Fractal Interrelationship in Field and Seismic Data

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Fifth Quarterly Technical Report - Contract No. DE-FG21-95MC32158

(21 Mar. 1995 to 21 June 1996)

TITLE: Fractal Interrelationship in Field and Seismic Data

BY

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5TH QUARTER STATUS

With the exception of some remaining field work and analysis of field data, the work under tasks 1 through 5 has been completed. With regard to field activities under Task 3, a topical report on these activities will be submitted during late August and discussed in the final report.

5th quarter action items:

1. Abstract accepted for the annual Baltimore meeting of the American Geophysical Union.
2. Abstract prepared and submitted for presentation at the forthcoming northeastern regional meeting of the American Association of Petroleum Geologists. Abstract accepted, and enclosed as Appendix 1.
3. Discussion submitted to the *Journal of Structural Geology* accepted (see 1st Quarterly report). Publication date has not been set as yet.
4. Paper titled *Some distinctions between compass and power spectral dimensions* was submitted to the journal *Mathematical Geology*. Paper included as Appendix 2.
5. Topical report regarding development of the compass walk program was completed. This topical report is enclosed as Appendix 3.
6. Final report outline prepared (see Appendix 4).

REQUEST FOR DATA: The primary goal of this study is to evaluate the possibility that the fractal characteristics of reservoir fracture systems might be inferred from the fractal characteristics of the reservoir reflector. Results discussed in the summary below provide support for such a view. The matter will, however, remain unresolved until fracture data acquired from core or FMS logs can be compared to reflection seismic data from the core areas.

If such data is available we would like to undertake its analysis and would consider undertaking the analysis at no additional cost to the contract.

SUMMARY: ADDITIONAL 5TH QUARTER STUDIES

A series of cross sections along the Middle Mountain syncline and Elkhorn Mountain anticline (Figure 1) constructed for the spring AGU meeting reveal significant differences in relief and relative abundances of different order folds. Structural relief across Section 1, for example, is nearly 2 km, and shortening is accommodated primarily by the long wavelength Middle Mountain/Elkhorn Mountain structure. To the southwest, shortening is accommodated more so by shorter wavelength folds, and structural relief

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across the first-order Middle Mountain/Elkhorn Mountain fold pair falls to approximately 1.2 km.

The fractal characteristics of topography were evaluated using 24 profiles spaced at approximately 600m intervals northeast to southwest through the area (locations referenced on Figure 2). Topographic profiles 1, 7, 13, 19, and 24 (Figure 2) correspond to locations of Sections 1 through 5 (Figure 1). Elevations were digitized at approximately 15m intervals along the profiles which were constructed from U. S. Geological Survey 7½ minute quadrangles. Roughness-length (Malinverno, 1990) and compass (Mandelbrot (1967) methods were used to compute fractal dimension. The roughness-length measure is based on the following relationship (Turcotte 1989, 1992)

$$\sigma = \tau^H \quad (1)$$

between the standard deviation (σ) and length of window (τ) over which σ is computed. H is referred to as the Hausdorf-Bessicovich dimension and is related to D through the relationship

$$D = 2 - H. \quad (2)$$

σ is computed from the average standard deviation of data in each of the τ -length subdivisions of the profile. Initially τ equals the length of the profile and it is subsequently diminished in size by a factor of 2 at each step. As τ decreases, σ is the average of the standard deviations computed from first 1, and then 2, 4, 8, etc. subdivisions of the profile. Log-log plots of σ versus τ are linear for fractal distributions (e.g. Figure 3A) and have slope H . In this study the standard deviation associated with the largest window was not included in the computation of H . Symmetry in the profile will lead to invariance in σ between the first and second values resulting in an anomalously high D .

The compass dimension is derived from the relationship

$$N = Cr^{-D}. \quad (3)$$

Here N represents the number of steps taken to traverse the profile, r is the step-length, and D , the fractal dimension. Equation (3) is proposed by Mandelbrot (1967) as a modification of Richardson's (1961) observations. Log-log plots of N versus r are linear with slope $-D$ when N has a fractal distributions (e.g. Figure 3B).

