DOE/ER/53199--T4

FINAL TECHNICAL REPORT ON

"Studies of Plasma Transport"

DOE DE-FG03-85ER53199

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MASTER

This document gives an overview of the scientific results obtained under the DoE grant, and references the journal articles which give more complete descriptions of the various topics.

Recently, the research has been focused on 2-dimensional vortices and turbulence: experiments using a new camera-diagnosed electron plasma apparatus (§I) have given surprising results which both clarify and challenge theories. Here, the cross-field $E \times B$ flow of the electron plasma is directly analogous to the 2-d flow of an ideal fluid such as water, and may also give insight into more complicated poloidal flows exhibited in toroidal plasmas.

The shear-flow instabilities, turbulence, and vortices can be accurately observed, and the free relaxation of this turbulence has been characterized. Surprisingly, fluctuations are not just "sheared apart" by the background flow (§II.A); rather, prograde "holes" are seen to be much more stable than retrograde "clumps." Under some conditions, the turbulence is observed to relax to a state of minimal enstrophy, rather than maximal entropy (§II.B), and this is an active area of theoretical development with implications to many fluid flows. Under some conditions, novel "vortex crystal" states (§II.C) have been observed to halt the relaxation of the turbulence; and some theoretical insight has been developed into the cause of this previously unpredicted effect.

The physical processes underlying the complicated turbulent evolution can also be studied in more controlled near-linear regimes. The "diocotron" shear-flow instabilities have been measured experimentally, including an $l=1$ instability which is still only partially understood theoretically (§III.1). The dynamics of one or two vortices in isolation has also been studied (§III.A and III.B), showing quantitative agreement with theory.

The original experimental focus of this program was on radial particle transport from applied external field asymmetries (§IV). Here, this

research program clearly identified the importance of the collective response of the plasma, giving smaller fields from shielding, or enhanced fields from resonant modes. Experiments clearly show enhanced transport due to collective modes, including modes which can be resonant with static field perturbations. As originally formulated, resonant particle transport theory considered only the applied fields, i.e. only half of the problem.

Experiments and theory work have also elucidated the flow of a plasma *along* the magnetic field (§V). Depending on the plasma temperature, the flow is described by either a single self-similar hydrodynamic model, or by a non-interacting gas model. This free expansion dynamics is closely related to electron hole and electron soliton dynamics, which has also been measured.

Finally, some theory was pursued for direct application to fusion plasmas, and to gravitating gas clouds in astrophysics (§VI). This work focused on Ion-Temperature-Gradient-Driven turbulence and on the dynamics of self-attracting phase space density fluctuations.

This program was highly successful in clarifying basic plasma transport processes. Two of the three principal investigators (Tom O'Neil and the late John Malmberg) received APS Maxwell Prizes for their work, and all three PIs shared the APS Excellence in Plasma Research Award. Furthermore, the relevance to fusion physics has increased with the recognition that sheared $E \times B$ flows strongly affect transport in toroidal fusion plasmas. If further funding is available, we believe this program will continue to produce new insights into transport from turbulence and sheared flows.

I. New Camera Apparatus

A major part of the effort during 1990—1993 was to design and construct a new camera-based electron confinement apparatus (called CamV). CamV is based on a 2 Tesla superconducting magnet and a low-noise CCD camera. Figure 1 shows an overview of the CamV apparatus, and Fig. 2 shows a

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FIG. 1. The new CamV apparatus and magnetic field profile.

conceptual schematic. The magnet has a horizontal room-temperature bore 12 3/4 inches in diameter and 68 inches long. Maximum field is 2 Tesla, with 2% uniformity over a length of 120 cm. The magnet enables scaling of electron plasma characteristics over more than 2 decades in field, i.e. 0.1–20 kG. As shown in Fig. 1, the CamV electrode stack has 10 containment cylinders with $2R_w = 7$ cm, for a total length of $L = 60$ cm. Two of the electrodes are divided azimuthally for sending and receiving waves: one electrode has 4 sectors, and the other has 8. We have also included analyzer electrodes for measuring $T_{||}$ and $T_{\perp}$.

The plasma density $n(r, \theta)$ is obtained by dumping the electrons onto a phosphor, and taking a picture of the resulting fluorescence. Plasmas with density $n \sim 10^7 \text{ cm}^{-3}$ over a column length of 50 cm give 5×10^8 electrons/cm² on the phosphor. The phosphor gives off 400 photons per electron, i.e. 2×10^{11} photons/cm² from the phosphor. The camera collection efficiency is $R \approx 2.5 \times 10^{-3}$, so the CCD receives about 1.3×10^{10} photons/cm², i.e.

FIG. 2. The cylindrical confinement geometry and phosphor/camera diagnostic.

9.1×10^4 photons per pixel on the 512×512 array. The CCD has a quantum efficiency of about 8.7%, giving 7.9×10^3 electrons/pixel. The CCD digitizer gain is 7.5 electrons/count, so the digitized signal is about 1000 counts. With proper filtration, we have reduced the "noise" from stray filament light to about 2 counts. Thus, the system can give a 500:1 signal-to-noise ratio with a spatial resolution of $R_w/512 \approx 0.13$ mm.

FIG. 3. Contours of the density (vorticity) during instability, vortex-driven transport, and relaxation to a meta-equilibrium state.

II. 2D Turbulence, Vortex Crystals

We study turbulence and relaxation by creating well-controlled but unstable initial conditions and then observing the free evolution. Shear-flow instabilities lead to the formation of vortices, which result in rapid cross-field particle transport to a state which is no longer globally unstable. The vortices then merge, shed filaments, and eventually relax to an axisymmetric, sheared meta-equilibrium state. This state can be treated as a sheared θ -averaged background profile, plus turbulent fluctuations on various spatial scales. An example of such an evolution is shown in Fig. 3.

