DEVELOPMENT OF AN IMPROVED PERMEABILITY MODIFICATION
SIMULATOR

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By
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National Petroleum Technology Office
U. S. DEPARTMENT OF ENERGY
Tulsa, Oklahoma

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Jon Elphnick

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TABLE OF CONTENTS

1.0 INTRODUCTION.....	1
2.0 DEVELOPMENT OF A RADIAL COORDINATE SYSTEM.....	3
2.1 Continuity, Darcy, and Pressure Equations in Rectangular Coordinates.....	3
2.2 Continuity, Darcy, and Pressure Equations in Cylindrical Coordinates.....	6
2.3 Finite Difference Forms of Flow Equations in Rectangular Coordinates	8
2.4 Finite Difference Forms of Flow Equations in Cylindrical Coordinates	11
2.5 Inclusion of the Radial Coordinate System into the PC-GEL Simulator.....	13
2.6 Validation of the Radial Model.....	13
3.0 THERMAL ENERGY EQUATION.....	17
3.1 Finite Difference Form of the Thermal Energy Equation in Rectangular Coordinates	18
3.2 Finite Difference Form of the Thermal Energy Equation in Cylindrical Coordinates	21
3.3 Thermal Properties of Reservoir Fluids and Rocks	23
3.4 Inclusion of the Thermal Energy Equation into the PC-GEL Simulator	27
3.5 Validation of Thermal Energy Equation.....	28
4.0 TEMPERATURE-DEPENDENT GEL CHEMISTRY AND FLUID RHEOLOGY OF A DELAYED GEL SYSTEM	31
5.0 WELLBORE SIMULATOR.....	35
5.1 Modification of the Wellbore Simulator to Include a 1D	37
5.2 Validation of the Modified Wellbore Simulator.....	39
6.0 FULLY IMPLICIT TIME STEPPING.....	43
6.1 Formulation of Jacobian Matrix.....	43
6.2 Singular Value Decomposition Method.....	45
6.3 Block Factorization Method	50
7.0 GEL PLACEMENT CALCULATIONS FOR WATER CONTROL	53
8.0 TECHNOLOGY TRANSFER	61
9.0 SUMMARY AND CONCLUSIONS	63
10.0 REFERENCES.....	65
11.0 NOMENCLATURE	69

LIST OF TABLES

5-1	Bottomhole temperatures and static head	36
6-1	The radial grid used in simulator.....	47
6-2	Comparison of CPU Times.....	48
7-1	Simulation results for reservoirs with layer permeability contrast equal to 10	58

LIST OF FIGURES

2-1	Effect of time-step size on the distribution of polymer concentration in the low-permeability layer when injecting at a constant rate of 50 bbl per day.....	14
2-2	Contours of polymer concentration from the radial coordinate system.....	15
2-3	Contours of polymer concentration from the rectangular coordinate system.....	16
3-1	Calculated gelant concentration and temperature profiles in a 1D rectangular porous medium	29
3-2	Calculated gelant concentration and temperature profiles in a 1D radial porous medium.....	29
4-1	Gelation time of DGS as a function of temperature and molar ratio of gelling agent to activator.....	32
5-2	Gelant concentration profile in a wellbore (dispersion coefficient = 0).....	38
5-3	Gelant concentration profile in a wellbore (dispersion coefficient = $1 \text{ E-5 cm}^2/\text{sec}$)	38
5-4	Temperature profiles of tubing fluid, annulus fluid, and formation in the wellbore	40
5-5	Concentration profiles of activator, gelling agent, and bridging agent in the wellbore.....	41
5-6	Temperature and chemical concentration profiles in the radial reservoir model before shut-in	41
5-7	Temperature and chemical concentration profiles in the rectangular reservoir model before shut-in	42
6-1	Comparison of cell pressure and oil saturation distributions with the IMPES and fully implicit methods.....	47
6-2	Comparison of average reservoir pressure, fractional oil recovery, and water-oil ratio with the IMPES and fully implicit methods.....	49
6-3	Calculated average reservoir pressures as a function of pore volume.....	51
7-1	Distribution of gelant concentration in ppm (single injection).....	54
7-2	Distribution of gelant concentration in ppm (dual injection).....	54
7-3	Distribution of low-viscosity (1.8 cp) gelant concentration in ppm (single injection).....	55
7-4	Distribution of low-viscosity (1.8 cp) gelant concentration in ppm (dual injection)	56
7-5	Distribution of low-viscosity gelant (ppm) after 0.3 day of single injection.....	58
7-6	Distribution of low-viscosity gelant (ppm) after 0.355 day of dual injection.....	59

ABSTRACT

This report describes the development of an improved permeability modification simulator performed jointly by BDM Petroleum Technologies and Schlumberger Dowell under a cooperative research and development agreement (CRADA) with the U.S. Department of Energy. The improved simulator was developed by modifying NIPER's PC-GEL permeability modification simulator to include a radial model, a thermal energy equation, a wellbore simulator, and a fully implicit time-stepping option. The temperature-dependent gelation kinetics of a delayed gel system (DGS) is also included in the simulator.

The radial model was developed by transforming all the governing equations from rectangular to cylindrical coordinates (r, θ, z). The phase transmissibility was reformulated in terms of cylindrical coordinates. The radial model was checked for the r and z components of the Darcy velocity vector under a single-phase fluid flow condition. Material balance checks were conducted under single- and two-phase fluid flow in one- and two-layer reservoir models. Good material balance (error less than 1%) was obtained for injection at constant pressure and a small time step during chemical injection. Compared with the rectangular coordinate system, the radial coordinate system gave less numerical dispersion and required less computing time.

To account for the change in temperature of the polymer gel system during pumping, a thermal equation that considers thermal convection/conduction was included in the PC-GEL simulator. Validation of the thermal model was conducted with a 1D rectangular and a 1D radial porous media.

Gelation chemistry studies on an inorganic DGS showed that the gelation occurred when the molar ratio of a bridging agent to a gelling agent reached a critical value. The concentration of the activator decreased exponentially in the first order reaction rate model. The reaction rate constant obeyed the Arrhenius equation. The gelation time decreased with an increase in the salt concentration. This system was modeled as an 8-component system in the simulator.

The wellbore simulator developed by Schlumberger Dowell was modified to include a 1D in-situ gelation model and to allow the use of a variable time-step size. The modified wellbore simulator was incorporated into the PC-GEL permeability modification simulator and validated with a DGS in both rectangular and radial grid reservoir models.

A fully implicit method was developed and incorporated into the PC-GEL simulator. The set of nonlinear equations consisting of the flow equations of the oil, water, and gas phases, and the equations of the well was linearized by using the Newton-Raphson method. Existing singular value decomposition method (SVD) and a block factorization method were used to solve the linearized equations. The SVD method did not have a numerical stability problem. Its use is recommended when the number of nonlinear equations is small. When the number of nonlinear

equations becomes large, the SVD method is recommended only when the IMPES (Implicit Pressure-Explicit Saturations) method has a numerical stability problem.

Simulation studies were conducted to investigate the influence of permeability anisotropy, thickness of the low-permeability layer, and polymer viscosity on the design of permeability modification treatments in two-layer rectangular and radial reservoir models. A dual injection method was investigated to control the undesired leakage of the gelant to the high-saturation oil-bearing zone (low-permeability layer) when injecting a gel system into the high-permeability layer. For low polymer viscosity, dual injection is recommended even when the high-saturation oil-bearing layer is relatively thick. For high polymer viscosity, dual injection is needed only when the layer permeability contrast is low, the formation permeability anisotropy is low, and the thickness of the oil zone is small. When the permeability anisotropy is very high, a single injection for the polymer solutions is adequate irrespective of the viscosities of the polymer solutions.

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1.0 INTRODUCTION

Permeability modification techniques have been used by oil producers to improve sweep efficiency, reduce water cut, and increase oil recovery. A simulator is needed to design the treatment based on reservoir characteristics, depth, temperature, and the wellbore configuration. Changes in fluid temperature during pumping and flow in the reservoir can significantly alter the gelation reaction rate because it depends exponentially on the temperature. For example, for a given concentration of a gelling agent currently used by Schlumberger Dowell, the gelation time can vary from 6 hr at 194°F to more than 250 hr at 130°F. Thus, the reaction rate of an injected slug of the gelling fluid will increase over several orders of magnitude, due to its heating upon contact with the warmer wellbore tubings and the reservoir rock and fluids as it travels from the wellhead to the designed setting point deep in the reservoir where it is intended to set. Ignoring this change can either lead to a premature gelation and a shut down of the injection before the fluid reaches the intended region of the reservoir, or the gel may not set sufficiently over the curing period allowed after the injection and simply flows back out during the subsequent production period. In other situations, a premature setting of the gel in a watered-out, high-permeability target layer will prevent the zone from taking additional fluid. As a result, the injected fluid will then be diverted to the adjacent oil-bearing low-permeability layers, where it will set and shut off the oil flow to the well.

In any of these scenarios, the treatment will provide little or no benefit in terms of improved oil recovery or reduced water cut and thus would be judged a failure. Proper considerations of temperature effects on both gelation rate and fluid rheology are needed in order to accurately describe the gel setting and the reservoir response to field gel treatments. The existing PC-GEL permeability modification simulator previously developed at BDM-Oklahoma/NIPER does not consider these effects and is only applicable to a constant temperature environment. An improved simulator that accounts for the temperature effects will help to correctly design the placement process and thus increase the success rate of gel treatments. The increased success rate in turn will accelerate the use of permeability modification techniques to economically recover the remaining oil in place.

This report describes the development of an improved permeability modification simulator performed under a cooperative research and development agreement (CRADA) established between BDM-Oklahoma, Inc. and Schlumberger Dowell. The improved simulator is developed through the modification of the existing PC-GEL permeability modification simulator to include a radial model, a thermal energy equation, a modified version of Schlumberger Dowell's wellbore simulator, and a fully implicit time-stepping option. The developed simulator describes the flow of the injected fluid in the wellbore, through the perforations, and in the reservoir. Flow in the reservoir is three dimensional and includes thermal conduction/convection among the injected fluid, the reservoir formation, the reservoir fluids, the overburden, and the underburden. The

coordinate system can be either rectangular or cylindrical. A fully implicit time-stepping option is included to overcome the numerical stability problem that occurred with the IMPES method currently used in the PC-GEL simulator. In addition, the temperature-dependent gelation kinetics and fluid rheology of an inorganic delayed gel system (DGS) also are included.

2.0 DEVELOPMENT OF A RADIAL COORDINATE SYSTEM

2.1 Continuity, Darcy, and Pressure Equations in Rectangular Coordinates

The PC-GEL permeability modification simulator (Gao et al. 1990, 1993, 1995) was developed by incorporating an in-situ gelation model into the BOAST simulator (Fanchi 1982). The flow equations used were derived for a rectangular coordinate system (x, y, z) for which the mass conservation equations for the three phases are (Fanchi 1982)

Oil

$$-\left[\frac{\partial}{\partial x}\left(\frac{\rho_{osc}}{B_o}V_{xo}\right) + \frac{\partial}{\partial y}\left(\frac{\rho_{osc}}{B_o}V_{yo}\right) + \frac{\partial}{\partial z}\left(\frac{\rho_{osc}}{B_o}V_{zo}\right)\right] - q_o = \frac{\partial}{\partial t}\left(\frac{\phi\rho_{osc}S_o}{B_o}\right) \quad (2-1)$$

Water

$$-\left[\frac{\partial}{\partial x}\left(\frac{\rho_{wsc}}{B_w}V_{xw}\right) + \frac{\partial}{\partial y}\left(\frac{\rho_{wsc}}{B_w}V_{yw}\right) + \frac{\partial}{\partial z}\left(\frac{\rho_{wsc}}{B_w}V_{zw}\right)\right] - q_w = \frac{\partial}{\partial t}\left(\frac{\phi\rho_{wsc}S_w}{B_w}\right) \quad (2-2)$$

and

Gas

$$\begin{aligned} &-\frac{\partial}{\partial x}\left(\frac{\rho_{gsc}}{B_g}V_{xg} + \frac{R_{so}\rho_{gsc}}{B_o}V_{xo} + \frac{R_{sw}\rho_{gsc}}{B_w}V_{xw}\right) \\ &-\frac{\partial}{\partial y}\left(\frac{\rho_{gsc}}{B_g}V_{yg} + \frac{R_{so}\rho_{gsc}}{B_o}V_{yo} + \frac{R_{sw}\rho_{gsc}}{B_w}V_{yw}\right) \\ &-\frac{\partial}{\partial z}\left(\frac{\rho_{gsc}}{B_g}V_{zg} + \frac{R_{so}\rho_{gsc}}{B_o}V_{zo} + \frac{R_{sw}\rho_{gsc}}{B_w}V_{zw}\right) - q_g \\ &= \frac{\partial}{\partial t}\left[\phi\rho_{gsc}\left(\frac{S_g}{B_g} + \frac{R_{so}S_o}{B_o} + \frac{R_{sw}S_w}{B_w}\right)\right] \end{aligned} \quad (2-3)$$

where ρ_{osc} , ρ_{wsc} , ρ_{gsc} = oil, water, and gas densities at standard conditions (usually 60°F and 14.7 psia); B_o , B_w , B_g = formation volume factors for oil, water, and gas phases; f = porosity; S_o , S_w , S_g = oil, water, and gas saturations; V_x , V_y , V_z = x, y, and z components of Darcy velocity vector; q_o , q_w , q_g = mass flow of oil, water, and gas into or out of a well; t = time; and R_{so} , R_{sw} = gas solubilities in oil and water phases.

Darcy velocities for the three phases (oil, water, and gas, respectively) are

$$V_o = -K \frac{k_{ro}}{\mu_o} \nabla(p_o - \frac{\rho_o g z}{144 g_c}) \quad (2-4)$$

$$V_w = -K \frac{k_{rw}}{\mu_w} \nabla(p_w - \frac{\rho_w g z}{144 g_c}) \quad (2-5)$$

and

$$V_g = -K \frac{k_{rg}}{\mu_g} \nabla(p_g - \frac{\rho_g g z}{144 g_c}) \quad (2-6)$$

where

$$\rho_o = \frac{1}{B_o} (\rho_{osc} + R_{so} \rho_{gsc}) \quad (2-7)$$

$$\rho_w = \frac{1}{B_w} (\rho_{wsc} + R_{sw} \rho_{gsc}) \quad (2-8)$$

$$\rho_g = \frac{\rho_{gsc}}{B_g} \quad (2-9)$$

K = permeability tensor, k_{ro} , k_{rw} , k_{rg} = relative permeabilities for oil, water, and gas phases; μ_o , μ_w , μ_g = oil, water, and gas phase viscosities; p_o , p_w , p_g = oil, water, and gas phase pressures; ρ_o , ρ_w , ρ_g = oil, water, and gas phase densities; g = acceleration of gravity; $g_c = 32.174 \text{ ft/sec}^2$; and ∇ = divergence operator.

