**Conceptual Design Analysis of An MHD Power
Conversion System for Droplet-Vapor Core
Reactors**

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CHAPTER 1

HISTORY AND INTRODUCTION TO MHD

1.1 Historical Background

The first idea of magnetohydrodynamic phenomenon was conceived by Michael Faraday in 1832, during his original investigation of electromagnetic induction [1]. Early in the 20th century a large number of magnetohydrodynamic (MHD) devices and machines were invented and patented. In 1907 the first MHD pump was designed by Northrup [2]. An MHD pump is a device which converts the electrical energy of the current supplied into mechanical energy of the pumped fluid. By inverting the operating principle of the MHD pump, an MHD generator was invented and patented by Karlovitz and Halacz in 1910 [3].

In 1930 Williams published the results of the first laboratory studies of MHD flows in pipes and ducts [4]. A very comprehensive theoretical study of this subject was carried out by Hartmann and Lazarus in 1936 and 1937 [5,6]. In the early 1940s a large and otherwise sophisticated Hall-current type MHD generator was built by Westinghouse Electric Corp. which failed due to insufficient knowledge of the properties of the ionized gases [7]. Plasma studies in the 1950s, created a sufficient databank of these properties.

In 1959, an experimental MHD generator was built at the Avco Everett Research Laboratory that produced 11.5 kw of power and obtained an appreciable pressure drop with the interaction between the gas and magnetic field [8]. In the early sixties many MHD generators operating at higher power levels were reported. The subsequent development of the MHD generators was pursued on basically three lines of approach: firstly, rare gas MHD generators, which used cesium seeded and thermally heated rare gas to get sufficient conductivity; secondly, liquid metal MHD generators, which suffered the inherent drawback of low flow rates; thirdly, combustion gas MHD generators, which utilized combustion products produced after burning the conventional type fuels. The earlier research work was concentrated on using the MHD generators to replace the conventional turbines. Although, the MHD generators have higher efficiencies (85-90%), but as the MHD systems are more expensive the earlier research efforts were curtailed. The MHD systems are not expected, as yet, to compete with the conventional generators in the commercial power plants.

But as the oil and natural gas reserves get consumed the MHD systems will become cost effective, due to their higher efficiency and lower fuel consumption. Added to this will be the environmental benefits due to considerably less pollution. The ever increasing applications in the special fields are giving further boost to the MHD research.

1.2 The Salient Features of MHD and its Special Applications

As an energy device, the MHD generator has very interesting characteristics which make it suitable for a variety of special applications. In an MHD generator the functions of both turbine and electric power generator are combined into a single unit with a simple compact geometry. There are no rotating parts which eliminates vibrations and noise and limits wear and tear. The MHD systems operate at higher temperatures than mechanical turbines with consequent saving in area, weight and cost of the radiator of the space power systems and it also leads to better efficiencies. High power turbines operating at high temperatures and speeds are less reliable than MHD systems due to high stress values. The DC type electric power output and the start-up time as low as a few milliseconds makes this power generation scheme very attractive.

The compactness and low weight-to-power ratio make MHD energy conversion a very promising option for space power applications. It also eliminates the possibility of any problems due to turbine vibrations and gyroscope effect. Efforts are being made to use MHD as a conversion system in a nuclear electric system. The major limitation on the design of nuclear reactor is to attain temperatures above 3000K to get sufficient ionization of the gases.

The MHD systems are widely used in a compact and mobile power source for military applications. Use of MHD on

a tactical aircraft for battlefield illumination was reported by Air Force Aero Propulsion Laboratory, Wright-Patterson Air Force Base, Ohio [9]. To escape from detection, submarines need a low noise and compact power source and closed-cycle liquid-metal MHD or nuclear-driven MHD are two possible schemes which are being looked into.

The principle of MHD pump can be utilized to accelerate a gas stream in a hypersonic wind-tunnel facility. The concept led to construction of the MHD pilot scale facility at the Arnold Engineering Development Center (AEDC) in Tullahoma, Tennessee [10]. The MHD generator, known as LORHO, was designed by AEDC and achieved peak powers of 18 MW for about 10 seconds. A series of multi-mega-watt MHD generators called Mark V, Mark VI, Mark VII were constructed by Avco Laboratory for the military research.

The MHD channel can be used as a flowmeter. The electric current output of a MHD gives a direct measurement of the flow rate of the fluid with known conductivity. The MHD channel can also cause population inversion in a flowing gas medium which is essential for the lasing action. The research is continuing in the field of MHD-laser systems.

The MHD systems have become a significant part of research and development in the field of space power and military applications. This research work concentrates on the study of proposed nuclear driven MHD system for the multi-mega-watt space power applications.

1.3 Outline of the Present Study

The objective of this research work is to develop a computer model to perform design analysis and study electrodynamics of the linear MHD generator for the proposed space power system. It is essential to study the basic operating principles and governing equations of the MHD generator before embarking on to the present research work. These principles are discussed in Chapter II of this report.

The Chapter III describes the proposed space power system and the constraints it imposes on the design of the MHD generator. The Chapter IV discusses the solution method incorporated in developing the quasi-one dimensional computer model for the linear MHD generator. A detailed parametric study is performed using this computer model.

The optimum designs are determined at 25, 100 and 400 MW power levels and the results are tabulated in Chapter V. The analysis of the variation of state parameters and basic electrodynamic phenomena is also performed. The Chapter V further suggests three design alternatives for operating at output power level of 1000 MW.

For more accurate studies it is essential to develop a three dimensional model. An innovative scheme for a 3-D model is formulated and discussed in details in Chapter VI.

CHAPTER 2

THE BASIC OPERATING PRINCIPLES OF THE MHD POWER GENERATOR

2.1 The MHD generator

MHD generator converts mechanical energy of the fluid directly into electric energy. In a MHD channel a conducting fluid is passed through a transverse magnetic field B . The free charge carriers inside the conducting fluid have a component of velocity U along the flow direction same as that of the neutral particles. The charge particles experience a force perpendicular to both U and B . This leads to flow of electric current in the perpendicular direction.

The Fig.2.1 shows a linear MHD channel with rectangular cross-section indicating the directions of magnetic field B , flow velocity U and current density J . The total current I passing through the load resistor R creates voltage drop V (and electric field E) between the electrode plates. This electric field acts in the direction opposite to the current density. The electric power output is obtained in the direction mutually perpendicular to both magnetic field and fluid flow as shown in the figure.

2.2 The Governing Equations in Electrodynamics

2.2.1 Maxwell's equations

The basic laws of electromagnetism are explained in terms of four vector relations stated below, which are known as the Maxwell's equations. These relations are expressed in the form of gradient and curl of the electric field E and magnetic field B vectors.

$$\nabla \cdot E = \frac{\Omega}{\epsilon} \quad \dots (2.1)$$

$$\nabla \cdot B = 0 \quad \dots (2.2)$$

$$\nabla \times E = - \frac{\delta B}{\delta t} \quad \dots (2.3)$$

$$\nabla \times B = \mu \cdot (J + \epsilon \cdot \frac{\delta E}{\delta t}) \quad \dots (2.4)$$

where, Ω is the free charge density (coul/m³),

J is the current density (amp/m²),

μ is the permeability of the medium (Henry/m),

ϵ is the permittivity of the medium (Farad/m).

Another important law, which is known as the conservation of electric charge, can be derived using Eq.2.1 and Eq.2.4. This law is given by the following equation.

$$\nabla \cdot J = - \frac{\delta \Omega}{\delta t} \quad \dots (2.5)$$

At steady state the Eq.2.5 gives, $\nabla \cdot J = 0$

2.2.2 Generalized Ohm's Law and parametric analysis

The most common form of Ohm's Law in terms of voltage V and current I is, $V = I.R$ (where, R is the resistance) or in terms of current density J and electric field E is, $J = \sigma.E$ (where, σ is the electrical conductivity of the medium). In the Generalized Ohm's Law there are certain additional terms which are significant only in some specific situations.

