Energy Technology Division

by V. L. Shah

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Energy Technology Division

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**ESTIMATION OF MAXIMUM TEMPERATURE IN A PACKAGE
SUBJECTED TO HYPOTHETICAL-ACCIDENT
THERMAL-TEST CONDITIONS**

by

V. L. Shah

Energy Technology Division
Transportation of Hazardous Materials Section

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ESTIMATION OF MAXIMUM TEMPERATURE IN A PACKAGE SUBJECTED TO HYPOTHETICAL-ACCIDENT THERMAL-TEST CONDITIONS

by

V. L. Shah

ABSTRACT

A simple reading-the-graph procedure has been developed to provide a quick estimate of maximum temperature in a package during hypothetical-accident thermal-test conditions. The procedure, based on applying the analytical solutions of a transient thermal-diffusion equation for temperatures in a semi-infinite solid and in an infinitely long circular cylinder subjected to step surface temperature boundary conditions, is applicable to all packages, with or without an internal heat source, that are protected by rectangular or cylindrical thermal insulating overpacks. Simple to follow, the procedure provides a conservative estimate of package maximum temperatures during hypothetical-accident thermal-test conditions. To demonstrate its applicability, the procedure is applied to the Mound 9859 tritium trap package and two values of the thermal diffusivity of the insulation in the overpack.

1 INTRODUCTION

To ensure that a package for the transportation of radioactive materials is designed adequately to meet the requirements specified in the Code of Federal Regulations, Title 10, Part 71 (10 CFR §71), we must know the maximum temperatures that the package may attain during normal transport and under hypothetical-accident thermal-test conditions. The test conditions for a hypothetical-accident thermal event, specified in 10 CFR §71.73, are

- Expose the whole package to a radiation environment at 800°C (1475°F) for 30 min, and
- Maintain the ambient temperature at 38°C (100°F) before and after the 30-min test.

To determine package temperature during a hypothetical-accident thermal event, designers and reviewers generally perform detailed analyses with computer codes. These detailed analyses and the required solution of thermal-diffusion equations are time-consuming and labor-intensive. Some simple conservative estimating procedures are therefore preferred because the very detailed and time-consuming numerical computation can be avoided if the results of the simple conservative estimation can show that a package meets the requirements of the Federal Code.

In this report, a simple procedure is presented that provides a quick and conservative estimation of package maximum temperatures $T_{\max}(x)$ at a distance x from the surface of the package under hypothetical-accident thermal-test conditions. The procedure, which is applicable to all packages, with or without an internal heat source, that are protected by rectangular or cylindrical thermal insulating overpacks, can be used to estimate package maximum temperatures before commencing a very lengthy and time-consuming computer analysis. In addition, the method can be used to check the results of a detailed numerical solution to ensure that no major modeling or input errors have been made in the analysis performed with a computer code.

The estimating procedure, described in Section 2, is based on applying the analytical solutions for temperature in a semi-infinite solid and in an infinitely long circular cylinder subjected to step surface temperature boundary conditions. These analytical solutions are presented in Sections 3 and 4, respectively.

To demonstrate the applicability of the procedure, the Mound 9859 tritium trap package and two values of thermal diffusivity of the insulation in the overpack are presented as examples in Section 5. The maximum temperatures estimated by the procedure are compared with those obtained by numerical simulations of a very detailed computer model of the package.

The analytical solution for temperature in an infinitely long, internal-heat-source-containing circular cylinder that has been subjected to step surface temperature boundary conditions is described in Section 6. In Section 7, the results of this analysis and other qualitative examples are used to show that the proposed method will provide a conservative estimate. Concluding remarks are presented in Section 8.

2 ESTIMATING PROCEDURE

The estimating procedure presented here is applicable to all packages with plane or cylindrical surface overpacks, as shown in Figs. 1(a) and 1(b), respectively.

Fig. 1. Package with (a) plane and (b) cylindrical surface overpacks

2.1 PLANE SURFACE

The procedure for estimating maximum temperature at location A in Fig. 1a at distance L from a plane surface of the overpack is

- Calculate the parameter L^2/α (s), where L (in.) is the distance from the surface of the overpack and α (in.²/s) is the thermal diffusivity of the overpack insulation.
- Read the maximum temperature $T_{A,\text{max-analytic},q=0}$ from a plot of the maximum temperature vs. the distance from the surface squared/thermal diffusivity (presented in Section 3.4).
- Calculate the rise in temperature

$$dT_{A,q\text{-steady}} = T_{A,t=0} - 100 \text{ (}^\circ\text{F)}$$

from the initial steady-state temperature distribution $T_{x,t=0}$.

- Estimate the maximum temperature from

$$T_{A,\text{max-estimation}} = T_{A,\text{max-analytic},q=0} + dT_{A,q\text{-steady}}.$$

2.2 CYLINDRICAL SURFACE

The procedure for estimating maximum temperature at location A, at a radial distance L from a cylindrical surface, in Fig. 1b is

- Calculate the parameters r^* ($= r/R = 1 - [L/R]$) and R^2/α (s), where L (in.) is the radial distance from the outer surface, R (in.) is the outer radius and α (in.²/s) is the thermal diffusivity of the overpack insulation.
- Read the maximum temperature $T_{A,\max\text{-analytic},q=0}$, from a plot of the maximum temperature vs. the outer radius squared/thermal diffusivity (presented in Section 4.4).
- Calculate the rise in temperature

$$dT_{A,q\text{-steady}} = T_{A,t=0} - 100 \text{ (}^\circ\text{F)}$$

from the initial steady-state temperature distribution $T_{r,t=0}$.

- Estimate the maximum temperature from

$$T_{A,\max\text{-estimation}} = T_{A,\max\text{-analytic},q=0} + dT_{A,q\text{-steady}}.$$

3 TEMPERATURE IN A SEMI-INFINITE SOLID

The analysis for temperature in a semi-infinite solid, without an internal heat source but with step changes in the surface temperature, is presented here.

3.1 GOVERNING EQUATION

The governing equation for temperature in a semi-infinite solid without an internal heat source is

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{\alpha} \frac{\partial T}{\partial t} = 0, \quad (1)$$

where, T, x, α , and t are the temperature, distance from the surface, thermal diffusivity, and time, respectively.

