.966	0.923	1.006	0.984	0.972	1.043
19.215	0.984	0.961	1.024	1.003	1.01	1.043
19.661	1.003	0.979	1.024	1.04	1.028	1.043
20.108	1.003	0.998	1.024	1.059	1.047	1.062
20.555	1.022	0.998	1.006	1.059	1.066	1.062
21.002	1.022	0.998	1.006	1.078	1.066	1.062
21.449	1.04	0.998	1.006	1.096	1.066	1.062
21.896	1.04	0.998	1.024	1.096	1.066	1.062
22.343	1.04	0.998	1.024	1.096	1.066	1.062
22.789	1.022	0.998	1.006	1.096	1.066	1.062
23.236	1.003	0.979	0.987	1.096	1.047	1.024
23.683	0.966	0.942	0.949	1.059	0.991	0.987
24.13	0.872	0.867	0.875	0.947	0.916	0.893

Table A.3. Density with radial position at three different axial positions of the ASH. $A^* = 1.00$, $Q^* = 4.55$, inlet pressure = 10.5 psi (ref. Figures 3.15 and 3.17).

Radius (mm)	5% Solids			15% Solids		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
0.447	0	0	0	0	0	0
0.894	0	0	0	0	0	0
1.341	0	0	0	0	0	0
1.787	0	0	0	0	0	0
2.234	0	0	0	0	0	0
2.681	0	0	0	0	0	0
3.128	0	0	0	0	0	0
3.575	0	0	0	0	0	0
4.022	0	0	0	0	0	0
4.469	0	0	0	0	0.236	0
4.915	0	0	0	0.218	0.292	0.084
5.362	0	0	0	0.33	0.367	0.215
5.809	0	0	0	0.386	0.423	0.327
6.256	0	0	0	0.461	0.479	0.402
6.703	0	0	0	0.498	0.535	0.477
7.15	0	0	0	0.554	0.573	0.533
7.596	0	0	0	0.592	0.592	0.57
8.043	0	0	0	0.629	0.61	0.608
8.49	0	0	0.255	0.666	0.648	0.645
8.937	0	0	0.349	0.704	0.666	0.701
9.384	0.18	0.18	0.461	0.741	0.704	0.739
9.831	0.255	0.236	0.573	0.76	0.704	0.795
10.278	0.311	0.292	0.666	0.779	0.723	0.869
10.724	0.367	0.367	0.76	0.797	0.741	0.926
11.171	0.405	0.461	0.853	0.816	0.76	0.963
11.618	0.461	0.554	0.91	0.835	0.779	0.982
12.065	0.498	0.629	0.947	0.835	0.797	1
12.512	0.535	0.704	0.966	0.853	0.816	1.019
12.959	0.592	0.779	0.984	0.853	0.835	1.019
13.406	0.629	0.835	0.984	0.872	0.853	1.019
13.852	0.666	0.872	0.984	0.872	0.891	1.019
14.299	0.685	0.891	1.003	0.891	0.91	1.019
14.746	0.723	0.91	1.003	0.91	0.928	1.038

Table A.3. (continued)

Radius (mm)	5% Solids			15% Solids		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
15.193	0.76	0.928	1.003	0.928	0.947	1.038
15.64	0.797	0.928	1.003	0.947	0.966	1.038
16.087	0.835	0.947	1.003	0.984	0.984	1.019
16.534	0.872	0.947	1.003	1.003	1.003	1.019
16.98	0.91	0.947	1.003	1.022	1.022	1.019
17.427	0.947	0.966	1.003	1.04	1.022	1.019
17.874	0.966	0.966	1.003	1.059	1.04	1.019
18.321	0.984	0.966	1.003	1.078	1.04	1.019
18.768	1.003	0.966	0.984	1.096	1.04	1.019
19.215	1.022	0.966	0.984	1.115	1.04	1.019
19.661	1.022	0.966	0.984	1.115	1.04	1.019
20.108	1.04	0.966	0.984	1.134	1.04	1.019
20.555	1.04	0.947	0.984	1.153	1.04	1.019
21.002	1.059	0.947	0.984	1.153	1.04	1.019
21.449	1.059	0.947	0.984	1.171	1.04	1.019
21.896	1.04	0.947	0.984	1.171	1.04	1.019
22.343	1.04	0.947	0.966	1.171	1.04	1.019
22.789	1.04	0.928	0.966	1.171	1.022	1
23.236	1.003	0.91	0.947	1.153	0.984	0.982
23.683	0.966	0.872	0.91	1.096	0.928	0.926
24.13	0.872	0.779	0.835	0.966	0.835	0.813