Equations 2-1 through 2-9 and the saturations constraint

$$S_o + S_w + S_g = 1 \quad (2-10)$$

are the basic fluid flow equations of the simulator. Combining Equations 2-4 through 2-6 with Equations 2-1 through 2-3 and using the capillary pressures of oil-to-water and gas-to-oil, p_{cow} and p_{cgo} , yield (Fanchi 1982)

Oil

$$\begin{aligned} \frac{\partial}{\partial x} (K_x \frac{k_{ro}}{\mu_o B_o} \frac{\partial p_o}{\partial x}) + \frac{\partial}{\partial y} (K_y \frac{k_{ro}}{\mu_o B_o} \frac{\partial p_o}{\partial y}) + \frac{\partial}{\partial z} (K_z \frac{k_{ro}}{\mu_o B_o} \frac{\partial p_o}{\partial z}) + CG_o \\ - \frac{q_o}{\rho_{osc}} = \frac{\partial}{\partial t} (\frac{\phi S_o}{B_o}) \end{aligned} \quad (2-11)$$

Water

$$\begin{aligned} \frac{\partial}{\partial x} (K_x \frac{k_{rw}}{\mu_w B_w} \frac{\partial p_o}{\partial x}) + \frac{\partial}{\partial y} (K_y \frac{k_{rw}}{\mu_w B_w} \frac{\partial p_o}{\partial y}) + \frac{\partial}{\partial z} (K_z \frac{k_{rw}}{\mu_w B_w} \frac{\partial p_o}{\partial z}) + CG_w \\ - \frac{q_w}{\rho_{wsc}} = \frac{\partial}{\partial t} (\frac{\phi S_w}{B_w}) \end{aligned} \quad (2-12)$$

and

Gas

$$\begin{aligned} \frac{\partial}{\partial x} [K_x (\frac{k_{rg}}{\mu_g B_g} + \frac{R_{so} k_{ro}}{\mu_o B_o} + \frac{R_{sw} k_{rw}}{\mu_w B_w}) \frac{\partial p_o}{\partial x}] + \frac{\partial}{\partial y} [K_y (\frac{k_{rg}}{\mu_g B_g} + \frac{R_{so} k_{ro}}{\mu_o B_o} + \frac{R_{sw} k_{rw}}{\mu_w B_w}) \frac{\partial p_o}{\partial y}] \\ + \frac{\partial}{\partial z} [K_z (\frac{k_{rg}}{\mu_g B_g} + \frac{R_{so} k_{ro}}{\mu_o B_o} + \frac{R_{sw} k_{rw}}{\mu_w B_w}) \frac{\partial p_o}{\partial z}] + CG_g - \frac{q_g}{\rho_{gsc}} \\ = \frac{\partial}{\partial t} [\phi (\frac{S_g}{B_g} + \frac{R_{so} S_o}{B_o} + \frac{R_{sw} S_w}{B_w})] \end{aligned} \quad (2-13)$$

where

$$CG_o = - \nabla \cdot K \cdot (\frac{k_{ro}}{\mu_o B_o}) \nabla (\frac{\rho_o z}{144}) \quad (2-14)$$

$$CG_w = - \nabla \cdot K \cdot \left(\frac{k_{rw}}{\mu_w B_w} \right) \nabla \left(\frac{\rho_w z}{144} + p_{cow} \right) \quad (2-15)$$

$$CG_g = \nabla \cdot \left\{ K \cdot \left[\frac{k_{rg}}{\mu_g B_g} \nabla \left(p_{cgo} - \frac{\rho_g z}{144} \right) - \frac{R_{so} k_{ro}}{\mu_o B_o} \nabla \left(\frac{\rho_o z}{144} \right) - \frac{R_{sw} k_{rw}}{\mu_w B_w} \nabla \left(p_{cow} + \frac{\rho_w z}{144} \right) \right] \right\} \quad (2-16)$$

K_x, K_y, K_z = permeabilities in the x, y , and z directions, and ∇ = divergence operator.

Using the saturations constraint, Equations 2-11 through 2-13 can be combined to give the following pressure equation (Fanchi 1982) for the oil phase, p_o :

$$\begin{aligned} (B_o - R_{so} B_g) \left(\nabla \cdot K \cdot \frac{k_{ro}}{\mu_o B_o} \nabla p_o + CG_o - \frac{q_o}{\rho_{osc}} \right) + (B_w - R_{sw} B_g) \left(\nabla \cdot K \cdot \frac{k_{rw}}{\mu_w B_w} \nabla p_o + CG_w - \frac{q_w}{\rho_{wsc}} \right) \\ + B_g \left[\nabla \cdot K \cdot \left(\frac{k_{rg}}{\mu_g B_g} + \frac{R_{so} k_{ro}}{\mu_o B_o} + \frac{R_{sw} k_{rw}}{\mu_w B_w} \right) \nabla p_o + CG_g - \frac{q_g}{\rho_{gsc}} \right] = \phi c_t \cdot \frac{\partial p_o}{\partial t} \end{aligned} \quad (2-17)$$

In the PC-GEL simulator, Equation 2-17 is first numerically solved for p_o . Equations 2-10 through 2-12 are then used to solve the phase saturations (Fanchi 1982).

2.2 Continuity, Darcy, and Pressure Equations in Cylindrical Coordinates

To include the cylindrical coordinates (r, θ, z) into the PC-GEL simulator, these equations were transformed from rectangular to cylindrical coordinates and incorporated into the simulator. Assuming cylindrical symmetry, Equations 2-11, 2-12, and 2-14 through 2-17 in cylindrical coordinates become

Oil

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r K_r \frac{k_{ro}}{\mu_o B_o} \frac{\partial p_o}{\partial r} \right) + \frac{\partial}{\partial z} \left(K_z \frac{k_{ro}}{\mu_o B_o} \frac{\partial p_o}{\partial z} \right) + CG_o - \frac{q_o}{\rho_{osc}} = \frac{\partial}{\partial t} \left(\frac{\phi S_o}{B_o} \right) \quad (2-18)$$

Water

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r K_r \frac{k_{rw}}{\mu_w B_w} \frac{\partial p_o}{\partial r} \right) + \frac{\partial}{\partial z} \left(K_z \frac{k_{rw}}{\mu_w B_w} \frac{\partial p_o}{\partial z} \right) + CG_w - \frac{q_w}{\rho_{wsc}} = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) \quad (2-19)$$

and

$$\begin{aligned}
& (B_o - R_{so}B_g) \left[\frac{1}{r} \frac{\partial}{\partial r} \left(rK_r \frac{k_{ro}}{\mu_o B_o} \frac{\partial p_o}{\partial r} \right) + \frac{\partial}{\partial z} \left(K_z \frac{k_{ro}}{\mu_o B_o} \frac{\partial p_o}{\partial z} \right) + CG_o - \frac{q_o}{\rho_{osc}} \right] \\
& + (B_w - R_{sw}B_g) \left[\frac{1}{r} \frac{\partial}{\partial r} \left(rK_r \frac{k_{rw}}{\mu_w B_w} \frac{\partial p_o}{\partial r} \right) + \frac{\partial}{\partial z} \left(K_z \frac{k_{rw}}{\mu_w B_w} \frac{\partial p_o}{\partial z} \right) + CG_w - \frac{q_w}{\rho_{wsc}} \right] \\
& + B_g \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left[rK_r \left(\frac{k_{rg}}{\mu_g B_g} + \frac{R_{so}k_{ro}}{\mu_o B_o} + \frac{R_{sw}k_{rw}}{\mu_w B_w} \right) \frac{\partial p_o}{\partial r} \right] \right. \\
& \left. + \frac{\partial}{\partial z} \left[K_z \left(\frac{k_{rg}}{\mu_g B_g} + \frac{R_{so}k_{ro}}{\mu_o B_o} + \frac{R_{sw}k_{rw}}{\mu_w B_w} \right) \frac{\partial p_o}{\partial z} \right] + CG_g - \frac{q_g}{\rho_{gsc}} \right\} = \phi c_t \cdot \frac{\partial p_o}{\partial t}
\end{aligned} \tag{2-20}$$

where

$$CG_o = -\frac{1}{r} \frac{\partial}{\partial r} \left[rK_r \frac{k_{ro}}{\mu_o B_o} \frac{\partial}{\partial r} \left(\frac{\rho_o z}{144} \right) \right] - \frac{\partial}{\partial z} \left[K_z \frac{k_{ro}}{\mu_o B_o} \frac{\partial}{\partial z} \left(\frac{\rho_o z}{144} \right) \right] \tag{2-21}$$

$$CG_w = -\frac{1}{r} \frac{\partial}{\partial r} \left[rK_r \frac{k_{rw}}{\mu_w B_w} \frac{\partial}{\partial r} \left(\frac{\rho_w z}{144} + p_{cow} \right) \right] - \frac{\partial}{\partial z} \left[K_z \frac{k_{rw}}{\mu_w B_w} \frac{\partial}{\partial z} \left(\frac{\rho_w z}{144} + p_{cow} \right) \right] \tag{2-22}$$

$$\begin{aligned}
CG_g = & \frac{1}{r} \frac{\partial}{\partial r} \left[rK_r \left(\frac{k_{rg}}{\mu_g B_g} \right) \frac{\partial}{\partial r} \left(p_{cgo} - \frac{\rho_g z}{144} \right) \right] + \frac{\partial}{\partial z} \left[K_z \left(\frac{k_{rg}}{\mu_g B_g} \right) \frac{\partial}{\partial z} \left(p_{cgo} - \frac{\rho_g z}{144} \right) \right] \\
& - \frac{1}{r} \frac{\partial}{\partial r} \left[rK_r \frac{R_{so}k_{ro}}{\mu_o B_o} \frac{\partial}{\partial r} \left(\frac{\rho_o z}{144} \right) \right] - \frac{\partial}{\partial z} \left[K_z \frac{R_{so}k_{ro}}{\mu_o B_o} \frac{\partial}{\partial z} \left(\frac{\rho_o z}{144} \right) \right] \\
& - \frac{1}{r} \frac{\partial}{\partial r} \left[rK_r \frac{R_{sw}k_{rw}}{\mu_w B_w} \frac{\partial}{\partial r} \left(p_{cow} + \frac{\rho_w z}{144} \right) \right] - \frac{\partial}{\partial z} \left[K_z \frac{R_{sw}k_{rw}}{\mu_w B_w} \frac{\partial}{\partial z} \left(p_{cow} + \frac{\rho_w z}{144} \right) \right]
\end{aligned} \tag{2-23}$$

K_r, K_z = permeabilities in the r and z directions, and c_t = total compressibility. Equations 2-18 through 2-23 differ from Equations 2-11, 2-12, and 2-14 through 2-17 only in the r components.

The conservation equations for each chemical species in the aqueous phase in rectangular (Gao et al. 1990, 1993, 1995) and cylindrical coordinates are

$$\begin{aligned}
\frac{\partial(\phi\rho_w S_w C_i)}{\partial t} + \frac{\partial[\phi\rho_R(1-\phi)C_{ir}]}{\partial t} = D\left\{\frac{\partial}{\partial x}\left[(\phi\rho_w S_w)\frac{\partial C_i}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\phi\rho_w S_w)\frac{\partial C_i}{\partial y}\right] \right. \\
\left. + \frac{\partial}{\partial z}\left[(\phi\rho_w S_w)\frac{\partial C_i}{\partial z}\right]\right\} - \frac{\partial(\rho_w C_i V_{xw})}{\partial x} - \frac{\partial(\rho_w C_i V_{yw})}{\partial y} - \frac{\partial(\rho_w C_i V_{zw})}{\partial z} \\
+ \phi\rho_w S_w R_i - Q_i \rho_w C_i / V
\end{aligned} \quad (2-24)$$

and

$$\begin{aligned}
\frac{\partial(\phi\rho_w S_w C_i)}{\partial t} + \frac{\partial[\phi\rho_R(1-\phi)C_{ir}]}{\partial t} = D\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[(r\phi\rho_w S_w)\frac{\partial C_i}{\partial r}\right] + \frac{\partial}{\partial z}\left[(\phi\rho_w S_w)\frac{\partial C_i}{\partial z}\right]\right\} \\
- \frac{1}{r}\frac{\partial(r\rho_w C_i V_{rw})}{\partial r} - \frac{\partial(\rho_w C_i V_{zw})}{\partial z} + \phi\rho_w S_w R_i - Q_i \rho_w C_i / V
\end{aligned} \quad (2-25)$$

respectively, where C_i = mass concentration of component i , C_{ir} = mass concentration of component i adsorbed on rock surface, f = fraction of rock surface contacted by the aqueous phase, ρ_R = rock density, D = a constant dispersion coefficient, V_{rw} , V_{xw} , V_{yw} , V_{zw} = superficial velocities of the aqueous phase in the r , x , y , and z directions, R_i = formation rate of component i in a unit volume of the porous medium, Q_i = well rate for component i , and V = well block volume.