In the MHD generator a working fluid with a limited conductivity (a partially ionized gas mixture for the proposed system) passes through the applied transverse magnetic field B with velocity U . Any charge q moving inside the magnetic field experiences a Lorentz force, F_1 , given by Eq.2.6.

$$F_1 = q.(U \times B) \quad \dots (2.6)$$

The motional emf corresponding to the Lorentz force can be written as $U \times B$, which adds up vectorially to the electric field E , applied by the electrodes. Thus the current density J in the MHD channel can now be written as,

$$J = \sigma.(E + U \times B) \quad \dots (2.7)$$

Once the current density distribution is established, as given by Eq.2.7, the charge particles move inside the medium with equilibrium velocity or the drift velocity W . This drift velocity, W , which is perpendicular to the

magnetic field further gives rise to motional emf, $\mathbf{W} \times \mathbf{B}$, known as Hall field. The drift velocity of the electrons $\mathbf{W}_e$ is usually far greater than that of the ions $\mathbf{W}_i$ and by neglecting the later we get,

$$\mathbf{J} = \sigma \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) + \sigma \cdot (\mathbf{W}_e \times \mathbf{B}) \quad \dots (2.8)$$

When the first term in the Eq.8 is the main term we can use the following relations.

$$\sigma \cdot \mathbf{W}_e = \mu \cdot \mathbf{J} = \frac{(\mathbf{w} \cdot \tau)}{B} \cdot \mathbf{J} \quad \dots (2.9)$$

where, μ is the mobility of the electrons in the medium,

w is the cyclotron frequency,

τ is the free flight time of the electrons,

$(w \cdot \tau)$ is known as the Hall parameter for the medium.

By combining Eq.2.8 and Eq.2.9 we get,

$$\mathbf{J} = \sigma \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) + \frac{w \tau}{B} \cdot (\mathbf{J} \times \mathbf{B}) \quad \dots (2.10)$$

The gas parameters are not uniform throughout the volume of the MHD channel and consequently the electron density is also nonuniform in the volume. The negative gradient of the partial pressure of the electrons, P_e , acts as a driving force for the electrons further modifying the current density pattern.

When the electric or magnetic field or the gas parameters are changed very rapidly the current density

pattern also tends to change accordingly. But as the charge carriers cannot change their velocities instantaneously they tend to lag behind at the extremely high frequencies. This phenomenon adds a transient term in the current density equation.

The Generalized Ohm's Law combines all the above phenomena into a single equation as given below.

$$\mathbf{J} = \sigma \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) + \frac{\omega \tau}{B} (\mathbf{J} \times \mathbf{B}) + \frac{\sigma}{n_e \cdot e} \cdot \nabla P_e - \tau \cdot \frac{D\mathbf{J}}{Dt} \dots (2.11)$$

where, n_e is free electron density (m^{-3}),

e is the electronic charge (coul).

2.2.3 Parametric Analysis

To determine the relative importance of each term in the Generalized Ohm's Law parametric analysis is performed taking into account the typical range of operating conditions for the proposed MHD system [11].

Velocity $U = 200 - 3000 \text{ m/s}$,

Magnetic field $B = 1 - 8 \text{ tesla}$,

Electric field $E \approx -0.5 \times (U \cdot B)$,

Conductivity $\sigma = 10 - 500 \text{ mhos/m}$,

First term $\rightarrow \sigma \cdot (\mathbf{E} + \mathbf{U} \times \mathbf{B}) = 1000 - 5 \times 10^7 \text{ amp/m}^2$,

Pressure $P = 2 \times 10^6 - 8 \times 10^6 \text{ Pa}$,

Particle density $n \approx 4 \times 10^{25} / \text{m}^3$,

Collision cross-section $Q \approx 2 \times 10^{-19} \text{ m}^2$,

Electron velocity $C_e \approx 4 \times 10^5$ m/s,

Free flight time for electron $\tau = \frac{1}{n \cdot Q \cdot C_e} \approx 3 \times 10^{-13}$ sec,

Electronic charge $e = 1.6 \times 10^{-19}$ coul,

Electronic mass $m_e = 9.1 \times 10^{-31}$ kg,

Hall parameter $w\tau = \frac{e \cdot B \cdot \tau}{m_e} = 0.05 - 0.5,$

$\frac{w \cdot \tau}{B} \approx 0.05$ (tesla⁻¹),

Second term $\rightarrow \frac{w \cdot \tau}{B} \cdot (J \times B) = 50 - 2.5 \times 10^7$ amp/m²

Third term $\rightarrow \frac{\sigma}{n_e \cdot e} \cdot \nabla P_e \approx \frac{1}{11606} \cdot \nabla T_e$

To get the magnitude of the third term comparable to the first two terms (i.e. above 10^3 amp/m²) the gradient of the electronic temperature must be at least 10^7 K/m. Such extremely high values of gradients are likely only in boundary layers and in waves. Temperature gradients in the boundary layers of a gaseous flow may be expected up to 10^5 K/m, but in the non-equilibrium state the temperature of electrons T_e can be a lot higher than the gas temperature and the gradient of T_e may have still higher values.

Fourth term $\rightarrow \tau \cdot \frac{DJ}{Dt} \approx 3 \times 10^{-13} \cdot \frac{DJ}{Dt}$

The fourth term will be significant only when the current density fluctuates with frequencies higher than 10^{13} Hz. If the magnetic field is switched on instantly or the electric circuit is connected then the current density

changes from 0 to 10^5 amp/m², but this process will be limited by this term and the time taken will be of the order of 10 nsec.

As can be seen from the above analysis, only the first two terms of the Generalized Ohm's Law are significant for the normal operation of the MHD generator. Therefore we will use Eq.10 for determining the current density J for the MHD channel.

2.2.4 Magnetic Reynolds Number R_m

For any MHD channel a dimensionless quantity known as Magnetic Reynolds Number is defined as follows:

$$R_m = \mu \cdot \sigma \cdot U \cdot L \quad \dots (2.12)$$

where, L is length of the MHD channel ($L = 0.5 - 3.0$ m).

The Magnetic Reynolds Number is a measure of an extent to which the gas motion can modify or deflect the applied magnetic field. When $R_m \approx 1.0$, then the induced magnetic field B_i is approximately equal to the applied magnetic field B_o . With the typical operating values considered earlier we get the range of R_m from 0.05 to 2.50. For the design of an 100 MW MHD generator, which is discussed in details later, the $R_m \approx 0.2$.

2.2.5 Induced magnetic field

The current density J set up inside the MHD channel induces its own magnetic field B_i . The component of induced magnetic field dB_i due to each current element $(I.ds)$ at any given point P is given by Eq.13, which known as the Biot-Savart Law. This law can be derived from the Maxwell's equation (Eq.4).

$$dB_i = \frac{\mu}{4\pi} \frac{(I.ds) \times R}{R^3} \quad \dots (2.13)$$

where, R is the displacement vector from the current element to the point P and R is its magnitude (m).

The net induced field B_i at point P is $\int dB_i$.

The induced magnetic field B_i adds up vectorially to the applied magnetic field B_o to give the total magnetic field B .

2.3 The Governing Equations in Fluid Mechanics

The standard fluid mechanics equations for flow through any channel also apply for the MHD channel. At any point inside the MHD volume the mass, momentum and energy of the fluid are conserved.

The conservation of mass law applied to a fluid passing through an infinitesimal, fixed control volume yields the equation of continuity, which is given by Eq.2.14.

$$\frac{\delta \rho}{\delta t} + \nabla \cdot (\rho \mathbf{U}) = 0 \quad \dots (2.14)$$

where, ρ is the density of the fluid (kg/m^3).

Newton's Second Law applied to a fluid passing through the control volume yields a following momentum equation:

$$\rho \frac{\delta \mathbf{U}}{\delta t} + \rho \mathbf{U} \cdot \nabla \mathbf{U} = \mathbf{f} + \nabla \cdot \boldsymbol{\pi}_{ij} \quad \dots (2.15)$$

where, $\boldsymbol{\pi}_{ij}$ is the stress tensor (N/m^2),

$\mathbf{f}$ is the body force per unit volume, which includes the gravitational force and the Lorentz force $\mathbf{f}_1$ terms, $\mathbf{f}_1 = \mathbf{J} \times \mathbf{B}$...from Eq.2.6

Substituting the values for each component of the stress tensor we get three scalar equations, which are known as Navier-Stokes equations [12]. By assuming incompressible flow with coefficient of viscosity μ , the equation reduces to a much simpler form given as follows:

$$\rho \frac{\delta \mathbf{U}}{\delta t} + \rho \mathbf{U} \cdot (\nabla \cdot \mathbf{U}) = \mathbf{F} - \nabla P + \mu \cdot \nabla^2 \mathbf{U} \quad \dots (2.16)$$

The First Law of Thermodynamics applied to a fluid through the control volume yields the energy equation.

$$\rho \frac{\delta e}{\delta t} + \rho \mathbf{U} \cdot \nabla e + P \cdot (\nabla \cdot \mathbf{U}) = \frac{\delta Q}{\delta t} + k \cdot \nabla^2 T + \mathbf{F} \cdot \mathbf{U} + \phi \dots (2.17)$$

where, e is the internal energy per unit mass ($e = C_v.T$),
 k is the thermal conductivity of the fluid ($W/^{\circ}K/m$),
 ϕ is the viscous dissipation function (W/m^2),
 $\frac{\delta Q}{\delta t}$ is the rate of heat produced per unit volume by
 external agencies such as Joule dissipation.

