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Error Propagation in Conductivity Measurements for the Isentropic Compression Experiment

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Abstract. — I calculate the error propagation for reference-resistor-based conductivity measurements in both the two-terminal and three-terminal case. I find no significant advantage to a three-terminal measurement in expanding dynamic range.

This has been a difficult study for me. Not only because it was algebraically tedious, but because I found the results to be counter-intuitive to the extent that I had to resort to Monte Carlo simulation before I would believe them. I thought we could devise a reference resistor arrangement to measure at least two different ranges of conductivity by using probes of different exposed area. With zero error in the measurements, this is certainly true, but if we had zero error, we would need only one probe.

Two-Terminal Measurement

To optimally design a conductivity probe for the isentropic compression experiments, we need to know how errors in the measurement of dips in potential relate to errors in measurement of conductivity. For the two terminal case, shown in Fig. 1a, error propagation is rather easy to understand. The conductivity from the probe to the wall is simply given by

$$s_w = \left(\frac{1}{d} - 1 \right) s, \quad (1)$$

where s is the conductivity of the reference resistor and

$$d = \frac{v}{v_0}, \quad (2)$$

the ratio of the voltage across the reference resistor when measuring s_w to the voltage across the reference resistor when $s_w = 0$. We would expect $1 > d > 0$.

Consider an error $\pm\delta$ in measuring d , simply in the form $d \rightarrow d \pm \delta$. The measurement of s_w could be at the extremes

$$s_w^{\pm} = \left(\frac{1}{d \pm \delta} - 1 \right) s. \quad (3)$$

The fractional error is therefore

$$\epsilon = \frac{s_w^- - s_w^+}{s_w} = \frac{2\delta s(s + s_w)^2}{s_w[s^2 - \delta^2(s + s_w)^2]}. \quad (4)$$

For small values of δ the fractional error can be approximated from the differential form as

$$\epsilon \simeq \frac{2\delta(s + s_w)^2}{ss_w}. \quad (5)$$

I mention this for completeness because in the three-terminal circuit, I can only treat the small- δ case without resorting to unbearably complex algebra.

The optimum value of the reference resistor s can be found by solving

$$\frac{d\epsilon}{ds_w} = \frac{2\delta s(s + s_w)[s_ws^2 - s^3 + \delta^2(s + s_w)^3]}{s_w^2[s^2 - \delta^2(s + s_w)^2]^2} = 0. \quad (6)$$

This results in a quartic with one positive real root, too complicated to be of much use. However, expanding the root in a McLaurin series gives

$$s = s_w + 8s_w\delta^2 - 32s_w\delta^4 + 512s_w\delta^6 + \dots \quad (7)$$

As expected, the optimal reference resistor is equal the resistance between the probe and the wall in the limit $\delta \rightarrow 0$. For an error $\delta = 0.01$, the optimal reference conductivity is $s \simeq 1.0008s_w$.

Figure 2 shows the fractional error in measuring s_w as a function of s_w with $\delta = 0.01$ and $s = 1.0\Omega^{-1}$. It is a plot of Eq. (4). The minimum error is $\epsilon = 0.080032$, which occurs at $s_w = 0.999212\Omega^{-1}$ ($s = 1.00078862s_w$). At $s_w = 10\Omega^{-1}$, we find $\epsilon = 0.244964$. At $s_w = 0.1\Omega^{-1}$, we find $\epsilon = 0.242029$. In other words, the error in measuring s_w is a factor of three larger when s_w and s are mismatched by a factor of 10.

Application to the 700 Gpa Argon Compression Experiment

From the oscillograph in Fig. 3 of the preprint¹, my guesstimation is that the dip can be ascertained within about $\pm 3\%$ of the full vertical range. If anyone has a better estimate, I would like to use it. This means $\delta \simeq 0.03$. The minimum measured resistance from the probe to the wall was 0.36Ω , so $s_w \simeq 2.8\Omega^{-1}$. The reference conductivity was $s = 1.0\Omega^{-1}$. From Eq. (4), this gives a fractional error $\epsilon \simeq 0.31$, an error of about $\pm 15\%$.

Three-Terminal Measurement

Figure 1b is from Carl Ekdahl's notes and shows the three-terminal unbalanced circuit, analogous to the reference-resistor circuit of Fig. 1a. In the three-terminal measurement, we measure two dips on the oscilloscope trace:

$$d_1 = \frac{v_1}{v_{10}} \quad (8)$$

across the reference conductor s_1 and

$$d_2 = \frac{v_2}{v_{20}} \quad (9)$$

¹ M. I. Dolotenko, A. I. Bykov, N. P. Kolokol'chikov, G. V. Borisov, N. I. Egorov, V. V. Asseeva, O. M. Tatsenko, S. M. Younger, and C. Ekdahl, "Conductivity of Argon Near 700 Gpa," submitted to *Phys. Rev. Lett.*

across the reference conductor s_2 . The conductivity between one of the probes and the wall is

$$s_w = \frac{(d_1 - d_2)s_1s_2}{d_2s_1 - d_1s_2} \quad (10)$$

and the conductivity between the two probes is

$$s_p = \frac{s_1s_2[d_1d_2(s_1 - s_2) + d_1s_2 - d_2s_1]}{(d_1s_2 - d_2s_1)(d_1s_2 + d_2s_1)}. \quad (11)$$

For simplicity, I assume that the error in measurement occurs on only one of the oscilloscopes — either one, but only one. Compounding of errors in both readings is straightforward, but not necessary for the central point I want to make.