2.3 Finite Difference Forms of Flow Equations in Rectangular Coordinates

In the PC-GEL simulator, Equations 2-11, 2-12, 2-17, and 2-24 were solved numerically. The finite difference forms used for Equations 2-11, 2-12, and 2-17 after being multiplied through by the bulk volume element, V_B , are (Fanchi 1982)

Oil

$$(\Delta A_o^n \Delta p_o^{n+1} + \text{GOWT} - \frac{q_o V_B}{\rho_{osc}})_{ijk} = \frac{1}{\Delta t} \left[\left(\frac{V_p S_o}{B_o} \right)^{n+1} - \left(\frac{V_p S_o}{B_o} \right)^n \right]_{ijk} \quad (2-26)$$

Water

$$(\Delta A_w^n \Delta p_o^{n+1} + \text{GWWT} - \frac{q_w V_B}{\rho_{wsc} \text{ijk}}) = \frac{1}{\Delta t} \left[\left(\frac{V_p S_w}{B_w} \right)^{n+1} - \left(\frac{V_p S_w}{B_w} \right)^n \right]_{ijk} \quad (2-27)$$

and

Pressure

$$\begin{aligned} & (B_o^n - B_g^n R_{so}^n)_{ijk} (\Delta A_o^n \Delta p_o^{n+1} + \text{GOWT} - \frac{q_o V_B}{\rho_{osc} \text{ijk}}) \\ & + (B_w^n - B_g^n R_{sw}^n)_{ijk} (\Delta A_w^n \Delta p_o^{n+1} + \text{GWWT} - \frac{q_w V_B}{\rho_{wsc} \text{ijk}}) \\ & + (B_g^n)_{ijk} (\Delta A_g^n \Delta p_o^{n+1} + \Delta R_{so}^n A_o^n \Delta p_o^{n+1} + \Delta R_{sw}^n A_w^n \Delta p_o^{n+1} \\ & + \text{GGWT} - \frac{q_g V_B}{\rho_{gsc} \text{ijk}}) = \left(\frac{V_p^n c_t^n}{\Delta t} \right) (p_o^{n+1} - p_o^n)_{ijk} \end{aligned} \quad (2-28)$$

where

$$\Delta A \Delta p = \Delta_x A_x \Delta p_x + \Delta_y A_y \Delta p_y + \Delta_z A_z \Delta p_z \quad (2-29)$$

$$\Delta_x A_x \Delta p_x = A_{i-1/2jk} (p_{i-1jk} - p_{ijk}) + A_{i+1/2jk} (p_{i+1jk} - p_{ijk}) \quad (2-30)$$

$$\Delta_y A_y \Delta p_y = A_{ij+1/2k} (p_{ij+1k} - p_{ijk}) + A_{ij-1/2k} (p_{ij-1k} - p_{ijk}) \quad (2-31)$$

$$\Delta_z A_z \Delta p_z = A_{ijk-1/2} (p_{ijk-1} - p_{ijk}) + A_{ijk+1/2} (p_{ijk+1} - p_{ijk}) \quad (2-32)$$

$$\text{GOWT} = -\Delta A_o^n \Delta \left(\frac{\rho_o z}{144} \right)^n \quad (2-33)$$

$$\text{GWWT} = -\Delta A_w^n \Delta \left(\frac{\rho_w z}{144} + p_{cow} \right)^n \quad (2-34)$$

$$GGWT = \Delta [A_g^n \Delta (p_{cgo} - \frac{\rho_g z}{144})^n - R_{so}^n A_o^n \Delta (\frac{\rho_o z}{144})^n - R_{sw}^n A_w^n \Delta (p_{cow} + \frac{\rho_w z}{144})^n] \quad (2-35)$$

$$V_p = \phi V_B \quad (2-36)$$

$A_{i-1/2,j,k}$, $A_{i+1/2,j,k}$ = the finite difference phase transmissibilities between blocks $i-1$ and i , and between blocks i and $i+1$, V_p = the pore volume element, V_B = the bulk volume element, and superscripts n , $n+1$ = the present and next future time level. The finite difference phase transmissibility, $A_{i-1/2,j,k}$ is defined as (Fanchi 1982)

$$A_{i-1/2,j,k} = \frac{2\Delta x_i}{\Delta x' + \Delta x''} A_{i-1/2,j,k}' \quad (2-37)$$

where

$$\Delta x' = x_i - x_{i-1} \quad (2-38)$$

$$\Delta x'' = x_{i+1} - x_i \quad (2-39)$$

$$A_{i-1/2,j,k}' = \frac{4k_{rp}(\text{upstream})}{(\mu_{i-1,j,k} + \mu_{i,j,k})(B_{i-1,j,k} + B_{i,j,k})} \left(\frac{2(kA)_{i-1,j,k}(kA)_{i,j,k}}{\Delta x_{i-1,j,k}(kA)_{i,j,k} + \Delta x_{i,j,k}(kA)_{i-1,j,k}} \right) \quad (2-40)$$

k_{rp} (upstream) is the permeability, μ is the phase viscosity, B is the phase formation volume factor, Δx = the gridblock size in the x direction, subscripts i and $i-1$ refer to blocks i and $i-1$, respectively, and $i-1/2$ and $i+1/2$ refer to the interfaces of blocks i and $i-1$, and i and $i+1$. Similar formulae also are defined for the other two dimensions.

For the conservation equation of the chemical species in the aqueous phase (Eq. 2-24), a central finite difference method of up to 10th order was used to approximate the spatial derivatives of species concentrations, $\partial C_i / \partial x$, $\partial C_i / \partial y$, $\partial C_i / \partial z$, $\partial^2 C_i / \partial x^2$, $\partial^2 C_i / \partial y^2$, and $\partial^2 C_i / \partial z^2$, and a forward difference form was used to approximate the spatial derivatives of superficial velocities in the x , y , and z directions (Gao et al. 1990).

2.4 Finite Difference Forms of Flow Equations in Cylindrical Coordinates

The finite difference forms used for Equations 2-18 through 2-20 after being multiplied through by the bulk volume element, V_B , are the same as those used in the rectangular coordinate system, Equations 2-26 through 2-28, except that the linear difference operator, $\Delta A \Delta p$, is defined as

$$\Delta A \Delta p = \Delta_r A_r \Delta p_r + \Delta_z A_z \Delta p_z \quad (2-41)$$

where

$$\Delta_r A_r \Delta p_r = A_{i-1/2, j, k} (p_{i-1, j, k} - p_{i, j, k}) + A_{i+1/2, j, k} (p_{i+1, j, k} - p_{i, j, k}) \quad (2-42)$$

and

$$\Delta_z A_z \Delta p_z = A_{i, j, k-1/2} (p_{i, j, k-1} - p_{i, j, k}) + A_{i, j, k+1/2} (p_{i, j, k+1} - p_{i, j, k}) \quad (2-43)$$

The finite difference phase transmissibilities in Equations 2-42 and 2-43, $A_{i-1/2, j, k}$, $A_{i+1/2, j, k}$, are different from those in the rectangular system (Eqs. 2-37 and 2-40). Using the same procedure as that used in BOAST (Fanchi 1982), the finite difference phase transmissibility between blocks $i-1$ and i in the cylindrical coordinate system was derived as

$$A_{i-1/2, j, k} = \frac{r_{i+1/2}^2 - r_{i-1/2}^2}{r_i(r_{i+1} - r_{i-1})} A'_{i-1/2, j, k} \quad (2-44)$$

where $A'_{i-1/2, j, k}$ is the Darcy phase transmissibility between blocks $i-1$ and i . It is defined as

$$A'_{i-1/2, j, k} = \frac{4k_{rp}(\text{upstream})}{(\mu_{i-1, j, k} + \mu_{i, j, k})(B_{i-1, j, k} + B_{i, j, k})} \left(\frac{2(kA)_{i-1, j, k}(kA)_{i, j, k}}{\Delta r_{i-1, j, k}(kA)_{i, j, k} + \Delta r_{i, j, k}(kA)_{i-1, j, k}} \right) \quad (2-45)$$

where

$$A_{i-1/2, j, k} = \pi \Delta z_{i, j, k} \frac{\Delta r_{i-1}}{\ln \frac{r_{i+1} + (\Delta r_{i-1})/2}{r_{i-1}}} \quad (2-46)$$

$$A_{i,j,k} = \pi \Delta z_{i,j,k} \frac{\Delta r_i}{\ln \frac{r_i}{r_i - (\Delta r_i)/2}} \quad (2-47)$$

k_{rp} (upstream) is the permeability, μ is the phase viscosity, B is the phase formation volume factor, Δr and Δz are gridblock sizes in the r and z directions, subscripts i and $i-1$ refer to blocks i and $i-1$, respectively, and $i-1/2$ refers to the interface of blocks i and $i-1$.

In the z direction, the finite difference phase transmissibility, $A_{i,j,k-1/2}$, between blocks $k-1$ and k of Equation 2-43, is

$$A_{i,j,k-1/2} = \frac{2 \Delta z_{i,j,k}}{z_{i,j,k+1} - z_{i,j,k-1}} A'_{i,j,k-1/2} \quad (2-48)$$

where

$$A'_{i,j,k-1/2} = \frac{4k_{rp}(\text{upstream})}{(\mu_{i,j,k-1} + \mu_{i,j,k})(B_{i,j,k-1} + B_{i,j,k})} \left(\frac{2(kA)_{i,j,k-1}(kA)_{i,j,k}}{\Delta z_{i,j,k-1}(kA)_{i,j,k} + \Delta z_{i,j,k}(kA)_{i,j,k-1}} \right) \quad (2-49)$$

$$A_{i,j,k-1} = \pi(r_{i+1/2,j,k-1}^2 - r_{i-1/2,j,k-1}^2) \quad (2-50)$$

$$A_{i,j,k} = \pi(r_{i+1/2,j,k}^2 - r_{i-1/2,j,k}^2) \quad (2-51)$$

$i,j,k-1/2$ refers to the interface of blocks i,j,k and $i,j,k-1$, and $i+1/2$ and $i-1/2$ refer to the interfaces of blocks i and $i+1$ and blocks i and $i-1$, respectively.

For the conservation equation of the chemical species in the aqueous phase in cylindrical coordinates (Eq. 2-25), a central finite difference method of up to 10th order also was used to approximate the spatial derivatives of species concentrations in the r and z directions, $\partial C_i / \partial r$, $\partial C_i / \partial z$, $\partial^2 C_i / \partial r^2$, and $\partial^2 C_i / \partial z^2$, and the forward difference form was used to approximate the spatial derivatives of superficial velocities in the r and z directions.

2.5 Inclusion of the Radial Coordinate System into the PC-GEL Simulator

To include the radial coordinate system into the PC-GEL simulator, six subroutines (GRID1, PARM1, TRAN1, NODES, SOLMAT, and SUBGEL) and the main program were modified. The geometric parts of Equations 2-45 and 2-49,

$$\frac{2(kA)_{i-1,j,k}(kA)_{i,j,k}}{\Delta r_{i-1,j,k}(kA)_{i,j,k} + \Delta r_{i,j,k}(kA)_{i-1,j,k}} \quad (2-52)$$

and

$$\frac{2(kA)_{i,j,k-1}(kA)_{i,j,k}}{\Delta z_{i,j,k-1}(kA)_{i,j,k} + \Delta z_{i,j,k}(kA)_{i,j,k-1}} \quad (2-53)$$

were incorporated into the subroutine TRAN1. Equations 2-44, 2-45, 2-48, and 2-49 were incorporated into the subroutine SOLMAT, which computes the pressure equation coefficients.

The conservation equation in cylindrical coordinates (Eq. 2-25) for each chemical species in the aqueous phase was incorporated into the subroutine SUBGEL. Radial grid block sizes are read by the subroutine GRID1. The radial position and pore volume of each radial grid are calculated in the subroutine PARM1. Superficial velocities in the r and z directions are calculated in the main program.

2.6 Validation of the Radial Model

After the PC-GEL simulator was modified to include the radial coordinate system, the modified simulator was debugged and checked for the r and z components of the Darcy velocity vector under a single-phase (water only) flow condition. Simulation runs also were conducted to check the material balance of polymer injected under single- and two-phase flow conditions. One- and two-layer reservoir models ($10 \times 1 \times 1$ and $10 \times 1 \times 2$) with an equal (10 ft) or variable gridblock sizes (2-75 ft) in the radial direction (Δr) were used. The thickness (h) of each layer was 15 ft. With a variable Δr grid, the ratio of Δr for adjacent gridblocks was kept at about 1.5.

Results showed that with a constant injection rate (Q) of 50 B/D and a time-step size varying between 0.01 and 1 day, the r component velocity (V_r) in each gridblock differed less than 0.3% from the calculated value for a simple radial flow equation, $V_r = Q / (2\pi r h)$. A material balance check showed that the material balance of the chemical injected was more sensitive to the time-step size. The r component velocity (V_r) was less sensitive. In a two-layer reservoir model with variable Δr , instability occurred when a constant injection rate was used. Both injection and

production rates diverged. This could be caused by the explicit calculation of the wellbore pressure. Good material balance (error less than 1%) was obtained with constant pressure at the well and with a small time-step size (0.01 day) during polymer injection. Figure 2-1 compares the polymer concentration profiles using two different variable time-step sizes in a two-layer reservoir model. As the figure shows, instability occurred when a maximum time-step size of one day was used. After 1,594 bbl of polymer solution (1,500 ppm) were injected, the amount of polymer in the reservoir was only 78% of that injected. When a time-step size of 0.01 day was used, no instability occurred. The amount of polymer in the reservoir was 99.7% of that injected. In both cases, no polymer had been produced.

The modified PC-GEL simulator was used to compare simulation results for a 15-layer reservoir model using the radial ($14 \times 1 \times 15$) and rectangular coordinate ($14 \times 14 \times 15$) systems. Figures 2-2 and 2-3 show the polymer concentration profiles for the two runs. The contour plots show polymer concentrations in a vertical radial plane through the treatment well after 1.0 day production and 0.5 day of injection at the same well at constant injection pressure. The contours for the highest concentration (800 ppm) are nearly identical in the two plots. The contours for concentrations of 600 and 400 ppm also are very close to a radial distance of 20 ft. The difference at greater radial distance may be attributed to numerical dispersion. The difference in the lowest concentration (200 ppm) contours is entirely attributable to the numerical dispersion. This comparison shows that the radial grid solution is superior to the rectangular grid solution. The unrealistic second bulge in the contour for 600 ppm concentration is present only in the rectangular grid solution. Also, the higher numerical dispersion causes the contour of 200 ppm concentration to spread out much farther from the injection well in the rectangular grid solution.

Computing times for the two runs cannot be compared directly as they were made on different computers (VAX for radial code and IBM RS6000 for the rectangular grid code). It is estimated that if both codes were run on the same machine, the ratio of the central processing unit (CPU) time requirement will be 25:1, the radial code being faster.

Figure 2-1 Effect of time-step size on the distribution of polymer concentration in the low-permeability layer when injecting at a constant rate of 50 bbl per day

Figure 2-2 Contours of polymer concentration from the radial coordinate system

Figure 2-3 **Contours of polymer concentration from the rectangular coordinate system**

3.0 THERMAL ENERGY EQUATION

To account for the change in temperature of the polymer gel system during pumping, a 3D, thermal energy equation was derived for three-phase (oil, water, gas) fluid flow through a reservoir, with solution gas in both oil and water phases. Applying the principle of conservation of energy to a volume element, we have

$$\frac{\partial[(1-\phi)\rho_R C_R T]}{\partial t} + (a \frac{\phi S_o}{B_o} + b \frac{\phi S_w}{B_w} + \frac{\phi S_g}{B_g} \rho_{gsc} C_v) \frac{\partial T}{\partial t} = -\nabla \cdot q -$$

$$(\frac{a}{B_o} V_o + \frac{b}{B_w} V_w + \frac{\rho_{gsc} C_v}{B_g} V_g) \cdot \nabla T - q_L - \sum_{i=1}^3 \frac{Q_i \rho_i H_i}{V} \quad (3-1)$$

where

$$a = \rho_{osc} C_{p,o} + \rho_{gsc} R_{so} C_v \quad (3-2)$$

$$b = \rho_{wsc} C_{p,w} + \rho_{gsc} R_{sw} C_v \quad (3-3)$$

B_o, B_w, B_g = formation volume factors for oil, water, and gas phases; $C_{p,o}, C_{p,w}, C_R, C_v$ = specific heats for oil, water, reservoir rock, and gas; H = enthalpy of fluid; Q = well rate; q = rate of heat conduction per unit area; q_L = rate of heat loss per unit volume to overburden and underburden; R_{so}, R_{sw} = gas solubilities in oil and water; S_o, S_w, S_g = oil, water, and gas saturations; T = temperature; t = time; V = well block volume; V_i = Darcy velocity vector for component i ; V_o, V_w , and V_g = Darcy velocities vectors for oil, water, and gas phases; $\rho_{osc}, \rho_{wsc}, \rho_{gsc}$ = oil, water, and gas densities at standard conditions (usually 60°F and 14.7 psia); and ∇ = divergence operator.

