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NAVAL REACTORS PHYSICS HANDBOOK,

VOLUME III

The
Physics of
Intermediate
Spectrum Reactors

J. R. Stehn, Volume Editor

KNOLLS ATOMIC POWER LABORATORY

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NAVAL REACTORS DIVISION OF REACTOR DEVELOPMENT

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FOREWORD

The Naval Reactors Physics Handbook, which is the seventh in a series of handbooks on the basic reactor technology developed under the sponsorship of the Naval Reactors Branch of the Atomic Energy Commission, presents the analytical and experimental physics techniques which have been developed for the reactors designed in the Naval and Shippingport (PWR) Reactor Programs. Reactor physics has been strongly emphasized in these programs, and extensive experimental and computational facilities have been developed. Further, particular attention has been paid to obtaining from reactor operation as much information as possible. Prototype reactors have been carefully instrumented and elaborate series of physics tests have been carried out with the two-fold objectives of confirming and refining the physics calculations used in each particular reactor design and of obtaining increased general understanding of reactors.

Power reactors must meet many stringent requirements such as size limitations, stability, and reliability. Above all, changes cannot be made in a finished core to compensate for design error. The fundamental importance of physics in establishing fuel loading, start-up, control, power distribution, and endurance is widely recognized. What is perhaps less well-realized is the constant interplay of mechanical, metallurgical, and plant considerations with physics to provide a well-balanced design. The requirements on physics are therefore especially severe in a power reactor. In order to provide precise and dependable physics information it has been necessary to develop many detailed methods of analysis. Many of these analytical techniques are applicable to other reactors of widely differing characteristics.

The physics methods reported in these volumes are characterized by their having been found useful in the design of actual power reactors. In addition, basic reactor theory developed in the laboratories participating in the Naval Reactors Program is also covered. This includes many areas of reactor physics which have long presented particular difficulty - for example, thermal neutron spectra, control rod effectiveness, self-shielding and transport effects, temperature coefficients of reactivity, and flux stability.

While I am pleased with the advances in physics which have been made in the course of the Naval Reactors and Shippingport programs, I know that there is a great deal more that must be done to improve our understanding of reactors and our ability to design them for longer life, more uniform power distribution, simpler control, and more effective utilization of the fuel. I hope that by publishing our available data and techniques in these volumes we will provide impetus toward the solution of many of the basic reactor physics problems still outstanding.

H. G. RICKOVER
Chief, Naval Reactors Branch
Reactor Development Division
U. S. Atomic Energy Commission

EDITORIAL BOARD PREFACE

The purpose of this handbook is to present the most pertinent parts of the body of physics knowledge which has been built up in the course of the Naval and Shippingport (PWR) Reactor Programs, with the aim of providing a background of understanding for those interested in nuclear core design. Much of the material has already been published either in scientific journals or topical laboratory reports; scientists engaged in the Naval Reactors Program, as elsewhere, are encouraged to report their work in the normal manner. However, as is usual in rapidly expanding programs, much data have remained in the form of internal memoranda and other informal communications. It was felt to be desirable and timely to review and recapitulate the work which has been done on the Program and to make it available in convenient and readily applicable form. Wherever practicable the chapters in these volumes have been prepared by authors who actively participated in the developments discussed.

Volume I of the handbook attempts to bring together the basic theoretical and experimental material which is of especially wide interest or is common to both thermal and intermediate-neutron-energy reactor types. The physics design of light-water-moderated and -cooled reactors is covered in Volume II, and that of intermediate-neutron-energy power reactors in Volume III.

One of the rewards of editing a work of this kind has been the discovery of many illuminating relationships between the development programs of different types of reactors. For example, in an intermediate reactor in which the physics characteristics in the core do not change greatly with temperature, it was natural in the critical assembly work to rely heavily upon a quite accurate mock-up of the core at room temperature. In light-water reactors, the philosophy was somewhat different. Since the physics properties of the core change greatly with temperature, an accurate mock-up at room temperature was not so important; the ability of calculational procedures to predict the results of the critical experiments at room temperature was reasonable assurance that the calculational procedures would also be valid under power operating conditions. This has indeed proved to be the case. Even so, as performance requirements have become more severe, it has been necessary to go to more and more faithful mock-ups of water-cooled reactors. Along with these developments, analytical techniques, aided by large digital computers, have been making great advances.

Reactor physics today is such a rapidly growing field that the book is necessarily uneven since it has been prepared over the course of several years by editors operating on a part-time basis. All members of the Editorial Board have participated fully in this work by reading all chapters and providing detailed comments. Criticism and suggestions by the reader will be welcomed.

The writer wishes to express his appreciation to Rear Admiral H. G. Rickover for suggesting the preparation of this handbook and for giving much encouragement at all times. Special thanks are also due to T. Pockwell, III, Technical Director of the Naval Reactors Branch, for many helpful suggestions.

A. RADKOWSKY
Chairman, Editorial Board
Naval Reactors Physics Handbook

EDITOR'S PREFACE

The present volume has been prepared for persons with some knowledge of the physics of nuclear reactors. It is intended to make available the accumulated physics experience of the Knolls Atomic Power Laboratory in its work on intermediate spectrum reactors. Only those portions have been selected which were deemed to be most useful and significant to other physicists concerned with the problems of reactor design.

The volume is divided into four parts which are more or less independent of one another. Part I (Chaps. 2-9), "Investigation of Reactor Characteristics by Critical Assemblies," reflects the importance of the properties of critical assemblies and of the techniques for obtaining experimental information about such assemblies. Part II (Chaps. 10-20), "Reactivity Effects Associated with Reactor Operation," details the use of both critical assemblies and reactor theory to make and test predictions of the manner in which the reactivity of an intermediate power reactor will vary during operation. Part III (Chaps. 21-26), "Heat Generation and Nuclear Materials Problems," considers how reactor heat generation is spread out in space and time, and what nuclear effects result from the presence of beryllium or sodium in the reactor. Part IV (Chaps. 27-38), "Reactor Kinetics and Temperature Coefficients," relates to the transient or near-transient behavior of intermediate reactors.

The persons who were in large part responsible for correlating the separate chapters in each part (R. T. Frost, P. L. Hofmann, J. S. King, H. B. Stewart, and M. L. Storm) had this responsibility by virtue of their particular experience with the subject of each part. I am indebted to them for much of the additional insight that the editorial modifications may provide. P. F. Zweifel also contributed as Associate Editor of the volume.

Further appreciation is due to Rear Admiral Rickover and Dr. Radkowsky of the Naval Reactors Branch of the U. S. Atomic Energy Commission for providing the stimulus for this handbook and for encouraging the work on this volume. The editorial advice of the A. E. C.'s Technical Information Service, Oak Ridge Extension, has been valuable. I particularly appreciate the performance of Miss M. J. Sears at KAPL as Technical Editor, translating the authors' demands into material practicable for the printed page.

Although the concepts and terminology of intermediate reactor physics may at times seem strange to the reader who has dealt exclusively with thermal reactor physics, they are necessary and natural outcomes of the differences between the two subjects. Being aware of the difference, we have attempted to indicate the development of the concepts and to explain the terminology as it is used.

J. R. STEHN
Theoretical Physicist
Knolls Atomic Power Laboratory

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Chapter 1

THE PHYSICS OF INTERMEDIATE SPECTRUM REACTORS

J. R. STEHN

1.1 INTERMEDIATE REACTORS IN GENERAL

An intermediate reactor is a nuclear reactor in which a majority of the fissions are produced by neutrons of energies intermediate between thermal and fast. A great range of intermediate reactors can be imagined, differing in neutron-energy spectrum distribution. One parameter useful in describing such a reactor is the median energy of the neutrons producing fissions, and values of this parameter between 1 and 10^5 electron volts are characteristic.

The neutron energy spectrum has a wide spread in intermediate reactors. The spectrum average of the neutron cross-section of each reactor constituent is therefore not strongly affected by the presence of one or a few low-energy resonances. Materials that are well-known neutron poisons in thermal reactors (cadmium, samarium, fission-product Xe^{135}) are not so markedly different from other materials in their tendency to absorb neutrons in an intermediate reactor. One consequence of this situation is that in intermediate reactors it is relatively easy to override xenon.

For a reactor to have an intermediate spectrum, it must have some moderation, but not too much. A water-cooled reactor can be intermediate only if water is present in proportions too small to provide adequate cooling for a power reactor. Other hydrogenous materials can be used (sparingly) for moderation if a coolant other than water is used; this is shown in the assemblies using metallic hydrides described in Chap. 2.

A. Beryllium as Moderator

Beryllium, beryllium oxide, and graphite were considered as moderators for the small-size intermediate reactors that were built at the Knolls Atomic Power Laboratory (KAPL). Beryllium was chosen from among these three materials because it had promise, as a metal, of greater strength than either of the others. Even though it was perhaps not to be relied upon as a structural material, beryllium could be shaped to fit snugly where it was needed, and it was expected to hold its shape better than graphite under irradiation.

In addition to its being a moderator, beryllium has certain less important nuclear properties which would be expected to influence the operation of reactors containing large amounts of this material. Beryllium is more liable than most elements to undergo three nuclear reactions that either produce or absorb neutrons: the $(n, 2n)$, the (n, α) , and the (γ, n) reactions.

In theoretical criticality calculations at KAPL, the $(n, 2n)$ and (n, α) effects have consistently been assumed to cancel each other. These two effects have accordingly been neglected when reactivities and power distributions were computed. Although the success of the results appears to justify neglecting these two effects, uncertainties in the other neutron cross-sections used in the calculations may have compensated for any error caused by neglecting them. Recent work by G. J. Fischer¹ suggests that the $Be(n, 2n)$ cross-section, at any rate, is quite appreciable for fast neutrons.

1.2 PHYSICAL STUDIES ON INTERMEDIATE REACTORS

Neutron-absorbing Li^6 is formed in the moderator as the decay product of the $\text{Be}^9(n,\alpha)\text{He}^6$ reaction. The resulting loss in reactivity is judged (according to Chap. 26) to be relatively unimportant. Helium gas (both He^4 and He^3) is generated in beryllium by $(n,2n)$, (n,α) , (γ,n) , and ensuing reactions; but the amounts of gas so produced are negligible for the reactors so far considered.

The threshold for the $\text{Be}^9(\gamma,n)\text{Be}^8$ reaction is only 1.67 Mev, lower than for any other element. Beryllium therefore acts as a neutron source in the presence of photons harder than 1.67 Mev. The photoneutrons that develop after high-power operation of a beryllium-moderated reactor contribute significantly to the effective neutron source strength for days and months, as long as Na^{24} and fission products are emitting hard gamma rays. Some observations on this appear in Chap. 25.

B. Sodium as Coolant

The choice of molten metal as the heat-transfer medium makes the intermediate power reactors discussed in this volume differ strikingly from water-cooled reactors, not only in nuclear characteristics but in engineering design as well. The choice of this type of coolant was a decision to take full advantage of the superior heat-transfer characteristics of fairly readily available materials (such as sodium, potassium, lead, or bismuth) that have slight affinity for neutrons, are monatomic and hence not likely to suffer radiation damage, and are liquids well below the boiling point at steam-turbine operating temperatures. The high thermal conductivities of liquid metals make it easy to avoid local overheating that may lead to boiling and resultant density fluctuations in the coolant. Their low vapor pressures at temperatures of interest for steam-turbine use enable the coolant system to be kept essentially at atmospheric pressure in a thin-walled piping system. A further advantage that accrues to the liquid-metal heat-transfer system is the possibility of using an electromagnetic pump, without bearings or moving parts, to circulate the coolant.

The choice of sodium as the particular metal to use was made primarily on the basis of its physical properties. It is relatively cheap, it conducts heat well, and its low density makes it easy to pump. Since it is an active alkali, it must be handled with care, and an elaborate technology had to be developed for keeping it out of contact with water and oxygen. Despite its chemical activity, it is not particularly corrosive to steels. Its worst drawback is the Na^{24} radioactivity that is generated in it by neutron capture; the resulting gamma radiation throughout the primary coolant system enlarges the portion of the reactor complex that must be shielded and prevents access to the coolant system both during operation and – unless the sodium is drained out – for as long as a week after shutdown.

Potassium has many of the same advantages and difficulties as sodium, but its neutron absorption cross-section is appreciably larger. Neutron absorption in a coolant is undesirable because of the consequent adverse positive effect on the temperature coefficient of reactivity (expansion of the coolant reduces the amount of neutron-absorbing material in the core). As a coolant, potassium therefore might not contribute negatively, as sodium in fact does (Chap. 28), to the temperature coefficient of the reactor. Both sodium and potassium are solids at room temperature. Although some mixtures of sodium and potassium have the advantage that they are liquid at room temperature, this advantage does not offset the possible temperature coefficient disadvantage accompanying the use of potassium. Use of the heavier molten metals was given little consideration, for their thermal conductivities are lower, their densities are higher, and little was known about their chemical properties at high temperatures.

1.2 PHYSICAL STUDIES ON INTERMEDIATE REACTORS

Much of the development effort on the reactors discussed in this volume was set by the decision to use an intermediate-energy neutron spectrum. Little was known about neutron cross-sections in such a spectrum, and no detailed theoretical methods were readily available which could connect the little that was known with the measurable properties of the reactor. Critical assemblies of the same nuclear composition as the new reactor designs being proposed had to be built and tested for their nuclear properties. Theoretical methods had to be devised for correlating the properties of one assembly with those of another, and the reliability of these correlations had to be checked

by experiment, before the effects of minor design changes could be accurately assessed. Special calculations and experiments had to be performed to check the hazards of various possible mishaps during reactor operation.

In general, much of the same kind of exploring had to be done for the intermediate reactors as had been done for the first nuclear reactors six years before. The intermediate enriched-fuel reactors considered here were much smaller than natural-uranium reactors, and the experimental and theoretical approaches that proved profitable here were quite different. Because these reactors had intermediate spectra, in addition, the exploratory approaches also differed considerably from those used with water-moderated enriched-fuel reactors. We now consider the uses made of the two principal tools which were developed for the study of these reactors: the critical assembly and multigroup age-diffusion theory.

A. Experimental Reactor Studies

A wide variety of nuclear designs was initially seen to be within the range of possibility. The critical assembly facility built to cover this range had to have considerable flexibility as to composition and geometry. Chapter 3 describes the flexibility in mechanical designs of the two facilities in which the assemblies were erected: the Preliminary Pile Assembly (PPA) and the Proof Test Reactor (PTR). The convenience and safety of the designs is indicated by the fact that in one year-and-a-half period, during which seven substantially different PPA's were built and tested, criticality was safely attained more than 3000 times, an average of over seven times per working day.

The PPA program, described in Chap. 2, had several functions. First, the critical mass and the effectiveness of the control scheme were established for each nuclear design considered. Then the power distribution was studied (Chap. 21) to learn how much variation in power density could be expected in the final reactor. To aid in forecasting the effect of minor design changes, the effects upon reactivity of inserting various materials (their reactivity coefficients) were determined, as in Chaps. 7, 8, and 9. For the most promising designs, more detailed information about the endurance of the reactor (the amount of energy in megawatt hours or equivalent units it could deliver, before refueling) was obtained by reducing the fuel load to represent various stages of depletion and by adding poison material to simulate the fission products (Chap. 19). A study of the possibility of using a burnable poison was made in a similar manner (Chap. 17). Measurements were made of the density of gamma radiation in the reflector and blanket (Chap. 23) as a check on rather detailed calculations of the amounts of gamma-ray heating throughout the reactor.

There was strong interplay between theory and experiment in the critical assembly program. The theory of reactivity coefficients used in Chap. 7 gave an understanding of the measured quantities and also guided an estimate of the isothermal temperature coefficient (Chap. 29), a quantity which in turn could be checked experimentally over a limited range of temperature (Chap. 28). A statistical nuclear theory approach to the neutron-absorbing properties of the fission products (Chap. 12), only a few of which were susceptible of direct measurement, was basic in deciding how to simulate fission products in the reactor endurance studies. In setting up suitable models for the partially depleted reactor, both the theoretical and the experimental endurance studies (Chaps. 10 and 19) were simplified by the fact that the power density distribution in these intermediate reactors varied relatively little with degree of burnup.

Because materials are in general more transparent to intermediate-energy neutrons than to thermal neutrons, the details of the inhomogeneities are less important in intermediate reactors. Hence an intermediate critical assembly can readily be made to be quite an accurate nuclear mock-up of a power reactor even though no individual part of the power reactor is simulated carefully. This enabled the over-all critical-assembly program to be considerably simpler than it might otherwise have been.

Another feature of these intermediate reactors that helped the critical assemblies to be reasonably close approximations to the power reactor at its operating temperature was their small temperature coefficients. Sodium, unlike water, has relatively little moderating effect upon neutrons; hence a temperature-induced change in coolant density makes relatively little difference to a sodium-cooled reactor. Even though the temperature effects were not known in detail until quite late in the program, it was clear that any critical assembly simulating a sodium-cooled reactor would be fairly close to the hot reactor in its nuclear properties.

1.3 CHARACTERISTICS OF INTERMEDIATE REACTORS

A characteristic of the PPA program was the use of the more conveniently handled solid metals, magnesium and aluminum, to simulate the nuclear properties of sodium. This substitution was initially a stop-gap measure, for it was difficult to enclose sodium in air-tight steel cans without using a solder containing a neutron poison (silver). Nevertheless the nuclear simulation of sodium by magnesium or aluminum proved to be remarkably close and was used thereafter in the PPA program and in part of the PTR program as well.

The Proof Test Reactor (PTR), a fairly exact room-temperature mock-up of power reactors at operating temperature, was a second type of critical assembly facility that was built as the engineering design of the power reactor approached completion. One of the objectives of the PTR was implied in its name: it was a facility for "proof testing" final components as to their nuclear properties before installing them in the reactor itself. Other objectives were to obtain more accurate fuel load and control estimations and to measure (at least over a limited range of attainable room temperatures) the temperature dependence of reactivity (Chap. 28).

A number of tests that could not be made on a critical assembly were made on each power reactor after it was completed. The temperature coefficient and the power coefficient, as well as the effects of xenon, were measured with the reactor at various temperatures and at various power levels. Kinetics studies were made by observing the reactor's response to oscillations in power level that resulted from purposeful oscillations of a control element. These tests, continued through the operating life of the reactor, are discussed in Chaps. 20, 30, and 35.

B. Multigroup Age-Diffusion Theory

A multigroup formulation of age-diffusion theory²⁻⁴ was developed into a powerful tool for the study of reactor statics. The neutron-energy spectrum is divided into a number of groups; over each group the neutron cross-sections are taken to be constants independent of energy in any given region of the reactor; and the diffusion equations for the separate groups are solved numerically by a rapidly converging iteration method. The basic approximation that the reactor may be considered to be spatially homogeneous in each region was fairly good even in markedly heterogeneous intermediate reactors, because the heterogeneities were short-range compared with the diffusion length for all except perhaps the thermal groups.

It is a strength of the multigroup method that the group widths can be adjusted, according to the refinement of the cross-section information available, to make allowance for significant variations of cross-section with neutron energy. The method is adaptable to large-scale digital computer use; indeed, it is quite cumbersome otherwise. Multigroup theory can be applied to many kinds of reactor problems, including those of water-moderated reactors. The distribution of fission and of non-fission events of various sorts, both in space and in energy, as well as the degree of criticality of an assembly, can be calculated by this method. A weakness of the method is that, like any method based on diffusion theory, it is not so accurate as transport theory in the treatment of the effects that occur in and near strong absorbers and at boundaries. It is possible, however, to introduce transport-theory corrections into the diffusion formulation. For example, self-shielding effects can be handled by special procedures such as those discussed in Chap. 11.

Four chapters of this volume are concerned primarily with applications of the multigroup calculational method: Chap. 10 tells how over-all surveys of reactor characteristics were made for various possible compositions; Chap. 11 describes a method of calculating self-shielding; Chap. 16 is an estimation of the effect of adding burnable poisons, and Chap. 18 gives details of reactivity calculations with two-dimensional models.

1.3 CHARACTERISTICS OF INTERMEDIATE REACTORS

A. Xenon and Fission-product Poisoning

The severe poisoning effect of fission-product Xe^{135} in thermal reactors is due to its having an absorption resonance in the thermal-energy region, with the peak of the cross-section at 0.065 ev.

The effect of this resonance is much less pronounced in a fast or intermediate reactor, where the neutron absorption of a material is a spectrum-weighted average over many absorption resonances in addition to those near thermal energy. Tests in intermediate reactors during and after power runs have shown the magnitude of absorption to be expected if there were no other major absorption resonances beside the well-known one in Xe^{135} . Thus xenon over-ride is not a major problem in the intermediate reactor.

In contrast to the behavior of Xe^{135} , neutron absorption in the remaining fission products increases in importance, relative to the neutron utilization in fission, as the reactor spectrum is changed from thermal to intermediate. In Chap. 12, fission-product poisoning is discussed on the basis of statistical theory. All nuclei are put on a more or less equal basis in an intermediate spectrum; for it is their average absorption in the resonance region, rather than their thermal absorption, which is important.

In a thermal reactor, neutron absorption by the fission products which are produced in the burnup of a given amount of fuel may typically cause only one-quarter as much loss in reactivity as is ascribable simply to the reduction in fuel load. In an intermediate reactor, on the other hand, the fission-product absorption may cause twice as much reactivity loss as does the corresponding reduction in fuel load. As a consequence, neutron absorption by fission products is a significant factor in determining the possible operating lifetime of an intermediate reactor.

B. Neutron Leakage

The intermediate reactors dealt with in this volume are small and thus have considerable neutron leakage. The relative transparency of matter to fast and intermediate-energy neutrons aggravates this leakage, but the reflector tends to reduce it. Neutrons may also stream out through channels in the reflector, an effect which should be particularly evident in the critical assemblies, where not even sodium fills up the interstices. The close agreement between critical assembly and power reactor criticalities confirmed that this streaming effect was not important.

C. Mechanism for Control

Because of its large neutron leakage a small intermediate reactor can be controlled completely by changes in the reflector. Reflector control offers the advantage that the design of the core of the reactor is free to be determined primarily by heat transfer considerations. Pure reflector control was suggested by the fact that the addition of a reflector to a bare assembly significantly reduces its critical mass. In one system that was tried, portions of a beryllium reflector could be slid parallel to the reactor axis into or away from positions adjacent to a cylindrical core. This scheme was tested in critical assemblies and found successful (Chap. 2).

A mixture of reflector and poison control was used in a second system. The core was surrounded by rotatable cylinders, each of which contained beryllium and boron concentrated on opposite faces of the cylinder. Rotation of each cylinder about its own axis displaces reflector beryllium and replaces it by boron, a good poison and a poor reflector material. The beryllium can be displaced a larger distance if the cylinders are larger (and accordingly few in number). On the other hand, a larger fraction of the reflector can be displaced if the cylinders are smaller (and more numerous). A choice of six for the number of cylinders was a reasonable compromise between the two alternatives, and this choice was tested in several critical assemblies (Chap. 15). This control system operated both by varying the neutron leakage and by changing the energy spectrum of the neutrons near the edge of the core.

A third system used pure poison control. This was combined with the second system in several reactors as a "secondary safety rod" which might contain boron, hafnium, or some other poison and which could be plunged into the center of the core.

D. Possibilities for Breeding

The Knolls Atomic Power Laboratory (KAPL) began its work on intermediate reactors in 1946, to investigate the possibility of breeding in an intermediate neutron spectrum. If true breeding is to occur, more than one new atom of fuel must be created by neutron capture in fertile material (U^{238}) for each atom of fuel (Pu^{239}) used up. The neutron economy of this process is very sensitive to the fraction of neutrons lost by non-

fission capture in the fuel, a fraction which, for plutonium in thermal reactors, is too high for breeding to be possible. Measurements showed that α , the neutron capture-to-fission cross-section ratio, was even higher for neutrons in the lower intermediate-energy region than it was for thermal neutrons, and that α decreased only at relatively high neutron energies. Hence true breeding of Pu^{239} can occur only in fast or nearly fast reactors.

After it was found that the goals of the intermediate power-breeder project were difficult to realize, the effort of the Laboratory was devoted to applying the desirable features of intermediate reactors to naval propulsion systems. The Submarine Intermediate Reactor (SIR) program was in some respects a continuation of the earlier effort to build a small-sized power reactor with highly enriched fuel and an intermediate-energy-neutron spectrum.

E. Temperature Coefficient

It was not evident a priori that an intermediate-type reactor would display a negative temperature coefficient of reactivity and thus be stable toward temperature fluctuations, under all conditions of operation. The primary effect of increasing the temperature of the coolant is to push coolant out of the core. If the coolant is a moderator like water, this expansion may make a large difference in reactivity; but if it is sodium, coolant expansion makes much less difference. When the reactivity coefficients of the various core materials became known (Chap. 7), the effects of expansion as a result of a uniform temperature rise all over the core could readily be calculated to be a reduction in reactivity (Chap. 29). The thermal base effect, a hardening of the thermal portion of the reactor neutron spectrum as the moderator temperature increases, was found to be fairly unimportant in the central region of the core (Chap. 31); but it makes a significant over-all contribution when integrated over the entire reactor (Chap. 28).

Rapid rises in fuel temperature can readily occur during transients without producing much of a temperature rise in the reactor core as a whole. Inasmuch as the fuel has appreciable thickness, self-shielding of its resonances occurs and the Doppler broadening of these resonances reduces the self-shielding as the temperature of the fuel increases. For a given resonance, the reduction in self-shielding has opposite effects on the reactivity, depending on whether it is a fission resonance or a capture resonance that is involved, or on what proportion of each occurs in the resonance. It was not possible by theory alone to predict with confidence whether the net effect of the Doppler reduction in self-shielding was a negative or a positive prompt temperature coefficient in the fuel (Chap. 32). A way was devised, however, to measure this net effect by periodically heating and cooling a portion of the fuel in an assembly (Chap. 33). The measurements showed that the net fuel Doppler coefficient contributed negatively to the temperature coefficient, and that the U^{238} residue in the enriched fuel contributed significantly to this result. Since the majority of the other effects also contribute negatively, the over-all temperature coefficient of these intermediate reactors is negative.

The safe direction in sign of the prompt temperature coefficient was confirmed by observations on the kinetics of the responses of a power reactor to sudden step-wise changes in reactivity (Chaps. 36 and 37), as well as by reactivity oscillator studies (Chap. 35).

The temperature-coefficient considerations just discussed were used in hazards calculations, as in Chap. 38, to aid in predicting the sequence of events that would occur if the reactivity controls were by some mischance to cause the reactivity to increase steadily beyond prompt criticality.

1.4 COMPARISON WITH THERMAL REACTORS

A. Control Requirements

One of the characteristics of the intermediate-energy-spectrum reactors is their relatively small control requirements. Table 1.1 compares typical control requirements of an intermediate reactor with those for a water-moderated thermal reactor having the same power and endurance.

TABLE 1.1--CONTROL REQUIREMENTS NECESSARY FOR APPROXIMATELY
2000 HR ENDURANCE AT 50 MW IN A TYPICAL INTERMEDIATE REACTOR
AND IN A COMPARABLE PRESSURIZED WATER REACTOR *

	Intermediate reactor	Equivalent pressurized- water reactor
Fuel burnup	\$ 4 †	\$12
Stable fission products	6	3
Saturated samarium	1	1
Peak xenon ‡	3	14
Temperature defect	1	5
Safety margin	5	5
TOTAL §	\$20	\$40

* The control requirements for both types of reactor depend on the size and composition of the reactor as well as on the fuel inventory and power distribution. The approximate values in the table refer to typical reactors using highly enriched uranium as fuel, having equivalent power and endurance characteristics and no burnable poison.

† A "dollar" of control is the amount of control needed to change the reactor from delayed critical to prompt critical. For SIR-type reactors it is taken to be 0.0085 in k. See Secs. 2.2 and 5.3 in subsequent chapters for details.

‡ "Peak xenon" refers to the total reactivity loss produced by the largest concentration of xenon after shutdown, as compared with a nonxenon condition. As used here, "peak xenon" is the sum of "equilibrium xenon" and "transient xenon."

§ The totals are obtained on the basis that requirements are independent and additive.

Although the control requirements for these reactors are only approximate, the difference in over-all control requirements between the two types of reactors is certainly quite large. The reactivity requirements indicated by the first three items in the table could, of course, be covered at least partially by the use of burnable poison in either reactor. If it is assumed that a burnable poison was used and that it could compensate for all but approximately one-fourth of these three reactivity requirements in each reactor, then the total control requirements for the intermediate reactor and the equivalent pressurized-water reactor would be about \$12 and \$28, respectively. Another factor which should be noted in comparing the two reactors is the relatively large peak xenon reactivity requirement in the water-cooled reactor. If one were willing to continue operation of the plant to a point in life where the over-ride of only the steady-state xenon is the limiting factor, then the addition to the endurance of the water-cooled reactor would be significantly larger than to that of the sodium-cooled reactor.

The smaller effect of xenon poisoning in the intermediate-energy reactor is due not only to the smaller average xenon cross-section in the intermediate spectrum (relative to the fuel cross-section) but also to the fact that the fuel inventory of the intermediate reactor is larger and its specific power is lower than the corresponding quantities for the equivalent pressurized-water reactor. The difference in fuel inventory also contributes to the difference in reactivity associated with the fuel burnups for the two reactors.

B. Fuel Requirements

Another illuminating way to compare two reactors is to note what portions of their fuel investments are required for what purpose. The total fuel requirements of an intermediate reactor and an equivalent water-cooled thermal reactor can be divided into fuel allowances for intrinsic mass and the fuel required to cover reactivity defects for peak xenon, samarium, stable fission-product poisons, and fuel burnup. The intrinsic mass is defined as the fuel required to establish criticality in the hot clean reactor. The effects of peak xenon and samarium remain relatively constant throughout the reactor life, while the fuel burnup and stable fission product accumulation grow throughout the reactor life to the final values which are assigned in this analysis.

By comparing the intrinsic critical masses, one can get some feeling for the basic fuel investments in the two reactors. By adding the fuel assignments for fuel burnup and stable fission-product poison accumulation, it is possible to arrive at some index for the efficiency of fuel utilization to cover reactivity losses associated with energy production. Table 1.2 shows typical fuel allocations for these two reactors.

TABLE 1.2--FUEL LOADINGS NECESSARY FOR APPROXIMATELY 2000 HR
ENDURANCE AT 50 MW IN A TYPICAL INTERMEDIATE AND IN A
COMPARABLE PRESSURIZED-WATER REACTOR

	Intermediate reactor*	Equivalent pressurized- water reactor*
Intrinsic critical mass	26 kg	10 kg
Peak xenon	4	7
Saturated samarium	2	1
Stable fission products	12	1
Fuel burnup	6	6
	50 kg	25 kg

* These fuel allocations are approximate and refer to typical highly enriched reactors having equivalent power and endurance, and no burnable poison.

Approximately three times as much fuel is required in the intermediate reactor simply to achieve criticality. The intermediate reactor requires a slightly smaller quantity of fuel to over-ride the combined xenon and samarium reactivity defects. The net result is that the intermediate reactor needs about twice as much fuel to cover the reactivity losses associated with fuel burnup in order to produce the same plant energy.

The fuel consumed in the two reactors was chosen to be the same in this particular comparison. There is some justification for this choice in the fact that the two phenomena influencing the fuel utilization act in opposite directions. For a given consumption of fuel, the total number of fissions in the intermediate reactor is somewhat smaller because of its higher value of α , the capture-to-fission ratio, for U^{235} (0.35 in the intermediate spectrum as compared to 0.18 in a thermal spectrum). This may be compensated by the potentially better thermodynamic efficiency of the sodium-cooled reactor, a result of its higher operating temperature.

1.5 TECHNIQUES AND RESULTS OF GENERAL APPLICABILITY

In many of the studies reported in this volume, devices and techniques (both experimental and theoretical) were developed and results were generated that have general use in reactor research. These are listed here for convenience, with references to chapters which give further details.

A. Devices

1. The mechanical design of the critical assemblies (Chap. 3) involving separable halves and a conveniently loaded horizontal tube matrix.
2. The pile oscillator (Chap. 9), with which reactivity effects of small samples of material are measured.
3. The fuel element "scanner" (Chaps. 6 and 21), which uses automatic devices to measure and record the spatial distribution of fissions in a critical assembly.
4. The apparatus for periodically heating and cooling a portion of the fuel to make a direct measurement of the Doppler contribution to the fuel temperature coefficient (Chap. 33).

B. Experimental Techniques

1. The use of the "foil-sandwich" technique (Chap. 6) to measure the absorption of neutrons in the most prominent resonance of a material - a technique that is helpful in determining the neutron energy spectrum of an assembly.

2. The schemes described in Chap. 5 for calibrating critical-assembly control elements in absolute units of reactivity.
3. The various methods discussed in Chap. 15 for using critical-assembly mock-ups to estimate the range of control to be available in a power reactor.
4. The ion-chamber for determining gamma-ray heating in a reactor (Chap. 23).

C. Theoretical Techniques

1. The detailed application of the multigroup age-diffusion theory. Although this application formed an essential background for many of the studies, it is ordinarily given only passing mention in the various chapters in this volume. For details of the method see Vol. I, Chap. 2, of this publication, and the work of H. Hurwitz, Jr., and R. Ehrlich.²⁻⁴
2. The systematic manner of surveying the nuclear properties of whole classes of reactors, by means of the multigroup calculational scheme (Chap. 10).
3. The technique of calculating the heating effects to be expected in the outer portions of the reactor (Chap. 22).
4. Forecasting with an analog simulator (Chap. 38) the sequence of occurrences to be expected in a reactor "runaway."

D. Conceptual Developments and Results

1. A broader understanding of how to use the inverse multiplication method during reactor loading to predict the critical fuel load (Chap. 4).
2. Experimental observations (Chap. 8) of the manner in which the "self-shielding" of layers of materials varies with their thickness.
3. The relatively simple recipes of Chap. 24 for estimating radioactivity and heat generation in the fission products after reactor shutdown.
4. The measurements of the shape of the thermal "tail" of the neutron energy spectrum, discussed in Chap. 31.
5. The use of both the pile oscillator and of the rod-drop technique to predict the kinetic behavior of a reactor. (Chaps. 34-37)

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Part I

**INVESTIGATION OF REACTOR CHARACTERISTICS
BY CRITICAL ASSEMBLIES**

H. B. STEWART and J. S. KING

Chapter 2

THE CRITICAL ASSEMBLY PROGRAM

H. B. STEWART and M. L. STORM

2.1 INTRODUCTION

This chapter is a brief account of the beryllium-moderated and/or-reflected assemblies which were studied at KAPL and their relation to the sodium-cooled reactor projects under consideration at the time. In addition, comparisons that were made between calculation and experiment for some of the more interesting assemblies are summarized.

The wealth of experimental data obtained from these assemblies found immediate application both as a test of theoretical methods and as a source of engineering design information. In this latter connection, it should be noted that the application of data from the critical assemblies to the sodium-cooled power reactors was relatively straightforward since temperature effects in the designs under consideration were small compared to those encountered in water-cooled reactors.

The first KAPL critical assembly, which was brought to criticality on April 22, 1948,¹ was constructed to assist in the design of a power-breeder reactor which was to operate in the intermediate neutron energy region. This Preliminary Pile Assembly (PPA), originally called "zero power assembly," was the first of about thirty beryllium- or hydrogen-moderated and beryllium-reflected critical assemblies whose U^{235} content varied from about 10 to 120 kg with consequent variation in their associated properties.

The Preliminary Pile Assembly is discussed in Secs. 2.2 - 2.4; the Proof Test Reactor is considered briefly in Sec. 2.5. The design, control, and instrumentation of these critical facilities are covered in Chap. 3.