1. Fluctuations in a Shear Flow

We find that measured fluctuations decay about 50 times slower than predicted by simple “passive tracer mixing” in the presence of shear, and that the measured “noise” is strongly skewed from Gaussian [9]. These effects are due to the longevity of “holes,” *i.e.* self-trapped regions of negative relative electron density or vorticity. These holes are clearly visible in the fourth and fifth frames of Fig. 3. Fig-

ure 4 shows the shot-to-shot density fluctuations $\tilde{n}$ measured at $r \equiv R/R_w = 0.33$. The fluctuations are observed to decay in hundreds of rotation times τ_R , whereas a passive tracer would be smeared out in a few τ_R . Here, $\tau_R = 10 \mu\text{s}$. The probability distribution p for measuring density n is strongly skewed towards low densities because the fluctuations are not random, but rather reflect the coherent holes.

A simple fluid model [Moore and Saffman, 1971] shows that elliptical vortices can be in equilibrium with a weak imposed shear, but are elongated and destroyed by strong shear. Specifically, if the shear is prograde (*i.e.* in the vortex rotation direction), elliptical equilibria exist for all shear strengths; however, strong shear gives unphysically large elongations. If the shear is retrograde, equilibria exist only for weak shear.

FIG. 4. Measured decay of fluctuations compared to passive tracer prediction, and measured non-Gaussian fluctuation distribution.

Thus, in flows with strong negative shear (*i.e.* $\partial\omega_R/\partial r < 0$), such as Fig. 3 after $\approx 30\tau_R$, density clumps and shallow holes are sheared apart by the background flow, while relatively intense holes remain self-trapped. Measurements of the aspect ratios of these elliptical holes agree with the equilibria predicted for the measured shear [9]. Similar results have been seen for the stability of vortices in an applied shear field [Eggleston, 1994]. However, we also observe that the holes slowly drift outward (due to unknown effects) and are eventually destroyed.

2. Minimum Entropy States

We find that the system relaxes to a low-noise meta-equilibrium state (MES). We generally observe that the MES is axisymmetric with a monotonically decreasing density profile, and lasts for about $10^4\tau_R$ before non-ideal effects cause it to evolve further. The total number of electrons N , scaled angular momentum P_θ , and scaled electrostatic energy H_ϕ are well conserved from the initial conditions to the MES [4]. However, less robust “ideal” invariants such as enstrophy z and mean-field entropy S vary significantly, due to the inevitable measurement coarse-graining or dissipation of small spatial scales.

We have compared the measured MES density profiles to various theories based on maximization of entropy or minimization of enstrophy [Kraichnan and Montgomery, 1980]. We find that minimization of enstrophy accurately predicts the meta-

equilibrium profiles for hollow initial conditions of moderate energy [4] such as shown in Fig. 3. These profiles are significantly different from the predictions of maximum entropy. The minimization is subject to constant N , P_θ , and H_ϕ , and to the physical constraint that density $n \geq 0$. The explicit P_θ dependence of the solution is removed by rescaling the enstrophy as $\hat{Z}_2 \equiv 4\pi(1-P_\theta)Z_2$, and considering the excess energy $H_\phi^{\text{exc}} \equiv H_\phi - H_\phi^{\text{min}}$. Here, H_ϕ^{min} is the minimum energy possible for given N and P_θ , *i.e.* the energy of a uniform density column.

FIG. 5. Predicted (curve) and measured (points) enstrophy $\hat{Z}_2$ vs excess energy H_ϕ^{exc} .

Figure 5 shows the measured MES $\hat{Z}_2$ (circles) and the predicted minimum $\hat{Z}_2$ (curve) for a range of excess energies. Also shown is the value of $\hat{Z}_2$ which would be obtained from a maximum entropy MES for one evolution. Here, the experimental measurements are typically 2-3 times closer to the minimum enstrophy predictions than to the maximum entropy predictions, both in the measured Z_2 and in the measured profiles $n(r)$. For higher excess energies, this class of symmetric, monotonic solutions no longer exist, and several off-axis and non-monotonic profiles are possible.

3. Vortex Crystal States

We have also studied highly filamented, high excess energy initial conditions, which form 50-100 vortices, and then freely relax toward a 2D meta-equilibrium. Here, chaotic mutual advection and vortex merger are clearly important dynamical processes, and the final meta-equilibrium is typically strongly peaked on center. Surprisingly, this relaxa-

FIG. 6. Measured $n(r, \theta)$ at 5 times for two slightly different initial conditions, one of which forms vortex crystals; and 5 selected patterns which were observed.

tion is sometimes halted when individual vortices settle into a stable, rotating crystalline pattern which lasts for thousands of rotation times.

Figure 6 shows the measured z -averaged electron density $n(r, \theta, t)$ at five times for two slightly different initial conditions: the upper sequence forms vortex crystals, whereas the lower sequence relaxes rapidly to a monotonically-decreasing profile [10]. The vortex crystal state consists of 5 to 11 individual vortices each 4 to 6 times the background vorticity, arranged in a lattice pattern which rotates with the background. That is, rods of enhanced electron density ($n \sim 7 \times 10^6 \text{ cm}^{-3}$) are maintaining self-coherence and positions relative to each other for several seconds, while $\mathbf{E} \times \mathbf{B}$ drifting with a diffuse background ($n_B \sim 2 \times 10^6 \text{ cm}^{-3}$). Vortex crystal states are repeatably observed over a range of filament bias voltages, but the characteristics of $n(r, \theta)$ required for these states are not yet understood. The lowest row in Fig. 6 shows selected patterns which have been observed.