The assumptions made in the derivation of Equation 3-1 are as follows:

1. Thermal equilibrium between fluids and rock matrix is instantly achieved within the grid concerned.
2. Nuclear, radiative, and electromagnetic energy are not considered.
3. Oil and water are immiscible.

4. Mass transfer of water or oil components from the liquid phase to the gas phase is not considered.
5. The heat losses through lateral reservoir boundaries are assumed to be zero.
6. The contribution of kinetic energy and the work done by viscous forces are negligible.

3.1 Finite Difference Form of the Thermal Energy Equation in Rectangular Coordinates

The finite difference form of the thermal energy equation in rectangular coordinates is

$$\begin{aligned}
 (V_B)_{i,j,k} \left[(1 - \phi) \rho_R C_R + a \frac{\phi S_o}{B_o} + b \frac{\phi S_w}{B_w} + \frac{\phi S_g}{B_g} \rho_{gsc} C_v \right]_{i,j,k} \frac{T_{i,j,k}^{n+1} - T_{i,j,k}^n}{\Delta t} = 24 \times \\
 (\Delta \tau \Delta T)_{i,j,k} + \Delta C_o \Delta \left(p_o - \frac{\rho_o z}{144} \right) + \Delta C_w \Delta \left(p_o - \frac{\rho_w z}{144} - p_{cow} \right) + \Delta C_g \Delta \left(p_o - \frac{\rho_g z}{144} + p_{cgo} \right) \\
 - \frac{(V_B)_{i,j,k} U_o^n}{\Delta t} \left[\left(\frac{\phi \rho_{osc} S_o}{B_o} \right)^{n+1} - \left(\frac{\phi \rho_{osc} S_o}{B_o} \right)^n \right] - \frac{(V_B)_{i,j,k} U_w^n}{\Delta t} \left[\left(\frac{\phi \rho_{wsc} S_w}{B_w} \right)^{n+1} - \left(\frac{\phi \rho_{wsc} S_w}{B_w} \right)^n \right] - \\
 \frac{(V_B)_{i,j,k} U_g^n}{\Delta t} \left[\phi \rho_{gsc} \left(\frac{S_g}{B_g} + \frac{R_{so} S_o}{B_o} + \frac{R_{sw} S_w}{B_w} \right)^{n+1} - \left(\frac{S_g}{B_g} + \frac{R_{so} S_o}{B_o} + \frac{R_{sw} S_w}{B_w} \right)^n \right] - \\
 (V_B)_{i,j,k} q_L - (Q_o \rho_o H_o + Q_w \rho_w H_w + Q_g \rho_g H_g)_{i,j,k}
 \end{aligned} \tag{3-4}$$

where

$$\Delta \tau \Delta T = \Delta_x \tau_x \Delta T_x + \Delta_y \tau_y \Delta T_y + \Delta_z \tau_z \Delta T_z \tag{3-5}$$

$$\Delta_x \tau_x \Delta T_x = \tau_{i-1/2,j,k} (T_{i-1,j,k} - T_{i,j,k}) + \tau_{i+1/2,j,k} (T_{i+1,j,k} - T_{i,j,k}) \tag{3-6}$$

$$\tau_{i-1/2,j,k} = \frac{2 \Delta x_i}{\Delta x' + \Delta x''} \tau'_{i-1/2,j,k} \tag{3-7}$$

$$\tau'_{i-1/2,j,k} = [w \lambda_{i-1,j,k} + (1-w) \lambda_{i,j,k}] \frac{2 A_{i-1,j,k} A_{i,j,k}}{\Delta x_{i-1,j,k} A_{i,j,k} + \Delta x_{i,j,k} A_{i-1,j,k}} \tag{3-8}$$

$$w = 1 \text{ if } T_{i-1,j,k} > T_{i,j,k}; w = 0 \text{ if } T_{i,j,k} > T_{i-1,j,k}$$

$$\Delta C \Delta p = \Delta_x C_x \Delta p_x + \Delta_y C_y \Delta p_y + \Delta_z C_z \Delta p_z \quad (3-9)$$

$$\Delta_x C_x \Delta p_x = C_{i-1/2,j,k} (p_{i-1,j,k} - p_{i,j,k}) + C_{i+1/2,j,k} (p_{i+1,j,k} - p_{i,j,k}) \quad (3-10)$$

$$C_{i-1/2,j,k} = \frac{2\Delta x_i}{\Delta x' + \Delta x''} C'_{i-1/2,j,k} \quad (3-11)$$

$$(C'_{i-1/2,j,k})_o = \frac{4(\rho_{osc} H_o + R_{so} \rho_{gsc} H_g) k_{ro(upstream)}}{(\mu_{i-1,j,k} + \mu_{i,j,k})(B_{i-1,j,k} + B_{i,j,k})} \times$$

$$\frac{2(kA)_{i-1,j,k}(kA)_{i,j,k}}{\Delta x_{i-1,j,k}(kA)_{i,j,k} + \Delta x_{i,j,k}(kA)_{i-1,j,k}} \quad (3-12)$$

$$(C'_{i-1/2,j,k})_w = \frac{4(\rho_{wsc} H_w + R_{sw} \rho_{gsc} H_g) k_{rw(upstream)}}{(\mu_{i-1,j,k} + \mu_{i,j,k})(B_{i-1,j,k} + B_{i,j,k})} \times$$

$$\frac{2(kA)_{i-1,j,k}(kA)_{i,j,k}}{\Delta x_{i-1,j,k}(kA)_{i,j,k} + \Delta x_{i,j,k}(kA)_{i-1,j,k}} \quad (3-13)$$

$$(C'_{i-1/2,j,k})_g = \frac{4(\rho_{gsc} H_g) k_{rg(upstream)}}{(\mu_{i-1,j,k} + \mu_{i,j,k})(B_{i-1,j,k} + B_{i,j,k})} \times$$

$$\frac{2(kA)_{i-1,j,k}(kA)_{i,j,k}}{\Delta x_{i-1,j,k}(kA)_{i,j,k} + \Delta x_{i,j,k}(kA)_{i-1,j,k}} \quad (3-14)$$

$$(V_B)_{i,j,k} q_L = \text{total rate of heat loss to the overburden or the underburden.}$$

The transconductivity, $\tau'_{i-1/2,j,k}$, defined in Equation 3-8 is the ability of heat to be transmitted between two adjacent gridblocks by conduction under the influence of a unit change in temperature. The first group on the right-hand side is an averaged thermal conductivity based on a simple weighting procedure. In the present study, an upstream weighting is used exclusively. The second group is a harmonic average of $(A/\Delta x)_{i-1,j,k}$ and $(A/\Delta x)_{i,j,k}$. A semi-analytical method developed by Vinsome et al. (1980) is used to calculate $(V_B)_{i,j,k} q_L$. This

method assumes that the temperature profile in the overburden or the underburden rock has the form

$$T(t,z) = (\theta + pz + qz^2)e^{-z/d} \quad (3-15)$$

where θ is the interface temperature between the reservoir and the overburden or underburden rock, p and q are fitting parameters, d is the diffusion length, z is vertical position, and t is the time measured from the instant at which the interface temperature first begins to change. Inserting the previous equation into the following condition at the interface,

$$\frac{\partial \theta}{\partial t} = \alpha \left. \frac{\partial^2 T}{\partial z^2} \right|_{z=0} \quad (3-16)$$

together with a finite-difference discretization of the time derivative, gives

$$\frac{\theta - \theta^n}{\Delta t} = \alpha \left(\frac{\theta}{d^2} - \frac{2p}{d} + 2q \right) \quad (3-17)$$

where θ^n is the interface temperature at the beginning of the time step. By integrating the heat conduction equation and using Equation 3-15, the following equation is obtained:

$$\frac{\theta d + p d^2 + 2p d^3 - I^n}{\Delta t} = \alpha \left(\frac{\theta}{d} - p \right) \quad (3-18)$$

where $I^n = q^n d^n + p^n (d^n)^2 + 2q^n (d^n)^3$. From Equations 3-17 and 3-18,

$$p = \frac{\frac{\alpha \Delta t \theta}{d} + I^n - \frac{d^3 (\theta - \theta^n)}{\alpha \Delta t}}{3d^2 + \alpha \Delta t} \quad (3-19)$$

Hence,

$$(V_B)_{i,j,k} q_L = \Delta y_{i,j,k} \Delta x_{i,j,k} \lambda \left. \frac{\partial T}{\partial z} \right|_{z=0} = \Delta y_{i,j,k} \Delta x_{i,j,k} \lambda \left(\frac{\theta}{d} - p \right) \quad (3-20)$$

3.2 Finite Difference Form of the Thermal Energy Equation in Cylindrical Coordinates

Equation 3-4 also represents the finite-difference form of the thermal energy equation in cylindrical coordinates. The parameters are defined as follows:

$$\Delta\tau\Delta T = \Delta_r\tau_r\Delta T_r + \Delta_z\tau_z\Delta T_z \quad (3-21)$$

$$\Delta_r\tau_r\Delta T_r = \tau_{i-1/2,j,k} (T_{i-1,j,k} - T_{i,j,k}) + \tau_{i+1/2,j,k} (T_{i+1,j,k} - T_{i,j,k}) \quad (3-22)$$

$$\tau_{i-1/2,j,k} = \frac{r_{i+1/2}^2 - r_{i-1/2}^2}{r_i (r_{i+1} - r_{i-1})} \tau'_{i-1/2,j,k} \quad (3-23)$$

$$\tau'_{i-1/2,j,k} = [w\lambda_{i-1,j,k} + (1-w)\lambda_{i,j,k}] \frac{2(A_c)_{i-1,j,k}(A_c)_{i,j,k}}{\Delta r_{i-1,j,k}(A_c)_{i,j,k} + \Delta r_{i,j,k}(A_c)_{i-1,j,k}} \quad (3-24)$$

$$w = 1 \text{ if } T_{i-1,j,k} > T_{i,j,k}; w = 0 \text{ if } T_{i,j,k} > T_{i-1,j,k}$$

$$(A_c)_{i-1,j,k} = \pi\Delta z_{i,j,k} \frac{\Delta r_{i-1}}{\ln \frac{r_{i-1} + (\Delta r_{i-1})/2}{r_{i-1}}} \quad (3-25)$$

$$(A_c)_{i,j,k} = \pi\Delta z_{i,j,k} \frac{\Delta r_i}{\ln \frac{r_i}{r_i - (\Delta r_i)/2}} \quad (3-26)$$

$$\tau_{i,j,k-1/2} = \frac{2\Delta z_{i,j,k}}{z_{i,j,k+1} - z_{i,j,k-1}} \tau'_{i,j,k-1/2} \quad (3-27)$$

$$(A_c)_{i,j,k-1} = \pi (r_{i+1/2,j,k-1}^2 - r_{i-1/2,j,k-1}^2) \quad (3-28)$$

$$(A_c)_{i,j,k} = \pi (r_{i+1/2,j,k}^2 - r_{i-1/2,j,k}^2) \quad (3-29)$$

$$\Delta C\Delta p = \Delta_r C_r \Delta p_r + \Delta_z C_z \Delta p_z \quad (3-30)$$

$$\Delta_r C_r \Delta p_r = C_{i-1/2,j,k} (p_{i-1,j,k} - p_{i,j,k}) + C_{i+1/2,j,k} (p_{i+1,j,k} - p_{i,j,k}) \quad (3-31)$$

$$C_{i-1/2,j,k} = \frac{r_{i+1/2}^2 - r_{i-1/2}^2}{r_i (r_{i+1} - r_{i-1})} C'_{i-1/2,j,k} \quad (3-32)$$

$$C_{i,j,k-1/2} = \frac{2\Delta z_{i,j,k}}{z_{i,j,k+1} - z_{i,j,k-1}} C'_{i,j,k-1/2} \quad (3-33)$$

$$(C'_{i-1/2,j,k})_o = \frac{4(\rho_{osc}H_o + R_{so}\rho_{gsc}H_g)k_{ro(upstream)}}{(\mu_{i-1,j,k} + \mu_{i,j,k})(B_{i-1,j,k} + B_{i,j,k})} \times$$

$$\frac{2(kA_c)_{i-1,j,k}(kA_c)_{i,j,k}}{\Delta r_{i-1,j,k}(kA_c)_{i,j,k} + \Delta r_{i,j,k}(kA_c)_{i-1,j,k}} \quad (3-34)$$

$$(C'_{i-1/2,j,k})_w = \frac{4(\rho_{wsc}H_w + R_{sw}\rho_{gsc}H_g)k_{rw(upstream)}}{(\mu_{i-1,j,k} + \mu_{i,j,k})(B_{i-1,j,k} + B_{i,j,k})} \times$$

$$\frac{2(kA_c)_{i-1,j,k}(kA_c)_{i,j,k}}{\Delta r_{i-1,j,k}(kA_c)_{i,j,k} + \Delta r_{i,j,k}(kA_c)_{i-1,j,k}} \quad (3-35)$$

$$(C'_{i-1/2,j,k})_g = \frac{4(\rho_{gsc}H_g)k_{rg(upstream)}}{(\mu_{i-1,j,k} + \mu_{i,j,k})(B_{i-1,j,k} + B_{i,j,k})} \times$$

$$\frac{2(kA_c)_{i-1,j,k}(kA_c)_{i,j,k}}{\Delta r_{i-1,j,k}(kA_c)_{i,j,k} + \Delta r_{i,j,k}(kA_c)_{i-1,j,k}} \quad (3-36)$$

3.3 Thermal Properties of Reservoir Fluids and Rocks

Thermal properties used in the thermal model include density, viscosity, thermal conductivity, heat capacity, and enthalpy.

Density

The density of rock is taken as a constant. For sandstone and carbonate, densities are 164.19 lb/ft³ and 167.31 lb/ft³, respectively (Butler 1991).

The densities of oil, water, and gas phases are considered as a function of temperature and pressure. The change of oil phase density with temperature is calculated from the following equation:

$$\rho_o(T) = \rho_{15^\circ C} (1 - 3.4917E-4 (T - 59) + 4.40123E-8 (T - 59)^2) \quad (3-37)$$

where

$$\rho_{15^\circ C} = 141.5 / (131.5 + ^\circ API) \times 62.36 \quad (3-38)$$

The previous equation represents very well the densities of petroleum fractions and crude oils (Butler 1991). A pressure equation is then applied to Equation 3-37 to account for the change in pressure. Alternately, the pressure effect is accounted for by using a formation volume factor.

$$\rho_o(T, P) = \rho_o(T) [1 + C_o(P - P_{init})] \quad (3-39)$$

The density of water at temperatures up to 554°F is calculated from Equation 3-40 (Butler 1991).

$$\rho_w(T) = 62.536 - 0.005605(T - 32) - 5.0484E-5(T - 32)^2 \quad (3-40)$$

A pressure equation is then applied to Equation 3-40. Alternately, the pressure effect is accounted for by using a formation volume factor.

$$\rho_o(T, P) = \rho_o(T) [1 + C_o(P - P_{init})] \quad (3-41)$$

The gas-phase density is computed from the gas law, $\rho = PM/[ZR(T + 460)]$. The gas deviation factor, Z , is estimated from Papay's equation (Takacs 1976):

$$Z = 1 - \frac{3.52 P_r}{10^{0.9813} T_r} + \frac{0.274 P_r^2}{10^{0.8157} T_r} \quad (3-42)$$

where P_r = reduced pressure and T_r = reduced temperature. Pressure effect is accounted for by using the formation volume factor for the gas phase.