2.2 THE PRELIMINARY PILE ASSEMBLIES (PPA'S)

As was mentioned previously, the first critical assembly was constructed as part of the power breeder project which had as an objective the investigation of the feasibility of breeding in intermediate neutron energy reactors. A bibliography of progress reports dealing with the power-breeder project as well as a chronological history of some key areas of study is given in KAPL-Memo-EAL-7². At the beginning of 1947, the following general specifications had been set for this power reactor³:

1. U^{235} - fueled (estimated critical loading, 40 kg)
2. Beryllium-moderated and beryllium-reflected
3. Sodium-cooled
4. Natural uranium breeding blanket
5. High intermediate spectrum, median fission energy of 10,000 ev
6. Steel container and structural material
7. 10-Mw heat output

In May 1947, when the feasibility report⁴ for the "zero power pile" was published, there was little available knowledge of neutron cross-sections in the intermediate-energy region and reactor calculations were especially difficult and uncertain. It thus appeared necessary to build a flexible critical assembly to obtain information on such items as:

critical mass and size; control effectiveness; power distributions; neutron spectrum; reactivity effects of structural materials, coolant and impurities; and plutonium production in the blanket. Design parameters to be studied were: moderator-to-fuel ratio, coolant-to-fuel ratio, self absorption of fuel, reflector thickness, blanket thickness, and coolant channel arrangement. The assembly could be used to obtain preliminary design information, check the details of the final design, and determine desirable parameters for future reactors.

The following are pertinent features of the critical facility, applicable to all the assemblies designated by PPA numbers. (More detailed descriptions are given in references 4, 5, 6, and in Chap. 3.)

The reactor consists of two right hexagonal prisms which rest on two horizontal beds such that the unique axes of the two prisms are colinear and parallel to the beds. One bed is stationary and the other is movable so that the two prisms can be separated or brought together by remote control. With the two halves apart, an internal face in each half is exposed to permit loading or unloading of the separated assembly. Figure 2.1 is a photograph of the stationary half of one of the earliest assemblies (PPA-3), showing the assembly in its normal shutdown condition with the reactor separated into two halves. Figure 2.2 shows both halves of a later PPA, partly loaded.

Each half of the core is enclosed in a hexagonal steel open-ended box which was surrounded by slabs of natural uranium (as in Fig 2.1) when it was desired to mock up a breeding blanket. These boxes are filled with a matrix of steel tubes arranged in a honeycomb fashion. Two-inch diameter rods containing the reactor components are inserted into the tubes. The core rods are made of component disks of fuel, moderator, structural material, or coolant mock-up. The assembled fuel slugs slide into the central tubes of the container, while the tubes in the peripheral region are occupied by moderator.

Fig. 2.1—Photograph of the stationary half of one of the earliest critical assemblies, PPA-3, encased in its uranium blanket. The ten holes in the core correspond to the particular slugs that were movable in the safety and control system.

Fig. 2.2—Photograph of a later critical assembly, partly loaded. In this assembly, blanket and reflector could be simulated by slugs of appropriate composition around the periphery of the hexagonal box.

The reactivity of the assembly is governed by the relative position of the two halves and, when these are together, by control and safety rods. These rods are similar in composition to the remainder of the core and their removal produces a decrease in reactivity.

* * *

The description of the PPA's follows. The composition of all pertinent assemblies is given in Section 2.6 (Tables 2.5 - 2.7), which was prepared by M. E. Way.

The first critical assembly¹ was used for orientation experiments. It was limited in resemblance to the proposed power breeder by the small amount of U^{235} on hand and the fact that the uranium blanket had not been completed. The core contained U^{235} and beryllium and was side-reflected with a beryllium reflector. There was no reflector at the ends of the hexagon and the core and reflector were surrounded by a lead blanket. Criticality was reached with about 10 kg of fuel, and it was estimated that half of the fissions were due to neutrons with energies below one electron volt. This reactor was dismantled when sufficient U^{235} was available for a faster spectrum.

The succeeding three assemblies, PPA-2, -3, and -4, had a complete natural uranium blanket.⁷⁻⁹ (It should be noted, as a matter of nomenclature, that the early critical facilities, located at the Sacandaga Atomic Power Laboratory, were referred to in the laboratory literature at the time as SAPL-2, etc., rather than PPA-2 etc.) These reactors, which culminated in PPA-5, were successively closer representations of the proposed Intermediate Power Breeder. Table 2.5 shows that from PPA-2 to PPA-5 the Be: U^{235} ratio decreased monotonically from 134:1 to 33:1 and the critical mass increased monotonically from 31 to 58 kilograms. The core of PPA-2 contained no sodium and it was directly surrounded by the natural uranium blanket. In PPA-3 sodium was included in the core and a beryllium reflector was added. More sodium was added in PPA-4 to allow for more heat transfer in the power reactor, and the reflector was made thicker for fuel economy. Beryllium was replaced by more sodium in the core of PPA-5¹⁰ which con-

sequently had a higher energy neutron spectrum. Experimental studies were carried out on PPA-5 for most of the year 1949.

Measurements on the capture-to-fission cross-section ratio of plutonium in various neutron spectra indicated that the breeding gain in a reactor similar to PPA-5 would be marginal, if possible at all,¹¹ and that it was desirable to go to a faster neutron spectrum for improved breeding. It was then proposed to design a "dual-purpose" reactor^{12,13} incorporating a large degree of flexibility, in which the dual objectives of high specific power and breeding could be investigated separately. The reactor core was to be made up of two interchangeable types of elements: beryllium rods, and fast reactor fuel elements which contained no beryllium. These fuel and moderator elements would be so designed that all would have nearly the same reactivity value. A wide variation of spectrum could thus be brought about by the substitution of fuel rods for beryllium rods or vice versa. With few fuel rods and a low intermediate spectrum the assembly could be tested at a high specific power, while the use of many fuel rods and a much faster spectrum would result in an assembly where the breeding gain could be optimized.

Various aspects of the dual-purpose reactor were studied in PPA-6, -7, -8, -9, and -10. The hexagonal prism in these assemblies was composed of aluminum tubes, replacing the steel tubes used in the previous assemblies. Assemblies PPA-6, 7, and -8¹⁴ were used to study the flexibility of the spectrum and the interchangeability of moderator and fuel rods. PPA-6 was completely loaded with 49 moderator rods and 27 fuel rods, Be:U²³⁵ ratio of 45:1. The U²³⁵ mass was 45 kg, but the pile was sub-critical by at least 5 kg. The spectrum was made faster in PPA-7 which contained 36 moderator rods and 40 fuel rods, Be:U²³⁵ ratio of 24:1. PPA-7 was critical with 66 kg of U²³⁵, but it had little excess reactivity for experimental flexibility. PPA-8 had a larger core volume than the previous two assemblies, but the loading pattern was similar to PPA-6 since the ratio of moderator to fuel slugs was two to one. The core contained 70 moderator rods and 35 fuel rods, Be:U²³⁵ ratio of 50:1, and had 85 cents excess reactivity with a fuel load of 58 kg of U²³⁵. The data obtained in PPA-6, -7, and -8 aided greatly in the nuclear design of the dual-purpose reactor, and assemblies PPA-9¹⁵ and -10¹⁶ respectively represented detailed mock-ups of the engineering design for the intermediate and fast loadings.

With PPA-11, an intense effort was made to investigate the fundamental characteristics of a fast intermediate energy reactor.¹⁷ PPA-11 contained 123 kg of U²³⁵ and had a Be:U²³⁵ ratio of only 1.7:1. Many of the experiments that had been performed in PPA-5 were repeated in PPA-11 in order to supplement the data obtained in the earlier slower piles.

PPA-12 and PPA-14 were¹⁸⁻²⁰ more nearly thermal assemblies, and gave information on the amount of control that could be attained by translating the side reflector beryllium parallel to the reactor axis. Table 2.6 gives the compositions of these and several subsequent assemblies. PPA-15 tested the concept of rotating drum control.²¹ The poison faces of the drums were simulated in the PPA by tubes filled with boron carbide. Modifications suggested by the work on PPA-15 were incorporated²² into PPA-18. A photograph of PPA-18, illustrating the control cylinder representation, is shown in Fig. 15.1.

On the basis of experiments conducted in PPA-18 an analysis was made of the fuel requirements and endurances for various contemplated loading patterns^{23,24}. The procedure consisted of making step-wise analytical corrections to the PPA geometry and composition, and of evaluating the reactivity changes associated with each step, until PPA-18 was converted conceptually to the loading pattern in question. (A complete description of the method appears in Chap. 19, especially Sec. 19.3.)

PPA-19 had 45 kg of fuel²⁵ and simulated control cylinders set at 106 deg. (See Sec. 15.2 for a discussion of control drum mock-ups.) PPA-20²⁶ was a mock-up of a clean reactor with about 50 kg of U²³⁵. Further information on reactors of this type was obtained in the Proof Test Reactor which will be discussed in Sec. 2.5.

PPA-21 designates seven assemblies, PPA-21A through PPA-21G.²⁷ These assemblies were used to study correlations between variations in reactor composition and other reactor parameters. PPA-21A (Table 2.7) was a clean reactor with a 62-kg fuel load and a Be:U²³⁵ ratio of about 93:1. Instead of a control cylinder mock-up it used a boron curtain mock-up employing core-length boron tubes distributed around the core-reflector boundary. PPA-21G represented the depleted version of PPA-21A. It had a 57-kg fuel load and employed cadmium uniformly distributed in the core to simulate fission-product poisoning. The boron curtain was removed from the core-reflector interface in PPA-21G. In assemblies PPA-21B through -21F were considered the effects of variations of the amounts of fuel and of beryllium and also of the presence of boron and cadmium, either in the core or at the core reflector boundary, on various reactor parameters. PPA-25 and -26 (Table 2.7) were the last of the beryllium-moderated and -reflected assemblies.²⁸ PPA-25 contained 47 kg of U²³⁵, 17 per cent of stainless steel by volume, and \$9 of boron poison. Bulk reflector studies were made in PPA-25 to determine the relative worth of such reflector materials as graphite, magnesium oxide, and stainless steel. In PPA-26 the boron was moved from the interstitial volume of the core to the core-reflector interface.

(The dollar, \$, is the unit of reactivity equal to the reactivity difference between delayed and prompt critical. Thus \$1 of reactivity is equivalent to the effective delayed neutron fraction in absolute reactivity units. A dollar value of 0.0085 in k has been used for many of the intermediate reactors discussed in this volume. A cent, ¢, is one per cent of a dollar.)

The use of zirconium hydride as a moderator, rather than beryllium, was also considered. In PPA-27 through -32 (Table 2.7) the core beryllium was replaced by an appropriate amount of zirconium and thin polyethylene disks to simulate zirconium hydride.^{29, 30} The assemblies in this group were beryllium-reflected and were contained in a stainless steel blanket. These studies considered variations of spectrum, power distribution, and reactivity coefficients for small-core zirconium hydride reactors as functions of moderator : fuel ratio (holding core volume constant) and of core volume (holding the amounts of fuel and moderator constant).

PPA-32 was the last assembly to be studied in connection with the sodium-cooled reactor program.

2.3 EXPERIMENT VS ONE-DIMENSIONAL MULTIGROUP THEORY

The theoretical work on the intermediate spectrum assemblies studied at this laboratory have been based on multigroup solutions of the age-diffusion equation. Methods for reducing the age-diffusion equation and its adjoint to multigroup form, as well as numerical and analytical techniques for solving these equations, have been described in detail by Hurwitz and Ehrlich, in Volume I, Chap. 2 of this Handbook and elsewhere.³¹⁻³⁴ More detailed discussions of the results of the multigroup calculations can be found in the references.

Criticality

Hurwitz and Ehrlich³¹⁻³³ have summarized pertinent results of multigroup calculations which performed from 1948 to 1952 for PPA-2, -3, -5, -11, -13 and -14. The multigroup formulation was not extended to two-dimensional problems until late 1953; hence all of these calculations were for spherical reactor models. Since the PPA geometry is hexagonal with different side- and end-reflector compositions, the question arose as to the equivalence between the spherical model and the PPA. A procedure for choosing spherical models which yields results in satisfactory agreement with experiment is as follows.³¹ It is assumed that a spherical core with the same percentage composition but with 10 per cent less core volume has the same k_{eff} as the hexagonal prism PPA. Appropriately averaged reflector and blanket regions are used. Table 2.1 summarizes the reactivities calculated for the six assemblies.

TABLE 2.1--CALCULATED k_{eff} OF VARIOUS ASSEMBLIES³¹

Assembly	PPA-2	PPA-3	PPA-5	PPA-11	PPA-13	PPA-14
k_{eff} calculated*	1.08(1.03)†	1.06(1.01)†	0.99	1.0	1.003	0.985‡
Core: U ²³⁵ , kg	31.5	42.4	58.1	123	32.5	36.3
Mols Be/mols U ²³⁵	131	46.1	33.8	1.74	185	197
Coolant	None	Na	Na	Na	Mg-Al	Mg-Al
Coolant vol %	0	9.8	28.4	40	24.5	29.1
Radius of equivalent sphere, cm	30	24	27.6	24	39	39
Reflector, thickness	U, 30 cm	Be, 30 cm	Be, 8 cm	Be, 8 cm	Be, 18 cm	Be, 18 cm
Blanket	None	U	U	U	SS §	SS
Energy groups	8	8	8	9	12	12
Date of calculation	1948	1948	1949	1952	1951	1951

* Each reactor was critical

† Parenthetical figures are estimates based on more recent cross-sections.

‡ Calculated without cadmium and boron mocking up fission products. Experimentally the cadmium and boron contributed $-\$10.80$ to the reactivity, which value was used in arriving at the calculated $k = 0.985$.

§ Stainless steel

Power Distribution and Fission Probabilities

During the year that PPA-5 was in existence, many detailed comparisons between theory and experiment were made for this assembly.³¹⁻³³ For example, in Figs. 2.3 and 2.4 we reproduce from reference 32 the power distribution and the "statistical weight" of fuel (see Sec. 7.1 for a definition of this term) as obtained by theory and experiment for PPA-5. A more detailed comparison of the calculated and measured power distributions would require two-dimensional multigroup calculations, but it is evident that there is qualitative agreement between the theoretical curve and the various experimental traverses. The theoretical and experimental values for the statistical weight of fuel are in good agreement in the central core region and the differences occurring at larger radii may be ascribed to the geometrical model employed in the calculations.

Fig. 2.3—Power distributions in PPA-5. The theoretical spherical model curve (dashed) is compared with the results of three experimental traverses. The traverse ending at A is the direction between two flats of the PPA prism. The traverse ending at B is along the axis of the assembly. The traverse ending at C is a diagonal one, though coplanar with the other two.

Fig. 2.4—Statistical weight (reactivity coefficient) of fuel in PPA-5 (theory vs experiment).

In Table 2.2 comparison is made³² between the calculated and experimental values of νF_H and F_L for gold foils in PPA-5. F_H is the fission-spectrum average of the iterated fission probability, $F(r, u)$; ν is the number of neutrons per fission; and F_L is the capture spectrum average of $F(r, u)$. (These terms are explained in Sec. 7.1.)

TABLE 2.2--ITERATED FISSION PROBABILITIES IN PPA-5³²

	Core center		Core edge	
	Theory	Experiment	Theory	Experiment
νF_H	162	163	83	107
F_L for 0.002-in. gold foil	102	104	51	75
F_L for 0.025-in. gold foil	98	97	49	69

All quantities are in dollar units.

The experimental values at the edge of the core appear too high because the data represent an average over the 2-in. diameter foils, which extended 2 in. into the core.

Representation of Control Cylinders

In representing the rotating-drum-type of reflector control by means of an equivalent spherical model the control drums were simulated by the introduction of uniformly distributed amounts of boron into the reflector. The boron concentration corresponding to the least reactive (0-deg) drum position was chosen to give agreement in reactivity with the safety margin (the amount of k_{eff} by which the reactor is subcritical at shutdown) which was predicted on the basis of PPA experiments; while the most reactive (146-deg) drum position, where the beryllium face is turned almost completely towards the reactor core, was represented by a reflector which contained no boron. Control drum rotation was represented by varying the amount of boron uniformly distributed in the reflector. For example, in the selection of a spherical model representation of a clean reactor (40 kg PPA-18) enough boron was introduced³⁵ in the reflector to make the representation critical; the amount bore no relation to the actual boron content in PPA-18.

Reactivity Coefficients

Experiments in PPA-18 were made the basis for estimating (as in Chap. 19) the fuel requirements and endurance for various contemplated loading patterns.²⁴ The analysis relied heavily upon a knowledge of the reactivity coefficients (defined in Sec. 7.1) of the reactor materials for the clean and depleted core conditions. Emphasis was placed on the perturbation theory evaluation of these reactivity coefficients for comparison with experimental values. As a representative example of this type of calculation, Fig. 2.5 shows a comparison³⁵ between the theoretical and experimental values of the boron reactivity coefficient in the clean PPA-18 as a function of sample thickness at the center and core edge. The experimental and calculated values are in good agreement at the core center, while the values calculated for the core edge lie consistently below the experimental values. The latter behavior reflects the inadequacies of the spherical model. The lower values of the calculated reactivity coefficient at the core edge is an indication of the hardening of the calculated core edge spectrum resulting from the use of a uniform reflector boron distribution to simulate the critical control drum settings.

Fig. 2.5—Reactivity coefficients of boron in PPA-18 (theory vs experiment).

Gamma-ray Heating

Much analytical and experimental effort was expended in evaluating the heating arising from gamma-ray interactions in a reactor. (See Chaps. 22 and 23.) The gamma-ray heating measurements were made in the reflector and blanket regions of PPA-18 and -19, and the agreement between experiment and calculation was found to be quite satisfactory.

2.4 EXPERIMENT VS TWO-DIMENSIONAL MULTIGROUP THEORY

The multigroup formulation was extended to two-dimensional problems in late 1953; and PPA-19 was the first critical assembly for which such calculations were performed. PPA-19 was a mock-up of a depleted reactor. The core loading was 45 kg of U^{235} , and the presence of accumulated fission products was simulated by the introduction in the core region of 55 moles of cadmium in the form of thin cadmium strips. The fuel load was such as to make the assembly just critical with all six control drums in the 106-deg position. The results of the two-dimensional (r, z) and/or (r, θ) calculations will now be compared with experimental information on reactivity as a function of control drum orientation, on power distribution, and on sodium activation.

 (r, z) Calculations

A suitable reactor for an (r, z) calculation³⁶ is obviously one in which angular variations are either nonexistent or relatively insignificant so that they can be ignored. In PPA-19 the presence of the control drums, and in particular the presence of the B_4C in the drums, introduces marked angular asymmetries. These asymmetries are minimized, so far as the core region is concerned, when the B_4C region is rotated away from the core-reflector interface. This means that the (r, z) formulation is most nearly applicable when the control drums are in the 180-deg setting. Nevertheless it was possible to apply the (r, z) formulation to derive useful information which could be compared with experimental data obtained when the control drums were in the 106-deg position, as will now be described. To eliminate angular dependence in the model, it was assumed that the drums were set in the 180-deg position and that in this setting the influence of their boron on the core could be ignored. The result was a side-reflector region comprised of the stationary reflectors and the non-boron region of the drums. Details of the construction of the reactor model are given in reference 36. A schematic diagram of the resulting model is shown in Fig. 2.6. The multigroup calculations were formulated for a reactor model with no boron in the reflector and with a core composition corresponding to an experimental loading that was critical with all control cylinders in the 106-deg setting. The eigenvalue, ν_c , of the multigroup calculation was 2.44, which is equivalent to an excess reactivity of \$2.8 (using \$1 = .0085 in k). This excess reactivity represents the reactivity change which would be brought about by a rotation of all drums from the 106-deg to the 180-deg setting, no change being made in the core composition. This \$2.8 excess reactivity is to be compared with an experimentally derived value of \$3.9.³⁶ The latter value was obtained by extrapolating the PPA-19 control cylinder calibration curve (which extended from 106-deg to 146-deg) to the 180-deg setting.

Fig. 2.6—Reactor model for (r, z) formulation. In this figure, $N_j \Delta r$ and $M_k \Delta z$ locate successive interfaces in the r and z directions. N_j and M_k are integers, while the Δr and Δz represent respectively the axial and radial mesh spacings.

2.4 EXPERIMENT VS TWO-DIMENSIONAL MULTIGROUP THEORY

(r, z) Calculation of Power Distribution

The power distribution obtained from the (r, z) multigroup calculation is shown in Fig. 2.7. Available experimental information³⁶ consists of a number of radial and longitudinal power traverses. In Fig. 2.8, a cross-section of PPA-19 (106-deg) at the equatorial plane, it is seen that these radial traverses extend from the central fuel slug O to peripheral slugs E, F, G, and H, respectively. In Fig. 2.9 the power distributions measured along these traverses are plotted on a radial scale which has been normalized to unity at the core-reflector interface for each traverse. In addition, the power itself has been normalized to unity at the core center. The calculated radial power distribution in the equatorial plane, similarly normalized, is also shown in Fig. 2.9. In Fig. 2.8 the fuel slug F is comparatively near the B₄C region, while slug H is located in a corner between two beryllium reflector surfaces, remote from the B₄C region. Accordingly the interface values of the corresponding radial traverses are noticeably lower and higher than the calculated value. As might be expected from Fig. 2.8, the best over-all agreement between calculated and experiment is obtained for the O-E traverse.

Measured values for longitudinal traverses represent the power average over the 2-in. thick slug cross-section. Hence in comparing longitudinal power traverses the multigroup results were appropriately averaged. In Figs. 2.10 and 2.11, calculated and experimental longitudinal traverses are compared at slugs O and T. The agreement is reasonable.

(r, z) Calculation of Coolant Radioactivity

It was important for the shield design to know the total Na²⁴ gamma activity. Hence experiments were performed in the PPA-19 to obtain sodium activation data. The measured activities in 30-min runs were corrected to saturation activities, and comparison with average regional values obtained from multigroup calculations is shown in Table 2.3.

Fig. 2.7—Power contours in one quarter of PPA-19 by (r, z) calculations (relative power values only). $\Delta r = 1.84$ cm, $\Delta z = 1.94$ cm.

TABLE 2.3--AVERAGE SATURATED SODIUM ACTIVATION IN PPA-19³⁸

Region	Experimental (10^4 disintegrations/sec/mol/watt)	Calculated
Core	27	24
Transition region	42	-
End reflector	40	30 (top reflector)
Stationary reflector	30	40 (side reflector)
Control drum	30	-

Fig. 2.8—A schematic diagram of the equatorial plane of PPA-19.

Fig. 2.9—Radial distributions of power in PPA-19.

2.4 EXPERIMENT VS TWO-DIMENSIONAL MULTIGROUP THEORY

Fig. 2.10—Axial distribution of power in PPA-19 along central slug O.

Fig. 2.11—Longitudinal power distributions along outer slug T.

Comparison of experimental and calculated values indicates best agreement for the core region. This reflects the more accurate representation of the core in the multigroup model. For shielding purposes, however, the agreement is satisfactory.

(r, θ) Calculation of Criticality and Power Distribution

The development of the (r, θ) multigroup formulation received impetus from the desire to create a technique which would effectively take into consideration the angular dependency introduced by the presence of the control cylinders. The application of the (r, θ) formulation to PPA-19 is described in reference 37. The reactor model used in this calculation is depicted by the solid lines in Fig. 2.12 while the actual PPA-19 median plane is indicated by broken lines and cross-hatching.

Fig. 2.12—Median plane of PPA-19 (106 deg). Regions of model used in calculations compared with actual PPA regions.

Fig. 2.13—Radial distribution of power in PPA-19.

The results of the (r, θ) calculation were in excellent agreement with experiment. The actual assembly was just critical for the 106-deg drum position and the calculated assembly was only 28 cents subcritical ($\nu_c = 2.506$). Comparison between calculated and experimental power distributions for the traverses from slug O to slugs E, F, and H are shown in Figs. 2.13, 2.14, and 2.15. The calculated power contours in the median plane, indicating the effect of the control cylinder boron on the power distribution, are presented in Fig. 2.16. Finally in Fig. 2.17, relative average sodium activation in the median plane as obtained by theory and experiment is compared.

(r, z) Calculations on Central Safety Rod

The (r, z) multigroup coding was also applied³⁸ to the determination of the effect of the B_4C secondary safety rod simulated in PPA-19. Two-dimensional calculations were performed for an (r, z) model in which the secondary safety was inserted and an (r, z) model in which the secondary safety was cocked. The calculated loss in reactivity in dollars resulting from the full insertion of the safety rod was obtained from these calculations as \$3.97. This is to be compared with an experimental value of \$2.14.

Fig. 2.14—Radial distribution of power in PPA-19.

Fig. 2.15—Radial distribution of power in PPA-19.

Fig. 2.16—Power contours in PPA-19 median plane by (r, θ) calculation (relative values only). This figure extends over a differently oriented and more restricted portion of the median plane than does Fig. 2.12.

Fig. 2.17—Radial distribution of sodium activation on PPA-19 median plane.

From a geometrical point of view the (r, z) multigroup formulation was well suited for this problem, for the safety rod was a cylinder located on the reactor axis. However, since the calculation involved the application of unmodified diffusion-theory boundary conditions to a centrally located strong absorber, it was to be expected that the calculated worth of the safety rod would be greater than the experimental value. In this regard it is of interest to note that (r, θ) calculations³⁹ of the safety margin in the clean reactor (0-deg control drum setting) yield a value of \$9.62 as compared to the experimental value of $\$6.3 \pm \1 .⁴⁰ These results are consistent with the comparison between theory and experiment obtained in the secondary safety calculation of the preceding paragraph.

16-Group Calculation for Hydride-Moderated Assemblies

Finally, some 16-group spherical model calculations, using the Selengut-Goertzel modification, were performed for three of the zirconium-hydrogen-moderated Preliminary Pile Assemblies.⁴¹ In particular, PPA-27, -28, and -30 were chosen for comparison purposes since these assemblies did not contain peripheral boron and thus were more amenable to the construction of the corresponding spherical models for preliminary calculations. In constructing the spherical model, the core volume was equated to that of the PPA core. The preliminary results are shown in Table 2.4.

TABLE 2.4--MULTIPLICATION FACTOR AND CENTRAL CADMIUM FRACTION FOR PPA-27, -28, AND -30⁴¹

	PPA-27 (Core volume 208 liters)		PPA-28 (Core volume 208 liters)		PPA-30 (Core volume 149 liters)	
	Calc	Exp	Calc	Exp	Calc	Exp
k	1.007	1.0081	0.981	1.0048	0.957	1.0043
Central cadmium fraction	0.25	0.23	0.29	0.30	0.22	0.23

The agreement in k between calculation and experiment for PPA-27 is probably fortuitous. The calculated multiplication factor is less than the experimental value, by an amount that increases as the core volume decreases. Inelastic scattering, which was neglected in all the calculations mentioned here, becomes more significant for smaller volumes and might account for part of the discrepancy.

The agreement in the values of the central cadmium fraction is satisfactory. In general, the cadmium fraction and normalized power distributions obtained from the spherical model were found to be in reasonable agreement with experiment. The greatest differences occur near the core-reflector interfaces, indicating the geometrical limitations of the model employed. The experimentally observed flat power distribution in the central core region was exhibited in the calculated results.

2.5 THE PROOF TEST REACTOR (PTR)

The PTR was a low-power critical assembly^{42,43} designed to furnish nuclear information for the power reactor development program which could not be obtained from the PPA.

The immediate objectives of the PTR program were to assure the safety of the power reactor loading and experimental programs, ensure the early and continued safe operation of the reactor, and to permit interpretations and evaluations of performance which would not be possible with the power reactor itself. The design, control, and instrumentation of the PTR is discussed in Sec. 3.4. It was a vertical assembly which was separable into two parts by means of a hydraulic elevator (see Fig. 3.5). About one-third of the fuel rods (half-length) were suspended above the elevator, while the rest of the core, the reflector, and the control cylinders were mounted in the elevator framework.

The limitations of the PPA in providing detailed design information stemmed from the following features:

1. The PPA has a fixed tube matrix for the entire reactor. Details of the exact geometry of the power reactor core-reflector interface associated with central cylinder rotation could not be reproduced in the PPA.
2. The core construction and fuel element arrangement in the PPA is different from that of the power reactor, and the PPA can only accommodate half-length rods.

The PTR was designed to eliminate these geometrical differences, and the assembly consisted of a full-scale mock-up of the power reactor expanded to operating temperature. It differed from the power reactors in that sodium was replaced by aluminum and magnesium in the PTR, and the PTR fuel rods were composed of disks and thus differed from the power reactor fuel assemblies.

Some of the provisions of the PTR which permitted a detailed study of the nuclear characteristics of each power reactor and allowed for the testing of fuel assemblies, control cylinders, and samples of reactor components were as follows:

1. The dimensions of the PTR core matrix, triffutes, core-reflector interface, and reflector components were selected to mock-up the power reactor core as closely as possible.
2. Core and reflector components were easily removed without disturbing the remainder of the reactor structure, and actual reactor components could be installed in the PTR. This included fuel slugs, reflector control cylinders, and stationary reflector assemblies.
3. The PTR fuel rod composition could be varied over a wide range to permit investigation of burnup and its effect on endurance.
4. The PTR control cylinders could be manually adjusted to any angular setting to allow detailed investigation of control cylinder effects.

The fully loaded critical condition of the PTR was achieved⁴⁴ on January 29, 1954. Following is a partial listing of some of the experiments which were performed:

1. Proof-testing of reactor control cylinders to ensure that their nuclear properties were uniform relative to each other and were consistent with other experiments⁴⁵.
2. Loading procedure test²⁷.
3. Calibration of ionization chambers²⁷.
4. Proof testing of secondary safety rod²⁷.
5. Measurement of Doppler temperature coefficients for SIR-type fuel elements⁴⁶.

The following chapters in this volume of the Handbook discuss experimental information obtained in the PTR:

1. Chapter 15 treats the evaluation of SIR-type reflector control in the PTR.
2. Chapter 17 discusses the experimental techniques used in the PTR investigation of burnable poisons.
3. Chapter 20 compares nuclear characteristics of the power reactor obtained from calculation, PTR experiments, and the actual operation of the reactor.
4. Chapter 21 discusses power distribution measurements in the PTR.
5. Chapter 28 discusses PTR isothermal-temperature-coefficient measurement.

Section 2. 6

TABLE 2.5--CRITICAL ASSEMBLY COMPOSITIONS (prepared by M. E. Way)

CORE	PPA-1	PPA-2	PPA-3	PPA-4	PPA-5	PPA-6†	PPA-7	PPA-8	PPA-9	PPA-10	PPA-11
No. of rings*	5	6	5	5.5	5.5	5.5	5.5	6.5	6.5	5.5	5
Length, in.	16.5	20.1	17.2	18.6	18.6	17.1	17.1	17.1	18.1	18.1	17.1
Volume, liters	-	~112	~62	85.0	85.1	78.0	78.0	111.9	118.6	86.0	62.6
% void	-	15.7	15.7	15.5	15.5	6.2	6.4	6.3	6.4	6.0	7.4
Be/U atom ratio	218	134	48	45	33	45.4	23.6	50.1	55.2	4.13	1.73
Be, kg	87.8	161.2	77.4	82.7	72.6	78.7	60.3	112.1	119.5	19.9	8.2
% Vol	-	-	-	-	-	54.6	41.8	54.1	54.4	12.5	7.1
Na, kg	0	0	6.2	19.2	24.2	12.0	17.7	18.5	18.2	32.9	24.3
% Vol	-	-	-	-	-	15.8	23.4	16.9	15.8	39.4	40.0
Al, kg	3.4	< 2	< 2	< 2	< 2	13.9	13.9	19.9	21.1	15.3	11.1
% Vol	-	-	-	-	-	6.5	6.5	6.5	6.4	6.4	6.4
Fe, kg	24.3	39.4	17.3	29.2	30.0	5.8	7.5	9.0	8.8	14.5	1.2
% Vol	-	-	-	-	-	1.0	1.4	1.0	1.0	2.2	2.2
Stainless Steel, kg	-	-	-	-	-	82.6	102.0	113.8	130.4	191.5	130.5
% Vol	-	-	-	-	-	12.6	15.8	12.4	13.4	25.4	25.8
U ²³⁵ , kg	10.5	31.3	42.4	47.5	58.1	45.2	66.6	58.3	56.4	125.4	123.6
% Vol	-	-	-	-	-	3.1	4.6	2.8	2.6	7.8	10.6
U ²³⁸ , kg	-	1.6	2.2	2.6	3.1	2.4	3.6	3.1	3.0	6.7	6.5
% Vol	-	-	-	-	-	0.2	0.2	0.2	0.1	0.4	0.6
Fuel thickness, in.	-	0.020	0.020	0.020	0.020	-	-	-	-	-	0.040
END-REFLECTOR											
Thickness, in.	- - - - (No end reflector)- - - -					7.6	7.6	-	7.2	7.2	9.7
Volume, liters						27.4	27.4	39.3	36.9	27.0	28.1
% Void						5.0	5.0	5.1	5.7	6.2	5.3
Be, kg						35.8	35.8	51.3	47.4	34.1	27.6
% Vol						70.3	70.3	70.4	69.2	68.2	52.9

* The rings refer to concentric hexagonal annuli of tubes containing the PPA elements. Ring 1 is the central tube, Ring 2 is the six tubes around the central tube, etc.; see Fig. 23.3, of Chap. 23.

† PPA-6 was subcritical.

TABLE 2.5 (Continued)

END-REFLECTOR (Continued)	PPA-1	PPA-2	PPA-3	PPA-4	PPA-5	PPA-6†	PPA-7	PPA-8	PPA-9	PPA-10	PPA-11
Na, kg						2.8	2.8	4.0	4.0	3.0	6.8
% Vol						10.6	10.6	10.6	11.2	11.6	24.7
Al, kg						5.0	5.0	7.2	6.7	4.9	5.2
% Vol						6.6	6.6	6.6	6.6	6.6	6.6
Fe, kg						1.4	1.4	2.0	2.0	1.5	3.3
% Vol						0.6	0.6	0.6	0.7	0.7	1.5
Stainless Steel, kg						14.9	14.9	21.3	20.0	14.6	20.4
% Vol						6.8	6.8	6.7	6.7	6.8	8.9
SIDE-REFLECTOR											
No. of rings	-	None	1	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
Volume, liters	-		-	-	-	52.4	52.4	61.6	65.3	52.2	49.3
% Void	-		21.7	21.0	21.0	5.8	5.8	5.7	5.7	5.7	5.6
Be, kg	76.8		71.3	143.4	143.4	72.7	72.7	88.0	90.6	74.2	67.2
% Vol	-		-	-	-	77.0	77.0	77.0	74.9	76.7	73.4
Na, kg	-		-	0.416	-	1.9	1.9	2.2	3.7	2.2	3.5
% Vol	-		-	-	-	3.7	3.7	3.7	5.8	4.2	7.4
Al, kg	-		1.9	3.0	6.6	9.3	9.3	10.9	11.6	9.3	9.0
% Vol	-		-	-	-	6.6	6.6	6.6	6.4	6.4	6.4
Fe, kg	-		25.4	19.0	46.9	0.91	0.91	1.08	1.79	0.99	1.7
% Vol	-		-	-	-	0.22	0.22	0.22	0.35	0.25	0.44
Stainless Steel, kg	-		-	-	-	28.5	28.5	33.4	35.4	28.2	10.9
% Vol	-		-	-	-	6.8	6.8	6.7	6.7	6.7	6.8
BLANKET											
Material	Pb	U	U	U	-	U	U	U	U	U	U
Thickness, in.	-	12	12	12	-	-	-	-	-	-	-
End, % Vol	-	-	-	-	-	17.0	25.2	17.1	18.8	48.9	53.6
Side, % Vol	-	-	-	-	-	69.5	69.5	56.3	55.1	66.7	56.5

TABLE 2.6--CRITICAL ASSEMBLY COMPOSITIONS (prepared by M. E. Way)

CORE	PPA-12	PPA-14	PPA-15	PPA-16	PPA-17	PPA-18	PPA-19	PPA-20
No. of rings	8	8	>9	*	*	*	*	*
Core length, in.	27.2	27.2	27.9	30.5	30.5	27.4	27.4	27.4
Core volume, liters	272	272	299.3	326.3	326.3	-	-	293.9
% Void	4.7	4.3	11.2	14.9	14.9	14.2	12.7	11.2
Be/U	178.5	195.8	187.2	206	206	187.2	157.4	137.5
Be, kg	272.1	272.1	294.0	317.2	317.2	287.8	272.8	278.8
% Vol	54.3	54.3	53.1	52.6	52.6	-	-	51.9
Mg, kg	89.3	95.1	89.2	84.5	84.5	73.0	88.5	77.2
% Vol	18.9	20.1	17.1	14.9	14.9	-	-	14.9
Al, kg	80.1	73.2	80.5	80.5	80.5	80.5	90.4	103.6
% Vol	10.9	10.0	10.0	9.1	9.1	-	-	13.2
Stainless steel, kg	221.5	221.5	179.3	204.2	204.2	165.4	183.7	183.4
% Vol	10.4	10.4	7.6	7.8	7.8	-	-	7.8
U ²³⁵ , kg	39.8	36.3	40.0	40.0	40.0	40.0	45.2	51.7
% Vol	0.8	0.7	-	0.7	0.7	-	-	0.9
U ²³⁸ , kg	2.9	2.7	2.7	2.7	2.7	2.7	3.0	3.5
% Vol	0.1	0.1	-	0.1	0.1	-	-	0.1
Fuel thickness, gm/cm ²	0.123	0.123	0.123	-	-	-	-	-
Cadmium, kg	-	1.7	-	-	6.4	7.1	5.9	1.0
SIDE-REFLECTOR								
Volume, liters Rot	†	367	401.3	402.6	402.6	402.6	402.6	402.6
Stat	-	-	216.8	216.8	216.8	216.8	216.8	216.8
% Void Rot†	†	9.85	12.9	12.3	12.3	11.7	10.6	11.3
Stat	-	-	11.9	11.9	11.9	10.3	7.9	7.9

* All cores beginning with PPA-15 had the rotating reflector control mock-up and an irregular core reflector boundary. (See Fig. 15.1.)