The roughness-length dimensions of topography (Figure 4A) rise gradually from 1.2 to 1.35, northeast to southwest along profiles 1 through 8. This rise is followed by smaller, irregular, increases of D from 1.35 to 1.4 between profiles 8 to 19. In the southwestern part of the area (profiles 20 through 24) fractal dimension drops to an average of approximately 1.33. Although there is greater variability in the results of the compass walk (Figure 4B) the pattern of variation along the structures is similar to that obtained using the roughness-length method (Figure 4A). The compass dimension is considerably smaller than the roughness-length dimension. The two procedures measure different but related quantities. The compass-walking measurements describe the length of the profile measured at different scales, while roughness-length measurements describe the relative variation of relief at different scales. Linear regression analysis yields a correlation coefficient of 0.81 between the compass and roughness-length dimensions shown in Figure 4.

Roughness-length and compass dimensions were computed for each of the cross sections across the Middle Mountain/Elkhorn Mountain structure. Variation in the fractal dimension of the structural profiles are compared to those of the associated topography

(Figures 5 and 6). Regression analysis of the compass and roughness-length measurements yield correlation coefficients of 0.82 and 0.91, respectively. Although the size of the data set is limited, the result suggests a linear relationship between the fractal characteristics of topography and structure.

Topography in the northeastern part of the area is dominated by the Middle and Elkhorn mountains. To the southwest, these mountains are dissected into a series of smaller hills and valleys. This change of landscape is described by increases of fractal dimension, which imply an increase in surface roughness. Roughness, in this context, is used to describe an increase in the amplitude of the shorter-wavelength relative to the longer-wavelength features in the profile. Structural cross sections 1 through 5 reveal similar increases in the abundance of smaller wavelength folds to the southwest. Again, these changes are represented by increased fractal dimension northeast to southwest through the area.

The linear relationship between the fractal characteristics of topography and structure quantifies the effect of deformation on the resistance to erosion of exposed rock formations. Observations used to construct the geologic cross sections are in many instances very sparse, particularly in this highly vegetated area. Information about topography, on the other hand, is quite detailed, and there is no need for interpolation. The fractal characteristics of topography provide a useful guide to locate and help quantify the extent of associated deformation intensity. The fractal characteristics of folds may also provide a useful tool for structural analysis since they estimate and allow one to incorporate the presence of small folds not represented on the regional scale cross sections (e. g. Wu 1993). Applications of fractal analysis in strain estimation should seek field verification for the presence of predicted short wavelength folds (e. g. Dunne and Ferrill, 1993).

Appalachian Plateau

Seismic data across the Plateau study area reveals the presence of normal faults in the basements underlying a relatively undeformed shallow section. Conversion to depth of a line from our data set (Figure 7) indicates that the basement faults have a combined offset of about 650m, and structural relief across the shallow Mississippian Greenbrier limestone reflector never exceeds 50m. The faults are associated with the east margin of the Rome Trough or Eastern Interior aulocagen. The seismic line crosses an oil field that produces from the Big Injun Sandstone which lies just below the Mississippian Greenbrier Limestone (Figure 7).

Seismic data across the area reveals that the rift-margin faults do not extend to the surface, which is covered by a flat lying blanket of Pennsylvanian sediments. Twenty four topographic profiles (Figure 8) were constructed across the area. Elevations were digitized from USGS 7½ minute topographic quadrangles at 15m intervals. Twelve profiles were constructed along a northeast-southwest trend of the subsurface structure and twelve profiles were constructed in the dip direction. The dip profiles (Figure 8) illustrate an almost random pattern of elevation variation. No significant differences in fractal dimensions occur in the area; however, an average D of 1.49 distinguishes the topography of this area significantly from that of the Valley and Ridge.

The fractal characteristics of the subsurface were analyzed and related to the surface using a set of six vibroseis seismic lines from the area. The fractal dimensions of interpreted reflection events (Figure 7) were computed for all seismic lines. Average values of fractal dimension and their standard error (Figure 9) reveal abrupt between the basement and Rutledge reflection events, and the Onondaga and Huron events. The basement and Rutledge events span the middle and late Cambrian period of geologic history. It was during this time that the main episode of rifting occurred (Shumaker and Wilson 1996). The change in fractal dimension quantifies the differences in reflector geometry. The lower fractal dimension of the basement suggests that larger differences in structural relief occur from scale-to-scale along the basement surface.