Viscosity

The viscosities of oil and water are considered as a function of temperature only. A two-parameter, double exponential equation is used to calculate the viscosity of oil phase as a function of temperature (Butler 1991):

$$\log [\log (m + 0.7)] = m \log [(T + 460)/1.8] + b \quad (3-43)$$

The two parameters, m and b , can be determined from two known viscosities at two different temperatures.

The water phase viscosity is calculated using the Bingham's equation (Ali 1970):

$$\mu^{-1} = 0.0119344\{(T - 47.183) + [26174.016 + (T - 47.183)^2]^{0.5}\} - 1.2 \quad (3-44)$$

The gas phase viscosity is treated as a function of temperature and pressure. The effect of temperature on gas-phase viscosity at low pressures (a few mm of Hg to several atm) is estimated from the Licht and Stechert's relationship (Gambill 1958):

$$\mu_1 = \frac{6.30 \text{ E-4 } M^{0.5} P_c^{0.667}}{T_c^{0.167}} \left(\frac{T_r^{1.5}}{T_r + 0.8} \right) \quad (3-45)$$

The viscosity obtained from the previous equation is then corrected for the pressure effect using the following equation for nonpolar gases:

$$\begin{aligned} \mu - \mu_1 = & [(0.10230 + 0.023364 \rho_r + 0.058533 \rho_r^2 - 0.040758 \rho_r^3 + \\ & 0.0093324 \rho_r^4) - 0.0001] / T_c^{0.16667} M^{-0.5} P_c^{-0.66667} \end{aligned} \quad (3-46)$$

and

$$\mu - \mu_1 = 0.0001656 \rho_r^{1.111} / T_c^{0.16667} M^{-0.5} P_c^{-0.66667} \quad (\rho_r \leq 0.1) \quad (3-47)$$

$$\begin{aligned} \mu - \mu_1 = & 0.00000607 (9.045 \rho_r + 0.63)^{1.739} / T_c^{0.16667} M^{-0.5} P_c^{-0.66667} \\ & (0.1 \leq \rho_r \leq 0.9) \end{aligned} \quad (3-48)$$

and

$$\log [-\log (\mu - \mu_1) T_c^{0.16667} M^{-0.5} P_c^{-0.66667}] = 0.6439 - 0.1005 \rho_r$$

$$(0.9 \leq \rho_r \leq 2.2) \quad (3-49)$$

for polar gases (Ali 1970).

Thermal Conductivity

Many factors including the temperature, porosity, fluid saturations, and types of fluids affect the formation thermal conductivity. The following equation recommended by Farouq Ali (Ali 1970) is used to estimate the formation thermal conductivity.

$$\lambda = 8.788 \exp(0.6 \rho_R / 62.422 + 0.6 S_w)(T + 460)^{-0.55} \quad (3-50)$$

For over- and underburden rocks, thermal conductivities are assumed to be independent of temperature (Ali 1974, Butler 1991).

Heat Capacity

The specific heat of sandstones and carbonate stones are calculated, respectively, from the following equations (Butler 1991):

$$C_p = 0.17077 + 2.265E-4 (T - 32) - 1.4065E-7 (T - 32)^2 \quad (3-51)$$

and

$$C_p = 0.19656 + 2.0049E-4 (T - 32) - 1.06E-7 (T - 32)^2 \quad (3-52)$$

For petroleum fractions and oils, Equation 3-53 is used.

$$C_p = (0.388 + 0.00045 T) / s^{0.5} \quad (3-53)$$

$$H = [0.388(T - 32) + 0.000225(T^2 - 1024)] / s^{0.5} \quad (3-54)$$

This equation was reportedly accurate to within 3% (Ali 1970).

For liquid water at saturation conditions (Butler 1991),

$$C_p = 0.9988 - 1.99E-5 (T - 32) + 2.536E-8 (T - 32)^2 + 1.7446E-9 (T - 32)^3$$

$$50^\circ F < T \leq 464^\circ F \quad (3-55)$$

$$H = 0.99941(T - 32) - 8.082E-6(T^2 - 1024) - 4.7373E-8(T^3 - 32768) + 4.3615E-10(T^4 - 1048576)$$

$$50^\circ F < T \leq 464^\circ F \quad (3-56)$$

and

$$C_p = 2.7585 - 8.5606E-3 (T - 32) + 1.1121E-5 (T - 32)^2$$

$$464^\circ F \leq T < 572^\circ F \quad (3-57)$$

$$H = 3.04383(T - 32) - 4.6362E-3(T^2 - 1024) + 3.707E-6(T^3 - 32768)$$

$$464^\circ F \leq T < 572^\circ F \quad (3-58)$$

For gas phase, a three-parameter equation is used (Butler 1991).

$$C_p = a + bT + c/(T - 460)^2 \quad (3-59)$$

$$H = a(T - 32) + \frac{b}{2}(T^2 - 1024) - c\left(\frac{1}{T - 460} + \frac{1}{428}\right) \quad (3-60)$$

where a, b, and c are parameters. Equations 3-59 and 3-60 apply to common gases such as air, hydrogen, nitrogen, oxygen, carbon dioxide, carbon monoxide, and methane.

Enthalpy

For calculating the oil-phase enthalpy, the correlation for the Mid-Continent oils in liquid form is used (Nelson 1936):

$$H = 15s - 26 - (0.465s - 0.811)T + 0.00029T^2 \quad (3-61)$$

The enthalpy of liquid water in state of saturation is calculated from the following expression (Burger et al. 1985):

$$H = 1.00315 (T - 32) \quad 32^\circ F \leq T \leq 266^\circ F \quad (3-62)$$

$$H = 1.00315(T - 32) + 9.2593E-4(T - 266)^2 \quad 266^\circ F \leq T \leq 590^\circ F \quad (3-63)$$

$$H = 1.00315(T - 32) + 9.2593E-4(T - 266)^2 + 1.3717E-4(T - 590)^3$$

$$590^\circ F \leq T \leq 703.4^\circ F \quad (3-64)$$

The gas phase enthalpy is computed as (Nelson 1936)

$$H = C_p(T - T_{ref}) \quad (3-65)$$

3.4 Inclusion of the Thermal Energy Equation into the PC-GEL Simulator

Two subroutines, THERMDATA and THERMO, were written and incorporated into the PC-GEL simulator. Subroutine THERMDATA reads the thermal properties of each phase, including the heat capacities, densities, and thermal conductivities of the overburden and underburden rocks, parameters of the reservoir rock heat capacity equation, parameters of the gas-phase heat capacity equation, API gravity of oil phase, parameters of the oil viscosity-temperature equation, and critical temperature and pressure of the gas phase. Initial fluid densities and viscosities are calculated in this subroutine. The subroutine THERMO solves the thermal energy equation and updates the fluid properties.

3.5 Validation of Thermal Energy Equation

Test of the thermal simulation capability was carried out by comparison with an analytical solution for a one-dimensional porous medium. In this test, it was assumed that a 140-ft-long slab (140 ft × 1 ft × 15 ft) of porous medium is initially filled with water at a uniform temperature of 100°F. A slug of gelant at 50°F is injected into it at a rate of 100 bbl/day. It can be shown that when the heat conduction in the direction of the fluid flow and heat loss to the surroundings are ignored, and that an instantaneous thermal equilibrium is established between the rock and the fluid, the thermal front will lag behind the fluid front. At the thermal front, the temperature will jump from the injection fluid temperature to the initial reservoir temperature. Using a heat balance, the locations of the two fronts in rectangular coordinates can be related by the following equation:

$$\frac{D_w - D_T}{D_T} = \frac{C_R \rho_R}{C_w \rho_w} \frac{1 - \phi}{\phi} \quad (3-66)$$

where D_w and D_T are the respective distances of the fluid and thermal fronts from the injection face. In radial coordinates, the locations of the two fronts can be related by

$$\frac{R_w}{R_T} = \sqrt{1 + \frac{C_R \rho_R}{C_w \rho_w} \frac{1-\phi}{\phi}} \quad (3-67)$$

where R_w and R_T are the respective distances of the fluid and thermal fronts from the injection well.

For $C_R/C_w = 0.1708$, $\rho_R/\rho_w = 2.641$, and $\phi = 0.3$, the theoretical ratio of the two distances in rectangular coordinates is 2.053. In radial coordinates, the ratio is 1.433. Figures 3-1 and 3-2 show calculated gelant concentration and temperature distributions along the porous medium in rectangular and radial coordinates, respectively, using the simulator. In these simulation runs, the heat conduction term and the heat loss to the surroundings were set to zero to mimic the assumptions made in the analytical analysis. Figure 3-1 shows that the middle points of the two fronts are located such that their distances from the injection face are in a ratio of about 2.067 in good agreement with the theoretical ratio of 2.053. In radial coordinates, the ratio of the two middle points is about 1.5, also in good agreement with the theoretical value of 1.433.

Figure 3-1 Calculated gelant concentration and temperature profiles in a 1D rectangular porous medium

Figure 3-2 Calculated gelant concentration and temperature profiles in a 1D radial porous medium

4.0 TEMPERATURE-DEPENDENT GEL CHEMISTRY AND FLUID RHEOLOGY OF A DELAYED GEL SYSTEM

The gelation kinetics and gelant rheology of a delayed gel system (DGS) developed by Schlumberger Dowell for water control were incorporated into the PC-GEL permeability modification simulator for simulation studies of the effect of temperature on the placement of the gel system in reservoirs. The DGS contains a gelling agent and one to two activators. Upon decomposition of the activators, a bridging agent is generated. Gelation occurs when the molar ratio of the bridging agent to the metal element of the gelling agent reaches a critical value of 2.75. This ratio monotonically increases with time, and increases through the critical value and continued to rise after gelation has occurred. It was found that the concentration of the activator decreased exponentially according to a first order reaction rate model. The reaction rate constant obeyed the Arrhenius equation, and thus was exponentially dependent on the reciprocal of the absolute temperature. The gelation time decreased with an increase in the salt concentration, and this dependence has been characterized through empirical correlations.

The rate equation for each of the two activators can be expressed as

$$\frac{dA}{dt} = -kA \quad (4-1)$$

where

$$k = e^a \cdot e^{-b/T} \quad (4-2)$$

where A = the molar concentration of the activator, k = the rate constant, a and b are empirical constants, and T = temperature in Kelvin. Four components in the aqueous phase (two for the activators, one for the bridging agent, and one for the gelling agent) and the same four components in the gelled phase were modeled in the simulator. Gelation occurs when the molar ratio of the bridging agent to the metal element reaches 2.75. Syneresis, which occurs when the molar ratio of the bridging agent to the metal element in the gelled phase is greater than 3.0, also was considered. With one activator, the gelation time (GT) can be expressed as

$$GT = -\ln(1 - 0.25 \times [M]/[A])/k \quad (4-3)$$

where $[M]$ = molar concentration of gelling agent, $[A]$ = molar concentration of activator, and k = reaction rate constant.

Figure 4-1 shows the gelation time as a function of the molar ratio of the gelling agent to the activator at temperatures of 37.8°C and 80°C for an activator having the parameters a equal to 35.02 and b equal to 14021. With this activator, the gelation time is 1000 hr at 37.8°C for

$[M]/[A] = 0.162$. However, at 80°C its gelation time decreases to 4.54 hr, a decrease of more than 220 times. Hence, proper consideration of the temperature effect is very important in the placement of the DGS. Ignoring the temperature effect on this system will result in a premature gelation and a shut down of the injection before the gel system reaches the intended region.

The empirical equation for the gelant viscosity as a function of temperature and gelant concentration is

$$\mu = 1.002 \times e^{[1899.2 (1/T - 1/293)]} \times (1 + 6.2209E-6 \times C - 5.4336E-11 \times C^2) \quad (4-4)$$

where μ = viscosity in cp, T = temperature in Kelvin, and C = metal element concentration in ppm. Equations 4-1 through 4-4 are included in the simulator. Simulation studies of the placement of the DGS are discussed in the wellbore simulator section.

Figure 4-1 Gelation time of DGS as a function of temperature and molar ratio of gelling agent to activator

5.0 WELLBORE SIMULATOR

Schlumberger Dowell's wellbore simulator was developed to calculate the change in temperature of the injected fluid as it travels from the wellhead to the bottom of the wellbore. Inclusion of the wellbore simulator in the PC-GEL permeability modification simulator will allow the proper consideration of the temperature effect on the gelation rate. Ignoring the temperature effect on the gelation rate may result in a premature gelation before the injected fluid enters into the formation. The consequences are the face plugging and the shut down of the injection.

Figure 5-1 shows the resistance to heat flow between the injection fluid and the formation. Resistance to heat flow includes the tubing wall, annulus, casing wall, and cement. With the wellbore simulator, temperatures of the tubing fluid, annular fluid, tubing wall, casing wall, and at the formation surface between rock and cement were calculated for all grid points. Required input data include injection rate, injection time, time-step size, well properties, rheological and thermal properties of the injected fluid, depth, density, and thermal properties of the formation.

Figure 5-1 Cross section of wellbore model

Prior to combination with the PC-GEL simulator, the code was modified to allow the use of a variable time-step size. The actual time-step size at each time step is determined from the PC-GEL simulator. To reduce the size of the executable file, the dimensions of the tubing fluid temperature profile, annular fluid temperature profile, and the formation temperature profile were reduced from (maximum number of grid points) $\times$ (maximum number of time steps) to (maximum number of grid points) $\times$ 3. After combination with the PC-GEL simulator, wellbore simulation calculations were conducted. Results were compared with those from the wellbore simulator itself. Comparison showed that temperature profiles in the wellbore were exactly the same before and after combination with the PC-GEL simulator. Table 5-1 shows calculated temperatures of the tubing fluid, annular fluid, and formation, and static head at the bottom of the wellbore as a function of time.

