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The Wolf effect and the Redshift of Quasars

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Abstract

We consider a simple model, based on currently accepted models for active galactic nuclei, for a quasi-stellar object (QSO or "quasar") and examine the influence that correlation-induced spectral changes ("The Wolf Effect") may have upon the redshifts of the optical emission lines.

1 Introduction

The Wolf effect is the name given to several closely related phenomena in radiation physics dealing with the modification of the power spectrum of a radiated field due to *spatial* fluctuations of the source of radiation. The first paper to demonstrate a connection between spatial source fluctuations (as characterized by the cross-spectral density function) appeared in 1987 [1]. Since then over 100 papers have appeared on this and related topics (for a recent review, see ref. [2]).

The discovery of coherence-induced spectral shifts grew out of work investigating the connection between optical coherence theory and the empirical laws of radiometry, in particular the dependence of radiative transfer on the wavelength of light. In 1986, Wolf introduced the scaling law [3], which is a condition upon the spatial coherence properties of a source under which the radiated field has the same normalized spectrum as the source; in other words, if the source obeys the scaling law, the radiation pattern is independent of wavelength. From this discovery, it was a comparatively short step to investigate the properties of a radiation source which does not obey the scaling law; it was found that for such a source there would be a small shift of the central frequencies of spectral lines, with shifts towards lower frequencies predominating [4]. This effect was soon confirmed experimentally [3, 4, 5, 6].

Because of the well known analogy between radiation and scattering, it is not surprising that analogous effects may occur in the scattering of light. Such spectral shifts were first investigated in ref.[7] for the case of static (i.e. time invariant) scattering media; the analysis of the case of time-dependent scatterers followed soon after [8]. This led to an important realization that scattering from time-dependent fluctuating random media could mimic the Doppler effect in many of its important features [9]. An example of such a scattering medium which could produce Doppler-like shifts was found in 1990: it involved a highly *statistically anisotropic*¹ scattering

¹we will use the term *statistically anisotropic* for such media to avoid confusion with other types of anisotropies, for example media which respond to different polarizations of light in different ways

medium in the sense that the coherence length of the fluctuations of the scatterer was much longer in one direction than in the others [10] (see also [11, 12, 13]).

It was speculated in ref.[4] that this effect might play a role in the interpretation of the spectra of certain extra-galactic astronomical objects such as quasars. Since then other applications, notably remote sensing, communications and filtering have since be proposed, however in this paper we will return to this first potential application. In the work on scattering from anisotropic media, it was pointed out that anisotropies of the type under consideration are compatible with current models of quasars [10]. Here we will investigate these points further by proposing a model for a quasar, closely related to a model currently favored [14], which predicts a sizable spectral shift due to the Wolf effect. The rest of the paper is organized as follows: in section 2 we discuss briefly various types of spectral shift phenomena and their importance to cosmology. In section 3 we give a qualitative discussion of the model quasar we are considering and a mathematical analysis of the spectrum of the light it radiates. Finally we assess the possible implications of the results derived.

2 Spectral shifts and Cosmology

In this section we will give a brief and non-exhaustive description of different physical phenomena which can give rise to shifts in the central frequency of a spectral line. The magnitude of such a shift is often measured in terms of the fractional shift or z -number, defined by the formula

$$z = \frac{\lambda'_0 - \lambda_0}{\lambda_0} \equiv \frac{\omega'_0 - \omega_0}{\omega'_0}, \quad (2.1)$$

where λ_0 is the *unshifted* central wavelength of the spectral line; λ'_0 is its *shifted* central wavelength; ω_0 is the angular frequency corresponding to λ_0 and ω'_0 is the angular frequency corresponding to λ'_0 . When z is a positive number the spectral line has been shifted to a lower frequency (i.e. a redshift) and when z is negative the shift is to a higher frequency (a blueshift). From eq.2.1 it is clear that z must always be in the range $-1 < z < \infty$.