For an MHD channel we can substitute,

$$\frac{\delta Q}{\delta t} = \frac{J^2}{\sigma} \quad \dots (2.18)$$

2.4 The Geometry of MHD Generator

MHD generators are designed in two entirely different geometries. The first geometry is known as linear MHD, which has a simple duct type shape with a mostly rectangular cross-section. The typical linear MHD generator is shown earlier in Fig. 2.1.

The other geometry consists of two concentric circular electrodes with magnetic field applied along the axis as shown in Fig. 2.2. The conducting fluid flows in between the two electrodes with radial velocity V_r and tangential velocity V_{θ} . This generator is called disc type MHD generator. The disc-MHD generator has a more compact geometry and its power density is higher than the typical line-MHD generator.

Linear MHD generators differ from each other in terms of the electrode geometry. The different types of electrode geometries are considered below.

2.4.1 Faraday generator

The Faraday generator is shown in Fig.2.1. Since the electrodes are continuous along the axis of the duct the electric field along the axis $E_x = 0$. Using Eq.2.10, we get the expressions for the Faraday current density J_y in the normal direction and the leakage Hall current density J_x as follows:

$$J_y = \frac{\sigma}{1 + w^2 \tau^2} \cdot (UB - E_y) j \quad \dots (2.19)$$

$$J_x = \frac{-w\tau\sigma}{1 + w^2 \tau^2} \cdot (UB - E_y) i \quad \dots (2.20)$$

As the electric field E_y has direction opposite to the current density J_y it appears with the negative sign. The Hall current finds a leakage path through each electrode plate completing the loop. The useful electric power can be obtained only from Faraday current and output power density is given by,

$$P_o = J \cdot E = J_y \cdot E_y = \frac{\sigma}{1 + w^2 \tau^2} \cdot (UB - E_y) \cdot E_y \quad \dots (2.21)$$

The approximate value for the overall power density for the MHD channel is given by Eq.2.22.

$$P_o = 0.2 \sigma U^2 B^2 \quad \dots (2.22)$$

2.4.2 Segmented Faraday generator

In this geometry the electrodes are segmented into

many separate pairs, each pair feeding a separate load as shown in Fig. 2.3. The basic advantage of segmenting is to block the Hall current and eliminate the Joule heat dissipation losses due to this component.

2.4.3 Hall generator

When Hall parameter ($w\tau$) of the medium has high value, i.e., $w\tau \geq 1.0$, it is advantageous to extract power from the Hall component of the current rather than the Faraday component. This is achieved by the type of geometry, called as the Hall generator, shown in Fig. 2.4. The Hall current density J_x and the output power density is given as follows,

$$J_x = \frac{-\sigma}{1 + w^2\tau^2} \cdot (w\tau UB - E_x) \quad \dots (2.23)$$

$$P_o = J \cdot E = J_x \cdot E_x = \frac{\sigma w\tau}{1 + w^2\tau^2} \cdot (UB - E_x) \cdot E_x \quad \dots (2.24)$$

The Hall generator has the advantage of high output voltage and the highest efficiency is obtained under heavy load conditions, i.e., just the opposite of a Faraday generator. The pressure gradient produced in the Hall generator is in the direction perpendicular to the fluid flow due to which it suffers the drawback of nonuniform fluid properties transverse to fluid flow.

2.4.4 Diagonally connected generator

The electrodes which would lie on the same potential surface in a normally operating Faraday generator are

connected. As a result no Hall current flows and obtains the efficiency advantage of the Faraday connection plus the High output voltage and reduced number of output circuits of the Hall connection. To get maximum efficiency the angle of the diagonal connection has to be optimized for every value of wr . For the intermediate values of wr , i.e., $wr \approx 3.0$, this generator is more efficient than both Hall generator and Faraday generator. The Fig. 2.5 shows the diagonally connected generator. The diagonally connected electrode configuration is further modified to obtain the frame type electrode configuration.

Although among the linear MHD generators the rectangular cross-section geometry is the most common, the MHD ducts with circular and oval cross-sections are also studied. A circular or an oval cross-section has structural and aerodynamic advantages over a rectangular one, but it causes distortions in the electric field and current flow pattern. The output power of a circular channel MHD as a function of an angular width of an electrode is studied by F. J. Fishman and the optimum value lies in between 30° to 40° [13]. The frame type MHD with oval cross-section is studied by V. A. Kirillin and its comparison with other geometries is reported [14].

Figure 2.3 The Segmented Faraday type MHD generator [11]

Figure 2.4 The Hall type MHD generator [11]

Figure 2.5 The Diagonally connected MHD generator [11]

CHAPTER 3

THE PROPOSED NUCLEAR DRIVEN MHD SYSTEM AND FORMULATION OF THE MODELING PROBLEM

3.1. Space Power System Description

The proposed system is an innovative space nuclear power concept for closed cycle operation for 100-400 MW_e output power levels. An Ultrahigh Temperature Vapor Core Nuclear Reactor (UTVR) combined with a linear MHD generator to operate on a direct, closed Rankine type cycle is being studied (Fig. 3.1).

The central vapor core region of the reactor contains a mixture of metal fluoride (e.g. potassium fluoride KF) and highly enriched (85%) uranium-tetrafluoride UF₄ vapor. The mole fraction of KF is about 0.90. The vapor fuel mixture exiting the reactor passes through the nozzle and enters the MHD generator at a temperature of about 4000°K and pressure of about 5 MPa. The mass flow rate of the closed cycle system at 200 MW_e power level is estimated to be 286 kg/s. The gas mixture after exiting MHD generator passes through a diffuser and a radiator heat exchanger. The metal fluoride condenses in the radiator and heat exchanger. Then it is pressurized and injected back into the reactor, where it is vaporized and mixed with the UF₄ gas.

Figure 3.1. Schematic Diagram of UTVR-MHD Rankine Power Cycle [15]

3.2 The Design Constraints on the MHD Generator

As discussed in Section §1.2, since the MHD convertor is compact and has a very low weight to power ratio, it is highly useful for space power applications. The use of MHD generator also eliminates the possibility of any problems due turbine vibrations and gyroscope effect in the space vehicle. To attain high power levels in the MHD channel the operating fluid, a gaseous mixture of UF_4 and KF , must have high electrical conductivity. The gas temperature in the excess of $3000^\circ K$ is necessary to get sufficient conductivity, which is achieved with the help of ultra-high temperature gas core reactor. The high operating temperatures of the proposed nuclear driven MHD system also ensure high efficiency of energy rejection in the radiators. The reduction in the area, weight and cost of the radiators further makes the proposed space power scheme more attractive.

There are two types of MHD generators, as discussed in Section §2.4, a disk-MHD and a line-MHD. Although the disk-MHD has higher power density, the proposed space power system is a closed-loop system and the gas at the exit of the MHD generator must be fully collected and recycled. This recycling design adds excessive volume and weight to the system and limits the utility of the disk type MHD generator. In this report a linear MHD is being looked into as the possible design option for the proposed system.

The linear MHD can have different types of electrode configurations as described in Section §2.4. The typical value of the Hall parameter for the proposed system lies between 0.15 to 0.8 (refer to Section §2.2), which means that the Faraday type MHD generator is the most efficient type of design in comparison with the Hall type and diagonally connected MHD generators. To utilize the maximum volume of the Faraday type MHD channel a continuous electrode configuration is preferred over the segmented type electrodes.

The basic requirement of compact geometry is met by operating the MHD in the medium and strong ($R_m = 0.2$ to 1.5) interaction regime. The higher the temperature of the gas mixture at the entrance of the MHD channel, the higher will be the electrical conductivity leading to the larger output power density. The average temperature at the entrance cross-section is limited by the temperature at the exit of the UTVR. The MHD system is being designed for the power levels from 25 MW_e up to 1000 MW_e and typical active volume of the MHD channel is of the order of 1.0 m^3 . The Eq.2.22 gives the approximate formula for the overall power density.

The proposed working fuel/fluid is a vapor mixture of uranium tetra-fluoride UF_4 ($\approx 10\%$) and potassium fluoride KF ($\approx 90\%$) in the partially ionized state. The gas mixture passing through the MHD channel also contains fission products. In addition to thermal excitation there also non-

equilibrium ionization induced because of the fission fragments.