Subsurface intervals coinciding with the Rutledge through Onondaga reflection events extend from the Late Cambrian through Lower Devonian Periods. The fractal dimensions of reflectors in this interval do not vary significantly and are on average approximately 1.29. Reactivation of basement faults in the area during this interval of time were relatively minor (Wilson et al. 1994). Another jump in fractal dimension occurs between the Lower Devonian Onondaga Limestone ($D=1.28$) and the Middle Devonian Huron Shale ($D=1.4$). Analysis of slickensides observed in cores from the Appalachian foreland (Evans 1980) suggest that the Middle Devonian shales are a decollement interval. Interpretation of the seismic data used in this study (Wilson et al. 1994; Hohn et al. 1994) reveal the presence of subtle detached structures above the Onondaga Limestone. The shallow Greenbrier and Huron reflection events have higher fractal dimension due to the presence of minor folds developed above a Middle Devonian decollement. Detached structures are concentrated in areas west of the Trough-margin.

The average fractal dimension of the shallow Greenbrier Limestone reflection event computed for portions of lines west of the margin (1.41) is slightly higher, but not statistically different from that for portions of the line east of the margin (1.36). The absence of significant differences in the fractal characteristic of surface topography are consistent with the lack of significant variation in near-surface structure.

As noted last quarter, one of the primary interests of this contract is to test the hypothesis that fractals might characterize structural differences within productive intervals. The preceding analysis supports the validity of that hypothesis. The fractal dimension increases across a detachment interval. The seismic studies reveal vertical variability that could be used diagnostically in the extent of detachment related deformation in different regions of a seismic data base.

The variation of fractal dimension of the Greenbrier/Big Injun reflection event was calculated (Figure 10). Although the line is short, and a smaller calculation window was used to compute D , the calculated fractal dimension is observed to rise across the central more productive areas of the field. It has already been shown (Wilson, 1987) that increased detachment produces increases of fracture intensity, hence, the results obtained thus far infer that the productive intervals above the decollement will be more intensely fractured.

Conclusions

In the plateau study area (Granny Creek) where surface deformation is subtle-to-absent, no change is observed in the fractal characteristics of topography across the area.

Large subsurface structures associated with the Rome Trough were largely inactive during and following deposition of near-surface intervals. Analysis of seismic data across the area reveals the presence of abrupt vertical changes in the fractal dimensions of reflection events. The largest differences occur between syn-and post-rift sediments and across a decollement zone. The increase of fractal dimension across the decollement zone implies that intervals above the decollement are rougher or more highly deformed. Variations in the fractal dimension of the producing interval across the Granny Creek field indicate that the producing area within the field is a rougher or more highly deformed region.

Near-surface deformation in the Middle Mountain and Elkhorn Mountain area of the Valley and Ridge province is significant. In this area the fractal dimension of topography is linearly related to the fractal dimension of underlying structure. Comparison of the fractal variability of Valley and Ridge structures with those observed in seismic data from the Plateau indicate that the increased fractal dimension of reflection events implies greater relative abundance of higher order or smaller wavelength structures.

Results from the seismic evaluation suggest that fractal analysis might provide a useful exploration tool in cases where one is interested in locating subtle detached structures or identifying fractured reservoirs.

Results from the Valley and Ridge area suggest that, in active tectonic areas, fractal analysis may provide a means to assess the relative frequency of earthquake activity over time periods that extend beyond the historical record.

References (Complete set of references will be provided in the final report).

FIGURE 1

FIGURE 3

A.

B.

FIGURE 4

FIGURE 5

Compass Walk

FIGURE 6

FIGURE 7

FIGURE 8

FIGURE 9

Mississippian Big Injun Sandstone

FIGURE 10

Appendix 1

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Applications of fractals in the characterization of geologic data in the central Appalachians of West Virginia

This study evaluates the fractal characteristics of topographic and structural relief, drainage, and topographic contours, and examines the possibility of using fractals to quantify interrelationships between them. Local studies in a total of four areas extending from the Valley and Ridge across the Appalachian Plateau reveal significant differences in the fractal dimension of surface topography and suggest that fractals may provide a quantitative measure of structural control on topographic development.