If I choose $d_1 \rightarrow d_1 + \delta$, where δ can have positive or negative values, but $|\delta|$ is small, then

$$\epsilon_w \simeq \frac{2d_2(s_1 - s_2)}{(d_1 - d_2)(d_2s_1 - d_1s_2)}\delta \quad (12)$$

and

$$\epsilon_p \simeq \frac{2(d_2^3s_1^3 + d_2^2s_1^2s_2 - d_2^3s_1^2s_2 - 2d_1d_2s_1s_2^2 + d_1^2d_2s_1s_2^2 + d_1^2s_2^3 - d_1^2d_2s_2^3)}{(d_1s_2 - d_2s_1)(d_2s_1 + d_1s_2)(d_2s_1 - d_1d_2s_1 - d_1s_2 + d_1d_2s_2)}\delta. \quad (13)$$

If I had chosen $d_2 \rightarrow d_2 + \delta$, the equations would be the same with the indices 1 and 2 interchanged. Note that a positive error in measuring the dip results in a negative fractional error in the pin-to-pin measurement. I have retained this sign convention throughout.

Some aspects of these expressions become immediately obvious upon substitution of realistic parameters. For example, if I substitute $s_w = 1\Omega^{-1}$ for the conductivity from the pins to the wall and $s_p = 0.001\Omega^{-1}$ for the conductivity between the pins, and use reference resistors $s_1 = 1\Omega^{-1}$ and $s_2 = 0.1\Omega^{-1}$ in the unbalanced circuit, I obtain $\epsilon_p = -641\delta$ and $\epsilon_w = 9.91\delta$. The error in reading the oscilloscope trace δ would have to be incredibly small to have the fractional error in the conductivity between the pins be acceptable. A question I have been asked is if the fractional error would be smaller if the reference resistors more closely matched the resistances we have been trying to measure. Certainly the fractional error was near minimum when the conductivity to the wall matched the reference conductivity in the two-terminal case. But this is not so in the three-terminal case. If I set $s_1 = s_w = 1\Omega^{-1}$ and $s_2 = s_p = 0.001\Omega^{-1}$, I obtain $\epsilon_p = -544\delta$ and $\epsilon_w = 8.14\delta$. This is some improvement, it is near the minimum values. But the value of ϵ_p remains unacceptable.

The problem of the great disparity in fractional error can be more fully understood by studying the ratio ϵ_w/ϵ_p purely as a function of the conductivities. To this end, I substitute the values of the oscilloscope dips,

$$d_1 = \frac{s_1(s_w + s_2)}{s_p(s_1 + s_2 + 2s_w) + (s_1 + s_w)(s_2 + s_w)} \quad (14)$$

and

$$d_2 = \frac{s_2(s_w + s_1)}{s_p(s_1 + s_2 + 2s_w) + (s_1 + s_w)(s_2 + s_w)}, \quad (15)$$

into Eqs. (12) and (13) to find

$$\frac{\epsilon_w}{\epsilon_p} = -\frac{s_p(s_1 + s_w)(s_1 + s_2 + 2s_w)}{s_w(s_1^2 + s_1s_p - s_2s_p + 2s_1s_w + s_w^2)}, \quad (16)$$

a relatively simple expression. Examination of Eq. (16) reveals the futility of the three-terminal-unbalanced-circuit approach to expanding the dynamic range of conductivity measurements. Looking at wall conductivities in the range of interest, specifically $s_w \sim 1\Omega^{-1}$, ϵ_w/ϵ_p is approximately proportional to s_p . If the conductance between the probes is three orders of magnitude less than the conductance from the probe to the wall, then the fractional error in measuring the conductance between the probes will be three orders of magnitude greater than the fractional error in measuring the conductance from the probe to the wall. Figure 3 shows $\log_{10}[\epsilon_w/\delta]$ and $\log_{10}[-\epsilon_p/\delta]$ for $s_1 = 1\Omega^{-1}$, $s_w = 1\Omega^{-1}$, and $s_p = 0.001\Omega^{-1}$. The curves are nearly parallel and the values of ϵ_w and ϵ_p are nearly three orders of magnitude apart.

The situation cannot be remedied by using impractically high reference conductors. Figure 4 shows the fractional errors for $s_1 = 10\Omega^{-1}$, $s_w = 1\Omega^{-1}$, and $s_p = 0.001\Omega^{-1}$, with s_2 ranging from 0 to $10\Omega^{-1}$.

(a)

(b)

Fig. 1

Fig. 2

Fig 3

Fig. 4