3.2 SOLUTION

The one-dimensional analytical solution of Eq. 1 for a semi-infinite solid, initially at temperature T_0 and subjected to step surface temperatures T_{s1} for a time $< t_1$, and T_{s2} for a time $> t_1$, i.e., is shown in Fig. 2 and given by Eq. 3 in Section 2.5 of Carslaw and Jaeger,² is

$$\frac{(T - T_0)}{(T_{s1} - T_0)} = \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t}}\right) \quad (2)$$

for $t \leq t_1$ and

$$\frac{(T - T_0)}{(T_{s1} - T_0)} = \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t}}\right) + \frac{(T_{s2} - T_{s1})}{(T_{s1} - T_0)} \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t-t_1}}\right) \quad (3)$$

for $t > t_1$. Here, $\operatorname{erfc}(x)$ is the complimentary error function, defined as

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt. \quad (4)$$

Fig. 2. Graphical representation of step surface temperature boundary condition

3.3 HYPOTHETICAL-ACCIDENT CONDITIONS

The solutions of Eqs. 2 and 3 for the hypothetical-accident surface temperature boundary conditions shown in Fig. 3 are

$$\frac{(T - 100)}{(1375)} = \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t}}\right) \quad (5)$$

for $t \leq 1800$ s (30 min) and

$$\frac{(T - 100)}{(1375)} = \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t}}\right) - \operatorname{erfc}\left(\frac{\sqrt{x^2/\alpha}}{2\sqrt{t - 1800}}\right) \quad (6)$$

for $t > 1800$ s (30 min). The temperatures at time = 30 min (1800 s), calculated from Eq. 5 for several different values of the parameter x^2/α , are presented in Fig. 4 and Table 1.

Fig. 3. Step surface temperature condition to represent a hypothetical-accident thermal test

Fig. 4. Temperature at time = 30 min in a semi-infinite solid subjected to 1475°F step temperature change at the surface

Table 1. Temperature in a semi-infinite solid at time = 30 min
for various values of x^2/α

x^2/α (s)	$x/\sqrt{4\alpha t}$	$\text{erfc}(x/\sqrt{4\alpha t})$	Temperature (°F)
72	0.1	0.887537	1320.0
288	0.2	0.777297	1169.0
648	0.3	0.671373	1023.0
1152	0.4	0.571608	886.00
1800	0.5	0.479500	759.30
2592	0.6	0.396144	644.70
3528	0.7	0.322199	543.00
4608	0.8	0.257899	454.60
5832	0.9	0.203092	379.30
7200	1.0	0.157299	316.30
8712	1.1	0.119795	264.70
10368	1.2	0.089686	223.30
12168	1.3	0.065992	190.70
14112	1.4	0.047715	165.60
16200	1.5	0.033895	146.60
18432	1.6	0.023652	132.50
20808	1.7	0.016210	122.30
23328	1.8	0.010909	115.00
25992	1.9	0.007210	109.91
28800	2.0	0.004678	106.43
31752	2.1	0.002979	104.10
34848	2.2	0.001863	102.56
38088	2.3	0.001143	101.57
41472	2.4	0.000689	100.95
45000	2.5	0.000407	100.56
48672	2.6	0.000236	100.32
52488	2.7	0.000134	100.18
56448	2.8	0.000075	100.10
60552	2.9	0.000041	100.06
64800	3.0	0.000022	100.03

3.4 MAXIMUM TEMPERATURE

The maximum temperature in a semi-infinite solid will occur when $\partial T / \partial t = 0$, or

$$\frac{\partial}{\partial t} \left[\operatorname{erfc} \left(\frac{\sqrt{x^2 / \alpha}}{2\sqrt{t}} \right) \right] - \frac{\partial}{\partial t} \left[\operatorname{erfc} \left(\frac{\sqrt{x^2 / \alpha}}{2\sqrt{t-1800}} \right) \right] = 0, \quad (7)$$

$$t_{T_{\max}}^{3/2} \exp \left(\frac{-x^2 / \alpha}{4t_{T_{\max}}} \right) = (t_{T_{\max}} - 1800)^{3/2} \exp \left(\frac{-x^2 / \alpha}{4(t_{T_{\max}} - 1800)} \right). \quad (8)$$

Here, $t_{T_{\max}}$ is the time when the temperature will be maximum. Equation 8 has been solved iteratively to obtain $t_{T_{\max}}$ for several different values of the parameter x^2 / α . The maximum temperature can then be obtained by substituting $t = t_{T_{\max}}$ in the solution of diffusion Eq. 6.

$$\frac{(T_{\max} - 100)}{(1375)} = \operatorname{erfc} \left(\frac{\sqrt{x^2 / \alpha}}{2\sqrt{t_{T_{\max}}}} \right) - \operatorname{erfc} \left(\frac{\sqrt{x^2 / \alpha}}{2\sqrt{t_{T_{\max}} - 1800}} \right). \quad (9)$$

The maximum temperatures, calculated from Eq. 9 for several values of x^2 / α , are presented in Fig. 5 and Table 2.

Fig. 5. Maximum temperature and time to attain maximum temperature in a semi-infinite solid subjected to 1475°F step temperature change at the surface

Table 2. Maximum temperature and time to attain maximum temperature for various values of x^2/α

x^2/α (s)	$t_{T_{\max}}$ (s)	$t_{T_{\max}}$ (min)	$T_{\max}$ (°F)
600	1822	30.37	1041
800	1832	30.53	981
1,000	1843	30.72	928
1,200	1855	30.92	882
1,400	1868	31.13	841
1,600	1881	31.35	805
1,800	1895	31.58	772
2,000	1910	31.83	742
2,500	1950	32.50	677
3,000	1994	33.23	624
3,500	2041	34.02	578
4,000	2091	34.85	539
4,500	2144	35.73	506
5,000	2200	36.67	476
5,500	2258	37.63	452
6,000	2319	38.65	429
7,000	2445	40.75	391
8,000	2577	42.95	361
9,000	2715	45.25	336
10,000	2856	47.60	314
11,000	3001	50.02	297
12,000	3149	52.48	282
13,000	3300	55.00	269
14,000	3452	57.53	258
15,000	3606	60.10	248
16,000	3761	62.68	240
17,000	3918	65.30	232
18,000	4075	67.92	226
19,000	4234	70.57	219
20,000	4393	73.22	213

4 TEMPERATURE IN AN INFINITE CIRCULAR CYLINDER ($Q = 0$)

The analysis of temperature in an infinite circular cylinder of radius R , with no internal heat source and step change in the surface temperature, is presented here.

4.1 GOVERNING EQUATION

The governing equation for temperature in an infinite circular cylinder is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{1}{\alpha} \frac{\partial T}{\partial t} = 0. \quad (10)$$

Where T , r ($= R - x$), x , α , and t are the temperature, radius, radial distance from the surface, thermal diffusivity, and time, respectively.