Table 5-1 Bottomhole temperatures and static head

Time (min)	Tubing Fluid (°C)	Annulus (°C)	Formation (°C)	Head (psi)
0.	37.8	37.8	37.8	4506.83
7.2	37.56	37.74	37.77	4506.9
14.4	37.38	37.68	37.74	4506.97
21.6	37.24	37.64	37.71	4507.04
28.8	37.14	37.59	37.68	4507.1
36.	37.06	37.56	37.65	4507.16
43.2	36.99	37.53	37.63	4507.22
50.4	36.94	37.5	37.61	4507.27
57.6	36.89	37.47	37.59	4507.32
64.8	36.84	37.45	37.57	4507.37
72.	36.81	37.43	37.55	4507.42
79.2	36.77	37.41	37.53	4507.46
86.4	36.74	37.39	37.52	4507.51
93.6	36.71	37.37	37.5	4507.55
100.8	36.68	37.36	37.49	4507.59
108.	36.65	37.34	37.47	4507.63
115.2	36.62	37.32	37.46	4507.67
122.4	36.6	37.31	37.45	4507.7
129.6	36.57	37.29	37.43	4507.73
136.8	36.55	37.28	37.42	4507.77
144.	36.53	37.27	37.41	4507.8
151.2	36.51	37.25	37.4	4507.83
158.4	36.49	37.24	37.39	4507.86
165.6	36.47	37.23	37.37	4507.89
172.8	36.45	37.22	37.36	4507.92

5.1 Modification of the Wellbore Simulator to Include a 1D In-Situ Gelation Model

To describe the changes in species concentrations of the gelant system in the wellbore, an in-situ gelation model was incorporated into the wellbore simulator. The in-situ gelation model consists of a 1D multicomponent transport equation (Eq. 5-1) and the gelation reaction kinetics:

$$\frac{\partial(\rho C_i)}{\partial t} = D \frac{\partial}{\partial z} \left(\rho \frac{\partial C_i}{\partial z} \right) - \frac{\partial(\rho C_i V_z)}{\partial z} + \rho R_i \quad (5-1)$$

where C_i = mass concentration of component i , D = a constant dispersion coefficient, V_z = fluid velocity in the z direction, ρ = fluid density, and R_i = formation rate of component i . Without the reaction term, Equation 5-1 becomes a dispersion-convection equation. With the gelation reaction term, the equation becomes nonlinear. An extended method of lines (MOL) (Ames 1977) was used to solve Equation 5-1. With MOL, the previous partial differential equation is expressed as a system of coupled nonlinear ordinary differential equations (ODES). The system of ODES was then solved using a Runge-Kutta-Gill method.

To determine how the gridblock size affects the shape of the fluid front, the gelation reaction term was neglected. Figures 5-2 and 5-3 show that increasing the number of grid points (decreasing the gridblock size) decreases the spreading of the fluid front when there was no dispersion (Fig. 5-2) and when the dispersion coefficient was low (Fig. 5-3). These indicate that when the fluid flow is dominated by convection, fine gridblock size is required. In the MOL, the spatial derivatives were approximated with 5th order central difference. Both figures also show that with the same gridblock size, MOL with 5th order central difference gave less spreading at the fluid front than did the Crank-Nicolson method (Carnahan 1969). Although the shape of the fluid front is sensitive to the number of grid points, temperature profiles are not affected. Calculated temperature profiles were the same regardless of whether 20 or 160 grid points were used.

Two gel systems, a polymer/dichromate/thiourea gel system and an inorganic DGS, were considered. The gelation kinetics of the polymer/dichromate/thiourea system is described in a previous report (Gao et al. 1990). The gelation kinetics of the DGS is described in Equations 4-1 and 4-2. The gelation time as a function of gelant concentration and temperature is described in Equation 4-3 and Figure 4-1.

The DGS contains a gelling agent and one to two activators. Upon decomposition of the activators, a bridging agent is generated. Gelation occurs when the molar ratio of the bridging agent to the metal element of the gelling agent reaches a critical value of 2.75. Syneresis occurs when the molar ratio of the bridging agent to the metal element in the gelled phase is greater than 3.0. Four components in the aqueous phase (two for the activators, one for the bridging agent, and one for the gelling agent) and the same four components in the gelled phase were modeled in the simulator. Once gel is formed, the gel is assumed to be immovable.

Figure 5-2 Gelant concentration profile in a wellbore (dispersion coefficient = 0)

Figure 5-3 Gelant concentration profile in a wellbore (dispersion coefficient = $1 \text{ E-5 cm}^2/\text{sec}$)

5.2 Validation of the Modified Wellbore Simulator

The simulator was validated by simulating a batch reaction in both the wellbore and the reservoir with the DGS. A 1D rectangular ($70 \times 1 \times 1$) reservoir model and a 1D radial ($14 \times 1 \times 1$) reservoir model were used. The depth of the top of the reservoir was 10,223 ft. The thickness of the rectangular model was 15 ft, and the thickness of the radial model was 3 ft. To mimic a batch reaction, both the wellbore and the reservoir temperatures were set at 37.8°C (100.04°F), and the initial $[M]/[A]$ in all gridblocks was set at 0.05. Injection rate was equal to 0. Calculated results with the PC-GEL simulator showed that the gelation time in the wellbore was 12.67 days and in the reservoir was 12.65 days. Both are in agreement with that (12.67 days) calculated from Equation 4-3.

Simulation of gel placement using the DGS also was conducted with 1D rectangular ($70 \times 1 \times 1$) and 1D radial ($14 \times 1 \times 1$) reservoir models. Injection rate was 100.8 bbl/day. The DGS contained 30,000 ppm of organic activator and 1,350 ppm (metal element) of the gelling agent. To place the gel in the reservoir, 504 bbl of DGS were injected into the wellbore followed by 241.9 bbl of water. The reservoir was then shut in until the gel was formed around the wellbore. Two scenarios were simulated: (1) a constant temperature (37.8°C) for injection fluid, formation, and reservoir, and (2) different temperatures for injection fluid (14°C), formation (14°C at top, 37.8°C at bottom), and reservoir (37.8°C).

Simulation results showed that under a constant temperature of 37.8°C (100.04°F), gelation was completed around the wellbore 17.54 days after the start of DGS injection in the radial model. This is equivalent to a gelation time of 12.58 days for the DGS in the gridblock around the wellbore, compared with the gelation time of 12.67 days calculated from Equation 4-3. In the rectangular model, it took 17.432 days to form gel within 97 ft from the wellbore. This is equivalent to a gelation time of 12.52 days for the DGS in the gridblock around the wellbore, compared with the gelation time of 12.67 days calculated from Equation 4-3.

When the DGS was injected at 14°C (57.2°F) and the reservoir was initially at 37.8°C (100.04°F), it took 25.02 and 24.98 days to complete gelation around the wellbore in the radial and rectangular reservoir models, respectively. Gel formed extended from the near wellbore to 42.5 ft from the wellbore in the radial model and 97 ft from the wellbore in the rectangular model. Figures 5-4 and 5-5 show calculated temperature and concentration profiles of the organic activator, bridging agent, and gelling agent in the wellbore, respectively, right before the reservoir was shut in. Figure 5-5 shows that at the end of fluid injection, there was still some organic activator and gelling agent left at the bottom of the wellbore. These chemicals need to be cleaned up before gel is formed. Figures 5-6 and 5-7 show calculated temperature and chemical concentration profiles in the radial and rectangular reservoir models, respectively, right before the reservoir was shut in. After shut-in, gel was progressively formed towards the wellbore. It took about 17.76 days after shut-in to complete gelation.

Figure 5-4 Temperature profiles of tubing fluid, annulus fluid, and formation in the wellbore

Figure 5-5 Concentration profiles of activator, gelling agent, and bridging agent in the wellbore

Figure 5-6 Temperature and chemical concentration profiles in the radial reservoir model before shut-in

Figure 5-7 Temperature and chemical concentration profiles in the rectangular reservoir model before shut-in

6.0 FULLY IMPLICIT TIME STEPPING

The numerical technique used in the PC-GEL simulator to solve the pressure and saturations of the entire reservoir grid system has been limited to IMPES (Implicit Pressure, Explicit Saturations) (Fanchi et al. 1982). The IMPES formulation results in the requirement of a very small time-step size to maintain the stability of calculations. To remedy this problem, the simulator was modified to include a strongly coupled, fully implicit method to solve simultaneously the cell pressures, water, oil, and gas saturations for the entire reservoir grid system, and the well flowing pressures.

6.1 Formulation of Jacobian Matrix

The finite-difference equations of the oil, water, and gas mass balance equations (Fanchi et al. 1982) for the fully implicit formulation are similar to those for the IMPES formulation, except for the time index of the transmissibility terms which were changed from the n to $n+1$ level.

Oil

$$\begin{aligned}
 (\Delta A_o^{n+1} \Delta p^{n+1})_{ijk} - [\Delta A_o^{n+1} \Delta (\frac{\rho_o z}{144})^{n+1}]_{ijk} - \frac{1}{\Delta t} [(\frac{V_P(1-S_w-S_g)}{B_o})^{n+1} - (\frac{V_P(1-S_w-S_g)}{B_o})^n]_{ijk} \\
 - \delta_w (\frac{WI k_{ro}}{\mu_o B_o})_{ijk}^{n+1} (p_{ijk}^{n+1} - p_{wf,m}^{n+1} - p_{h,ijk}^n) = 0
 \end{aligned} \tag{6-1}$$

Water

$$\begin{aligned}
 (\Delta A_w^{n+1} \Delta p^{n+1})_{ijk} - [\Delta A_w^{n+1} \Delta (\frac{\rho_w z}{144} + p_{cow})^{n+1}]_{ijk} - \frac{1}{\Delta t} [(\frac{V_P S_w}{B_w})^{n+1} - (\frac{V_P S_w}{B_w})^n]_{ijk} \\
 - \delta_w (\frac{WI k_{rw}}{\mu_w B_w})_{ijk}^{n+1} (p_{ijk}^{n+1} - p_{wf,m}^{n+1} - p_{h,ijk}^n) = 0
 \end{aligned} \tag{6-2}$$

and

Gas

$$\begin{aligned}
& (\Delta A_g^{n+1} \Delta p^{n+1})_{ijk} + (\Delta R_{so}^{n+1} A_o^{n+1} \Delta p^{n+1})_{ijk} + (\Delta R_{sw}^{n+1} A_w^{n+1} \Delta p^{n+1})_{ijk} \\
& + [\Delta A_g^{n+1} \Delta (p_{cgo} - \frac{\rho_g z}{144})_{ijk}^{n+1}] - [\Delta R_{so}^{n+1} A_o^{n+1} \Delta (\frac{\rho_o z}{144})_{ijk}^{n+1}] - [\Delta R_{sw}^{n+1} A_w^{n+1} \Delta (p_{cow} + \frac{\rho_w z}{144})_{ijk}^{n+1}] \\
& - \frac{1}{\Delta t} \left[\left\{ V_P \left(\frac{S_g}{B_g} + \frac{R_{so}(1-S_w-S_g)}{B_o} + \frac{R_{sw}S_w}{B_w} \right) \right\}^{n+1} - \left\{ V_P \left(\frac{S_g}{B_g} + \frac{R_{so}(1-S_w-S_g)}{B_o} + \frac{R_{sw}S_w}{B_w} \right) \right\}^n \right]_{ijk} \\
& - \delta_w \left(\frac{WI}{\mu_g B_g} \right)_{ijk}^{n+1} (p_{ijk}^{n+1} - p_{wf,m}^{n+1} - p_{h,ijk}^n) = 0
\end{aligned} \tag{6-3}$$

where terms with a linear difference operator, Δ , are defined similarly to Equations 2-29 through 2-51 except for the time index. The single phase mobility used in each of the previous equations is for production wells. For water or gas injection wells, a total mobility instead of a single phase mobility must be used.

The well equations (production/injection) are written in general as,

$$P_m(p, S_w, S_g, P_{wf}) = 0 \tag{6-4}$$

For example, the specification of constant water injection (QW) in a vertical well is written as,

$$P_m = \sum_k \left[\left(\sum_p \frac{WI}{\mu_p B_p} \right)_{ijk}^{n+1} (p_{ijk}^{n+1} - p_{wf,m}^{n+1} - p_{h,ijk}^n) \right] - QW = 0 \tag{6-5}$$

Equations 6-1 through 6-5 define the nonlinear finite-difference system. For N gridblocks and M wells, there are $3N + M$ unknowns in a three-phase system or $2N + M$ unknowns in a two-phase system to be determined simultaneously. To solve the nonlinear, coupled system of finite-difference equations, a Newton-Raphson method (Carnahan et al. 1969) is used. Let

$$R_l(X) = 0 \tag{6-6}$$

represent the set of Equations 6-1 through 6-4 where X is the vector of all unknowns, l denotes the l^{th} gridblock when $1 \leq l \leq N$ and the $(l-N)^{\text{th}}$ well when $N < l \leq (N + M)$.

Let J denote the Jacobian matrix,

$$J = \begin{bmatrix} & \text{A} & \text{B} \\ \text{C} & & \text{D} \end{bmatrix} \quad (6-7)$$

where elements of the Jacobian matrix consist of the appropriate derivatives of Equation 6-6 with respect to the reservoir variables and wellbore pressures. The upper left portion (A) of the Jacobian matrix contains the derivatives of the flow equations with respect to the reservoir variables (cell pressures, water, and gas saturations). The border of columns on the upper right (B) contains derivatives of the source terms with respect to the well flowing pressures. The border of rows on the lower left (C) contains derivatives of the well equations with respect to the reservoir cell pressures. The block on the lower right (D) contains derivatives of the well equations with respect to the well flowing pressures. The submatrix A has the usual banded structure. Though its incidence matrix is symmetric, the incidence matrix of the border is not necessarily symmetric (Nghiem et al. 1982). The number of columns and rows outside the banded area depends on the number of fully coupled wells.

Let δX represent the changes in values of all unknowns from the k to $k+1$ iterate, and R denote the residual vector. The Newton-Raphson iteration is then defined as

$$J^k \delta X^k = -R^k \quad (6-8)$$

A singular value decomposition and a direct method were used to solve the previous set of simultaneous linear equations.

6.2 Singular Value Decomposition Method

The singular value decomposition (SVD) method (Press et al. 1987) deals with matrices that are either singular or numerically very close to singular. Prior to using this method, a solver called LINPACK (Dongarra et al. 1979) was first used. The LINPACK subroutines use Gaussian elimination with partial pivoting to compute LU factorization of a matrix $A = LU$, where L is a unit lower triangular matrix and U is an upper triangular matrix. The LU factorization is then used to solve the linear system $Ax = b$ if the factors are not singular, where A is the coefficient matrix of the linear system, and x is the solution vector. Unfortunately, this method failed because of a singularity problem. Therefore, the SVD method was then tested.

With the SVD method, the Jacobian matrix (Eq. 6-7) is first decomposed into the product of an orthogonal matrix U , a diagonal matrix W with positive or zero elements, and the transpose of an orthogonal matrix V . Mathematically,

$$J = U \cdot W \cdot V^T \quad (6-9)$$

A subroutine SVDCMP (Press et al. 1987) is first called to decompose the Jacobian matrix into U , W , and V . A back substitution routine, SVBKSJ (Press et al. 1987), is then used to solve $Ax = b$ by calculating

$$x = V \cdot [diag(1/w_i)] \cdot (U^T \cdot b) \quad (6-10)$$

where w_i 's are diagonal elements of the diagonal matrix W . After each iteration, values of all unknowns are updated. New residuals of all nonlinear equations (6-6) are then calculated, and the root mean square (RMS) residual is evaluated. Nonlinear iteration continues until the RMS residual is sufficiently small. This criterion has also been used by Ponting et al. (1983).