One of the most famous physical phenomenon which can produce spectral shifts is the Doppler effect. When a source of radiation is in motion with respect to the observer, there is a shift of the wavelength of spectral lines with respect to the wavelength that would be observed if the source were at rest. In this case the z -number is given by the the following formula [15, 16]:

$$\begin{aligned} z &= \frac{1 - \beta \cos \theta}{\sqrt{1 - \beta^2}} \\ &\approx -\beta \cos \theta, \end{aligned} \quad (2.2)$$

where $\beta = v/c$, v being the speed of the source relative to the observer and c the speed of light, and θ is the angle between the velocity of the source and the line joining the source and the observer. The approximate expression is valid when $\beta \ll 1$. For example, if the source is moving directly away from the observer, so that $\theta = \pi$, the z -number has the property $z > 0$, i.e. a redshift is observed, whereas if the source is heading directly toward the observer (so that $\theta = 0$) then the z -number is negative and a blueshift will be observed. An important property of the Doppler effect is that the z -number is *independent* of the wavelength of the line in question. The fact that redshifts with z -numbers approximately independent of wavelength are observed in the spectra of all galaxies, thereby suggesting that the spectral shift is due to the Doppler

effect associated with receding motion, is the single most compelling piece of evidence in favor of the expanding universe. Hubble's law, the empirical linear relationship between distances and redshifts of close by galaxies, confirms the predictions of cosmological models based on general relativity, and so it has become widely accepted that the redshift of the radiation emitted by an object is an indicator of its cosmological distance. However there is observational evidence that seems to contradict these assumptions. A well known example is the close pairing of the galaxy NGC-4319 (with redshift $z = 0.006$) and the quasar Mk-205 ($z = 0.07$), which, despite having drastically different redshifts (and therefore being at completely different distances according to the usually accepted interpretation of redshifts), have a distinct luminous connection between them. If we abandon this conventional interpretation, we must then ask what phenomena, other than the recessional motion of an object due to cosmological expansion, may be the cause of the observed spectral shifts.

Another important type of spectral shift is the gravitational shift discovered by Einstein [17]. This occurs when there is a difference between the gravitational potential in the vicinity of the source and the gravitational potential in the vicinity of the observer. In this case the z -number is given by

$$z = \frac{\Delta\Phi}{c^2} \quad (2.3)$$

where $\Delta\Phi$ is the difference in gravitational potential. This shift, like the Doppler shift, has a z -number which is independent of wavelength. However, it can be shown that this shift is small for quasars.

Another type of shift, which is sometimes suggested as a possible source of spectral shifts in quasars, is the Compton effect, due to scattering by free electrons [18, 19]. The scattered radiation has a spectral shift with respect to the incident radiation given by the formula:

$$z = \frac{2\hbar\omega_0 \sin^2(\Theta/2)}{m_e c^2} \quad (2.4)$$

where m_e is the mass of an electron and Θ is the scattering angle. However, shifts due to the Compton effect have z -numbers which are dependent on the wavelength of incident radiation (i.e. ω_0 , the angular frequency of the incident radiation, appears in eq.2.4), and therefore the Compton effect cannot account for all of the large shifts observed in the spectra of extra-galactic objects.

Other important optical effects which produce spectral shifts are the closely related scattering phenomena known as Brillouin scattering [20] and Raman scattering [21, 22]. Both of these effects produce changes in the wavelength of light due to interactions with excitations of internal degrees of freedom of a scattering medium, such as rotational excitations of molecules or vibrations of a crystal. However these effects do not produce simple spectral shifts: the spectrum of scattered monochromatic radiation contains various discrete frequencies (a triplet in the case of Brillouin scattering, multiplets in the case of Raman scattering).

One other hypothesized type of spectral shift should also be mentioned, namely the redshift due to fundamental particles having variable masses [23, 24, 25]. Although the effect has never been confirmed experimentally, it can be put on a sound theoretical basis, and it does seem to offer a possible explanation for many otherwise contradictory observations of the universe [26].

To these possibilities, we now add the Wolf effect, already discussed briefly in the introduction. In the following section we discuss a model of a quasar which displays a redshift of its spectral lines due to this effect.

