The Steering of a Satellite by Ion Propulsion

R. Fox

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I. INTRODUCTION:

One of the possible military applications for an ion propulsion system is the steering of reconnaissance satellites for extended periods of time. Two reasons for doing this are immediately apparent. First, the orbit could be perturbed in a random pre-programmed manner. Such perturbations, if sufficiently large, would greatly increase the difficulty of interception and destruction of the vehicle. Second, if the orbit could be made to rotate continuously in space at a rate of one revolution per day then a single vehicle could keep under surveillance a given area of interest. This turns out to be a much more difficult requirement in terms of required ion rocket performance than does the first requirement.

II. ORBIT DYNAMICS:

Consider now the perturbation problem. In order to define the problem we assume that the earth is spherical and that the satellite orbit is initially circular with an altitude of about 200 miles. Further, assume constant thrust per unit mass is applied by the propulsion system. Assume that at time $t = 0$ (and earlier times) the enemy makes the necessary observations for determining the orbit parameters and plans to attempt an intercept one period later ($t = \tau_0$). The problem is now to introduce a perturbation so that at $t = \tau_0$ the satellite vector position is given by

$$\underline{r}(\tau_0) = \underline{r}_0(\tau_0) + \underline{\Delta r} \quad (1)$$

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THE STEERING OF A SATELLITE BY ION PROPULSION

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$$\mathbf{r}(T_0) = \mathbf{r}_0 + \Delta \mathbf{r} \quad (1)$$

$r_0(\gamma)$ is the unperturbed position at $t = \gamma_0$. We now consider separately three distinct modes of application of the thrust vector:

1. Perpendicular to the orbital plane (transverse thrust)
2. Parallel to the orbital plane and perpendicular to the radius vector from the center of the earth (circumferential thrust)
3. Parallel to both the orbital plane and radius vector (radial thrust)

These three cases are shown schematically in Figures 1, 2, 3. Solution is achieved by an approximate method which assumes that the perturbations are small.

Consider first the case of transverse thrust. The equation for the vector $\underline{h}$ representing the angular momentum per unit mass is

$$\dot{\underline{h}} = \underline{r} \times \underline{f} \quad f = \text{force per unit mass} \quad (2)$$

In component form,

$$\begin{aligned} \dot{h}_x &= fr \sin \theta \\ \dot{h}_y &= fr \cos \theta \end{aligned} \quad (3)$$

For an observer at $x = r_0$, $y = z = 0$, variations in h_x result in tilting of the orbit about the y - axis and hence a miss distance after one revolution

$$\frac{\Delta z}{r_0} = \frac{\Delta h_x}{h_0} \quad (4)$$

Variations in h_y result in a tilting about the x - axis and hence a miss distance of zero. Only the direction of approach of the satellite to the interception point is perturbed and the perturbation is

$$\Delta \phi = \frac{\Delta h_y}{h_0} \quad (5)$$

as we shall see, the latter effect is certainly insignificant.

The projection of $\underline{h}$ in the x, y plane for constant positive f is shown in Figure 4 (a). If f is made negative for $\pi < \theta < 2\pi$, the result is shown in Figure 4 (b). Clearly, this mode of operation gives maximum Δh_x /turn.

We have for one revolution,

$$\Delta h_x = fr \gamma_o \sin \theta \quad 0 < \theta < \pi \quad (6)$$

$$= \frac{2}{\pi} fr \gamma_o .$$

Therefore, since $r(t) \simeq r_o(0)$,

$$\frac{\Delta z}{r_o} = \frac{\Delta h_x}{h_o} = \frac{2}{\pi} \frac{fr_o \gamma_o}{r_o v_o} = 4f/g_o . \quad (7)$$

g_o = gravitational attraction at $r = r_o$

Now consider the case of circumferential thrust. Since the thrust is applied parallel to the velocity vector a small change in orbit energy will occur, appreciable changes in both γ , $r(\gamma_o)$ will occur.