The typical values of the working parameters are determined by the required power level and the constraints due to overall system design. The superheated vapor mixture at the exit of the UTVR has a temperature of 3000 to 4500°K and a pressure of 3 to 8 MPa, which sets limits on the values of the operating ranges of temperature and pressure for the MHD generator. To achieve the power levels of few hundreds of megawatts the typical operating ranges for the applied field, inlet velocity, inlet temperature and inlet pressure are selected to be from 2 to 8 tesla, 800 to 1800 m/s, 2800 to 4500°K and 3 to 8 MPa, respectively. The electric conductivity of the gaseous mixture is a function of gas temperature, pressure, composition. The conductivity is further enhanced due to the presence of fission fragments and high values of current density and for the proposed system it lies between 10 to 500 mhos/m.

These design constraints define the ranges of operating conditions for the MHD generator. Within the boundaries of these constraints, the system parameters are adjusted so as to optimize the performance of the overall system. Certain additional criteria such as keeping velocity more or less constant along the MHD channel or a constant power density or a specified temperature or pressure drop across the channel may also be imposed on the system.

3.3 Formulation of the Problem

The aim of this study is to perform preliminary analysis of the MHD channel for the proposed system. An attempt is made to develop a simplified computer model incorporating both the electrodynamics and fluid mechanics aspects of the MHD. This computer model is intended to predict the basic electrodynamic phenomena in the MHD channel and to estimate the variation of temperature, pressure, velocity and current density in the flow direction and velocity and current density variation across the flow.

The proposed system is being designed to operate at power levels ranging from 25 MW up to 1000 MW. The operating values of different parameters are adjusted to keep the axial velocity more or less constant along the flow direction. This computer model is intended to calculate the net change in pressure, temperature and velocity and also the total output current, voltage and electric power. The geometry of the channel plays an important role in the performance of the system and the optimum geometry has to be determined at each power level.

To develop a computer model for the linear MHD channel for the prediction of electrodynamic phenomena and parametric study, the fuel / working fluid mixture is simulated by helium gas. This is mainly due to a lack of available data on thermal, electrical and transport properties of uranium tetrafluoride and metal fluorides at such high temperatures.

The electrical conductivity of the fuel / working fluid mixture is assumed to follow the variation as that of seeded helium. Non-thermal ionization of the MHD gas mixture by the fission fragments has been hypothesized to significantly enhance the electrical conductivity. The actual functional dependence of the conductivity for the UF_4 and KF mixture at temperatures above 2500°K is being investigated. For developing this computer model it is essential to know the value of electrical conductivity at each point inside the MHD channel. Fully seeded helium gas at similar temperature and pressure is used to substitute the working fluid mixture for determining the electrical properties. However, it should be noted that these assumptions are justified only for the evaluation of the electrodynamic phenomena in the MHD generator. The thermal and transport properties of uranium fluoride and metallic fluorides are expected to be considerably different from inert gases.

To perform this preliminary analysis a quasi-one dimensional computer model is developed, which solves the fluid mechanics equations discussed in Section §2.3 taking into account the electro-magnetic force and energy terms. The detailed solution method is described in the next chapter.

CHAPTER 4

SOLUTION METHOD

A quasi-one dimensional computer model is developed to perform phenomenological study and to simulate the interplay between the system state variables and different design parameters. The model takes into account the major physical aspects of the system involving both electrodynamics and fluid mechanics.

In this incompressible quasi-one dimensional approach, the component of the fluid velocity in the transverse direction (Y, Z directions) is assumed to be very small and is neglected. But the axial velocity, U, temperature, T, and pressure, P, are described as functions of position (x,y,z). The solution proceeds in the axial direction (X direction) from one axial node to the next, using a finite difference technique. The first step in this model is to calculate the average values of the state parameters for the node cross-section. The governing equations in fluid mechanics as given in Section 2.3 are converted into one dimensional form. The one dimensional continuity equation can be written as follows:

$$\frac{d}{dx}(\rho \cdot U \cdot A) = 0 \quad \dots (4.1)$$

The equations of conservation of momentum and energy are further modified to include the effects of electric and magnetic fields. The momentum equation includes Lorentz force per unit volume, f_1 and the energy balance equation includes the energy consumed to do work against f_1 and energy converted from electrical to thermal because of Joule heating. These equations can be written as follows:

$$\frac{d}{dx} \{ (\rho U^2 + P)A \} = -(F_f + f_1 \cdot A) \quad \dots (4.2)$$

where, μ is the viscosity of the gas (N.sec/m²),

A is the cross-sectional area of the channel (m²),

F_f is the frictional drag force per unit length (N/m)

f_1 is the Lorentz force per unit volume ($f_1 = J_y \cdot B_z$).

For the Reynolds number Re in the range of 10^6 , the frictional drag force can be determined by the following empirical relationship, known as the Blasius relation:

$$F_f = C_f \cdot \frac{\rho U^2}{2D_h} \quad \dots C_f = 0.184 Re_D^{-1/5} \quad \dots (4.3)$$

where, C_f is the turbulent friction factor,

D_h is the hydraulic diameter of the channel, which is

$$\text{given by } D_h = \frac{4 \times \text{Area}}{\text{Perimeter}}$$

The Reynolds Number is given by, $Re_D = \frac{\rho \cdot U \cdot L}{\mu}$

The energy balance equation is given by Eq.4.4.

$$\frac{d}{dx} \left(UA \cdot \left(\rho C_V T + \frac{\rho U^2}{2} + P \right) \right) = \frac{d}{dx} \left(kA \cdot \frac{dT}{dx} \right) + \frac{J^2 A}{\sigma} - U(F_f + f_l \cdot A) - H_w \quad \dots (4.4)$$

where, k is the coefficient of thermal conductivity,

H_w is the rate of heat loss to the sidewalls and electrodes per unit length.

The gas flowing through the MHD channel is at very high temperatures (above 3000°K) and the Ideal Gas Law relation is assumed to hold and is used to substitute for the gas density, as given by Eq.4.5.

$$P = \rho \cdot R \cdot T \quad \text{or} \quad \rho = \frac{P}{R \cdot T} \quad \dots (4.5)$$

The electrical conductivity of the gas mixture is a strong function of its temperature and it also depends on the gas pressure, gas composition, current density and density of fission fragments and hence on neutron flux. The exact variation of the conductivity of the working fluid in the temperature range of 2500 to 4500°K is not yet known. The conductivity of the gas mixture is enhanced due the presence of the fission fragments, which induce non-equilibrium ionization. As discussed in Section §3.3, the electrical properties of the gas mixture are simulated by helium seeded with cesium (He + 1.41% Cs). An approximate function for the gas conductivity, σ , is used, which is given as follows [11]:

$$\sigma = 80 \cdot \left[\frac{T}{3000} \right]^{4.0} \cdot \left[\frac{3}{P} \right]^{0.5} \text{ mhos/m} \quad \dots (4.6)$$

where, T is the gas temperature in $^{\circ}\text{K}$,

P is the gas pressure in MPa.

The Reynolds Number at the typical operating conditions is 10^6 or higher, which implies a turbulent type flow. At the inlet of the MHD channel a fully-developed velocity profile is assumed which can be given by the power-law approximation as follows:

$$\frac{U}{U_{\max}} = \left[\frac{2 \cdot Y}{W} \right]^{1/n}, \quad n = 7 \quad \dots (4.7)$$

where, U is the velocity at a distance Y from the wall,

$U_{\max}$ is the maximum value of the gas velocity,

W is the total width of the channel.

The temperature profile in the transverse direction is given by the Reynolds Analogy for the fully-developed incompressible turbulent flow through a pipe. It assumes a thermal boundary layer equal to the momentum thickness, δ and assumes linear variation of temperature across the boundary layer.

$$\delta = \int_{y=0}^{W/2} u/U_0 \cdot (1 - u/U_0) \cdot dy \quad \dots (4.8)$$

where, u is the axial velocity at distance y from the wall

or the electrode (m/s),

U_0 is the axial velocity at the center ($y = W/2$).

For the velocity profile expressed by Eq. 4.7, $\delta = 0.097 W/2$.