Figure 7 shows the number of distinct vortices N_v , and the enstrophy Z_2 for the two sequences. In both sequences, the unstable filamentary initial condition forms $N_v = 50$ –100 vortices of roughly equal

FIG. 7. Number of discrete vortices N_v and total enstrophy Z_2 versus time for two evolutions, one of which forms vortex crystals.

circulation, after which N_v initially decreases as $N_v \propto t^{-\xi}$, with $\xi \approx 1$. This relaxation is generally consistent with a dynamical scaling based on conserved quantities in repeated vortex merger [Carnvale *et al.*, 1993]. The observed ξ range from 0.4 to 1.1 as the initial conditions are varied, with 0.8 being commonly observed.

In the evolution of the top sequence in Fig. 6, the relaxation is arrested by the “cooling” of the chaotic vortex motions, with formation of vortex crystals by $10\tau_R$. The diamonds in Fig. 7 show that 8 to 10 distinct vortices survive for about $10^4\tau_R$. When the vortices all have about the same circulation, the patterns are quite regular, as seen at $600\tau_R$ in Fig. 6. After $10^4\tau_R$, N_v decreases to 1 as the individual vortices decay away in place. Other experimental images show that as N_v decreases, the remaining vortices re-adjust to a new rigidly rotating, symmetrically spaced pattern.

The measured integral quantities for both sequences are consistent with 2D inviscid motion on large scales and dissipation on fine scales. Experimentally, the circulation, angular momentum, and energy are robust invariants. In contrast, the enstrophy Z_2 is a “fragile” invariant, and initially decays a factor of 2 in both sequences. For the crystals sequence, Z_2 is constant from $10\tau_R$ until $10^4\tau_R$, at which time the individual vortices decay in place.

Reduction of the chaotic advective motions of the individual vortices is required to form the vortex crystal states. The average magnitude of the random velocities of the individual vortices, $|\delta V|$, decreases a factor of 6 between $2\tau_R$ and $100\tau_R$ for the crystals sequence, whereas only slight cooling is seen before $N_v=1$ (and $|\delta V|=0$ by definition) for the monotonic sequence [22]. This cooling is shown in Fig. 8.

We believe this cooling and cessation of relaxation through mergers is a near-inviscid 2D fluid effect, *i.e.* independent of the details of the fine-scale dissipation [22]. It appears that the vortex cooling occurs due to an interaction between the individual vortices and the boundary of the background vorticity. A weak interaction would be described as the excitation of surface waves on the background, and these waves could be damped by direct or beat-wave spatial Landau damping as described above. For strong interactions and short wavelengths, this would correspond to entrainment and mixing of low vorticity

FIG. 8. Evolution of the average chaotic velocity $|\delta V|$ of the vortices for the two sequences, normalized by $V_{\text{edge}} \equiv 2\pi R_p / \tau_R = 5.5 \times 10^4$ cm/sec.

city regions from the edge of the column. A similar process may cause the negative vorticity “holes” to become symmetrically situated, as discussed above.

Many different symmetric crystal patterns have been observed, with 3–10 vortices. Apparently, there are many different “meta-equilibria” to which the system can evolve under near-inviscid 2D dynamics. Because of these attractors, the system does not evolve ergodically, and the final state cannot be predicted from statistics alone. Nor can this system be adequately approximated as point vortices punctuated by occasional merger events: the discrete vortex motion is non-Hamiltonian due to interaction with the background vorticity. In contrast, experiments on vortex dynamics without a background have shown frequencies and instability rates closely corresponding with point vortex theory [17], as discussed below.

III. 2D Waves and Transport

In addition to the fully non-linear turbulent evolutions, we have been studying the simpler case of radial transport produced by isolated vortices, or by electric fields varying as $e^{il\theta + ikz}$ with $l \neq 0$ but $k = 0$. Here, the plasma $\mathbf{E} \times \mathbf{B}$ drifts in the azimuthal and radial directions, and radial transport can be on the same time scale as the plasma rotation. We have theoretically and experimentally performed experiments on the transport from naturally occurring diocotron (shear-flow) instabilities; established the characteristics of a single vortex interacting with a cylindrical wall (the $l=1, k=0$ diocotron mode);

FIG. 9. Measured density $n(r, \theta)$ at 5 times in the evolution of a hollow column which had a small $l = 1$ perturbation at $t = 0$.

and performed experiments on the dynamics of two isolated but interacting vortices.

1. $l=1$ Shear-Flow Instability

When the charge density profile $n_0(r)$ is non-monotonic (*i.e.* at least partially "hollow"), unstable "Kelvin-Helmholtz, or "diocotron," modes may appear. Experimentally, these unstable modes are found to be distinct from the stable modes [13,14]. That is, for any given l , the stable and unstable modes coexist, with distinct frequencies and radial eigenfunctions. This distinction has been missed historically due to reliance on step profile approximations. We observe instabilities with $l \geq 2$, as expected [13], and these results are basically consistent with 2D theory predictions when realistic, non-step profiles are considered. Furthermore, as discussed below, we observe a robust exponential

instability for $l=1$ where none is predicted [14]. These instabilities give rise to large θ -asymmetries in $n(r, \theta)$ and $\phi(r, \theta)$, which then results in radial transport on the drift motion time scale.