From the angular momentum equation we have,

$$\dot{h} = fr$$

so

$$h(t) = h_o + frt \quad (8)$$

We can estimate the time T required for one revolution as follows:

$$T = \bar{\gamma}(t) \quad \text{where } \gamma = \text{instantaneous period}$$

Now

$$\frac{\gamma}{\gamma_o} = \frac{h^3}{h_o^3} \quad (9)$$

Therefore from (8), (9)

$$\frac{\bar{\gamma}(t)}{\gamma_o} = \frac{1}{\gamma_o h_o^3} \int_0^{\gamma_o} h^3(t) dt \simeq 1 + 3/2 \frac{fr \gamma_o}{h_o} = 1 + 3\pi f/g_o \quad (10)$$

Thus

$$\frac{T - \gamma_o}{\gamma_o} = 3\pi f/g_o \quad (11)$$

The corresponding position perturbation is

$$\frac{\Delta y}{r_o} = 2\pi \frac{(T - \gamma_o)}{\gamma_o} = 6\pi^2 f/g_o . \quad (12)$$

The instantaneous period is

$$\frac{\tau(\tau_o)}{\tau_o} = \frac{h^3(\tau_o)}{h_o^3} = \left(\frac{r}{r_o}\right)^{3/2} \quad (13)$$

Therefore

$$\frac{\Delta r}{r_o} = \frac{2fr_o\tau_o}{h_o^3} = 4\pi f/g_o \quad (14)$$

Finally, we consider the case of radial thrust. H.S. Tsien⁽¹⁾ has given an exact integral for the equation of motion in the form

$$t = \frac{\tau_o}{2\pi} \int_0^p \frac{dp}{(p-1)(2\mu p^2 + p + 1)} , \quad p = \frac{r}{r_o} , \quad \mu = f/g_o \quad (15)$$

Let

$$p = 1 + \delta$$

and assume

$$\delta \ll 1 .$$

Then, to 1st order the integral is

$$\frac{\pi(1+\mu)}{2\sqrt{1-2\mu}} = \sqrt{\delta(2\mu-\delta)} + \frac{(1+\mu)}{\sqrt{1-2\mu}} \sin^{-1} \left[\left(\frac{\delta}{\mu} - 1 \right) - 2\delta \right] . \quad (16)$$

After one period $t \approx \tau_o$ this expression must have the value 2π . For $\mu \ll 1$ this can only be the case if

$$\frac{\Delta r}{r_o} = \delta \ll \mu = f/g_o . \quad (17)$$

In fact, even after a very long time $t \gg \tau_o$, we find that the satellite reaches an equilibrium orbit characterized by the force equilibrium

$$g_o \left(\frac{r_o}{r} \right)^2 = f + \frac{v^2}{r} \quad (18)$$

also,

$$vr = v_o r_o \quad \text{since } h = \text{constant} \quad (\underline{f} \times \underline{r} = 0) .$$

The resulting displacement is:

$$\left(\frac{\Delta r}{r_o} \right) \approx f/g_o \quad (19)$$

Consequently, radial thrust is of no interest.

Let us now summarize the results for the orbit perturbation achieved after one turn by the various modes of applying thrust. In order to illustrate the magnitudes involved, values are calculated for $f = 0.2 \text{ cm/sec}^2$ and $r_0 = 4200 \text{ miles}$.

<u>Mode of applying thrust</u>	<u>Displacement after one revolution</u>	<u>If $f=0.2 \text{ cm/sec}^2$, $r_0=4200 \text{ miles}$</u>
<u>Circumferential</u>	$\frac{\Delta y}{r_0} = 6\pi^2 f/g_0$	= 50 miles
	$\frac{\Delta r}{r_0} = 4\pi f/g_0$	= 10.6 "
<u>Transverse</u>		
$\left(\begin{array}{l} \text{thrust reversal at} \\ \theta = 0, \pi \end{array} \right)$	$\frac{\Delta z}{r_0} = 4 f/g_0$	= 3.36 "
$\left(\begin{array}{l} \text{thrust reversal at} \\ \theta = \pi/2, 3\pi/2 \end{array} \right)$	$\Delta \phi = 4 f/g_0$	= 0.46°
<u>Radial</u>	$\frac{\Delta r}{r_0} \ll f/g_0$	$\ll .84 \text{ miles}$

III. CHARACTERISTICS OF THE ION PROPULSION SYSTEM:

Now it is necessary to relate these results to the specific power capabilities of an ion rocket. In this way we can assess the technological feasibility of steerable satellites. It has been shown⁽²⁾ that the following relation exists for an ion rocket:

$$\gamma_a \geq \frac{(\Delta V)^2 \text{ eff}}{2K} \frac{\mu (1+\delta)}{(\mu-1)} \quad (20)$$

$$\gamma_a = \text{total acceleration time}$$

$$K = \text{specific power, } \frac{\text{exhaust power}}{\text{total propulsion plant mass}}$$

$$(\Delta V) \text{ eff} = \text{effective velocity change} = f \gamma_a$$

$$\mu = \text{ion rocket mass ratio, } \frac{\text{initial total mass}}{\text{"burnout" mass}}$$

$$\delta = \frac{\text{payload mass}}{\text{propulsion equipment mass}}$$

The equality sign applies for the "optimum" exhaust velocity program

$$v_{ex}(t) = \frac{(\Delta V)_{eff}}{\mu - 1} \frac{M_{initial}}{M_{initial} - m(t)} \quad (21)$$

$m(t)$ = propellant consumed in time interval $0 < t' < t$

Since $(\Delta V)_{eff} = f \gamma_a$,

$$\gamma_a \leq \frac{2K}{f^2} \frac{\mu - 1}{\mu(1 + \delta)} \quad (22)$$

This expression relates the "lifetime" γ_a to the system parameters.