Tests with the strongly coupled, fully implicit method were conducted with a waterflood in 1D and 2D radial reservoir models, and 1D, 2D, and 3D rectangular reservoir models. Results also were compared with that using the IMPES (implicit pressure-explicit saturation) method. In all cases, the initial oil saturation was 0.75 and water saturation was 0.25. Residual oil and water saturations were 0.22 and 0.78, respectively. Porosity was 0.2. The permeability in all 1D reservoir models is 100 mD. In all 2D and 3D reservoir models, the permeability in the top layer was 100 mD and that in the bottom layer was 1000 mD. The ratio of vertical permeability to horizontal permeability was 0.01, except in the 2D rectangular models where the ratio of vertical permeability to horizontal permeability was 0.005. The gridblock size used in radial models ranged from 3 ft to 86.5 ft (Table 6-1). In 1D and 2D rectangular models, the gridblock

size in the x-direction was 15 ft. In 3D models, the gridblock sizes in both x and y directions were 100 ft. The thickness of each layer was 15 ft.

Figure 6-1 compares calculated results with the IMPES (time-step size = 0.0001 day) and the fully implicit (time-step size = 0.1 day) method for a waterflood in a 1D ($12 \times 1 \times 1$) radial reservoir model after 1 year of waterflooding. With the IMPES method, a very small time-step size had to be used because of the numerical stability problem. However, even using a very small time-step size, calculated reservoir pressures kept increasing. As a result, the cell pressures were much higher with the IMPES method than with the fully implicit method. Figure 6-1 also shows that the oil saturation distribution was less numerically dispersed with the fully implicit than with the IMPES method. Under simulated conditions, the IMPES method used 8,454 sec of central-processing-unit (CPU) time with a 200 MHz Pentium Pro PC, while the fully implicit method only used 120 sec (Table 6-2).

Figure 6-1 Comparison of cell pressure and oil saturation distributions with the IMPES and fully implicit methods

Table 6-1 The radial grid used in simulator

Wellbore radius, ft	Gridblock size in radial direction, ft											
	3.0	4.2	5.9	8.2	11.5	16.	22.6	31.6	44.3	62.	86.5	30.
0.25												

Tests with the IMPES method revealed that certain parameters, such as the slope of oil formation volume factor versus pressure curve for pressure above oil bubble-point pressure (BSLOPE) and rock compressibility, could also affect the numerical stability. For example, by changing the BSLOPE in the previous example from 3E-6 to 0 RB/STB/psi, where RB and STB stand for the reservoir barrel and standard condition barrel, respectively, a time-step size of 0.01 day did not cause a stability problem with the IMPES method. Under this condition, the IMPES took 84 sec of CPU time while the fully implicit method took 38 sec to complete calculations for 1-year waterflooding as shown in Table 6-2.

Table 6-2 Comparison of CPU times

Reservoir Model	BSLOPE, RB/STB/psi	FULLY IMPLICIT		IMPES	
		Time-Step Size, day	CPU, sec	Time-Step Size, day	CPU, sec
1D Radial	0	0.1-0.5	38	0.01	84
1D Radial	3E-6	0.1	120	0.0001	8454 [†]
2D Rectangular	0	0.5	110	0.0001	15840
2D Rectangular	3E-6	0.5	110	0.0001	[†]
3D Rectangular	0	2	15770	0.01-10	12
3D Rectangular	3E-6	2	15846	0.01-10	12 [†]

[†]= Pressure Unstable

For a 2D rectangular model ($10 \times 1 \times 2$) with a gridblock size of 10 ft in the y direction, an injection rate of 20 bbl/day, and BSLOPE equal to 0, a time-step size of greater than or equal to 0.0005 day always caused a numerical stability problem with the IMPES method. To maintain numerical stability, the time-step size had to be reduced. With a time-step size of 0.0001 day, the IMPES method took 15,840 sec of CPU time to complete the calculations for 1-year waterflood as shown in Table 6-2. With the fully implicit method, a time-step size of 0.5 day did not cause a numerical stability problem. It spent only 110 sec to complete the calculations. However, when the value of BSLOPE was set to 3E-6 RB/STB/psi, there was a numerical stability problem even with a very small time-step size (0.0001 day) when the IMPES method was used. No numerical stability problem occurred with the fully implicit method. The CPU time for the fully implicit method was 110 sec with a time-step size of 0.5 day. This comparison indicates that the fully implicit method rather than the IMPES method should be used in 1D and 2D reservoir models.

In 3D ($10 \times 10 \times 2$) calculations, there was no numerical stability problem with the IMPES method when the parameter BSLOPE was equal to 0. Under this condition, the IMPES method only took 12 sec of CPU time to complete the calculation of 1-year waterflooding with a variable time-step size of 0.01 to 10 days. Calculated average reservoir pressure was very stable. No numerical stability problem occurred with the fully implicit method either; however,

the fully implicit method required much longer CPU time than did the IMPES method to complete calculations as shown in Table 6-2. This suggests that when the number of nonlinear equations becomes large, there is no benefit using the fully implicit method if the IMPES does not cause any numerical stability problem.

With BSLOPE was equal to $3E-6$ RB/STB/psi, calculated average reservoir pressure with the IMPES method increased from 8,709 psi at the start of waterflooding to 9,057 psi at the end of 1 year of waterflooding. With the fully implicit method, calculated average reservoir pressures were stable, indicating that the fully implicit method provides more reasonable results than the IMPES method. Results from these two methods were almost the same except for the cell pressures (Fig. 6-2). The fully implicit method took 4 hr and 26 min of CPU time with a time-step size of 2 days to finish calculations, compared with 14 sec for the IMPES method. Hence, the fully implicit method is recommended when the number of nonlinear equations is small. When the number of nonlinear equations is large, it is recommended only when the IMPES method has a numerical stability problem. A faster solver to calculate the linear system is also desirable for the fully implicit method. The next section describes evaluation of another solver to reduce CPU time.

Figure 6-2 Comparison of average reservoir pressure, fractional oil recovery, and water-oil ratio with the IMPES and fully implicit methods

6.3 Block Factorization Method

In an attempt to find a solver to accelerate the calculation of the bordered Jacobian system resulting from fully coupled multiblock wells, a method based on a block factorization was tested. This method is the second factorization of the three possible block factorizations of sparse, linear systems discussed by George (1974). This method also has been used by Behie et al. (1985) to solve fully implicit, fully coupled multiblock wells. With this method, the bordered Jacobian matrix is mathematically expressed as

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} \quad (6-11)$$

Submatrix $\mathbf{A}$ contains the derivatives of the flow equations with respect to the reservoir variables (cell pressures, water, and gas saturations). The border of column $\mathbf{B}$ contains derivatives of the source terms with respect to the well flowing pressures. The border of rows on the $\mathbf{C}$ contains derivatives of the well equations with respect to the reservoir cell pressures. The block on the lower right $\mathbf{D}$ contains derivatives of the well equations with respect to the well flowing pressures. $\mathbf{x}_1$ contains unknowns connected with flow in the reservoir (cell pressures, water saturations, etc.), and $\mathbf{x}_2$ contains unknowns connected with wells (wellbore flowing pressures).

The system in the previous equation is factored as

$$\begin{aligned} \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} &= \\ \begin{bmatrix} \mathbf{LU} & \mathbf{0} \\ \mathbf{C} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & (\mathbf{LU})^{-1} \mathbf{B} \\ \mathbf{0} & \mathbf{D} - \mathbf{C} (\mathbf{LU})^{-1} \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} &= \\ \begin{bmatrix} \mathbf{LU} & \mathbf{0} \\ \mathbf{C} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} &= \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} \end{aligned} \quad (6-12)$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{L}$ and $\mathbf{U}$ are the factors of $\mathbf{A}$.

The solution of the entire system can be calculated according to the following algorithm:

$$\text{Solve} \quad \mathbf{LU} \mathbf{y}_1 = \mathbf{b}_1 \text{ for } \mathbf{y}_1 \quad (6-13)$$

$$\text{Calculate} \quad \mathbf{y}_2 = \mathbf{b}_2 - \mathbf{C} \mathbf{y}_1 \quad (6-14)$$

$$\text{and} \quad \mathbf{E} = \mathbf{D} - \mathbf{C} (\mathbf{LU})^{-1} \mathbf{B} \quad (6-15)$$

$$\text{Solve} \quad \mathbf{E} \mathbf{x}_2 = \mathbf{L}_E \mathbf{U}_E \mathbf{y}_2 \text{ for } \mathbf{x}_2 \quad (6-16)$$

$$\text{and then calculate} \quad \mathbf{x}_1 = \mathbf{y}_1 - (\mathbf{LU})^{-1} \mathbf{B} \mathbf{x}_2 \quad (6-17)$$

Tests with this method were conducted for a waterflood in a 3D rectangular reservoir model used in Section 6.1. Results were compared with those calculated by using an SVD and the IMPES methods. When the value of BSLOPE was set to equal to 0, the block factorization method took 4,456 sec of CPU time to finish 1 year of waterflooding calculations. Compared with the SVD method (15,770 sec), the block factorization method is more than three times faster. However, it is still much slower than the IMPES method (12 sec). Though the block factorization method is faster than the SVD method, the average cell pressure calculated from this method is not as stable as that from the SVD method or the IMPES method as shown in Figure 6-3. When BSLOPE was equal to $3\text{E-}6$ RB/STB/psi, the block factorization method became worse. It could not converge. Although this method is easy to implement, it suffers from the numerical stability problem too and is not better than the SVD method. The SVD method, though it requires more CPU time when the number of nonlinear equations become large, does not have a numerical stability problem.

Figure 6-3 Calculated average reservoir pressures as a function of pore volume

 

7.0 GEL PLACEMENT CALCULATIONS FOR WATER CONTROL

The NIPER PC-GEL simulator was used to determine where the gelant would travel when injected into a two-layer reservoir. A 3-ft-thick watered-out zone of high permeability (10 Darcy) is adjacent to an oil-bearing layer of thickness h with a permeability of 100 mD. Two values of h (7 and 28 ft) were investigated. The permeability anisotropy (k_r/k_z) in both layers was the same and set equal to 1, 10, or 100.

Water production was controlled by placing a gel plug in the high-permeability zone in the neighborhood of the production well. PC-GEL was used to study how much of the injected gelant, when injected into the high-permeability layer, would leak off into the low-permeability zone. A different placement process called dual injection was studied as a method of controlling the undesired leakage of the gelant into the lower-permeability zone. In that process, a protective fluid (usually water) is simultaneously injected into the low-permeability zone, leading to an increased pressure that reduces or entirely eliminates a flow from the high-permeability zone.

A 3D rectangular grid of either $16 \times 16 \times 10$ cells or $16 \times 16 \times 15$ cells, depending on the thickness of the reservoir, simulated a block of the reservoir with a production well at the corner, which was treated by gelant injection. Both single and dual injection were simulated for each set of model parameters. In this initial study, the gelant was assumed to be chemically inactive during pumping. Two different types of polymers were employed, one 100 times more viscous than the other. The less viscous polymer solution had a viscosity comparable to that of water. In dual injection, the influence of the relative values of the injection pressures in the two zones was studied. In each case with a different set of parameters, three scenarios of constant pressure injection were simulated: (1) when the water and polymer injection pressures were equal, (2) when the water injection pressure was higher by 100 psi, and (3) when the water injection pressure was lower by 100 psi.

With single injection of a low-viscosity polymer (~ 1 cp) into the high-permeability layer, the leakoff was not objectionably high with permeability anisotropy of 100; however, when the anisotropy was low ($k_r/k_z \leq 10$), the dual injection process was necessary for a satisfactory placement of the gel. These results agree with previous findings (Gao et al. 1990, 1993, 1995). In cases where dual injection was required, a satisfactory placement was obtained with the protective fluid injection pressure somewhat lower (50 to 100 psi) than that of the gelant. Figures 7-1 and 7-2 show the distribution of the gelant in a vertical cross-section through the injection well. The injection well is located on the left edge of the plot, and the high-permeability zone is located at the top. Figure 7-1 shows that a single injection would significantly spread the gelant into the low-permeability zone in a reservoir with the anisotropy of 10. When dual injection was employed, as shown in Figure 7-2, the gelant was confined entirely to the high-permeability zone.

Figure 7-1 Distribution of gelant concentration in ppm (single injection)

Figure 7-2 Distribution of gelant concentration in ppm (dual injection)

It also was found that when the polymer viscosity was high (100 cp), the dual injection was not required except when the thickness of the low-permeability layer was very small. The highly viscous polymer invaded only a few feet into the oil layer even when the permeability anisotropy was small. It was concluded that such an invasion can be tolerated if the low-permeability zone was greater than 15 ft thick. On the other hand, when injecting a polymer solution with low viscosity (e.g., 1.8 cp), an unacceptably large polymer invasion of the low-permeability layer occurred (Fig. 7-3) unless it was protected by simultaneous injection of water (Fig. 7-4). Hence, dual injection is recommended even when the low-permeability zone layer is relatively thick and when the viscosity of the injected polymer solution is low; however, when the permeability anisotropy is very high ($k_r/k_z = 1,000$), single injection may be adequate for polymer solutions with any viscosity.

Figure 7-3 Distribution of low-viscosity (1.8 cp) gelant concentration in ppm (single injection)

Figure 7-4 Distribution of low-viscosity (1.8 cp) gelant concentration in ppm (dual injection)

The injection strategy for gelant placement in the high permeability layer of a reservoir with crossflow also was investigated using a radial grid model. The reservoir model used is assumed to have two layers: a thicker, low-permeability, oil-bearing layer and a thinner, high-permeability thief layer. Each layer is fully perforated and is assumed to have a competent cement sheath behind the casing. The two layers are in communication, and the crossflow between them is determined by the vertical permeabilities; no resistance (skin) is present at the interface between the two layers. The reservoir model is radially symmetric around the production well, which is also referred to as the treatment well. The exterior boundary of the domain is at a radius (R_e) of 526 ft from the treatment well. This boundary is assumed either to be of the no-flow type or to have active wells situated symmetrically along it. The reservoir is assumed to be occupied by only two fluid phases: oil and water. Initial oil saturations are 0.6 in the oil zone and 0.4 in the thief zone. The fluid viscosity is 0.8 cp for water and 3.0 cp for oil. The rock porosity is 0.2.

A radial grid of $14 \times 1 \times 10$ or $14 \times 1 \times 15$ cells, depending on the thickness of the oil-bearing layer, simulated a block of the reservoir with the treatment well at the corner. The dimensions of the 14 gridblocks in the radial direction ranged from 1 ft to 160 ft, and those in the vertical direction ranged from 1 ft to 3 ft. The placement well radius was 0.25 ft. The thickness of the high-permeability thief layer was held constant at 3 ft. The thickness of the oil zone was varied

over two values: 10 ft and 28 ft. The other parameters that were varied are the viscosity of the gelant, the permeability anisotropy (equal in both layers), and the permeability contrast between the two layers. To a limited extent, the injection pressure of the gelant and the protective fluid also were varied.