Table A.4. Density with radial position at three different axial positions of the ASH. 15% solids, $A^* = 1.00$, $Q^* = 4.55$ (ref. Figure 3.20).

Radius (mm)	6.5 psi			10.5 psi		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
0.447	0	0	0	0	0	0
0.894	0	0	0	0	0	0
1.341	0	0	0	0	0	0
1.787	0	0	0	0	0	0
2.234	0	0	0	0	0	0
2.681	0	0	0	0	0	0
3.128	0	0	0	0	0	0
3.575	0	0	0	0	0	0
4.022	0	0	0	0	0	0
4.469	0	0	0	0	0.236	0
4.915	0	0	0	0.218	0.292	0.084
5.362	0	0.284	0.187	0.33	0.367	0.215
5.809	0.218	0.302	0.199	0.386	0.423	0.327
6.256	0.236	0.34	0.218	0.461	0.479	0.402
6.703	0.274	0.377	0.255	0.498	0.535	0.477
7.15	0.311	0.433	0.292	0.554	0.573	0.533
7.596	0.33	0.489	0.349	0.592	0.592	0.57
8.043	0.367	0.527	0.405	0.629	0.61	0.608
8.49	0.405	0.583	0.442	0.666	0.648	0.645
8.937	0.442	0.62	0.498	0.704	0.666	0.701
9.384	0.479	0.658	0.535	0.741	0.704	0.739
9.831	0.535	0.676	0.592	0.76	0.704	0.795
10.278	0.573	0.714	0.629	0.779	0.723	0.869
10.724	0.629	0.733	0.685	0.797	0.741	0.926
11.171	0.685	0.751	0.723	0.816	0.76	0.963
11.618	0.723	0.77	0.741	0.835	0.779	0.982
12.065	0.741	0.77	0.779	0.835	0.797	1
12.512	0.76	0.789	0.797	0.853	0.816	1.019
12.959	0.76	0.789	0.835	0.853	0.835	1.019
13.406	0.76	0.789	0.853	0.872	0.853	1.019
13.852	0.76	0.807	0.872	0.872	0.891	1.019
14.299	0.779	0.826	0.891	0.891	0.91	1.019
14.746	0.797	0.845	0.891	0.91	0.928	1.038

Table A.4. (continued)

Radius (mm)	6.5 psi			10.5 psi		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
15.193	0.816	0.863	0.91	0.928	0.947	1.038
15.64	0.816	0.882	0.928	0.947	0.966	1.038
16.087	0.835	0.901	0.947	0.984	0.984	1.019
16.534	0.853	0.919	0.966	1.003	1.003	1.019
16.98	0.891	0.938	0.984	1.022	1.022	1.019
17.427	0.91	0.957	1.003	1.04	1.022	1.019
17.874	0.947	0.976	1.003	1.059	1.04	1.019
18.321	0.966	0.994	1.022	1.078	1.04	1.019
18.768	0.984	0.994	1.022	1.096	1.04	1.019
19.215	1.003	1.013	1.04	1.115	1.04	1.019
19.661	1.022	1.013	1.04	1.115	1.04	1.019
20.108	1.04	1.013	1.04	1.134	1.04	1.019
20.555	1.04	1.013	1.04	1.153	1.04	1.019
21.002	1.059	1.013	1.04	1.153	1.04	1.019
21.449	1.059	1.013	1.04	1.171	1.04	1.019
21.896	1.078	1.013	1.04	1.171	1.04	1.019
22.343	1.078	1.013	1.04	1.171	1.04	1.019
22.789	1.096	1.013	1.022	1.171	1.022	1
23.236	1.078	0.994	1.003	1.153	0.984	0.982
23.683	1.022	0.938	0.947	1.096	0.928	0.926
24.13	0.91	0.845	0.853	0.966	0.835	0.813