A detailed study in the Valley and Ridge reveals systematic changes in the fractal dimension of topography along major structures where they cross the Parsons CSD. The fractal dimension of structural relief was computed for several cross sections through the area, and a linear correlation was obtained between the fractal dimensions of topographic and structural relief.

The east margin of the Rome trough is defined by a major normal fault with several thousand feet of displacement. Fractal analysis of seismic profiles across the east margin of the trough reveal abrupt increases in fractal dimension between Precambrian basement and top of the Rome Formation, and between the Devonian Onondaga Ls. and Huron Shales. These differences are associated with Cambrian rifting and Alleghanian detachment, respectively. The east-margin fault was relatively inactive following the early Paleozoic and variation in the fractal dimension of surface topography is not observed locally across it; however, a significant difference is observed between the east and west margins of the trough suggesting the presence of possible regional influence of trough structure on topography.

In general our research indicates that fractals are a useful tool for quantifying interrelationships between various geological variables. More comprehensive studies are in progress.

Appendix 2:

Some distinctions between compass and power spectral dimensions

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Compass and power-spectral estimates of fractal dimension are widely used to evaluate the fractal characteristics of geological and geophysical variables. Compass measurements establish the self-similarity of a profile, and the fractal interrelationship can be used to estimate profile length at a certain scale. Linear scale transformations alter the compass dimension and distort profile length estimated from the compass-walk. The power spectral method provides a truly scale-invariant measure of relief variations along a profile since the fractal dimension obtained from the log-log slope of the power spectrum is invariant to linear scale transformations. This property of the power spectrum makes it a useful tool for comparing surfaces of diverse origin.

KEY WORDS: fractal dimension, compass method, power spectral method.

INTRODUCTION

This paper discusses three issues of interest in the application of compass and power spectral methods of fractal analysis to geological problems, including; 1) basic differences between the compass (Mandelbrot 1967, and 1985) and power-spectral (Mandelbrot and van Ness 1968) methods of fractal analysis; 2) the effect of magnification (Brown, 1987) to

compensate for large compass openings or undersampling of profiles, and 3) the scale invariant nature of the power spectral estimate of fractal dimension.

SELF-SIMILAR AND SELF-AFFINE FRACTALS

The physical significance of compass derived fractal interrelationships is well known (Mandelbrot 1985). N steps taken with compass opening r to cover the length of a curve, implies that the curve at that scale has length $L = Nr$. If the relationship between N and r is

$$N = \frac{C}{r^D} \quad 1)$$

then the curve is defined to be self-similar with fractal dimension D . The length of the curve L is Nr or Cr^{1-D} , and remains invariant to a rotation of coordinate axes. D derived from the compass walk is referred to as the compass dimension (D_C). It is this characteristic of self-similar fractals that allows one to predict curve length at a certain scale. Use of the compass method to estimate shortening (Wu, 1993), for example, assumes that the structural profile is self similar. This assumption can be made when a high correlation between N and r is obtained from equation (1).

Fractal curves may also be classified as self-affine. The path of an object as a function of time or the temporal variation of the earth's magnetic field at some point on the earth's surface are considered to be self-affine rather than self-similar (e.g. Turcotte, 1992). The notion of length is meaningless for such curves since the dependent and independent variables have different units. Klinkenberg (1994), however, notes that treatment of a curve as self-similar or self-affine may depend on the objectives of the research. Termonia and Meakin (1986), for example, use the compass method to evaluate the fractal dimension of cracks in models of polymeric systems and experimental data from crushed glass and fractured steel. The assumption of self-similarity is

implicit in their analysis. On the other hand, sea-floor topography is often characterized as self-affine (Fox, 1989, 1985; Malenverno, 1989; Mareschal, 1989). Topographic contours are described as self-similar (Mandelbrot, 1983; Gilbert, 1989; Turcotte, 1989, 1992), while vertical topographic relief may be considered self-affine (Turcotte, 1992).

