UNIVERSAL CORRELATIONS OF NUCLEAR OBSERVABLES 
AND THE STRUCTURE OF EXOTIC NUCLEI

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Despite the apparent complexity of nuclear structural evolution, recent work has shown a remarkable underlying simplicity that is unexpected, global, and which leads to new signatures for structure based on the easiest-to-obtain data. As such they will be extremely valuable for use in experiments with low intensity radioactive beams. Beautiful correlations based either on extrinsic variables such as $N_pN_n$ or the $P$-factor or correlations between collective observables themselves have been discovered. Examples to be discussed include a tri-partite classification of structural evolution, leading to a new paradigm that discloses certain specific classes of nuclei, universal trajectories for $B(E2; 2^+ \rightarrow 0^+)$ values and their use in extracting hexadecapole deformations from this observable alone, the use of these $B(E2)$ values to identify shell gaps and magic numbers in exotic nuclei, the relationship of $\beta$ and $\gamma$ deformations, and single nucleon separation energies. Predictions for nuclei far off stability by interpolation will also be discussed.

1 Introduction

The physics of unstable nuclei is entering a new era with the development and proliferation of radioactive nuclear beam (RNB) accelerators. Nuclear structure studies of such nuclei will especially flourish with the coming of second generation ISOL-type RNB facilities with capabilities similar to those envisioned in the IsoSpin Laboratory (ISL) concept\cite{1}.

The opportunities offered by the technological advances embodied in advanced RNB facilities are great and enticing but they must be tempered by the realization that RNB intensities will typically be orders of magnitude lower than the beam intensities nuclear physics has been accustomed to.

This presents two companion challenges. The first is to develop new experimental techniques and instruments that are much more efficient than existing ones. Many of these new approaches exploit one particular advantage of most RNB experiments, namely inverse kinematics in which the roles of target and projectile are reversed relative to traditional experiments. In inverse kinematics all the reactions products are forward focussed. This has the dual advantage of getting rid of unwanted radioactivity and of enabling the use of detectors that can measure total rather than partial cross sections. (Of course, measur-
ing angular distributions is correspondingly more difficult.) One approach to studying the lowest levels of exotic nuclei in inverse kinematics with RNBs — low energy Coulomb excitation — has recently been the focus of experimental work by the Yale-BNL-Clark group. (See elsewhere in these Proceedings[2].)

The second challenge is to extract more structure information from less data. It is our purpose here to focus on recent advances in this quest.

Our general approach exploits the notion of nuclear systematics — however, not in the traditional sense of that word. Systematics have been used for decades to estimate properties of unknown nuclei and to try to understand how structure evolves as a function of proton and neutron number. Generally, nuclear observables are plotted vs. $A, Z$, or $N$. In an effort to develop more powerful and more efficient signatures of structure for unknown nuclei that may soon be accessible with RNB’s, we have taken a rather different approach. We have sought quantities — either extrinsic variables such as the valence nucleon number product $N_pN_n$, as a measure of the valence residual $p-n$ interaction, or intrinsic variables, this is, other nuclear observables themselves [e.g., $E(2^+_2)$] that lead to remarkably simple and compact correlations of nuclear observables that will be easy to measure with RNB intensities.

These correlations of data produce new paradigms for structural evolution, allow deviant behavior to be more easily spotted, and give clues to underlying physics. The examples we will present will show new ways in which the $4^+_1$ and $2^+_1$ energies can give evidence for closed shells (magic numbers) and for spherical-deformed transition regions, how a new tri-partite classification of structural evolution emerges from the correlation of $4^+_1$ and $2^+_1$ energies, how $B(E2)$ values provide evidence for hexadecapole deformations, how $\gamma$ deformation values behave relative to $\beta$ values, and how separation energies follow simple linear trajectories in $N_p$ and $N_n$. Finally, we note how these techniques allow predictions for unknown nuclei by interpolation rather than extrapolation. Some of this work has recently been reviewed [3]. Other interesting work on odd-$A$ nuclei is discussed in Ref. [4].

2 Correlations of Observables: New Signatures of Structure

Nuclear systematics often appear extremely complex. However, when the data are viewed in alternate ways, remarkable correlations emerge. One is given in Figure 1 which shows that, for the entire region from $Z = 50-82$, $E(4^+_1)$ is correlated with $E(2^+_2)$ in a tri-linear fashion with physically meaningful slopes. Rotor nuclei occur at low $2^+_1$ energies with a slope of 3.33. The second region, with slope of 2.00 and finite intercept, is characteristic of an anharmonic vibrator (AHV), while the third, corresponding to “pre-collective” nuclei with $R_{4/2}$
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Figure 1: Tri-partite classification of structure and structural evolution for Z = 50-82.

$=\frac{E(4^+)}{E(2^+)} < 2.00$, has a slope of unity, characteristic of the addition or subtraction of $0^+$ coupled pairs of protons or neutrons.

Together the three regimes define a tri-partite classification of structure [5] and show that the nuclear observable $E(2^+_1)$ itself acts as a monitor and efficient guide to structure, at least within a given major shell region.

The existence of a well-defined correlation of any sort also defines a new benchmark or paradigm and acts as a magnifier of structural differences when data are plotted relative to the new paradigm. Let us consider rotor and AHV regions – the collective nuclei. For these, the correlation is even more global than Fig. 1 suggests: Both the $Z = 38-50$ and 50-82 regions can be combined in a single plot. (See inset to Fig. 2.) Then the harmonic limit of $E(4^+_1) = 2E(2^+_1)$ defines a new standard. Deviations from this standard, defined as $\epsilon_4 = E(4^+_1) - 2E(2^+_1)$, are plotted in the main part of Fig. 2[6].

