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PARTICLE PRESSURES IN FLUIDIZED BEDS

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SUMMARY

This is a largely experimental project to make detailed measurements of the particle pressures generated in fluidized beds. The focus lies in two principle areas: (1) the particle pressure distribution around single bubbles rising in a two-dimensional gas-fluidized bed and (2) the particle pressures measured in liquid-fluidized beds.

This year, we have begun to make significant progress in all areas. We have developed image processing and data analysis software that will allow us to determine the particle pressures around rising bubbles in the gas fluidized bed and have begun to take significant data. Surprisingly, it appears that the largest particle pressures are a result of the bed material becoming defluidized in the bubble wake.

For the liquid fluidized bed, we have developed a third version of the particle pressure transducer for the surprisingly small particle pressures encountered in liquid-fluidized beds and have begun measurements. Towards the same ends, we have developed an alternate, quasi-theoretic approach to resolve the particle pressures and other quantities required to evaluate the stability of liquid-fluidized beds. The first step involved the development of a "generic" stability model. The generic character arises from leaving undetermined all terms that are usually modeled, such as those that involve fluid-particle drag and interparticle forces. The analysis indicates that the unknown quantities may be determined by observation of the growth and propagation of voidage disturbances. Consequently, it should be possible to evaluate these quantities by fitting the model to experimental results. This should not only yield insight into particle pressures, but also allow evaluation of the appropriateness of the various models used for other terms.

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1.0 INTRODUCTION

The particle pressure may be thought of as the force per unit area exerted by the particulate phase of a multiphase mixture and, as such, reflects the total momentum transport that can be attributed to the motion of particles and their interactions. It has a direct analog in the kinetic theory of gases in which the pressure acting on a surface is visualized as a result of the impacts of molecules. The same picture can be applied to particle-fluid situations with the particles taking the place of molecules. The only difference between the two cases is that solid particles may, in addition to short-duration collisional impacts, transmit a force via long duration contacts. (E.g. the weight of a particle, or an assembly of particles, resting on a surface).

The difficulty in measuring the particle pressure is that the total pressure exerted on a surface - the pressure that would be measured with a standard flush mounted pressure transducer - is the sum of the particle pressure and the pressure exerted by the fluid that resides in the interstices between the particles. Furthermore, in many cases - for example, fluidized beds or slurry flows - in which the motion is driven by fluid pressure - the particle pressure may be a small fraction of the total. Conceptually, such a measurement is not complicated, nor is the measurement terribly difficult. Essentially all one has to do is measure the total force acting on a surface and then let that fraction due to the fluid pressure balance itself out. Campbell & Wang (1990) described a very simple transducer for this purpose. That probe consisted of a solid diaphragm flush mounted into the wall. Small holes on either side of the diaphragm admit fluid, but no particles into a chamber behind the diaphragm. The face of the diaphragm experiences the total pressure exerted by both the particles and the fluid, while the rear experiences only fluid forces. Thus, the net deflection of the diaphragm reflects the contribution of the particle forces only. That probe has been used to make the particle pressure measurements on the vertical side walls of gas-fluidized beds that were presented in Campbell & Wang (1991) (although it could be used in many other flow situations). Campbell & Rahman (1992) have developed an improved form of the particle pressure transducer with better response characteristics.

Campbell & Wang (1991) showed that the particle pressures in gas-fluidized beds were largely generated by the passage of bubbles. In particular, they showed that the average particle pressure exerted on the side walls scaled with the average size of the bubble. This immediately brings to mind two questions: (1) what is it about bubbles

that leads to particle pressure generation and (2) would there be measurable particle pressures in liquid-fluidized beds which, while unstable, do not bubble? This project is largely aimed at answering these two questions. To attack the first problem, we have built a two-dimensional gas-fluidized bed into which bubbles may be injected and the distribution of particle-pressure measured. For the latter, other experiments are being performed in liquid fluidized beds.

However, it soon became apparent that the particle pressures generated in the liquid beds are extremely small. This has pointed that phase of the research in two directions. The first is the design and construction of a third, and more sensitive, form of the particle pressure transducer. The second approach arose from reflection on what ultimately was the utility of the current research. To a large extent, this research was motivated by interpenetrating continua multiphase-flow models which employ separate, but coupled, equations for the particle and fluid phases. The particle-phase equations have terms that involve the particle pressures and other interparticle forces: appropriate models for these terms have only been speculated upon and it was hoped that this research would yield insight into their modeling. Furthermore, the classic fluidization problem to which these models have been applied is the stability of a uniform state of fluidization. The only way in which a fluidized bed has been shown to be stable is through a particle-phase elasticity - i.e. it is stabilized through the particle pressure. Furthermore, once instabilities develop, the particle pressures will have significant effect on the growth and propagation of voidage disturbances. Naturally, of course, the instabilities will also be influenced by the manner in which the fluid-particle drag and other terms are modeled. This, led to the development of a "generic" stability model, in which all modeled terms are left unspecified. From analyzing this model, we have developed an experimental plan that, by measuring the characteristics of voidage disturbances and comparing with the theory, will allow us to back out appropriate values for the modeled terms. The results will, not only, yield insight into the particle pressure, but also of the fluid drag. The latter results may be used to evaluate common models for these terms.