The gelant viscosity had either of two values: 2.0 cp and 100 cp. The first value is typical of a gelant that is generally placed deep in the reservoir; the higher value is typical of a gelant that is used to seal a zone (several feet) around the wellbore. The simulations followed the placement of either 20 bbl of the high-viscosity gelant (100 cp) or 500 bbl of the low-viscosity gelant (2.0 cp).

Table 7-1 summarizes the simulation results of injecting 500 bbl of a low-viscosity gelant (2 cp) into reservoirs with a layer permeability contrast equal to 10. Under the simulated conditions, the radial penetration of the gelant in the thief layer is about 27.2 ft. For reservoirs with $k_h/k_v = 1$, the gelant penetrates through the entire thickness (10 ft) of the oil zone. In this type of reservoir and for this type of gelant, dual injection is absolutely required to protect the oil zone. When the permeability anisotropy is 10, the lower vertical permeability in each layer leads to a smaller depth, 4 ft, of oil-zone invasion. The contour plot of the gelant profile in the reservoir for this case is shown in Figure 7-5. Because 4 ft of invasion into a 10-ft-thick oil zone may be considered unacceptably high, dual injection may be recommended for this situation. When the permeability anisotropy is 100, the amount of invasion into the oil zone is 1.5 ft. It may be considered acceptable, and dual injection is not required. Table 7-1 also shows the results of dual injection with two different values of protective fluid injection pressure. These values are, respectively, 100 psi above and 100 psi lower than the injection pressure (4023.8 psi) of the gelant. Both cases are considered successful. No gelant penetration into the oil zone was observed. Figure 7-6 shows the gelant concentration distribution for the case when the gelant pressure was higher. Similar distribution also was obtained for the case when the protective fluid injection pressure was higher, indicating that a practical tolerance of 100 psi in either direction is permissible.

Similar conclusions also can be drawn from Table 7-1 for the situations when the oil zone thickness is 28 ft. The only difference is that, for such a thick oil zone, an invasion of 5 ft may be acceptable, and hence the dual injection is even less often required.

Simulation studies also show that the need for dual injection is generally less when the same low-viscosity gelant (2.0 cp) is placed in the thief zone of reservoirs with a layer permeability contrast equal to 100. When small quantities (20 bbl) of a high-viscosity gelant (100 cp) is placed in the thief zone of reservoirs with a lower layer permeability contrast (10), dual injection placement is generally not required. Dual injection also is not required when placing small quantities of the high-viscosity gelant into the thief zone of reservoirs with a layer permeability contrast equal to 100.

Table 7-1 Simulation results for reservoirs with layer permeability contrast equal to 10

Permeability Anisotropy, k_h/k_v	Thickness of Oil Zone, ft	Injection Method	Protective Fluid Injection Pressure, psi	Duration of Injection, D	Volume of Protective Fluid, bbl	z = Penetration of Oil Zone Near Wellbore, ft
1	10	Single		0.258		10
10	10	Single		0.300		4
10	10	Dual	4123.8	0.390	285.0	0
10	10	Dual	3923.8	0.355	185.5	0
100	10	Single		0.312		1.5
1	28	Single		0.233		13
10	28	Single		0.270		5
10	28	Dual	4123.8	0.380	652.3	0
10	28	Dual	3923.8	0.350	476.2	0
100	28	Single		0.304		2

Figure 7-5 Distribution of low-viscosity gelant (ppm) after 0.3 day of single injection

The previous studies were conducted before the fully implicit method was developed. More detailed results were reported in an SPE paper 35382, "Necessity of Dual Injection for Gelant Placement in the High Permeability Layer of a Reservoir with Crossflow" (Shah and Gao 1996).

Figure 7-6 Distribution of low-viscosity gelant (ppm) after 0.355 day of dual injection

8.0 TECHNOLOGY TRANSFER

PC-GEL simulator is a PC-based version of NIPER's permeability modification simulator which was developed and initially released to the public through DOE in 1990. The PC-GEL simulator is currently available on the Web site of the DOE's National Petroleum Program Office (NPTO). Since the development of this simulator, many requests for this simulator from major oil companies have been received and honored. The addition of a graphics package to NIPER's permeability simulator has resulted in a commercial product sold as BEST-GEL. This product has been used by oil producers and university researchers.

Since the development of the simulator, three reports, three presentations, and two publications were generated.

1. Gao, H. W., and M. M. Chang. 1990. *A Three-Dimensional, Three-Phase Simulator for Permeability Modification Treatments Using Gelled Polymers*. U.S. DOE Report NIPER-388, March. NTIS Order No. DE90000227.
2. Gao, H. W., M. M. Chang, T. E. Burchfield, and M. K. Tham. 1990. *Studies of the Effects of Crossflow and Initiation Time of a Polymer Gel Treatment on Oil Recovery in a Waterflood Using a Permeability Modification Simulator*. Paper SPE/DOE 20216 presented at the SPE/DOE 7th Symposium on Enhanced Oil Recovery, Tulsa, Oklahoma, April 22-25.
3. Gao, H. W., and T. E. Burchfield. 1993. The Effects of Layer Permeability Contrast and Crossflow on the Effectiveness of Polymer Gel Treatments in Polymer Floods and Waterfloods. Paper SPE 25453 presented at the 1993 SPE Production Operations Symposium, Oklahoma City, Oklahoma, March 21-23.
4. Gao, H. W., M. M. Chang, T. E. Burchfield, and M. K. Tham. 1993. Permeability Modification Simulator Studies of Polymer Gel-Treatment Initiation Time and Crossflow Effects on Waterflood Oil Recovery. *SPE Reservoir Engineering* Vol. 8, No. 3, August: p. 221-227.
5. Chang, M. M., and H. W. Gao. 1993. *User's Guide and Documentation Manual for PC-GEL Simulator*. DOE Report NIPER-705, Oct. NTIS Order No. DE94000104.
6. Gao, H. W., and T. E. Burchfield. 1995. *Effects of Crossflow and Layer Permeability Contrast on the Effectiveness of Gel Treatments in Polymer Floods and Waterfloods*. *SPE Reservoir Engineering* Vol. 10, No. 2, May: p. 129-135.

7. Gao, H. W., and P. C. Shah. 1995. *Development of an Improved Permeability Modification Simulator*. DOE Report NIPER/BDM-0156, July.
8. Shah, P. C., and H. W. Gao. 1996. Necessity of Dual Injection for Gelant Placement in the High Permeability Layer of a Reservoir with Crossflow. Paper SPE 35382 presented at the 1996 SPE/DOE Tenth Symposium on Improved Oil Recovery, Tulsa, Oklahoma, April 21–24.

The results gained through research on the permeability modification simulator are very useful to the oil industry, including both producers and service companies. Further improvements to the simulator to include radial coordinates, temperature effect, a wellbore simulator, gelation kinetics of a second gel permeability modification system, and a fully implicit method have been described in this report. The simulator with these improvements will be submitted to the DOE for distribution to the petroleum industry.

9.0 SUMMARY AND CONCLUSIONS

The NIPER PC-GEL permeability modification simulator was modified to accommodate a cylindrical coordinate system (r, θ, z), a thermal energy equation, a wellbore simulator, and a fully implicit time-stepping option. The radial model was developed by transforming all the governing equations from rectangular to cylindrical coordinates (r, θ, z). The phase transmissibility was reformulated in terms of cylindrical coordinates. The radial model was tested against the r and z components of the Darcy velocity vector under a single-phase fluid flow condition and material balance under single- and two-phase fluid flow in one- and two-layer radial reservoir models. Good material balance (error less than 1%) was obtained for injection at constant pressure and with a small time-step size. Compared with the rectangular coordinate system, the radial coordinate system gave less numerical dispersion and required less computing time.

To account for the change in temperature of the gel system during pumping, a 3D three-phase thermal energy equation that considers thermal conduction/convection between the injected fluid, the reservoir formation, the reservoir fluids, the overburden, and the underburden was derived and incorporated into the PC-GEL permeability modification simulator. The thermal energy model was validated by comparing a calculated ratio of the fluid front to the thermal front with that from analytical analysis in both a 1D rectangular and a 1D radial porous medium.

Gelation studies of an inorganic delayed gel system (DGS) showed that gelation occurred when the molar ratio of a bridging agent to a gelling agent reached a critical value. The concentration of the activator decreased exponentially according to a first order reaction rate model. The reaction rate constant conformed to the Arrhenius equation. The gelation time decreased with an increase in the salt concentration. Four components in the aqueous phase—two for the activators, one for the bridging agent, and one for the gelling agent—and the same four components in the gelled phase were modeled in the simulator. Gelation occurs when the molar ratio of the bridging agent to the metal element of the gelling agent reaches 2.75. Syneresis, which occurs when the molar ratio of the bridging agent to the metal element in the gelled phase is greater than 3.0, also was considered.

A wellbore simulator developed by Schlumberger Dowell was modified to include a 1D in-situ gelation model and to allow the use of a variable time-step size. The modified wellbore simulator was incorporated into the PC-GEL permeability modification simulator to account for the change of gelation rate with temperature as the gel system travels down the wellbore. Calculated temperature profiles in the wellbore before and after the wellbore simulator was incorporated into the PC-GEL simulator were exactly the same. With the modified wellbore

simulator, calculated gelation times of a DGS in both rectangular and radial grid reservoir models agreed with those from the gelation rate equation.

To implement a fully implicit time-stepping option in the PC-GEL permeability modification simulator, the finite difference forms of the flow equations for oil, water, and gas phases, and equations of wells completed in more than one zone were solved simultaneously for the entire grid. A Newton-Raphson method was used to linearize the strongly coupled nonlinear equations. An existing singular value decomposition method (SVD) and a block factorization method were used to solve the linearized equations. The block factorization method required less CPU time than the SVD method. Calculated cell pressures and wellbore flowing pressures with the block factorization method were not as stable as those with the SVD method. The SVD method did not have a numerical stability problem. The SVD method is recommended when the number of nonlinear equations is small. When the number of nonlinear equations becomes large, the SVD method is recommended only when the IMPES method has a numerical stability problem. A solver that is faster than the SVD method in solving the linear system is desirable for the fully implicit method.

Simulation studies with rectangular and radial grid models were conducted to investigate the effects of permeability anisotropy, thickness, and polymer viscosity on the amount of gelant injected into the high-permeability layer that would leak off into the low-permeability layer for a two-layer reservoir. These studies were conducted before the fully implicit method was developed. A dual injection method was studied as a method for controlling the undesired leakage of the gelant into the low-permeability zone. When single injection was used, the leakoff increased with a decrease in the permeability anisotropy (k_r/k_z). When dual injection was required, a satisfactory placement was obtained with the protective fluid injection pressure somewhat lower than that of the gelant.

When small quantities of high-viscosity gelant are to be placed, dual injection is needed only when the layer permeability contrast is low, the formation permeability anisotropy is low, and the thickness of the oil zone is small. When the gelant viscosity is low, single injection will suffice when the permeability contrast is 100 or greater or when the permeability anisotropy is greater than 100. When the permeabilities are nearly isotropic, dual injection is usually needed unless a small volume of high-viscosity gelant is being placed and the oil zone thickness is relatively large (15 ft or greater).

The development and inclusion of the radial model, thermal model, wellbore model, and the fully implicit method within the existing PC-GEL permeability modification simulator have greatly improved its capability.

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11.0 NOMENCLATURE

A_c	cross-sectional area, ft^2
A	finite-difference phase transmissibility or molar concentration
B	formation volume factor or bbl
C	species concentration
C_o	oil compressibility, psi^{-1}
C_p	heat capacity at constant pressure, $\text{Btu/lb-}^\circ\text{F}$
C_R	heat capacity of rock, $\text{Btu/lb-}^\circ\text{F}$
C_v	heat capacity at constant volume, $\text{Btu/lb-}^\circ\text{F}$
C_w	water compressibility, psi^{-1} or heat capacity of water, $\text{Btu/lb-}^\circ\text{F}$
D	day
D_T	linear distance of thermal front from the injector
D_w	linear distance of fluid front from the injector
d	diffusion length
g	acceleration of gravity
g_c	32.174 ft/sec^2
H	enthalpy, Btu/lb
K	permeability tensor
K	permeability
k	gelation rate constant

k	absolute permeability, md
k_r	relative permeability
M	molecular weight or molar concentration
P	well production (injection) equation
P	pressure
p	oil phase pressure
P_c	critical pressure, atm
P_r	reduced pressure
p_h	well hydrostatic head correction
p_{cgo}	gas-oil capillary pressure
p_{cow}	oil-water capillary pressure
p_{wf}	bottom hole flowing pressure at a reference datum
Q	volumetric rate of injection/production
q	mass flow rate into or out of a well
q_L	rate of heat loss per unit volume to over- or underburden
R	residual finite-difference equation for oil, water or gas phase
R	gas constant = 10.731 psi-ft ³ /lb-mole-°R
R_e	Radius of exterior boundary of reservoir
R_T	radial distance of thermal front from the injector
R_w	radial distance of fluid front from the injector
R_{so}	gas solubility in oil phase

R_{sw}	gas solubility in water phase
r	r direction
S	saturation
s	specific gravity
T	temperature
T_c	critical temperature, K
T_r	reduced temperature
t	time
U	internal energy, Btu/lb
$\mathbf{V}$	Darcy velocity vector
V	Darcy velocity
V_B	bulk volume of a grid block, ft ³
V_p	pore volume of a grid block, ft ³
WI	well index
w	weighting factor
x, y, z	rectangular coordinates
Δx	gridblock size in the x direction
$\Delta x'$	$x_i - x_{i-1}$
$\Delta x''$	$x_{i+1} - x_i$
Δy	gridblock size in the y direction
Δz	gridblock size in the z direction

Z	gas deviation factor
Δt	time-step size
Δr	gridblock size in the r direction
Δz	gridblock size in the z direction

Symbols

n	old time level
$n+1$	new time level
ρ	density, lb/ft ³
ρ_c	density at critical temperature and pressure
ρ_r	reduced density = ρ/ρ_c
μ	viscosity, cp
λ	thermal conductivity, Btu/hr-ft-°F
α	thermal diffusivity
ϕ	porosity
θ	interface temperature between formation and overburden or underburden rock
∇	divergence operator
Δ	linear difference operator
τ	finite-difference transconductivity
τ'	transconductivity
δ_w	Kronecker delta function signifying the presence or absence of a well

Subscripts

B	bulk
c	critical conditions
g	gas phase
o	oil phase
R	rock
r	r direction
ref	reference
sc	standard condition (usually 60°F and 14.7 psia)
x	x direction
y	y direction
z	z direction
w	water phase