For the $l=1$ diocotron instability, our experiments strongly disagree with fluid theory. Figure 9 shows the evolution of a partially hollow electron column when a small $l=1$ seed exists at $t=0$ [14]. The initial seed asymmetry has two components: the center-of-mass (CM) of the plasma column is displaced off the cylindrical axis, and the central density minimum is not centered in the plasma. We observe that the CM orbits about the axis with constant orbit size (this is the stable mode). Similarly, the low density region orbits about the CM but it is spiraling outward with time (this is the unstable mode). The growth of the unstable mode causes the

high density ring to collapse in θ into a high density region, as shown at $t = 150 \mu\text{s}$. Next, large scale radial transport occurs: the high density region moves to the center and the low density region spreads out in θ at an appropriate radius, as shown at $t = 200 \mu\text{s}$. Most θ -variations (with respect to the CM) have been eliminated by $t = 1000 \mu\text{s}$.

The initial stages of this evolution can be analyzed from the perspective of linear modes, and our data for $n(r, \theta, t)$ characterizes the $k = 0$ modes rather completely [9]. The unstable mode arises only for density profiles which are at least partially hollow. The unstable mode frequency f_{1u} equals the maximum of $f_E(r)$, to within the experimental accuracy of $\pm 3\%$ in determining $f_E(r)$. Interestingly, this mode is largely self-shielding: the electric field arising from the mode is essentially zero outside the plasma. That is, no wall-sector signal is received from the growing mode, and the electric field eigenfunction $\delta\phi_{1u}(r)$ calculated from the measured $\delta n_{1u}(r)$ is essentially zero outside the plasma.

Figure 10 shows the magnitude of the unstable mode amplitude A_{1u} versus time for 2 initial seed perturbations. The instability e-folds in about 6 wave periods, and appears to be a linear process. Further, the mode saturates at the expected level, representing θ -variations from the minimum to the maximum density. Also shown is the predicted $l = 1$ growth from a new 2D fluid theory analysis [Smith and Rosenbluth, 1990]. This work supported by other grants established that a subtle, non-exponential instability may occur. The curve was obtained by numerically integrating a linear Laplace transform solution which started from initial conditions similar to the experimental initial conditions. Asymptotically, the initial value integration gives a perturbation proportional to $t^{1/2}$, i.e., algebraic growth.

Although the differences between experiment and theory here are large, there are intriguing similarities. The early time instability growth is seen to be similar. Further, the time-asymptotic theory perturbation is self-shielding, as is observed experimentally at all times. Further theory work [Smith, 1992] has established that non-2D effects such as finite Larmor radius and column end effects can cause exponential instabilities to occur. However, this fundamental fluid instability is still only partially understood.

FIG. 10. Amplitude of the unstable $l = 1$ mode vs. time for two different amplitudes of initial $l = 1$ perturbation.

2. One Vortex Equilibrium ($l = 1$ Diocotron Mode)

The $l = 1$ diocotron mode is the dynamical state of an off-axis electron column which is orbiting around the containment axis at the $\mathbf{E} \times \mathbf{B}$ drift velocity. When the electron column is pushed a long distance off center, linear theory is inapplicable, and the motion is more properly referred to as a nonlinear dynamical state of the system. The electron column is observed to be oval rather than circular, equivalent to the distortion of a fluid vortex interacting with a wall. Thus, the column is stationary in a frame rotating at the wave frequency f_{1s} . This implies that the density contours coincide with the potential contours, since in $\mathbf{E} \times \mathbf{B}$ flow the density is convected along potential contours. The nonlinear dynamical state is therefore determined by

$$\nabla^2 \phi_f(r, \theta) = 4\pi en(r, \theta) = F(\phi_f),$$

where ϕ_f is the potential in the rotating frame, and F is an unknown function.

When the column is off-axis, it distorts into an oval shape, which is then in equilibrium with the straining field from its image charges. We developed a simple perturbation theory [16] which predicts that the distortion varies as the offset

FIG. 11. Experimental measurements of vorticity vs. (r, θ) during two vortex merger.

squared, giving rise to shifts in the mode frequency which also vary as offset squared. These predictions are in good agreement with experiments. In this case, the simple theory based on body forces and geometric distortions is much more useful than the standard theoretical approaches based on surface charges and Fourier expansions on a centered column.

3. Two Vortex Merger and Orbits

The two vortex merger process can be studied as an isolated process, independent of any shear-flow instability processes. Straightforward manipulation techniques allow us to form an initial condition consisting of two electron columns of chosen profile and placement.

When two equal columns are placed symmetrically on either side of the cylindrical axis, separated by a distance $2D$, we observe that the behavior of the two vortices depends dramatically on their separation to diameter ratio, *i.e.* $2D/2R_p$ [111]. If the vortices are separated by more than 1.8 diameters, we observe that they orbit around each other relatively unperturbed for up to 10^4 orbits.

If the vortices are initially separated by 1.6 diameters, their mutual interaction quickly results in filamentary tail formation, but the vortices still orbit around each other for about 100 orbits before merging at the center. If the vortices are initially separated by 1.5 diameters, they merge within one orbit period, as shown in Fig. 11. This may represent the cleanest experimental measurement to

date of this fundamental vortex interaction process, due to the low inherent viscosity of the plasma, and due to the total absence of boundary layers at the wall. These results are in close agreement with theory and computational results, and with a more qualitative experiment in a water tank [Griffiths and Hopfinger, 1987].

FIG. 12. A plot of the time required to merge vs. the initial separation of the two vortices.