2.0 PARTICLE PRESSURE MEASUREMENTS AROUND SINGLE BUBBLES IN A TWO-DIMENSIONAL GAS-FLUIDIZED BED

The results of Campbell and Wang (1991) indicate that particle pressures, measured along the side wall of a gas-fluidized bed, are primarily generated by the passage of bubbles. The primary evidence lies in the observation that the average values of the particle pressure scales with the equivalent diameter of bubbles. However, it is clear that the pressure obviously cannot be uniform across the bed. In particular, the particle pressure must go to zero in the particle-free region inside the bubble. Furthermore, the side wall of the bed may be a peculiar region as the particles pushed aside by the passage of a bubble cannot cross the wall, and therefore, the walls must affect the bubble motion in their immediate neighborhood.

The easiest way to determine the distribution of the particles pressure around a bubble is to perform the experiment in a "two-dimensional" fluidized bed. This is a term used to describe beds that are extremely thin in one dimension. In such a situation, bubbles span the small breadth of the bed so that it is possible to make a measurement of the particle pressure across a bubble without actually inserting a probe in the bed. The position of bubbles may be localized by artificially injecting bubbles into a bed held near minimum fluidization.

Figure 2.0.1 is a schematic of the two-dimensional bed. The test section is 60" high, 18" wide, but only 1" deep. It is fed by an air supply system that passes air through a ten inch packed bed and several of the Buckbee-Mears (2-2-8) 70 μ m etched screens to assure a uniform airflow. The air flow is set so that the bed is at minimum fluidization conditions. Then bubbles may be injected through a porous plate covered port, located 9 inches above the distributor. The bubble injector consists of a series of plenums which are pressurized through a precision pressure regulator. Firing a solenoid valve admits the additional air to discharge into the bed, causing a bubble to form. Different plenum pressures will discharge different quantities of gas. Finally 25 ports are cut into the face of the channel to admit the particle pressure transducers. Sixteen of the ports are configured in 4 lines of 4, spaced at 6 inch intervals above the injection port; in each line, the ports are spaced 2 inches apart, spanning the area from the center of the bed to one wall. The other 9 ports (not shown) are located at intermediate locations across the bed, to allow a finer spatial resolution. Four particle pressure probes are available to be inserted in any of the ports.

Figure 2.1: Schematic of the two-dimensional fluidized bed.

Bubbles are tracked and followed by an image processing system. The image originates in an Image Technology Methods, Datavision 262, video camera. The image is sampled by a Data Translation 3851 frame grabber with 512K of memory that is mounted inside an IBM clone computer with a 50MHz 486 processor. The 3851 board possesses an external trigger which allows the acquisition of the images to be synchronized with the acquisition of the particle pressure data. (The data acquisition is performed in a separate 33MHz 486 IBM clone; digital outputs from that computer are used to trip a relay which injects the bubble and to fire the external trigger of the 3851 as well as to sample the particle pressure information. This tightly synchronizes the entire experimental process.) The 512K of memory permit 25 frames to be acquired at a rapid rate and stored on the board.

Both the acquisition and interpretation of the images is controlled by Data Translation's Global-Lab Image[®] Software package. This package provides many powerful image processing options. The most useful for this phase of the project is the ability to detect and analyze "particles" (which in this context refers to bubbles). In particular, it locates the center-of-area of the bubble which we use as a reference for the bubble location, calculates the area of the bubble, the average radius from the center-of-area, and so on.

Figures 2.0.2-2.0.4 illustrate some of the utility of this software in locating and analyzing bubbles. These were taken for relatively large bubbles injected into a bed of 1mm glassbeads. Figure 2.0.2 shows the position of the bubble's center-of-area, measured above the injection point, as a function of time. (Note: to provide more detail of the bubble's location in the neighborhood of the particle pressure probes, the camera is placed so that the injection point is slightly out of frame; the points shown here, start when the lower edge of the bubble is just visible within the frame and thus allows an accurate evaluation of the center-of-area.) In this figure, as in those below, the points represent the data taken from the individual frames; the lines are cubic spline fits to that data.