Table A.5. Density with radial position at three different axial positions of the ASH. 15% solids, $A^* = 1.00$, inlet pressure = 10.5 psi (ref. Figure 3.22).

Radius (mm)	$Q^* = 2.28$			$Q^* = 6.83$		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
0.447	0	0	0	0	0	0
0.894	0	0	0	0	0	0
1.341	0	0	0	0	0	0
1.787	0	0	0	0	0	0
2.234	0	0	0	0	0	0
2.681	0	0	0	0	0	0
3.128	0.349	0	0	0	0	0
3.575	0.461	0	0	0	0	0
4.022	0.573	0.162	0	0	0	0
4.469	0.685	0.349	0	0	0.162	0
4.915	0.779	0.517	0	0.236	0.218	0
5.362	0.853	0.685	0	0.274	0.292	0
5.809	0.891	0.816	0	0.33	0.349	0
6.256	0.928	0.91	0	0.405	0.405	0
6.703	0.928	0.966	0.367	0.461	0.442	0
7.15	0.928	1.022	0.442	0.517	0.479	0
7.596	0.928	1.04	0.535	0.573	0.498	0.218
8.043	0.928	1.059	0.629	0.61	0.535	0.292
8.49	0.928	1.078	0.741	0.648	0.554	0.349
8.937	0.928	1.096	0.816	0.666	0.592	0.405
9.384	0.928	1.096	0.891	0.704	0.61	0.461
9.831	0.947	1.096	0.947	0.723	0.648	0.535
10.278	0.947	1.096	1.003	0.741	0.666	0.61
10.724	0.947	1.096	1.04	0.76	0.685	0.666
11.171	0.966	1.096	1.078	0.779	0.685	0.741
11.618	0.984	1.096	1.096	0.779	0.704	0.816
12.065	0.984	1.096	1.096	0.797	0.723	0.853
12.512	0.984	1.096	1.115	0.797	0.741	0.91
12.959	0.984	1.096	1.115	0.816	0.76	0.928
13.406	0.984	1.115	1.115	0.816	0.779	0.966
13.852	1.003	1.115	1.115	0.835	0.797	0.984
14.299	1.003	1.115	1.115	0.835	0.835	0.984
14.746	1.003	1.115	1.115	0.853	0.853	1.003

Table A.5. (continued)

Radius (mm)	$Q^* = 2.28$			$Q^* = 6.83$		
	Density (g/cc)			Density (g/cc)		
	top	middle	bottom	top	middle	bottom
15.193	1.003	1.115	1.115	0.872	0.872	1.003
15.64	1.022	1.115	1.115	0.891	0.891	1.003
16.087	1.022	1.115	1.115	0.91	0.91	1.022
16.534	1.04	1.115	1.115	0.928	0.928	1.022
16.98	1.04	1.115	1.115	0.947	0.947	1.022
17.427	1.04	1.115	1.115	0.984	0.966	1.022
17.874	1.059	1.115	1.115	1.003	0.984	1.022
18.321	1.059	1.115	1.115	1.022	0.984	1.022
18.768	1.059	1.115	1.115	1.04	0.984	1.022
19.215	1.078	1.115	1.115	1.059	1.003	1.022
19.661	1.078	1.134	1.115	1.078	1.003	1.022
20.108	1.078	1.134	1.115	1.078	1.003	1.022
20.555	1.096	1.134	1.115	1.096	1.003	1.022
21.002	1.096	1.134	1.096	1.115	1.003	1.022
21.449	1.115	1.134	1.096	1.115	1.003	1.022
21.896	1.115	1.134	1.096	1.134	1.003	1.022
22.343	1.115	1.134	1.096	1.134	0.984	1.003
22.789	1.115	1.115	1.096	1.134	0.984	0.984
23.236	1.096	1.078	1.059	1.115	0.947	0.966
23.683	1.04	1.003	1.003	1.059	0.891	0.891
24.13	0.928	0.891	0.891	0.928	0.816	0.816