We have also studied the dynamics of the circular equilibrium orbits of two vortices too far apart to merge [17]. Here, each vortex is moving in the flow field from the other vortex and from the image charges in the wall. A common assumption has been that the advection can be well described by treating the spatially extended vortices as Hamiltonian point vortices unless they are close enough to merge. This is often called "punctuated Hamiltonian dynamics," i.e. integrable motion punctuated by discrete vortex merger events. However, prior experimental studies of vortices in water, in electrolyte, and in superfluid helium have typically found interactions quite different from that predicted for point vortices, apparently because of unwanted viscous and/or boundary effects. Our experiments demonstrate the validity of the Hamiltonian dynamics approximation in most regimes we can access.

In equilibrium, each vortex orbits around the center of the cylinder at constant radius. We find that some of these equilibria are linearly stable, others are unstable. We have measured both oscillations about stable equilibria and exponential divergence from unstable equilibria [17]. As shown in the measured equilibrium positions, orbit frequencies and instability rates are well predicted by treating the spatially extended vortices as if they were point vortices. We find that departures from this point vortex approximation occur only when the separation between vortices is less than 1.6 times their diameter and merger is observed to occur, and when the vortices "scrape" the cylindrical wall and circulation (charge) is lost.

The 2 point vortex system is fully integrable, having 2 degrees of freedom and 2 constants of the motion. However, this approximation neglects the effects of surface waves and shape distortions: the fields outside an extended vortex are the same as the fields outside a point vortex only if the extended vortex is circular. Experimentally, we observe elongations away from circularity of $\leq 10\%$ in general, and up to 30% for $r_1 = r_2 \sim 0.23$ (near merger, since $r_p \sim .15$). These time-varying eccentricities have not, however, been observed to cause experimentally noticeable departures from the predictions of the point vortex model. Thus, it is a non-trivial result to find that the overall stability of this obviously non-integrable system is well described by the integrable point vortex approximation. This is apparently because the internal degrees of freedom associated

with $r - \theta$ shape distortions do not significantly couple to the center-of-mass motion.

IV. Resonant Particle Transport

We have studied radial transport produced by externally applied electric and magnetic field asymmetries with Fourier components characterized by $k \neq 0, l \neq 0$. Here, we consider perturbations varying as $\exp \{ikz + il\theta - i\omega t\}$. For the case of electric field asymmetries, we could apply the fields at non-zero frequency (i.e., $\omega \neq 0$), a procedure with considerable experimental advantage and which conceptually broadens the transport problem in an important way.

Conceptually, it is useful to divide the transport problem into two parts. First it is necessary to determine the collective response of the plasma to an externally applied asymmetry; that is, to determine the asymmetric fields which exist in the plasma. Then, it is necessary to determine the transport produced by these fields.

In its original form, resonant particle transport theory seriously addressed only the second half of the problem. Transport equations were obtained by using mappings (that is, by single particle studies), but the collective response of the plasma was not taken into account. In contrast, our analysis has been based on a solution of a drift-kinetic version of the Fokker-Planck equation coupled to Poisson's equation, and the collective response is included. Of course, a complete solution of the coupled Fokker-Planck-Poisson system is not practical; we approach the solution as a perturbation expansion in the strength of the field asymmetry.

1. Linear Response

We find that the fields in the plasma can be very different in magnitude than the applied field [22]. For example, an applied electric field can be shielded out of the plasma, or it can be greatly enhanced in the plasma. The enhancement occurs when the applied field contains wave number and frequency components (k, l, ω) which correspond to a normal mode of the plasma. We have demonstrated that the enhancement can occur even for static asymmetries ($\omega = 0$), since the plasma rotates [Koenigs, 1981]. That is, a normal mode which propagates backward (upstream) on the column can have zero frequency in the laboratory frame and be driven resonantly by an applied field asymmetry.

As expected from theory, we have observed experimentally that the radial transport is enhanced when the applied perturbation frequency is near a normal mode frequency of the plasma column [22]. Figure 13 shows the perturbation induced density change at the center of the plasma as a function of applied frequency. We have observed up to a twenty-five fold increase in $\delta n/n_0$ at frequencies where normal modes were present.

FIG. 13. Induced density change δn ($r=0$) vs. the frequency ω of the applied perturbation.

We have also established the scaling of radial transport with the amplitude of an applied electrostatic asymmetry [22]. We can measure the asymmetry amplitude in the plasma by applying the perturbation at a normal mode frequency and then monitoring the amplitude of the mode with a second wall probe. Using this technique we find that 1) after an initial transient, the mode amplitude is linearly proportional to the applied perturbation and 2) the radial transport scales as the square of this amplitude. These observations are consistent with the predictions of plateau regime transport theory.

2. Nonlinear Responses

We have generalized this work by considering various nonlinear resonances and have shown that an applied field asymmetry can couple to collective modes through all of the wave-wave and wave-particle interactions of weak turbulence theory [19,23]. A detailed theory has been developed for the two lowest order processes, namely, the three-wave interaction and induced scattering. For the three-wave interaction, an applied field asymmetry

acts as a pump which excites the exponential growth of two daughter modes through the decay instability. For induced scattering, one of the daughter modes is effectively replaced by resonant particles; that is, the applied asymmetry scatters off of the resonant particles into a mode, and the mode grows exponentially. In both cases, the instability threshold is set by effects such as Landau damping of the daughter modes.