Figure 2.0.3 show the effective bubble radius as a function of the time since injection (Figure 2.0.3a) or as a function of position above the injector (Figure 2.0.3b). Two plots are shown for two different ways of computing the bubble radius. The circular points (solid line) represents the effective radius (in cm) which is computed from the area of the bubble (i.e. $\text{Radius} = \sqrt{\text{Area}/\pi}$) while the crosses (dashed line) are

Figure 2.0.2: The position above the injections point (in cm) versus time (in secs) of the center-of-area of a relatively large bubble in a two-dimensional fluidized bed of 1mm glassbeads.

the average of the distance from the bubble's center-of-area to the bubble edge (provided by the image processing software). The most interesting feature of this figure is that the bubble radius initially decreases. This is somewhat surprising as one expects the bubble to grow monotonically as it rises due to the decreasing pressure in its surroundings. But this is not too surprising as these are injected bubbles. Initially the gas in the bubble will flow radially away from the injection point and it apparently takes some time to set up the gas-flow pattern that is characteristic of bubbles in a fluidized bed. Thus, the bubble's shape will change (in this case, the bubble both distorts and shrinks) until the bubble has had sufficient time to develop. This development is evident in the small slope changes that are visible in the position vs. time of the center-of-area shown in Figure 2.0.2. These may be interpreted as the results changes in the location of the center-of-area due to bubble shape changes. As a result of this figure, we have decided to lengthen our bed by adding a section between the injector and the probes to allow the bubble to develop more fully.

Some the effects of this change in shape can be seen in the plot of bubble velocity

(a)

(b)

Figure 2.0.3: The bubble size as a function of time (a) and position (b).

Figure 2.0.4: The bubbles velocity as a function of position. Its wildly fluctuating character is due to the changes in the bubble shape.

vs. position which is shown in Figure 2.0.4. These are found by taking numerical derivatives of the data shown in Figure 2.0.2. Wild velocity fluctuations are seen in the region where the bubble is initially shrinking. Afterwards, the velocity increases as the bubble grows, as is commonly observed. Apparently, the huge variations in velocity are a byproduct of the barely noticeable slope changes seen in Figure 2.0.2. This also indicates that the probes should be placed at least 70cm above the injection point which will require extension of the current channel. (We attempted to measure the bubble velocity by analyzing the motion of the front and back of the bubble captured in the video images. This is easily done using the Global-Lab software by subtracting successive frames from one another which reveals the difference between the frames, i.e. the region through which the bubble has moved. However, this method is even more strongly affected by shape changes and leads to even wilder velocity fluctuations.)

Figure 2.0.5 shows a time history of the particle pressures measured by the four probes which are located eighteen inches above the injection point and 0, 5, 10 and 15cm from the bed centerline (the corresponding plots are shown in order starting at the top of the figure). The figure is also labeled with the locations where the top and

Figure 2.0.5: The time history of the particle pressures in a bed of 1mm glassbeads. The four plots come from 4 probes mounted 46cm above the injection point. From top to bottom, the probes are mounted 0, 5, 10, 15cm from the bed centerline.

Figure 2.0.6: The same particles pressures as functions of the position of the bubble's center-of-area.

bottom of the bubble cross the probe, as well as the locations where the bubble erupts from the surface of the bed and the point when the bed, at least visually, returns to its original state. At the time it crosses the probes, the bubble has a equivalent radius of about 10cm and thus, the bulk of the bubble crosses the positions of the first two probes.

Naturally, the particle pressure is zero when the bubble passes the probes as there are no particles present. Surprisingly though, the particle pressure is also zero above the bubble. Particle pressures may first be seen along the sides of the bubble. (The bottom two figures, which represent the two probes outside of the bubble, show an elevated particle pressure starting at about $t=0.4s$, which the top two, over which the bubble passes, show zero particle pressure.) The largest particle pressure are observed in the particle wake and furthermore, are seen in the far wake. In fact, the largest measurements occur just before the bubble erupts from the bed. To show this in more detail, Figure 2.0.5 shows the particle pressures plotted versus the position of the center-of-area of the bubble. It does show that the largest particle pressures occur but a full 40cm (almost two bubble diameters) below the center-of-area of the bubble. It is possible that the large pressures are a byproduct of eruption. But it seems more likely that the passage of a bubble causes severe disruption in the structure of a bed which seems to persist long after the bubble passes. This is even more apparent in the data from probes 3 and 4 that lie furthest from the path of the bubble. There a slightly elevated particle pressure is preserved in the bed, long after the bed has settled. Remember that the bed is initially at minimum fluidization so that the structure will have limited mobility. Most likely, this disruption would be brief in a fully fluidized bed, in which the bed would quickly return to a random state due to the passage of subsequent bubbles.

While requiring further study, this is a potentially important observation with regards to the manner in which particle pressure is understood in a gas-fluidized bed. Definitely, the particle pressure is very different from a gas pressure which would relax very quickly. It is doubtful that the pressure would persist for so long, if it were generated by random particle fluctuations. This result suggests that the pressures are the result of changes in the bed structure which could only be held for so long if the particles are in intimate contact. This indicates that the passage of the bubble defluidizes the material in its wake. The fact that the largest particle pressures are observed just before eruption, suggests that these elastic structures are supporting

progressively more material, as if they were supporting a large percentage of the material above the probe that is defluidized by the bubble's passage.