Note that

$$\frac{\text{payload mass}}{\text{initial total mass}} = \frac{\delta}{\mu - (1 + \delta)} = h$$

Thus, (22) becomes

$$\gamma_a \leq \frac{2K}{f^2} \frac{(\mu - 1)(1 - h\mu)}{\mu} \quad (23)$$

If we want to maximize the lifetime γ_a subject to the requirement that h , the ratio of payload mass to initial mass is fixed, then we must have

$$\mu_{opt}^2 = 1/h$$

so

$$\gamma_a \leq \frac{2K}{f^2} h \left(\sqrt{\frac{1}{h}} - 1 \right)^2 \quad (24)$$

We may now combine (12), (24) in order to relate lifetime, specific power, and payload. We have:

$$\gamma_a \leq 72\pi^4 K \left(\frac{r_o}{\Delta y g_o} \right)^2 h \left(\sqrt{\frac{1}{h}} - 1 \right)^2 \quad (25)$$

Results are given in Figure 5 for $h = 0$ (no payload), $r_o = 4200$ miles. The dependence of γ_a upon the payload term is shown in Figure 6. We see that the lifetime is strongly dependent on the payload fraction h when K , Δy are fixed. One could alternatively inquire about the sensitivity of Δy to the payload when γ_a , K are fixed. The result is shown in Figure 7. Clearly,

$$\frac{\Delta y}{(\Delta y)_{h=0}} = \sqrt{\frac{\gamma_a}{(\gamma_a)_{h=0}}} \quad (26)$$

IV. CONCLUDING REMARKS

The effect of an orbit perturbation upon the interception probability depends upon the detailed nature of the observation system used for obtaining orbit parameters and the intercept vehicle performance. Suppose final fixes can be made when the satellite first passes over the boundary of a country and utilized for final adjustment of firing time and orientation of the intercept vehicle within the next few minutes. In this case, perturbations like 50 miles per turn are probably not very significant. If firing times must be computed from the previous pass must be used, then a significant reduction in intercept probability may result unless the terminal guidance capabilities are considerably augmented. Note that a 50 mile perturbation corresponds to an angular deviation of about 15° as seen from the earth's surface (200 mile orbit). It is clear that perturbations like several hundred miles per turn together with reasonable propulsion system lifetime cannot be achieved without a specific power considerably in excess of 0.1 k w/k g .

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- (1) H. S. Tsien, "Take-off From Satellite Orbit", Journal of the American Rocket Society", 23, 233, July - August 1953.
- (2) R. H. Fox, (" The Payload Capabilities of Ion Propulsion Rocket System") UCRL - 5199. (to be issued)

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Figure 1

Perturbation of an Orbit by
Transverse Thrust

Figure 2 Perturbation of an orbit by
Circumferential Thrust

Figure 3 Perturbation of an Orbit by
Radial Thrust

Figure 4 Projection on the $x-y$ plane of the precessing $\underline{L}$ resulting from transverse thrust

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(a) Continuous thrust parallel to z -axis

(b) Thrust reversed when $\pi < \theta < 2\pi$

Figure 5 Relationship between specific power, orbit perturbation per turn, and propulsion plant lifetime. Lifetime is determined by fuel supply. No payload

$(\Delta y)_{turn}$ miles

Figure 6 Effect of inclusion of payload upon propulsion plant lifetime. Perturbation per turn is constant.

$$h = \frac{\text{payload mass}}{\text{total initial mass}}$$

$$\frac{\tau_a}{(\tau_a)_{h=0}} = h \left(\frac{1}{\sqrt{h}} - 1 \right)^2$$

Figure 7 Effect of inclusion of payload upon orbit perturbation per turn. Propulsion plant lifetime is held constant.

$$h = \frac{\text{payload mass}}{\text{total initial mass}}$$

$$\frac{\Delta y}{(\Delta y)_{L=0}} = 1 - \frac{1}{\sqrt{2}}$$