We have observed such an induced scattering instability in our plasma [24]. The experiment involves the unstable growth of an $l=0$, $k=1$ wave, and a sevenfold increase in the loss of plasma from radial transport is observed. The threshold and growth rate of the instability exhibit dependencies on asymmetry amplitude and wave damping that are in qualitative agreement with theory. The plasma loss rate increases with both asymmetry amplitude and mode amplitude, as expected from theory. This instability is observed over a wide range of plasma parameters, but can be suppressed by a trick which increases the damping of the unstable wave.

3. Induced Transport

The second part of the transport problem is to calculate the particle flux produced by a known asymmetric field in the plasma. The basic idea of resonant particle transport theory is that this flux is largely associated with particles which experience a resonance with the asymmetric field. Depending on the strength and nature of the resonance, the vicinity of additional resonances, and the relative importance of collisions, the transport equation assumes various forms. In other words, the name resonant particle transport theory should really be understood as a collection of several different but related theories.

The simplest case corresponds to the case of an isolated wave-particle resonance which is sufficiently weak that the trapping frequency is small compared to the effective collision frequency (the plateau regime). In this case, the transport equation follows from a simple quasi-linear analysis of the Fokker-Planck equation and is a generalization (for $\omega \neq 0$) of the plateau regime transport equation, which was obtained earlier by mapping techniques.

A related transport equation is obtained for the case of an isolated wave-particle-wave resonance [21]. Such a resonance typically dominates when the fields in the plasma are produced through the nonlinear process of induced scattering. A particle

experiences a resonance with beats between the field asymmetry and the nonlinearly excited mode. The transport equation is second order in the product of the mode amplitude and the field asymmetry amplitude and can be interpreted as a higher order quasi-linear equation.

For both these types of resonances, the nature of the transport equation is modified when the strength of the interaction is such that the trapping frequency is larger than the effective collision frequency (banana regime). Also, the transport can become stochastic when neighboring resonances overlap. Finally, one must realize that the two parts of the transport problem, determination of the asymmetric fields in the plasma and calculation of the transport due to these fields, are in fact coupled. As the plasma distribution evolves due to transport in configuration space and velocity space, the collective response of the plasma is modified.

IV. Free Plasma Expansion, Solitons, Holes

We have performed experimental and theoretical investigations of the free expansion of a magnetized pure electron plasma column along the axial magnetic field [26,27]. The expansion occurs when the containment voltage is removed from one end of the electron column. The free expansion dynamics is closely related to electron hole and electron soliton dynamics, which we have characterized on the same plasma columns.

Measurements of the radial average density made at several positions along the plasma column indicate that during plasma free expansion, a rarefaction disturbance propagates into the plasma at a speed which is within $\pm 10\%$ of the phase speed of the long wavelength electron plasma waves [7]. The plasma density behind the disturbance decreases to $\sim 1/2$ of the initial density. When the disturbance reaches the confined end of the plasma column it is reflected, causing the remaining plasma density to decrease, eventually emptying the confinement region of plasma. A new hydrodynamic theory has been developed, showing that the initial flow is completely self-similar, with no characteristic scale length. This is shown schematically in Fig. 14(a).

At low to moderate temperatures, where the Debye length is small compared to the plasma radius, the dynamics is determined primarily by collective electrostatic effects, and the data is predicted by the new hydrodynamic model, as shown in

Fig. 14(b). At moderate temperatures where the Debye length is comparable to the plasma radius, the dynamics is determined by both kinetic and electrostatic effects. In this regime the measurements can be approximately predicted both by the hydrodynamic model as well as a kinetic model which describes the plasma as a neutral noninteracting gas.

FIG. 14. a) Self-similar density evolution from new hydrodynamic theory. b) Measured image charge signals on 3 rings compared to theory.

The rarefaction wave can be considered an azimuthally symmetric ($l=0$) large amplitude electron plasma wave. Theory demonstrates that a wave propagating from low to high density spreads: the low density part cannot keep up with the faster moving high density part. This is what occurs in the rarefaction wave which forms during the plasma free expansion described in this experiment. The opposite situation in which a wave propagates from high to low density leads to a shock wave. This description of the nonlinear evolution of the plasma density is analogous to the description of the spreading or steepening of pressure waves in a compressible fluid.

The same phenomena of wave steepening and dispersion are also important in describing a soliton

in a bounded cylindrical plasma [27]. This positive density perturbation travels with a speed greater than the local plasma wave phase speed, and can be described well by considering only the hydrodynamic properties of the plasma. Short wavelength plasma modes present in the soliton are subject to dispersion, but this is balanced by the nonlinear effect of wave steepening. What results is a steady state object which retains its shape. Figure 15 shows a soliton which decays in ~ 6 axial transits of the column.

A full description of an electron hole must take into account the kinetic properties of the plasma. A hydrodynamic description alone predicts the evolution of the hole into a wavetrain, but this is not what is observed: the hole in Fig. 15 is stable for more than 50 transits of the column. The hole consists of a localized negative density perturbation containing a group of electrons trapped in the hole potential. These trapped electrons can be described as a hollow vortex in phase space. Plasma wave dispersion does not play an important role in the evolution of the electron hole.

VI. Other Theory

Several papers on Ion-Temperature-Gradient-Driven Turbulence and on the dynamics of self-attracting phase space density fluctuations were also supported by the DoE grant.