It is the final intent of this phase of the research to produce maps of the particle pressure that surrounds a bubble. This will be done by taking many observations on many similarly shaped bubbles of different sizes and with the probes in different positions so as to accumulate enough data from which such a map may be drawn. The required software is currently under development.

3.0 AN EXAMINATION OF DAVIDSON'S PARTICLE PRESSURE ANALYSIS

Davidson & Harrison (1963) present just about the only predictions available of the particle pressure about a rising bubble. Other than its basis in Davidson's (1961) bubble analysis, the origins of this analysis is a little obscure. Davidson & Harrison use a Bernoulli-type equation based on the total (fluid+particle) pressure without justifying its validity. We decided to develop our own particle pressure analysis based on adding a particle-phase equation to Davidson's (1961) bubble model. (We chose Davidson's model as it is the most accurate in its predictions of the gas-pressure field around bubbles.) However, when Davidson's assumptions were included, the equations may be integrated to find exactly the Bernoulli-type equation presented in Davidson & Harrison (1963). (This analysis is rather simple and it is surprising that it does not appear, nor is referred to in either Davidson & Harrison or in the only other place that the model has been put to use, Stewart & Davidson (1967).)

The major problem with applying this model is that it predicts largely negative particle pressures. When resolved for the particle pressure the Davidson & Harrison equation becomes:

$$P_p = C - \left(\frac{\rho u_p^2}{2} + \rho \mathbf{g} \cdot \mathbf{z} + P_f \right)$$

Where P_p is the particle pressure, C is the integration constant, ρ is the bulk density, u_p is the particle velocity, $\mathbf{g}$ is the gravitational acceleration vector, $\mathbf{z}$ is the position, and P_f is the fluid pressure (given by the Davidson (1961) analysis). Davidson & Harrison argue that the particle pressure should be zero at the forward stagnation point (this is consistent with the measurements made in last year's report) of the bubble which fixes the constant C . The problem with this analysis is that it predicts that the particle pressures will often be negative which, is counter to any intuitive idea of how the particle pressure might be generated. While explicit forms for u_p , $\mathbf{z}$ and P_f would be necessary to see this directly, it is easy to guess why these negative pressures appear; beyond the forward stagnation point, the particle velocities, u_p increase rapidly from zero, pulling the particle pressure negative.

Now, as originally presented, Davidson's model was derived for spherical or cylindrical bubbles. As the shape of the bubble governs the particle velocities, we thought that, if a more realistic particle shape were used, these negative pressures might

CONTOUR GRAPH OF PARTICLE PRESSURE

Figure 3.0.1: A contour map of the particle pressure around a distorted bubble using the modified Davidson model.

be avoided. As a result, we looked at the model of Collins (1965) who employed conformal mapping to distort a cylindrical bubble into one with a more realistic shape. It was hoped that, if a distorted shape were used, these negative particle pressures could be avoided. Furthermore, we had the feeling that the dimple that is characteristically found at the rear of bubbles might be a result of the large particle pressures generated by the collision of the two particle streams meeting at the stagnation point. (We know from the last section that that is a region of large particle pressures.) Unfortunately, we found that no distortion of the shape that would yield pressures that were always positive. A contour plot of the pressures that we found is shown in Figure 3.1.

We feel that this analysis points out other problems with Davidson's (1961) model. The negative pressures arise because of the many assumptions, other than shape that Davidson made. In particular, we feel that the source lies in his assumption of an

incompressible particle phase, which is clearly not valid in the cloud region surrounding the bubble. In fact, it is quite evident that Davidson was well aware of the limitations of his model. For one thing, he only presented the particle pressure on the bubble centerline. Furthermore, his argument that the particle pressure had to be zero at the stagnation point is largely that the particle pressure should be zero on the bubble boundary as there are no particles beyond. But, he cannot satisfy this condition at the rear stagnation point, where, as the particle velocities are zero, the particle pressure must be positive as a result of the hydrostatic pressure changes across the bubble. All in all, we feel that Davidson's assumption that the particle pressure must be zero at the forward stagnation point, while obviously true at the bubble boundary, may not be a good approximation in light of the assumptions in his model, even though it does seem to agree with our preliminary measurements. (I.e. Davidson assumes an abrupt discontinuity in the particle concentration, therefore, one might also expect an abrupt discontinuity in the particle pressure.) As a result, we understand that the model will not be valid very close to the bubble boundary and, in plotting Figure 3.1 have set the constant C according to the constraint that no unphysical negative particle pressures are generated. It will be interesting to see if the measured particle pressure field look anything like that predicted above. However, this seems unlikely, as the results presented in the last section seem to indicate that the particle pressures are concentrated in the far wake, while the Davidson model predicts that they will be concentrated in the immediate neighborhood of the bubble.