The fractal dimension of a self-affine profile is often obtained from the power spectrum of the profile. Application of this method assumes that the profile is single valued (e.g. no overhangs or overthrusts). The power spectrum provides information about the relative amount and amplitude of different wavelength features in a profile, or of different frequency components in a time series. The power spectral dimension (D_P) is derived from the log-log slope of the power spectrum (β), and $\beta = (5-2D_P)$.

In shortening applications (e.g. Wu 1993), length of a geologic horizon must be isotropic and self-similar. It cannot vary with changes of reference frame or coordinate axis rotation. At the same time, meaningful application of compass measurements require that the compass openings be small enough to measure profile irregularity.

COMPASS DIMENSION OF LOW RELIEF PROFILES

Estimation of compass dimension from equation (1) is sensitive to the range of compass openings used to measure curve length. If the compass opening is too large, the compass walk will not follow irregularities of a certain scale. A fracture surface for example, is essentially flat, or one dimensional, when measured with compass openings that are large with respect to relief variations along the surface. Measurements of profile length with large compass openings remain the same for all walks, and $D_C=1$ (Mandelbrot 1985).

Structural cross sections portray structural relief associated with a limited range of scales or fold orders. Local observations may suggest that smaller scale structures (higher order folds) are present beneath the resolution of the section. The fractal interrelationship (equation 1) may be used to estimate the length of the profile assuming that larger scale irregularities defined by (1) continue through a range of smaller scales (Wu, 1993). •

Brown (1987) uses the relationship developed by Wong (1987) to investigate conditions under which D will be underestimated by the compass method when the profile is undersampled or the compass openings are too large. Wong (1987) models the variation of relief along a profile as a fractional Brownian process, and expresses profile length (λ) in the following form (Brown, 1987)

$$\lambda = \frac{\lambda_0}{\Delta x} \left[\Delta x^2 + \sigma^2 \right]^{\frac{1}{2}} \quad 2)$$

In this relationship λ_0 represents the horizontal distance covered by the profile, Δx is the horizontal sample interval, and λ is the measured length of the curve after $\lambda_0/\Delta x$ steps. σ is the standard deviation of relief variations at intervals of Δx along the profile. Wong (1987) uses the idea of a crossover length (Mandelbrot, 1985), and notes that if Δx is much larger than the crossover length, the measured length of the profile (λ) will be approximately equal to the horizontal distance spanned by the profile (λ_0), and D_C will be underestimated (Brown, 1987). For this case, compass steps occur mainly along the horizontal baseline (Mandelbrot, 1985), and D_C will be approximately 1.

Figure 1 is a profile generated by a fractional Brownian process and has power-spectral dimension D_P of 1.1. The length of the profile (λ) estimated directly from equation (2) is 11905m. σ , the standard deviation of the differences in relief between adjacent samples is 6.46m. λ_0 equals

10000 m and Δx is 10 m. The value of λ obtained from equation 2 is 19% greater than λ_0 and suggests that the profile, in this case, is not significantly undersampled. The estimated length λ of the profile based on the statistical relationship defined by equation (2) is very close to the length of the segment determined simply as $\sum_{i=2}^N (\Delta x^2 + \Delta h_i^2)^{\frac{1}{2}}$ to be 11696 m, where i refers to individual samples, N is the total number of samples, and Δh_i is the change in vertical relief ($h_i - h_{i-1}$). This observation suggests that useful measurements of profile length can be determined for the range of scales depicted in the profile. The range of compass openings used in the present context diverge significantly from the horizontal and incorporate measurement of lengths across the detailed features in the profile.

Brown (1987) suggests that more accurate estimates of D can be obtained by magnifying profile relief. Although magnification increases D , the physical significance of the result is distorted. A simple example is used to illustrate the effects of magnification on shortening estimates. In Fig. 2, a folded layer is represented by the inclined sides of a simple isosceles triangles. The base of Triangle 1 (Fig. 2) has a length and height of 2 meters. The combined length of the inclined sides of Triangle 1 is 4.47 m, and the shortening is 55%. The process of magnification proposed by Brown (1987) multiplies profile relief by some constant. The vertical relief of Triangle 1 has been increased by a factor of 2 to obtain Triangle 2 (Fig. 2). The height of Triangle 2 is 4 m, the length of its inclined sides is 8.25 m, and the shortening is 76%. In Triangle 3 both the base and height have been scaled by a factor of 1/2. The 55% shortening is preserved in this case, but the length of the inclined sides is only 2.24 m.