† Per cent voids apply only to the Be-Mg-SS portions of the control cylinders.

Rot = portion of reflector simulating rotating control cylinder.

Stat = stationary portion of reflector.

‡ PPA-12 had a partial side reflector. See Sec. 2.2 of text.

TABLE 2.6 (Continued)

SIDE-REFLECTOR (Continued)		PPA-12	PPA-14	PPA-15	PPA-16	PPA-17	PPA-18	PPA-19	PPA-20
Be, kg	Rot	‡	-	599.7	601.6	601.6	610.2	618.4	613.1
	Stat	-	-	325.9	325.9	325.9	334.5	343.8	343.8
Mg, kg	Rot	‡	30.6	14.1	14.1	14.1	14.1	14.1	14.1
	Stat	-	-	7.3	7.3	7.3	7.3	7.3	7.3
Stainless Steel, kg	Rot	‡	126.1	178.0	179.5	179.5	138.5	138.5	138.5
	Stat	-	-	74.8	74.8	74.8	74.2	74.2	74.2
Control Cylinder Position		-	-	66 deg	-	-	146 deg	106 deg	66 deg
END-REFLECTOR									
Thickness, in.	Fixed	14.8	14.8	14.1	8.7	8.7	8.0	7.8	7.8
	Movable	-	-	-	8.3	8.3	7.8	7.7	7.7
Volume, liters	Fixed	148	148	130.6	69.7	69.7	85.2	85.6	83.6
	Movable	-	-	-	68.2	68.2	84.1	82.9	82.9
% Void	Fixed	4.9	4.9	30.2	23.3	23.3	16.2	11.8	11.8
	Movable	-	-	-	25.6	25.6	20.7	8.3	8.3
Be, kg	Fixed	108.8	180.8	153.1	81.7	81.7	94.9	79.0	79.0
	Movable	-	-	-	63.8	63.8	80.2	64.7	64.7
Mg, kg	Fixed	63.4	63.4	27.0	11.0	11.0	26.4	47.6	47.6
	Movable	-	-	-	23.4	23.4	33.8	64.2	64.2
Stainless steel, kg	Fixed	51.2	51.2	53.3	24.1	24.1	29.2	32.1	32.1
	Movable	-	-	-	23.3	23.3	31.4	32.5	32.5

‡ PPA-12 had a partial side reflector. See Sec. 2.2 of text.

Fixed = Fixed half of PPA.

Movable = Movable half of PPA.

TABLE 2.7--CRITICAL ASSEMBLY COMPOSITIONS (prepared by M. E. Way)

CORE	BERYLLIUM-MODERATED			HYDRIDE-MODERATED					
	PPA-21A*	PPA-25	PPA-26	PPA-27	PPA-28	PPA-29	PPA-30	PPA-31	PPA-32
No. of rings	8	7	7	7	7	7	6	6	6
Length, in.	27.3	27.7	27.7	27.7	27.7	27.7	27.7	20.5	20.5
Volume, liters	273.3	208.2	208.2	208.2	208.2	208.2	149.2	110.4	110.4
% Void	13.1	15.1	15	14.3	12.0	14.3	14.2	15.2	14.2
Be/U atom ratio	92.9	113.7	113.7	-	-	-	-	-	-
Be,									
kg	220.6	206.6	206.6	-	-	-	-	-	-
% Vol	43.7	53.8	53	-	-	-	-	-	-
Mg,									
kg	98.4	-	-	-	-	-	-	-	-
% Vol	20.7	-	-	-	-	-	-	-	-
H/U	-	-	-	12.0	15.3	10.4	12.1	12.1	21.9
Polyethylene,									
kg	-	-	-	17.0	21.6	17.0	17.1	17.1	21.8
% Vol	-	-	-	9.1	11.4	9.1	12.7	17.2	22.0
Zr,									
kg	-	-	-	217.4	217.4	217.4	218.4	63.5	115.5
% Vol	-	-	-	16.4	16.4	16.4	23.0	9.0	16.4
Al,									
kg	124.2	95.4	95.4	244.9	244.9	253.3	123.0	100.8	73.0
% Vol	16.5	17.3	17.3	45.7	45.7	45.7	31.0	34.5	26.0
SS,									
kg	93.9	211.3	211.3	211.3	211.3	211.3	203.3	189.5	181.9
% Vol	4.3	13.3	16	13.3	13.3	13.3	17.1	21.6	20.7
U ²³⁵ ,									
kg	61.9	47.4	47.4	47.4	47.4	54.2	47.2	47.2	33.5
% Vol	1.2	2.0	2.0	1.2	1.2	1.2	1.7	2.4	1.7

* The first of a series of minor modifications of PPA-21.

TABLE 2.7--(Continued)

CORE	BERYLLIUM-MODERATED			HYDRIDE-MODERATED					
	PPA-21A*	PPA-25	PPA-26	PPA-27	PPA-28	PPA-29	PPA-30	PPA-31	PPA-32
U ²³⁸ , kg	4.38	3.30	3.3	3.3	3.3	3.8	3.3	3.3	2.3
% Vol	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.1
Poison †	B	-	-	-	B	B	B	B	B
SIDE REFLECTOR	BERYLLIUM-MODERATED			HYDRIDE-MODERATED					
	PPA-21A*	PPA-25	PPA-26	PPA-27	PPA-28	PPA-29	PPA-30	PPA-31	PPA-32
No. of Rings	4	4	4	4	4	4	4	4	4
Volume, liters	518.1	463.5	463.5	463.5	463.5	463.5	409.0	409.0	409.0
% Void	13	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8
Be, kg	793.4	710.3	710.3	710.3	710.3	710.3	627.4	627.4	627.4
SS, kg	184.1	159.2	159.2	159.2	159.2	159.2	141.8	141.8	141.8
Poison †	B ₄ C †	-	B ₄ C	-	-	B †	-	B, B ₄ C	B
Poison Distribution	curtain	-	interface	-	-	curtain	-	distributed	distributed
\$ of Poison	~ 12	-	8.4	-	-	3.0	-	5.6	2.5
END REFLECTOR									
Thickness, in. ‡	17.5	17.4	17.4	17.4	17.4	17.4	17.4	17.4	17.4
Volume, liters	175	130.8	130.8	130.8	130.8	130.8	93.7	93.7	93.7
% Void	13.5	12.8	12.8	12.8	12.8	12.8	12.8	12.8	12.8
Be, kg	140.5	104.7	104.7	104.7	104.7	104.7	75.2	75.2	75.2
Mg, kg	117.6	-	-	-	-	-	-	-	-
Al, kg	-	134.7	134.7	134.7	134.7	134.7	97.3	97.3	97.3
SS, kg	61.3	45.3	45.3	45.3	45.3	45.3	30.8	30.8	30.8

* The first of a series of minor modifications of PPA-21.

† B indicates boron in pyrex rods; B₄C, stainless steel tubes filled with B₄C powder.

‡ Total of 2 equal reflectors.

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Chapter 3

DESIGN OF CRITICAL ASSEMBLIES

F. G. LaVIOLETTE

3.1 OBJECTIVES AND REQUIREMENTS OF CRITICAL ASSEMBLIES

Critical experiment programs have been found to be valuable during at least three distinct stages in the design and construction of a power reactor. These stages are:

1. The exploratory study of new reactor design concepts,
2. The development of design details,
3. The proof-testing of the completed reactor components.

Different types of assemblies are needed at the different stages. A critical assembly designed for development studies or for proof-testing may be completely inadequate for exploratory studies, and vice versa.

In general, the design choice for an exploratory critical experiment program should emphasize economy, flexibility, and speed for setting up and changing experimental configurations. During the early phase of a reactor design program, it may be necessary to investigate reactor designs having radically different sizes, geometries, and compositions. Different types of control systems may be studied, and the gross power distribution characteristics may be of interest. Not only must the assembly be able to accommodate this variety of design conditions, but it must be able to do it easily and quickly. In a reactor design program, reactor engineers are frequently waiting for physics results in order to make design decisions, a fact which places a premium on speed of experimentation. As a rule, the fine details of these design problems are relatively unimportant, so that the accuracy of the mock-up can be sacrificed for versatility and flexibility.

Experience has indicated that an assembly consisting entirely of solid material is superior to a liquid-containing assembly for these investigations, even though the coolant in the power reactor is usually a liquid. The use of solid materials is not only advantageous in achieving flexibility, but it also makes it easier to reproduce the reactivity in successive experiments. Reactivity-coefficient experiments which require accurate reactivity measurements are usually very valuable in the exploratory program, and thus reactivity reproducibility is very important.

The size of the components in the flexible critical assembly is often chosen as a compromise between reliable experimental results and a minimum number of components to handle in the assembly operations. Large reactor components (e.g., fuel elements, moderator components, poison elements) make assembly operations easy and fast. However, the larger components make it more difficult to simulate fine geometry details carefully. In particular, self-shielding effects and flux inhomogeneities can be large if the components become too large. These difficulties can be eliminated somewhat by investigating the reactivity- or flux-distribution effect of substituting some more precisely designed fuel rods for a few of the large component rods.

In general, the less accurate results from coarse mock-ups were justifiable to achieve flexibility and versatility in the exploratory program for intermediate reactors.

The Preliminary Pile Assembly (PPA), which will be described in Sec. 3.3, is an example of a critical assembly which has been used for exploratory reactor design programs.

The development program demands more precise information on design details. Hence the critical assembly for this program must have design features which approximate the power reactor design quite closely wherever the nuclear physics characteristics may be important. The control elements, in particular, must be simulated quite accurately in the critical assembly which is used for development studies. However, unlike the mock-up of water-cooled reactors in which the use of water critical assemblies is dictated for this phase of the program, sodium-cooled reactors can be simulated by all-solid assemblies, with magnesium or aluminum, for example, to mock the sodium. This is possible because the sodium has a relatively small effect on both the reactivity and the flux distribution in the reactor.

The Proof Test Reactor (PTR), which will be described in Sec. 3.4, and which was used in the design studies of the power reactors as well as for nuclear proof testing of power reactor components, is typical of a critical assembly which was designed for development investigation. In this assembly, reflector and control elements could be mocked in considerable detail, but there was still some flexibility maintained in the core. An essential requirement for the PTR design was that it provide a means for substituting power reactor components for critical assembly components to ensure design and fabrication procedures.

Since critical experiment operations involve frequent design changes which usually result in reactivity changes, the problem of safety is important. For this reason, careful consideration must be given to the design of the critical assembly to ensure safe operations. Some of the requirements for safety are as follows:

1. Loading - The reactivity margin below critical during shutdown operations should be substantially larger than any reactivity changes which might conceivably result from assembly rearrangements.
2. Instrumentation - Neutron detector systems should be ready for monitoring and for activating alarm and automatic shutdown mechanisms when either the power level or the reactor period becomes abnormal.
3. Interlocks - The control and safety system should be designed to limit operations to a safe sequence by interlocks in the system.
4. Rate of reactivity increase - Provision should be made for limiting the rate of reactivity increase to values which permit adequate time for operator action and automatic shutdown in the event of an experimental error or equipment malfunction.
5. Fast shutdown - The automatic monitoring and scram system should be capable of reducing the reactivity rapidly and reliably in the event of an unusual power level or reactor period condition.
6. Fail-safe design - All equipment, particularly that involving the control system, should be designed so that no single failure can lead to an increase in reactivity or an unsafe condition.

3.2 APPLICATIONS AND GENERAL FEATURES

As is implied in the preceding section, the PPA is a flexible reactor with respect to changes in composition and geometry. It is therefore ideally suited to basic design investigations leading to determinations of critical mass, control range, endurance, spectrum, and power distribution as well as for materials testing. The PPA has been used for the investigation of 37 different assemblies ranging in spectrum from thermal to fast intermediate. Details of the composition of these assemblies are given in Chap. 2.

The PTR was constructed to the dimensions of a hot power reactor with provisions for installation of actual power reactor components in the core and reflector regions. The uses of the PPA and the PTR assemblies in the reactor design program are told in Chap. 2.

Both the PPA and PTR facilities embody the following general features:

1. A mechanism separating the reactor into two roughly equal portions far enough apart to reduce the reactivity of the assembly far below criticality. This ensures a safe margin for manual loading procedures.

2. Rapid-action safety rods.
3. Control rods for fine adjustments of reactivity during approach to, and maintenance of, criticality.
4. A shielded cell or enclosure to protect operating personnel both during normal operations and in the event of a power surge.
5. A remote control panel for operation of the assembly.
6. An instrumentation and control system for reactor monitoring, for safety interlocks that delineate operating procedure, and for automatic safety shutdown.

The normal procedure for a reactor experiment consists of two steps: loading and achievement of criticality.

The loading step involves manual repositioning of the reactor fuel to prepare for the particular test under way. This is done with the reactivity fully reduced and with interlock circuits arranged to prevent operation of reactor controls. In this condition, it is safe from the standpoint of reactivity effects for personnel to work at the core interface, between the two halves. However, the normal operating power level of the assembly must be kept as low as possible to minimize the residual radiation from the core during loading. Power levels of about 1 watt are sufficiently low to permit personnel access 10 min after shutdown.

Achievement of criticality is carried out by remote operation of the reactor. This is done with the shield cell closed and with interlock circuits arranged to permit only a predetermined sequence of reactivity control changes. During this step the critical position of calibrated control elements is determined, or else a constant power level is held for sample activation, depending on the test being run.

3.3 THE PRELIMINARY PILE ASSEMBLY (PPA)

Structure

The PPA, Fig. 3.1, consists of two similar tube bundles mounted on a horizontal axis, each containing one-half of the core and reflector regions of the reactor. One

Fig. 3.1—The Preliminary Pile Assembly (PPA) with halves separated.

Fig. 3.2—PPA slug, showing disks of various reactor materials. Scale is in inches.

bundle, mounted on a screw-driven plate, can be moved toward or away from the other. The separation of the two halves effects a reactivity change of about \$30 for loading safety. (See Sec. 2.2 for the definition of the unit of reactivity called the dollar.)

The reactor materials (uranium-aluminum alloy fuel, beryllium, magnesium, stainless steel) are in the form of disks of various thickness placed on a steel rod to form rods or slugs, Fig. 3.2, which fit into the aluminum or stainless steel tube matrix. There are in addition beryllium triflutes, Fig. 3.3, which fit into the interstices of the tube matrix.

Control

The drive screw mechanism for moving the two halves of the reactor together is powered by an amplidyne-controlled d-c gear-motor (lower left of Fig. 3.1) with slow-down limit switches built into the movable bed mechanism. This system automatically reduces the rate of increase of reactivity as the assembly is closed. The rate of increase is kept at 5¢/sec per dollar below critical. (A logarithmic reactivity increase

Fig. 3.3—PPA beryllium triflute.

rate of 0.05 has been adopted in KAPL critical-assembly operating practice as a rough safety criterion. This rate produces a subcritical reactor period of about 20 sec.)

Control, safety, and source rod drive mechanisms are mounted on support plates at the rear faces of the tube bundles, Fig. 3.1. The control and safety drive mechanisms can be arranged to move either regular PPA-type fuel rods or boron or cadmium poison rods.

The number and location of safety rods in the two matrices are selected to provide a total reactivity change of \$1.25 to \$1.50. Rates of reactivity increase on the safety rods are limited to about 20¢/sec by interlock circuits and by mechanism cocking speeds. The safety mechanisms are designed to provide a reduction of reactivity at rates as rapid as \$5 to \$10 per second.

Two to four control mechanisms moving one or two rods each are used to provide control from 60¢ to 90¢ of reactivity. The rate of reactivity increase on these control rods is limited to 1 or 2¢/sec. Control rod positions are indicated at the control panel by a vernier dual-selsyn system. The precision of setting and reading the control rod positions corresponds to a reactivity of the order of 0.01¢. Usually other factors, such as errors in observing exact criticality, perturbation of control rod calibration by experimental changes, and buildup of photoneutron sources, actually limit the accuracy of reactivity determinations to about 0.1¢.

All control mechanisms are designed to fail in a safe manner. A storage battery supply furnishes power to drive the PPA halves apart in case of power failure. The safety rods are driven to their least reactive position by mechanical springs when the current to the holding coils is interrupted.

A polonium-beryllium neutron source of about 10^6 neutrons/sec is used to provide an initial neutron flux in the reactor even with the reactivity fully reduced by the loading safety mechanism. The source may be withdrawn from the core to the back of the reflector by remote control to avoid errors which it may produce in the determination of criticality.

Interlock circuits incorporated in the control system impose the following limitations on the operating sequence:

1. Before the halves of the assembly can be moved together to increase reactivity
 - a. the source must be in the core (to be pulled out later),
 - b. the control rods must be in their least reactive position,
 - c. the safety rods must be cocked in the most reactive position, and
 - d. the reactor shield cell must be closed.
2. Reactivity can be increased with the control rods only after the halves of the assembly are completely closed.

Instrumentation

The following nuclear control instrumentation is used on the PPA:

1. Three BF_3 -filled current ionization chambers with associated electronic current amplifiers and trip circuits. These provide meter readings and electrical output signals proportional to the neutron flux or power level of the reactor. Signals exceeding twice the full-scale meter reading operate the trip circuit and initiate reactor scram.
2. A fourth BF_3 -filled current ionization chamber with an electronic log response current amplifier operating in conjunction with a differentiating amplifier. This provides a meter reading and output signal proportional to the inverse pile period. This also drives a trip circuit set to cause reactor scram on periods of less than 20 sec.
3. Four neutron pulse counter sets that have either B^{10}F_3 -filled pulse chambers or B^{10} -lined proportional counters. These counters are used in approaching criticality and for subcritical reactivity measurements.

Experimental Considerations

The uranium-aluminum alloy fuel disks for the PPA contain 35 per cent by weight of uranium enriched to ~90 per cent U^{235} . Two 0.040-in. thick disks (of diameter 1.96 in.) are located at each fuel position to provide about the same self-shielding as exists in the power reactor fuel elements.

The PPA does not contain sodium; the sodium coolant is mocked by aluminum and magnesium disks. These are used in the proper amount to provide a reactivity contribution equal to that of the power reactor sodium as determined by tests of sodium samples in the PPA.

The reactor control cylinders are mocked up in the PPA by beryllium reflector rods and solid boron carbide rods arranged in six hexagonal patterns at the corners of the core lattice, as can be seen in Fig. 3.4. These are similar in size and position to the control cylinders. The detailed characteristics of control cylinders were studied in a special "single-cylinder" mock-up (the hexagon that is farthest to the right in Fig. 3.4) having B C poison rods.

Study of the depleted reactor is facilitated by mocking fission-product poisons with thin cadmium strips inserted in the interstices of the tube bundle and by removing some fuel disks to simulate fuel depletion.

Fig. 3.4—Simulation of reactor control cylinders in the PPA. On the hexagonal face of the PPA midplane are outlined six smaller hexagons, each of which encloses a control cylinder mock-up. One "control cylinder" (farthest to the right, about half-way up the PPA face) contains poison rods similar in size and composition to those to be used in the reactor being mocked-up. The disks labelled MD are ends of "moderator" (i.e., beryllium and steel) slugs. Those labelled R are the ends of "reflector" slugs (mostly beryllium). The core slugs are numbered according to a positional scheme in use at the time.

3.4 THE PROOF-TEST REACTOR (PTR)

Structure

The entire PTR reflector and core structure and most of the fuel rods are mounted vertically on a 25-ton hydraulic elevator, Fig. 3.5. Fuel rods containing one-sixth of

1. Elevator	7. Alignment plate
2. Bottom (movable) portion of PTR	8. Control rods
3. Matrix plate support	9. Control drive mechanism
4. Top support plate	10. Reactor control cylinder
5. Suspended fuel rods	11. Beryllium reflector
6. Core matrix plate	12. Core rods

Fig. 3.5—The Proof Test Reactor (PTR) facility sketched in a cutaway view.

the fuel and core materials are suspended from a tower above the elevator, Fig. 3.6. As the elevator is raised the suspended fuel rods enter the core and complete the as-

Fig. 3.6—Top (stationary) portion of PTR core, hanging from the top-support plate and containing one-sixth of the core. The three longer rods are full-core-length fuel rods that fit into three rodless tubes in the lower portion of the assembly. As the elevator is lowered, three B₇C rods are driven into these three tubes as an additional safety mechanism (see text on Control).

sembly, Fig. 3.7. The reactor is subcritical by about \$20 when the elevator is down, so that manual changes may be made safely.

Fig. 3.7—PTR in operating position, the two "halves" together. Dials indicate PTR control rod positions.

Control

Safety during PTR loading is ensured by two separate mechanisms. The first is that provided by the relative motion of the suspended fuel rods and core matrix described above; this contributes approximately \$10 of reactivity change. The second loading safety mechanism contributes an additional \$10; it involves three core-length B_4C rods which are mechanically driven into the core, as the elevator is lowered, by a linear geneva mechanism coupled to a control rail on the elevator shaft. As the elevator rises, the elevator speed is automatically reduced by the operation of a series of limit switches and run-off trolleys which successively reduce the pumping capacity of the elevator hydraulic drive system. In this way the rate of reactivity increase is limited to 5¢/sec per dollar subcritical.

Control rod, safety rod, and source drive mechanisms are mounted in a compartment immediately below the reactor lower support plate. Five groups of fuel-containing safety rods (a total of 16 rods of half-core length) provide a total reactivity change of about \$2. The mechanisms limit the rate of safety-rod reactivity increase to about 15¢/sec and provide reactivity reduction at a rate of \$10/sec.

Three groups of fuel-containing control rods (total of 10 half-core-length rods) provide a total fine control range of about 90¢. The associated mechanisms and interlocks limit the rate of reactivity increase to about 2¢/sec. The precision of setting for the control rod positions is about 0.002¢. The same factors which were mentioned for the PPA usually limit the accuracy of reactivity determinations to about 0.1¢. The control rod positions are indicated by three large revolution counters mounted on the elevator below the reactor, Fig. 3.7. This counter indication is visible from the control room through a shield window when the reactor is raised.

The PTR has a Po-Be neutron source which emits about 10^7 neutrons/sec. This source is sufficiently strong to activate period-measuring instruments when the reactivity is far below critical. Thus the period trip when set to operate at a 20-sec period establishes a limit on the subcritical rate of increase of reactivity at a value of about 5¢/sec per dollar subcritical.

Interlock circuits impose the same limitations on the operating sequence in the PTR control system as were described for the PPA.

Instrumentation

The PTR nuclear instrumentation is similar to that of the PPA with the exception that individual channel relay trip circuits are replaced by a single multichannel dual-bus electronic-trip circuit. The normal operation of this system is independent of any relay functions and is therefore highly reliable.

The neutron-sensitive instrument complement includes:

1. three zero-stabilized amplifier systems (providing power level trip) amplifying the current from three BF^3 ion chambers,
2. two log and period amplifier systems (providing period trip) similarly activated,
3. four neutron pulse counter systems.

Experimental Considerations

All PTR core and reflector components are manufactured to the size of the corresponding cold power reactor components. However, the PTR matrix tubes are mounted in an expanded array corresponding to the size of the hot reactor core. This provides clearances between tube and triffute components sufficient to permit easy removal and exchange of parts for experiments.

The PTR fuel rods are stacks of flat disks of beryllium, stainless steel, fuel, and magnesium (to simulate sodium), held in thin-walled stainless steel tubes. The fuel is an alloy of 35 percent (by weight) of uranium in aluminum, the uranium being enriched to ~90 percent in U^{235} . The fuel disks, which have a thickness of 0.040 in. and a diameter of 1.60 in., are generally stacked in layers three disks thick to provide self-shielding similar to that in the power reactors.

The disk-type PTR fuel rods can be replaced by regular reactor fuel slugs. In addition, fuel elements may be installed with sodium contained in a sealed matrix tube.

3.4 THE PROOF-TEST REACTOR (PTR)

Provision has been made for insertion of samples in certain triflute spaces and in special fuel rods with axial sample holes. Cadmium-plated sleeves can be installed in the 0.03-in. annular spaces between PTR fuel rods and the matrix tubes to mock fission-product poisons.

Chapter 4

INVERSE MULTIPLICATION TECHNIQUES

H. B. STEWART, F. G. LaVIOLETTE, and T. J. KRIEGER

4.1 REACTOR MULTIPLICATION

The technique of measuring reactor multiplication is an important one in reactor operation since it is frequently used to indicate the approach to criticality during reactor loading. Multiplication techniques also provide an important means for measurement of reactivity changes in critical experiment work.

A subcritical reactor will produce a neutron flux at a measurable level only if a steady source of neutrons is supplied to the core. The reactor assembly then serves as a multiplying medium to enhance the neutron flux produced by the source. The neutron flux is measured with detectors placed in or near the assembly. In any quantitative measurement of flux level the source distribution, the geometry of the assembly, and the properties of the detectors are important factors which must be taken into account.

The multiplication of an assembly, M , is defined as the ratio of the total number of fission energy neutrons appearing per unit time in the assembly to the number which would appear in a fictitious assembly identical in every respect to the assembly in question except that the fuel is replaced by a completely inert material which neither scatters nor absorbs neutrons. From the definition of M it follows that

$$M = (F + S)/S$$

where S is the source strength, F/ν is the fission rate in the assembly in question, and ν is the number of neutrons emitted per fission.

In practice, however, it is not the reactor fission rate that is measured, but the neutron flux response of a particular detector at a particular location. There are thus disturbing effects due to core leakage, to the spectrum and spatial distribution of the source, and to the geometry, location, and spectral response of the detector. A more nearly practical quantity, therefore, is the apparent multiplication, M_{app} , which is defined as the ratio of the responses of a particular neutron detector in the actual and in the fictitious assemblies, the same source being present during both measurements. The ratio of M_{app} to M is denoted as:

$$p = M_{app}/M$$

The value of the quantity p depends upon the disturbing effects mentioned above. Some of these effects are discussed further in Secs. 4.2 and 4.3.

In actuality, of course, a measurement in the fictitious assembly is impossible; but this difficulty is circumvented by measuring differences in $1/M_{app}$ rather than absolute values (see Sec. 4.3). In principle, if the fictitious assembly could be built, absolute measurements of both F and S could be made which would relate the flux in the actual assembly to that in the fictitious assembly.

The ultimate aim of the multiplication measurements is to determine the k_{eff} of the assembly in question as a function of some core parameter, such as the amount of fuel loaded, and to extrapolate accurately to the value of that parameter which makes the

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assembly just critical ($k_{eff} = 1$). It is thus necessary to obtain the functional relationship between k_{eff} and M_{app} ; this is done in Sec. 4.2. The relationship is particularly simple when the source produces neutrons having the same spatial and energy distributions as the neutrons produced by the fundamental mode of the fission distribution. In that case, it can be shown from simple neutron conservation arguments that $1/M_{app}$ is directly proportional to $(1 - k_{eff})$. In the general case, where the source neutrons do not have the same distribution as the fundamental mode of fission neutrons, and where the detector characteristics must be considered, the relationship between M_{app} and k_{eff} is more complicated. Nevertheless it will appear in Sec. 4.2 that in the limit as $k_{eff} \rightarrow 1$, $1/M_{app} \rightarrow (1 - k_{eff})$; so that the usual procedure of plotting $1/M_{app}$ as a function of the amount of fuel loaded, and extrapolating to a zero value to find the critical load, is valid. The complete shape of the loading curve depends of course upon the actual relationship between $1/M_{app}$ and k_{eff} . Some approximate formulae for this relationship will be derived in Sec. 4.2. These are useful in extrapolating $1/M_{app}$ to zero.

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This concerns the multiplication in intermediate reflected reactors. In these cases, unfortunately, the usual normal mode analysis is not legitimate, for the large neutron energy range, together with the rapid spatial variation of neutron spectrum, render invalid the concepts of spectrum-averaged parameters (such as the diffusion constant and the various cross-sections) which are independent of position. The results to be derived below, while of considerable interest in themselves, are based on the use of such concepts and thus are applicable to intermediate reactors only in a qualitative way.

Consider for mathematical convenience a bare, subcritical, homogeneous reactor, containing a source of fast neutrons whose distribution over the reactor is that of the n th normal mode, $\Psi_n(\underline{r})$ and whose strength is S_n . This may be considered to be a component of an arbitrary source. Let E_ℓ denote the neutron energy above which the likelihood of producing fissions may be neglected. (In intermediate reactors, E_ℓ may be very high, of the order of kilovolts.) Let $P(E_\ell, |\underline{r}' - \underline{r}|)$ denote the infinite-medium slowing-down kernel to energy E_ℓ . A diffusion equation for the flux of neutrons, Φ , in the 0 to E_ℓ energy range is:

$$D \nabla^2 \Phi - \Sigma_a \Phi + \int d\underline{r}' [k \Sigma_a \Phi(\underline{r}') + S_n \Psi_n(\underline{r}')] P(E_\ell, |\underline{r}' - \underline{r}|) = 0 \quad (4.2)$$

where D is the diffusion constant averaged over the energy range 0 - E_ℓ ,

Σ_a is the absorption cross-section similarly averaged,

k is the ratio of neutrons born in one generation to those born in the preceding generation (in an infinite medium).

Taking the Fourier transform of Eq. 4.2

$$-D b^2 \bar{\Phi}(b) - \Sigma_a \bar{\Phi}(b) + [k \Sigma_a \bar{\Phi}(b) + S_n \bar{\Psi}_n(b)] \bar{P}(E_\ell, b^2) = 0 \quad (4.3)$$

where b is the Fourier transform variable and the bar indicates a transformed function. Since $\bar{\Psi}_n(\underline{r})$ is defined as the n th normal mode of the source it obeys the wave equation

$$\nabla^2 \Psi_n + B_n^2 \Psi_n = 0 \quad (4.4)$$

where B_n^2 is the n th buckling eigenvalue. Taking the transform of Eq. 4.4,

$$-b^2 \bar{\Psi}_n(b) + B_n^2 \bar{\Psi}_n(b) = 0$$

which implies that $\bar{\Psi}_n(b) \equiv 0$ unless $b^2 = B_n^2$. Hence, if $b^2 \neq B_n^2$, Eq. 4.3 becomes

$$[D b^2 + \Sigma_a - k \Sigma_a \bar{P}(E_\ell, b^2)] \bar{\Phi}(b) = 0 \quad (4.5)$$

It follows that $\bar{\Phi}(b) \neq 0$ only when $b^2 = B_n^2$ and possibly also when $b^2 = B_M^2$, where B_M^2 , the material buckling, is the positive zero of the expression in the square brackets in Eq. 4.5. Now it will be shown that $\Phi(B_M^2) = 0$.

Since the functions Ψ_n form a complete set, expand ϕ in terms of the Ψ_n :

$$\begin{aligned}\text{whence } \Phi &= \sum_j a_j \Psi_j \\ \bar{\Phi} &= \sum_j a_j \bar{\Psi}_j\end{aligned}$$

Now $\bar{\Psi}$ differs from 0 only when $b^2 = B_n^2$ and, as the reactor is considered to be subcritical, $B_n^2 < B_M^2$ for all n . Consequently, $\bar{\Phi}(B_M^2)$ is equal to 0.

Thus the flux in the presence of a normal-mode source distribution has the same normal mode distribution as the source. To determine the amplitude of the flux distribution, set

$$\Phi = A_n \Psi_n$$

in Eq. 4.3, which gives

$$\left[Db^2 + \Sigma_a - (k\Sigma_a + \frac{S_n}{A_n}) \bar{P}(E_\ell, b^2) \right] \bar{\Psi}_n(b) = 0$$

Now $\bar{\Psi}_n(b) \neq 0$ when $b^2 = B_n^2$. Hence,

$$DB_n^2 + \Sigma_a - (k\Sigma_a + \frac{S_n}{A_n}) \bar{P}(E_\ell, B_n^2) = 0$$

or

$$\frac{A_n}{S_n} = \frac{\bar{P}(E_\ell, B_n^2) / \Sigma_a}{1 + B_n^2 L^2 - k\bar{P}(E_\ell, B_n^2)} \quad (4.6)$$

where $L^2 \equiv D/\Sigma_a$.

When the fuel is replaced by inert material, the value of k is zero. Let the added subscript 0 denote this condition. Then,

$$\frac{A_{n0}}{S_n} = \frac{\bar{P}_0(E_\ell, B_n^2) / \Sigma_{a0}}{1 + B_n^2 L_0^2} \quad (4.7)$$

Dividing Eq. 4.6 by Eq. 4.7,

$$M_n = \frac{A_n}{A_{n0}} = \left[\frac{\Sigma_{a0} \bar{P}(E_\ell, B_n^2)}{\Sigma_a \bar{P}_0(E_\ell, B_n^2)} \right] \left[\frac{1 + B_n^2 L_0^2}{1 + B_n^2 L^2 - k\bar{P}(E_\ell, B_n^2)} \right] \quad (4.8)$$

Here M_n is the apparent multiplication in the presence of the n th normal mode source distribution. In the case of highly enriched reactors, $\bar{P}(E_\ell, B_n^2) = \bar{P}_0(E_\ell, B_n^2)$, for which case Eq. 4.8 reduces to

$$M_n = \left[\frac{\Sigma_{a0}}{\Sigma_a} \right] \left[\frac{(1 + B_n^2 L_0^2)}{(1 + B_n^2 L^2)(1 - k_n)} \right] \quad (4.9)$$

where

$$k_n = \frac{k \bar{P}(E_\ell, B_n^2)}{1 + B_n^2 L^2} \quad (4.10)$$

If the source distribution is a single normal mode, it must be the fundamental ($n=0$) mode since this is the only one which is physically realizable (all others have negative values at some interior points). Since criticality corresponds to $k_0 = 1$, it follows from Eq. 4.9 that the multiplication becomes infinite as criticality is approached, a result in agreement with the general concept of reactor multiplication.

For an arbitrary source distribution, the apparent multiplication is not defined independent of detector position, as was found to be the case (Eq. 4.9) when only a single normal mode is present. By Eqs. 4.6, 4.7 and 4.10:

4.2 DERIVATION OF MULTIPLICATION FORMULA

$$\begin{aligned}
 M(\underline{r}) &= \frac{\Phi(\underline{r})}{\Phi_0(\underline{r})} = \frac{\sum_n A_n \Psi_n(\underline{r})}{\sum_n A_{n0} \Psi_n(\underline{r})} \\
 &= \frac{\sum_n \left[\frac{S_n \bar{P}(E_l, B_n^2) / \Sigma_a}{1 + B_n^2 L^2 - k \bar{P}(E_l, B_n^2)} \right] \Psi_n(\underline{r})}{\sum_n \left[\frac{S_n \bar{P}(E_l, B_n^2) / \Sigma_{a0}}{(1 + B_n^2 L_0^2)} \right] \Psi_n(\underline{r})} \\
 &= \frac{\Sigma_{a0} \sum_n \left[\frac{S_n k_n}{(1 - k_n)} \right] \Psi_n(\underline{r})}{\Sigma_a \sum_n \left[\frac{S_n k_n (1 + B_n^2 L^2)}{(1 + B_n^2 L_0^2)} \right] \Psi_n(\underline{r})}
 \end{aligned} \tag{4.11}$$

As criticality is approached with increasing core loading, the $n = 0$ term in the numerator of the last form of Eq. 4.11 becomes progressively more predominant. If the higher-order terms in the numerator are neglected, we have

$$M(\underline{r}) \approx \frac{\Sigma_{a0} [S_0 k_0 / (1 - k_0)] \Psi_0(\underline{r})}{\Sigma_a \sum_n [S_n k_n (1 + B_n^2 L^2) / (1 + B_n^2 L_0^2)] \Psi_n(\underline{r})} \tag{4.12}$$

Thus, at any given point $\underline{r}$, as k approaches unity M approaches infinity as $1/(1 - k_0)$, or $1/M$ approaches zero like $(1 - k_0)$. If $1/M$ is plotted as a function of the fuel loading L , the curve will cross the L axis at $L = L_c$ where $k_0(L_c) = 1$. In general, $1/M$ is not a linear function of L except near $k_0 = 1$ so that care must be exercised in extrapolating the curve to the L axis from too far below criticality.