4.0 PARTICLE PRESSURES IN LIQUID FLUIDIZED BEDS

Last year, we presented measurements of the particle pressures generated in a liquid fluidized bed using the same type of probe described in Campbell & Rahman (1992). Unfortunately, we were unable to find an appropriate displacement transducer that could accurately measure the deflection of the diaphragm and could also operate within a liquid environment. Originally, we had thought to measure the displacement by sensing changes in the resistance in the circuit through the liquid between the moving diaphragm and a stationary electrode. (After all, we use a capacitance measurement in the gas-fluidized bed.) However, we found that this was too unstable. Then we discovered an eddy current device that we used to make the measurements presented in last year's report. Unfortunately that transducer was only available with a 1mm range, forcing us to use only a few percent of the available range for our measurements. Those results showed that the particle pressures were essentially zero after reaching minimum fluidization. This was somewhat surprising as it indicated that the particle pressures are significantly smaller than those inferred from impact velocities by Kumar et al. (1990) (data that, during the design and calibration process, we used to estimate the range of forces). A noticeable, but still small, improvement was found by using a much looser screen.

4.1 A THIRD DESIGN FOR THE PARTICLE PRESSURE TRANSDUCER

Our inability to discover an appropriate displacement sensor necessitated a significant redesign of the particle pressure probe. The new design is shown in Figure 4.1.1. Instead of using the displacement of a diaphragm, we make a direct measurement of the force applied to a rigid plate using a sensitive load cell. The plate is perforated with holes to permit the flow of fluid through to the backside. The surface is covered with a loose fine screen to prohibit the passage of particles; as the screen is loose, it does not absorb any of the applied force, leaving it, instead to be measured by the load cell.

It took a while to find an appropriate load cell as it had to be invulnerable to the liquid environment. We decided on a Wagner Instruments, LPM 530 load cell with a 50 gmf range. This was the smallest range cell that we could find that could be appropriately configured for this purpose. Using a 4.5cm diameter plate, the full range

Figure 4.1.1: A schematic of the newest version of the particle pressure transducer.

scale is approximately $30.8\text{mmH}_2\text{O}$, (although we have found that it is able to withstand slightly higher pressures). Assuming 1% resolution, we should be able to resolve about $0.3\text{mmH}_2\text{O}$.

Some preliminary measurements made with the new probe are shown in Figure 4.1.2. The measurements were made on a $13 \times 13\text{cm}$ square bed of 1mm glassbeads. They are compared with identical measurements made with the older design. For these experiments, the two probes were mounted at the same level on opposite sides of the bed. At zero velocities, the particle pressure is large as all of the weight of the bed is supported across interparticle contacts. At low velocities, before the bed becomes fluidized, the particle pressure decreases monotonically as progressively more of the material in the bed is supported by fluid forces. Both probes exactly follow one another during this phase. At minimum fluidization, however, the particle pressure drops dramatically. The new design registers the minimum particle pressure as about $0.8\text{mmH}_2\text{O}$. The old design cannot follow (in fact, it will register almost the same value of the particle pressure for all larger superficial velocities). However, the particle pressure rises quickly, returning (interestingly enough) to roughly the same value measured by the old design. However, at the highest velocities, when the bed is extremely expanded and there are very few particles present. This seems to indicate two competing processes. The first is the agitation of the bed, either as individual particles reacting to a turbulent flow, or by the larger scale motion of voidage instabilities.

Figure 4.1.2: Measurements of the particle pressures exerted on the sidewalls of a 13×13 cm water-fluidized bed of 1mm glassbeads. The measurements with the newest probe are compared against those of the older design.

Increasing the superficial velocity will increase both contributions. However, at the same time, the bed expands, reducing the local concentration of particles which should work to reduce the particle pressure. At moderate velocities, these two competing processes seem to pull to a draw, but, at the largest velocities, the reduced concentration wins the tug-of-war and draws down the particle pressure.

4.2 ALTERNATIVE QUASI-THEORETIC APPROACH:

It is most likely that, given the small magnitudes of the particle pressures in a liquid bed, it will be nearly impossible to directly measure the small pressures that

occur near the neutral stability point. This is unfortunate, as those are the particle pressures that affect the stability of the bed. It has been shown many times (e.g. Anderson & Jackson (1968), Garg & Pritchett (1975), Batchelor (1988)) that an "elasticity" of the particle phase can stabilize the bed. (Here, the elasticity is defined as $E_p = -\frac{dp_p}{d\epsilon}$, where ϵ is the void fraction.) Jackson (1985) argues that the large value required of the elasticity could not be generated by particle fluctuations. However, we came up with another idea to indirectly measure the particle pressures.