The value of the compass method to strain and shortening estimation is in its ability to predict the length of a profile that includes variations of relief at scales beneath the resolution limit of the geologic cross section. The process of magnification distorts bed-length and results in erroneous prediction of bed-length incorporating finer scales of deformation.

INVARIANCE OF THE POWER-SPECTRAL DIMENSION

In the following discussion, the power-spectral dimension is shown to be invariant to arbitrary linear rescaling of the input. The influence of axis rescaling on β and D is evaluated using the scaling property of the Fourier transform (Brigham, 1974). In the following discussion, $h(x)$ is the structural relief at point x along the profile. $H(k)$ is the Fourier transform of $h(x)$ at wavenumber k . The Fourier transform ($H(k)$) of $h(x)$ is

$$H(k) = \int_{-\infty}^{\infty} h(x) e^{-i2\pi kx} dx, \quad 3)$$

where k is the wavenumber.

If the x -axis is expanded or contracted by an arbitrary factor l , then $h(x)$ becomes $h(lx)$.

The Fourier transform of $h(lx)$ becomes

$$\int_{-\infty}^{\infty} h(lx) e^{-i2\pi kx} dx. \quad 4)$$

This is evaluated by substituting x' for lx so that dx becomes dx'/l , yielding

$$\int_{-\infty}^{\infty} h(x') e^{-i2\pi \frac{k}{l} x'} \frac{dx'}{l} \quad 5)$$

$$= \frac{1}{l} \int_{-\infty}^{\infty} h(x') e^{-i2\pi \frac{k}{l} x'} dx' \quad 6)$$

$$= \frac{1}{l} H\left(\frac{k}{l}\right). \quad 7)$$

The power spectrum, $S(k)$, equals the product of the amplitude spectrum with its complex conjugate, $H(k)H(k)^*$, which equals $|H(k)|^2$. Given the above, the power spectrum of $h(lx)$

$$= \frac{1}{l^2} \left| H\left(\frac{k}{l}\right) \right|^2 = l^{-2} S(k'), \quad 8)$$

where $k' = k/l$. When $h(x)$ is fractal $S(k) = Ck^\beta$, and the power spectrum of $h(lx)$ becomes

$$Cl^{-2} (k')^{-\beta}. \quad 9)$$

Taking the log of this expression yields

$$\log(Cl^{-2} (k')^{-\beta}) = \log(Cl^{-2}) - \beta \log(k'). \quad 10)$$

The intercept C and wavenumber k are both scaled by l^{-2} and l^{-1} , respectively, but the slope of the power spectrum in log-log space remains β .

For the case where $h(x)$ is scaled by l , $lh(x)$ has Fourier transform

$$\int_{-\infty}^{\infty} lh(x) e^{-i2\pi kx} dx = l \int_{-\infty}^{\infty} h(x) e^{-i2\pi kx} dx. \text{ This is just } lH(k). \text{ Hence, the power spectrum } S(k) = Clk^\beta,$$

which, again, has slope $-\beta$ in log-log space. The above results, including that for arbitrary scaling of both x and $h(x)$, are easily verified by direct computation.

Arbitrary scaling of x and $h(x)$ leaves D_p invariant. This makes power spectral analysis useful for portraying similarity in the distribution of relative heights of features in a profile, and a useful method of comparing the distributions of relative heights between data sets of diverse origins (structural relief versus fracture-surface relief). But it is the relative heights in all these profiles that remain invariant after scaling, and not the length of the profile.

CONCLUSIONS

The compass and power spectral methods are not directly related to each other. The compass method provides estimates of profile length at a certain scale for self-similar profiles. Self-similarity is established by showing that N and r satisfy equation (1) over several orders of magnitude. Rescaling distance along the profile or profile relief changes D and distorts profile length obtained from equation (1). D_P , on the other hand, does not vary with linear rescaling of profile relief or length. The power-spectral estimate of fractal dimension is related solely to the relative heights of different wavelength features in the profile, and in this sense, provides a truly scale-invariant measure of fractal characteristics. However, D_P is not uniquely related to structural variables such as length, shortening, or strain.