3 A model for active galaxies with discordant redshifts

Quasars are star-like objects which have many unusual properties. They emit a non-thermal continuum spectrum of radiation and emission lines that are highly redshifted (z -numbers of up to 4 or 5 have been reported). Because of this high redshift, they are conventionally thought to be very distant objects, which therefore can reveal important information about the structure of the universe on very large scales. However, given their observed intensity, if this interpretation of their redshifts were correct, then quasars must be very powerful sources of radiation indeed. Since the discovery of quasars over 30 years ago, very elegant theories have evolved to explain their properties and account for this peculiar intensity of radiation. The possibility that the observed intensity might be the correct indicator of distance (rather than redshift), and that therefore these object might have a peculiar non-cosmological redshift is usually implicitly rejected.

The basic features of currently fashionable models of quasars [14], and other types of peculiar extra-galactic object (collectively known as active galactic nuclei or AGNs), are shown in figure 1. The most important point is that these objects have axial rather than spherical symmetry. At the center is a very dense object (the 'central engine', sometimes characterized as a black hole, although no direct confirmation of the predictions of general relativity have been seen) which, due to gravitational attraction, sucks in matter from the dust torus, causing it to radiate with a broad, non-thermal power spectrum. This radiation excites gaseous clouds which will emit line radiation. Usually there are thought to be two types of clouds, the broad line regions (BLR) and the narrow line regions (NLR), which emit spectral lines of different widths. (In our diagram we have lumped the BLR and NLR together). The beauty of this model is that the properties of different types of AGN can be explained quite simply by the fact that one is observing the same type of object from different directions. The dust torus blocks out certain wavelengths in certain directions.

Our model differs from that usually considered in two important ways. First, the line emitting clouds are considerably closer to the central engine (or alternatively the dust torus has a considerably larger bulge), so that the line radiation is columnated as well as the continuum radiation. Secondly we assume the existence of a time fluctuating scattering medium, whose properties will be discussed below, which causes the line radiation to be scattered into the direction of the observer. A schematic version of this model, suitable for mathematical analysis, is shown in fig.2

3.1 spectrum of scattered radiation

Scattering by a time-dependent medium is described by the following inhomogeneous partial differential equation:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) U(\mathbf{r}, t) = -4\pi\eta(\mathbf{r}, t)U(\mathbf{r}, t). \quad (3.1)$$

In this formula, $U(\mathbf{r}, t)$ is the radiation field, treated in the scalar approximation, and $\eta(\mathbf{r}, t)$ is the time-dependent dielectric susceptibility of the scattering medium. We have used a simplified model for the medium, in which $\eta(\mathbf{r}, t)$ has only a single time argument; more generally, it should have two time arguments [27], in order to account for the possibilities of resonant frequencies of the scatterer.

We will analyze the scattering in the space-frequency domain. The Fourier transforms ² of

²Since both $U(\mathbf{r}, t)$ and $\eta(\mathbf{r}, t)$ are stationary random processes, generalized harmonic analysis is required.

$U(\mathbf{r}, t)$ and $\eta(\mathbf{r}, t)$ are defined by the formulas:

$$\begin{aligned}\tilde{U}(\mathbf{r}, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} U(\mathbf{r}, t) \exp(i\omega t) dt, \\ \tilde{\eta}(\mathbf{r}, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \eta(\mathbf{r}, t) \exp(i\omega t) dt.\end{aligned}\quad (3.2)$$

Substituting these definitions into eq. (3.1) we obtain the following partial differential equation:

$$(\nabla^2 + k^2) \tilde{U}(\mathbf{r}, \omega) = -4\pi \int_{-\infty}^{\infty} \tilde{\eta}(\mathbf{r}, \omega - \omega') U(\mathbf{r}, \omega') d\omega', \quad (3.3)$$

where $k = \omega/c$. This can be solved using the standard Green's function:

$$\begin{aligned}\tilde{U}(\mathbf{r}, \omega) &= \tilde{U}_0(\mathbf{r}, \omega) \\ &+ \int \int d^3 r' d\omega' \tilde{\eta}(\mathbf{r}', \omega - \omega') \tilde{U}(\mathbf{r}', \omega') \frac{\exp(ik|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|},\end{aligned}\quad (3.4)$$