The idea arose from a class project performed by Chengzhen Jin, one of the students supported by this grant. He performed a survey of the various stability analysis and (although not required for the assignment) derived one of his own. This analysis, like its predecessors, models the particle and fluid phases as separate, but coupled continua, makes assumptions about the forms of the constitutive behavior, the interfacial drag, particle elasticity, bulk viscosity, etc. and predict the onset of instability, the initial linear growth rate and velocities of voidage disturbances. Of the results depend on the various assumptions. We realized that this process could be reversed. I.e. One could create a generic stability analysis (i.e. one that make as few assumptions as possible). Then by comparing the theory with measured properties such as the wave growth and velocity and from comparison with the theory, back out appropriate values for the modeled terms. As will be shown in the following, the model may be reduced to different wave equations in the near and far field. (The measurements of El-Kaissy & Homsy (1976) confirm different near and far-field behaviors.) Presumably, for each measurement made, one could back out the corresponding value of an assumed quantity. For example, measurements made of the growth rate and velocity of instability waves, in the near and far fields, (four measurements) should yield values for four modeled quantities such as the fluid-particle drag and the particle phase elasticity. The measurements can be made using the same light attenuation method as Anderson & Jackson (1969) and El-Kaissy & Homsy (1969). (Here though, as described in the proposal, we will analyze the results using a video camera and frame grabber.)

A similar idea was used by Ham et al. (1990). They measured the location of the onset of instability in a small fluidized bed and extracted the particle-phase elasticity from an analysis based on Batchelor's (1988) theory. By measuring additional properties we hope to extend this to determine the other modeled properties of the stability analyses. Measurements of the drag and viscosity will be valuable in

themselves as they allow the evaluation of various models that are in common use.

4.2.1 The Generic Stability Model

The model is based on standard interpenetrating continuum techniques. Its development will be briefly outlined below, omitting many of the details common to other models of this type. The equations appropriate to the one-dimensional propagation of plane voidage waves in the z-direction are (in dimensionless form):

Conservation of Mass:

$$\text{fluid phase: } \frac{\partial \epsilon}{\partial t} + V_f \frac{\partial \epsilon}{\partial z} + \epsilon \frac{\partial V_f}{\partial z} = 0 \quad (4.2.1)$$

$$\text{solid phase: } - \frac{\partial \epsilon}{\partial t} - V_p \frac{\partial \epsilon}{\partial z} + (1 - \epsilon) \frac{\partial V_p}{\partial z} = 0 \quad (4.2.2)$$

A combined phase equation (describing the motion of the bulk mixture) is used for conservation of momentum:

$$\begin{aligned} & \epsilon \left(\frac{\partial V_f}{\partial t} + V_f \frac{\partial V_f}{\partial z} \right) + (1 - \epsilon) \rho \left(\frac{\partial V_p}{\partial t} + V_p \frac{\partial V_p}{\partial z} \right) = \\ & - \frac{1}{R_e} \frac{\partial p_f}{\partial z} + E \frac{\partial \epsilon}{\partial z} + \frac{1}{R_{e\eta}} \frac{\partial^2 \epsilon}{\partial z \partial t} - [\epsilon + (1 - \epsilon) \rho] \frac{1}{F_r} \end{aligned} \quad (4.2.3)$$

where, $E = \frac{E_{p0}}{J_0^2 \rho_f}$, $p_f = \frac{P_f}{\mu_f J_0 / d_p}$, $R_e = \frac{\rho_f J_0 d_p}{\mu_f}$, $R_{e\eta} = \frac{\rho_f J_0 d_p}{\eta_p}$, $F_r = \frac{J_0^2}{g d_p}$, $\rho = \frac{\rho_p}{\rho_f}$. d_p is the particle diameter. J_0 is the superficial fluid velocity ($J_0 = \epsilon V_f$), and E_p represents the effective elasticity of the particulate phase and η_p represents an effective viscous resistance to voidage growth. In writing (4.2.3) it is assumed that the resistance of the particle phase to a voidage disturbance has both an elastic and viscous component. I.e:

$$\sigma_p = E_p \epsilon + \eta_p \frac{\partial \epsilon}{\partial t}$$

Anderson & Jackson (1968) come to the same conclusion using a viscous interparticle stress term.

4.2.2 The Stability of the State of Uniform Fluidization and What can be Learned from It.

Let ε_0 , V_{f0} , V_{p0} , P_{f0} , P_{p0} , describe the void fraction, velocities and pressures that correspond to a state of uniform fluidization. Under those conditions: $\varepsilon_0 = \text{const}$, $V_{f0} = J_0/\varepsilon_0$, $V_{p0} = 0$, $\partial P_{p0}/\partial z = 0$. Let it then be disturbed by ε' , v_f' , v_p' , p_f' , p_p' . I.e:

$$\begin{aligned}\varepsilon &= \varepsilon_0 + \varepsilon' \\ V_f &= V_{f0} + v_f' \\ V_p &= V_{p0} + v_p' \\ p_f &= P_{f0} + p_f' \\ p_p &= P_{p0} + p_p'\end{aligned}$$