Until now, to this point in the chapter, reactor multiplication has been considered from a purely theoretical viewpoint. Questions arising from deviations of the reactor from the ideal one discussed above will now be discussed, as well as with those arising from the practical difficulties encountered in measuring reactor multiplication. Clearly, these questions can generally be treated only in a semiempirical, phenomenological manner, and with even less rigor than was possible in the development thus far. We now discuss M_{app} for some typical cases.

Consider first the influence of detector location on the measurement of reactor multiplication. If the detector is sufficiently small and is such that it can be introduced into the reactor core without greatly perturbing the conditions therein, then the multiplication measured will be that given by Eq. 4.11 or Eq. 4.12 above. If the detector is distributed over a volume Δ of the core, then, provided the flux is negligibly perturbed by the presence of the detector, the multiplication is given by

$$\begin{aligned}
 M(\Delta) &= \frac{\int_{\Delta} d\underline{r} \Phi(\underline{r})}{\int_{\Delta} d\underline{r} \Phi_0(\underline{r})} \\
 &= \frac{\Sigma_{a0} \sum_n \left[\frac{S_n k_n}{(1 - k_n)} \right] \int_{\Delta} d\underline{r} \Psi_n(\underline{r})}{\Sigma_a \sum_n \left[\frac{S_n k_n (1 + B_n^2 L^2)}{(1 + B_n^2 L_0^2)} \right] \int_{\Delta} d\underline{r} \Psi_n(\underline{r})}
 \end{aligned} \tag{4.13}$$

as follows from Eq. 4.11. From Eq. 4.13 $M(\Delta)$ becomes infinite as criticality is approached, provided that

$$\int_{\Delta} d\underline{r} \Psi_0(\underline{r}) \neq 0$$

But the last inequality follows directly from the positive nature of $\Psi(\underline{r})$ inside the reactor. In the special case when Δ is the entire volume of the reactor core, the integrals $\int_{\text{core}} d\underline{r} \Psi_n(\underline{r})$ become smaller with increasing n as does k_n , with the result that the higher-order terms in the numerator in Eq. 4.13 may be neglected as in Eq. 4.12.

When the detector is outside the reactor, the situation is more difficult to analyze, owing in part to the preferential attenuation of the thermal leakage neutrons by the reflector. Let F_n denote the fraction of neutrons escaping from the reactor which originate in the n th mode of the source. Roughly, F_n is given by

$$F_n = (1 - \beta) \left[1 - \frac{\bar{P}(E_\ell, B_n^2)}{(1 + B_n^2 L^2)} \right] \quad (4.14)$$

where β is of the nature of an effective albedo factor for all the neutrons. Thus the external detector is expected to indicate a quantity proportional to $F_n \Phi_n$ or to

$$\frac{F_n S_n \bar{P}(E_\ell, B_n^2) / \Sigma_a}{1 + B_n^2 L^2 - k \bar{P}(E_\ell, B_n^2)}$$

when the source has an n th mode distribution. The multiplication $(M_n)_{\text{ext}}$ for such a source as measured by an external detector is

$$(M_n)_{\text{ext}} = (F_n / F_{n_0}) (M_n)_{\text{int}} \quad (4.15)$$

where $(M_n)_{\text{int}}$ is the multiplication measured by an internal detector. When the source is arbitrary, the externally indicated multiplication is

$$M_{\text{ext}} = \frac{\sum_n \frac{F_n S_n \bar{P}(E_\ell, B_n^2) / \Sigma_a}{1 + B_n^2 L^2 - k \bar{P}(E_\ell, B_n^2)}}{\sum_n \frac{F_{n_0} S_n \bar{P}(E_\ell, B_n^2) / \Sigma_{a_0}}{1 + B_n^2 L_0^2}} \quad (4.16)$$

from which it again follows that $M_{\text{ext}} \rightarrow \infty$ as criticality is approached, since the fraction escaping in the fundamental mode is not zero. Thus, even with an externally situated detector, the approach to criticality can be gained by plotting $1/M$ against core loading and extrapolating to the L axis.

The analysis of this section has given a qualitative understanding of those portions of p (the ratio of M_{app} to M) which are generated by the source distribution and detector location. Other aspects of the situation are best discussed from an empirical point of view, as is done in the next section.

4.3 INVERSE MULTIPLICATION MEASUREMENTS IN REACTOR LOADING

The apparent multiplication, M_{app} , defined in Sec. 4.1, was related to the true multiplication, M , by a factor p . In the present section the true multiplication will be considered to be that obtained in the presence of a normal mode source, so that

$$1/M_{\text{app}} = 1/pM = (1 - k_{\text{eff}})/p$$

The factor p is of course a function of k_{eff} , but it is slowly varying when k_{eff} is near unity, as was seen in Sec. 4.2. The factor $(1 - k_{\text{eff}})$, on the other hand, rapidly changes its magnitude when k_{eff} is near unity; so that it becomes dominant in determining the value of $1/M_{\text{app}}$ near criticality.

In the region where p varies relatively slowly, the loading required to achieve criticality can be calculated approximately. If two successive multiplication measurements M_1 and M_2 (where $M_2 > M_1$) are taken for values of k_{eff} equal to k_1 and k_2 , then

$$1 - k_2 = \frac{1}{M_2} \frac{(k_2 - k_1)}{(1/M_1 - 1/M_2)} \quad (4.17)$$

Let θ be a measure of the reactivity change made by the addition of equal-valued fuel rods or by the motion of a linear control element such that the values θ_1 and θ_2 corre-

4.3 INVERSE MULTIPLICATION MEASUREMENTS IN REACTOR LOADING

spond to the reproduction factors k_1 and k_2 . Then the value of θ which will make the system critical will be

$$\theta_c = \theta_2 + \frac{1}{M_2} \frac{\theta_2 - \theta_1}{(1/M_1 - 1/M_2)} = \theta_2 + \frac{1}{M_2} \frac{\Delta\theta}{\Delta(1/M)} \quad (4.18)$$

or, in terms of detector counting rates,

$$\theta_c = \theta_2 + \Delta\theta \frac{C}{C_2 - C_1} \quad (4.19)$$

The inverse multiplication as a function of k_{eff} is indicated for three examples in Fig. 4.1. The curves represent the reciprocal of the apparent multiplication as given by a leakage detector (Eq. 4.16) for cases where the leakage decreases, remains constant, or increases as the fuel load increases to criticality. It may be shown from the analysis of Sec. 4.2 that the general shape of the curves would be reversed if the detectors were located in the core instead of outside the core boundary. Hence, if the character of a loading curve is unknown prior to the loading program, the most cautious procedure is to place multiplication detectors both in and outside the reactor core.

The extremes in loading characteristics are illustrated in Fig. 4.2, which shows some data obtained in loading experiments on the PTR. The abscissa in this figure is the number of fuel rods loaded rather than k_{eff} . The fuel was distributed in such a

Fig. 4.1—Dependence of apparent inverse multiplication on k_{eff} for three types of leakage variation when an external detector is used.

Fig. 4.2—Two loading curves: inverse multiplication vs amount of fuel loaded. Curve A was obtained when fuel and moderator were loaded simultaneously into the PTR. Curve B was found when fuel alone was loaded into the assembly after the moderator was completely installed. Both curves were obtained with an external detector.

broad pattern, however, that the statistical weight of successive increments could be considered to be constant. The top curve, labelled A, was obtained when complete fuel slugs containing both fuel and beryllium were used in the loading (the details appeared on pp. 71-83 of KAPL-1207¹). The bottom curve, labelled B, was obtained when fuel alone was added to a core already complete in its beryllium content (as described on pp. 77-79 of KAPL-1180²). Both curves show external (end) counter data.

The optimum loading curve would be a straight line. Of the two examples in Fig. 4.2, Curve B is safer to use for extrapolations than Curve A, despite the fact that the latter is closer to a straight line, for the following reasons:

1. The actual critical mass is smaller than the critical mass suggested by the slope at any given point of Curve A, for Curve A is concave downwards.
2. As fuel is added, the counting statistics improve rapidly for case B, while for case A the statistics show no significant improvement until about 90 per cent of the rods are loaded.

4.4 INVERSE MULTIPLICATION MEASUREMENT OF k

The multiplication method is useful in comparing reactivity changes in a reactor provided that the value of p remains suitably constant.

Consider a critical assembly in which (1) a small reactivity increment R can be calibrated by rod drop or reactor period measurements, and (2) the value of p is constant over the range of interest.

Assume that the reactor is subcritical by an amount v so that

$$1/M_v = v/p \quad (4.20)$$

If k is then reduced by the known amount R ,

$$1/M_R = (v + R)/p \quad (4.21)$$

Similarly, if the reactor is subcritical by an amount w such that

$$1/M_w = w/p \quad (4.22)$$

prior to an unknown reduction X in k_{eff} , then

$$1/M_X = (X + w)/p \quad (4.23)$$

From Eqs. 4.20 and 4.21,

$$p = \frac{R}{(1/M_R - 1/M_w)} = \frac{R}{\Delta(1/M)_R} \quad (4.24)$$

The reactivity X is, similarly,

$$X = p(1/M_X - 1/M_w) = p\Delta(1/M)_X \quad (4.25)$$

The value of X can also be expressed directly in terms of R :

$$X = R \left[\frac{M_R}{M_X} \right] \left[\frac{1 - M_X/M_w}{1 - M_R/M_w} \right] \quad (4.26)$$

Here detector counting rates may be substituted for corresponding multiplication values so that

$$X = R \left[\frac{C_R}{C_X} \right] \left[\frac{1 - C_X/C_w}{1 - C_R/C_w} \right] \quad (4.27)$$

Where large reactivity changes are to be measured, it may be necessary to determine the value of p as a function of reactivity. This can be done if a mechanism is available for changing reactivity by an increment R whose value is either constant or varies in a known manner as the reactivity under measurement is altered.

In this case the reactivity change to be measured is made in a number of more or less equal steps, X_1, X_2 , etc. Initially and after each step X_1 , measurements are made of the incremental change in inverse multiplication due to the reference change R . This will provide a set of values for p :

$$\begin{aligned} p_0 &= R_0 / \Delta(1/M)_{R_0} \\ p_1 &= R_1 / \Delta(1/M)_{R_1} \\ p_2 &= R_2 / \Delta(1/M)_{R_2} \\ &\text{etc.} \end{aligned} \quad (4.28)$$

The values of the incremental steps X_1 may then be calculated using average values of p from Eq. 4.28. The unknown reactivity change X is then

$$X = X_1 + X_2 + \dots = \Delta \left[\frac{1}{M} \right]_{X_1} \left[\frac{p_0 + p_1}{2} \right] + \Delta \left[\frac{1}{M} \right]_{X_2} \left[\frac{p_1 + p_2}{2} \right] + \dots \quad (4.29)$$

The values of p measured in Eq. 4.28 must be determined as the changes are made in X , the particular reactivity increment to be measured. The values of p associated with any other large reactivity change may be different even though such a change may be similar to X but in a different part of the core region.

An example of this method is illustrated by the measurement of the safety margin in PPA-20. A central PPA safety rod (a core fuel rod) was selected as the reference rod to provide the known reactivity R . Since this rod is quite far from the control cylinders in the reflector, it was assumed that its value R would remain constant as the control cylinders are rotated or at most decrease only as fast as the reactivity of the assembly.

Some values of p measured by J. A. Bistline are plotted in Fig. 4.3, as a function of reactivity in dollars. Since the control cylinders in effect control neutron leakage from the core, the leakage should increase as boron is rotated into the core, and therefore p might be expected to increase. Actually p decreased as can be seen in Fig. 4.3. This indicates that although core leakage increased as a result of the boron poison at the core periphery, the attenuation of the neutron flux in the control cylinders and reflector also

Fig. 4.3—Values of the ratio of apparent to true multiplication, $p = \Delta k / \Delta (1/M_{app})$, as a function of the degree of subcriticality.

increased. The value of p changed by about 30 per cent over a range of \$8. Standard deviations of the counting data from six counters are indicated by the vertical lines. At the point \$8 below critical the standard deviation was 15 per cent. This large error value is obtained because the measurement of p involves the difference of two relatively small counting rates. The dashed line indicates the values of p calculated with the assumption that the safety rod value decreased at the same rate as the reactivity of the assembly.

The inverse multiplication measurements have all been based on the assumption that the reactivity value of the central safety rods remains constant, independent of the control cylinder positions. It is not easy to devise an experiment which will test the validity of this assumption. However, an independent calibration of p was carried out by measuring the reactivity value of B^{10} disks in the clean critical reactor, and using this Δk to find p subsequent to a $\Delta(1/M)$ measurement of the disks in the subcritical pile. The justification for considering this an independent calibration was based on (a) the fact that the boron sample involved neutron absorption instead of beryllium scattering and U^{235} fissions as in the safety rod, and (b) the boron sample was located at a point instead of along a line.

The relative results of the B^{10} and the safety rod calibration experiments are outlined in Table 4.1. In general, the two calibration constants agree quite well. Some of the deviations might be accounted for by shadow effects between the three samples when they are close together. The $\Delta(1/M)$ resulting from the B^{10} disks located near the end of the reactor core depend strongly on the particular end chosen, since the counting sets were located predominately at one end of the assembly, viz, the end of the stationary half.

4.5 INDICATION OF RATE OF REACTIVITY INCREASE

One purpose for the experiments summarized in Table 4.1 was to examine the effect of the point source distribution on the $\Delta(1/M_{app})$ measurements. Since the apparent attenuation of the boron reactivity effect along the symmetry axis is comparable at $k = 1.0$ and $k = 0.9$, as measured both by critical experiments and by multiplication experiments, it is concluded that the point source distribution is not a significant problem for subcritical experiments where $k \geq 0.9$.

In contrast to the measurements along the symmetry axis, it is noted that the $\Delta(1/M)$ value of the boron falls off severely in the subcritical reactor for positions near the control cylinder. This, however, is interpreted as a real change in the reactivity effect of the samples near the control cylinders due to a large change in the neutron flux and neutron value in this region of the core.

TABLE 4.1--COMPARISON OF $\Delta(1/M)$ CALIBRATION CONSTANTS USING FUEL SAFETY-ROD STANDARD AND B^{10} DISK STANDARD

Location of B^{10} Disk	Reactivity Value of Three B^{10} Disks* (control cylinders at 66 deg)	$p_{SR} \Delta(1/M)$ of Three B^{10} Disks† (control cylinders at 6 deg)	$p_B / p_{SR} \ddagger$
Ring 2, center	\$1.24	\$1.17	1.06
Ring 2, 10 inches from center in stationary half	0.66	0.72	0.92
Ring 2, 13 inches from center in stationary half	---	0.49	---
in movable half	0.40	0.31	1.00
Ring 3, center	1.13	1.02	1.10
Ring 9, center	0.26	0.11	0.42

* Three B^{10} disks were used in the multiplication experiments while one disk was used in the critical experiment. The quoted reactivity value is three times the single disk reactivity value.

† $\Delta(1/M)$ reactivity values are based on the safety rod calibration constant, p_{SR} .

‡ p_B is the B^{10} calibration constant.

The results of the examination indicate that the inverse multiplication technique is quite good for applications where $k \geq 0.9$. However, the accuracy and ease of experimentation is considerably poorer than that associated with critical experiments.

4.5 INDICATION OF RATE OF REACTIVITY INCREASE

The routine approach to criticality in a critical assembly involves making large reactivity increases in a relatively short time. The shutdown margin for a critical assembly may be in the order of \$10 to \$30 of reactivity and usually this is recovered in about 5 to 10 min. Although the drive speed may be controlled to decrease the reactivity rate as the assembly approaches critical, normal monitoring with neutron counters or ion chambers does not provide information to the operator which can be readily interpreted in terms of rate of reactivity increase.

The use of a sensitive period meter permits direct monitoring of the approximate logarithmic reactivity rate.

If the counting rate for a neutron detector is

$$C = \frac{C' p_S}{1 - k} \quad (4.30)$$

the use of a logarithmic response circuit produces an output signal

$$V = \ln C = \ln (C' p_S) - \ln (1 - k) \quad (4.31)$$

Differentiation of this signal with respect to time gives

$$\frac{dV}{dt} = - \frac{dk/dt}{(1-k)} = \frac{1}{T} \quad (4.32)$$

where T is the reading of the reciprocal scale of the period meter and the value of $C' p S$ is assumed to be constant.

A constant reading of the period meter indicates that reactivity is increasing at a rate proportional to the reactivity margin below critical. Thus a 20-sec period indication would result if the reactivity margin below critical were decreasing at a rate of 5 per cent per second. In this case the rate of increase of reactivity would be 50¢/sec when the reactor is \$10 below critical and would decrease to 1¢/sec for a reactivity of 20¢ below critical.

This treatment does not consider the effect of transients introduced by the delayed neutrons, which become significant near critical. However, these effects produce an increase in the period meter reading (shorter indicated periods), which errs on the safe side.

The period meter can be arranged to operate the reactor shutdown system in the event that the percentage rate of reactivity increase exceeds a predetermined value. For such a trip system to provide adequate protection it is necessary to use a sensitive detector or a strong neutron source. The log-period circuit must receive a signal in excess of its minimum threshold value for a sufficient time to compute the period and operate the shutdown system before dangerous reactivity levels are achieved.

REFERENCES

- | | <u>Referred to
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| 2. Report of the Physics Section for December 1953, January, February 1954, pp. 77-79, Report KAPL-1180. | 4.3 |

Chapter 5

METHODS OF CALIBRATING CONTROL RODS

F. FEINER and P. G. KLANN

5.1 INTRODUCTION

Many aspects of present-day critical assembly techniques hinge upon the use of a well-calibrated set of control rods. We may mention such widely differing examples as the determination of the adequacy of reactor shutdown systems, and the comparison of calculations with experimentally determined reactivity and temperature coefficients. Considerable effort has been expended in the development of calibration methods. It is the purpose of this chapter to describe and compare some of the techniques presently used at the Knolls Atomic Power Laboratory.

To calibrate any control system, it is necessary to obtain a yardstick or "ruler" whose value can be found in terms of some natural reactivity unit, such as inhours or dollars. This may be either an element of the control system itself or a safety rod whose size or mechanical arrangement makes it more convenient for this purpose. We shall now discuss ways of evaluating the reactivity worth of such a "ruler," or any readily removable or insertable amount of fuel or absorbing material.

5.2 ROD-DROP METHOD

A. Basic Equations

The rod-drop method makes use of the fact that the consequence of a sudden reduction in multiplication from criticality by an amount Δk is to reduce the flux by a factor $1/[1 + (\Delta k/\beta_{\text{eff}})]$. Here β_{eff} is the "effective" delayed neutron fraction, larger than the actual fraction β because the delayed neutrons are less energetic than the prompt ones and hence less likely to leak out from the core. That is to say, if n_0 is the steady-state neutron density prior to the reduction in multiplication (henceforth called the "drop") and n_1 is the neutron density a few prompt generation times after the reduction

$$\frac{n_1}{n_0} = \frac{1}{[1 + (\Delta k/\beta_{\text{eff}})]} \quad (5.1)$$

This expression can be derived as follows from the one-group space-independent kinetic equations for the neutron density n as a function of time t :

$$\frac{dn}{dt} = \frac{(k_p - 1)}{\tau} n + \sum_i \lambda_i c_i \quad (5.2)$$

$$\frac{dc_i}{dt} = \frac{\beta_i n}{\tau} - \lambda_i c_i \quad (5.3)$$

Here k_p = prompt multiplication constant,
 τ = prompt generation time,
 λ_i = decay constant of i th group of delayed neutron precursors,
 c_i = spatial density of i th group of delayed neutron precursors,
 β_i = effective yield of i th group of delayed neutron precursors,
 $\sum_i \beta_i = \beta_{eff}$

Prior to the drop, when the precursor formation rate and the flux are not changing,

$$n_0 = \frac{\tau \sum_i \lambda_i c_i}{(1 - k_p^0)} = \frac{\tau \sum_i \lambda_i c_i}{\beta_{eff}} \quad (5.4)$$

where k_p^0 denotes the prompt multiplication constant before the drop when the assembly is at delayed critical. Any external neutron source has been removed before this equilibrium situation is attained. Subsequent to the drop, three distinct time intervals may be considered:

Interval 1., the transient time, within a few prompt generation times of the drop, say from 0 to 10^{-4} sec;

Interval 2., the interval between the transient time and a time somewhat less than the half-life of the shortest-lived significant delayed group, i. e. from 10^{-4} sec to 10^{-2} sec;

Interval 3., times greater than 10^{-2} sec after the drop.

During Interval 2 the precursor density is unchanged and a quasi-steady-state situation exists in which the neutrons resulting from precursors established prior to the drop are multiplied by the reactor which now has a prompt multiplication constant k_p^1 . Thus

$$n_1 = \frac{\tau \sum_i \lambda_i c_i}{(1 - k_p^1)} \quad \text{and} \quad \frac{n_1}{n_0} = \frac{\beta_{eff}}{(1 - k_p^1)} \quad (5.5)$$

Writing $k^1 = k_p^1 + \beta_{eff}$ and $\Delta k = 1 - k^1$, Eq. 5.5 becomes

$$\frac{n_1}{n_0} = \frac{1}{[1 + (\Delta k / \beta_{eff})]}$$

B. Rod-Drop Technique

If the quasi-stationary flux in the post-drop Interval 2 described above could be attained and measured accurately, we would then have a simple means of evaluating $(\Delta k / \beta_{eff})$. This, however, is not generally possible since the recording trace of any flux-measuring detector will reflect unwanted transient effects (including those of the rod motion) in the time interval of interest. To evaluate n_1 , it is thus necessary to make use of the trace for times during Interval 3 and extrapolate it back to Interval 2 by using the delayed neutron fractions and their lifetimes. This technique has been used with success and good reproducibility, but it is cumbersome and time-consuming.

A much more convenient alternative was developed by H. F. Schulz. He found that if experimental observations on the post-drop flux are used to calculate the ratio

$$R \equiv \frac{\int_{-\tau}^0 n_0(t) dt}{\int_{\epsilon}^{\epsilon+\tau} n_1(t) dt} = \frac{n_0 T}{\int_{\epsilon}^{\epsilon+T} n_1(t) dt} \quad (5.6)$$

then

$$(\Delta k / \beta_{eff}) = C(R-1) \quad (5.7)$$

Here T is a time of the order of 15 sec, and ϵ is a short time well into Interval 3, i.e., perhaps 10^{-1} sec after the drop; in practice, ϵ may be the instant at which the rod or rods have reached the end of their stroke. The quantity C is a constant, generally called the Schulz constant.

The value of C was determined by performing a series of drops in an assembly having an intermediate energy spectrum and evaluating $(\Delta k/\beta_{\text{eff}})$ by the extrapolation method described in the preceding paragraph. The reactivity changes were in the range from \$.60 to \$1.20, and they led to a least squares value for C of 0.417 if T equals 15 sec. It was subsequently shown by H. Hurwitz¹ that C should indeed be quite insensitive to the magnitude of Δk ; its values should range from 0.37 to 0.42, the exact limits being determined by the parameters of the delayed neutron fractions. On the basis of the delayed neutron information available in 1952,² Hurwitz made the calculations represented in Figs. 5.1 and 5.2. Figure 5.1 shows the post-drop flux as a function of time for a number of values of Δk ; Fig. 5.2 displays the "constant" calculated from the $n_1(t)$ curves of Fig. 5.1. The measured value, 0.417, agrees reasonably well with the calculations at the smaller values of R , for which the experiments were done.

The equipment used in the rod-drop measurements consists of 15-sec counting circuits gated by the complete insertion of the rod, i.e., the post-drop counting interval starts after the transient effects associated with the drop have died out. The pre-drop intervals are started manually and repeated until satisfactory duplication indicates a

Fig. 5.1—Reactor neutron level decay (calculated) for rod drops producing various reactivity changes.

5.2 ROD-DROP METHOD

Fig. 5.2—Theoretical dependence of rod-drop calibration "constant" C on magnitude of drop.

steady power level. The equipment permits simultaneous measurement to be made on several counters positioned around the reactor.

C. Limitations of the Rod-Drop Method

The foregoing discussion implicitly assumes that modal changes associated with the rod drop are negligible, i.e., that the detecting system samples the pre- and post-drop fluxes equally. Unequal flux sampling may arise from two causes: (1) Local perturbations of the flux shape in the counter vicinity due to the dropped rod; (2) Real changes in the fundamental mode of the reactor between the pre- and post-drop configurations.

Regarding the former effect, it is found in practice that unless considerable care is exercised, variations with counter position of 15 per cent in the measured value of Δk may be observed for drops as small as 10¢ in a thermal reactor. In intermediate spectrum reactors (used in the determinations of the Schulz constant) such sensitivity was not observed. However, even in thermal reactors judicious placement of counters can prevent this complication and permit sampling of the flux as a whole.

The modal effect is most clearly encountered when a rod drop is performed on a whole bank of rods, as in an effort to determine the reactivity of the reactor in a shut-down condition. A counter located near the peak of the pre-drop flux distribution will see a larger sample of the pre-drop flux than of the post-drop flux; it will thus indicate a larger apparent Δk than that measured from other counter locations. Such behavior has been observed experimentally (on the Bettis Pressurized Water Reactor flexible assembly, for example). In experiments of this nature, the assumption of an unchanged fundamental mode is clearly violated and some modification of the theory is required.

D. Application of Rod-Drop Method to Large Δk

When the reactivity change due to the dropping of a rod bank is large (of the order of \$5 or more) the modal changes may be taken into account in a simple way. Inspection of Eq. 5.1 shows that for drops of \$5 or more ($\Delta k/\beta_{eff} \geq 5$), the flux decreases by a factor of six or more within a few prompt generation times. It thus becomes reasonable to consider as negligible the number of additional delayed neutron precursors formed during the early part of the post-drop interval (say 10 or 20 sec, i.e., one or two precursor decay "periods"). In this situation we can approximate Eq. 5.3 by

$$\frac{dc_1}{dt} = -\lambda_1 c_1$$

whose solution is

$$c_1(t) = c_1(0)e^{-\lambda_1 t} = \frac{\beta_1 n_0}{\lambda_1 \tau} e^{-\lambda_1 t} \quad (5.8)$$

where the $c_1(0)$ are the precursor concentrations prior to the drop. The delayed neutrons emitted per unit time,

$$\sum_1 \lambda_1 c_1(t) = n_0 \sum_1 \left(\frac{\beta_1}{\tau} \right) e^{-\lambda_1 t},$$

then constitute the source which is multiplied by the reactor. The spatial distribution of this source is that of the pre-drop flux, $n_0(\underline{r})$, so that in order to apply the general theory for a multiplying system³ we must expand this source in terms of the eigenmodes of the post-drop reactor. Denoting these as $\Psi_n(\underline{r})$, we have

$$n_0(\underline{r}) = \sum_{n=0}^{\infty} a_n \Psi_n(\underline{r}) \quad (5.9)$$

and thence,

$$n_1(\underline{r}, t) = \sum_{n=0}^{\infty} \frac{a_n \Psi_n(\underline{r})}{(1 - k_{pn})} \sum_1 \left(\frac{\beta_1}{\tau} \right) e^{-\lambda_1 t} \quad (5.10)$$

where k_{pn} is the prompt multiplication constant of the n th mode neutrons in the post-drop reactor.³ The higher mode multiplication constants are related to that of the fundamental mode by³

$$\frac{k_{pn}}{k_{p0}} = \frac{1 + M^2 B_0^2}{1 + M^2 B_n^2}$$

where B_0^2 and B_n^2 are total bucklings for the fundamental and n th mode respectively and M^2 is the migration area. As expressed in Eq. 5.10, $n_1(\underline{r}, t)$ is thus a function of k_{p0} , both explicitly and implicitly through the k_{pn} .

To apply the foregoing formalism, let us consider a one-dimensional problem, i.e., $n_0(\underline{r}) = n_0(z)$. We then have

$$n_1(z, t) = \sum_{n=0}^{\infty} \frac{a_n \Psi_n(z)}{(1 - k_{pn})} \sum_1 \left(\frac{\beta_1}{\tau} \right) e^{-\lambda_1 t} \quad (5.11)$$

$$\Psi_n(z) = \sin(n\pi z/L)$$

where L = post-drop reactor height plus reflector savings, and

$$\frac{k_{pn}}{k_{p0}} = \frac{1 + M^2 (B_t^2 + B_{z0}^2)}{1 + M^2 (B_t^2 + B_{zn}^2)} \quad (5.12)$$

where B_t^2 is the transverse buckling and $B_{zn}^2 = (n\pi/L)^2$.

To get an estimate of the Δk due to the dropped bank we would then proceed as follows: (1) measure $n_0(z)$, (2) measure $\int n_1(z, t) dt$ for an interval of say 15 sec after the drop, and (3) compare this measurement with calculations of $\int n_1(z, t) dt$ for a family of values of k_{p0} and the same time interval and choose the value giving the best fit. In practice, an experimental determination of $\int n_1(z, t) dt$ may be difficult. In this case it may be possible to make use of the difference in shape of $n_1(z)$ for different values of k_{p0} and make measurements at a small number of critical positions.

5.3 PERIOD MEASUREMENTS

The fundamental expression relating reactivity and period may be written as⁴

$$\frac{(k-1)}{k\epsilon\beta} - \frac{\tau\omega}{k\epsilon\beta} = 1 - \sum_1 \frac{\epsilon_1 \beta_1}{\epsilon\beta} \frac{\lambda_1}{\omega + \lambda_1} \quad (5.13)$$

where $\frac{(k-1)}{k\epsilon\beta} = \rho$, the excess reactivity in dollars,
 ω = the inverse e-folding time (0.693/doubling time),
 β_1 = the fraction of all fission neutrons which result from the i th precursor group,
 $\beta = \sum_1 \beta_1$ = the fraction of all fission neutrons which are delayed,
 ϵ_1 = a factor giving the relative importance of the i th delayed neutron group. All the ϵ_1 will be taken as being equal in this discussion; $\epsilon_1 = \epsilon$, where $\epsilon = \sum_1 \epsilon_1 \beta_1 / \beta$
 λ_1 = the decay constant of the i th precursor group,
 τ = the prompt generation time.

For $\tau < 10^{-3}$ sec and $\rho < 20\%$, the second term on the left hand side is about 2 per cent of the first. For intermediate reactors τ is of the order of 10^{-4} sec so the term $\tau\omega/k\epsilon\beta$ may be neglected altogether. Assuming $\epsilon_1 = \epsilon$, Eq. 5.13 then becomes:

$$\rho = 1 - \sum_1 \frac{\beta_1}{\beta} \frac{\lambda_1}{\omega + \lambda_1} \quad (5.14)$$

It should be noted here that one may assign an ordering to the directness of our experimental knowledge of the different units of reactivities, i.e., inhours, dollars, and per cent Δk . Inhours are, of course, direct experimental observations of the rate of power level rise of a reactor and thus require no interpretation. Dollars are determined by Eq. 5.13. Under the constraint that the second term of the left-hand side of Eq. 5.13 is negligible compared to the first (i.e., $\tau\omega/(k-1) \ll 1$), and the assumption that $\epsilon_1 = \epsilon$, the evaluation of reactivity in dollars depends only on a knowledge of the β_1/β , i.e., the relative yields (without effectiveness factors) of the various delayed groups. To convert dollars to per cent Δk , however, requires actual knowledge of the effectiveness factor ϵ , which differs from one reactor to another because of different leakage conditions.

From the expression in Eq. 5.14 we may construct a curve of reactivity versus ω (or doubling-time since this is more convenient). Figure 5.3 is such a curve for U^{235} based upon the delayed neutron data of Keepin, Wimett and Zeigler.⁵

Doubling-time calibration measurements are generally made by withdrawing the rod in question while the reactor is critical at low power. The time required for the power to double is then found by observing the current from several ionization chambers located around the reactor. In the course of a number of such doublings the reactor power increases by one or two decades. As the reactor power increases, the current fluctuations decrease so that, in general, more weight is put on the later determinations, provided there is no evidence of saturation of the chamber currents. By inspecting a series of five or so measurements one should satisfy oneself that a true stable period has been reached and observed. Experimental aspects of the period-measurement method are discussed in Vol. II, Chap. 8 of this Handbook (in Sec. 8.17 (e) "Rod-Bump Period Method").

The use of period measurements in calibrating rods has in principle somewhat more severe restrictions than the rod-drop method. A practical lower limit for reactivity calibrations by means of period measurements is set by the possibility that for doubling times of longer than 2 min pile drift may introduce uncertainties into the measurements. An upper limit for calibrations is set by the fact that reactivities greater than 20% result in periods short enough to trip the safety circuits. In addition it should be noted that between successive period measurements time must be allowed for the delayed precursors to reach equilibrium. This time is of the order of 5 or 10 min.

A. Comparison of Period and Rod-Drop Measurements

Considerable use has been made of the two methods described so that it may be of interest to compare them. While good period measurements are restricted to a range of reactivity between 5% and 20%, rod-drop measurements may in principle be made over a much wider range. However, rod-drop measurements do in practice suffer from the

Fig. 5.3—Excess reactivity versus reactor period, calculated under restrictions given in text of Sec. 5.3.

counter location effects previously alluded to even for small drops, while the agreement between different chambers in a period measurement is much better, ~3 per cent. Thus it has been found that, in the 10¢ to 20¢ range, period measurements are easier to make than rod-drop measurements. By paying close attention to counter positioning, it is possible to obtain agreement between rod-drop measurements and period measurements of reactivity to within 6 per cent.

5.4 ADDITIONAL METHODS

A. Pulse Methods

Recently the pulse method has been used⁶ with good success in the determination of reactivity. It may be shown that under certain conditions (for example, long enough after a pulse that all the modes but the fundamental have died down, and yet soon enough that the delayed neutrons are not dominant) the neutron density decays as $e^{-(\lambda - k'_{po})t/\tau}$ following the introduction of a short burst of neutrons into a subcritical assembly at $t=0$. The slope of the (log flux) vs (time) curve is thus $-\Delta k_p/\tau$. By determining τ by means of a pulsing experiment at delayed critical, and then making another experiment after the unknown Δk has been introduced, Δk may be found. Good agreement between pulse measurements and a combination of period and rod-drop determination has been obtained for values of Δk as large as \$15.

B. Rod Oscillator

Calibration technique based on the periodic oscillation of a section of rod has been used successfully at Bettis and is described in Vol. II, Sec. 8.17 (d) of this Handbook and elsewhere.⁷ In this method the oscillating component of the power level resulting from the rod oscillation is measured and related to the Δk of the rod by one of two methods: (1) absolutely by an integration of the pile kinetic equations, or (2) in terms of a measurement of one of the previously described methods. That is to say, if we denote the oscillating component and the steady-state neutron density by δn and n respectively, we have $\delta n/n = A(\omega)\Delta k$ where ω is the frequency of oscillation. $A(\omega)$ may be calculated from the pile kinetic equations or it may be found empirically by oscillating a known Δk .