The ITGDT work derives a unified theory which ties the long-wavelength trapped-ion mode to the moderate wavelength hydrodynamic mode in toroidal geometry [32,35]. A general dispersion relation, taking full account of ion resonance, is derived from kinetic theory. It is found that although perpendicular compressional effects have a stabilizing influence for toroidal geometry above a critical temperature gradient scale size, they are unlikely to explain the improved bulk confinement in plasmas with seemingly flat density (so-called H-mode regime). Moreover, results indicate that a saturation of the heretofore experimentally-observed favorable current scaling of the energy confinement time may occur in low ion-collisionality regimes characteristic of JET and CIT-grade plasmas. Finally, it is shown that a population of energetic trapped particles, such as those that may be produced during auxiliary heating, can significantly exacerbate the instability. In all cases, mixing-length expressions for the fluctuation amplitudes and transport coefficients are

derived.

The work on self-attracting phase-space density fluctuations has applications both to "clumps" and "holes" in turbulent, collisionless plasmas, and to self-gravitating clouds in astrophysics [33,34]. A simple physical model is derived to describe the dynamics of the turbulence and the spectrum of these fluctuations. In both systems, the fluctuations tend to self-organize into hierarchical structures via the mechanism of Jeans collapse. The essential physical ingredient of the model is a cascade, constrained by the physical requirements of quasi-virialization. The theory demonstrates that the model exhibits interesting geometric properties such as intrinsic intermittency and anisotropy.

FIG. 15. Image charge signals showing a soliton and a hole repeatedly transiting the apparatus. (These were two separate experiments.)

Publications

2D Fluid Turbulence

1. C. F. Driscoll, J. H. Malmberg, K. S. Fine, R. A. Smith, X.-P. Huang and R. W. Gould, "Growth and Decay of Turbulent Vortex Structures in Pure Electron Plasmas," in *Plasma Physics and Controlled Nuclear Fusion Research 1988*, Vol. 3, 507 (Intl. Atomic Energy Agency, Vienna, 1989).

2. C. F. Driscoll, R. A. Smith, X.-P. Huang and J. H. Malmberg, "Growth and Decay of Vortex Structures in Pure Electron Plasmas," in *Structures in Confined Plasmas—Proc. of Workshop of U.S.-Japan Joint Institute for Fusion Theory Program*, Report No. NIFS-PROC-2, 69 (National Institute for Fusion Science, Nagoya, 1990).
3. C. F. Driscoll, "Wave and Vortex Dynamics in Pure Electron Plasmas," in *Research Trends in Physics: Nonlinear and Relativistic Effects in Plasmas*, V. Stefan, ed., 454 (American Institute of Physics, New York, 1992).
4. X.-P. Huang and C.F. Driscoll, "Relaxation of 2D Turbulence to a Metaequilibrium Near the Minimum Enstrophy State," *Phys. Rev. Lett.* **72**, 2187 (1994).
5. C.F. Driscoll, K.S. Fine, X.-P. Huang, T.B. Mitchell, A.C. Cass, and T.M. O'Neil, "Vortices, Holes, and Turbulent Relaxation in Sheared Electron Columns," *Proc. of 15th Intl. Conf. on Plasma Physics and Controlled Nuclear Fusion Research*, 79 (Intl. Atomic Energy Agency, Vienna, 1995).
6. C.F. Driscoll, T.B. Mitchell, X.-P. Huang, and K.S. Fine, "Turbulence and Relaxation in 2D Non-Neutral Plasmas," *Non-Neutral Plasma Physics II*, J. Fajans and D.H.E. Dubin, eds., AIP Conf. Proc. 331, 38 (American Institute of Physics, New York, 1995).
7. C.F. Driscoll, K.S. Fine, X.-P. Huang, T.B. Mitchell, and B.P. Cluggish, "Vortices and Turbulent Relaxation in Magnetized Electron Columns," *Transport, Chaos, and Plasma Physics 2*, S. Benkadda, F. Doveil and Y. Elskens, eds., 19 (World Scientific, Singapore, 1996).
8. C.F. Driscoll, "Electron Plasmas as Simple Fluids: From Turbulence to Vortex Crystals to Thermal Equilibrium," *Bull. Am. Phys. Soc.* **40**, 1746 (1995).
9. X.-P. Huang, K.S. Fine, and C.F. Driscoll, "Coherent Vorticity Holes from 2D Turbulence Decaying in a Background Shear Flow," *Phys. Rev. Lett.* **74**, 4424 (1995).
10. K.S. Fine, W.G. Flynn, A.C. Cass, and C.F. Driscoll, "Relaxation of 2D Turbulence to Vortex Crystals," *Phys. Rev. Lett.* **75**, 3277

(1995).

11. K.S. Fine, C.F. Driscoll, A.C. Cass, T.B. Mitchell, and X.-P. Huang, "Instabilities, Turbulence and Organized Structures in Magnetized Electron Columns," to appear in *Proc. of Crossed-Field Devices Workshop* (Univ. of Michigan, 1996).

2D Waves and Isolated Vortices

12. K. S. Fine, "Experiments with the $l=1$ Diocotron Mode,"
13. C. F. Driscoll, "Experiments on Vortex Dynamics in Pure Electron Plasmas," *Bull. Am. Phys. Soc.* **34**, 2001 (1989).
14. C. F. Driscoll, "Observation of an Unstable $l=1$ Diocotron Mode on a Hollow Electron Column," *Phys. Rev. Lett.* **64**, 645 (1990).
15. T.M. O'Neil and R.A. Smith, "Stability Theorem for Off-Axis States of a Non-neutral Plasma Column," *Phys. Fluids B* **4**, 2720 (1992).
16. K.S. Fine, "Simple Theory of a Nonlinear Diocotron Mode," *Phys. Fluids B* **4**, 3981 (1992).
17. T.B. Mitchell, C.F. Driscoll, and K.S. Fine, "Experiments on Stability of Equilibria of Two Vortices in a Cylindrical Trap," *Phys. Rev. Lett.* **71**, 1371 (1993).
18. T.M. O'Neil and R.A. Smith, "Stability Theorem for a Single Species Plasma in a Toroidal Magnetic Configuration," *Phys. Plasmas* **1**, 2430 (1994).

