PREDICTING MULTIDIMENSIONAL ANNULAR FLOW WITH A
WITH A LOCALLY BASED TWO-FLUID MODEL

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Predicting Multidimensional Annular Flows with a Locally Based Two-Fluid Model

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**Goal of Work**

- Develop a methodology to predict annular flows using a multidimensional four-field, two-fluid Computational Fluid Dynamics (CFD) computer code.

- Develop closure models which use the CFD predicted local velocities, phasic volume fractions, etc...

- Implement a numerical method which allows the discretized equations to have the same characteristics as the differential form.

- Compare predicted results to local flow field data taken in a R-134a working fluid test section.
Conservation of Phasic Mass

\[ \frac{\partial}{\partial t} (\alpha_i \rho_i) + \nabla \cdot (\alpha_i \rho_i \mathbf{u}_i) = \sum_{j \neq i} \left( \Gamma_{ji} - \Gamma_{ij} \right) \]

Conservation of Phasic Momentum

\[ \frac{\partial}{\partial t} (\alpha_i \rho_i \mathbf{u}_i) + \nabla \cdot (\alpha_i \rho_i \mathbf{u}_i \mathbf{u}_i) = -\alpha_i \nabla p + \nabla \cdot \mathbf{\tau}_i + M_i + \sum_{j \neq i} D_{ij} (\mathbf{u}_j - \mathbf{u}_i) + \Gamma_{ji} \mathbf{u}_{int} - \Gamma_{ij} \mathbf{u}_{int} \]

- Field Indicator, \( i = \text{cl} \) (continuous liquid), \( i = \text{dl} \) (dispersed liquid)
  \( i = \text{cv} \) (continuous vapor), \( i = \text{dv} \) (dispersed vapor)

- Non-drag Interfacial Momentum Forces

- Mass Transfer from phase “i” to phase “j”

- Interfacial Drag Coefficient between phase “i” and phase “j”

- Interfacial Velocity

- Phasic volume fraction, density, velocity and shear stress
Three Field Model of Annular Flows

Vapor Core Flow (cv field)

\[ \nabla \cdot (\alpha_{cv} \rho_{cv} u_{cv}) = 0 \]

\[ \nabla \cdot (\alpha_{cv} \rho_{cv} u_{cv} u_{cv}) = -\alpha_{cv} \nabla p + \nabla \cdot \tau_{cv} \]

\[ + M_{cv} + D_{cv} - dl (u_{dl} u_{cv}) + D_{cv} - cl (u_{cl} u_{cv}) \]

Liquid Film Flow (cl field)

\[ \nabla \cdot (\alpha_{cl} \rho_{cl} u_{cl} u_{cl}) = \Gamma_{cl} - cl - \Gamma_{cl} - dl \]

\[ \nabla \cdot (\alpha_{cl} \rho_{cl} u_{cl} u_{cl}) = -\alpha_{cl} \nabla p + \nabla \cdot \tau_{cl} \]

\[ + M_{cl} + D_{cl} - cv (u_{cv} u_{cl}) + \Gamma_{dl} - cl u_{dl} + \Gamma_{cl} - dl u_{cl} \]

Droplet Flow (dl field)

\[ \nabla \cdot (\alpha_{dl} \rho_{dl} u_{dl} u_{dl}) = \Gamma_{cl} - dl - \Gamma_{dl} - cl \]

\[ \nabla \cdot (\alpha_{dl} \rho_{dl} u_{dl} u_{dl}) = -\alpha_{dl} \nabla p + \nabla \cdot \tau_{dl} \]

\[ + M_{dl} + D_{dl} - cv (u_{cv} u_{dl}) + \Gamma_{cl} - dl u_{cl} + \Gamma_{dl} - cl u_{dl} \]
Two-Fluid Models Applied on Local Basis

- Entrainment and Deposition of Droplets

- Interfacial Area Density

- Interfacial Shear (cv-cl)

- Dispersive Annular Flow Force

- Collective Annular Flow Force
Entrainment and Deposition of Droplets

- Apply Model from Kataoka and Ishii (1983) on a Local Basis

\[ \Gamma_{cl-dl} = G_E^{\prime\prime\prime} \]
\[ \Gamma_{dl-cl} = G_D^{\prime\prime\prime} \]
\[ \frac{G_E D_h}{\mu_{cl}} = 6.6 \cdot 10^{-7} Re_{cl}^{0.74} Re_l^{0.185} We^{0.925} \left( \frac{\mu_{cv}}{\mu_{cl}} \right)^{0.26} \]
\[ \frac{G_D D_h}{\mu_{cl}} = 0.022 Re_{dl}^{0.74} \left( \frac{\mu_{cv}}{\mu_{cl}} \right)^{0.26} \]
\[ Re_{dl} = \frac{\rho_{dl} \alpha_{dl} u_{dl} D_h}{\mu_{dl}} \]
\[ Re_{cl} = \frac{\rho_{cl} \alpha_{cl} u_{cl} D_h}{\mu_{cl}} \]
Interfacial Area Density