where $\tilde{U}_0(\mathbf{r}, \omega)$ is the incident field. This is a Fredholm integral equation of the second kind. It can be solved using the standard Born series. Retaining the first two terms only, we obtain the following expression for the scattered field:

$$\begin{aligned}\tilde{U}_s(\mathbf{r}, \omega) &\equiv \tilde{U}(\mathbf{r}, \omega) - \tilde{U}_0(\mathbf{r}, \omega) \\ &\approx \int \int d^3 r' d\omega' \tilde{\eta}(\mathbf{r}', \omega - \omega') \tilde{U}_0(\mathbf{r}', \omega') \frac{\exp(ik|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|}.\end{aligned}\quad (3.5)$$

In the far zone (i.e. in the limit $|\mathbf{r}| \gg |\mathbf{r}'|$), the scattered field is given by the formula:

$$\tilde{U}_s^{(\infty)}(r\mathbf{u}, \omega) \approx \frac{\exp(ikr)}{r} \int \int d^3 r' d\omega' \tilde{\eta}(\mathbf{r}', \omega - \omega') \tilde{U}_0(\mathbf{r}', \omega') \exp(-iku \cdot \mathbf{r}'). \quad (3.6)$$

where $\mathbf{u}$ is the unit vector in the direction of observation (fig.2). The power spectrum of the far-zone scattered field is therefore:

$$\begin{aligned}S^{(\infty)}(r\mathbf{u}, \omega) &= \frac{1}{r^2} \int \int d^3 r'_1 d^3 r'_2 \int d\omega' W_\eta(\mathbf{r}'_1, \mathbf{r}'_2, \omega - \omega') \\ &\times W_0(\mathbf{r}'_1, \mathbf{r}'_2, \omega') \exp[iku \cdot (\mathbf{r}'_1 - \mathbf{r}'_2)]\end{aligned}\quad (3.7)$$

where we have used the following identities involving the cross-spectral density functions for the incident field, $W_0(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$, and for the dielectric fluctuations $W_\eta(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$ and the spectrum of the scattered field $S^{(\infty)}(r\mathbf{u}, \omega)$:

$$\begin{aligned}\langle \tilde{U}_s^{(\infty)*}(r\mathbf{u}, \omega) \tilde{U}_s^{(\infty)}(r\mathbf{u}, \omega') \rangle &= S^{(\infty)}(r\mathbf{u}, \omega) \delta(\omega - \omega') \\ \langle \tilde{U}_0^*(\mathbf{r}'_1, \omega) \tilde{U}_0(\mathbf{r}'_2, \omega') \rangle &= W_0(\mathbf{r}'_1, \mathbf{r}'_2, \omega) \delta(\omega - \omega') \\ \langle \tilde{\eta}^*(\mathbf{r}'_1, \omega) \tilde{\eta}(\mathbf{r}'_2, \omega') \rangle &= W_\eta(\mathbf{r}'_1, \mathbf{r}'_2, \omega) \delta(\omega - \omega'),\end{aligned}\quad (3.8)$$

the chevron brackets denoting an average over ensembles of *both* the field and dielectric fluctuations.

3.2 The Incident Field

Assume that the incident field is radiated by a three dimensional primary source distribution $Q(\mathbf{r}, t)$. Further, assume that the scattering region is in the far zone of the source. The cross-spectral density of the field is then given by

$$W(r_1 \mathbf{s}_1, r_2 \mathbf{s}_2, \omega) = \frac{\exp[ik(r_2 - r_1)]}{r_1 r_2} \int \int d^3 r_1'' d^3 r_2'' W_Q(\mathbf{r}_1'', \mathbf{r}_2'', \omega) \times \exp[ik(\mathbf{s}_1 \cdot \mathbf{r}_1'' - \mathbf{s}_2 \cdot \mathbf{r}_2'')], \quad (3.9)$$

where the cross-spectral density function $W_Q(\mathbf{r}_1', \mathbf{r}_2', \omega)$ of the source distribution is defined by

$$\langle \tilde{Q}^*(\mathbf{r}_1', \omega) \tilde{Q}(\mathbf{r}_2', \omega') \rangle = W_Q(\mathbf{r}_1', \mathbf{r}_2', \omega) \delta(\omega - \omega') \quad (3.10)$$

where $\tilde{Q}(\mathbf{r}', \omega)$ is the generalized Fourier transform of the source distribution $Q(\mathbf{r}, t)$.