In the core of the flow, the stagnation temperature, T_o , is assumed to be constant.

$$T_o = T + \frac{U^2}{2C_p} \quad \dots (4.9)$$

The rate of heat loss to the wall per unit area H_w' , is given by the following equation:

$$H_w' = h.(T_\delta - T_w) \quad \dots (4.10)$$

where, h is the heat transfer coefficient at the wall,

$$h \approx k/\delta \quad (W/m^2/^{\circ}K),$$

T_δ is the temperature at the edge of the boundary layer,

$$H_w = H_w'.(\text{wall perimeter})$$

The mass, momentum and energy balance equations in the one dimensional strong conservation form are transformed into matrix equation as represented by Eq.4.11.

$$\left[A(I) \right] \cdot \frac{d}{dx} \left[U(I) \right] = \left[B(I) \right] \quad \dots (4.11)$$

where, $[A]$ -- Coefficient matrix (3x3)

$[B]$ -- Value matrix (3x1)

$[U]$ -- State parameter matrix (3x1)

The individual elements of each of these matrices are given below:

$$[A] = \begin{bmatrix} \rho & \frac{U \cdot R}{T} & -\frac{S \cdot U}{T} \\ 2\rho U & 1 + \frac{U^2}{RT} & -\frac{U^2 S}{T} \\ \rho C_v T + P + \frac{3}{2} \rho U^2 & \frac{U + C_v U + \frac{3}{2} U^3}{R} & -\frac{\rho U^3 + k d T}{T dx} \end{bmatrix}$$

$$[B] = \begin{bmatrix} -\frac{\rho \cdot U \cdot dA}{A dx} \\ -\frac{\rho U^2 \cdot dA}{A dx} - C_f \frac{\rho U^2}{2 D_h} - J_y \cdot B_z \\ -\frac{U \cdot dA}{A dx} \cdot (\rho C_v T + \frac{3}{2} \rho U^2 + P) - H_w - J_y E_y + \frac{k d^2 T}{dx^2} \end{bmatrix} \quad \frac{d[U]}{dx} = \begin{bmatrix} \frac{dU}{dx} \\ \frac{dP}{dx} \\ \frac{dT}{dx} \end{bmatrix}$$

The average values of state parameters are updated and advanced along the flow direction (X direction) using a finite difference method. The values of all the elements in the [A] and [B] matrices are calculated at the axial node (I) and the matrix equation is solved to get the axial gradient of the [U] matrix. The finite difference method incorporates a predictor-corrector scheme similar to the Explicit MacCormack Method. The predictor step uses forward differencing and a corrector step with central differencing is used to eliminate the one-sided bias of the forward differencing step.

The steps used for advancing the solution from axial node (I) to node (I+1) are described below:

Step-I Evaluate each element in the [A] and [B] matrices using the values of the state parameters at the node (I) and form the matrix equation,

$$\left[A(I) \right] \cdot \frac{d}{dx} \left[U(I) \right] = \left[B(I) \right]$$

Step-II Solve this equation using matrix inversion and evaluate $\frac{d}{dx} \left[U(I) \right]$

Step-III Estimate the values of the state parameters at the node (I+1) using their differential values along the axial direction. (This is a predictor step with forward differencing. The asterix mark above the state parameter U indicates that it is only a first guess at the value.)

$$U^*(I+1) = U(I) + \delta x \cdot \frac{d}{dx} U(I) + \frac{\delta x^2}{2} \cdot \frac{d^2}{dx^2} U(I) \quad \dots (4.12)$$

Step-IV Calculate the average value of the state parameter for the interval from node (I) to node (I+1)

$$U(I+\frac{1}{2}) = [U(I) + U^*(I+1)] / 2.0 \quad \dots (4.13)$$

Step-V Redefine [A], [B] using $U(I+\frac{1}{2})$ to give,

$$\left[A(I+\frac{1}{2}) \right], \left[B(I+\frac{1}{2}) \right]$$

and substitute in the matrix equation (Eq.4.11).

Step-VI Solve for $\frac{d}{dx} U(I+\frac{1}{2})$ using matrix inversion

Step-VII Repeat step-III to calculate $U(I+1)$ using the new differential values of the state parameters at the intermediate node $(I+\frac{1}{2})$. (This is a corrector step incorporating central differencing.)

This method is stable only in the subsonic range of flow velocities as the solution advances only in the forward direction. The typical operating velocities are chosen to be from 800 to 1800 m/s, which are well below the sonic velocity in helium at the operating temperatures (about 3500 m/s).

This method estimates the variation of average values of pressure, temperature, density and velocity at each axial node. The local values of velocity, current density and temperature at each cross-section are different from the average values. By incorporating the local values of state parameters in the momentum balance equation (Eq. 4.2) the variations of axial velocity and current density in the transverse directions (z -directions) at each axial node are calculated. The average values of the thermal and flow parameters which are estimated at earlier steps, serve to calculate normalizing factors.

Another important feature of the medium and strong interaction MHD generator is the significant modification in the applied magnetic field due to fluid motion. Each current element in the MHD channel induces its own magnetic field in the surrounding space which is given by the Biot-Savart law

(Eq. 2.13). The total induced magnetic field due to each current segment, adds vectorially to the applied field. This modification in the magnetic field changes the current density distribution and magnetic force values, which in turn change the values of the state parameters in the channel. The changes in the current density pattern further change the induced magnetic field values. To incorporate these coupled flow and electrodynamic phenomena, the computer model uses the following iterative scheme,

Figure 4.1 The Iterative Scheme

The rate of convergence in this iterative scheme is very fast for low values of the Magnetic Reynolds Number R_m . For a wide range of medium and strong interaction MHD channel problems which were modeled, the convergence was achieved within 4 iterations for $R_m \leq 0.4$ and within up to 10 iterations for $R_m \geq 0.7$ (to give better than 1% accuracy).

CHAPTER 5

RESULTS AND DISCUSSION

5.1 Design of 25, 100 and 400 MW MHD Generators

Using the quasi-one dimensional computer model a detailed study was performed so as to select typical operating parameters for the 25, 100 and 400 MW_e MHD power generator. The operating ranges of the gas temperature, pressure and flow velocity and the strength of the applied magnetic field are chosen to enable achieving the desired power levels of the MHD channel and to suit the design of the overall system.

The variation of the cross-sectional area of the MHD channel plays a major role in determining the variation in other parameters of the system and the performance of the MHD generator. A 25 MW_e MHD generator with a constant cross section area is selected and the operating conditions are specified in Table 5.1. The variation of the axial velocity along the length of the channel is plotted in Fig. 5.1. The drop in the pressure in the axial direction is faster than the drop in temperature which leads to decrease in the gas density along the flow direction. This results in monotonic

Figure 5.1. Variation of the Axial Velocity of the Gas with the Distance along the MHD Channel.

increase in the flow velocity along the channel length as illustrated by the graph.

The increase in the gas velocity at constant area is extremely fast at higher power levels. The current density and power density also increase along with the gas velocity. For effective utilization of the MHD channel volume and available operating conditions, it is desirable to choose a diverging type MHD channel so as to maintain the velocity almost constant.

The two electrode plates apply the electric field E_y across the MHD channel, which can be given by $E_y \approx V/W$, where V is the potential difference between the electrodes in volts and W is the width (in the Y direction) in meters. To keep the current density constant, the electric field has to be maintained at the same value along the MHD channel (Eq.2.19). Hence the width W of the MHD channel is maintained constant and the breadth along the Z direction is increased to get a diverging type channel geometry.

The results were obtained using the computer model under various operating conditions and a comparative study was performed. The angle of divergence of the cross section area was adjusted to match the inlet and outlet velocities. The reason for keeping the axial velocity more or less constant along the channel is to have a uniform distribution of current density and hence, the power density, ensuring a full utilization of the channel volume. A set of typical

operating values for each parameter are selected for the output power levels of 25, 100 and 400 MW_e. The operating conditions and performance results of the MHD system are given in Table 5.1.

Table 5.1 Design of 25, 100 and 400 MW MHD Generators

Channel Geometry -

Length (X-direction) = 1.5 m, Width (Y-direction) = 0.7 m,

Working Fluid - Partially ionized helium

<u>Case</u> →	I 25 MW	II 100 MW	III 400 MW
Flow rate (Q)	275 kg/sec	275 kg/sec	270 kg/sec
Inlet Temperature (T _i)	2800 °K	2800 °K	3500 °K
Inlet Pressure (P _i)	4.00 MPa	4.00 MPa	4.00 MPa
Inlet Velocity (U _i)	800 m/sec	800 m/sec	980 m/sec
Inlet Area (A _i)	0.5 m ²	0.5 m ²	0.5 m ²
Exit Temperature (T _e)	2775.4 °K	2730.3 °K	3216.3 °K
Exit Pressure (P _e)	3.77 MPa	3.37 MPa	2.35 MPa
Exit Velocity (U _e)	842.7 m/sec	797.8 m/sec	985.7 m/sec
Exit Area (A _e)	0.5 m ²	0.58 m ²	0.77 m ²
Magnetic Field (B)	2.2 tesla	4.0 tesla	4.0 tesla
Mag. Reynolds No. (R _m)	0.10	0.10	0.24
Variation in B (ΔB)	0.06	0.12	0.41
Wall Temperature (T _w)	2200 °K	2200 °K	2200 °K
Heat loss to walls (H)	+0.34 MW	+0.32 MW	+0.55 MW
Internal Energy (C _v T)	-21.1 MW	-59.9 MW	-242.2 MW
Work done (W _o)	-13.3 MW	-39.9 MW	-152.7 MW
Kinetic Energy (P _{KE})	+9.6 MW	-0.5 MW	+1.5 MW
Voltage (V)	448 V	896 V	1097.6 V
Current (I)	55.1 KAmper	110.6 KAmper	364.7 KAmper
Output Power (P _{out})	24.3 MW	99.7 MW	400.3 MW

5.2 Variation of Parameters along the Flow Direction

Using the computer model, the variation of various parameters along the flow direction is estimated and the detailed results for Case III (output power = 400 MW_e) are presented here. The analysis of this variation enhances the basic understanding of the physics of the problem and can be used for improving the performance of the MHD system.