ACKNOWLEDGMENTS

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Figure 1: Profile obtained through simulation of a fractional Brownian process with fractal dimension of 1.1.

Figure 2: Three isosceles triangles illustrate the affect of magnification on profile length and shortening

Appendix 3:

TASK 4: TOPICAL REPORT

Program Development - *The compass walk*

Prepared for DoE Contract # DE-FG21-95MC32158

Compass-walk Algorithm **PROGRAM COMPASS**

The following FORTRAN program (*COMPASS*) undertakes a compass walk along a given profile. The program was developed specifically for the analysis of profiles and has not been tested on topographic contour data sets, although the program should work on such data.

The program computes the number of steps (N) required to traverse the profile using compass-openings of size (r). The program also computes the length of the profile, or its perimeter (P), where P is Nr . The increment in compass-opening (Δr) is defined by the user. The compass opening starts at a minimum value defined as the minimum distance between any two consecutive points in the profile. To reduce minimum value of r it is best to interpolate between points. This must be done by the user. The improvement in accuracy through the interpolation may be limited or biased depending upon the manner in which the interpolation is undertaken. Interpolation may not be necessary depending upon the format used to digitize the input profile. The opening is successively increased by (Δr) until a maximum opening, specified by the user, is reached.

The remainder method is used to compute N and P . Thus, N will be non-integer, and the fractional portion of one step is computed as the straight-line distance between last point of the compass and the last point on the profile.

There are several approaches to computing compass-walk data. This program provides a large number of data points from which base-2 decrements of r (i.e. successive compass-openings of r_1 , $r_1/2$, $r_1/4$, $r_1/8$, etc.) can be selected. Compass-openings that result in near-integer numbers of steps can also be extracted from the output data-file.

Computational Approach

The distance between two points along the line is computed and compared with the size of the current compass opening. When that distance exceeds the compass opening, the location of the pivot-point is computed. The point is assumed to lie on a straight line between the last two points inspected. The intersection of a circular arc having radius equal to the current ruler length with the line connecting the last two data points is then computed. The point of intersection then serves as the starting point for the next step in the compass-walk.

In some instances an error may occur during the operation of the program. The error can often be eliminated by moving the starting point up one sample value. The source of the error most likely arises when the slope of the line connecting the last two inspected points is either very large or very small. Hopefully this error can be resolved in the final report.

FORTRAN code for program *COMPASS* is listed below.

COMPASS - FORTRAN CODE

```
      real*4 x(3000),y(3000),xb(3000),yb(3000)
      real*8 rl
      character in*20
      write(*,1)
1      format(' Enter perimeter filename:',$)
      read(*,2)in
2      format(a20)
      open(1,file=in,status='old')
      open(2,file='perim.dat',status='new')
      open(3,file='error.dat',status='new')
      flength=0.0
      do 3 i=1,3000
      read(1,*,end=100)x(i),y(i)
3      continue
100     n=i-1
      do 4 i=1,n-1
      gap=((x(i+1)-x(i))**2+(y(i+1)-y(i))**2)**0.5
      flength=flength+gap
      if(gap.gt.rl) rl=gap
4      continue
      write(*,*)
      write(*,105) rl
105     format(' The maximum distance between points is ',f10.4)
      write(*,106)flength
106     format(' Total curve length is ',f12.4)
      del=x(2)-x(1)
      write(*,*)
      write(*,102)
102     format(' Enter increment in ruler length. Increment should')
      write(*,103)del
103     format(' be greater than the sample interval',f10.4,')
      write(*,104)
104     format(' Enter:',$)
      read(*,*)delx
      rl=0.0
      write(*,*)
      write(*,38)
38     format(' Enter maximum ruler length:',$)
      read(*,*) rlmax
      c      if(d.gt.rlmax)rlmax=d
```

```

6      1
      nrl=0
      xb(1)=x(1)
      yb(1)=y(1)
      j=1

7      do 8 i=1start,n-1
      d=((x(i+1)-xb(j))**2+(y(i+1)-yb(j))**2)**0.5
      if(abs(rl-d).le.0.1) then
          j=j+1
          xb(j)=x(i+1)
          yb(j)=y(i+1)
          nrl=nrl+1
          1start=i+1
          go to 71
      endif
      if(d.ge.rl) then
          call point(rl,x(i),x(i+1),y(i),y(i+1),xb(j),yb(j),xnew,ynew)
          j=j+1
          xb(j)=xnew
          yb(j)=ynew
          nrl=nrl+1
          1start=i+1
          go to 71
      endif
71     if(i.eq.(n-1)) go to 9
8      continue

9      res=((x(n)-xb(j))**2+(y(n)-yb(j))**2)**0.5
      dnrl=nrl+res/rl
      plen=nrl*rl+res
      write(2,101)rl,dnrl,plen
      write(*,*)rl,dnrl,plen
      rl=rl+delx
      if(rl.gt.rlmax) go to 20
      go to 6

101    format(1x,f12.5,1x,f12.5,1x,f12.5)

20     stop
      end

      subroutine point (rl,x1,x2,y1,y2,xb,yb,xn,yn)
      real*8 rl,dely2,dely1,delx1,delx2,slope,bint,a,b,c,d
      dely2=y2-yb