The agreement between results of this method and the ones previously described is good, and there are some practical advantages to using it rather than the period measure-

REFERENCES

ments for calibration of a control system. A large number of differential rod worths may be obtained much more quickly since (1) the steady-state power is constant so that no time is lost in waiting for the delayed precursors to return to equilibrium, and (2) it is not necessary for the reactor power to be exactly constant prior to starting a measurement since it is only the alternating component which is being observed. The use of this technique, of course, requires the availability of the necessary equipment to oscillate rods, preferably with variable period and amplitude.

C. Extension of Single-Rod Calibration

In the foregoing we have been primarily concerned with the calibration of a single rod or part of a rod. The extension to the calibration of a control system is fairly straightforward and may be accomplished in essentially two ways, i.e., by using the calibrated rod as a "ruler," or by repeated use of the methods described. The former technique involves the repeated substitution of portions of the as-yet-uncalibrated rods for the "ruler" with suitable intermediate compensating changes on other rods. Care must be exercised in regard to coupling between the rods, i.e., the extent to which the value of the ruler depends on the configuration of the other rods. It may thus be necessary to determine the reactivity value of the "ruler" several times in the course of calibrating the whole control system.

Alternatively, the direct calibration methods described may be used repeatedly. It has already been mentioned that the rod oscillator technique lends itself well to the determination of a large number of differential rod worths. Similarly, a series of period measurements and rod drops may be performed in principle, although in practice this may be more time consuming than use of the "ruler."

The question of the total value of the rods of a control system again involves the coupling between the rods. In general, the sum of the individually calibrated rods is greater than their bank worth, the quantity of interest in connection with shutdown reactivity. Perhaps the most convenient way of determining the bank worth is by means of the pulse technique. The bank worth compared with the sum of the individual rods then gives some insight into the coupling effects.

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Chapter 6

ACTIVATION EXPERIMENTS

H. B. STEWART and G. B. GAVIN

6.1 INTRODUCTION

Because the absorption of neutrons is readily detectable in those materials which become radioactive upon exposure to neutron irradiation, activation experiments represent a very important part of any critical experiment program. Many of the activation techniques which have been used in the KAPL critical experiment program are familiar to nuclear physicists but are included in this discussion for completeness. Some of the activation techniques which are described were developed as part of the KAPL critical experiment program, and in some cases the measurements were important principally because of the interest in intermediate energy spectrum reactors. Applications of activation experiments include: 1. neutron economy studies, 2. power distributions, and 3. the determination of the neutron energy spectrum.

1. Neutron economy experiments are those which determine the neutron absorption in each of the reactor materials. Although reactivity coefficient measurements (such as those described in Chaps. 7, 8, and 9) are also important in neutron economy studies, particularly where the neutron absorption in a reactor material does not lead to a radioactive isotope whose activation can be measured, most neutron economy experiments are based fundamentally on activation measurements. By means of activation experiments it is possible to measure the neutron absorption in various poison materials relative to the fission rate in the fuel, even though the effective cross-sections for these materials may not be known accurately for the reactor spectrum under study.
2. The determination of the relative heat generation rates in different parts of the reactor core is crucial to the engineering design of power reactors. In order to allow access to the experimental facility, it is necessary to limit the operation of critical assemblies to power levels so low (of the order of one watt) that the heat generation cannot be measured directly by temperature-sensing equipment. Nevertheless, the relative amounts of fission-product activity induced in the fuel elements within the critical assembly core allows the power distribution to be estimated in a corresponding power reactor. Furthermore, by calibrating in a standard flux the induced fission-product activity per unit fission rate, activation experiments can be used to make an absolute determination of the total reactor fission rate and, consequently, of the reactor power. The details of these calibrations and some examples of applications will be discussed in Secs. 6.2-6.4.
3. Two methods for determining the energy distribution of the neutron flux have been used for investigating reactor spectra. One method is to measure the fraction of fissions in fuel which are caused by neutrons having energies below a certain cut-off energy by comparing the fission-product activity induced in a bare fuel disk to that in a similar fuel disk which is shielded by some absorbing material (such as cadmium) which absorbs all neutrons below a cut-off energy. The other technique for measuring the energy distribution of the neutron flux in a reactor is to observe the activity induced in resonance absorbers having a

6.2 NEUTRON ABSORPTION MEASUREMENT BY ACTIVATION

single strong resonance. Because the resonance absorption is quite often significant in other resonances as well as the particular resonance of interest, special techniques are usually needed to distinguish the absorption in the single strong resonance from the total resonance activation. These techniques and the calibration procedures will be described in Sec. 6.5.

6.2 NEUTRON ABSORPTION MEASUREMENT BY ACTIVATION

The radioactivity induced in a sample during a reactor exposure is a measure of the neutron absorption in the sample. In this section techniques for measuring the activation are described and the interpretation of the activation measurements are discussed.

In most activation experiments, which involve thin samples or foils, beta counting with proportional counters has been found to be the most satisfactory method for measuring the activity.

However, it has been the practise in this laboratory to count the gamma activity of the fission products in the enriched uranium fuel in order to obtain a measure of the fission rate that has occurred in the fuel. The natural beta background of the U^{235} in the PPA fuel disks tends to be relatively large; hence, the amount of statistical accuracy required in beta counting is usually so large that counting losses become appreciable. Gamma counting has the additional advantage over beta counting in that there is less self-absorption in the sample. Beta counting tends to emphasize surface activation effects, while gamma counting is more indicative of the volume activation.

A third activation technique which has been used occasionally in fission rate measurements is that of counting the beta activity of fission fragments which have been caught by aluminum foils in contact with the fuel disk. The most important advantage of this technique is that purely surface effects can be measured if desired since the range of the fission fragments in fuel is very small. The actual counting technique is the same as that for counting the beta activity induced in foils.

The efficiency of a particular counting arrangement depends on the geometry of the counting tube and the sample. In order to assign a quantitative measure to the counting efficiency, a quantity known as the "counting geometry" is defined as the ratio of the events counted by the counter to the number of disintegrations which occurred in the sample during the same time interval. The counting geometry as defined here depends not only on the arrangement of the counting tube and sample, but also on the absorption and scattering of the beta particles by the sample and surroundings.

A. General Theory of Activation Measurements

The rate of absorption of a neutron flux in a foil can be described by the following general relations:

Let p = number of neutrons absorbed/cm²/sec in a foil
 x = thickness of foil in cm
 Σ = absorption cross-section in cm²/cm³
 nv = thermal neutron flux in neutrons/cm²/sec
 Nv = epithermal neutron flux in neutrons/cm²/sec per unit lethargy interval
 $\mu = \cos \phi$, where θ = angle of incident neutron flux relative to surface normal

For an isotropic thermal neutron flux the absorption rate in a thin flat foil is

$$p_{th} = nv \int_0^1 d\mu \mu [1 - e^{-x\Sigma_{th}/\mu}] \quad (6.1)$$

where Σ_{th} is an appropriately averaged thermal absorption cross-section which will be discussed in greater detail somewhat later.

For an isotropic epithermal neutron flux, the absorption rate is

$$p_{res} = \int_{E_{thermal}}^{\infty} \frac{dE}{E} Nv \int_0^1 d\mu \mu (1 - e^{-x\Sigma/\mu}) \quad (6.2)$$

Some elements are known to have an unusually strong resonance in their absorption cross-section in the low-energy region, so that they capture neutrons predominantly within a small energy interval including this resonance. If this neutron capture leads to a radioactive isotope, then the activity induced in such a foil is due largely to the incident neutron flux which lies in this resonance energy region. If Nv is constant or a slowly

varying function of energy in the region where Σ is appreciable, then Nv can be removed from the integral in Eq. 6.2, or

$$p_{\text{res}} = (Nv)_{\text{res}} \int_0^1 \frac{dE}{E} \int_0^1 d\mu \mu (1 - e^{-x\Sigma/\mu}) \quad (6.3)$$

Thus the rate of absorption of epithermal neutrons in the foil is proportional to the neutron flux in the energy region of the resonance.

The intensity of a thermal flux can be determined from the thermal activation of a foil provided that: (1) the relation between p_{th} and the foil activation is known and (2) the value of the integral in Eq. 6.1 is known.

Likewise, if the resonance absorption in a foil is due primarily to a single strong resonance, then the intensity of an epithermal flux can be determined at the energy E_{res} from the resonance activation of the foil provided that: (1) the relation between p_{res} and the foil activity is known and (2) the value of the integral in Eq. 6.3 is known.

The relation in item (1) depends on the exposure time and decay time of the foil and the counting geometry. This relation is the same for thermal and epithermal activations. Item (2) differs for the two cases and will be discussed independently later.

B. Relation between Absorption Rate and Measured Activity of an Activation Foil

If r is the number of radioactive nuclei per cm^2 present at time t in the foil and if they all are of one species having a decay constant λ , then this number depends on the rate of absorption of neutrons and the rate of decay of active nuclei according to

$$\frac{dr}{dt} = p - \lambda r \quad (6.4)$$

After the foil is removed from the neutron flux the rate of decay is

$$\frac{dr}{dt} = -\lambda r \quad (6.5)$$

The solution of these equations with boundary conditions appropriate to a reactor exposure time t_1 , followed by a decay time t_2 which ends at the middle of a counting interval Δt , is well approximated by

$$\begin{aligned} p &= \frac{\lambda \Delta r}{(1 - e^{-\lambda t_1}) e^{-\lambda t_2} (1 - e^{-\lambda \Delta t}) e^{\lambda \Delta t/2}} \\ &\approx \frac{\Delta r / \Delta t}{(1 - e^{-\lambda t_1}) e^{-\lambda t_2}} \end{aligned} \quad (6.6)$$

if $\lambda \Delta t \ll 1$. Here Δr is the number of disintegrations per cm^2 that occur in the foil during the counting interval Δt . Immediately after an infinite exposure, the disintegration rate is equal to the rate of absorption of neutrons, p , and is known as the saturation disintegration rate; the corresponding activity of the foil as measured by a counter is known as the saturation activity, A , of the foil. Then it may be said that

$$A = K p a \quad (6.7)$$

where K is the counting geometry and a is the area of the foil. If the foil has been exposed to a neutron flux for time t_1 and is counted at a time t_2 following the exposure, giving Δc counts in a time Δt , the saturation activity is found from

$$A \approx \frac{K a (\Delta c / \Delta t)}{(1 - e^{-\lambda t_1}) e^{-\lambda t_2}}, \quad (6.8)$$

if $\lambda \Delta t \ll 1$. If the decay time t_2 is measured from the termination of the exposure to the middle of the counting period, then the error in the approximation used in Eqs. 6.6 and 6.8 is particularly small. For example, for $\Delta t = 0.2 T_{1/2}$ the error is less than 0.1 per cent, and for $\Delta t = 0.5 T_{1/2}$ the error is 0.5 per cent, where $T_{1/2}$ is the half-life of the activity.

6.2 NEUTRON ABSORPTION MEASUREMENT BY ACTIVATION

Tables were prepared for the 54-minute indium, the 2.7-day gold, and the 2.59-hour manganese activities (those used in this work) so that values of $e^{-\lambda t}$ and $(1 - e^{-\lambda t})$ could easily be found for any time t .

C. Relation between Fission Rate and Measured Activity in a Fission Foil

The activity which is induced in fissionable material by neutron absorption is the result of the superposition of a large number of fission-product decays. Therefore, it does not follow a simple exponential decay law and the saturation activity cannot be determined. However, if the exposure time t_1 , the decay time t_2 , and the counting period Δt are the same in all experiments and the counting geometry is always the same, the activity is nevertheless proportional to the fission rate which occurred in the foil. For this reason U^{235} and Pu^{239} were always exposed for a standard time of 30 min, and a decay table was available for correcting the activity to a fixed decay time chosen for convenience to be 20 min. The counting interval was standardized at 3 min and the counting geometry was always the same. The decay curve for Pu has been found to be slightly different from that for U^{235} , hence separate decay tables were prepared for the two fissile elements. Within the limits of experimental error the decay curve has been found to be the same for fissions induced by thermal neutrons and intermediate-energy neutrons. Figure 6.1 indicates a typical decay curve obtained for a U^{235} fuel disk under the standard conditions outlined above.

An "apparent counting geometry" for the fuel disks is defined, K_{app} , in such a way that

$$A_{20} = K_{app} p a \quad (6.9)$$

where A_{20} = the fission-product gamma activity of a fuel disk corrected to a 20-min decay time as described above,

and p = fission rate per cm^2 which occurred in the fuel disk during the 30-min exposure.

It should be noted, however, that the definition of a counting geometry as applied to fission foil counting loses some of its significance; for in this case the induced activity is a result of the decay of fission products, and the ratio of the activation to the neutron absorption in the foil is more complicated than for the case of neutron absorption leading to only one active isotope.

Fig. 6.1—Decay of gamma activity of U^{235} fission products following 30-min exposure.

The shielding and scattering effects of the foil itself and of parts of the beta counting equipment make the direct evaluation of the counting geometry practically impossible. Nevertheless, in many neutron economy experiments and in the calibration of the critical assembly power level it is necessary to know the absolute neutron absorption rate in particular absorber samples. In intermediate-energy-spectrum reactors where neither the neutron flux spectrum nor the resonance cross-sections are known accurately, the calculation of the neutron absorption rate is particularly difficult. The resonance absorption rate, p_{res} , can, however, be measured providing the counting geometry K (or K_{app}) can be determined using a known thermal flux.

Since the decay constants and beta energy characteristics of radioactive nuclei are independent of the way in which the nuclei are produced, the counting geometry associated with a radioactive foil resulting from resonance neutron absorption is the same as the counting geometry associated with the same foil activated by a thermal neutron flux. Hence, by irradiating the foil in a calibrated thermal neutron flux where both the neutron flux intensity and the thermal absorption cross-section are accurately known, the counting geometry can be determined. This same counting geometry can then be used to evaluate the absorption rate in the same foil irradiated in the intermediate energy neutron spectrum.

Having calibrated a foil of one material in a standard neutron flux, the counting geometry of foils of another material can be evaluated by exposing the standard foil and the secondary foil simultaneously to the same thermal neutron flux. The absorption rate in the secondary foil can then be evaluated from its thermal cross-section and the measured thermal neutron flux. All of the types of foils which were used in the critical assembly activation experiments were calibrated in this manner relative to a standard foil.

D. Absorption of Thermal Neutrons in Foils

The thermal calibration experiments are done both in a collimated beam of neutrons and in a neutron flux which is isotropic in angular distribution. For the case of a collimated neutron flux nv falling on a unit area of foil which has an absorption cross-section Σ , the absorption rate is given by

$$p = nv (1 - e^{-x\Sigma}) \quad (6.10)$$

while for the case of a neutron flux having an isotropic distribution, the absorption rate is

$$p = nv \int_0^1 d\mu \mu (1 - e^{-x\Sigma/\mu}) \quad (6.11)$$

where $nv/4\pi$ is the flux per unit solid angle. In the case of absorption at thermal energies, the effective value of Σ_{th} depends upon the thickness of the foil, because of the distribution in velocity of the neutrons in the thermal region. If the thermal neutrons are assumed to obey a Maxwellian velocity distribution, then the following interpretation can be placed on nv and Σ_{th} .

Let n = total number of neutrons of thermal energies per unit volume
 v = velocity of a group of neutrons, not necessarily the most probable velocity
 v_0 = most probable velocity at temperature T_0 ; $v_0 = (2k T_0/m)^{1/2}$
 $v_0 = 2200$ m/sec at 20°C
 m = mass of neutron
 k = Boltzmann constant
 T = effective temperature of the neutron flux
 $T_0 = 293^\circ\text{K}$

then, defining a dimensionless velocity variable,

$$y = \left(\frac{mv^2}{2kT} \right)^{1/2} \quad (6.12)$$

the effective neutron flux in thermal equilibrium is

$$nv_{\text{th}} = \frac{4}{\sqrt{\pi}} nv \int_0^\infty dy y^3 e^{-y^2} = \frac{2}{\sqrt{\pi}} nv_0 \quad (6.13)$$

If the neutron flux falls on a foil which absorbs neutrons according to a $1/v$ law in the thermal region, then the absorption cross-section of the foil can be expressed by

$$\Sigma = \frac{\Sigma_0 v_0}{v} \quad (6.14)$$

where Σ_0 is the cross-section for neutrons having velocity v_0 . The value of Σ_0 for most elements is well known at the standard thermal (2200 m/sec) neutron velocity.¹ From the definitions of v_0 and y it follows that

$$\frac{y}{v_0} = \left(\frac{v}{v_0}\right) \left(\frac{T_0}{T}\right)^{1/2} \quad (6.15)$$

and therefore that

$$\Sigma = \frac{\Sigma_0 y_0}{y} \left(\frac{T_0}{T}\right)^{1/2} \quad (6.16)$$

Defining

$$s \equiv \Sigma_0 y_0 \left(\frac{T_0}{T}\right)^{1/2} \quad \text{so that } \Sigma = s/y \quad (6.17)$$

then the number of neutrons absorbed per second by a unit area of the foil in a collimated neutron beam is given by

$$p_{th} = nv_{th} \left[2 \int_0^\infty dy (1 - e^{-sx/y}) y^3 e^{-y^2} \right] \quad (6.18)$$

while for a flux which is isotropic in angular distribution the absorption rate per unit area of the foil is

$$p_{th} = nv_{th} \left[2 \int_0^1 d\mu \mu \int_0^\infty dy (1 - e^{-sx/\mu y}) y^3 e^{-y^2} \right] \quad (6.19)$$

Fig. 6.2—Neutron absorption in thick foils, $1/v$ cross-section.

The factors in the brackets of Eqs. 6.18 and 6.19 have been evaluated by Zahn² for values of $sx = 0.001$ to $sx = 1.0$. The results have been extended to larger values of sx by numerical integration and are indicated in Fig. 6.2. The curve marked A represents the absorption rate per unit flux for a foil of unit area exposed to a monoenergetic flux isotropic in angular distribution, plotted as a function of sx . Curves B and C represent the absorption rate in a $1/v$ detector per unit effective thermal flux for the cases of a collimated thermal neutron flux and a thermal flux isotropic in angular distribution, respectively, each as a function of sx . These curves are the results of the evaluation of the bracketed parts of Eqs. 6.18 and 6.19. The temperature T in Eq. 6.12 is the effective temperature of the neutron flux and is influenced both by the temperature of the diffusion medium and the components of flux in the epithermal energy region. The effective temperature of the neutron flux must be determined by independent experiments.

Equations 6.18 and 6.19 are based on the assumption that the cross-section of the absorber obeys a $1/v$ law in the thermal energy region. The cross-sections of U^{235} and Pu^{239} deviate somewhat from a $1/v$ relation at thermal energies because of their low energy resonances. In order to calculate the absorption rate in U^{235} and Pu^{239} , it is necessary to solve equations similar to 6.18 and 6.19, but with sx/y replaced by $x\Sigma$ where Σ is a more complicated function of y and T than previously. In these cases the solutions must be obtained by numerical integration.

6.3 EVALUATION OF ABSOLUTE POWER AND POWER DISTRIBUTION IN CRITICAL EXPERIMENTS

The rate of fissioning in the entire assembly can be determined if the absolute number of fissions per second occurring in one fuel disk is measured and the power distribution relative to this fuel disk is determined. The relative power distribution can be measured accurately by comparing the relative gamma activity induced in various fuel disks distributed throughout the reactor core. To measure the absolute number of fissions per second occurring in one fuel disk involves the determination of the apparent counting geometry for the fission product activity, as indicated by Eq. 6.9. It can be measured by exposing the fuel disk to a calibrated thermal flux, for which the fission rate can be calculated from the neutron flux and the fission cross-section. Then the gamma activity associated with a known fission rate can be measured.

A. Automatic Neutron-Flux Scanner

As an addition to the gamma counting of individual fuel elements, a flux-scanning instrument has proved to be very useful in the critical experiment program. To obtain radial distributions of power or flux, strips of U^{235} or other foil materials are attached in convenient locations in the interface between the two halves of the PPA. To obtain distributions in the axial direction, the strips are inserted in matrix tubes adjacent to the fuel slugs. The assembly is brought to criticality and operated at a constant power level to give the strips the standard 30-min exposure. After shutdown the strips are removed from the assembly and are scanned by a travelling radiation counter.

If the experiment is carried out in high neutron fluxes, the induced activity in a long sample of uranium can be measured as a function of position along the sample simply by scanning it with a counting-rate meter. With low neutron fluxes, on the other hand, the relatively low counting rate dictates a slow scanning speed, lest statistical fluctuations in the recorded curve be excessive. On the other hand, the low counting rate also requires that the counting be done soon after the sample irradiation, when the induced activity is still decaying too rapidly to remain sensibly constant during the time required to scan. It is possible, of course, to correct the data obtained with a slow scanning speed for the effects of the decay. Since the decay of the fission-product activity depends on the duration of irradiation, both the decay and the irradiation must be timed accurately.

The scanning device which has been used in this laboratory³ corrects automatically for any smooth time decay of radioactivity during scanning without recourse to an empirical decay curve. Exact knowledge of the duration of pile irradiation and the moment of shutdown is not required.

B. Principle of Scanner Operation

When a long thin sample is irradiated in an inhomogeneous neutron flux, the induced radioactivity A , after irradiation has ceased, is given as a function of position z along its length and of time t by

$$A(z,t) = f(z) \times g(t) \quad (6.20)$$

where f gives the variation with position and g represents the time decay of the radioactivity.

Let n be the total number of points along a strip whose relative activities are to be compared. Suppose the scanner starts at one end of the strip and counts at successive points down to the other end and back again. If the activity at $x = x_j$ is measured at time t_j and again at time t_{2n-j+1} , the sum of the two counts recorded will be proportional to

$$f(x_j) \times [g(t_j) + g(t_{2n-j+1})]$$

Comparing this to the counts recorded at the starting point, $x = x_1$, the ratio is

$$\frac{f(x_j)}{f(x_1)} \left[\frac{g(t_j) + g(t_{2n-j+1})}{g(t_1) + g(t_{2n})} \right] \equiv \frac{f(x_j)}{f(x_1)} R \quad (6.21)$$

If the intervals $(t_{2n} - t_{2n-j+1})$ and $(t_j - t_1)$ are chosen to be equal, Fig. 6.3 shows that the ratio R is unity to at least the same degree of accuracy that a linear approxima-

Fig. 6.3—Validity of linear approximation to decay curve.

tion to $g(t)$ is valid over the interval from t_1 to t_{2n} . (R is equal to the ratio of lengths $\overline{BD}/\overline{AD}$ in Fig. 6.3, and this is larger than $\overline{CD}/\overline{AD}$ for all pairs of points.) The linear approximation involved here is not the first term of a Taylor expansion about any point, but rather the chord that intersects the curve $g(t)$ at t_1 and t_{2n} . If the scanning cycle is made short enough, it may be assumed that $R = 1$ and the ratio of the sums of pairs of counts in the two channels may be taken as a measure of the ratio $f(x_1)/f(x_2)$.

When counting uranium strips, the scanner is set to read at ten separate points, counting for 30 sec at each point. Thus a complete back-and-forth scan requires $2 \times 10 \times 30 \text{ sec} = 10 \text{ min}$, plus the small additional time consumed in moving between points. Examination of the empirical decay curve has shown that the linear approximation to $g(t)$ introduces an error of only 0.5 per cent in the relative activity of the two extreme ends of a fuel sample when a single 10-min scan is taken 35 min after shutdown following an irradiation time of 30 min. Of course, the relative error introduced on subsequent scans or for points closer together in the sample is less.

In measuring activities that decay exponentially with time, the requirements on the scanning cycle (so that the correction for the time decay during scanning be sufficiently accurate) are particularly simple. Here the ratio R as applied to the two ends of the sample becomes

$$(1 + e^{-\Delta/\tau})/2e^{-\Delta/2\tau} = \cosh(\Delta/2\tau) \quad (6.22)$$

where $\Delta = t_{2n} - t_1$ is the time required for a complete scanning cycle and τ is the mean life of the activity being observed. Thus, to attain a time decay correction accuracy of δ or less, the condition is

$$\cosh(\Delta/2\tau) \leq (1 + \delta), \text{ or, approximately, } (\Delta/\tau) \leq 2\sqrt{2\delta} \quad (6.23)$$

That is, for an accuracy of 0.5 per cent, the scanning cycle should be no more than $1/5$ of the mean life of the activity measured.

To ensure that the counts recorded in the j th channel represent a measure of the relative radioactivity only at the position x_j , it is necessary that the counter be well shielded and collimated. For this reason beta counting is preferable in this application to gamma counting.

The scanner has been used extensively for measurements of neutron flux in low-power experimental reactors, particularly for determining the central-to-average power ratio. An absolute measurement of central neutron flux by conventional methods (with standard foils), together with a number of traverses through the reactor, permits calculation of the total number of fissions in the reactor.

6.4. POWER DISTRIBUTION IN SODIUM-COOLED INTERMEDIATE-ENERGY REACTORS

Knowledge of the power distribution is important both for thermal design and for nuclear design of a power reactor.

A convenient figure of merit for indicating the evenness of power distribution in a reactor is the ratio (P_o/P_{ave}) , where P_o is the central specific power and P_{ave} is the average specific power in the reactor core. On this basis, a reactor with a perfectly flat power distribution would, of course, have a power ratio of $P_o/P_{ave} = 1$. Table 6.1 shows typical values of P_o/P_{ave} measured for a number of the preliminary pile assemblies discussed in Chap. 2, Sec. 2.2. Figures 6.4 and 6.5 indicate axial and radial power distribution curves for two of these intermediate-neutron-energy critical assemblies. The power distributions have the following characteristics which are typical of intermediate energy spectrum reactors:

TABLE 6.1--RATIO OF CENTRAL SPECIFIC POWER TO AVERAGE SPECIFIC POWER IN CRITICAL ASSEMBLIES

PPA	-2	-3	-4	-5	-11	-12	-14	-19	-20	-21	-25	-28	-30
P_o/P_{ave}	1.92	1.61	1.19	1.12	1.09	1.77	1.56	1.45	1.60	1.38	1.36	1.11	1.11

Fig. 6.4—Traverses of power distribution in PPA-4.

Fig. 6.5—Traverses of power distribution in PPA-3 and PPA-5

1. In general, the peak-to-average power ratios in intermediate spectrum reactors have values between 1.2 and 2.0. These ratios are distinctly smaller than the corresponding ratios in thermal reactors for two reasons: First, the effectiveness of the beryllium reflector allows the axial and radial fluxes to decrease only slightly near the edge of the core. Second, the variations and gradients in flux produced by core inhomogeneities tend to be small because of the relatively large effective diffusion lengths for neutrons in intermediate energy spectra.
2. With reflector control, changes in control settings produce rather large changes in the power generation at the edge of the reactor core. Even here, the gradients in the neutron flux are not so severe as in a thermal reactor — again because the intermediate energy neutrons have larger diffusion lengths.
3. The neutron flux distribution can be significantly different from the power distribution. This difference arises because the reflector not only scatters neutrons back to the reactor core but also has a moderating effect which tends to increase the intensity of the low-energy neutron flux at the edge of the core relative to the fast neutron flux.

4. Neutron channelling or streaming, as through coolant channels, can be relatively important in the intermediate energy spectrum because the gradients in neutron flux may be significantly different for different neutron energy groups. Since channels usually exist in axial directions through the core and end-reflectors of a reactor, neutron streaming tends to enhance axial leakage and make the end reflectors less effective than the side reflectors. Furthermore, the channels provide a path for the relatively low-energy neutron flux in the reflector region to stream back into the central core region where the neutron flux tends to be somewhat higher in energy. These effects were particularly noticeable in the relatively fast assemblies with small core dimensions, PPA-2 to PPA-11.

6.5 DETERMINATION OF NEUTRON ENERGY SPECTRUM BY FOIL ACTIVATION TECHNIQUES

The absorption rate in a foil having a predominant resonance at one neutron energy is proportional to the neutron flux at that energy (Eq. 6.3). Furthermore, since the saturation activity A_{res} induced in the resonance foil is proportional to the absorption rate,

$$(Nv)_{\text{res}} = C_{\text{res}} A_{\text{res}} \quad (6.24)$$

It must be emphasized, however, that this relation is valid only when no significant absorption occurs outside the region of the single predominant resonance.

A. Calibration of Resonance Foils in the Sigma Pile

The constant, C_{res} , for the resonance foils can be calibrated by some activation experiments in a sigma pile (also called a "standard pile"). The sigma pile used in these calibrations was a 6 ft by 6 ft by 10 ft rectangular stack of graphite blocks, with the long dimension of the stack lying in a horizontal plane. Two of the graphite blocks which were slightly undersized could be removed from the longitudinal axis of the sigma pile. At a distance 5 cm from the end of one of these blocks a cylindrical hole contained a 3-curie radium-beryllium neutron source which was normally located at a position 3 ft from the end of the pile. The other block, which was located collinear with and against the end of the source block, had eight lateral slots beginning at 5 cm from the end of the block and spaced 10 cm apart. Hence, foils could be placed between 10 to 80 cm from the source for activation experiments.

The maximum epithermal flux which was available in the sigma pile was of the order of $Nv = 3 \times 10^3$ neutrons/cm²/sec per unit lethargy interval. The maximum thermal flux which was available was approximately $nv = 4 \times 10^4$ neutrons/cm²/sec.

The theory of the calibration of the constant C_{res} is as follows:

- Let q = the slowing-down density of the neutrons, i.e., the density of neutrons crossing energy E per second, per unit volume,
 Q = the number of neutrons per second emitted by the source,
 λ = the mean free scattering length for the neutrons at E_{res} ,
 ξ = the average logarithmic energy loss per collision.

Then, from age theory, and Eq. 6.24,

$$Nv = \lambda q / \xi = C_{\text{res}} A_{\text{res}} \quad (6.25)$$

If the source is in an infinite moderating medium (approximated by the large graphite sigma pile), then the slowing-down rate of the neutrons integrated over the complete volume of the medium must equal the number of neutrons being produced per second in order that equilibrium be maintained. In other words,

$$4\pi \int_0^\infty dr \, r^2 q = Q \quad (6.26)$$

From Eqs. 6.25 and 6.26, the value of C_{res} is found to be

$$C_{res} = \frac{\lambda}{\xi} \frac{Q}{4\pi \int dr r^2 A_{res}} \quad (6.27)$$

A direct calibration of the strength Q of the radium-beryllium source has not been attempted in this laboratory. However, if the activation of resonance foils is measured both in a standardized sigma pile and in an uncalibrated sigma pile, the source strength of the uncalibrated source can be evaluated by means of Eq. 6.27. This was done using the Argonne sigma pile and gold foils. The source strength of the 0.5-curie Ra-Be source in the Argonne sigma pile was taken to be 5.7×10^6 neutrons per second (A. Wattenberg, private communication). From this value, and the results of activation experiments with gold foils, the strength of the 3-curie Ra-Be source in the KAPL sigma pile was calculated to be $Q = 3.1 \times 10^7$ neutrons per second. In order to evaluate C_{res} , cadmium-covered foils were exposed in a number of slots in the sigma pile, so that a value of $\int dr r^2 A_{res}$ could be obtained by numerical integration, and then C_{res} was found by Eq. 6.27.

B. Self-Absorption, or Foil Sandwich, Technique for Measuring Epithermal Neutron Fluxes

The derivation of Eqs. 6.3 and 6.24 involved the assumption that the neutron absorption was appreciable in an energy region which contained only one resonance. If the epithermal neutron flux is approximately constant with energy, this assumption seems to be fairly good for indium, gold, and manganese foils. However, if the neutron flux is appreciably greater in the higher intermediate-energy region (from 10^3 to 10^5 ev) than it is between 1 to 100 ev, an appreciable fraction of the absorption may occur at resonances in the higher energy region where the flux is more intense. In such a case, the foil activation is not a measure of the neutron flux at the predominant resonance energy of the foil.

A more accurate method for measuring the neutron flux at E_{res} , suggested by T. M. Snyder, is based on measuring the self-absorption of neutrons in a foil which has a thickness such that $x\Sigma \approx 1$ at E_{res} and $x\Sigma \ll 1$ at all energies outside the predominant resonance region.

Figure 6.6 illustrates the general principle of this self-absorption or foil sandwich method. The dashed horizontal line at the top of Fig. 6.6a indicates the neutron energy spectrum in the reactor (assumed constant in the region of the two resonances) as it impinges on a single foil. The solid line in Fig. 6.6a shows the absorption of this incident flux that takes place in such a foil.

If a foil is covered on both sides by similar foils, then Fig. 6.6a represents what occurs in either one of the outer foils. Figure 6.6b indicates the fraction of the flux transmitted by the outer foil, assuming again that the epithermal neutron flux spectrum is constant. This same fraction transmitted appears in Fig. 6.6c as the flux impinging on the inner foil.

The inner foil, exposed to a flux reduced in the region of each resonance, absorbs a smaller fraction of the original flux in that region, as is indicated by the solid curve on Fig. 6.6c. The amount by which it is reduced, as compared to the single foil, is indicated by the shaded area on Fig. 6.6d. This difference between the single and inner foil, the self-absorbed part of the activation, is larger for the more pronounced resonance.

The advantage which is realized by using the self-absorption technique rather than the resonance-absorption method can be seen from the following analysis. (It is assumed that the neutron flux is perpendicular to the plane of the foil in order to simplify the analysis. The demonstration can be extended to the case of an isotropic flux distribution.)

If the complete energy region of neutron absorption is divided into sub-regions of width Δ_1 within which the flux is approximately constant, then the neutron absorption in an outer foil can be represented by

$$P_{single} = \phi_1 \int_{\Delta_1} \frac{dE}{E} [1 - e^{-x\Sigma}] + \sum_{i \neq 1} \phi_i \int_{\Delta_i} \frac{dE}{E} [1 - e^{-x\Sigma}] \quad (6.28)$$

Fig. 6.6—Self-absorption (foil sandwich) method for enhancing resonance flux.

Here ϕ_1 is the neutron flux in the energy sub-region that includes the predominant resonance at E_1 , and $\phi_i (i \neq 1)$ represents the flux in a typical sub-region, including regions where resonances might occur at higher energies. p_{res} , which is the first term on the right, must be determined.

Since the foils are sufficiently thin that $x\Sigma \ll 1$ in all regions except that of the predominant resonance, the exact expression can be expanded in the region of E_1 to the form

$$p_{single} = p_{res} + \sum_{i \neq 1} \phi_i \int_{\Delta_i} \frac{dE}{E} [x\Sigma - (x\Sigma)^2/2 + \dots] \quad (6.29)$$

The energy interval Δ_1 is sufficiently large to include all the significant contribution from any one resonance. The interval Δ_1 is therefore of the order of Γ , where Γ is the total neutron width for absorption. In general, the absorption width Γ_a is much larger than the scattering width Γ_n below 10 kev, and Γ_a is approximately independent of the resonance energy; so the intervals Δ_i for the resonances at higher energies will be at least of the same order of magnitude as the interval Δ_1 for the predominant resonance energy.