The cross-sectional area of the MHD channel increases along the flow direction and the angle of divergence is so adjusted to match the inlet and outlet velocities. The variation of the axial velocity of the gas along the channel is plotted in Fig.5.2. As can be seen from the graph the velocity does not remain constant but varies along the length of the channel. The velocity keeps decreasing until the mid-section of the channel and then increases back to the original value.

This typical variation of the velocity is the result of interplay between all the parameters, but it can be partially understood by analyzing the variation of pressure along the length of the channel (Fig.5.3) and the work input due to the pressure drop. The linear variation of pressure and area lead to a peaking of their product at the mid-section as can be seen from the results shown in Table 5.2.

Figure 5.2. Variation of the Axial Velocity U of the Gas with the Distance X along the MHD channel

Figure 5.3. Variation of the Pressure P with the Distance X along the MHD Channel

Table 5.2 Calculation Table

Cross-Section	Area A (m ²)	Pressure P (MPa)	P.A (N)	Velocity U (m/s)	P.A.U Wo(MW)
Inlet	0.50	4.00	2.00	980.0	1960.0
Middle	0.64	3.175	2.032	930.0	1874.9
Exit	0.78	2.35	1.832	985.7	1805.8

The net input of power due to the work associated with the pressure is 154.2 MW, which is the difference between the inlet and outlet W_o values. It is noticed from the table that the product P.A peaks at the center. The lower value of velocity at the mid-section ensures that the value of W_o decreases more or less uniformly along the channel, which means a continuous input of the work associated with the pressure.

The current density J_y is directly proportional to the axial velocity of the conducting gas and hence, its variation along the channel, as shown in Fig.5.4, is similar to that of the velocity. As can be seen from the graph, the current density increases more rapidly near the exit of the MHD channel. This rapid increase can be attributed to two factors. First, the percentage drop in the pressure along the flow direction is far higher than the percentage drop in temperature, which results in increased electrical conductivity towards the exit. Second, the value of the net magnetic field keeps rising along the channel and reaches

maximum at the exit (reference Fig.5.6). Since the current density is directly proportional to the conductivity and magnetic field, the rise is very fast at the exit.

The graph of temperature versus axial distance, as shown in Fig.5.5, is more or less linear, but it has slight drooping nature near the exit. More energy is converted into MHD output towards the exit end due to high values of current density. The kinetic energy also is again built up in the latter half of the MHD channel. This energy is extracted from the internal energy of the gas leading to a faster drop in temperature near the exit.

5.3 Different Designs for the 1000 MW MHD Generator

At higher power levels (of the order of 1000 MW_e) maintaining the volume of the MHD channel almost constant requires very high power densities. This can be achieved either by increasing the axial velocity or the temperature or the strength of magnetic field. These three alternatives were analyzed using the computer model and the results are tabulated in Table 5.3. The values of the temperature drop, pressure drop, flow rate, heat loss rate and output voltage can be compared using the table. A comparative study of these designs is helpful in selecting a set of operating conditions best suited for the design of the overall system.

It should be noted that in Case IV the flow rate of the gas is considerably higher as compared to other cases due

Figure 5.4. Variation of the Current Density J_y with the Distance X along the MHD Channel

Figure 5.5. Variation of the Temperature T with the Distance X along the MHD Channel

to high value of the inlet velocity. The high operating temperature in Case V leads to higher electrical conductivity and hence, the output current is larger.

Table 5.3 Designs for 1000 MW MHD Generator

Channel Geometry - Length $L = 1.5$ m, Width $W = 0.7$ m,

Working Fluid - Partially ionized Helium

<u>Case</u> →	IV High Vel.	V High Temp.	VI High Mag. B
Flow rate (Q)	705 kg/sec	418 kg/sec	496 kg/sec
Inlet Temperature (T_i)	3500 °K	4155 °K	3500 °K
Inlet Pressure (P_i)	6.00 MPa	6.00 MPa	6.00 MPa
Inlet Velocity (U_i)	1707 m/sec	1200 m/sec	1200 m/sec
Inlet Area (A_i)	0.5 m ²	0.5 m ²	0.5 m ²
Exit Temperature (T_e)	3233.7 °K	3686.5 °K	3105.5 °K
Exit Pressure (P_e)	3.54 MPa	2.94 MPa	2.91 MPa
Exit Velocity (U_e)	1693 m/sec	1212 m/sec	1221 m/sec
Exit Area (A_e)	0.795 m ²	0.905 m ²	0.90 m ²
Magnetic Field (B)	4.0 tesla	4.0 tesla	5.65 tesla
Mag. Reynolds No. (R_m)	0.41	0.44	0.31
Variation in B (B)	0.81	0.92	0.85
Wall Temperature (T_w)	2200 °K	2200 °K	2200 °K
Heat loss to walls (H)	+0.56 MW	+0.93 MW	+0.54 MW
Internal Energy ($C_v T$)	-591.8 MW	-609.4 MW	-615.3 MW
Work done (W_o)	-369.8 MW	-377.1 MW	-381.5 MW
Kinetic Energy (P_{KE})	-14.8 MW	+6.1 MW	+12.3 MW
Voltage (V)	1915.2 V	1344.0 V	1883.2 V
Current (I)	524.0 KAmper	745.3 KAmper	531.2 KAmper
Output Power (P_{out})	1003.6 MW	1001.4 MW	1000.7 MW

5.4 Analysis of the Electrodynamic Phenomena

Three different types of electrodynamic phenomena are analyzed here, which are peculiar to the MHD generator. The analysis of these phenomena give us a deeper insight into the characteristics of the system.

5.4.1 The Modification in Applied Magnetic Field

As discussed in Section §2.2.5, the current density pattern in the MHD channel volume induces its own magnetic field at each point. The induced magnetic field adds vectorially to the applied uniform magnetic field, resulting in a nonuniform net magnetic field. This modification in the magnetic field depends upon the electrical conductivity of the fluid, its axial velocity, the strength of the applied magnetic field and the MHD channel length.

The variations of the net magnetic field along the length of the MHD channel are plotted for cases II, III, IV and V in Fig. 5.6. Each graph shows that the the induced magnetic field opposes the applied magnetic field in the section near the inlet of the MHD channel resulting in decreased values of the net magnetic field. Incontrast, in the section near the exit both the fields add to give higher net magnetic field values.

Figure 5.6. Variation of the Net Magnetic Field with the Distance X along the MHD channel

Figure 5.7. Percentage Variation ($B/B \times 100$) in the Magnetic Field Expressed as a Function of Magnetic Reynolds Number R_m

The net change in the magnetic field (B) in the axial direction can be expressed in terms of the percentage value of the applied magnetic field. The resulting percentage value is plotted as a function of the Magnetic Reynolds number (R_m), which is defined earlier in Section §2.2.4, in Fig. 5.7. The graph shows that as the system operates in the high Magnetic Reynolds number regime, i.e. stronger magnetic interaction with the fluid flow, the effect of induced magnetic field becomes more significant.

5.4.2 Analysis of the Transverse Profiles of Axial Velocity and Current Density

Fig. 5.8 shows the axial velocity profiles in the transverse direction (Z -direction) for case III (400 MW_e) at the inlet and exit of the MHD channel. The results show that in the presence of the magnetic field, the original velocity profile gets modified.

The current density near the sidewalls is lower as compared to that at the center due to the lower conductivity and the smaller velocity. The current density may have an opposite direction at the two edges. This results in smaller value of opposing Lorentz force near the sidewalls and the velocity tends to build up here. The axial velocity profile changes its shape as the fluid progresses in the flow direction. The typical velocity profile at the exit shows

Figure 5.8. The Axial Velocity U Profile in Transverse Z Direction (along the Magnetic Field B)

Note: The breadth along the Z -direction is normalized to 2m.