4.2 SOLUTION

The one-dimensional analytical solution of Eq. 10 for an infinite circular cylinder, initially at temperature T_0 and subjected to step surface temperatures T_{s1} for a time $\leq t_1$, and T_{s2} for a time $> t_1$, i.e.,

$$T = T_0 \quad \text{for } t = 0, \quad (11)$$

$$T_{r*=1} = T_{s1} \quad \text{for } 0 < t \leq t_1,$$

$$T_{r*=1} = T_{s2}; \quad \text{for } t > t_1, \quad (12)$$

as given by Eq. 8 in Section 7.6 of Carslaw and Jaeger,² is

$$\frac{(T - T_0)}{(T_{s1} - T_0)} = 1 - 2 \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}, \quad (13)$$

for $t \leq t_1$, and

$$\frac{(T - T_0)}{(T_{s1} - T_0)} = 1 - 2 \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}$$

$$+ \frac{(T_{s2} - T_{s1})}{(T_{s1} - T_0)} \left(1 - 2 \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha (t-t_1)}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)} \right) \quad (14)$$

for $t > t_1$. In Eqs. 13 and 14, R , r^* ($= r/R$), and $J_n(\beta)$ are the radius of the cylinder, dimensionless radial distance, and Bessel function of the first kind of order n , respectively, and $\pm(\beta_n)$, $n = 1, 2, \dots$, are the roots of the Bessel function $J_0(\beta) = 0$. Let

$$\psi_n = \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}, \quad (15)$$

and

$$\psi'_n = \exp\left(\frac{-\beta_n^2 \alpha (t-t_1)}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}. \quad (16)$$

Eqs. 15 and 16 simplify to

$$T = T_{s1} - 2(T_{s1} - T_0) \sum_{n=1}^{\infty} \psi_n \quad (17)$$

for $t \leq t_1$ and

$$T = T_{s2} - 2(T_{s1} - T_0) \sum_{n=1}^{\infty} \psi_n - 2(T_{s2} - T_{s1}) \sum_{n=1}^{\infty} \psi'_n \quad (18)$$

for $t > t_1$.

4.3 HYPOTHETICAL-ACCIDENT CONDITIONS

The solution of Eq. 10 for surface temperature conditions during the hypothetical-accident shown in Fig. 2, i.e., $T = T_0 = 100^\circ\text{F}$, for $t = 0$, $T_{r^*=1} = T_{s1} = 1475^\circ\text{F}$ for $0 < t \leq 1800$ s, and $T_{r^*=1} = T_{s2} = 100^\circ\text{F}$ for $t > 1800$ s is

$$T = 1475 - 2(1375) \sum_{n=1}^{\infty} \psi_n \quad (19)$$

for $t \leq 1800$ s (30 min) and

$$T = 100 - 2(1375) \sum_{n=1}^{\infty} \psi_n + 2(1375) \sum_{n=1}^{\infty} \psi'_n \quad (20)$$

for $t > 1800$ s (30 min).

The temperatures at time = 30 min (1800 s), calculated from Eq. 19 for several values of the parameters R^2/α and r^* ($= r/R$), are presented in Fig. 6 and Table 3.

Fig. 6. Temperature at time = 30 min in an infinite cylinder subjected to 1475°F step temperature change at the surface

4.4 MAXIMUM TEMPERATURE

The maximum temperature in a semi-infinite cylinder will occur when $\partial T/\partial t = 0$ or

$$\sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha (t_{T_{\max}} - 1800)}{R^2}\right) \frac{\beta_n \alpha J_0(r^* \beta_n)}{R^2 J_1(\beta_n)} = \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t_{T_{\max}}}{R^2}\right) \frac{\beta_n \alpha J_0(r^* \beta_n)}{R^2 J_1(\beta_n)}. \quad (21)$$

Here, $t_{T_{\max}}$ is the time when the temperature will be maximum. Equation 21 has been solved iteratively to obtain $t_{T_{\max}}$ for several values of the parameters r^* ($= r/R$) and R^2/α .

The maximum temperature can be obtained by substituting $t = t_{T_{\max}}$ in Eq. 20. The maximum temperature and time when the maximum temperature occurs have been calculated for several values of the parameters r^* ($= r/R$) and R^2/α . The results are presented in Figs. 7 and 8 and Tables 4 and 5.

Table 3. Temperature in an infinite cylinder at time = 30 min
for various values of R^2/α and r^* ($= r/R$)

R^2/α (s)	$\alpha t/R^2$	r^*				
		0.3	0.4	0.5	0.6	0.65
0	∞	1475	1475	1475	1475	1475
2250	0.8000	1456	1458	1461	1463	1465
3000	0.6000	1415	1421	1429	1438	1442
5000	0.3600	1235	1260	1291	1326	1344
18000	0.1000	424.7	516.9	635.9	779.6	859.4
24000	0.0750	290.7	376.4	495.1	648.0	736.7
30000	0.0600	214.9	287.7	397.0	548.8	641.1
36000	0.0500	170.5	229.4	326.1	471.3	563.9
40000	0.0450	151.3	201.6	289.5	428.4	520.1
50000	0.0360	123.5	156.3	223.2	344.3	430.9
60000	0.0300	111.0	131.7	181.2	283.7	363.3
72000	0.0250	104.5	116.1	149.8	231.8	301.9
120000	0.0150	100.1	101.2	107.6	137.3	174.2
180000	0.0100	100.0	100.0	100.8	108.3	122.8
360000	0.0050	100.0	100.0	100.0	100.0	100.8

R^2/α (s)	$\alpha t/R^2$	r^*				
		0.7	0.75	0.8	0.85	0.9
0	∞	1475	1475	1475	1475	1475
5000	0.3600	1363	1382	1401	1420	1439
18000	0.1000	943.4	1031	1120	1211	1301
30000	0.0600	743.6	855.2	974.1	1098	1225
40000	0.0450	626.9	748.0	881.7	1025	1175
50000	0.0360	536.8	662.1	805.2	963.1	1131
60000	0.0300	465.3	591.1	740.0	908.9	1093
72000	0.0250	397.2	520.7	673.0	851.7	1051
90000	0.0200	320.7	437.0	589.6	777.9	996.2
120000	0.0150	237.4	337.3	482.7	677.7	918.4
144000	0.0125	195.3	281.3	417.3	612.3	865.1
180000	0.0100	155.9	222.7	342.3	531.5	795.9
225000	0.0080	129.2	176.5	275.3	452.0	722.7
300000	0.0060	110.2	135.8	204.5	355.2	624.1
360000	0.0050	104.4	119.8	170.0	299.5	560.3