APPENDIX B

FLOTATION RECOVERY AND WATER SPLIT

Table B.1. Recovery of solids and water split to the overflow for specified experimental conditions with 100 x 200 mesh quartz particles. Collector : 800 g/ton, pH = 7.4, porous tube length = 12" (ref. Figures 4.3, 4.4, 4.5 and 4.6).

A*	Q*	Inlet pressure (psi)	% Solids	Recovery of solids in the overflow (%)	Water split to the overflow (%)
0.74	4.55	10.5	5	68.4	1.2
0.85	4.55	10.5	5	85.5	4.4
1.00	4.55	10.5	5	94.2	17.1
0.74	4.55	10.5	15	15.5	2.1
0.85	4.55	10.5	15	50.9	5.3
1.00	4.55	10.5	15	97.3	19.9
1.00	2.28	6.5	5	41.5	4.4
1.00	4.55	6.5	5	69.3	9.0
1.00	6.83	6.5	5	80.1	14.2
1.00	7.80	6.5	5	73.4	15.3
1.00	2.28	6.5	15	42.2	5.0
1.00	4.55	6.5	15	69.9	7.8
1.00	6.83	6.5	15	80.6	16.3
1.00	7.80	6.5	15	73.2	18.4
1.00	2.28	8.5	5	47.0	10.9
1.00	4.55	8.5	5	82.6	17.0
1.00	6.83	8.5	5	90.6	18.1
1.00	7.80	8.5	5	87.4	20.0
1.00	2.28	8.5	15	47.9	14.3
1.00	4.55	8.5	15	83.1	18.1
1.00	6.83	8.5	15	91.1	22.3
1.00	7.80	8.5	15	88.5	20.4
1.00	2.28	10.5	5	50.1	12.9
1.00	4.55	10.5	5	91.2	17.8
1.00	6.83	10.5	5	95.1	18.4
1.00	7.80	10.5	5	93.0	19.4
1.00	2.28	10.5	15	51.9	16.2
1.00	4.55	10.5	15	92.4	20.6
1.00	6.83	10.5	15	95.6	22.5
1.00	7.80	10.5	15	93.5	23.9

Table B.1. (continued)

A*	Q*	Inlet pressure (psi)	% Solids	Recovery of solids in the overflow (%)	Water split to the overflow (%)
1.00	2.28	13.5	5	44.9	13.1
1.00	4.55	13.5	5	58.4	14.6
1.00	6.83	13.5	5	68.1	19.4
1.00	7.80	13.5	5	64.3	21.7
1.00	2.28	13.5	15	46.0	17.0
1.00	4.55	13.5	15	58.7	18.3
1.00	6.83	13.5	15	68.9	22.4
1.00	7.80	13.5	15	65.3	25.9
1.00	2.28	16.5	5	38.0	14.7
1.00	4.55	16.5	5	37.6	16.1
1.00	6.83	16.5	5	40.3	20.4
1.00	7.80	16.5	5	34.0	21.9
1.00	2.28	16.5	15	38.8	16.2
1.00	4.55	16.5	15	41.8	17.3
1.00	6.83	16.5	15	41.1	20.0
1.00	7.80	16.5	15	34.7	23.6

Table B.2. Recovery of solids and water split to the overflow for specified experimental conditions with 200 x 325 mesh quartz particles. Collector : 800 g/ton, pH = 7.4, porous tube length = 12" (ref. Figure 4.6).