The following definitions will simplify the notation:

$$\begin{aligned} \frac{\Delta_1 B_1}{E_1} &\equiv \int_{\Delta_1} \frac{dE}{E} [1 - e^{-x\Sigma}] \\ \frac{\Delta_1 c_1}{E_1} &\equiv \int_{\Delta_1} \frac{dE}{E} (x\Sigma) \\ \frac{\Delta_1 d_1}{E_1} &\equiv \int_{\Delta_1} \frac{dE}{E} (x\Sigma)^2 \\ \frac{\Delta_1 J_1}{E_1} &\equiv \int_{\Delta_1} \frac{dE}{E} [1 - e^{-x\Sigma}]^2 \end{aligned}$$

The factor B_1 is some mean value of $(1 - e^{-x\Sigma})$ over the energy interval Δ_1 and is of the order of 1 since $x\Sigma \approx 1$ in this interval. Likewise c_1 is some mean value of $x\Sigma$ and d_1 is a mean value of $(x\Sigma)^2$ over the interval Δ_1 ; but $c_1 \ll 1$ and $d_1 \ll c_1$ since $x\Sigma \ll 1$ away from the predominant resonance. The quantity J_1 , to be used later, is a mean value of $(1 - e^{-x\Sigma})^2$ in the interval Δ_1 and is, like B_1 , of the order of unity. By these definitions,

$$\begin{aligned} p_{single} &= p_{res} + \sum_{i \neq 1} \frac{\phi_i \Delta_i c_i}{E_i} - \sum_{i \neq 1} \frac{\phi_i \Delta_i d_i}{2E_i} + \dots \\ p_{res} &= \frac{\phi_1 \Delta_1 B_1}{E_1} \end{aligned} \quad (6.30)$$

The ratio of the total outer foil absorption to that in the predominant resonance energy region is given by

$$\frac{p_{single}}{p_{res}} = 1 + \frac{E_1}{\phi_1 \Delta_1 B_1} \sum_{i \neq 1} \frac{\phi_i \Delta_i c_i}{E_i} - \dots \quad (6.31)$$

Similar reasoning is applied to express the neutron absorption in the inner foil, upon which impinges a flux at each energy that is smaller by a factor $e^{-x\Sigma}$ than the flux impinging upon the outer foil.

$$p_{inner} = \phi_1 \int_{\Delta_1} \frac{dE}{E} [1 - e^{-x\Sigma}] e^{-x\Sigma} + \sum_{i \neq 1} \phi_i \int_{\Delta_i} \frac{dE}{E} [1 - e^{-x\Sigma}] e^{-x\Sigma} \quad (6.32)$$

The difference between the absorptions in the two foils is

$$\begin{aligned}
 p_{\text{single}} - p_{\text{inner}} &= \Phi_1 \int_{\Delta_1} \frac{dE}{E} [1 - e^{-x\Sigma}]^2 + \sum_{i \neq 1} \Phi_i \int_{\Delta_i} \frac{dE}{E} [1 - e^{-x\Sigma}]^2 \\
 &= \frac{\Phi_1 \Delta_1 J_1}{E_1} + \sum_{i \neq 1} \Phi_i \int_{\Delta_i} \frac{dE}{E} (x\Sigma)^2 - \dots \\
 &= \frac{J_1}{B_1} p_{\text{res}} + \sum_{i \neq 1} \left(\frac{\Phi_i \Delta_i d_i}{E_i} \right) - \dots
 \end{aligned} \quad (6.33)$$

Here the expression for p_{res} given in Eq. 6.30 has been used. Finally,

$$\frac{p_{\text{single}} - p_{\text{inner}}}{p_{\text{res}}} = \frac{J_1}{B_1} \left\{ 1 + \left(\frac{E_1}{\Phi_1 \Delta_1 J_1} \right) \sum_{i \neq 1} \left(\frac{\Phi_i \Delta_i d_i}{E_i} \right) - \dots \right\} \quad (6.34)$$

The relative error made by ignoring any of the summation terms above is considerably smaller in Eq. 6.34, where it is proportional to d_i/J_1 , than in Eq. 6.31, where it is proportional to c_i/B_1 . (Remember that d_i and c_i are of the order of magnitude of $(x\Sigma)^2$ and $(x\Sigma)$, respectively, while J_1 and B_1 are of the order of unity.) Typical order-of-magnitude values for a resonance might be these:

$$c_i \sim 10^{-3}, \quad d_i \sim 10^{-6}, \quad \frac{E_1}{E_i} \frac{\Phi_i \Delta_i}{\Phi_1 \Delta_1} \sim 10^2$$

Substitute these values into Eqs. 6.31 and 6.34, as if no other resonance contributes, and remember that B_1 and J_1 are of the order of unity

$$\begin{aligned}
 \frac{p_{\text{single}}}{p_{\text{res}}} &= 1 + (\sim 10^{-1}) \\
 \frac{p_{\text{single}} - p_{\text{inner}}}{p_{\text{res}}} &= \frac{J_1}{B_1} \left\{ 1 + (\sim 10^{-4}) \right\}
 \end{aligned} \quad (6.35)$$

This comparison indicates the advantage that lies in the foil sandwich technique for estimating resonance absorption.

A quantity calculable directly from observations on foils activated in an assembly is the ratio

$$\begin{aligned}
 \frac{p_{\text{single}} - p_{\text{inner}}}{p_{\text{single}}} &= \frac{\left\{ \frac{J_1}{B_1} 1 + \left(\frac{E_1}{\Phi_1 \Delta_1 B_1} \right) \sum_{i \neq 1} \left(\frac{\Phi_i \Delta_i d_i}{E_i} \right) - \dots \right\}}{\left\{ 1 + \left(\frac{E_1}{\Phi_1 \Delta_1 B_1} \right) \sum_{i \neq 1} \left(\frac{\Phi_i \Delta_i c_i}{E_i} \right) - \dots \right\}} \\
 &\equiv \frac{(J_1/B_1)}{(1+f)}
 \end{aligned} \quad (6.36)$$

The correction term f defined here has the value, if smaller order terms involving the d_i are neglected,

$$f \approx \left(\frac{E_1}{\Phi_1 \Delta_1 B_1} \right) \sum_{i \neq 1} \left(\frac{\Phi_i \Delta_i c_i}{E_i} \right) \quad (6.37)$$

Although the ratio (J_1/B_1) would be difficult to compute, it can be approximated experimentally by the ratio $(p_{\text{single}} - p_{\text{inner}})/p_{\text{single}}$ found on foils exposed in a neutron flux for which $f = 0$. A neutron spectrum having Φ_1/Φ_i close to zero is found in the sigma pile at positions remote from the source, at least for indium ($E_1 = 1.46$ ev) and for gold ($E_1 = 4.9$ ev).

Table 6.2 gives values of (J_1/B_1) and of f found for indium and gold foils in three different piles. According to the last column of the table, single foil activation measurements to determine $(Nv)_{res}$ by Eq. 6.24 would in PPA-5 give results too high, by 24 per cent in the case of indium, by 77 per cent in the case of gold.

TABLE 6.2--FRACTIONAL RESONANCE ABSORPTIONS
IN OUTER FOIL, BY SELF-ABSORPTION METHOD (EQ. 6.36)

Foil	E_1	In Predominant Resonance	In Secondary Resonances		
		(J_1/B_1) Sigma Pile	f Sigma Pile*	f PPA-4	f PPA-5
Indium	1.46 ev	0.46	0.01 ± 0.04	0.14	0.24
Gold	4.91 ev	0.53	0.04 ± 0.04	0.47	0.77

*The correction term f measured 10 cm from source in sigma pile.

Using Eqs. 6.7 and 6.33 it is found that for the self-absorption technique

$$(Nv)_{res} \approx C'_{res} (P_{single} - P_{inner}) \quad (6.38)$$

Fig. 6.7—Epithermal neutron energy spectra in three assemblies.

The constant C'_{res} can be obtained from sigma pile calibrations just as C_{res} was obtained using Eq. 6.27. In this case the calibration is more difficult, however, because errors in counting rates are magnified by using differences

The effect of scattering resonances has been neglected in this discussion. If the foils are sufficiently thin, however, that $x\Sigma \ll 1$ at all energies other than the absorption resonance energy, then the essential features of the theory are not changed.

C. PPA Spectrum Measurements

Figure 6.7 shows the energy distributions of the epithermal fluxes at the center of PPA-3, -4, and -5.

The points indicate the measured values of the flux at 1.46, 4.9, and 340 eV, using the resonance self-absorption technique for indium, gold, and manganese foils. The vertical lines indicate the experimental uncertainties in the measurements. The solid curves are the neutron flux distributions obtained from multigroup calculations.

The foils used were annular disks having an outer diameter of 1.5 in. and a hole of 0.5 in. diameter. The indium foils ($E_1 = 1.46$ eV) had to be diluted with a non-absorber in order to attain a small enough effective thickness; they were an alloy of indium and aluminum, containing 10 per cent indium. They weighed 0.74 g each and their effective thickness was 0.0004 in. The gold foils ($E_1 = 4.91$ eV) were pure gold, 0.002 in. thick,

Fig. 6.8—Spatial distribution of epithermal neutral flux—axial traverse in PPA-5.

weighing 1.10 g. The manganese foils ($E_1 = 337$ ev) included 3 per cent copper to make them ductile; they were 0.010 in. thick and weighed 1.83 g.

In epithermal flux measurements some foils were exposed bare and others were shielded by cadmium shield foils (0.020 in. thick) to absorb out the neutron flux below 0.5 ev. These cadmium shield foils had an outer diameter of 2.0 in. and a central hole of diameter 0.2 in., and they covered as much area as was possible for foils stacked in a PPA fuel slug.

Figure 6.8 shows the axial distribution of the neutron flux in PPA-5 as measured by the resonance foils. The curves show quite strikingly the moderating effect of the reflector, which degrades the energies of the faster neutrons even as it returns them to the outer part of the core.

6.6 CADMIUM FRACTION EXPERIMENTS

The cadmium fraction is defined as

$$CF = \frac{p_{\text{bare}} - p_{\text{Cd}}}{p_{\text{bare}}} = \frac{A_{\text{bare}} - A_{\text{Cd}}}{A_{\text{bare}}} \quad (6.39)$$

where p_{bare} = fission rate in a bare fuel disk

p_{Cd} = fission rate in a cadmium-covered fuel disk

$p_{\text{bare}} - p_{\text{Cd}}$ = fission rate in a fuel disk caused by neutrons having energies below the cadmium cut-off

The cadmium fraction of a fuel disk represents the fraction of the fissions which result from neutrons having energies below the cadmium cut-off energy, relative to the total number of fissions in the disk. A large cadmium fraction in a fuel disk would indicate that it had been exposed to a relatively slow neutron energy spectrum. (The cadmium fraction must not be confused with the cadmium ratio, which is defined as $A_{\text{bare}}/A_{\text{Cd}}$ and is equal to $1/(1 - CF)$.)

If the cross-section for cadmium were large below energy E_c and became zero abruptly above energy E_c , then the cut-off energy would be E_c . However, since the cut-off region for cadmium extends from 0.2 to approximately 2.0 ev, the effective cut-off energy will depend on: (1) the thickness of the cadmium shield, (2) the neutron flux distribution in this energy region, and (3) the absorption cross-section of the fuel disk in the same region. It is significant to know the cadmium cut-off energy in the different spectra as a function of cadmium thickness. The cut-off energy, E_c , is defined as the energy above which the absorption in bare fuel equals the total absorption in fuel covered by cadmium. Assuming that no absorptions occur in cadmium-covered fuel below 0.2 ev and that the cadmium transmits all neutrons above 2.0 ev, then the definition of cadmium cut-off can be expressed as:

$$\int_{E_c}^{2\text{ev}} (dE/E) \phi \int d\mu \mu [1 - e^{-x_f \Sigma_f / \mu}] = \int_0^{2\text{ev}} (dE/E) \phi \left\{ \int d\mu \mu [1 - e^{-(x_f \Sigma_f + x_{\text{Cd}} \Sigma_{\text{Cd}}) / \mu}] - \int d\mu \mu [1 - e^{-x_{\text{Cd}} \Sigma_{\text{Cd}} / \mu}] \right\} \quad (6.40)$$

Here ϕ is the epithermal neutron flux and in general is a function of the energy. x_f and Σ_f are the thickness and total absorption cross-section of the fuel; x_{Cd} and Σ_{Cd} are similar quantities for cadmium. If ϕ is known in the energy region from 0.2 to 2.0 ev, then Eq. 6.40 can be solved for E_c by numerical integration.

Flux experiments in PPA-5, using indium, gold, and manganese foils, indicate that the neutron flux in this assembly below 340 ev can be represented by the empirical relations:

$$\phi(E) = a E^{1/2} \quad (\text{at center of core}) \quad (6.41)$$

and

$$\phi(E) = b E^{1/4} \quad (\text{at edge of core}) \quad (6.42)$$

Eq. 6.40 has been solved for both types of spectrum and for a number of cadmium shield thicknesses. The cadmium cut-off energy as a function of cadmium thickness is shown in Fig. 6.9. The cut-off energy curves for the two flux distributions Eqs. 6.41 and 6.42 are practically identical. A similar calculation with ϕ assumed to be independent of E gave cut-off energies as a function of cadmium thickness within 5 per cent of the curve shown in Fig. 6.9. Hence, it can be assumed that the cadmium cut-off energy for a particular cadmium shield is practically independent of the energy distribution of the flux within the limits of variation in the intermediate reactor. The cadmium cut-off energy for an 0.020 in. cadmium shield covering an 0.020 in. U^{235} fuel disk is $E_c = 0.53$ ev.

Fig. 6.9—Effect of cadmium shield thickness on cadmium cutoff energy.

Fig. 6.10—Cadmium fraction distribution in three assemblies.

A. Results of Some PPA Cadmium Fraction Measurements in Preliminary Pile Assemblies

The average cadmium fraction for an assembly is defined as

$$\overline{CF} = \frac{\sum_n A_n (CF)_n}{\sum_n A_n} \quad (6.43)$$

where n refers to an individual fuel disk in the reactor core and the summation is over all the fuel disks. Table 6.3 gives the cadmium fractions at the core center and the edge of some typical assemblies. The average cadmium fraction is also given.

Figure 6.10 shows the variation of the cadmium fractions along an axial traverse for several preliminary pile assemblies. The moderating effect of the beryllium reflectors is again quite apparent. The effect is, of course, most apparent in the higher energy spectrum reactors having relatively small dimensions.

TABLE 6.3-- CADMIUM FRACTIONS IN INTERMEDIATE ASSEMBLIES

PPA	CF at Core Center	CF at Core Edge	$\overline{CF}$
-2	0.18	0.16	0.19
-3	0.04	0.10	0.17
-4	0.03	0.18	0.11
-5	0.02	0.15	0.08
-12	0.20	0.45	0.28
-14	0.20	0.45	0.27
-20	0.10	0.55	-
-25	0.04	0.48	(0.24)
-26	0.17	0.50	(0.20)
-27	0.22	0.55	(0.31)
-28	0.30	0.49	(0.40)
-29	0.22	0.48	(0.31)
-30	0.23	0.44	(0.30)
-31	0.26	0.37	-
-32	0.39	0.49	(0.36)

Values in () are estimated.

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in section
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Chapter 7

REACTIVITY COEFFICIENT SYSTEMATICS

H. B. STEWART and M. L. STORM

7.1 INTRODUCTION

An integral part of the critical assembly program at KAPL has been the experimental determination of the reactivity changes associated with the introduction of samples of various materials into the assemblies. This chapter considers the reactivity effects that were observed in several widely different intermediate energy critical facilities, all using beryllium for moderator and reflector. The observed spectral dependence of the reactivity coefficient data is interpreted in terms of the reactivity coefficient theory presented by Hurwitz¹. Many of the basic concepts arising in reactor design can be understood, at least qualitatively, by adopting this approach to the interpretation of the experimental information.

The assemblies under discussion encompass a wide range of compositions, for they were used to simulate the clean and depleted conditions in reactors of interest to the Intermediate Power Breeder and Submarine Intermediate Reactor programs. For example, their fuel loadings ranged from about 40 to 125 kg of U^{235} , while the associated beryllium-to-uranium atom ratios varied from about 180 to 1.7. Some of the cores contained as much as nine dollars of absorber, and the median fission energy at the center of the core ranged from about 40 to 2.4×10^5 ev. Further description of these critical assemblies appears in Chap. 2.

As discussed by Hurwitz¹, the reactivity effects arising from the introduction of a given material into a critical assembly are a function of the associated changes in macroscopic fission cross-section $\Delta\Sigma_f$, absorption cross-section $\Delta\Sigma_a$, diffusion coefficient ΔD , and slowing-down power and inelastic scattering cross-section $\Delta\Sigma(E \rightarrow E')$. The resultant reactivity change is

$$\frac{\Delta k}{k} = \frac{\int d\mathbf{r} \int dE \phi(\mathbf{r}, E) \left\{ \nu F_H(\mathbf{r}) \Delta\Sigma_f - F(\mathbf{r}, E) \Delta\Sigma_a + \int dE' [F(\mathbf{r}, E') - F(\mathbf{r}, E)] \Delta\Sigma(E \rightarrow E') \right\}}{\nu \int d\mathbf{r} \int dE \phi(\mathbf{r}, E) F_H(\mathbf{r}) \Sigma_f} \quad (7.1)$$

$$- \frac{\int d\mathbf{r} \int dE [\nabla \phi(\mathbf{r}, E) \cdot \nabla F(\mathbf{r}, E)] \Delta D}{\nu \int d\mathbf{r} \int dE \phi(\mathbf{r}, E) F_H(\mathbf{r}) \Sigma_f}$$

where $\phi(\mathbf{r}, E)$ is the neutron flux per unit energy at position $\mathbf{r}$ and neutron energy E , $F(\mathbf{r}, E)$ is the iterated fission probability (the "favorableness" of a neutron), $F_H(\mathbf{r})$ is the fission spectrum average of $F(\mathbf{r}, E)$ (the high energy neutron "favorableness"), and ν is the number of neutrons emitted per fission. The total fission rate for the entire reactor, f_{tot} , is related to $F_H(\mathbf{r})$ by

$$f_{tot} = \int d\mathbf{r} \int dE \phi(\mathbf{r}, E) \Sigma_f = \nu \int d\mathbf{r} \int dE \phi(\mathbf{r}, E) F_H(\mathbf{r}) \Sigma_f \quad (7.2)$$

In many cases only one or two of the four effects covered by Eq. 7.1 account for the major part of the reactivity change produced by the sample, and it is possible to classify the experimental data accordingly. In addition, matters are further simplified when the sample is placed at the center of the (symmetrical) assembly, where the reactivity effect due to a change in diffusion coefficient is zero.

Section 7.2 considers first the central reactivity coefficients (RC's) of those elements which are predominantly neutron absorbers. Using data such as those in Table 7.2, one can evaluate the average capture cross-sections for each material at the center of each assembly. These are expressed as a ratio $\bar{\sigma}_a/\bar{\sigma}_f$ of the effective absorption cross-section per absorber atom to the effective fission cross-section per fuel atom, for this ratio is more significant in reactor design than is either cross-section by itself. While these central coefficients have been measured in only a few reactors having significantly different neutron spectra, there are sufficient data to discern over-all trends in the average resonance cross-sections. Section 7.2 then analyzes the central RC's of some typical moderators (H, Be, C). Although it is not possible to obtain average slowing-down cross-sections (analogous to the previously mentioned average capture cross-sections) on the basis of experimental data alone, the central reactivity coefficients for these moderators do correlate somewhat with their slowing-down powers.

Section 7.3 takes up the distributed RC's (i.e., the values averaged over the core volume) of fuel, moderator, and absorber. Some general observations are made regarding the distributed RC of fuel in various reactors. Inasmuch as the principal concern with poisons in reactor design is the amount of additional fuel required to compensate a given amount of poison, the ratio of the distributed poison RC to the distributed fuel RC ($\overline{RC}_a/\overline{RC}_f$) is taken as a measure of the effect of a poison. The reactivity effects of neutron slowing-down and neutron scattering are difficult to separate explicitly in the observed distributed RC values, but some systematics can be presented.

Section 7.4 ends the chapter with a mention of transformation theorems and their application to problems in reactor design. With these theorems it is possible to predict the reactivity effect of volume expansions, with or without density changes, if one knows the reactivity contributions of all the reactor materials.

The reactivity coefficient data presented here are the product of the labors of many people over a period of about eight years. It is practically impossible to acknowledge all their efforts; the interested reader is referred to the bibliography in Ref. 2 for a summary of literature pertinent to each assembly. The compilation of the experimental information was performed by Miss M. E. Way.

7.2 CENTRAL REACTIVITY COEFFICIENTS

A. Absorbers

According to Eqs. 7.1 and 7.2 the reactivity change associated with the placement of a pure absorber at position $\underline{r}$ in a critical assembly is

$$\frac{\Delta k}{k} = - \frac{N_a \int dE \phi(\underline{r}, E) \sigma_a F(\underline{r}, E)}{f_{\text{tot}}} \quad (7.3)$$

where N_a is the number of atoms in the sample, so that $\Delta \Sigma_a = N_a \sigma_a$. Define $F_{La}(\underline{r})$, the average favorableness of the neutrons lost by absorption, by

$$F_{La}(\underline{r}) = \frac{\int dE \phi(\underline{r}, E) \sigma_a(E) F(\underline{r}, E)}{\int dE \phi(\underline{r}, E) \sigma_a(E)} \quad (7.4)$$

Consider one mole of material and express the reactivity change in units of cents, then, the reactivity coefficient of the absorbing sample, in cents per mole, is

$$RC_a(\underline{r}) = - \frac{\Delta a(\underline{r}) F_{La}(\underline{r})}{f_{\text{tot}} c} \quad (7.5)$$

Here $\Delta a(\underline{r})$ is the absorption rate in one mole of the sample, and c is one hundredth of the reactivity difference between prompt and delayed critical.

As mentioned in Sec. 7.1, the value needed is the ratio of flux-weighted average absorption and fission cross-sections at position $\underline{r}$, i.e.,

$$\frac{\bar{\sigma}_a(\underline{r})}{\bar{\sigma}_f(\underline{r})} \equiv \frac{\int dE \phi \sigma_a}{\int dE \phi \sigma_f} = \frac{\Delta a}{\Delta f} \quad (7.6)$$

Here Δf , analogous to Δa , is the fission rate per mole of fuel. Division of both sides of Eq. 7.5 by $\Delta f(\underline{r})/f_{\text{tot}}$ leads to

$$\frac{RC_a}{\Delta f/f_{\text{tot}}} = - \frac{\Delta a}{\Delta f} \frac{F_{La}}{c} = - \frac{\bar{\sigma}_a(\underline{r})}{\bar{\sigma}_f(\underline{r})} \frac{F_{La}}{c} \quad (7.7)$$

For U^{235} as a fuel,

$$\frac{\Delta f(\underline{r})}{f_{\text{tot}}} = \frac{235}{M} \frac{P(\underline{r})}{P_{av}} \quad (7.8)$$

where M is the total fuel load in grams and $P(\underline{r})/P_{av}$ is the ratio of the specific power at $\underline{r}$ to the average reactor specific power. Thus, it may be stated that

$$\frac{\bar{\sigma}_a(\underline{r})}{\bar{\sigma}_f(\underline{r})} = \frac{-RC_a}{\frac{235}{M} \frac{P(\underline{r})}{P_{av}} \frac{F_{La}(\underline{r})}{c}} \quad (7.9)$$

Now the results of central reactivity coefficient measurements performed in PPA-5, -11, -12 and -20 will be considered in some detail. Table 7.1 gives the central median fission energy, the ratio of central to average specific power, and $F_{La}(0)/c$ for each of these assemblies. In addition, the beryllium-to-uranium atom ratio, the fuel inventory, the fraction of fissions at the center occurring below the cadmium cut-off energy, and the factor used to convert RC_a to $\bar{\sigma}_a(0)/\bar{\sigma}_f(0)$ are also indicated.

TABLE 7.1--CENTRAL DATA FOR PPA-5, -11, -12, -20

PPA	Be:U Atom Ratio	Fraction of Fissions Below Cd Cutoff	Median Fission Energy (ev)	$\frac{F_{La}(0)}{c}$	$\frac{P(0)}{P_{av}}$	Fuel Load, M (gm)	$\left[\frac{P(0)}{P_{av}} \frac{235}{M} \frac{F_{La}(0)}{c} \right]^{-1}$
11	1.73	<0.01	235,000	0.86×10^4	1.09	123,600	0.05613
5	33	0.02	1,500	0.97×10^4	1.12	58,100	0.02276
20	137.5	0.09	82	1.45×10^4	1.60	51,700	0.009479
12	178.5	0.24	40	1.42×10^4	1.77	39,800	0.006739

The values of $F_{La}(0)$ listed for PPA-5, -11, and -20 were obtained from Eq. 7.5 using simultaneous RC and activation measurements on the same gold foil. The same technique was used with indium to find $F_{La}(0)$ for PPA-12. The results for the two absorbers should be comparable; for it has been found in practice that F_{La} is not a sensitive function of the absorption spectrum in a given reactor (see Chap. 8).

TABLE 7.2--CENTRAL REACTIVITY COEFFICIENTS

Assembly (and Median Fission Energy)	PPA-12 (40 ev)			PPA-20 (82 ev)			PPA-5 (1,500 ev)			PPA-11 (235,000 ev)		
	RC ¢/mol	Thickness 10 ⁻³ mol/cm ²	$\bar{\sigma}_a/\bar{\sigma}_f$	RC ¢/mol	Thickness 10 ⁻³ mol/cm ²	$\bar{\sigma}_a/\bar{\sigma}_f$	RC ¢/mol	Thickness 10 ⁻³ mol/cm ²	$\bar{\sigma}_a/\bar{\sigma}_f$	RC ¢/mol	Thickness 10 ⁻³ mol/cm ²	$\bar{\sigma}_a/\bar{\sigma}_f$
1	H	-	-	-	-	-	+5.24	100.	-	-	-	-
3	Li	-	-	-	-	-	-1.98	39.	0.045	-	-	-
4	Be	-	-	+0.37	~51.	-	+0.68	52.	-	+0.86	51.	-
5	B	-152	0.28	1.02	-79.3	0.505	0.75	-24.6	1.7	0.56	-3.0	7.4
6	C	-	-	-	-	-	-	+0.35	143.	-	+0.65	159.
7	N	-	-	-	-	-	-	-0.02	33.	0.00045	-	-
8	O	-	-	-	-	-	-	+0.29	82.	-	-	-
9	F	-	-	-	-	-	-	+0.49	10.4	-	-	-
11	Na	-	-	+0.12	54.	-	-	+0.24	56.	-	+0.20	43.
12	Mg	-	-	+0.12	115.	-	-	+0.24	74.	-	-	-
13	Al	-	-	-	-	-	-	+0.14	102.	-	-	-
14	Si	-	-	-	-	-	-	+0.12	34.	-	-	-
15	P	-	-	-	-	-	-	+0.32	25.	-	-	-
16	S	-0.19	-	0.0013	-	-	-	-0.18	71.	0.0041	-0.18	-
17	Cl	-	-	-	-	-	-	-0.75	34.	0.017	-0.023	-
20	Ca	-	-	-	-	-	-	-0.22	15.	0.50	-	-
22	Ti	-	-	-	-	-	-	-0.05	10.	0.00114	-	-
23	V	-	-	-	-	-	-	-0.17	-	0.00387	-	-
24	Cr	0.68	-	0.0044	-0.38	36.	0.0036	-0.043	61.2	0.0010	-	-
25	Mn	3.3	143.0	0.022	-4.4	2.1	0.042	-3.1	0.83	0.071	-	-
25	Fe	-	-	-	-	-	-	-0.02	44.	0.00046	-	-
27	Co	-	-	-	-13.0	4.0	0.123	-6.3	2.01	0.14	-	-
28	Ni	-	-	-	-0.69	153.	0.0065	-0.28	152.0	0.0063	-	-
29	Cu	-	-	-	-	-	-	-1.24	0.6	0.0282	+0.01	14.
30	Zn	-	-	-	-	-	-	-0.38	110.	0.00865	-	-
31	Ga	-	-	-	-	-	-	-	-	-	-0.24	-
32	Ge	-1.8	-	0.012	-	-	-	-0.84	8.	0.019	-	-
33	As	-	-	-	-	-	-	-6.22	7.	0.142	-	-
34	Se	-4.0	1.5	0.027	-	-	-	-1.4	15.6	0.031	-	-
35	Br	-	-	-	-	-	-	-12.2	4.5	0.278	-	-
37	Rb	-	-	-	-	-	-	-2.11	3.7	0.048	-	-
41	Nb	-5.0	0.59	0.034	-3.8	12.0	0.0362	-4.0	1.1	0.092	-0.57	-
42	Mo	-6.4	5.0	0.043	-5.1	4.6	0.048	-3.2	5.0	0.072	-0.73	4.8
44	Ru	-	-	-	-	-	-	-11.	3.3	0.25	-	-
45	Rh	-133.	0.68	0.90	-82.	0.74	0.78	-18.8	7.0	0.43	-2.08	3.8
46	Pd	-15.	5.7	0.10	-	-	-	-7.9	5.6	0.18	-	-
47	Ag	-67.1	2.4	0.45	-122.	0.027	1.21	-25.2	0.22	0.57	-	-
48	Cd	-330.	0.23	2.22	-193.	0.0177	1.83	-9.7	0.23	0.22	-	-
49	In	-	-	-	-	-	-	-33.8	0.071	0.769	-	-
50	Sn	-	-	-	-	-	-	-1.67	3.3	0.038	-	-
51	Sb	-	-	-	-	-	-	-8.74	4.5	0.20	-	-
52	Te	-	-	-	-	-	-	-2.13	3.6	0.0485	-	-
53	I	-	-	-	-	-	-	-22.1	1.1	0.503	-	-
55	Cs	-93.	-	0.63	-32.	1.39	0.30	-11.8	5.4	0.264	-	-
56	Ba	-	-	-	-	-	-	-0.58	4.0	0.013	-	-
58	Ce	-	-	-	-	-	-	-0.20	5.3	0.0046	-	-
60	Nd	-	-	-	-	-	-	-0.88	3.0	0.02	-	-
62	Sm	-	-	-	-440.	0.0075	4.2	-40.	0.6	0.9	-	-
62	Sm	-	-	-	-180.	(Epi Cd)	1.7	-	-	-	-	-
62	Sm	-	-	-	-260.	(Sub Cd)	2.46	-	-	-	-	-
63	Eu	-	-	-	-	-	-	-137.	0.50	3.12	-	-
70	Yb	-	-	-	-	-	-	-15.3	4.0	0.348	-	-
72	Hf	-	-	-	-182.	0.40	1.73	-	-	-	-	-
73	Ta	-70.	-	0.47	-120.	0.071	1.14	-17.2	2.4	0.39	-3.1	-
74	W	-	-	-	-42.	0.59	0.40	-8.7	5.3	0.20	-	-
75	Re	-	-	-	-	-	-	-50.7	0.97	1.15	-	-
76	Os	-	-	-	-	-	-	-18.	-	0.418	-	-
77	Ir	-	-	-	-	-	-	-78.	0.19	1.77	-	-
78	Pt	-	-	-	-	-	-	-12.5	1.4	0.285	-	-
79	Au	-	-	-	-173.	0.026	1.64	-33.	0.56	0.76	-3.1	-
80	Hg	-	-	-	-	-	-	-4.72	11.0	0.107	-	-
81	Tl	-	-	-	-	-	-	-2.29	3.6	0.052	-	-
82	Pb	-	-	-	-	-	-	-0.013	18.	0.000296	-	-
83	Bi	-	-	-	-	-	-	-0.008	4.7	0.000182	+0.11	47.
90	Th	-	-	-	-	-	-	-9.18	2.6	0.209	-	-
92	U	-	-	-	-4.13	80.7	0.039	-8.65	0.23	0.197	-	-
92	U ²³³	-	-	-	+23.5	0.55	-	+19.	4.	-	+15.3	8.08
92	U ²³⁸	-	-	-	-13.3	2.07	0.126	-	-	-	+0.13	80.
94	Pu ²³⁹	-	-	-	-	-	-	-	-	-	+28.4	7.7

Table 7.2 summarizes central reactivity coefficients that were measured in the same four assemblies and the corresponding values for $\bar{\sigma}_a(0)/\bar{\sigma}_f(0)$, as computed from Eq. 7.9 and the last column of Table 7.1. (It should be noted that scattering contributions to the measured reactivity coefficient may be significant for weak absorbers.)

The median fission energy has been selected as the parameter to be used in depicting the spectrum dependency of the values of $(\bar{\sigma}_a/\bar{\sigma}_f)$ given in Table 7.2. This, of course, is not a unique choice and, for example, a median absorption energy as defined in KAPL-Memo-HBS-2² could have been employed. In this alternative approach Eq. 7.3 would be written in the form

$$\frac{\Delta k}{k} = - \frac{N_a (\sigma_a)_{\text{eff}} \int dE \phi(r, E) F(r, E)}{f_{\text{tot}}} \quad (7.10)$$

with the effective absorption cross-section defined as

$$(\sigma_a)_{\text{eff}} \equiv \frac{\int dE \phi F \sigma_a}{\int dE \phi F} \quad (7.11)$$

The median absorption energy would then be defined as the energy E_m for which

$$\int_0^{E_m} dE \phi \sigma_a F = \int_{E_m}^{\infty} dE \phi \sigma_a F \quad (7.12)$$

Figure 7.1, based on the data in Table 7.2, indicates the variation of $\bar{\sigma}_a/\bar{\sigma}_f$ with median fission energy for a few absorber samples which were measured in more than one of the assemblies. The thermal values of $\bar{\sigma}_a/\bar{\sigma}_f$ have been included. In the case of the non-1/v absorbers, the Maxwellian - averaged cross-sections were used assuming a neutron temperature of 293°K. As indicated in Fig. 7.1, data on samples placed at the outer edge of the core of PPA-12 were also used in drawing the curves for boron, cadmium, and gold. In addition, values of $\alpha = \sigma_c/\sigma_f$ for thick and thin U²³⁵ samples³ have been indicated in the figure. The thin sample value at 235,000 ev was obtained from PPA-11 experiments. Three classes of absorbers can be identified:

1. the 1/v absorbers
2. the typical resonance absorbers
3. the resonance absorbers having a large resonance near the thermal energy region.

It is evident both in Table 7.2 and in Fig. 7.1 that the value of $(\bar{\sigma}_a/\bar{\sigma}_f)$ for a 1/v absorber (boron) decreases monotonically as the median fission energy increases. This means that the U²³⁵ fission cross-section decreases less rapidly than 1/v with neutron energy. Additional measurements at lower values of the median fission energy would be required to show the increase of $\bar{\sigma}_a/\bar{\sigma}_f$ with increasing median fission energy which occurs in thermal reactors.

Resonance absorbers typically show a maximum in the value of $(\bar{\sigma}_a/\bar{\sigma}_f)$ as the neutron energy increases. In the case of the lighter atom absorbers (e.g. niobium and molybdenum) this maximum appears to occur above 1000 ev, whereas with heavier atom absorbers (e.g., tantalum and gold) the maximum occurs below 1000 ev. The different behavior of the two groups of resonance absorbers reflects the difference in level structure between them.

The over-all behavior of these two groups is probably representative of that of the stable fission products with small and with large level spacings. This indicates in general that the stable fission products are relatively more important absorbers in intermediate spectra reactors than they are in either thermal or fast reactors.

Since the capture resonances in U²³⁵ are probably typical of the heavy element resonance absorbers, the values of the capture-to-fission cross-section ratio in thin samples, α_{thin} behave in a manner similar to $(\bar{\sigma}_a/\bar{\sigma}_f)$ for gold and tantalum. The values for α_{thick} indicate that the capture resonances were more self-shielded than the fission resonances, a consequence of which is that the values of α measured on the thicker samples are somewhat lower.

The third type of absorber is typified by cadmium and samarium (also Xe^{135}). In this case the value of $(\bar{\sigma}_a/\bar{\sigma}_f)$ decreases rapidly as the median fission energy increases, since the principal resonance for these absorbers occurs in the thermal energy region.

B. Moderators

The fractional change in reactivity brought about by the introduction of moderating material which does not absorb neutrons significantly is given by Eq. 7.1 as:

$$\frac{\Delta k}{k} = \frac{\int d\mathbf{r} \int dE \phi(\mathbf{r}, E) \int dE' [F(\mathbf{r}, E') - F(\mathbf{r}, E)] \Delta \Sigma(E \rightarrow E')}{f_{\text{tot}}} - \frac{\int d\mathbf{r} V \int dE \nabla \phi(\mathbf{r}, E) \cdot \nabla F(\mathbf{r}, E) \Delta D}{f_{\text{tot}}} \quad (7.13)$$

For the case of a centrally located specimen containing N_m atoms, Eq. 7.13 becomes

$$\frac{\Delta k}{k} = \frac{N_m}{f_{\text{tot}}} \int dE \phi(0, E) \int dE' [F(0, E') - F(0, E)] \sigma(E \rightarrow E') \quad (7.14)$$

Thus, if RC_m is the moderator's reactivity coefficient in cents/mole, an equation, analogous to Eq. 7.5 for RC_a is:

$$RC_m(0) = \frac{\Delta m(0)}{f_{\text{tot}}} \frac{\Delta F_{E \rightarrow E'}}{c} \quad (7.15)$$

where $\Delta m(0)$ is the rate of occurrence of moderating events at the center of the reactor for a mole of the sample and $\Delta F_{E \rightarrow E'}$ is the average change in neutron value as a result of the moderation. However, since the total number of moderating events cannot be determined experimentally, $\Delta F_{E \rightarrow E'}$ cannot be determined on the basis of experiment alone. It is convenient for purposes of correlation to divide both sides of Eq. 7.15 by $\Delta f(0)/f_{\text{tot}}$ as defined by Eq. 7.8. Then

$$\frac{\Delta m(0)}{\Delta f(0)} \frac{\Delta F_{E \rightarrow E'}}{c} = \frac{RC_m}{\frac{235}{M} \frac{P(0)}{P_{av}}} \quad (7.16)$$

The quantity $[\Delta m(0)/\Delta f(0)] \Delta F_{E \rightarrow E'}/c$ for hydrogen, beryllium, and carbon is shown as a function of central median fission energy in Fig. 7.2. The relative effectiveness of H, Be, and C in scattering neutrons from high energy regions of low importance to lower energies of greater importance is clearly seen, the ordering being in accordance with their relative values of $\xi \sigma_s$. In addition, as would be expected, the moderating effectiveness increases as the assembly spectrum gets harder, in accord with the fact that σ_s/σ_f increases with energy.