Fig. 7. Maximum temperature in an infinite cylinder subjected to 1475°F step temperature change at the surface

Fig. 8. Time to attain maximum temperature in an infinite cylinder subjected to 1475°F step temperature change at the surface

Table 4. Maximum temperature in an infinite cylinder
for various values of R^2/α and r^* ($= r/R$)

R^2/α (s)	r^*					
	0.3	0.36	0.4	0.5	0.6	0.65
0	1475.00	1475.00	1475.00	1475.00	1475.00	1475.00
2250	1457.59	1458.41	1459.07	1461.08	1463.54	1464.89
3000	1420.48	1422.89	1428.87	1431.03	1438.69	1442.96
5000	1263.00	1271.11	1278.10	1300.93	1330.44	1347.15
18000	603.08	619.58	637.02	707.13	815.42	882.67
24000	488.30	502.80	518.85	586.81	698.08	770.40
30000	415.02	427.62	441.95	504.78	612.45	685.42
36000	364.59	375.59	388.33	445.55	547.12	618.51
40000	338.95	349.06	360.87	414.53	511.59	581.26
50000	292.18	300.54	310.45	356.35	442.36	506.77
60000	260.62	267.71	276.19	315.95	392.23	451.14
72000	234.12	240.11	247.31	281.38	347.98	400.80
120000	180.72	184.38	188.82	210.14	253.19	289.05
180000	153.86	156.32	159.31	173.32	203.14	228.14
360000	126.95	128.18	129.68	136.95	151.89	164.75

	r^*				
	0.7	0.75	0.8	0.85	0.9
0	1475.00	1475.00	1475.00	1475.00	1475.00
5000	1466.31	1467.77	1469.24	1470.71	1471.83
18000	1447.42	1453.02	1456.72	1461.42	1465.72
30000	1364.80	1383.13	1401.83	1420.63	1438.92
40000	957.41	1038.30	1123.77	1212.02	1300.94
50000	771.84	871.19	981.88	1101.24	1225.62
60000	705.56	808.64	926.64	1056.77	1194.73
72000	667.76	772.12	893.71	1029.83	1175.80
90000	626.22	731.18	856.13	998.62	1153.65
120000	589.92	694.65	821.92	969.76	1133.92
144000	529.60	632.16	761.77	917.79	1094.96
180000	473.22	571.48	701.00	863.47	1054.26
225000	341.37	419.18	535.20	702.47	925.47
300000	265.61	324.15	418.97	573.32	809.14
360000	184.40	216.33	272.45	380.13	595.31

Table 5. Time (min) to attain maximum temperature in an infinite cylinder for various values of R^2/α and r^* ($= r/R$)

R^2/α (s)	r^*					
	0.3	0.36	0.4	0.5	0.6	0.65
0	30.00	30.00	30.00	30.00	30.00	30.00
2250	30.58	30.48	30.41	30.27	30.16	30.12
3000	30.95	30.78	30.67	30.43	30.25	30.18
5000	32.22	31.80	31.53	30.96	30.55	30.39
18000	46.23	42.84	40.63	36.11	33.16	32.16
24000	54.20	49.22	45.89	39.12	34.71	33.20
30000	62.53	55.94	51.47	42.41	36.44	34.38
36000	71.03	62.88	57.28	45.91	38.33	35.66
40000	76.78	67.57	61.22	48.34	39.66	36.57
50000	91.28	79.51	71.31	54.63	43.17	39.01
60000	105.93	91.61	81.57	61.14	46.92	41.64
72000	123.59	106.24	94.04	69.15	51.62	44.98
120000	194.71	165.39	144.63	102.19	71.61	59.58
180000	283.98	239.80	208.45	144.30	97.68	79.05
360000	552.24	463.64	400.67	271.79	177.60	139.55

R^2/α (s)	r^*				
	0.7	0.75	0.8	0.85	0.9
0	30.00	30.00	30.00	30.00	30.00
5000	30.08	30.05	30.03	30.02	30.01
18000	30.13	30.08	30.05	30.03	30.02
30000	30.26	30.17	30.10	30.05	30.03
40000	31.41	30.86	30.48	30.23	30.10
50000	32.83	31.70	30.93	30.44	30.18
60000	33.64	32.17	31.18	30.55	30.22
72000	34.22	32.52	31.36	30.63	30.25
90000	34.99	32.97	31.60	30.74	30.28
120000	35.79	33.44	31.84	30.85	30.31
144000	37.51	34.45	32.37	31.08	30.39
180000	39.71	35.77	33.06	31.38	30.48
225000	49.66	41.88	36.31	32.79	30.93
300000	63.37	50.70	41.20	34.95	31.59
360000	107.04	80.01	58.59	43.19	34.18

5 EXAMPLE

To illustrate the simplicity of the estimating procedure, the Mound 9859 tritium trap package is used as an example. To further show that the procedure provides a conservative prediction, the results obtained from the example are compared with results obtained with a computer solution of a very detailed numerical model. Although the example selected has no internal heat source, i.e., $T_{\text{max-estimation}} = T_{\text{max-analytic, } q=0}$, the qualitative arguments and mathematical analysis, presented later, will show that the estimating procedure is also applicable to a package with an internal heat source.

5.1 MOUND 9859 TRITIUM TRAP PACKAGE

The Mound 9859 tritium trap package, shown in Fig. 9a, is a cylindrical package designed to transport Type-B quantities of solid uranium tritide. The package is constructed to the requirements of the U.S. Department of Transportation Specification 6M for metal packaging. The trap provides the primary containment of the tritium. The inner vessel provides the secondary containment. The outer container is a stainless steel drum, 15 in. in diameter and 18 in. high, with lid, gasket, and bolted locking ring. The Celotex spacers between the drum and the inner container provide impact and thermal protection.

5.2 TEMPERATURE ESTIMATION

Two locations, shown in Fig. 9b, are selected as the positions on the surface of the inner vessel for estimating the maximum temperature during a hypothetical-accident thermal test. Location A, which requires the plane surface solution (presented in Section 3), is 3.1 in. away from the top plane surface. Location B, which requires the cylindrical surface solution (presented in Section 4), is 3.7 in. away from the outer cylindrical surface. The radius of the outer cylindrical surface is 5.83 in. The thermal diffusivity values of Celotex for the example are taken to be 3.77×10^{-4} and 1.17×10^{-3} (in.²/s). These two values, one very high and one very low, were selected to demonstrate that the method is applicable over a wide range of parameter values.