It will be convenient for the later development to express this result in terms of "sum-and-difference" coordinates, as follows:

$$W(R\mathbf{s}, \mathbf{b}, \omega) \approx \frac{\exp[ik\mathbf{s} \cdot \mathbf{b}]}{R^2} \int \int d^3 R'' d^3 r'' W_Q(\mathbf{R}'', \mathbf{r}'', \omega) \times \exp \left[-ik \left(\mathbf{s} \cdot \mathbf{r}'' + \frac{\mathbf{b}_\perp \cdot \mathbf{R}''}{R} \right) \right] \quad (3.11)$$

where

$$W(\mathbf{R}, \mathbf{b}, \omega') \equiv W \left(\mathbf{R} - \frac{1}{2}\mathbf{b}, \mathbf{R} + \frac{1}{2}\mathbf{b}, \omega \right) \quad (3.12)$$

and

$$\begin{aligned} \mathbf{R} \equiv R\mathbf{s} &= \frac{r_1 \mathbf{s}_1 + r_2 \mathbf{s}_2}{2}; & \mathbf{b} &= r_2 \mathbf{s}_2 - r_1 \mathbf{s}_1 \\ \mathbf{R}'' &= \frac{\mathbf{r}_1'' + \mathbf{r}_2''}{2}; & \mathbf{r}'' &= \mathbf{r}_2'' - \mathbf{r}_1''. \end{aligned} \quad (3.13)$$

If we assume that the source is quasi-homogeneous and has a normalized spectrum that is independent of position, i.e.

$$W_Q(\mathbf{R}'', \mathbf{r}'', \omega) = v_{coh} s_0(\omega) I_Q(\mathbf{R}'') \delta(\mathbf{r}'') \quad (3.14)$$

where $I_Q(\mathbf{R}'')$ is the intensity distribution, v_{coh} is the coherence volume (which is formally infinite) and $s_0(\omega)$ is the normalized spectrum then eq.(3.11) can be re-written as follows

$$W(R\mathbf{s}, \mathbf{b}, \omega) \approx \frac{\exp(i k \mathbf{s} \cdot \mathbf{b})}{R^2} v_{coh} s_0(\omega) \times \int d^3 R'' I_Q(\mathbf{R}'') \exp \left(\frac{-i k \mathbf{b}_\perp \cdot \mathbf{R}''}{R} \right) \quad (3.15)$$

If we assume that the source is a spherically symmetric Gaussian function, viz.,

$$I_Q(\mathbf{R}'') = I_0 \exp \left(-\frac{R''^2}{2a^2} \right), \quad (3.16)$$

where a can be considered to be the effective radius of the source, then we obtain the following expression for the cross-spectral density function of the field in the far zone of the source:

$$\mathcal{W}(Rs, \mathbf{b}, \omega) \approx \frac{v_{coh} V_0 I_0}{R^2} s_0(\omega) \exp(i\mathbf{k}s \cdot \mathbf{b}) \exp\left(-\frac{a^2 k^2 b_{\perp}^2}{2R^2}\right) \quad (3.17)$$

where $V_0 = (2\pi)^{3/2} a^3$ is the volume of the source. We will treat the baffle shown in fig.2, which represents the blocking effect of the dust torus, with a purely geometric approximation. Thus the cross-spectral density function for the field incident upon the scatterer is given by

$$\mathcal{W}_0(Rs, \mathbf{b}, \omega) = \begin{cases} \frac{v_{coh} V_0 I_0}{R^2} s_0(\omega) \exp(i\mathbf{k}s \cdot \mathbf{b}) \exp\left(-\frac{a^2 k^2 b_{\perp}^2}{2R^2}\right), & \text{lit region} \\ 0, & \text{shadow region.} \end{cases} \quad (3.18)$$