As. in the uniform state the fluid-particle drag, F_f , results in the bulk pressure gradient:

$$F_f = - \frac{\partial(p_f)}{\partial z} = F_f(\varepsilon, V_f - V_p)$$

so:

$$- \frac{\partial(p_f')}{\partial z} = \left(\frac{\partial F_f}{\partial(V_f - V_p)} \right)_0 (v_f' - v_p') + \left(\frac{\partial F_f}{\partial \varepsilon} \right)_0 \varepsilon' = A(v_f' - v_p') - C \varepsilon'$$

Where:

$$A = \left(\frac{\partial F_f}{\partial(U_f - U_p)} \right)_0, \quad C = - \left(\frac{\partial F_f}{\partial \varepsilon} \right)_0$$

After substituting into (4.2.3), using (4.2.1) and (4.2.2) to replace v_f' and v_p' in favor of ε , and eliminating higher order terms one finds:

$$A \frac{\partial^2 \varepsilon'}{\partial t^2} + B \frac{\partial^2 \varepsilon'}{\partial z \partial t} + C \frac{\partial^2 \varepsilon'}{\partial z^2} + F \frac{\partial \varepsilon'}{\partial t} + G \frac{\partial \varepsilon'}{\partial x} = S \frac{\partial^3 \varepsilon'}{\partial z^2 \partial t} \quad (4.2.4)$$

where

$$\begin{aligned}\mathcal{A} &= \rho - 1, \\ \mathfrak{B} &= -2 V_{f0}, \\ \mathfrak{C} &= -[V_{f0}^2 + E], \\ \mathfrak{T} &= \frac{A}{Re \varepsilon_0 (1 - \varepsilon_0)} \\ \mathfrak{G} &= \left[\frac{A V_{f0}}{Re \varepsilon_0} + \frac{C}{Re} - \frac{\rho - 1}{Fr} \right]\end{aligned}$$

and:

$$\mathfrak{S} = \frac{1}{Re \eta}$$

Then equation (4.2.4) can be rewritten in the form of wave equation (Whitham, 1974, Liu, 1982):

$$\tau \left(\frac{\partial}{\partial t} + c_1 \frac{\partial}{\partial x} \right) \left(\frac{\partial}{\partial t} + c_2 \frac{\partial}{\partial x} \right) \varepsilon' + \left(\frac{\partial}{\partial t} + a_1 \frac{\partial}{\partial x} \right) \varepsilon' = \nu_e \tau \frac{\partial^3 \varepsilon'}{\partial z^2 \partial t} \quad (4.2.5)$$

where:

$$\begin{aligned}\tau &= \frac{\mathcal{A}}{\mathfrak{T}} \\ \nu_e &= \frac{\mathfrak{S}}{\mathcal{A}}\end{aligned}$$

$$\begin{aligned}c_1, c_2 &= \frac{1}{2} \frac{\mathfrak{B}}{\mathcal{A}} \left\{ 1 \pm \sqrt{1 - 4 \frac{\mathfrak{C} \mathcal{A}}{\mathfrak{B}^2}} \right\} \\ &= -\frac{1}{2} \frac{2 V_{f0}}{\rho - 1} \left\{ 1 \pm \sqrt{1 + 4 \frac{(V_{f0}^2 + E)(\rho - 1)}{4 V_{f0}^2}} \right\}\end{aligned} \quad (4.2.6)$$

and:

$$a_1 = \mathcal{G}/\mathcal{F} = \frac{[\frac{A V_{f0}}{R_e \varepsilon_0} + \frac{C}{R_e} - \frac{\rho - 1}{F_r}]}{\frac{A}{R_e \varepsilon_0 (1 - \varepsilon_0)}}$$

The quantities, c_1 and c_2 (note that, as $\rho > 1$, c_1 is positive and c_2 is negative) represent the dynamic wavespeeds and a_1 represents the kinematic wavespeed in the material. Stability requires $a_1 < c_1$ and instability arises when: $a_1 > c_1$.

It is possible to obtain simplified forms of these equations in the region near and far from the distributor using techniques derived from Whitham (1974) and Liu (1982). Near the distributor, waves will move approximately with the dynamic wavespeed, c_1 (as c_1 represents the wavespeed moving away from the distributor) we can approximately have:

$$\frac{\partial}{\partial t} \doteq -c_1 \frac{\partial}{\partial x}$$

except for the term that already involves c_1 , (Whitham (1974)) apply this to Eq(4.2.5), get:

$$\left(\frac{\partial}{\partial t} + c_1 \frac{\partial}{\partial x}\right) \varepsilon' + \frac{c_1 - a_1}{c_1 - c_2} \frac{1}{\tau} \varepsilon' = \bar{\nu}_e \frac{\partial^2 \varepsilon'}{\partial z^2} \quad (4.2.7)$$

where $\bar{\nu}_e = \frac{\nu_e c_1}{(c_1 - c_2)}$ is a dimensionless effective viscosity. Now, assume a plane wave disturbance of the form:

$$\varepsilon' = \alpha_0 e^{a_h t + i\omega_h(t-z/U_h)}$$

Substituting into Eq(4.2.6) yields the local growth rate:

$$a_h = \frac{a_1 - [c_1 + \tau \bar{\nu}_e k_h^2 (c_1 - c_2)]}{(c_1 - c_2) \tau} \quad (4.2.8)$$

and local wavespeed:

$$U_h = c_1 = \frac{V_{f0}}{\rho - 1} \left\{ \sqrt{1 + \frac{(V_{f0}^2 + E)(\rho - 1)}{V_{f0}^2}} - 1 \right\} \quad (4.2.9)$$

where k_h is the wave number, $k_h = \omega_h/U_h$.