Figure 5.9. The Current Density Profile in Transverse Z Direction (along the Magnetic Field B)

Note: The breadth along the Z-direction is normalized to 2m.

thinning of the boundary layers near the sidewalls and slight overshoots at the boundary layers.

It is important to analyze the physical cause of these overshoots at the sidewalls. When the value of axial velocity in the boundary layer becomes equal to that at the center, the current density is still smaller due to the lower temperature and hence the lower conductivity. Thus, the velocity can further increase leading to overshoots. The resulting thinning of the boundary layer increases the temperature restricting the overshoot. The viscous drag force at the walls also serves to limit the overshoot. The effects are more pronounced at higher power levels.

Fig. 5.9 shows the current density profile at the inlet and at the exit of the MHD channel for the Case III. The current density profile is a combination of the axial velocity profile and the electrical conductivity profile. The current density at the exit is more uniformly spread out and almost flat.

5.4.3 The Presence of Space-charge in the MHD Channel

For the Faraday type MHD channel, the axial current density J_x is given by

$$J_x = -w\tau J_y \quad \dots \text{Section } \$2.4$$

At standard operating conditions the Hall parameter ($w\tau$) is less than 0.2. When a diverging type channel is used

the current density J_y is maintained nearly constant along the axis. Hence,

$$\frac{\delta J_x}{\delta x} \approx 0.2 \frac{\delta J_y}{\delta x} \approx 0 \quad \dots (5.1)$$

At steady state, the conservation of electric charge implies that,

$$J = 0 \implies \frac{\delta J_x}{\delta x} + \frac{\delta J_y}{\delta y} + \frac{\delta J_z}{\delta z} = 0 \quad \dots (5.2)$$

Hence, $\frac{\delta J_y}{\delta y} = 0$, or the J_y is constant in Y -direction.

$$J_y(y) = \text{constant} = \sigma \cdot (B_z \cdot U_x(y) - E_y(y)) \quad \dots (5.3)$$

Thus the electric field E_y adjusts itself according to the variation of axial velocity U_x in the Y -direction, so as to maintain the value of the current density J_y constant. The electric field E_y has an integral boundary condition given by,

$$\int_{-W/2}^{W/2} E_y \cdot dy = V = I \cdot R \quad \dots (5.4)$$

where, I is the total current through the external load resistance R and V is the voltage drop across it.

In the boundary layer near the electrodes the partial derivative of the axial velocity with respect to distance in Y -direction is very high. High value of $\frac{\delta U_x}{\delta y}$ implies that

the value of $\frac{\delta E_y}{\delta y}$ is large. Also in the neighbourhood of the metallic electrode the tangential components of the electric field E_x and E_z are zero.

The Maxwell's equation (Eq.2.1) reduces to,

$$\frac{\delta E_y}{\delta y} = \frac{\Omega}{\epsilon} \quad \dots \text{For } E_x = E_z = 0 \quad \dots (5.5)$$

where, ϵ is the permittivity of the medium,

Ω is the free space-charge density.

Thus the equations predict the presence of space-charge near the electrodes and its density directly reflects the variation of the axial velocity in the Y -direction. An experimental method can be devised to measure the variation in the electric field (E_y) values along the Y -direction and from this variation the profile of the axial velocity in the Y -direction can be calculated.

5.4.4 The Effect of Changing the Load Factor

The load factor, L_f , of an MHD generator is defined as the ratio of the value of the transverse electric field (E_y) applied by the electrodes to the emf generated by the velocity of the conducting fluid transverse to the magnetic field ($U.B$). In the present study the load factor is defined using the inlet velocity and the applied magnetic field.

$$L_f = \frac{E_y}{U_i \cdot B} \quad \dots (5.6)$$

Figure 5.10. Variation of the Output Power with the Load Factor L_f

The net power output of the MHD generator depends on the value the load factor. Theoretically the power output of the MHD system is maximum for $L_f = 0.5$. The variation of the power output of the system, estimated using the computer model, with the load factor is plotted in Fig.5.10. The graph shows that the maximum of the output power is at $L_f = 0.4$. The reason for this discrepancy is due to the average value of the axial velocity being lower than the value at the inlet of the MHD channel. Hence, the average value of the load factor is higher than what is defined here.

5.5 The Effect of Changing the Length of the MHD Channel

It is important in the design analysis of the MHD generator to study the effect of changing the length of the MHD channel. By increasing the length of the channel, the Magnetic Reynolds Number of the system also increases. As a result, the variation in the magnetic field (B) is induced to a greater extent, as discussed in Section §5.4.1. Four different cases are studied here by changing the length of the channel ($L = 1.5, 2.25, 3, 4\text{m}$) and net magnetic field results are plotted in Fig.5.11. In each case in the cross-sectional area at the inlet is 0.2 m^2 and the flow rate is maintained at 86 kg/s .

As the length of the MHD channel is increased, the characteristics of the axial variation of the temperature, pressure, axial velocity and current density, as discussed in

Section §5.2, become more pronounced. These characteristics can be compared from the graphs in the Figs.5.12-5.15.

The net output power verses the length of the channel is plotted in the Fig.5.16 and the variation is almost linear. But the average value of the cross-sectional area is higher for the channels with longer length, hence, the average power density is not the same for all the cases. The variation of the average power density with the length of the MHD channel is shown in Fig.5.17. For longer length channels the temperature drop is more which lowers the electrical conductivity towards the exit; consequently the current density cannot rise back to its entrance level (refer to Fig.5.15). The lower values of the average current density lead to lower average power density for the longer channels.

Figure 5.11. Variation of the Net Magnetic Field with the Normalized Distance X_n along the MHD channel for Different Values of the Channel Length L .

Figure 5.12. Variation of the Temperature T with the Normalized Distance X_n along the MHD channel for Different Values of the Channel Length L .

Figure 5.13. Variation of the Pressure P with the Normalized Distance X_n along the MHD channel for Different Values of the Channel Length L .

Figure 5.14. Variation of the Axial Velocity U with the Normalized Distance X_n along the MHD channel for Different Values of the Channel Length L .

Figure 5.14. Variation of the Current Density J with the Normalized Distance X_n along the MHD channel for Different Values of the Channel Length L .

Figure 5.16. Variation of the Power Output P_o with the Length L of the MHD Channel.

Figure 5.17. Variation of the Power Density PD_o with the Length L of the MHD Channel.

CHAPTER 6

THREE-DIMENSIONAL COMPUTER MODEL

6.1 Extension of the Work to a 3-D Model

A quasi-one dimensional model reported in the earlier chapters is very useful for the preliminary studies. The model is based on a simplified phenomenological approach and the results obtained give an overall view of the response of the system to various types of operating conditions. For more accurate and reliable results, a three dimensional model encompassing all the fluid mechanics, electrodynamic and thermal phenomena has to be developed. A detailed 3-D model will be an extension of this work.

A scheme for the development of the 3-D model for the MHD channel is cited here. In this scheme the fluid mechanics equations and electrodynamic equations are decoupled from each other and solved separately. The fluid mechanics model and electrodynamics model are coupled externally and the solution is iterated back and forth to satisfy both sets of equations. An additional input to the electrodynamics model is provided by the electrical properties package. The electrical conductivity of the gas is a function of temperature, pressure and various other

factors. The local value of the conductivity at every point in the MHD channel volume is essential. The coupling between the models is indicated by the diagram (Fig. 6.1) of the overall scheme of the proposed 3-D model.

Figure 6.1. Overall Scheme of the 3-D Model

An experimental work has to be done to express the electrical conductivity of the gas mixture in an exact functional form. The Lorentz force term f acts as a body force term and energy term Q acts like a external source term for the fluid mechanics model. In both fluid mechanics and electrodynamics model the solution is advanced and updated in finite time steps. The solution converges to the steady state solution as it progresses in time. This approach of solving the equations is very common in fluid mechanics and a large number of advanced techniques such as Mac Cormack method or Beam Warming method are available. In Section §6.2, the efforts are made to formulate the solution scheme for the electrodynamics model.

6.2 Electrodynamics Model

6.2.1 Basic Equations

The proposed model assumes an initial solution and as it differs from the steady state solution it generates transient terms which update the solution as it advances in time. As the time progresses the transient terms tend to restore the balance and the solution converges to a steady state solution. As the steady state solution approaches the transient terms tend die out and all the variables reach a constant values in time.

The basic equations in electrodynamics are the Maxwell equations, which are discussed earlier in Section §2.2 :

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon} \quad \dots (2.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad \dots (2.2)$$

$$\nabla \times \mathbf{E} = - \frac{\delta \mathbf{B}}{\delta t} \quad \dots (2.3)$$

$$\nabla \times \mathbf{B} = \mu \cdot (\mathbf{J} + \epsilon \cdot \frac{\delta \mathbf{E}}{\delta t}) \quad \dots (2.4)$$

The Eq.2.1 & 2.2 are the scalar equations. The Eq.2.3 & 2.4 are the vector equations and lead to three scalar equations each. This forms a set of eight equations. The first two equations do not have differential terms with

respect to time and hence cannot generate any transient terms and it is advantageous to satisfy them separately.