3.3 The Spectral Scattering Kernel

We will introduce a model to describe the correlation properties of the scattering medium. Let us assume that we can approximate the cross spectral density of the scatterer by the following formula:

$$W_{\eta}(\mathbf{r}'_1, \mathbf{r}'_2, \omega) \approx \rho(R) \tilde{g}(\mathbf{b}, \mathbf{s}, \omega), \quad (3.19)$$

where

$$\begin{aligned} \mathbf{R} &\equiv R\mathbf{s} = \frac{\mathbf{r}'_1 + \mathbf{r}'_2}{2} \\ \mathbf{b} &= \mathbf{r}'_2 - \mathbf{r}'_1. \end{aligned} \quad (3.20)$$

This model is somewhat analogous to the well-know quasi-homogeneous model, with the added wrinkle that the coherence properties can be dependent on orientation (as characterized by the unit vector $\mathbf{s}$). What I have in mind specifically is that the coherence length in the radial direction (i.e. along $\mathbf{s}$) is different from the coherence length in the transverse direction (i.e. perpendicular to $\mathbf{s}$).

Therefore the spectrum is

$$S^{(\infty)}(r\mathbf{u}, \omega) = \frac{1}{r^2} \int \int d^3 R d^3 b \int d\omega' \rho(R) \tilde{g}(\mathbf{b}, \omega) \mathcal{W}_0(\mathbf{R}, \mathbf{b}, \omega') \exp(i\mathbf{k}\mathbf{u} \cdot \mathbf{b}) \quad (3.21)$$

where On substituting from eq.(3.18) into eq.(3.21) we obtain the following expression for the spectrum of the scattered radiation

$$S^{(\infty)}(r\mathbf{u}, \omega) = \frac{1}{r^2} d\omega' \kappa(\mathbf{u}, \omega, \omega') s_0(\omega') \quad (3.22)$$

where the *spectral scattering kernel* $\kappa(\mathbf{u}, \omega, \omega')$ is given by the formula

$$\kappa(\mathbf{u}, \omega, \omega') \approx \kappa_0 \int_{\Omega_0} d^2 \Omega_s \int d^3 b \tilde{g}(\mathbf{b}, \omega - \omega') \exp\left(-\frac{k'^2 a^2 b_{\perp}^2}{2R_0^2}\right) \exp[i(\mathbf{k}'\mathbf{s} - \mathbf{k}\mathbf{u}) \cdot \mathbf{b}] \quad (3.23)$$

where $k' = \omega'/c$ and

$$\kappa_0 = v_{coh} V_0 I_0 \int_{R_0}^{\infty} dR \rho(R). \quad (3.24)$$

In these formulas, R_0 represents the distance from the source to the baffle and Ω_0 is the solid angle of the lit region (see fig.2).

3.4 The Scattering Medium

We shall assume that the scattering medium is anisotropic, in the sense that the correlation length in the radial direction (i.e. along the vector $\mathbf{s}$) is different from the correlation in the transverse direction (i.e. perpendicular to $\mathbf{s}$). For simplicity, we will assume a Gaussian model, i.e.

$$g(\mathbf{b}, \tau) = \exp \left[-\frac{1}{2} \left(\frac{b_{\perp}^2}{\sigma_{\perp}^2} + \frac{b_{\parallel}^2}{\sigma_{\parallel}^2} + \frac{\tau^2 c^2}{\sigma_{\tau}^2} \right) \right] \quad (3.25)$$