Resonant Particle Transport

19. J.D. Crawford, T.M. O'Neil and J.H. Malmberg, "Effect of Nonlinear Collective Processes on the Confinement of a Pure-Electron Plasma," *Phys. Rev. Lett.* **54**, 697 (1985).
20. J.H. Malmberg, D.L. Eggleston, T.M. O'Neil, and C.F. Driscoll, "Experiment on Radial Transport," *Intl. Atomic Energy Agency Tech. Comm. Meeting on Mirror Fusion Research*, 49 (Univ. of Tsukuba, Japan, 1985).
21. J.D. Crawford, S. Johnston, A.N. Kaufman, and C. Oberman, "Theory of Beat-Resonant Coupling of Electrostatic Modes," *Phys. Fluids* **29**, 3219 (1986).

22. D.L. Eggleston, J.D. Crawford, T.M. O'Neil, and J.H. Malmberg, "Enhancement of Radial Transport by Collective Processes," *Plasma Physics and Controlled Nuclear Fusion Research 1986*, Vol. 2, 337 (Intl. Atomic Energy Agency, Vienna, 1987).
23. J.D. Crawford and T.M. O'Neil, "Nonlinear Collective Processes and the Confinement of a Pure-Electron Plasma," *Phys. Fluids* **30**, 2076 (1987).
24. D.L. Eggleston and J.H. Malmberg, "Observation of an Induced Scattering Instability Driven by Field Asymmetries in a Pure Electron Plasma," *Phys. Rev. Lett.* **59**, 1675 (1987).
25. D. L. Eggleston, J. H. Malmberg, C. F. Driscoll, B. Beck and A. Hyatt, "Parallel Energy Analyzer for Pure Electron Plasma Devices," *Phys. Fluids B* **4**, 3432 (1992).

Free Expansion, Solitons, Holes

26. J. D. Moody and J. H. Malmberg, "Free Expansion of a Pure Electron Plasma Column," *Phys. Rev. Letters* **69**, 3639 (1992).
27. J.D. Moody and C.F. Driscoll, "Rarefaction Waves, Solitons, and Holes in a Pure Electron Plasma," *Phys. Plas.* **2**, 4482 (1995).

General

28. C.F. Driscoll, "Pure Electron Plasma Experiments," *Third Workshop on EBIS Sources and Their Applications* (Cornell Univ., Ithaca NY, 1985).
29. C.F. Driscoll, "Containment of Single-Species Plasmas at Low Energies, *Low Energy Antimatter*, David Cline, ed., 184 (World Scientific, Singapore, 1986).
30. T. M. O'Neil, "Plasmas with a Single Sign of Charge," *Nonneutral Plasma Physics*, C. W. Roberson and C. F. Driscoll, eds., AIP Conf. Proc. **175**, 1 (American Institute of Physics, New York, 1988).
31. J. H. Malmberg, C. F. Driscoll, B. Beck, D. L. Eggleston, J. Fajans, K. S. Fine, X-P. Huang and A. W. Hyatt, "Experiments with Pure Electron Plasmas," *Nonneutral Plasma Physics*, C. W. Roberson and C. F. Driscoll, eds., AIP Conf. Proc. **175**, 28 (American Institute of Physics, New York, 1988).

32. H. Biglari, P. Diamond and M. Rosenbluth, "Toroidal Ion-Pressure-Gradient-Driven Drift Instabilities and Transport Revisited," *Phys. Fluids B* **1**, 109 (1988).
33. H. Biglari and P. Diamond, "Cascade and Intermittency Model for Turbulent Compressible Self-Gravitating Matter and Self-Binding Phase-Space Density Fluctuations," *Phys. Rev. Lett.* **61**, 1716 (1988).
34. H. Biglari and P. Diamond, "Clouds and Holes: Self-Organization in Compressible Fluid and Collisionless Plasma Turbulence," *Physica D* **37**, 206 (1989).
35. H. Biglari *et al.*, "Advances in the Theory of Ion-Temperature-Gradient-Driven Turbulence," *Proc. of Twelfth Int. Conf. on Plasma Physics and Controlled Nuclear Fusion Research*, Vol. 2, 261 (Intl. Atomic Energy Agency, Nice, 1989).
36. J. H. Malmberg, "Some Recent Results with Nonneutral Plasmas," *Plasma Physics and Controlled Fusion* **34**, K. Lackner and W. Lindinger, editors, 1767 (Pergamon, New York, 1992).
37. T.M. O'Neil, "Plasmas with a Single Sign of Charge (An Overview)," *Physica Scripta* **T59**, 341 (1995).
38. T.M. O'Neil, "Thermal Equilibria and Thermodynamics of Trapped Nonneutral Plasmas," *Bull. Am. Phys. Soc.* **41**, 1529 (1996).

Ph.D. Theses

39. K. S. Fine, "Experiments with the $l=1$ Diocotron Mode," Ph.D. Dissertation (1988).
40. B. R. Beck, "Measurement of the Magnetic and Temperature Dependence of the Electron-Electron Anisotropic Temperature Relaxation Rate," Ph.D. Dissertation (1990).
41. X.-P. Huang, "Experimental Studies of Relaxation of Two-Dimensional Turbulence in Magnetized Electron Plasma Columns," Ph.D. Dissertation (1993).