For the long wave region (low frequency waves, far way from the distributor), waves will move with the kinematic wavespeed, a_1 (Liu (1982)). We then approximately have:

$$\frac{\partial}{\partial t} \doteq - a_1 \frac{\partial}{\partial x}$$

except for the term that already involves a_1 , (Whitham (1974)) and the viscous term, η_p can be neglected. Applying this to Eq(4.2.5), yields:

$$\left(\frac{\partial}{\partial t} + a_1 \frac{\partial}{\partial x} \right) \varepsilon' = \mathfrak{D} \frac{\partial^2 \varepsilon'}{\partial z^2} \quad (4.2.10)$$

where $\mathfrak{D} = \tau(c_1 - a_1)(a_1 - c_2)$, is a dimensionless diffusivity. Inserting:

$$\varepsilon' = \alpha_0 e^{a_l t + i\omega_l(t - z/U_l)}$$

into Eq(4.2.10). reveals the local growth rate:

$$a_l = - \mathfrak{D} k_l^2 = - \tau(c_1 - a_1)(a_1 - c_2) k_l^2 \quad (4.2.11)$$

(where $k_l = \omega_l/U_l$) and then wavespeed:

$$U_l = a_1 \quad (4.2.12)$$

From Eq(4.2.8) , (4.2.9). (4.2.10) and (4.2.11), one finds:

$$E = U_h^2 (\rho - 1) + 2 V_{f0} U_h - V_{f0}^2 \quad (4.2.13)$$

and:

$$A[V_{f0}(1 - \varepsilon_0) - U_h] + C(1 - \varepsilon_0)\varepsilon_0 - \bar{\nu}_e k_l^2 \left[2 U_h + \frac{2 V_{f0}}{\rho - 1} \right] (\rho - 1) \varepsilon_0 (1 - \varepsilon_0)$$

$$= \frac{R_e}{F_r} \varepsilon_0 (1 - \varepsilon_0) (\rho - 1) + a_h \left[2 U_h + \frac{2 V_{f0}}{\rho - 1} \right] (\rho - 1) \varepsilon_0 (1 - \varepsilon_0) \quad (4.2.14)$$

and:

$$A[V_{f0} (1 - \varepsilon_0) - U_l] + C (1 - \varepsilon_0) \varepsilon_0 = \frac{R_e}{F_r} \varepsilon_0 (1 - \varepsilon_0) (\rho - 1) \quad (4.2.15)$$

So:

$$A = - \frac{R_e \varepsilon_0 (1 - \varepsilon_0) (\rho - 1)}{a_l} (U_h - U_l) \left(U_h + U_l + \frac{2 V_{f0}}{\rho - 1} \right) k_l^2 \quad (4.2.16)$$

with the four equations. (4.2.13), (4.2.14), (4.2.15) and (4.2.16), we can solve for the four unknowns:

$$A = \left(\frac{\partial F_f}{\partial (V_f - V_p)} \right)_0, \quad C = - \left(\frac{\partial F_f}{\partial \varepsilon} \right)_0, \quad E = \frac{E_p}{J_0^2 \rho_f} \quad \text{and} \quad \bar{\nu}_e = \frac{\nu_e c_1}{(c_1 - c_2)}.$$

The last two, E and $\bar{\nu}_e$ reveal the elasticity and viscosity of the particle phase. The first two, A and C , are derivatives the the fluid force with respect to the velocity difference between the phases, $V_f - V_p$, and the void fraction, ε . Both derivative are easily computed from existing drag laws. Comparison of the predictions with the measured values should yield insight into the appropriateness of the various models.

These measurements will be made in a two-dimensional liquid fluidized bed. When illuminated from behind, the voidage disturbances will be visible as variations in the intensity of light transmission through the bed. In the last year, such a two-dimensional bed was built. However, test reveal that a smaller bed should be constructed for these tests to facilitate even illumination and to make it easier to construct a uniform distributor. The one difficult term to measure will be the wave number, $k_l = \omega_l / U_l$, that appears in equations (4.2.14) and (4.2.16). The equations have been solved for k_l instead of k_h as the results of El-Kaissy and Homsy (1976) suggest that k_l is nearly constant for naturally occurring waves, while k_h is not. We will test to see if such natural waves may be used, but suspect that it will be necessary to force the bed at a specified frequency.

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