- Assume Thin Smooth Films

\[ A_{cl-cl} = \| \nabla \alpha_{cl} \cdot \hat{n}_{int} \| \]

Interfacial Shear Force

- Apply Model from Wallis (1969) on a Local Basis

\[ \tau_{cv-cl}^{int} = \frac{1}{2} f_i \rho_{cv} |u_{cv} - u_{cl}| (u_{cv} - u_{cl}) \]

\[ f_i = 0.005 \left( 1 + 300 \frac{y_{wall}}{D_h} \right) \]
Dispersive Annular Flow Force

- Thin Film Dynamics Controlled by Collective and Dispersive Interfacial Forces

- Apply Simplified Kelvin-Helmholtz Stability Analysis to Thin Film to show form for Dispersive Force

Unsteady Flow Equations \( u = \bar{u} + u' \)

for Vapor (i=CV) and Film (i=CL)

\[
\begin{align*}
\frac{\partial u_i'}{\partial x} + \frac{\partial v_i'}{\partial y} &= 0 \\
\frac{\partial u_i'}{\partial t} + u_i \frac{\partial u_i'}{\partial x} + v_i \frac{\partial u_i'}{\partial x} &= \frac{1}{\rho_i} \frac{\partial p_i'}{\partial x} \\
\frac{\partial v_i'}{\partial t} + u_i \frac{\partial v_i'}{\partial x} &= \frac{1}{\rho_i} \frac{\partial p_i'}{\partial y} \\
p'_{CV} + \sigma \frac{\partial^2 \xi}{\partial y^2} &= p'_{CL}
\end{align*}
\]
Dispersive Annular Flow Force

- Solve in terms of Potential Functions and Wave Function

\[ u' = \frac{\partial \phi_i'}{\partial x} \]
\[ v' = \frac{\partial \phi_i'}{\partial y} \]
\[ \xi(x, t) = \xi e^{ikx + st} \]

- Solution takes the form

\[ \phi_i' = \left( A_i e^{-ky} + B_i e^{ky} \right) e^{ikx + st} \]

- Growth Factor, \( s = is_i + s_r \), Evaluated by Substitution (Normalized Results)

\[ \tilde{S} = -ik \frac{\tilde{\rho} \tilde{u} c_v \coth(\tilde{k}(1 - \tilde{h})) + \tilde{u}_{cl} \coth(\tilde{k}\tilde{h})}{\tilde{\rho} \coth(\tilde{k}(1 - \tilde{h})) + \coth(\tilde{k}\tilde{h})} \]

\[ \pm \sqrt{\frac{\tilde{\rho} \tilde{k}^2 \coth(\tilde{k}(1 - \tilde{h})) \coth(\tilde{k}\tilde{h})}{(\tilde{\rho} \coth(\tilde{k}(1 - \tilde{h})) + \coth(\tilde{k}\tilde{h}))^2} - \frac{\tilde{\rho}^2 k^3}{W_e}} \]
**Dispersive Annular Flow Force**

- Film becomes unstable when Radicand is greater than zero
  Stability Condition Becomes,

\[
\tilde{k} > We \frac{\coth(\tilde{k}(1 - \tilde{h})) \coth(\tilde{k}\tilde{h})}{\tilde{\rho} \coth(\tilde{k}(1 - \tilde{h})) + \coth(\tilde{k}\tilde{h})}
\]

- Short Wave Length Limit \( \tilde{k}\tilde{h} \ll 1 \)

\[
\tilde{k} > We \\
\lambda < \frac{2\pi t}{We}
\]

Short Wave length can be Stable

- Long Wave Length Limit \( \tilde{k}\tilde{h} \gg 1 \)

\[
\tilde{k} > \sqrt{We} \\
\lambda < \frac{2\pi t}{\sqrt{We}}
\]

Long Wave Length can be Unstable (Bernoulli Effects)

- Model Long Wave Length Dispersive Effects
  Proportional to a Bernoulli Pressure Force

\[
M_{cl-cv}^{Disp} = C_{Disp} \rho_{cv} |u_{cv} - u_{cl}|^2 A_{cl-cv} \hat{n}_{int}
\]
Collective Annular Flow Force