The first two equations (Eq.2.1 & 2.2) can be eliminated because of the following reasons:

(i) The Eq.2.3 & 2.4 form a set of six scalar equations and six unknowns (3 components of E and B each). Where as the Eq.2.1 is the seventh equation and introduces an additional variable, which is the local free charge density $\Omega(x,y,z,t)$. Hence it does not help in solving the equations for the desired variables. This equation can be used at the end of the solution to calculate the local space charge density as a function of position inside the MHD channel.

(ii) The Eq.2.2, in its special case form, can be derived from the Eq.2.3 as follows:

$$\nabla \times E = - \frac{\delta B}{\delta t} \quad \dots (2.3)$$

$$\nabla \cdot (\nabla \times E) = - \nabla \cdot \frac{\delta B}{\delta t}$$

$$\frac{\delta}{\delta t} (\nabla \cdot B) = 0$$

$$\nabla \cdot B = \text{constant (not a function of time)} \quad \dots (6.1)$$

In this model, the solution is updated using very small but finite time steps, till the transient terms wane out and the solution reaches a steady state. The Eq.6.1 implies that as time changes the local value of the $\nabla \cdot B$

remains constant. Hence if we ensure the continuity of the magnetic field ($\nabla \cdot \mathbf{B} = 0$) all across the volume in the beginning as the initial condition, it will be maintained as the solution progresses in time. But any small errors may keep accumulating and the continuity of the magnetic field will be eventually lost. To eliminate this possibility a rebalancing of the magnetic field has to be performed after a fixed number of time steps.

The six scalar equations from the remaining two equations, Eq.2.3 & 2.4 are converted into a matrix equation as follows [18]:

$$[Q]_t + [E]_x + [F]_y + [G]_z = [S] \quad \dots (6.2)$$

The subscript to each matrix in the above equation indicates the partial derivative with respect to that quantity. The individual elements of each matrix are given below:

$$[Q] = \begin{bmatrix} B_x \\ B_y \\ B_z \\ \mu_e E_x \\ \mu_e E_y \\ \mu_e E_z \end{bmatrix}, \quad [E] = \begin{bmatrix} 0 \\ -E_z \\ E_y \\ 0 \\ B_z \\ -B_y \end{bmatrix}, \quad [F] = \begin{bmatrix} E_z \\ 0 \\ -E_x \\ -B_z \\ 0 \\ B_x \end{bmatrix}, \quad [G] = \begin{bmatrix} -E_y \\ E_x \\ 0 \\ B_y \\ -B_x \\ 0 \end{bmatrix}, \quad [S] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\mu J_x \\ -\mu J_y \\ -\mu J_z \end{bmatrix} \quad \dots (6.3)$$

$$\text{where, } J_x = \sigma \cdot (E_x + vB_z - wB_y) + \mu_e \cdot (J_y B_z - J_z B_y)$$

$$J_y = \sigma \cdot (E_y + wB_x - uB_z) + \mu_e \cdot (J_z B_x - J_x B_z)$$

$$J_z = \sigma \cdot (E_z + uB_y - vB_x) + \mu_e \cdot (J_x B_y - J_y B_x)$$

...From Eq.2.8

where, μ_e is the mobility of the electrons in the medium.

$$\mu_e = \frac{e \cdot r}{m_e} \approx 0.05 \text{ (for typical operating conditions)}$$

Refer to Section §2.2 for more details about μ_e .

6.2.2 Input of the Model

To define the value of current density ($J \rightarrow J_x, J_y, J_z$) we need the velocity field at every point, which is obtained as an input from the fluid mechanics model.

$$\text{Velocity Field } V = ui + vj + wk$$

where, $u = u(x, y, z, t)$, $v = v(x, y, z, t)$, $w = w(x, y, z, t)$

6.2.3 Output of the Model

The following quantities are sent as an output to the fluid mechanics model:

(i) Lorentz Force f (force per unit volume)

$$f = J \times B = f_x i + f_y j + f_z k \quad \dots (6.4)$$

$$\text{where, } f_x = f_x(x, y, z, t) = J_y \cdot B_z - J_z \cdot B_y,$$

$$f_y = f_y(x, y, z, t) = J_z \cdot B_x - J_x \cdot B_z,$$

$$f_z = f_z(x, y, z, t) = J_x \cdot B_y - J_y \cdot B_x$$

(ii) Energy term Q (energy per unit volume per unit time)

$$Q = Q(x, y, z, t) = Q_J + Q_f$$

$$\text{Joule dissipation } Q_J = J^2 / \sigma \quad \dots (6.5)$$

$$\text{Work done } Q_f = f \cdot V = f_x \cdot u + f_y \cdot v + f_z \cdot w \quad \dots (6.6)$$

6.2.4 Solution Method

The whole MHD volume is divided into a three dimensional mesh. At the beginning an initial state is assumed by assuming the values of B , E and J at each point.

As the assumed values do not correspond to the steady state values the partial derivatives of these variables with respect to time are non-zero. These partial derivatives can be calculated by solving the matrix equation (Eq. 6.2) for the $[Q]_t$ values. By assuming a finite time step the values of the variables are updated. The matrix equation is redefined and solved again for $[Q]_t$. The values of the variables are continuously updated with this iterative scheme till the values of the variables assume an invariant state at every point in the volume. This invariant state corresponds to a steady state solution of the problem. A precaution has to be taken to be taken to ensure the continuity of the magnetic field. The field rebalancing has to be performed after a fixed number of time steps each, as discussed earlier.

6.2.5 Initial Conditions

The magnetic field B everywhere at the beginning is assumed to be equal to the applied magnetic field B_0 . The velocity field V is provided as an input by the fluid mechanics model. The electric field E and the current density J are considered to be only in Y -direction. The values E_y and J_y are calculated from the strength of the magnetic field B_0 by solving simultaneously the following equations (Eqs. 6.7 & 6.8):

$$E_y = \frac{V}{W} = \frac{I \cdot R}{W} = \frac{R}{W} \cdot (J_y \cdot A) \quad \dots (6.7)$$

$$J_y = \sigma_o \cdot (u_o B_o - E_y) \quad \dots (6.8)$$

where, A is the projection of the area of the electrode in the XZ-plane,

σ_o is the average value of the electrical conductivity over the entire MHD volume,

u_o is the average axial component of the velocity.

Outside the MHD volume the electric field E_y and current density J_y are zero everywhere.

6.2.6 Boundary Conditions

The exact boundary conditions for the MHD problem can only be defined at the infinity. But solving Eq.6.6 for infinitely large volume is impossible and hence a set of approximate boundary conditions, which matches closely to reality is defined. The boundary conditions are specified on the enclosure as shown in the diagram given below. The larger the size of the enclosure the more accurate will be the boundary conditions.

The boundary conditions are shown by the diagram in Fig. 6.1. They are expressed in terms of B_o , B_I .

Where, B_o is the externally applied magnetic field,

B_I is the magnetic due to current I.

B_I is given by the Biot Savart Law (Eq. 2.13).

$$B_I = \int dB_I = \int \frac{\mu}{4\pi} \cdot \frac{(I \cdot ds) \times R}{R^3} \quad \dots (2.13)$$

Figure 6.2 The Boundary Conditions for the MHD Channel

The total current I flowing through the external circuit is given by Eq. 6.9.

$$I = \int_{A \text{ (electrode)}} \mathbf{J} \cdot d\mathbf{A} \quad \dots (6.9)$$

The voltage drop V_L at the external load resistance R_L appears across the two electrodes. This gives the additional boundary condition as follows:

$$V_L = I.R_L = - \int_{\text{Cathode}}^{\text{Anode}} \mathbf{E} \cdot d\mathbf{l} = - \int E_y \cdot dy \quad \dots (6.10)$$

6.3 The Interaction with the Fluid Mechanics Model

Initially a steady state velocity field V is calculated using the fluid mechanics model assuming the magnetic force and energy terms to be zero. This acts as a first input to the electrodynamics model. After reaching steady state in the electrodynamics model a force field f and energy field Q are sent back to fluid mechanics model as an input. This introduces new transient fluid mechanics terms and after certain time steps again steady state is reached. Now a different steady state velocity field V appears as an input to electrodynamics model and the previous steady state values act as the initial conditions. This iterative scheme continues till both the models reach a steady state to give the final results.

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