```



```

dely1=y1-yb
delx1=x1-xb
delx2=x2-xb
if(x1.eq.x2) then
    dely=(r1**2-delx1**2)**0.5
    xn=x1
c   y1 and y2 greater than yb, or y1 less than yb and y2 greater
c   than yb
    if(dely.gt.dely1.and.dely.lt.dely2) yn=yb+dely
c   y1 and y2 less than yb, or y1 greater than yb and y2 less
c   than yb
    if(dely.gt.dely1.and.dely.gt.dely2) yn=yb-dely
    go to 1
endif
if(y1.eq.y2) then
    delx=(r1**2-(y1-yb)**2)**0.5
    yn=y1
    if(x2.gt.x1) xn=xb+delx
    if(x2.lt.x1) xn=xb-delx
    go to 1
endif

slope=(y2-y1)/(x2-x1)
bint=y1-slope*x1
a=1+slope**2
b=2.0*slope*bint-2.0*xb-2.0*yb*slope
c=xb**2+yb**2+bint**2-2.0*yb*bint-r1**2
d=b**2-4.0*a*c

if(d.lt.0.0) then
    xn=-b/2*a
write(3,*)r1,x1,y1,x2,y2
    call neg(a,b,c,d,xb,yb)
    go to 2
endif
sol1=(-b+d**0.5)/(2.0*a)
sol2=(-b-d**0.5)/(2.0*a)
if(sol1.lt.x1.and.sol1.gt.x2)xn=sol1
if(sol1.gt.x1.and.sol1.lt.x2)xn=sol1
if(sol2.lt.x1.and.sol2.gt.x2)xn=sol2
if(sol2.gt.x1.and.sol2.lt.x2)xn=sol2

2   yn=slope*xn+bint
3   format(1x,f17.7,1x,f17.7,1x,f17.7,1x,f17.7)

```

```
1      return
      end

      subroutine neg(a,b,c,d,xb,yb)
      write(*,1)
1      format(' b-square minus 4.a.c is negative ')
      write(*,11)xb,yb
11     format(' at xb = ',f17.5,' yb = ',f17.5)
      write(*,2)a
2      format(' a = ',f20.5)
      write(*,3)b
3      format(' b = ',f20.5)
      write(*,4)c
4      format(' c = ',f20.5)
      write(*,5)d
5      format(' d = ',f20.5)
      write(*,6)
6      format(' This should not happen! Please check input data.')
      return
      end
```

Appendix 4:

Fractal Interrelationships in Field and Seismic Data

Preface

Chapter 1: Basic Concepts

Introduction

Computation of the Fractal Dimension

Chapter 2: Methods

Roughness-Length

Power spectral

Box counting

Compass walk

Edge effects

Chapter 3: Topographic Analysis

Topographic relief

Topographic Contours

Chapter 4: Drainage Analysis

Chapter 5: Seismic Analysis

Chapter 6: Fracture Analysis

Chapter 7: Fractal Interrelationships: Summary and Conclusions

Appendix A: Compass Walk Algorithm