Substituting this formula into eq.(3.23), we obtain the following formula for the scattering kernel

$$\begin{aligned} \kappa(\mathbf{u}, \omega, \omega') &\approx \kappa_0 \frac{\sigma_{\tau}}{c\sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{\sigma_{\tau}^2}{c^2} (\omega - \omega')^2 \right] \\ &\times \int_{\Omega_0} d^2\Omega_s \int d^3b \exp \left[-\frac{1}{2} \left(\frac{b_{\perp}^2}{\sigma_{\perp}^2} + \frac{b_{\parallel}^2}{\sigma_{\parallel}^2} \right) \right] \exp(i\mathbf{f} \cdot \mathbf{b}) \end{aligned} \quad (3.26)$$

$$(3.27)$$

where

$$\mathbf{f} = k'\mathbf{s} - k\mathbf{u} \quad (3.28)$$

and

$$\frac{1}{\sigma_{\perp}^2} = \frac{1}{\sigma_{\perp}^2} + \frac{a^2 k'^2}{R^2} \approx \frac{1}{\sigma_{\perp}^2}. \quad (3.29)$$

The approximation made in the last formula is valid provided that the traverse coherence area of the incident light is much larger than that of the scattering medium. Given that the propagation distance R is of the order on tens or even hundreds of parsecs, this is not an unreasonable assumption. Performing the integration over $\mathbf{b}$ we obtain the following formula for κ :

$$\begin{aligned} \kappa(\mathbf{u}, \omega, \omega') &\approx \kappa_0 \frac{2\pi\sigma_{\tau}\sigma_{\perp}^2\sigma_{\parallel}}{c} \exp \left[-\frac{1}{2} \frac{\sigma_{\tau}^2}{c^2} (\omega - \omega')^2 \right] \\ &\times \int_{\Omega_0} d^2\Omega_s \exp \left(-\frac{1}{2} \left[\sigma_{\perp}^2 k^2 u_{\perp}^2 + \sigma_{\parallel}^2 (k' - k u_{\parallel})^2 \right] \right) \\ &= \kappa_0 \frac{(2\pi)^2 \sigma_{\tau} \sigma_{\perp}^2 \sigma_{\parallel}}{c} \exp \left[-\frac{1}{2} \frac{\sigma_{\tau}^2}{c^2} (\omega - \omega')^2 \right] \\ &\times \int_{\cos\theta_0}^1 dx \exp \left(-\frac{1}{2} \left[\sigma_{\perp}^2 k^2 (1 - x^2 \cos^2 \theta') \right. \right. \\ &\left. \left. + \sigma_{\parallel}^2 (k' - kx \cos \theta')^2 \right] \right), \end{aligned} \quad (3.30)$$

where θ' is the scattering angle, as shown in fig.2. The integral in eq.(3.30) can be evaluated approximately using the following formula:

$$\begin{aligned} \int_{\cos\theta_0}^1 f(x) dx &\approx (1 - \cos\theta_0) f \left(\frac{1 + \cos\theta_0}{2} \right) \\ &= \frac{\Omega_0}{2\pi} f \left(1 - \frac{\Omega_0}{4\pi} \right). \end{aligned} \quad (3.31)$$

Then we obtain the following

$$\kappa(\mathbf{u}, \omega, \omega') \approx \kappa_0 \frac{2\pi\sigma_\tau\sigma_\perp^2\sigma_\parallel\Omega_0}{c} \exp\left(-\frac{1}{2}[\alpha'\omega'^2 - 2\beta\omega\omega' + \alpha\omega^2]\right) \quad (3.32)$$

where

$$\begin{aligned} \alpha &= \frac{1}{c^2} [\sigma_\tau^2 + \sigma_\perp^2 + q^2(\sigma_\parallel^2 - \sigma_\perp^2)] \\ \beta &= \frac{1}{c^2} [\sigma_\tau^2 + q\sigma_\parallel^2] \\ \alpha' &= \frac{1}{c^2} [\sigma_\tau^2 + \sigma_\parallel^2] \\ q &= \left(1 - \frac{\Omega_0}{4\pi}\right) \cos \theta' \end{aligned} \quad (3.33)$$