Reactivity effects of materials in which the effects of absorption, slowing down, and inelastic scattering are of comparable significance are usually difficult to analyze because information is lacking on the effects of inelastic scattering. However, as a specific illustration of these effects, consider the central RC of zirconium (-0.16¢/mole) measured in PPA-20. An analysis of this experimental value, which employed adjoint functions and available information on its inelastic scattering properties, indicated that the negative reactivity contribution due to absorption was about -0.53¢/mole while the positive contribution due to slowing down and inelastic scattering combined was about $+0.37\text{¢/mole}$. The contribution of the inelastic scattering was 3.5 times as large as that of the slowing-down.

7.3 DISTRIBUTED REACTIVITY COEFFICIENTS

Fig. 7.2—Moderating effectiveness versus neutron energy.

7.3 DISTRIBUTED REACTIVITY COEFFICIENTS

In this section the factors affecting the RC of fuel are considered from a qualitative point of view: the variation of the distributed fuel and beryllium reactivity coefficients in a wide variety of intermediate spectrum reactors, and the fuel increment required to compensate for reactor poisons.

There are fewer experimental data for distributed absorber RC's, since the distributed coefficient is more laborious to measure. In addition, the scattering contributions tend to obscure the interpretation of the distributed RC for the weak absorbers. However, ratios of averaged absorption to fission cross-sections obtained from distributed RC measurements are important parameters in studying burnable poisons, fission-product poisoning, and the poisoning effects of structural materials. Some experimental values obtained in conjunction with the burnable poison program are given in Chap. 17 of this volume. Calculated values of these parameters, as well as applications to burnable poison studies, are discussed in Chap. 16.

Referring to Eqs. 7.1 and 7.2, and neglecting sample-introduced changes other than in absorption and fission cross-sections, one sees that the reactivity change associated with the introduction of a sample of U^{235} at position $\underline{r}$ in a critical assembly is

$$\begin{aligned} \frac{\Delta k}{k} &= \frac{N_f \nu F_H(\underline{r}) \int dE \phi(\underline{r}, E) \sigma_f(E) - N_f \int dE \phi(\underline{r}, E) F(\underline{r}, E) \sigma_a(E)}{f_{tot}} \\ &= \frac{N_f \nu F_H(\underline{r}) \int dE \phi \sigma_f - N_f \int dE \phi F(1+\alpha) \sigma_f}{f_{tot}} \end{aligned} \quad (7.17)$$

Here N_f is the number of atoms in the fuel sample. The second form of this relation arises because the total absorption (i.e., neutron loss) cross-section involves both fission and capture: $\sigma_a = \sigma_f + \sigma_c = \sigma_f(1+\alpha)$. Considering one mole of fuel, and expressing the reactivity change in units of cents,

$$\begin{aligned}
 RC_f \left(\frac{\text{cents}}{\text{mole}} \right) &= \frac{\Delta f(\underline{r})}{cf_{\text{tot}}} \left\{ \nu F_H(\underline{r}) - [F_{Lf}(\underline{r}) + \alpha(\underline{r}) F_{Lc}(\underline{r})] \right\} \\
 &= \frac{\Delta f(\underline{r})}{cf_{\text{tot}}} \left\{ \nu F_H(\underline{r}) - [1 + \alpha(\underline{r})] F_L(\underline{r}) \right\}
 \end{aligned} \quad (7.18)$$

where $\Delta f(\underline{r})$ is the fission rate in a mole of fuel; $F_{Lf}(\underline{r})$ and $F_{Lc}(\underline{r})$ are respectively the average of $F(\underline{r}, E)$ for neutrons producing fission in U^{235} and for neutrons absorbed by radiative capture in U^{235} , $\alpha(\underline{r})$ is the average value of α at $\underline{r}$ for neutrons producing fission in U^{235} and $F_L(\underline{r})$ is defined by its use in Eq. 7.18.

Now consider the extrapolated total value of fuel, which is defined as the average RC (cents/mole) of the fuel times the number of moles of fuel in the assembly. This extrapolated total value can be obtained by imagining that fuel is added to the reactor in such a manner that the spatial distribution of the added fuel is the same as the distribution of fuel already in the assembly. If, say, one percent of the fuel load in the assembly were considered to be added in this manner, then 100 times the resultant reactivity change would be the extrapolated total fuel value.

From Eq. 7.18 the following expression is obtained for the extrapolated total value of fuel

$$\overline{RC}_f M_f = \frac{1 - (1 + \alpha) \overline{F}_L}{c} \quad (7.19)$$

where $\overline{RC}_f$ is the distributed fuel reactivity coefficient and M_f is the fuel load in moles. In obtaining Eq. 7.19, use has been made of the relation in Eq. 7.2 that the power-weighted average of $\nu F_H(\underline{r})$ over the whole core is unity. The quantity $(1 + \alpha) \overline{F}_L$ represents an average of the indicated parameters also weighted by the power distribution. Thus the distributed fuel reactivity coefficient is a function of M_f and $(1 + \alpha) \overline{F}_L$.

It is evident that a decrease in fuel load M_f , as might be obtained by going to a more thermal assembly, tends to increase the distributed fuel reactivity coefficient. Since α is near its peak value in intermediate energy assemblies, $\overline{RC}_f$ also tends to be increased by a reduction in α which can be achieved in an intermediate assembly by going either to a more nearly thermal or to a much faster spectrum. Finally $\overline{RC}_f$ tends to be increased by a decrease in $\overline{F}_L$, which can be achieved by decreasing the leakage occurring during the slowing-down process. (To justify the statement just made, consider a neutron introduced at high energy in an intermediate reactor. If the probability of leakage during the slowing-down process is decreased, then the iterated fission probability for the energy region where most of the fissions are occurring must also decrease -- so long as the reactor remains just critical -- in order that criticality be maintained.) A change in leakage probability accompanies rotations of control cylinders, located at the core-reflector interface, which contain an absorbing face and a reflecting face. Initially, when the core is clean, the reactors achieve criticality with the poison side facing the core. As fuel is depleted and fission products accumulate, criticality is maintained by rotating the drums so that the reflecting sides face the core and the leakage probability is decreased. Hence $\overline{F}_L$ diminishes, and $\overline{RC}_f$ grows larger, as the fuel is burned up. The decrease in M_f as the fuel is depleted has a similar effect on $\overline{RC}_f$.

In Table 7.3 are summarized the distributed reactivity coefficients of U^{235} and the total extrapolated values of U^{235} for twelve critical assemblies. In addition, the corresponding values of $(1 + \alpha) \overline{F}_L$ as obtained from Eq. 7.19, as well as the pertinent core volumes and Be:U ratios, are listed. The condition of core and reflector can be seen in the Comment column. More detailed descriptions of these reactors are given in Chap. 2.

Although at first glance there is no obvious over-all correlation of the U^{235} data, a definite pattern is discernable. Considering the reactors in pairs [PPA-21A vs PPA-21G; PPA-19 (66 deg vs 106 deg); PPA-18 (66 deg vs 146 deg)] note that the distributed fuel coefficient for the clean core (high leakage) condition is considerably smaller than the corresponding coefficient for depleted (low leakage) condition where there are no reflector poisons at the core-reflector interface. The extrapolated total values of fuel for these assemblies follow the same pattern, and accordingly the values of $(1 + \alpha) \overline{F}_L$ are higher for the clean condition than for the depleted condition. Since

TABLE 7.3--DISTRIBUTED CORE DATA FOR U^{235} AND BERYLLIUM

Assembly	Core Volume, liters	Be:U Atom Ratio	U^{235}				Be			Comments
			$\overline{RC}$, cents/mole	M_F , kg	Extrapolated Total Value, \$	$(1+\alpha)F_L$	$\overline{RC}$, cents/mole	M_{Be} , kg	Extrapolated Total Value, \$	
PPA-11	63	1.73	9.1	124	48	0.60	0.45	8.2	4.1	Clean core and clean reflector; the beryllium $\overline{RC}$ is an estimated value
PPA-5	85	33	10.8	58	27	0.77	0.47	73	38	Clean core and clean reflector
PPA-4	85	45	12.0	48	24	0.79	0.45	83	41	Clean core and clean reflector
PPA-21A	273	93	8.0	62	21	0.82	0.23	220	57	Clean core, boron curtain mock-up at core-reflector interface
PPA-21G	273	101	12.5	57	30	0.74	0.16	220	39	Depleted version of PPA-21A. No boron curtain, poisoned core
PPA-20 (66 deg)	294	138	9.1	52	20	0.83	0.19	279	59	Clean core, boron in reflector
PPA-19 (66 deg)	293	157	10.4	45	20	0.83	0.21	273	64	Clean core, boron in reflector
PPA-19 (106 deg)	293	157	17.2	45	33	0.72	0.15	273	44	Partially depleted and poisoned core
PPA-18 (66 deg)	293	187	11.2	40	19	0.84	0.20	288	65	Clean core, boron in reflector
PPA-18 (146 deg)	293	187	18.6	40	32	0.73	0.12	288	38	Depleted reactor, clean reflector, poisoned core
PPA-15	299	187	11.7	40	20	0.83	0.20	294	65	Clean core, boron in reflector
PPA-17	326	206	20.2	40	35	0.71	0.11	317	39	Poisoned core, clean reflector

$(1+\alpha)$ does not vary greatly with position for a given reactor, the decrease in $\overline{F}_L$ with decreasing leakage is reflected by an increase in total fuel value. The values of $\overline{(1+\alpha)F}_L$ for these reactors vary from 0.82 to 0.84 in the clean condition and from 0.71 to 0.74 in the depleted condition. On the other hand, the value of $\overline{(1+\alpha)F}_L$ for PPA-11 is 0.60. This lower value reflects the smaller α obtaining in that faster assembly. However, it should be pointed out that in a fast assembly like PPA-11, which contains a considerable amount of fuel and little moderator, the degradation and transport properties of fuel may be significant. The validity of calculating $\overline{(1+\alpha)F}_L$ for PPA-11 from Eq. 7.19, which neglects these effects, has not been considered in detail.

The reactivity change associated with the introduction of beryllium in a critical assembly is described by Eq. 7.13. Since it is difficult to separate the various effects experimentally a qualitative discussion follows. Both the moderation and the diffusion terms in Eq. 7.13 contributing to a distributed beryllium RC can be expected to decrease in proceeding from the clean to depleted condition; this goes with the decrease in value of the iterated fission probability F as the neutron leakage is reduced by rotating the control drums.

Also consider the extrapolated total value of beryllium which is obtained as the product of the distributed beryllium RC by the total number of moles of beryllium in the region. Table 7.3 summarizes the distributed RC and the extrapolated total value of beryllium for the cores of the twelve critical assemblies. Again considering the reactors in pairs, both the distributed RC and the extrapolated total value for beryllium decrease as the leakage decreases. In addition the distributed beryllium coefficient is noticeably larger in the high leakage assemblies.

As an indication of the relative contributions of the moderation and diffusion terms in Eq. 7.13 to the distributed beryllium RC, Table 7.4 summarizes the values of these effects, as obtained by P. L. Hofmann⁴ using perturbation theory, for several assemblies. The contribution of absorption to the reactivity effect is also listed.

TABLE 7.4--EXPERIMENTAL AND CALCULATED VALUES FOR THE BERYLLIUM DISTRIBUTED REACTIVITY COEFFICIENT

Assembly	Calculation			Experiment	
	Contribution of Degradation Term, ϕ/mole	Contribution of Transport Term, ϕ/mole	Contribution of Absorption Term, ϕ/mole	Total $\overline{RC}$, ϕ/mole	Measured $\overline{RC}$, ϕ/mole
PPA-18 (66 deg)	+0.136	+0.067	-0.001	+0.202	+0.204
PPA-18 (146 deg)	+0.082	+0.036	-0.001	+0.117	+0.118
PPA-20 (66 deg)	+0.131	+0.074	-0.0006	+0.204	+0.192
PPA-19 (106 deg)	+0.091	+0.048	-0.0006	+0.138	+0.146

The calculations were performed for spherical models in which the control drum settings were simulated by the introduction of uniformly distributed amounts of reflector boron. The calculation of the $\overline{RC}$ in PPA-18 at 66 deg was specifically performed for a PPA model while the remaining values were obtained using reactor models whose fuel load and composition were comparable to the experimental assemblies. It is seen that both the transport and degradation effects decrease as the leakage decreases. The contribution of absorption to the total $\overline{RC}$ is negligible.

Consider next the amount of fuel required to compensate the reactivity loss produced by the distributed addition of a given amount of poison material. The RC of a mole of absorber at position r is

$$RC_a = - \frac{\Delta a(r) F_{La}(r)}{c f_{tot}} \quad (7.5)$$

or the total value of the poison is

$$\overline{RC}_a \Delta M_a = - \frac{a_{\text{tot}} \overline{F}_{La}}{c f_{\text{tot}}} \quad (7.20)$$

where $\overline{RC}_a$ is the distributed absorber RC (ϕ /mole), ΔM_a is the total number of moles of poison added, a_{tot} is the total absorption rate in the poison, and $\overline{F}_{La}$ is the average value of $F_{La}(r)$ for f_{tot} neutrons absorbed by the poison.

Let this poison addition be compensated by the distributed addition of ΔM_f moles of U^{235} . The worth of fuel added is obtained from Eq. 7.19 as

$$\overline{RC}_f \Delta M_f = \frac{[1 - (1 + \alpha) \overline{F}_L]}{c} \frac{\Delta M_f}{M_f} \quad (7.21)$$

Adding Eqs. 7.20 and 7.21 and setting this sum equal to zero for compensated additions of fuel and absorber,

$$\frac{a_{\text{tot}} \overline{F}_{La}}{f_{\text{tot}}} = \left[1 - (1 + \alpha) \overline{F}_L \right] \frac{\Delta M_f}{M_f} \quad (7.22)$$

However,

$$\frac{a_{\text{tot}}}{f_{\text{tot}}} = \frac{\Delta M_a}{M_f} \frac{\overline{\sigma}_a}{\overline{\sigma}_f} \quad (7.23)$$

where the double bar indicates averages over both the neutron flux spectrum and the core volume. Combining Eqs. 7.22 and 7.23 yields

$$\frac{\Delta M_f}{\Delta M_a} = \frac{\overline{\sigma}_a}{\overline{\sigma}_f} \frac{\overline{F}_{La}}{1 - (1 + \alpha) \overline{F}_L} \quad (7.24)$$

Thus the fuel increase required to compensate for a given poison addition depends explicitly not on the fuel loading but on $\overline{F}_{La}$, $\overline{\sigma}_a/\overline{\sigma}_f$, and $(1 + \alpha) \overline{F}_L$, and a reduction in any of these tends to lower the additional fuel requirements.

Since $\frac{\Delta M_f}{\Delta M_a} = \left| \frac{\overline{RC}_a}{\overline{RC}_f} \right|$ for a compensated addition of fuel and absorber, it is of interest to examine the ratio of distributed fuel and absorber reactivity coefficients for some of the reactors considered previously. For example, in Table 7.5 the distributed RC's of U^{235} and cadmium as well as their ratios are listed for PPA-18 and PPA-19 for each of the drum positions considered previously. The cadmium used for these measurements was in the form of slivers whose approximate dimensions were 0.03 in. $\times$ 0.06 in. $\times$ 13.75 in. The values given in Table 7.5 are taken from more detailed compilations by Kerr⁵ and Schulz⁶.

TABLE 7.5--DISTRIBUTED RC'S OF U^{235} AND Cd

	PPA-18 66 deg	PPA-18 146 deg	PPA-19 66 deg	PPA-19 106 deg
$\overline{RC}_{235}$ (ϕ /mole)	11.2	18.6	10.4	17.2
$\overline{RC}_{Cd}$ (ϕ /mole)	20.2	19.4	16.5	14.6
$\frac{\overline{RC}_{Cd}}{\overline{RC}_{235}}$	1.80	1.04	1.59	0.85

For each reactor as one proceeds from the condition of higher leakage to lower leakage, the previously discussed reduction of $(1-\alpha)F_L$ with decreasing leakage is manifested in the decrease in $\overline{RC}_{Cd} / \overline{RC}_{235}$. In this example the change in the fuel RC is more significant than that of the cadmium.

7.4 TRANSFORMATIONS

In many instances it is not possible to simulate in a critical assembly the engineering details of the power reactor under investigation. However, the determination of the reactivity coefficients of the reactor's constituents in a given critical assembly provides a means for determining the difference in criticality between the two reactors. This is accomplished by means of conceptual transformations of the critical facility into the power reactor, whereby the reactivity changes associated with the transformation are evaluated via the appropriate reactivity coefficients. In fact, as Hurwitz and Ehrlich⁷ have discussed, by assuming linear addition of reactivity changes it is possible to predict the net reactivity difference between two assemblies to the order of 50 cents when the sum of the absolute values of the various reactivity changes is of the order of 10 dollars.

With the knowledge of all the reactivity coefficients for the materials in a given assembly, it is also possible to introduce the concept of the total extrapolated value of all reactor materials. The total value of all materials in the intermediate assemblies studied at KAPL has ranged from about 70 to 120 dollars. In this connection, Hurwitz has shown⁸ that although there is no general rule regarding the total value of all materials in an assembly, there is a general relation of a similar nature. According to this relation, the sum of all reactivity values arising from fission, absorption, and moderation, minus the total value due to scattering, is equal to zero. However, as discussed by Hurwitz, this relationship cannot be used as a check on reactivity coefficient measurements unless the individual reactivity coefficients can be resolved into transport and non-transport components.

Specific examples of reactor transformations have been described in detail in the literature. For example, in KAPL-837⁹, on the basis of experiments conducted in PPA-18, an analysis was made of the fuel requirement and endurance for three reactor loading patterns. In another work,¹⁰ reactivity coefficients measured in the core and reflector of PPA-4 were used to extrapolate to the criticality of the reactor which PPA-4 mocked up. Since the core volume of the reactor of interest was not the same as that of PPA-4, a transformation was made in which the product of linear dimensions and density was held constant. The transformed reactor had the same size as PPA-4 and the reactivity differences between the latter and the transformed assembly were then evaluated.

The constancy of reactivity, for transformations where the product of linear dimensions and density is held constant, can be coupled with a knowledge of the total extrapolated value of all reactor materials to provide a simple means for estimating the effects of various transformations. Considering uniform volume (V) and density (ρ) changes,

$$\frac{dk}{k} = R \left(\frac{dV}{V} + \frac{3d\rho}{\rho} \right) \quad (7.25)$$

since

1. an increase in volume with constant density increases reactivity
2. an increase in density with constant volume increases reactivity
3. a change in which $\rho V^{1/3}$ is constant, or $(d\rho/\rho) + (1/3)(dV/V) = 0$, results in no change in reactivity.

The constant R is obtained by a standard reactivity coefficient evaluation of all core and reflector materials with the volume held constant. Thus, if K_T is the total extrapolated value of all materials in the reactor, $K_T = 3R$ and

$$\frac{dk}{k} = \frac{K_T}{3} \left(\frac{dV}{V} + \frac{3d\rho}{\rho} \right) \quad (7.26)$$

REFERENCES

For example, the transformation which maintains a constant inventory of materials in the reactor is useful in the determination of the temperature coefficient associated with expansion. In this case

$$\frac{d\rho}{\rho} + \frac{dV}{V} = 0 \quad (7.27)$$

and

$$\frac{dk}{k} = \frac{K_T}{3} \left[\frac{dV}{V} - \frac{3dV}{V} \right] = -\frac{2}{3} K_T \frac{dV}{V} \quad (7.28)$$

Applying Eq. 7.28 to the clean SIR where $K_T \approx 0.85$ yields $dk/k = -0.57 dV/V$ in reasonable agreement with the value $0.5 dV/V$ obtained by Hofmann and Siegel¹¹ via multigroup calculations.

As an extension to the preceding discussion it is quite often of interest to evaluate the fuel increase necessary to compensate the above change. In this case, if K_F is the total extrapolated fuel value,

$$K_F \frac{dM_F}{M_F} = \frac{2}{3} K_T \frac{dV}{V} \quad (7.29)$$

or, for constant K_T and K_F ,

$$M_F = A V^{2K_T/3K_F} \quad (7.30)$$

where A is a constant of integration.

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Chapter 8

REACTIVITY COEFFICIENTS AND SELF-SHIELDING MEASUREMENTS

J. S. KING

8.1 INTRODUCTION

Measurements of reactivity coefficients have found extensive use in the design of intermediate energy reactors. The reactivity worth for samples of each core component was obtained in practically all the critical assemblies which were examined experimentally. These have been of great value in the predictions of the appropriate fuel load, endurance, effects of variations in composition, and the worth of control systems, as well as in the study of the neutron economy of the reactors. As an important part of the reactivity coefficient program, the self-shielding characteristics of thick absorber and fuel foils were measured in some reactors of low intermediate energy spectra. These measurements and their implications form a major part of the subject of the present chapter.

The interpretation and application to critical experiments of reactivity coefficient measurements is based on the danger coefficient theory developed by Hurwitz.¹ The concepts and definitions of that theory in terms of measurable quantities are summarized in Chap. 7 of this volume, which considers particularly the dependence of the reactivity coefficients upon reactor characteristics such as neutron energy spectrum.

The reactivity coefficient (RC) of a material is the change in reactivity per unit amount of material (ideally, for infinitesimal samples) inserted into a void (or air) space in the reactor. Another term for the reactivity coefficient has been "danger coefficient." Yet another is "statistical weight," particularly when the dependence of the RC of fuel upon position in the reactor is being discussed. The RC is most directly measured by observing the displacement of a calibrated control rod necessary to compensate the reactivity change brought about by the introduction of the sample. Some care must be exercised to avoid extraneous reactivity effects which can complicate the interpretation of the experiment. For example, large samples of strong absorbers or moderators can cause local flux perturbations which affect the neutron absorption in the sample and may even cause a change in the sensitivity of the control rod.

In heterogeneous reactor core designs, it is quite common for absorber or fuel materials to be located in components which have relatively large dimensions so that self-shielding effects can be significant. Furthermore, it is often convenient in critical experiment facilities to use components having different dimensions and possibly different geometries from those in the power reactor; therefore the self-shielding factors may be somewhat different in the critical experiment from those in the power reactor. As a result, self-shielding measurements are an important part of the critical experiment program.

In general, the reactivity coefficient of an absorber sample having a thickness x can be expressed, as a consequence of Eqs. 7.4 and 7.5 and an extension of the thinking therein to a thicker sample, in the form

$$RC_a(x, \underline{r}) = - \frac{L F_{La} \int dE \phi_o(x, E) \sigma_a(E) f(N_a \sigma_t x)}{c f_{tot}} \quad (8.1)$$

where ϕ_0 is the magnitude of the flux (assumed isotropic) at the surface of the absorber,
 $\sigma_a(E)$ is the total cross-section for atoms of the material,
 $f(N_a \sigma_a x)$ is a self-shielding factor for thickness x of the absorber,
 L is the number of atoms per mole (0.6025×10^{24}).
 Also refer to the quantities defined in Chap. 7.

The calculation of the proper self-shielding function f is discussed in detail³ in Volume I, Chap. 2, of this Handbook. It is sufficient here to remark that because of the predominance of resonance absorption the incident flux external to the sample is essentially unperturbed. Then the simple escape probability function of Placzek adequately describes the self-shielding effect and the correct expression is assumed to be

$$f(N_a \sigma_a x) = \frac{1}{N_a \sigma_a x} \int_0^1 d\mu \mu (1 - e^{-N_a \sigma_a x / \mu})$$

or

$$f(\xi) = \frac{1}{\xi} \int_0^1 d\mu \mu (1 - e^{-\xi/\mu})$$

(8.2)

where $\xi(E) \equiv N_a x \sigma_a(E)$

The value of F_{La} in Eq. 8.1 can depend somewhat on the self-shielding of the sample if the self-shielding is sufficiently large to change the neutron absorption spectrum substantially. In practice it has been found that this change in F_{La} is insignificant.

8.2 SELF-SHIELDING MEASUREMENTS ON ABSORBERS IN PPA-20

The effects of self-shielding on the reactivity effectiveness of absorber samples can be observed and extended reliably into the region of infinite dilution if very thin absorber samples are used. A program of experiments was conducted on a number of absorber materials located at the center of the PPA-20 core. The intermediate energy spectrum at the center of the PPA-20 core is shown in Fig. 8.1. The higher energy portion of the curve was determined (see Sec. 6.5, especially Fig. 6.7) by activation measurements on indium, gold, and manganese foils, the points for which appear as squares with associated ranges of error at energies of 1.46, 4.91, and 337 ev. The lower energy portion of the curve was established by a trial-and-error fitting of RC measurements on very thin boron, cadmium, and cadmium-covered samarium (the last two behave as non-activating

Fig. 8.1—Neutron energy spectrum near center of PPA-20.

Fig. 8.2—Iterated fission probability (neutron favorableness function) as a function of neutron energy, at center of PPA-20.

isotopes with strong low energy resonances). Equation 7.5 was used and a multigroup calculation estimate of the variation of F_{La} below the gold or indium resonance energies. A more recent time-of-flight spectrometer measurement of the shape of the spectrum from a lattice of this same assembly² gave the numerous points shown as circles (without their fairly large errors) on Fig. 8.1. The absolute values for the neutron flux were adjusted to match the results obtained from the resonance foil measurements.

Independent measurements of the reactivity coefficient and the activation of gold foils provided an absolute determination of F_{La} at the prominent gold resonance energy of 4.9 ev. This determination was used to normalize a multigroup calculation of the dependence of $F(r,E)$ upon E at the center of the reactor core. The result of this calculation is indicated in Fig. 8.2. In principle, this curve can be checked at other energies by similar measurements with other resonance foils. Since the function is slowly varying and rather difficult to measure, only the single value of F_{La} was attempted with the gold foil activation.

The reactivity coefficients and activations associated with different thicknesses of gold foils were measured in order to examine the possible variation of F_{La} as a function of sample thickness and, therefore, neutron absorption spectrum. The results of the experiment indicated that the value of F_{La} was independent of the foil thickness within the limits of uncertainty for the experiment.

A foil-sandwich technique was used in the self-shielding experiments in order to emphasize the difference in reactivity coefficients which arise from the self-shielding only. In this technique, for example, 12 similar foils would be located at 12 symmetrical positions in the reactor core and the reactivity of the assembly would be measured by observing the control rod position for steady-state power equilibrium. Following this test the foils would be then arranged in six doublets at a symmetrical set of six of the same geometrical positions. The reactivity of the reactor for this condition would again be measured from control rod position and the difference in reactivity relative to the initial test determined. Since the same total amount of material would be used in each test, the difference in reactivity would be due only to the difference in self-shielding (aside from shadowing effects involving the locations of the foils relative to the controls). In subsequent steps the foils might be arranged in three groups of four, two groups of six, and finally one group of 12, and the appropriate reactivity measurements then made.

The results of self-shielding measurements for boron, cadmium, gold, silver, tantalum, hafnium, and tungsten are shown in Fig. 8.3--8.10. Measurements were made

8.2 SELF-SHIELDING MEASUREMENTS ON ABSORBERS IN PPA-20.

Fig. 8.3—Self-shielding of reactivity coefficient for boron near center of PPA-20.

Fig. 8.4—Self-shielding of reactivity coefficient for cadmium near center of PPA-20.

Fig. 8.5—Self-shielding of reactivity coefficient for cadmium at outer edge of PPA-20 core.

Fig. 8.6—Self-shielding of reactivity coefficient for gold near center of PPA-20.

Fig. 8.7—Self-shielding of reactivity coefficient for silver near center of PPA-20.

with samples sufficiently thin ($\sim 2 \times 10^{-5}$ moles/cm²) to permit reasonably accurate extrapolation to zero thickness in most cases. Figure 8.11 indicates the self-shielding curves for the different absorbers on a common ordinate scale on which the reactivity coefficient is unity for zero thickness.

8.3 SELF-SHIELDING MEASUREMENTS OF FUEL

Self-shielding measurements for various fuel sample thicknesses have been investigated in some detail in PPA-20 and in PPA-21 (an assembly with quite a different neutron

Fig. 8.8—Self-shielding of reactivity coefficient for tantalum near center of PPA-20.

Fig. 8.9—Self-shielding of reactivity coefficient for hafnium near center of PPA-20.

energy spectrum). The results of the fuel measurements in PPA-20 are shown in Fig. 8.12. The values of z indicate distances in an axial direction from the central plane bisecting the reactor core. The interface between the core and the end reflector occurs at $z = 13$ in. In PPA-20, Ring 9 is the ring of fuel slug positions adjacent to the beryllium reflector and therefore has a relatively low energy neutron spectrum. The position indicated as $z = 0.2$ in. in Ring 4, on the other hand, is fairly representative of the neutron flux at the center of the reactor core and has a somewhat higher energy spectrum. The activation measurements of Fig. 8-1 were actually performed in Ring 4 at the position $z = 0.2$ in. It should be kept in mind that "fuel" here denotes uranium containing ~90 per cent U^{235} and ~10 per cent U^{238}

Fig. 8.10—Self-shielding of reactivity coefficient for tungsten near center of PPA-20.

Fig. 8.11—Comparison of self-shielding curves normalized to unity.

In general, it can be seen from a comparison of the measurements made at various positions in the reactor core that the fuel self-shielding decreases as the neutron spectrum becomes faster. This effect can be explained by assuming that the self-shielding characteristics of the fission resonances in U^{235} vary with neutron energy somewhat differently from those of the capture resonances in U^{235} and U^{238} . A systematic study of this self-shielding characteristic for fuel has not been made, but scattered measurements in assemblies possessing somewhat faster neutron energy spectra have shown that the reactivity coefficient of fuel can actually increase as the sample thicknesses increase. In this case, the self-shielding of the parasitic capture resonances must be more significant than the self-shielding of the fission resonances. An example of such a condition is

8.4 INTERPRETATION OF REACTIVITY COEFFICIENTS FOR RESONANCE ABSORBERS

Fig. 8.12—Self-shielding of reactivity coefficient for fuel at various locations in PPA-20. Explanation of locations is given in Sec. 8.3 of text.

Fig. 8.13—Self-shielding of reactivity coefficient for U^{235} and for fuel near center of PPA-20 and PPA-21. Fuel contains ~ 10 per cent of U^{235} .

shown in Fig. 8.13 which compares the reactivity coefficient values for fuel samples measured near the center of the PPA-21 reactor core. In this figure the self-shielding curves both for fuel and U^{235} (corrections have been made for the U^{238} contribution) which were measured in PPA-20 and PPA-21 are shown. Although some self-shielding effect can be seen for the U^{235} in PPA-21, the curves are extremely flat over almost two decades of thickness.

8.4 INTERPRETATION OF REACTIVITY COEFFICIENTS FOR RESONANCE ABSORBERS

In the intermediate energy spectrum reactor, the resonance region contributes heavily to the absorption integral and hence to the reactivity coefficient. In the spectrum of Fig. 8.1, for example, an unshielded $1/v$ absorber (boron) has a median absorption energy of 2.0 ev, whereas U^{235} has a median fission energy of about 40 ev. Because of the complexity of the multiple resonance cross-sections as well as the incompleteness of cross-section data, calculation of reactivity coefficients

from nuclear data is in general a difficult problem when self-shielding is present. The reactivity coefficients of simple absorbers such as boron, cadmium, and gold can be successfully computed, for any degree of self-shielding, by straightforward integrations of Eqs. 8.2 and 8.1. As is evident in Figs. 8.3-8.6, the deviations between calculations and experiments were about ± 6 per cent for these cases. However, the uncertainty is much greater for elements with multiple resonances, even for unshielded samples. An attempt was made to calculate the self-shielded reactivity coefficient of silver and the zero-thickness reactivity coefficients for tantalum and hafnium.

The method of attack was to consider the absorption cross-section as the sum of a $1/v$ term and a multi-resonance term. The first term involves merely a renormalized boron integration. The second term is assumed to be a sum of resonance integrals, each with the cross-section obeying the single-resonance Breit - Wigner formula:

$$\sigma_{aj} = \frac{4\pi\lambda^2 g \Gamma_{nj}/\Gamma_j^2}{1 + \left(\frac{E-E_j}{\Gamma_j/2}\right)^2} = \frac{\sigma_{aj}^{\max}}{1 + \left(\frac{E-E_j}{\Gamma_j/2}\right)^2} \quad (8.3)$$

Here $\sigma_{aj}^{\max}$ is the maximum value of the (n, γ) cross-section, and Γ_{nj} , Γ_{aj} , Γ_j are respectively the neutron width, the gamma width, and the total width for the j th resonance. Eq. 7.3, when modified as was Eq. 8.1 to include self-shielding, becomes

$$RC_a(x, r) = - \frac{L}{f_{\text{tot}}} \int dE \phi(r, E) \sigma_a(E) F(r, E) f(\xi) \quad (8.4)$$

This expression is broken up, in order to obtain the multi-resonance term, into a summation over the individual resonances:

$$RC_a(x, r) = - \frac{L}{f_{\text{tot}}} \sum_j \int_{\text{res } j} dE \phi(r, E) \sigma_a(E) F(r, E) f(\xi) \quad (8.5)$$

It is assumed that $F(r, E)$ does not change appreciably over a single resonance, so that

$$RC_a(x, r) = - \frac{L}{f_{\text{tot}}} \sum_j F_{Lj} \int_{\text{res } j} dE \phi(r, E) \sigma_a(E) f(\xi) \quad (8.6)$$

Then, defining

$$\mathcal{J}_j(x, r) = \frac{\int_{\text{res } j} dE \phi(r, E) \sigma_a(E) f(\xi)}{\int_{\text{res } j} dE \phi(r, E) \sigma_a(E)} \quad (8.7)$$

which is an absorption-spectrum-weighted average of $f(\xi)$ over the resonance, Eq. 8.5 becomes

$$RC_a(x, r) = - \frac{L}{f_{\text{tot}}} \sum_j F_{Lj} \mathcal{J}_j(x, r) \int_{\text{res } j} dE \phi(r, E) \sigma_a(E) \quad (8.8)$$

Use of the functional form of $\sigma_a(E)$ in Eq. 8.3, together with the assumption that $\phi(r, E)$ varies only slightly from a value ϕ_j over a single resonance, leads to

$$\int_{\text{res } j} dE \phi(r, E) \sigma_a(E) \approx \phi_j \sigma_{aj}^{\max} \int_0^\infty \frac{dE}{1 + \left(\frac{E-E_j}{\Gamma_j/2}\right)^2} \approx \phi_j \sigma_{aj}^{\max} \Gamma_j \pi/2 \quad (8.9)$$

Thus Eq. 8.5 becomes

$$RC_a(x, \bar{r}) = -\frac{I}{\pi} \sum_j I_j \sigma_{aj} \phi_j F_{L_j}(x, \bar{r}) \quad (8.10)$$

where $\phi_j(x, \bar{r})$ can be expressed, from the slowly varying nature of $\phi(\bar{r}, E)$ in Eq. 8.7, as

$$\phi_j(x, \bar{r}) \approx \frac{\pi I_j}{2} \int_0^\infty \frac{dE f(\bar{r})}{(E - E_j)^2 + \left(\frac{\Gamma_j}{2}\right)^2} \quad (8.11)$$

When the special form of the function $f(\bar{r})$, given in Eq. 8.2, and the Breit - Wigner expression for $\sigma_j(E)$ in $\bar{r}$ are substituted into Eq. 8.11, one obtains

$$\phi_j(x, \bar{r}) = \frac{\pi I_j (N_a x \sigma_{aj}^{\max})}{2} \int_0^\infty dE \int_1^\infty d\mu \left[1 - \exp \left\{ -\frac{\mu \left[1 + \left(\frac{E - E_j}{\Gamma_j/2} \right)^2 \right] \right\}}{(N_a x \sigma_{aj}^{\max})} \right] \quad (8.12)$$

If ξ_j is $(N_a x \sigma_{aj}^{\max})$, the maximum value of ξ in the j th resonance, then $\phi_j(x, \bar{r})$ depends on x only in the combination $(N_a x \sigma_{aj}^{\max})$. It does not really depend on I_j either, and essentially not on E_j . Accordingly $\phi_j(x)$ may be treated as a universal function $F(\xi_j)$. Graphs of both $f(\xi)$ and $F(\xi)$ appear in Fig. 8.14. The curve for $F(\xi)$ is strictly true only in the limit of no Doppler broadening. (When Doppler corrections are included, the true curve for $F(\xi)$ depends upon the absolute temperature of the absorber, as G. M. Roe showed in KAPL-1241.³ The $F(\xi)$ is Roe's $f_1(0, \xi)$ on his Figs. VII and pp. 39ff. See also Vol. I, Chap. 2 of this Handbook.)