The geometrical and physical parameters, (x^2/α) for Location A, and (R^2/α) and $r^*(= r/R)$ for Location B, are calculated first. The maximum temperatures $T_{\text{max-estimation, } q=0}$ for these locations are then read directly from the constructed curves shown in Figs. 10 and 11. The estimated maximum temperatures $T_{\text{max-estimation, } q=0}$ are presented in Table 6.

Fig. 9. Mound 9859 tritium trap package
(a) actual, (b) computer model

Fig. 10. Maximum temperature at Location A in the Mound 9859 tritium trap package, estimated from Fig. 5

Fig. 11. Maximum temperature at Location B in the Mound 9859 tritium trap package, estimated from Fig. 7

Table 6. Maximum temperatures at Locations A and B in Fig. 9b

Location	Selected High and Low Thermal Diffusivity α (in. ² /s)	x^2/α (s) or R^2/α (s) ^a	Estimated $T_{\max}$ (°F) ^b	Predicted $T_{\max}$ (°F) ^c
A ^d	3.77×10^{-4}	2.55×10^4	188	170
	1.17×10^{-3}	8.21×10^3	355	339
B ^d	3.77×10^{-4}	1.18×10^5	186	158
	1.17×10^{-3}	3.80×10^4	362	289

^a x^2/α applies to Location A, R^2/α applies to Location B.

^bEstimated from Fig. 5 for Location A, from Fig. 7 for Location B.

^cPredicted by numerical simulation.

^dFor Location A, $x = 3.1$ in.; for Location B, $r = 2.41$ in.; and $r/R = 0.36$.

5.3 NUMERICAL SOLUTION

To show that the procedure provides a conservative estimate, a very detailed numerical simulation of the Mound 9859 package under hypothetical-accident thermal-test conditions was performed. The computer model of the package for the numerical simulation is shown in Fig. 9b. The HEATING module (version 6.0) of the SCALE 3.0 computer code was used for the simulation. The maximum temperatures at Locations A and B, predicted with the computer solution, are presented in Table 6.

5.4 COMPARISON OF PREDICTED AND ESTIMATED MAXIMUM TEMPERATURES

The maximum temperatures at Locations A and B predicted by the estimating procedure and by the numerical simulation are presented in Table 6. Comparison shows that the temperatures of the estimating procedure are higher than those of the numerical simulation. Therefore, the present procedure provides conservative estimates when compared with values that are predicted by a very detailed numerical simulation.

6 TEMPERATURE IN AN INFINITE CIRCULAR CYLINDER ($Q \neq 0$)

The analytical solution for temperature in a cylinder with an internal heat source, presented here, is not used explicitly in the estimating procedure. However, the solution provides a mathematical argument to affirm that the estimating procedure is conservative.

6.1 GOVERNING EQUATION

The governing equation for temperature in an infinite circular cylinder is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} - \frac{1}{\alpha} \frac{\partial T}{\partial t} + \frac{q'''}{K} = 0, \quad (22)$$

where T , r , α , t , q''' , and K are the temperature, radial distance, thermal diffusivity, time, heat generation rate, and thermal conductivity, respectively. The initial condition that corresponds to a steady-state solution for temperature in a cylinder with a constant heat source q''' , is

$$\begin{aligned} T &= T_0 + (T_{si} - T_0) + \frac{q''' R^2}{4K} (1 - r^{*2}) \\ &= T_0 + (T_{si} - T_0) + \Phi (1 - r^{*2}) \end{aligned}$$

$$= T_0 + dT_{q\text{-steady}}. \quad (23)$$

Here, $(T_{si} - T_0)$ is the rise in surface temperature due to the heat source, T_0 is the initial temperature in a package without a heat source, R is the radius of the cylinder, r^* ($= r/R$) is the dimensionless radial distance, $dT_{q\text{-steady}}$ is the temperature rise in a package due to the heat source, and

$$\Phi = \frac{q'''R^2}{4K} = \frac{dT_{q\text{-steady}} - (T_{si} - T_0)}{(1.0 - r^{*2})}. \quad (24)$$

The boundary condition for a cylinder subjected to step surface temperatures T_{s1} for time $< t_1$, and T_{s2} for time $> t_1$ is

$$T_{r^*=1} = T_{s1} \text{ for } 0 < t \leq t_1, \text{ and}$$

$$T_{r^*=1} = T_{s2} \text{ for } t > t_1. \quad (25)$$

Because Eq. 22 is a linear differential equation, the solution can be obtained by summing the solutions, $T = T_1 + T_2 + T_3$, of three separate problems of T_1 , T_2 , and T_3 .

6.2 PROBLEM 1

Consider the case of a cylinder with a constant internal heat source, zero initial temperature, and zero surface temperature. The governing equation to be solved is

$$\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} - \frac{1}{\alpha} \frac{\partial T_1}{\partial t} + \frac{q'''}{K} = 0. \quad (26)$$

The corresponding initial and boundary conditions are

$$T_1 = 0 \text{ for } t = 0, \quad (27)$$

and

$$T_1 = 0 \text{ for } r^* = 1, \quad (28)$$

respectively. The solution of Eq. 26 for temperature T_1 with the initial and boundary conditions expressed by Eqs. 27 and 28, as given by Eq. 1 in Section 6.9 Carslaw and Jaeger,² is

$$T_1 = \frac{q'''R^2}{4K} \left[(1 - r^{*2}) - 8 \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n^3 J_1(\beta_n)} \right]. \quad (29)$$

In Eq. 29, $J_n(\beta)$ is the Bessel function of the first kind of order n , and $\pm(\beta_n)$, $n = 1, 2, \dots$, are the roots of the Bessel function $J_0(\beta) = 0$. Equation 29 can be rewritten as

$$T_1 = \Phi(1 - r^{*2}) - 8\Phi \sum_{n=1}^{\infty} \frac{\Psi_n}{\beta_n^2}, \quad (30)$$

where

$$\Phi = \frac{q'''R^2}{4K} = \frac{dT_{q-\text{steady}} - (T_{si} - T_0)}{(1.0 - r^{*2})} \quad (31)$$

and

$$\Psi_n = \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}. \quad (32)$$

6.3 PROBLEM 2

Consider the case of a cylinder with no internal heat source, a prescribed initial parabolic temperature distribution, and zero surface temperature.