- Apply Wall Force Model to Vapor Core to show form for Collective Force

- The Wall Force in Dispersed Flows has the Form

\[ M_{cl - dv}^{Wall \ Force} = \frac{C_W^{\alpha_{dv}} \rho_{cl} |u_{cv} - u_{cl}|^2}{R_{bubble}} \hat{n}_{wall} \]

- The Collective Force uses Same Form Scaled by Duct Thickness

\[ M_{cl - cv}^{Collective} = \frac{C_{coll}^{\alpha_{cl} \alpha_{cv}} \rho_{cv} |u_{cv} - u_{cl}|^2}{t} \hat{n}_{wall} \]
Model Implementation in CFD Code

- Finite Volume Discretization Scheme

- Segregated, Pressure Based Algorithm

- Non-Staggered Grid

- Subcooled and Saturated Boiling Flows

- Bubbly, Transitional, and Annular Flows
Numerical Implementation

- Improved Pressure-Velocity Coupling for Volume Fraction Terms

\[ U_E = \overline{U_E} + B_E(\overline{\nabla p_E} - \nabla p_E) + A_E(\overline{\nabla \alpha_E} - \nabla \alpha_E) \]

Numerical Implementation

• Fully Developed without Velocity-Volume Fraction Coupling

- Without Velocity-Volume Fraction Coupling

• Fully Developed with Velocity-Volume Fraction Coupling

- With Velocity-Volume Fraction Coupling
Numerical Implementation

- Discretization of Volume Fraction Gradient Term

\[ \frac{\partial \alpha}{\partial y} \bigg|_p = \frac{\alpha_E - \alpha_W}{\Delta y} \]

\[ \alpha_E = \frac{F_e}{F_e + F_p} \alpha_e + \frac{F_p}{F_e + F_p} \alpha_p \quad \alpha_W = \frac{F_w}{F_w + F_p} \alpha_w + \frac{F_p}{F_w + F_p} \alpha_p \]

\[ F_w = \frac{1}{\alpha_w (1 - \alpha_w)} \quad F_e = \frac{1}{\alpha_e (1 - \alpha_e)} \quad F_p = \frac{1}{\alpha_p (1 - \alpha_p)} \]

- Limiting Conditions for Volume Fraction Gradient Term

| $\alpha_w$ | $\alpha_p$ | $\alpha_e$ | $\frac{\partial \alpha}{\partial y} |_p$ |
|------------|------------|------------|-----------------|
| 1          | 1          | $1 > \alpha > 0$ | 0               |
| $1 > \alpha > 0$ | 1          | 1          | 0               |
| 1          | $1 > \alpha > 0$ | 0          | $1/\Delta y$    |
| $1 > \alpha > 0$ | $1 > \alpha > 0$ | $1 > \alpha > 0$ | $\frac{\partial \alpha}{\partial y} < 1/\Delta y$ |
Numerical Implementation

- Require Model to Predict Fully Developed Annular Flows

  - Under Fully Developed Conditions the Collective and Dispersive Forces Must Balance

  
  \[ M_{cl-cv}^{Disp} + M_{cl-cv}^{Coll} = 0 \]

- Substitute Collective and Dispersive Models

  
  \[
  \frac{C_{coll}^\alpha_{cl}^\alpha_{cv} p_{cv} u_{cv}^2 - u_{cl}^2}{t} - C_{Disp} p_{cv} u_{cv}^2 u_{cl}^2 \frac{\partial \alpha}{\partial y} = 0
  \]

  Simplify,

  
  \[
  \frac{C_{coll}^\alpha_{cl}^\alpha_{cv}}{t} - C_{Disp} \frac{\partial \alpha}{\partial y} = 0
  \]

  May Not Sum to Zero For Fully Developed Flows

- Numerical Testing Confirmed Force Imbalance
**Numerical Implementation**

- Require Model to Predict Fully Developed Annular Flows

  - Strategy: Develop Sensor(s) for Fully Separated Flows

**Condition 1:**

Fully Separated Conditions Imply: \( \frac{\partial \alpha}{\partial y} = \frac{1}{\Delta y} \)

\[
\frac{M_{Disp}}{1 - \frac{M_{cl-cv}}{\frac{M_{Disp}}{M_{cl-cv}}}} < 1
\]