3.5 The z-number of the scattered radiation

The scattering kernel given in eq.(3.32) is equivalent to one we have already investigated in a previous paper [see ref.[12], eq.(12)]. In that paper, we showed that such a spectral scattering kernel causes Doppler-like spectral shifts with z -numbers given by the formula:

$$z = \frac{\alpha}{|\beta|} - 1. \quad (3.34)$$

Substituting the expressions for α and β from eq.(3.33), we obtain the following result:

$$z = \frac{1-q}{\sigma_\tau^2 + \sigma_\parallel^2 q} [\sigma_\perp^2 + q(\sigma_\perp^2 - \sigma_\parallel^2)] \quad (3.35)$$

Let us assume a 'white noise limit' for the scattering medium: i.e. its time fluctuations are effectively uncorrelated: $\sigma_\tau \rightarrow 0$. Then the formula for the spectral shift of the scattered radiation is:

$$z(\varepsilon, q) = \frac{\varepsilon}{q} + (1 - \varepsilon)q - 1, \quad (3.36)$$

where

$$\varepsilon = \left(\frac{\sigma_\perp}{\sigma_\parallel}\right)^2 \quad (3.37)$$

$$q = \left(1 - \frac{\Omega_0}{4\pi}\right) \cos \theta'. \quad (3.38)$$

This formula is our final result. It demonstrates that, for the specific model of active galactic nucleus we are considering, the spectral shift observed will be dependent on the viewing angle θ_0 , the solid angle of the radiation cone Ω_0 and the statistical anisotropy of the scatterer, as represented by the parameter ε . When $\varepsilon = 1$, the scattering medium is statistically isotropic: the coherence lengths are the same in all directions. In the limit $\varepsilon \rightarrow 0$ the longitudinal coherence length is much greater than the transverse coherence length, i.e. one has a cigar shaped coherence volume. In the limit $\varepsilon \rightarrow \infty$, one has the opposite case: the longitudinal coherence length is much smaller than the transverse coherence length and the coherence volume is like a pancake. Note that $z(\varepsilon, q) > 0$ provided that $\varepsilon > 0.5$, so that one will have a preference redshifts over a large area of the (ε, q) plane.

4 Discussion and Conclusions

Our results, derived in the last section, demonstrate the plausibility of the existence of extra galactic objects with discordant redshifts. One can easily spot potential problems with the theory presented here. For example, we have been deliberately vague about the underlying physical nature of the scattering medium, other than specifying its anisotropic coherence properties. The scatterer, which is assumed to have a 'white noise' power spectrum (implying that its fluctuations are very energetic), is situated further away from the central engine than the line emitting clouds (which cannot be too hot, otherwise they would have completely ionized, making line radiation impossible). Although our results apply for a broad spectral range, we have not considered the shifts of absorption lines or of the 21cm radio line. We have ignored the unscattered radiation and the efficiency of the scattering process (although it is possible a stimulated version of the spontaneous scattering process considered here is applicable). Also the issue of spectral linewidths has not been addressed. It should be emphasized that our theory is based entirely on established principles of optics: no hypothetical new physics was introduced. Furthermore certain aspects of the underlying phenomenon, the Wolf effect, have been extensively tested in the laboratory. The formula (3.36) gives the relationship between the z -number of the observed radiation and certain variables dependent on the object, namely the angle of vision, the degree of columnation due to the dust torus, and the degree of anisotropy of the scatterer. As it may be possible to measure some of these quantities for certain objects, it therefore may be possible to test these predictions.

Acknowledgments

This paper, like the others in this special issue, is presented in honor of Prof. Emil Wolf on the occasion of his 75th birthday. It was my great good fortune to be Prof. Wolf's research student between 1987 and 1992, and I learned a tremendous amount from him. His enormous knowledge of all aspects of optical physics and his care and attentiveness as a mentor are outstanding. A lot of the ideas presented in this paper originate from his work.

Figure Captions

Fig.1: A schematic illustration of the model AGN under consideration.

Fig. 2: A simplified version of Fig.1, for mathematical analysis, showing the symbols used.

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Scattering Medium

Dust Torus

L i t C o n e

scattered light

Line emitting
clouds

“Central Engine”

Axis of
symmetry

Incoherent primary
source (representing
the line emitting clouds)