A*	Q*	Inlet pressure (psi)	% Solids	Recovery of solids in the overflow (%)	Water split to the overflow (%)
1.00	6.83	8.5	5	85.4	16.1
1.00	6.83	13.5	5	91.2	19.4
1.00	6.83	16.5	5	88.3	20.6
1.00	6.83	18.5	5	89.5	22.5
1.00	6.83	23.0	5	87.2	23.8
1.00	6.83	26.0	5	88.4	25.9
1.00	4.55	8.5	5	86.5	14.2
1.00	4.55	13.5	5	89.4	14.8
1.00	4.55	16.5	5	87.4	14.3
1.00	4.55	18.5	5	87.3	14.9
1.00	4.55	23.0	5	86.8	15.8
1.00	4.55	26.0	5	87.9	15.4
1.00	4.55	30.0	5	86.3	16.1
1.00	4.55	33.0	5	85.9	16.4
1.00	4.55	36.0	5	78.5	17.2
1.00	2.28	10.5	5	64.3	14.9
1.00	2.28	13.5	5	69.6	15.6
1.00	2.28	16.5	5	69.1	16.8
1.00	2.28	18.5	5	68.0	16.1
1.00	2.28	23.0	5	69.1	17.2
1.00	2.28	26.0	5	68.7	17.1
1.00	2.28	30.0	5	67.9	16.7
1.00	2.28	33.0	5	68.1	16.1
1.00	2.28	36.0	5	65.8	16.2
1.00	2.28	38.0	5	52.4	14.9
1.00	2.28	40.0	5	39.2	13.2

APPENDIX C

AXIAL FLOW REVERSAL - TRACER SPLIT

Table C.1. Tracer concentrations in the product streams at various radial positions with 100 x 200 mesh quartz particles, 800 g/ton collector, 12" long porous tube and a pH of 7.4. The distances are measured from the porous tube wall. 5% solids, $A^* = 0.74$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

Overflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	--	--	--	--	401	391	202	165	134
65	--	--	--	376	371	192	166	128	100
105	--	--	--	329	308	171	139	91.3	60.1
150	--	--	287	282	144	109	64.1	28.8	15.3
195	--	--	136	79.8	60.0	32.6	18.3	8.2	2.1
230	--	--	39.8	20.2	11.6	7.3	2.7	0	--
275	--	--	8.5	2.4	1.8	0	0	--	--

Underflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	--	--	--	--	10.2	11.0	26.5	29.6	32.1
65	--	--	--	12.2	12.6	27.4	29.5	32.6	34.9
105	--	--	--	16.1	17.8	29.1	31.7	35.6	38.2
150	--	--	19.5	20.0	31.3	34.1	37.9	40.8	41.9
195	--	--	32.0	36.6	38.2	40.5	41.6	42.5	43.0
230	--	--	39.8	41.5	42.2	42.5	42.9	43.1	--
275	--	--	42.5	42.9	43.0	43.1	43.0	--	--

Table C.2. Tracer concentrations in the product streams at various radial positions with 100 x 200 mesh quartz particles, 800 g/ton collector, 12" long porous tube and a pH of 7.4. The distances are measured from the porous tube wall. 5% solids, $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

Overflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	--	--	152	141	143	136	129	75.4	61.3
65	--	--	133	128	124	117	77.2	68.4	54.2
105	--	--	102	98.3	92.6	58.0	47.3	31.3	24.8
150	--	--	68.3	57.1	28.2	22.6	12.3	7.9	7.5
195	--	--	31.2	22.4	8.9	4.1	2.7	1.6	1.2
230	--	--	9.8	3.2	2.6	2.4	1.7	1.4	0
275	--	--	4.1	2.9	0.8	0.8	0	0	0

Underflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	--	--	12.1	14.6	14.1	15.7	17.3	29.5	32.7
65	--	--	16.4	17.5	18.4	20.0	29.1	31.0	34.3
105	--	--	23.4	24.3	25.6	33.4	35.8	39.5	41.0
150	--	--	31.1	33.6	40.2	41.5	43.8	44.8	44.9
195	--	--	39.5	41.5	44.6	45.7	46.0	46.2	46.3
230	--	--	44.4	45.9	46.0	46.0	46.2	46.3	47.0
275	--	--	45.7	45.9	46.4	46.8	47.0	--	--