**Condition 2:**

Collective Force Must Be Larger than Dispersive Force Evaluate with: \( \frac{\partial \alpha}{\partial y} = \frac{1}{\Delta y} \)

\[
\frac{M_{Disp}}{M_{cl-cv}} \frac{\partial \alpha}{\partial y} = \frac{1}{\Delta y}
\]

\[
\frac{M_{Disp}}{M_{cl-cv}} < 1
\]
Numerical Implementation

- Require Both Conditions to be Met

\[
\zeta = \text{MIN} \left[ 1, \text{MAX} \left( \frac{M_{cl}^{\text{disp}} \left| \frac{\partial \alpha}{\partial x_i} = 1/\Delta y \right|}{M_{cl}^{\text{coll}}}, 1 - \frac{M_{cl}^{\text{disp}} \left| \frac{\partial \alpha}{\partial x_i} = 1/\Delta y \right|}{M_{cl}^{\text{coll}}} \right) \right]
\]

\[
M_{cl - cv}^{\text{Collective}} = \frac{C_{coll} \zeta \alpha_{cl} \alpha_{cv} \rho_{cv} |u_{cv} - u_{cl}|^2}{t \hat{n}_{wall}}
\]
Results

• Compare Against Local Volume Fraction Measurements in Lockheed Martin Corporation’s R-134a Test Facility

- Adiabatic Flow Conditions
- SUVA\(^1\) working fluid
- Duct
- GDS Measurements ~170 Hydraulic Diameters Downstream of Inlet
- Homogenous Mixture Entering Test Section

\[\text{Gamma Densitometer Measurements (GDS)}\]

1. SUVA\(^\circ\) is a trademark of Dupont Chemicals for R-134a
Results

- Compare over Range of Mass Flows, Pressures and Inlet Qualities

Table 1: Test Conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>Inlet Mass Flux (Kg/m²-s)</th>
<th>Inlet Pressure kPa</th>
<th>Inlet Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>512.</td>
<td>2380.</td>
<td>0.5998</td>
</tr>
<tr>
<td>2</td>
<td>1032.</td>
<td>1361.</td>
<td>0.1639</td>
</tr>
<tr>
<td>3</td>
<td>1994.</td>
<td>1361.</td>
<td>0.4360</td>
</tr>
<tr>
<td>4</td>
<td>2040.</td>
<td>2380.</td>
<td>0.4905</td>
</tr>
</tbody>
</table>

- Predicted Phasic Volume Fraction for Case 2
Results

- Predicted Velocities and Volume Fraction
  (x/L = 170 hydraulic diameters)
Results

- Predicted Film Thickness and Average Volume Fraction

![Graph showing film thickness and axial distance for cases 1 to 4.]

- Collective and Dispersive Lateral Force for Case 2

![Graph showing lateral force for collective and dispersive forces.]

0.6 - 0.045

Case 2 - Case 1

E 0.035 = am. B 0.025

Case 1 - Case 4

0.0 a2 a4 0.6

Axial Distance (x/L)

Predicted Average Void Fraction

Dispersive Foree

Collective Force

Lateral Force (N/m^3)

Collective and Dispersive Lateral Force for Case 2

Flow Direction
Results

- Predicted Void Distribution and Film Thickness for Inclined Flow (Inclined $45^\circ$ from Vertical).
  (Inlet Flow Conditions are same as Case 2)

![Graph showing predicted void distribution and film thickness](image)

- Uniform, Homogenous Inlet Flow

Liquid Volume Fraction Phase Distribution for $45^\circ$ Inclined Annular Flow
Results

- Predicted Void Distribution and Film Thickness for Horizontal Flow. (Inlet Flow Conditions are same as Case 2)

Liquid Volume Fraction Phase Distribution for Horizontal Annular Flow
Conclusions

- With Proper Modeling and Numerical Treatment, Annular Flows can be Predicted with a Multidimensional, Two-Fluid Model

- Annular Flow Models Developed From Simplified Analysis of Identified Force Mechanisms

- Numerical Treatment of Annular Flow Models
  - Void Gradient Treatment
  - Improved Pressure-Velocity Treatment
  - Requirements for Fully Developed Annular Flows

- Comparisons against Local Volume Fraction Data
  - Good Predictions for Local Volume Fraction
  - Good Predictions for Average Volume Fraction

- Model Predictions for Inclined Orientations
  - 45° Inclined Annular Flows
  - Horizontal Orientation Annular Flows
Additional supporting information on what will be discussed

Topic: Dispersive Annular Flow Force

The approved paper states, “Simple inviscid (Kelvin-Helmoltz) analysis of vertical separated flows indicate that while surface tension is able to stabilize small wavelength disturbances, Bernoulli pressure effects tend to make the long wavelength disturbances unstable by increasing the pressure along the wave trough and decreasing the pressure along the wave crest.” The presentation will include three overheads summarizing the supporting analysis. A summary of the planned discussion is:

- Apply Simplified Kelvin-Helmholtz Stability Analysis to Thin Film to Show Form for Dispersive Force
  - Discuss wavy thin film flowing on a wall.
  - Amplitude of thin film may grow (unstable) or decay (stable) for various wave lengths.
- Unsteady Flow Equations for Vapor (i=CV) and Film (i=Cl)
  - Discuss writing single phase Navier-Stokes equation on each side of the thin film.
  - Expand equations by dividing the velocity and pressure into steady and unsteady terms.
  - Crank through math and arrive at unsteady flow equations (as done in many text books).
- Solve in terms of Potential Functions and Wave Function
  - Discuss use of potential function and wave function to solve for film amplitude growth factor.
- Film becomes unstable when Radicand is greater than zero
  - If growth factor has positive real value then amplitude will grow with time.
  - If growth factor has negative real value then amplitude will decay (stable interface).
- Short Wave and Long Wave Length Limit
  - Discuss looking at limiting conditions, short and long wave lengths. Analysis shows that short wave length will be stable but long wave lengths can be unstable (i.e., can grow with time)
- Model Long Wave Length Dispersion Effects Proportional to a Bernoulli Pressure Force
  - Discuss long wave length growth are related to pressure difference on each side of the film interface. This effect can be modeled as a Bernoulli pressure force times the interfacial area density.

Topic: Collective Annular Flow Force

The approved paper states, “This effect, which is generalized to take the form of an aerodynamic lift force applied to the interface can be written as.” The presentation will include some supporting information. A summary of the planned discussion is:

- Apply Wall Force Model to Vapor Core to show form for Collective Force
  - Discuss wall having significant influence on location of vapor between parallel walls
  - Simplify process to vapor slug traveling between parallel walls
- The Wall Force in Dispersed Flows has the Form
  - Discuss modeling vapor slug as large bubble having the influence of each wall
  - Use wall force model (available in open literature) to model effects of walls on the vapor field
The Collective Force uses Same Form Scaled by Duct Thickness
- Discuss use of wall force model to prescribe form of collective force model

Topic: Model Implementation in CFD Code:

The paper states, “The multifield, two-fluid models have been implemented into a segregated, pressure based algorithm to solve the system of equations and to predict the phase distribution and velocity profiles (see Siebert et al., 1994).” To better describe the code the following will be summarized.
- Finite Volume Discretization Scheme
- Segregated, Pressure Based Algorithm
- Non-Staggered Grid
- Subcooled and Saturated Boiling Flows
- Bubbly, Transitional, and Annular Flows
  - These are general attributes of the code and will better convey the results to the audience.
  - The code uses a standard finite volume discretization scheme on a non-staggered grid.
  - The code has capability to predict flows from single phase heating, through high volume fraction annular flows. The code has capability to model subcooled and saturated boiling flows and can model bubbly flows coalescing into annular flows.

Topic: Improved Pressure-Velocity Coupling for Volume Fraction Terms

The paper states, “The requirement to conform to the realizable limits (0<volume fraction<1) and their restrictions on implementation into a discretized computational model is a process by which any model(s) must endure. However, most other models do not produce sharp void fraction gradient; therefore, a more general numerical treatment is adequate.” Included in the discretized computational model is the need to improve the pressure-velocity coupling. This is an important ingredient to adequately predicting annular flows. The discussion will summarize the need to include numerical treatment to improve annular flow predictions. A example of fully developed adiabatic annular flows with and without the treatment is included in the presentation. In addition, an approved paper (A Coupled Phase Exchange Algorithm for Three-Dimensional Multi-Field Analysis of Heated Flows with Mass Transfer, RF Kunz et al.) has been submitted to the Journal of Computer and Fluids for publication which will also present and discuss improved pressure-velocity coupling. A reference to this submitted paper will be included in the presentation.

Topic: Results

The results were limited in the paper by the conference restriction of a maximum length of eight pages. The presentation will include results presented in the paper with additional supporting information. All results are taken from the analysis of the four cases included in the approved paper. The planned presentation of the results will include:
- Predicted Phasic Volume Fraction for Case 2 - color contours (included in paper)
- Predicted Velocities and Volume Fraction - line plots for each of 4 cases (included in paper)
- Predicted Film Thickness and Average Volume Fraction - line plots for each of 4 cases (included in paper)
- Collective and Dispersive Lateral Force for Case 2 - color contour plot (included in paper)
• Predicted Void Distribution and Film Thickness for inclined Flow (Inclined 45° from Vertical). - line plots of film thickness (included in paper) and color contour plot of liquid fraction distribution.
• Predicted Void Distribution and Film Thickness for Horizontal Flow. - line plot of film thickness and color contour plot of liquid film fraction