The governing equation to be solved is

$$\frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r} \frac{\partial T_2}{\partial r} - \frac{1}{\alpha} \frac{\partial T_2}{\partial t} = 0. \quad (33)$$

The corresponding initial and boundary conditions are

$$T_2 = T_{si} + \frac{q'''R^2}{4K} (1 - r^{*2}) \quad \text{for } t=0, \quad (34)$$

and

$$T_2 = 0 \quad \text{for } r^* = 1, \quad (35)$$

respectively. The solution of Eq. 33 with the initial and boundary conditions defined in Eqs. 34 and 35, as given by Eq. 7 in Section 6.6 of Carslaw and Jaeger,² is

$$T_2 = 2 \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n^2 J_1^2(\beta_n)} \left[T_{si} \beta_n J_1(\beta_n) + \frac{q''' R^2}{2K} J_2(\beta_n) \right]. \quad (36)$$

Because, from Eq. 26 of Gray, Mathews, and Macrobert,³

$$\frac{2 J_1(\beta_n)}{\beta_n} = J_0(\beta_n) + J_2(\beta_n), \quad (37)$$

and, because $\pm(\beta_n)$ are the roots of the Bessel function $J_0(\beta_n) = 0$,

$$J_2(\beta_n) = \frac{2 J_1(\beta_n)}{\beta_n}. \quad (38)$$

Therefore, Eq. 36 simplifies to

$$\begin{aligned} T_2 &= 2 \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n^2 J_1^2(\beta_n)} \left[T_{si} \beta_n J_1(\beta_n) + \frac{q''' R^2}{K} \frac{J_1(\beta_n)}{\beta_n} \right] \\ &= 2 \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)} \left[T_{si} + \frac{q''' R^2}{K} \frac{1}{\beta_n^2} \right] \\ &= \sum_{n=1}^{\infty} \psi_n \left[2 T_{si} + \frac{8 \Phi}{\beta_n^2} \right]. \end{aligned} \quad (39)$$

6.4 PROBLEM 3

Consider the case of a cylinder with no internal heat source, zero initial temperature, and a step surface temperature boundary condition. The governing equation to be solved is

$$\frac{\partial^2 T_3}{\partial r^2} + \frac{1}{r} \frac{\partial T_3}{\partial r} - \frac{1}{\alpha} \frac{\partial T_3}{\partial t} = 0. \quad (40)$$

The corresponding initial and boundary conditions are

$$T_3 = 0 \quad \text{for } t = 0, \quad (41)$$

and

$$T_3 = T_{s1} \quad \text{for } 0 < t \leq t_1 \text{ at } r^* = 1 \text{ and}$$

$$T_3 = T_{s2} \quad \text{for } t > t_1 \text{ at } r^* = 1, \quad (42)$$

respectively. The solution of Eq. 40 for T_3 with the initial and boundary conditions defined in Eqs. 41 and 42, as given by Eq. 8 in Section 6.6 of Carslaw and Jaeger,² is

$$T_3 = T_{s1} - 2 T_{s1} \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)} \quad (43)$$

for $t \leq t_1$ and

$$T_3 = T_{s2} - 2 T_{s1} \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)} \\ - 2(T_{s2} - T_{s1}) \sum_{n=1}^{\infty} \exp\left(\frac{-\beta_n^2 \alpha (t-t_1)}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)} \quad (44)$$

for $t > t_1$. The solution can be rewritten as

$$T_3 = T_{s1} - 2 T_{s1} \sum_{n=1}^{\infty} \psi_n \quad (45)$$

for $t \leq t_1$ and

$$T_3 = T_{s2} - 2 T_{s1} \sum_{n=1}^{\infty} \psi_n - 2(T_{s2} - T_{s1}) \sum_{n=1}^{\infty} \psi_n' \quad (46)$$

for $t > t_1$. Here, ψ_n' is defined as

$$\psi_n' = \exp\left(\frac{-\beta_n^2 \alpha (t-t_1)}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}. \quad (47)$$

6.5 SOLUTION FOR TEMPERATURE T

The final solution of Eq. 22 is

$$T = T_1 + T_2 + T_3,$$

i.e.,

$$T = \Phi(1 - r^{*2}) - 8\Phi \sum_{n=1}^{\infty} \frac{\psi_n}{\beta_n^2} + \sum_{n=1}^{\infty} \psi_n \left(2T_{si} + \frac{8\Phi}{\beta_n^2} \right) + T_{s1} - 2T_{s1} \sum_{n=1}^{\infty} \psi_n \quad (48)$$

for $t \leq t_1$ and

$$\begin{aligned} T = & \Phi(1 - r^{*2}) - 8\Phi \sum_{n=1}^{\infty} \frac{\psi_n}{\beta_n^2} + \sum_{n=1}^{\infty} \psi_n \left(2T_{si} + \frac{8\Phi}{\beta_n^2} \right) \\ & + T_{s2} - 2T_{s1} \sum_{n=1}^{\infty} \psi_n - 2(T_{s2} - T_{s1}) \sum_{n=1}^{\infty} \psi_n' \end{aligned} \quad (49)$$

for $t > t_1$. Eqs. 48 and 49 further simplify to

$$T = T_{s1} + \Phi(1 - r^{*2}) - 2(T_{s1} - T_{si}) \sum_{n=1}^{\infty} \psi_n \quad (50)$$

or

$$T = T_0 + dT_{q\text{-steady}} + (T_{s1} - T_{si}) \left(1 - 2 \sum_{n=1}^{\infty} \psi_n \right), \quad (51)$$

for $t \leq t_1$ and

$$T = T_{s2} + \Phi(1 - r^{*2}) - 2(T_{s1} - T_{si}) \sum_{n=1}^{\infty} \psi_n - 2(T_{s2} - T_{s1}) \sum_{n=1}^{\infty} \psi_n' \quad (52)$$

or

$$T = T_0 + dT_{q\text{-steady}} + (T_{s2} - T_{si}) - 2(T_{s1} - T_{si}) \sum_{n=1}^{\infty} \psi_n - 2(T_{s2} - T_{s1}) \sum_{n=1}^{\infty} \psi_n' \quad (53)$$

for $t > t_1$.