Table C.3. Tracer concentrations in the product streams at various radial positions with 100 x 200 mesh quartz particles, 800 g/ton collector, 12" long porous tube and a pH of 7.4. The distances are measured from the porous tube wall. 15% solids, $A^* = 0.74$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

Overflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	--	--	--	239	230	219	122	101	83.1
65	--	--	--	220	204	136	114	91.3	72.8
105	--	--	201	189	179	108	89.7	71.0	53.6
150	--	192	184	176	100	89.3	74.1	47.9	21.0
195	--	135	132	68.5	49.7	24.5	11.0	5.1	1.3
230	--	53.5	48.2	18.4	9.6	5.3	1.6	1.2	--
275	--	10.3	8.6	2.5	1.3	1.0	0	--	--

Underflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	--	--	--	11.6	12.8	14.2	26.9	29.7	32.1
65	--	--	--	14.1	16.2	25.1	28.0	30.9	33.4
105	--	--	16.6	18.2	19.5	28.8	31.2	33.6	35.9
150	--	17.8	18.8	19.9	29.8	31.2	33.2	36.7	40.2
195	--	25.2	25.6	33.9	36.4	39.7	41.5	42.3	42.8
230	--	35.9	36.6	40.5	41.7	42.3	42.7	42.8	--
275	--	41.6	41.8	42.6	42.8	42.8	43.0	--	--

Table C.4. Tracer concentrations in the product streams at various radial positions with 100 x 200 mesh quartz particles, 800 g/ton collector, 12" long porous tube and a pH of 7.4. The distances are measured from the porous tube wall. 15% solids, $A^* = 1.00$, $Q^* = 4.55$ and 10.5 psi inlet pressure.

Overflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	190	179	182	173	172	166	158	79.3	68.8
65	162	158	149	152	143	141	82.3	67.6	49.3
105	112	109	101	104	99.6	69.0	53.3	45.2	31.1
150	92.0	90.3	86.2	84.6	41.3	32.8	26.5	14.6	6.3
195	52.8	43.3	45.7	41.3	15.2	9.7	2.2	3.8	1.6
230	22.0	18.6	19.8	7.6	4.0	4.2	1.8	1.1	0
275	8.3	5.2	5.5	1.0	1.8	0	0	0	0

Underflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	10.2	12.1	11.6	13.2	13.4	14.5	16.0	30.4	32.3
65	15.2	16.0	17.6	17.1	18.7	19.1	29.8	32.5	35.9
105	24.4	25.0	26.4	25.9	26.7	32.3	35.2	36.6	39.2
150	28.1	28.4	29.1	29.4	37.4	38.9	40.1	42.3	43.8
195	35.3	37.0	36.5	37.3	42.2	43.2	44.5	44.3	44.6
230	40.9	41.5	41.3	43.5	44.2	44.6	44.7	45.0	--
275	43.4	43.8	43.2	44.8	44.6	45.3	--	--	--

Table C.5. Tracer concentrations in the product streams at various radial positions with 100 x 200 mesh quartz particles, 800 g/ton collector, 12" long porous tube and a pH of 7.4. The distances are measured from the porous tube wall. 15% solids, $A^* = 1.00$, $Q^* = 2.28$ and 10.5 psi inlet pressure.

Overflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	212	210	195	188	186	179	177	99.3	79.1
65	190	188	170	166	158	156	78.5	68.1	49.9
105	133	140	116	100	98.9	42.2	35.6	28.1	26.6
150	101	99.4	91.3	88.8	41.9	33.2	26.5	21.7	16.0
195	72.6	71.3	71.7	70.1	35.8	24.2	18.1	11.0	5.2
230	29.0	28.6	24.9	11.5	7.8	6.6	2.8	1.7	1.0
275	12.2	7.8	6.2	4.0	2.3	1.2	0	0	0

Underflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	14.2	14.5	16.5	17.4	17.7	18.6	18.9	29.6	32.3
65	17.2	17.5	19.9	20.5	21.6	21.8	32.4	33.8	36.3
105	25.0	24.1	27.3	29.5	29.6	37.4	38.3	39.3	39.5
150	29.3	29.6	30.7	31.0	37.4	38.6	39.5	40.2	41.0
195	33.2	33.4	33.3	33.6	38.3	39.8	40.7	41.6	42.5
230	39.2	39.2	39.8	41.6	42.1	42.2	42.8	42.9	--
275	41.5	42.1	42.3	42.6	42.8	43.0	--	--	--

Table C.6. Tracer concentrations in the product streams at various radial positions with 100 x 200 mesh quartz particles, 800 g/ton collector, 12" long porous tube and a pH of 7.4. The distances are measured from the porous tube wall. 15% solids, $A^* = 1.00$, $Q^* = 4.55$ and 6.5 psi inlet pressure.

Overflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	372	343	336	323	312	307	195	162	131
65	302	297	281	266	248	123	103	93.2	71.0
105	259	268	241	229	118	94.2	77.5	49.2	38.6
150	178	174	153	161	86.7	67.5	49.9	28.3	11.6
195	128	130	123	71.6	57.2	43.6	28.0	14.9	4.2
230	47.6	41.5	32.9	18.4	13.1	8.3	5.1	1.3	0
275	11.4	9.3	3.9	1.3	2.1	1.5	0	0	0

Underflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	12.6	16.1	17.0	18.6	19.9	20.5	34.1	38.1	41.9
65	21.1	21.7	23.6	25.5	27.6	42.8	45.2	46.4	49.1
105	26.3	25.2	28.5	30.0	43.4	46.3	48.3	51.8	53.1
150	36.1	36.6	39.2	38.2	47.3	49.6	51.7	54.3	56.3
195	42.2	42.0	42.8	49.1	50.8	52.5	54.3	55.9	57.2
230	52.0	52.7	53.8	55.5	56.2	56.7	57.1	57.6	--
275	56.4	56.6	57.2	57.7	57.5	57.6	57.2	--	--

Table C.7. Tracer concentrations in the product streams at various radial positions with 100 x 200 mesh quartz particles, 800 g/ton collector, 12" long porous tube and a pH of 7.4. The distances are measured from the porous tube wall. 15% solids, $A^* = 1.00$, $Q^* = 6.83$ and 10.5 psi inlet pressure.

Overflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	183	176	170	164	161	160	150	74.1	65.3
65	151	142	140	142	133	128	78.3	62.9	47.5
105	103	104	97.3	100	90.6	64.2	54.3	41.2	27.2
150	90.1	90.3	79.8	72.6	38.7	30.8	24.6	18.3	9.2
195	48.8	42.3	42.7	38.7	13.2	9.9	3.4	3.8	1.9
230	20.1	19.2	19.8	8.2	4.6	4.0	2.1	0	0
275	6.2	3.2	3.5	1.0	1.8	0	0	0	0

Underflow

Distance from top (mm)	Tracer (ppm)								
	Radial distance from porous tube wall (mm)								
	18	16	14	12	10	8	6	4	2
25	11.3	12.5	11.9	14.0	14.4	14.5	16.1	31.6	32.0
65	15.8	16.2	18.1	18.1	18.5	18.9	30.1	32.2	34.8
105	25.1	25.1	26.3	27.1	28.2	34.4	35.7	37.6	39.1
150	28.7	28.4	29.8	30.5	38.3	38.9	40.0	42.1	42.9
195	37.2	38.1	36.9	38.3	43.9	44.2	44.5	44.2	44.9
230	42.0	43.1	42.3	45.6	45.2	45.4	45.7	--	--
275	44.2	45.8	45.3	45.8	45.4	45.7	--	--	--

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