Even though $E(4^+_1)$ varies from 270 to 1800 keV, over 85% of the $\epsilon_4$ points are contained within the simple trajectory defined by the dashed lines: $\epsilon_4$ varies only from $\sim 70$ to 210 keV. This compact correlation immediately discloses a few deviant nuclei that would otherwise never have seemed anomalous. Interestingly, the deviant cases are not random but, as the different symbols show, correspond to specific classes of nuclei. The points below the main envelope of points are nuclei in spherical-deformed transition regions, while those above
are mostly nuclei with two holes relative to a magic number. Clearly, such an analysis can be useful in spotting nuclear transition regions or identifying magic numbers in nuclei that will become accessible with radioactive beams.

Correlations of nuclear observables with extrinsic variables are also useful. Here we discuss a couple of Valence Correlation Schemes (VCS), that is, approaches to phenomenology that focus on the variations of structure as a function of the number of valence nucleons $N_p$ and $N_n$.

The $N_pN_n$ scheme[7], which provides excellent correlations that reflect the equilibrium structure, is founded on the importance of the valence residual p-n interaction. Figs. 3a,b) shows $B(E2;2^+_1 \to 0^+_1)$ values (in W.u.) for all nuclei with $Z > 38$, plotted against $Z$ and $N_pN_n$. The $N_pN_n$ correlation is an improvement but is hardly compact. However, the branches in Fig. 3b are a result of varying $\beta_4$ deformations which raise these $B(E2)$ values early in a shell and reduce them later on because of the dependence of the quadrupole moment $Q$ [and hence the $B(E2)$] on $\beta_4$. When these $B(E2)$ values are corrected for $\beta_4$, the resulting values, denoted $B(E2)_Q$ since they result from the quadrupole deformation alone, lie on the compact linear correlation in Fig. 3c [8].

Aside from the intrinsic interest in such a compact envelope of points, it
has a practical utility, as shown in Fig. 4. Here we focus on the actinides. The B(E2) values seem to follow a smooth curving trajectory without any apparent anomalies. However, the linear trajectory of the B(E2)Q values (Fig. 4b) immediately shows that the two heaviest nuclei $^{250,252}$Cf lie clearly below the main trend. The $\beta_4$ values for these two nuclei are not known and the predictions of Ref. 9 were used. Fig. 4b actually allows us to extract semi-quantitative estimates of $\beta_4$ defined as those values needed to raise the B(E2)Q values to the trendline. These $\beta_4$ values are $\sim -0.05 \pm 0.02$, opposite in sign to predictions of Ref. 9. This points to different binding of such nuclei and has implications for the shapes and stability of the heaviest elements.

Recent calculations suggest that shell structure [magic numbers, j-shell sequences] as we know it may be substantially different near the neutron drip line where the outermost neutrons are very weakly bound and spatially extended[10]. Thus, in RNB studies of these nuclei, it will be very important to identify shell gaps and magic numbers. Normally, this requires extensive spectroscopy but B(E2: $2^+_1 \rightarrow 0^+_1$) values, which are easy to measure, and the $N_pN_n$ scheme, provide simple clues to shell structure. The basic idea is that $N_p$ and $N_n$ values implicitly entail a choice of closed shells. Hence $N_pN_n$ plots made under different assumptions will look very different. A choice that leads to a scattering of points is likely to be wrong. This is illustrated in Fig. 5 for the $A \sim 130$ mass region. The left panel uses the well known $Z=50$ and $N=82$ magic numbers, resulting in a compact phenomenology. The other panels use (known-to-be-incorrect) choices of shell gaps and result in chaotic plots.

Since $\beta$ deformation values are closely connected with quadrupole moments and B(E2: $2^+_1 \rightarrow 0^+_1$) values, it is hardly surprising that they are smooth against
Figure 4: B(E2) and B(E2)_Q values for the actinides. From Ref. 8.

Figure 5: B(E2) values for A ~ 130 for various choices of shell gaps (i.e., of N_p and N_n).
What is perhaps unexpected, however, is how smoothly behaved $\gamma$ values are, at least in specific regions. Nuclear $\gamma$ values can be extracted, using the Davydov and Fillipov model [11], from B(E2) values and energies involving the $\gamma$ bands. The results for the Hg-Hf nuclei are shown in Fig. 6, based on Ref. 12. They are striking. First, not only are the $\gamma$ values smooth against $N_pN_n$ [see also Ref. 13 for results in the $A = 130$ region] but the $\beta$ and $\gamma$ values themselves are correlated, as also shown in Fig. 6. In this region, and perhaps elsewhere, this suggests that the evolution of nuclear shapes, at least for known nuclei, might be described in terms of a single variable, related to both $\beta$ and $\gamma$.

Finally, we illustrate a different type of VCS. Nuclear separation energies also reflect valence residual interactions. For example, $S_n$ increases with proton number, on account of the attractive p-n interaction, but decreases with neutron number since the average like-nucleon interaction is repulsive. While showing regularities, a plot of $S_n$ values [see Fig. 7 for an example] is hardly simple. However, when $S_n$ is plotted against $\alpha N_p - N_n$, as in Fig. 7, a very compact linear trajectory results for $\alpha = 2.6$ [14].

One interesting application, both of Fig. 7 and of $N_pN_n$ plots, is the special feature that new nuclei, far from stability, often have $\alpha N_p - N_n$ or $N_pN_n$. The figures show that $\beta$ and $\gamma$ values are smooth against $N_pN_n$.
$N_pN_n$ values within the range of values already applicable to known nuclei.
Hence, properties of such exotic nuclei can be predicted, not via extrapolation as hitherto necessary, but rather by interpolation along existing curves.

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