6.6 EFFECT OF HEAT SOURCE

The temperature rise in a cylinder due to an internal heat source can be determined by comparing the solutions of Eqs. 50-53 and 17 and 18, for the two cases, with and without heat source, respectively. Therefore,

$$dT_q = T_{q \neq 0} - T_{q=0} = \Phi(1 - r^{*2}) + 2(T_{si} - T_0) \sum_{n=1}^{\infty} \psi_n \quad (54)$$

or

$$T_{q \neq 0} - T_{q=0} = dT_{q\text{-steady}} - (T_{si} - T_0) \left(1 - 2 \sum_{n=1}^{\infty} \psi_n\right). \quad (55)$$

6.7 HYPOTHETICAL-ACCIDENT CONDITIONS

6.7.1 Case when $T_{si} \neq 100^\circ\text{F}$

The solution of Eq. 22 for the hypothetical-accident surface temperature boundary conditions:

$$T_0 = 100^\circ\text{F}, T_{s1} = 1475^\circ\text{F}, \quad T_{s2} = 100^\circ\text{F}, \text{ and } t_1 = 1800 \text{ s (30 min)},$$

as shown in Fig. 3, is

$$T = 100 + dT_{q\text{-steady}} + (1475 - T_{si}) \left(1 - 2 \sum_{n=1}^{\infty} \psi_n\right) \quad (56)$$

for $t \leq 1800 \text{ s (30 min)}$ and

$$T = 100 + dT_{q\text{-steady}} - (T_{si} - 100) - 2(1475 - T_{si}) \sum_{n=1}^{\infty} \psi_n + 2(1375) \sum_{n=1}^{\infty} \psi_n' \quad (57)$$

for $t > 1800 \text{ s (30 min)}$. The effect of a heat source, obtained by comparing the solutions of Eqs. 56 and 57 and 19 and 20, is

$$T_{q \neq 0} - T_{q=0} = dT_{q\text{-steady}} - (T_{si} - 100) \left(1 - 2 \sum_{n=1}^{\infty} \psi_n\right). \quad (58)$$

6.7.2 Case when $T_{si} = 100^\circ\text{F}$

For a cylinder with no internal heat source, the initial surface temperature T_{si} is equal to the initial ambient temperature $T_0 (= 100^\circ\text{F})$. Equations 56-58 simplify to

$$T = 100 + 1375 \left(1 - 2 \sum_{n=1}^{\infty} \psi_n\right) + dT_{q\text{-steady}} \quad (59)$$

for $t \leq 1800$ s and

$$T = 100 + 2(1375) \left(\sum_{n=1}^{\infty} \psi'_n - \sum_{n=1}^{\infty} \psi_n \right) + dT_{q\text{-steady}} \quad (60)$$

for $t > 1800$ s. Comparison of Eqs. 59 and 60 with the solutions to Eqs. 19 and 20, obtained for the case of no internal heat source, show that

$$T_{q \neq 0} - T_{q=0} = dT_{q\text{-steady}} \quad (61)$$

when $T_{si} = 100^\circ\text{F}$.

6.8 MAXIMUM TEMPERATURE

The maximum temperature in a cylinder will occur when

$$\partial T / \partial t = 0$$

or

$$2(1375) \sum_{n=1}^{\infty} \psi'_n \frac{\beta_n^2 \alpha}{R^2} = 2(1475 - T_{si}) \sum_{n=1}^{\infty} \psi_n \frac{\beta_n^2 \alpha}{R^2}. \quad (62)$$

This equation can be solved iteratively for various values of T_{si} , r^* , and R^2/α to compute time $t_{T_{\max}}$ when the temperature will be maximum. Time $t_{T_{\max}}$ can then be substituted in the solution to obtain the maximum temperature.

No attempt has been made to solve Eq. 62 because the estimating procedure presented in this paper does not require the solution of Eq. 62, and the use of Eq. 62, with three independent parameters, will make the estimating procedure more difficult.

7 CONSERVATIVE PROCEDURE

A procedure is considered conservative when the procedure-estimated maximum temperature is greater than or equal to the maximum temperature that a package will experience during a hypothetical-accident thermal-test condition, i.e.,

$$T_{\text{max-estimation}} \geq T_{\text{max-package}}.$$

The present procedure can, therefore, be considered conservative if

$$T_{\text{max-estimation}} = T_{\text{max-analytic}, q=0} + dT_{q\text{-steady}} \geq T_{\text{max-package}}.$$

The following sections explain why the proposed procedure is expected to provide conservative estimates.

7.1 NO INTERNAL HEAT SOURCE

For the case of a package with no heat source, the term $dT_{q\text{-steady}}$ is zero; thus, the procedure is conservative if

$$T_{\text{max-estimation}, q=0} = T_{\text{max-analytic}, q=0} \geq T_{\text{max-package}, q=0}. \quad (63)$$

The example presented in Section 5 has already shown that the temperature $T_{\text{max-estimation}, q=0}$, estimated for the mound package by the proposed procedure, is higher than $T_{\text{max-package}, q=0}$, the temperature obtained from a detailed numerical model of the package and the HEATING6 computer code. The following discussion will further explain why the maximum temperature in the analytical model $T_{\text{max-analytic}, q=0}$ will be higher than $T_{\text{max-package}, q=0}$, the maximum temperature that a package will experience under hypothetical-accident thermal-test conditions.

1. The surface temperature conditions that a package will experience under a hypothetical-accident thermal test, specified in 10 CFR §71.73 of the Code of Federal Regulations and imposed in the analytical model, are presented in Fig. 12. This figure shows that the heat inflow rate during the first 30 min and the total heat flow into a package will be higher in the analytical model than in a package under the conditions specified in 10 CFR §71.73. Therefore,

$$T_{\text{analytic}, q=0} \geq T_{\text{package}, q=0} \quad \text{for } t \leq 30 \text{ min}. \quad (64)$$

Fig. 12. Surface temperature conditions experienced by package under hypothetical-accident thermal test specified in 10 CFR §71.73 and imposed in the analytical model

For time > 30 min, some of the heat that penetrated during the first 30 min will continue to flow inward to raise the temperature of the inner region, even though the heat flow at the surface is now in the outward direction. Because

$$Q_{\text{Total-Inflow,analytic}} \geq T_{\text{Total-Inflow,package}} \quad (65)$$

and the heat inflow in the internal region continues until all regions of a package have attained their maximum temperature, the estimated temperature of the inner region will generally be higher, even though the heat outflow rate at the surface for time >30 min is higher in the case of the analytical model.

2. To provide thermal protection during a hypothetical-accident fire event, the outer section (overpack) of a transportation package is generally packed with an insulating material. The thermal diffusivity of the inner region of a package is, therefore, generally higher than that of the outer region. The analytical model, in contrast, assumes one uniform thermal diffusivity for the entire region of a package.

To illustrate the effect of thermal diffusivity on temperature, one-dimensional, steady-state temperature distributions in two solids are shown in Fig. 13. The solid with a uniform thermal diffusivity represents the analytical model. The solid with two thermal diffusivities, with the higher value in the inner region, represents a package. It can be seen from this figure that the temperature in a solid with a higher thermal-diffusivity material in the inner region will be lower than that in a solid with a uniform thermal diffusivity. The reason for this lower temperature is that the higher conducting inner region allows the heat to penetrate deeper and faster and thus lowers the temperature. Although, the figure shows a steady-state case, the temperature behavior will be similar even if the heat flow conditions are transient.

For a package with no internal heat source, it can be concluded that

$$T_{\text{max-analytic,q=0}} \geq T_{\text{max-package,q=0}} \quad (66)$$

7.2 INTERNAL HEAT SOURCE $Q \neq 0$

For the case of a package with an internal heat source, the procedure is conservative if

Fig. 13. Steady-state temperature distributions in two solids, one with uniform diffusivity, the other with two diffusivities

$$T_{\max\text{-estimation}, q \neq 0} \geq T_{\max\text{-package}, q \neq 0}, \quad (67)$$

i.e.,

$$T_{\max\text{-analytic}, q=0} + dT_{q\text{-steady}} \geq T_{\max\text{-package}, q=0} + dT_{\max\text{-package}, q \neq 0}. \quad (68)$$

Here,

$$dT_{\max\text{-package}, q \neq 0} = T_{\max\text{-package}, q \neq 0} - T_{\max\text{-package}, q=0} \quad (69)$$

is the maximum temperature rise in a package due to an internal heat source. Because

$$T_{\max\text{-analytic}, q=0} \geq T_{\max\text{-package}, q=0}, \quad (70)$$

from Eq. 66, the procedure is conservative if

$$dT_{q\text{-steady}} \geq dT_{\max\text{-package}, q \neq 0}. \quad (71)$$

The following discussion shows why the term $dT_{q\text{-steady}}$ will be greater than or equal to $dT_{\max\text{-package}, q \neq 0}$.

1. Suppose the ambient temperature for two identical packages, one with a constant internal heat source and the other with no heat source, is raised from T_0 (assume 100°F) to T_{s1} (assume 1475°F) and then kept constant at T_{s1} for an infinite time period. The final asymptotic maximum temperature in these packages, as shown in Fig. 14, will be

$$T_{\text{max-package}, q \neq 0, \text{asymptotic}} = T_{s1} + (T_{\text{package-steady}} - T_0)$$

$$= T_{s1} + dT_{q, \text{steady}}, \quad (72)$$

and

$$T_{\text{max-package}, q=0, \text{asymptotic}} = T_{s1}, \quad (73)$$

respectively. Therefore,

$$(T_{\text{max-package}, q \neq 0} - T_{\text{max-package}, q=0})_{\text{asymptotic}} = dT_{q, \text{steady}} \quad (74)$$

or

$$(dT_{\text{max-package}, q \neq 0})_{\text{asymptotic}} = dT_{q, \text{steady}} \quad (75)$$

or

$$dT_{\text{max-package}, q \neq 0} \leq dT_{q, \text{steady}}. \quad (76)$$

Fig. 14. Asymptotic temperature distribution in two packages, one with, the other without an internal heat source, as ambient temperature is raised from T_0 to T_{s1} and held at T_{s1} for infinite time

2. The comparison of the analytical solutions for temperature in two infinite cylinders subjected to step surface temperature boundary conditions, one with an internal heat source (Section 6) and one without heat source (Section 7), shows that

$$T_{\text{cyl}, q \neq 0} - T_{\text{cyl}, q=0} = dT_{q\text{-steady}} - (T_{\text{si}} - T_0) \left(1 - 2 \sum_{n=1}^{\infty} \psi_n \right). \quad (77)$$

Because the argument $(\beta_n^2 \alpha t / R^2)$ is always positive, the value of the exponential function $\exp(-\beta_n^2 \alpha t / R^2)$ ranges between zero and one. Therefore, the maximum value of the term ψ_n is

$$(\psi_n)_{\text{max}} = \left(\exp\left(\frac{-\beta_n^2 \alpha t}{R^2}\right) \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)} \right)_{\text{max}} = \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)}. \quad (78)$$

The term $(T_{\text{si}} - T_0)$ is positive because the initial steady-state surface temperature T_{si} in a package with an internal heat source will always be greater than the initial ambient temperature T_0 . Also, from the properties of the Bessel functions,

$$\sum_{n=1}^{\infty} \frac{J_0(r^* \beta_n)}{\beta_n J_1(\beta_n)} = 0.5. \quad (79)$$

Therefore,

$$0 \leq \left(1 - 2 \sum_{n=1}^{\infty} \psi_n \right) \leq 1, \quad (80)$$

and

$$0 \leq (T_{\text{si}} - T_0) \left(1 - 2 \sum_{n=1}^{\infty} \psi_n \right) \leq (T_{\text{si}} - T_0). \quad (81)$$

It can be seen from Eqs. 77 and 81 that

$$T_{\text{cyl}, q \neq 0} - T_{\text{cyl}, q=0} \leq dT_{q, \text{steady}} \quad (82)$$

or

$$dT_{\text{cyl}, q \neq 0} \leq dT_{q, \text{steady}} \quad (83)$$

or

$$dT_{\max, \text{cyl}, q \neq 0} \leq dT_{q, \text{steady}}. \quad (84)$$

Inasmuch as the hypothetical-accident thermal test condition is less severe than the step surface temperature condition imposed in the analytical solution for temperature in an infinite circular cylinder, the concluding Eq. 84, valid for a cylinder, can also be considered valid for a package undergoing a hypothetical-accident thermal test, i.e.,

$$dT_{q, \text{steady}} \geq dT_{\max, \text{package}, q \neq 0}. \quad (85)$$

Therefore, for a package with an internal heat source, it can be concluded that

$$T_{\max, \text{estimation}, q \neq 0} \geq T_{\max, \text{package}, q \neq 0}. \quad (86)$$

8 CONCLUSIONS

We have presented a simple, easy-to-use estimating procedure that provides a conservative prediction of the maximum temperature of an inner container under conditions of a hypothetical-accident thermal event. The procedure is applicable to all packages, with or without an internal heat source, that are protected by rectangular or cylindrical thermal insulating overpacks. Designers and reviewers of transportation packages will find it beneficial to use this estimating procedure before commencing a very lengthy and detailed computer simulation.

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