Equation 8.10 was applied to the ten reported resonances of Ag^{107} and Ag^{109} below 100 ev. Self-shielding factors, including Doppler corrections, were calculated. The result is shown as the lower curve of Fig. 8.7, whose zero thickness value of the reactivity coefficient is 105¢/mol. The $1/v$ term gives an additional 7¢/mol. Evidently there are not enough resonance terms included in this summation. Renormalization of this curve at zero thickness, to allow for possible errors in parameters, does not give a good fit at large thickness, which indicates that the missing resonances are on the average much smaller than the lowest resonances. By adding enough resonances, each

Fig. 8.14—Self-shielding functions.

resonance with $F_j(x)$ equal to that of the small resonance at 40.8 ev, to give the correct magnitude at $x = 0$, one obtains the excellent fit shown by the upper curve in Fig. 8.7.

It would appear from the above analysis either that resonances above 100 ev are important in the PPA-20 spectrum, or else that many small resonances below 100 ev have not been reported in the cross-section literature. On the assumption that the former was the case, the average level strength and level density of the reported resonances for silver, tantalum, and hafnium were used to extend the summation of Eq. 8.10 from 100 ev to ∞ , a region with practically no measured resonances reported at the time. This was done for $x = 0$, that is, for all $F_j = 1.0$. The results are shown in Table 8.1. It is evident that in each case the calculated reactivity coefficient is still too small. (The resonance data used are shown in Table 8.2 and represent unpublished Brookhaven data as of July 1954.)

TABLE 8.1--COMPUTATION OF ZERO-THICKNESS REACTIVITY COEFFICIENTS FROM CROSS-SECTION RESONANCE DATA

Isotope	$2g\Gamma_n^0$, mv	$\bar{D}$, ev	$2g\frac{\Gamma^0}{D}n$ $\times 10^4$	Calculated RC, β /mol				RC of Natural Element, β /mol	
				$1/v$	0-100 ev	100 ev- ∞	Total	Calc.	Observed
Ag ¹⁰⁷	2.1	25.0	0.84	4	24	8	36	125	138 ($\pm 5\%$)
Ag ¹⁰⁹	3.0	16.5	1.82	11	191	17	219		
Ta ¹⁸¹	3.4	6.0	5.6	2	72 (0-50 ev)	63 (50 ev- ∞)	137	137	140 ($\pm 8\%$)
Hf ¹⁷⁷	2.11	4.35	4.85	38	742	49	829		
Hf ¹⁷⁸	17.8	~ 100	1.78	11	224	1	236	230	295 ($\pm 10\%$)
Hf ¹⁷⁹	2.32	11.1	2.09	8	58	19	85		
Hf ¹⁸⁰	4.07	~ 100	0.41	2	5	0	7		

TABLE 8.2--NEUTRON CROSS-SECTION RESONANCE DATA USED IN COMPUTATIONS

Isotope	% Abundance	E_0 , ev	$\sigma_a \frac{\Gamma^2}{b \cdot \text{ev}^2}$	$2g\Gamma_n$, mv	Γ_γ , ev
Ag ¹⁰⁷	51.4	16.6	60 ± 7	5.6 ± 0.8	0.13 (assumed)
		42.4	45 ± 15	10 ± 4	0.13 (assumed)
		45.6	8 ± 2	2.1 ± 0.6	0.13 (assumed)
		52.2	150 ± 30	36 ± 10	0.13 (assumed)
Ag ¹⁰⁹	48.6	5.22	775 ± 25	12.2 ± 0.5	0.156
		31.1	52 ± 6	9.0 ± 0.2	0.13 (assumed)
		40.8	34 ± 10	8 ± 3	0.13 (assumed)
		56.8	95 ± 25	24 ± 6	0.13 (assumed)
		72.5	150 ± 30	47 ± 15	0.13 (assumed)
Ta ¹⁸¹	100	88.5	21 ± 5	10 ± 3	0.13 (assumed)
		4.29	59 ± 5	1.9	0.10 (assumed)
		10.36	32 ± 5	2.5	0.10 (assumed)
		13.9	12 ± 2	1.3	0.10 (assumed)
		20.7	7 ± 2	1.1	0.10 (assumed)
		24.2	27 ± 5	5.0	0.10 (assumed)
		36.7	250	71.0	0.10 (assumed)
Hf ¹⁷⁷	18.4	39.4	180	54.0	0.10 (assumed)
		1.08	110	2.0	0.043
		2.3	280	4.5	0.078
		5.9	80	5.5	0.055 (assumed)
		3.5	170	11	0.049

REFERENCES

TABLE 8.2--(Continued)

Isotope	% Abundance	E_0 , ev	$\sigma_a \frac{\Gamma^2}{b \cdot \text{ev}^2}$	$2g\Gamma_n$, mv	Γ_γ , ev
Hf ¹⁷⁷		9.0	41	3.5	0.076 (assumed)
		14.1	42	4	0.116
		22.5	17	2.4	0.100 (assumed)
		23.6	3.9	0.7	0.100 (assumed)
		27.3	7.6	1.6	0.100 (assumed)
		33.2	5.3	1.3	0.100 (assumed)
		37.0	46	11	0.100 (assumed)
		46.8	62	18	0.100 (assumed)
		49.1	220	53	0.100 (assumed)
		56.0	63	22	0.100 (assumed)
		57.2	20	8.4	0.100 (assumed)
		64.6	140	46	0.100 (assumed)
		68.4	91	36	0.100 (assumed)
		71.3	14	7.1	0.100 (assumed)
		75	38	18	0.100 (assumed)
		85.8	12	7.2	0.100 (assumed)
		87.4	15	8.9	0.100 (assumed)
		94.4	12	7.7	0.100 (assumed)
		100.1	32	20	0.100 (assumed)
Hf ¹⁷⁸	27.1	7.8	1500	50	0.100 (assumed)
Hf ¹⁷⁹	13.8	5.7	70	4.5	0.060 (assumed)
		17.7	11	1.4	0.100 (assumed)
		23.9	29	46	0.100 (assumed)
		26.5	5.6	1.0	0.100 (assumed)
		31.5	18	3.9	0.100 (assumed)
		36.9	75	18	0.100 (assumed)
		40.5	80	21	0.100 (assumed)
		42.6	26	7.5	0.100 (assumed)
		69.7	16	8.0	0.100 (assumed)
Hf ¹⁸⁰	35.3	74	200	35	0.100 (assumed)

The conclusion is that the errors are more easily accounted for by missing resonances in the low energy region (below 100 ev) than by errors in extending the sum to high energies. Since the latter region accounts for only a small fraction of the total reactivity coefficient, a much more drastic revision in the assumed level strengths and densities would have to be made than would be necessary in the low energy region. These calculations have not been repeated with more recent cross-section data.

It is of interest to note that the calculation and measurements, if refined, can in principle be used as a means of detecting and measuring effective level densities and strengths in regions not resolved by cross-section study techniques.

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	Referred to
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2. R. S. Stone and R. E. Slovacek, work described by M. S. Nelkin, The Thermal Spectrum of an Intermediate Assembly, Nuclear Science and Engineering 2:373 (May 1957). See also Chap. 31, this Volume.	8.2
3. G. M. Roe, The Absorption of Neutrons in Doppler-Broadened Resonances, Report KAPL-1241, October 1954. See also Vol. I Chap. 2 of this Handbook.	8.4

Chapter 9

APPLICATION OF PILE OSCILLATION TECHNIQUES TO THE MEASUREMENT OF REACTIVITY COEFFICIENTS

F. FEINER and R. T. FROST

9.1 INTRODUCTION

Knowledge of the zero-thickness reactivity coefficients of the separated isotopes would be useful in many phases of reactor design and analysis. In principle these coefficients could be calculated for materials of any desired thickness if the resonance parameters were known from cross-section measurements and if the spectrum of the reactor were available. For many elements or isotopes of elements which might be of use as control-rod materials or burnable poisons, however, insufficient cross-section information exists to permit such calculations. It is then useful to use the reactor itself to measure the coefficients. Report KAPL-1703¹ discusses the principle underlying the use of the pile oscillator for this purpose.

Two factors tend to complicate the experimental determination of isotopic reactivity coefficients. The first is that, in general, one is not able to obtain samples of single isotopes in quantity (it is for this reason, of course, that the cross-sections have not been measured), and the second is the uncertainty as to how strongly the samples are self-shielded. By virtue of its great sensitivity, the pile oscillator technique is useful in overcoming both of these difficulties. Small quantities of materials, even though diluted so as to minimize self-shielding effects, still give measurable signals. As a specific application of the oscillator technique, this chapter will discuss a survey of elements for possible use as burnable poisons in an intermediate spectrum reactor. The measurements were made in PPA-31 (whose composition appears in Chap. 2, Table 2.7).

9.2 CORRECTIONS FOR SELF-SHIELDING

In a survey of the kind here under consideration, isotopes are encountered with extremely high cross-sections. The interpretation of reactivity coefficient measurements even on very small amounts of such materials may be clouded by self-shielding effects. The samples must be made thin enough, or diluted sufficiently, to render unimportant two types of effects:

- Type 1. For a homogeneous mixture of infinitesimally small grains of absorber in a matrix, or "solvent," of less absorbing material, the central grains may be shielded from the neutron flux by the outer grains. This is termed configuration shielding.
- Type 2. When the grains are of finite size, self-shielding effects may be a factor within each grain.

It is possible to calculate the size of a sphere (having the density of the material in question) whose self-shielding is equal to the configuration shielding of the sample in its capsule. In order to eliminate effects of Type 2, then, it is necessary to make the actual particle size much smaller than the size which gives the equivalent of the configuration shielding. (In an extreme case this requirement meant grain sizes of less than 10 microns in diameter. To this end the samples were ground down finer and finer until microscopic analysis indicated that the criterion had been met. After thorough mixing with aluminum powder, the "solvent," the mixture was again checked under the microscope for signs of clumping.)

As regards the configuration shielding, it is possible to obtain a zero-thickness reactivity coefficient value in either of two ways. The self-shielding factor, f , is expressible in the form²

$$f = 1/(1 + a\xi) \quad (9.1)$$

Here a is a constant for a given isotope

$$\xi = m\rho/AV \text{ (mol/cm}^2\text{)}$$

$$m = \text{mass of sample (g)}$$

$$\rho = \text{"hydraulic radius" of capsule} \equiv 2V/S$$

$$A = \text{atomic weight of sample}$$

$$V = \text{volume of absorber (cm}^3\text{)}$$

$$S = \text{surface area of sample (cm}^2\text{)}$$

From this expression, and the definition of f as $f = RC/RC_0$, it follows that

$$RC = RC_0/(1 + a\xi)$$

where RC is the reactivity coefficient for a given thickness, and

RC_0 is the zero-thickness reactivity coefficient.

From a plot of RC vs ξ it is thus possible to determine RC_0 and a . This is the first way of obtaining RC_0 .

The second way of obtaining RC_0 depends upon the empirical fact that for self-shielding factors greater than 0.6, ($\ln RC$) varies linearly with ξ . That is to say, within the accuracy of obtainable data, $RC = RC_0 e^{-a\xi}$. It may easily be verified that the plots of $1/(1 + a\xi)$ and of $e^{-a\xi}$ lie within 5 per cent of each other when $a\xi \leq 0.7$ and $a/a = 0.77$. When only a few measurements of RC as a function of ξ are available, and there is good reason to believe that $a\xi \leq 0.7$ (i.e., $f \geq 0.6$), it is easier to use the second method and extrapolate a straight-line semi-log plot of RC vs ξ to obtain RC_0 .

Frequently the limitations of time and lack of enriched isotopes in larger amounts make it impossible to measure more than two samples of any given isotopic mixture. The second method must then be used and extrapolated from two points to determine a value of RC_0 . Needless to say, if a meaningful extrapolation is to be made from just two experimental points, a good idea of the experimental error to assign to them must be established.

A complication arises from the fact that enriched isotopic samples are rarely obtained in which other isotopes are really present only in negligible quantities. Therefore, in going from one quantity of this sample to another the self-shielding change is not only that of the isotope under study but involves the contaminants as well. As an example, suppose five different mixtures of the five principal hafnium isotopes, i.e., samples with five different isotopic abundance ratios, are available. From measurements on these it is desired to deduce the zero-thickness reactivity coefficient value for as many of the isotopes as possible.

In order to obtain the zero-thickness RC 's of the five isotopes, RC 's for the five mixtures must be measured as a function of thickness. If only a single RC measurement on each of the mixtures were obtained, there would be, in addition to the five unknown zero-thickness reactivity coefficients, twenty-five unknown self-shielding factors. However, if RC measurements on each of the five isotope mixtures are carried out for several thicknesses, the following applies: Let $\overline{RC}_0^i$ denote the measured reactivity coefficient value of sample i ($i = 1 \dots 5$) extrapolated to zero thickness. Let RC_0^i be the zero thickness reactivity coefficient of hafnium isotope i ($i = 1 \dots 5$, corresponding to hafnium 176, 177, 178, 179, 180, the other isotope being neglected). If $C_{\nu 1}$ represents the atom fraction of isotope ν in sample number i , then $\overline{RC}_0^i = \sum_{\nu} C_{\nu 1} RC_0^{\nu}$. These five equations can be solved for the zero-thickness reactivity coefficients of the five isotopes, RC_0^{ν} .

In practice, an approximate solution can sometimes be obtained for the RC 's of the isotopes with the larger cross-sections without the necessity for performing measurements on more than a single thickness of the samples enriched in the isotopes with the smaller cross-sections, if the single sample is thin enough that self-shielding is relatively unimportant.

9.3 EQUIPMENT

The apparatus used in the measurements made by the oscillator technique consists of (1) a mechanical pile oscillator (i.e. a device for periodically moving into and out of the active region of the assembly a sample of the material to be measured), and (2) a detection device which measures the effect of these periodic changes on the pile reactivity.

The mechanical oscillator is made up of the following parts:

1. The tube in which the sample slides into and out of the assembly core. This tube is generally the center slug of the assembly.
2. A 10-chamber cylinder which may be "loaded" with sample capsules prior to a run. Any sample may then be introduced into the oscillator tube by remote control.
3. A gas pressure system to propel the sample capsules both into and out of the reactor core. The motion is such as to produce a square-wave reactivity modulation.
4. A time-base generator and selsyn system which permits adjustment of the oscillating period to equal that of the galvanometer (see below) to within a few tenths of one per cent.

The detecting system makes use of a high-Q detector with no electronic amplification, in preference to the more standard integrators. The elimination of all such electronic elements and their accompanying stability difficulties has proved to be a valuable innovation. The detecting system (see Fig. 9.1) consists of:

Fig. 9.1—Block diagram of detecting equipment.

5. An ion chamber located near the region of maximum flux.
6. A micro-ammeter for monitoring the d-c level of the reactor.
7. A high-pass (pile drift) filter which enables the subsequent equipment to look only at the a-c component of the neutron level.
8. An attenuator and Q selector.
9. A moving-coil galvanometer in an evacuated bell jar. Removal of air from the jar to the point where the mean free path is greater than the dimensions of the galvanometer makes it possible to reduce air damping to the point where the Q of the galvanometer may be made to exceed 100. In general, Q values of about 30 were found to be most useful.

Some of these components will now be described in greater detail.

The drive mechanism (3) consists of a synchronous motor with a coupling arrangement which permits accurate and stable adjustment of the output speed over a limited range. Synchronization with the line frequency was found to be excellent for times as long as a day, and the line frequency is thought to be stable to a few tenths of a per cent.

The ion chamber (5) was made sufficiently large that

- a. no amplification was necessary,
- b. chamber noise was negligible compared to pile noise. In practice this means a d-c chamber current of the order of a microampere when the pile power is a few watts.

A diagram of the filter (7) is shown in Fig. 9.2 along with a plot of the magnitude of its transfer function vs frequency. The important features of the filter are:

- For all settings of the Q-selector, its resistance r must be much less than the filter resistance R , otherwise the filter attenuation would not be independent of the attenuator setting.
- Analysis of the filter network shown in Fig. 9.2 shows that the magnitude of the high-pass filter transfer function, G , varies as ω^3 near $\omega = 0$, so that slow pile drifts are really strongly suppressed. It should be pointed out that the first resistor of the filter should be either wire-wound or of a type with similar low-noise characteristics.

The attenuator and Q-selector are diagrammed in Fig. 9.3. For given Q-selector setting the attenuator switch may only be set at positions on the level or below the Q-selector switch. It may then be shown that with the resistance values indicated, the

$|G|$ vs. ω

Fig. 9.2—High pass (pile drift) filter characteristics. G is the transfer function of the filter, shown in its dependence upon the frequency.

Fig. 9.3—Attenuator and Q-selector. Values of the high-pass filter resistance r are shown in ohms.

Fig. 9.4—Control panel and Q-selector switch. Lights indicate which sample is being oscillated. Galvanometer is in background.

Fig. 9.5—Oscillator tube with the 1-chamber sample. Chamber is on the left. In the foreground is the square-wave calibrating oscillator. A poison half-cylinder that rotates inside a stationary poison half-cylinder is visible at the left end of the tube.

Fig. 9.6—Close-up view of the 10-chamber sample cylinder.

attenuator switch cannot change the Q by more than 10 per cent, nor can the Q -selector switch affect the attenuator by more than 0.5 per cent. The position commonly used was C ($Q \approx 30$); in this case the effect of the attenuator on Q is ≤ 2 per cent.

The moving-coil galvanometer has a very high sensitivity. It was adjusted to have the desired natural period of about 15 sec.

Figures 9.4–9.6 are photographs of the equipment. Figure 9.4 shows the control panel including attenuator and Q -selector switch. The lights indicate which sample is being oscillated. Figure 9.5 shows the oscillator tube with the ten-chamber cylinder at the left and a calibrating square-wave oscillator of conventional rotating design.

Figure 9.6 is a close-up view of the ten-chamber cylinder and associated pneumatic tubing.

9.4 EXPERIMENTAL PROCEDURE

During a typical day's running, approximately five sample capsules can be measured along with a standard and a blank capsule. The standard consists of a solid cylinder of phenolic resin in the shape of the capsules, whose reactivity value is sufficiently large to be measurable on the control rods. It thus serves to calibrate the oscillator detection equipment. The capsules are bullet-shaped shells of aluminum 2.4 in. long, with an outer diameter of 0.35 in. and an internal diameter of 0.25 in. In cases where the volume of the sample to be measured was smaller than that of the capsule, aluminum powder was used to "dilute" the sample.

In order to obtain measurements in which the effects of self-shielding are reduced to a minimum under the restraint of constant sample volume, it is clear that the material should be diluted as much as is consistent with obtaining reactivity signals of measurable size. The sample was diluted until the signal was so small that the error introduced by pile level fluctuations became of the same order of magnitude as the uncertainties in the self-shielding. For most samples, this led to carrying out the measurements with samples giving signals of the order of 0.05¢, which is equivalent to 15 mg of highly diluted natural boron. In cases where insufficient material was available, naturally, smaller signals had to be used.

For most of the capsules, the galvanometer deflections were recorded for 100 oscillator cycles. The deflections were read by the experimenter at times corresponding to a given point of the oscillation cycle and punched into an automatic point plotter.

A typical trace so obtained is shown in Fig. 9.7. At frequent intervals during a run the power level was read on the d-c ammeter. The power level was generally held constant to within 2 parts in 150 during a 100-cycle (~25 min) measurement. It should be noted that to achieve such stability, it was in general necessary to wait for about 15 minutes after a given sample was started oscillating before beginning a measurement; an additional period of 10 minutes was needed for the galvanometer to come to equilibrium.

9.5 ANALYSIS OF DATA

For each sample measured, two numbers were desired: the RC of the sample and the accuracy with which it was known. To determine the RC, straight lines were drawn on the data traces corresponding to the average values of the left and right deflections, as judged by eye. The peak-to-peak average deflection so obtained was then corrected for the aluminum powder and capsule and compared with the standard.

The objection may be raised that a more systematic way of averaging the traces should be found than just an estimate by eye. Checks on the accuracy of the eye method were made by

1. having different observers make the estimate,
2. finding the average by using a planimeter to measure the area included by the galvanometer trace and calculating the width of the equivalent rectangle.

Agreement between observers was within 1 per cent and agreement between the eye and planimeter method was about 2 per cent for peak-to-peak deflections of roughly 3 cm. In view of the errors due to noise (see below) which amount to ~10 per cent for a 3-cm peak-to-peak deflection, it was considered unprofitable to expend the extra effort re-

Fig. 9.7—Typical trace obtained from point plotter.

Fig. 9.8—Reactivity coefficient vs sample thickness.

quired by the planimeter method. (The reproducibility of planimeter measurements for such erratic figures is probably not better than 1 per cent.)

Reactivity coefficient values in cents per mole were calculated for each capsule and the results plotted vs the "thickness" of the sample material. These curves were then extrapolated to the zero-thickness reactivity coefficient values for the various samples measured. Measurements were made only on elements containing not more than two naturally occurring isotopes. For those with just one isotope, the zero-thickness value obtained from the extrapolation was obviously the isotopic value wanted. For those elements with two isotopes, two samples with different isotopic abundances were measured so that the two zero-thickness values obtained from the graphs made it possible to solve for the isotopic zero-thickness reactivity coefficient values.

9.6 ERROR ANALYSIS

The advantage of the oscillator technique is its high sensitivity, making it possible to measure very small signals. While this feature is very desirable it necessitates a thorough investigation of the noise associated with the experiment. In addition, the use of a high-Q detecting device (the galvanometer) complicates the error analysis.

The errors to be expected have been examined in considerable detail in Report KAPL-1703.¹ Here only the approach taken and the significant results will be given. Use is made of the fact that noise fluctuations of the pile are representable by an oscillating source. For measurements at 10-w reactor power lasting for 25 min, it is shown that a signal-to-noise ratio of 1.0 is realizable for signals as small as 0.002¢.

It is also shown that the signal-to-noise ratio of the detector is independent of its Q. While the signal-to-noise ratio for a single cycle increases with Q, the number of independent measurements made in a given period of time decreases with Q, and the two effects cancel.

The traces obtained by means of the technique described in Sec. 9.4 bear out the results of the noise analysis, i.e., the spread of what can reasonably be called the average value of the deflections agrees well with 0.002¢.

9.7 EXPERIMENTAL RESULTS

The experimental results obtained are plotted in Figs. 9.8-9.10 for those instances in which two capsules of different thickness were measured. The label on each curve indicates the isotope predominantly present in the sample. The errors indicated were a consequence of the pile noise as discussed in Sec. 9.6. The isotopic zero-thickness reactivity coefficients (obtained directly from the graphs in the case of single-isotope elements, and solved for in the case of two-isotope elements) are shown in Table 9.1. The errors indicated in the table for the two-isotope elements were calculated as follows:

1. A standard deviation is computed for the zero-thickness reactivity coefficient,

$\overline{RC}_0^1$ for each isotopic mixture measured. Let $\overline{RC}_1^1$ be the measured reactivity coefficient for the sample of effective thickness ξ_1 and let $\delta \overline{RC}_1^1$ be the standard deviation in $\overline{RC}_1^1$. The form assumed is:

$$\overline{RC}_1^1 = \overline{RC}_0^1 e^{-\alpha \xi_1}$$

Since the $\overline{RC}_1^1, \overline{RC}_2^1, \dots$ are independent measurements on different thicknesses $\xi_1, \xi_2, \dots$ we may combine the errors in the standard Gaussian manner and obtain (for the case of two thicknesses)

$$\left[\frac{\delta \overline{RC}_1^1}{\overline{RC}_0^1} \right]^2 = \left(\frac{\xi_2}{\xi_1} \right)^2 \left(\frac{\delta \overline{RC}_1^1}{\overline{RC}_1^1} \right)^2 + \gamma^2 \left(\frac{\delta \overline{RC}_2^1}{\overline{RC}_2^1} \right)^2$$

where

$$\gamma = \frac{\xi_1}{\xi_2 - \xi_1}$$

Fig. 9.9—Reactivity coefficient vs sample thickness.

Fig. 9.10—Reactivity coefficient vs sample thickness.

The $\xi_1, \xi_2, \dots$ are assumed to have negligible error.

2. The $\overline{RC}_0^{-1}$ values corresponding to different isotopic mixtures are then combined, again on the basis of independence, to obtain the quoted standard deviations on the isotopic reactivity coefficients. In this case the isotopic abundances in the different mixtures are assumed to be known to within negligible error.

9.8 COMPARISON WITH ESTIMATES BASED UPON CROSS-SECTION RESONANCE PARAMETERS

Expected zero-thickness reactivity coefficient values are calculated from the expression for absorbers discussed in Chap. 7. (Eq. 7.3):

$$RC_a = - \frac{L}{f_{\text{tot}}} \int dE \phi(E) \sigma_a(E) F(E)$$

RC is in cents/mole

L = Avogadro's number

f_{tot} = total fission rate in the reactor per watt

$\sigma_a(E)$ = absorption cross-section per atom

$\phi(E)$ = neutron flux per unit energy interval per watt

$F(E)$ = neutron value function or favorableness (see Sec. 7.1). This is assumed to be a slowly varying function over each resonance.

Since the coefficient of the integral is the same for all samples, the quantity of interest is

$$\int_0^{\infty} dE \phi(E) \sigma_a(E) F(E)$$

For multi-resonance absorbers, reasoning analogous to that leading to Eq. 8.10 gives as the resonance contribution:

$$\int_0^{\infty} dE \phi(E) \sigma_a(E) F(E) = \frac{\pi}{2} \sum_j \Gamma_j \sigma_{a,j}^{\text{max}} \phi_j F(E_j)$$

Here

Γ_j is the total width at half-maximum of the j th level

$\sigma_{a,j}^{\text{max}}$ is the peak absorption cross-section at the j th resonance

ϕ_j is the neutron flux at the j th resonance energy

$F(E_j)$ is a mean value of $F(E)$ at the j th resonance

It has been assumed here that $\phi(E) F(E)$ varies a negligible amount over the width of one resonance. The contribution of the nonresonance processes such as $1/v$ absorption must be added when significant.

In many instances, due to the inadequacy of the cross-section data, these estimates are rather crude. Every effort was made to use the latest published data for widths and peak cross-sections. It should be noted that only known resonances are included in these estimates, as opposed to those quoted in Chap. 8 where an effort was made to extrapolate to higher energies.

The flux used was measured in PPA-20 by G. B. Gavin using the methods described in Chap. 6.

The estimates so made are generally useful from two points of view: (1) Comparison with subsequently measured values indicates the degree to which predictions of reactivity coefficient values may be made. (2) The estimates serve to determine the amount of the sample material to be put into the capsules. It should also be noted that in some instances comparison of the calculated and measured reactivity coefficients may make it possible to assign to an isotope a resonance which is only known to exist for the element.

The values calculated according to the above prescription are shown in Table 9.1. Inspection of the results in Table 9.1 indicates that in many cases the uncertainties of the experimental data prevent meaningful comparison with the calculated values. In these cases where comparison is possible, discrepancies may be due to a lack of resonance parameter data at the higher energies which are still significant in intermediate reactors, and to uncertainty in assigning resonances to a particular isotope. These factors are especially applicable to the iridium results.

The data of Table 9.1 show reasonable agreement in 12 out of the 18 isotopes tested. Of the six showing disagreement, one difference was in an inexplicable direction, for the measured RC was lower than that calculable from the known resonances.

In general, although the data accumulated in the very short survey here reported are not definitive, the method itself has been shown to be a very sensitive means of obtaining reactivity coefficients on small quantities of essentially unshielded isotopic samples.

TABLE 9.1--REACTIVITY COEFFICIENT DATA

Element	RC from Resonances, ¢/mol	1/v Contrib., ¢/mol	Total Calculated RC ¢/mol	Experimental RC, ¢/mol
Single Isotopes				
Mn	3.9	1.6	5.5	24.0 ± 2.6
Co	11	4.3	15.3	28.4 ± 4.0
As	7.9	0.48	8.4	28.2 ± 2.8
*Y	0.16	0	0.16	0.1 ± 0.3
Rh	102	17.5	120	135 ± 14
Ho (13% Eu)	110	7.5	118	180 ± 42
Tm (1% Lu)	220	13.8	234	165 ± 30
Ta	112	2.5	115	140 ± 15
Pairs of Isotopes				
*Ag ¹⁰⁷	34	3.5	37.5	33 ± 6
Ag ¹⁰⁹	218	9.8	228	290 ± 50
In ¹¹³	55	7.4	62	670 ± 670
In ¹¹⁵	330	23	353	390 ± 50
*Sb ¹²¹	51	0.66	52	48 ± 6
*Sb ¹²³	4.7	0.45	5.2	37 ± 4
Re ¹⁸⁵	252	11.7	264	290 ± 50
*Re ¹⁸⁷	26	6.3	32	23 ± 5
*Ir ¹⁹¹	240	117	357	430 ± 80
Ir ¹⁹³	130	14	144	390 ± 60

* Only one sample thickness was measured.

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in section

9.1, 9.6

9.2

Part II

REACTIVITY EFFECTS ASSOCIATED WITH REACTOR OPERATION

P. L. HOFMANN and M. L. STORM

Chapter 10

REACTOR SURVEY CALCULATIONS

P. L. HOFMANN and M. L. STORM

10.1 INTRODUCTION

The purpose of conducting reactor survey calculations is to investigate the interaction of a relatively large number of nuclear parameters (such as reactor size, type of moderator, type of coolant, and amount of fissionable material required) in order to predict reactor behavior and to obtain an optimum nuclear design. Survey calculations can be very simple if only one or two parameters are to be varied. They may become quite complex if a large number of parameters are to be considered.

In this discussion survey calculation techniques employed at the Knolls Atomic Power Laboratory (KAPL) in the investigation of SIR-type reactors are primarily considered. The term SIR-type reactor refers to an intermediate spectrum reactor that is reflector-controlled by means of rotating control drums located at the core-side reflector interface. One side of each drum has a neutron absorber face; the other face consists of moderating material.

The determination of the nuclear characteristics of intermediate spectrum reactors depends heavily on the so-called "multigroup approach" of Hurwitz and Ehrlich, described in Volume I¹. At KAPL various multigroup schemes were in use, most of them employing about 15 lethargy groups. This comparatively large number of groups makes it impractical to perform calculations without the aid of some sort of automatic computing equipment. Until fairly recently IBM machines like the CPC were employed, and with their aid it was possible to complete 15-group multigroup calculations for comparatively simple reactor models (for instance: three concentric spheres, each of uniform composition, representing reactor core, reflector, and blanket, respectively) in about 5 hr. Similar calculations by means of desk calculators might occupy one person about two weeks. Recently, fast computers like the UNIVAC have become more generally available, and it is now possible to perform these calculations in a matter of minutes.

The results of a reactor survey can be presented graphically by mapping a series of "reactor points." For example, a typical map might consist of a critical mass contour as a function of moderator volume fraction and core size. In a simple survey each reactor point can be mapped on the basis of the information derived from a single multigroup calculation. In a more complex survey involving, for instance, the determination of reactor endurance, it might be necessary to conduct a whole series of multigroup calculations to fix a single reactor point. In the following paragraphs various survey techniques employing one- and two-dimensional multigroup formulations will be described.

10.2 SPHERICAL GEOMETRIES: CLEAN AND DEPLETED REACTORS

A survey technique for determining reactor endurance is described in Report KAPL-975². This was the first time that the large capacity and high computing speed of the UNIVAC was utilized to survey the nuclear properties of a large variety of re-

actors. In this approach two reactor conditions were examined in the calculation of each reactor point: the clean-core critical condition and the completely depleted reactor condition. For calculation of the depleted condition fission products other than xenon were lumped together, and boron was used to simulate this group of fission products. The mathematical treatment of the xenon followed standard techniques similar to those described in Chap. 13.³ A reactor life in full-power hours was assumed, and the necessary mass in the depleted condition was calculated with the reactor reflected by a beryllium-sodium-steel reflector.

In the clean condition boron was removed from the core, and a specified amount of U^{235} was added to account for the burnout for the required full-power hours. The reflector for this clean calculation was the same as the one used to determine the critical mass in the depleted condition. Thus the difference in multiplication constant between these two calculations is due to fission-product poisoning and fuel depletion. The addition of the peak xenon reactivity defect and of a suitable allowance for safety margin yields the total reactivity range which must be supplied by the control system for safe operation at the specified power level and for the specified number of hours.

The next step was to examine the amount of shim control which would be supplied by the SIR-type control system. Another calculation was performed for a clean core and a reflector containing some specified uniformly distributed amount of boron, characteristic of the zero-degree control-cylinder position (B_4C face toward the core). This amount was based on experimental information obtained in the preliminary pile assembly (PPA). The reactivity difference between the reactor containing the initial U^{235} loading and having a clean reflector (no boron) and the reactor with the boron reflector is a measure of the reactivity available for control. A comparison of this reactivity difference with the required reactivity difference provides an estimate of the reactor endurance.

The results of these calculations are presented in Report KAPL-975² as a series of contours of fuel loading, endurance, and xenon reactivity defects as a function of core beryllium volume fraction and core size. Such mappings facilitate the selection of optimum nuclear design regions. Representative calculation times per reactor point are of the order of 15 min.

The results shown in Fig. 10.1 (reproduced from Fig. 1 of Report KAPL-975²) pertain to a hypothetical sodium-cooled beryllium-moderated 120-Mw reactor. Endurance values given in Fig. 10.1 are normalized to 900 hr operation at 120 Mw. For example, an endurance of 2.0 corresponds to 1800 full-power hours. As described in Report KAPL-975, if the indicated endurance is greater than 900 hr, suitable adjustments must be made in the depleted loading to account for the increased fission-product accumulation.

10.3 SPHERICAL GEOMETRIES: STEPWISE BURNOUT

Another and somewhat more elaborate technique to obtain similar contour maps is described in references 4 to 7. This technique was specifically developed to treat the application of burnable poisons to SIR-type reactors. However, it can also be used for other reactor endurance surveys. As before, all calculations were patterned on three-media spherical models consisting of homogeneous core, reflector, and blanket regions. The basic purpose of all calculations was to obtain the endurance in full-power hours for various SIR-type reactors. The following procedure is used. It is assumed that the reactor operates at full power over its entire life span. This time span is divided into arbitrary time increments. An initial reflector boron concentration is chosen to represent the critical control-cylinder position, and a multigroup calculation is performed. From the results of this first calculation, a rate of U^{235} burnout and fission-product accumulation (simulated by boron) is obtained. With these rates the uniform U^{235} loading and fission-product concentration at the end of the time interval can be determined. If the burnup of fuel over the entire life cycle is not too great, the assumption of uniformity of fuel depletion and stable fission-product accumulation is adequate. After adjustments are made for the presence of xenon, criticality is re-established by the removal of some reflector boron to simulate control-cylinder rotation. This entire procedure is repeated until the reflector boron concentration is reduced to zero (control cylinders in their most reactive position) which indicates the end of the reactor life.

Examples of the results obtained from this type of survey are presented in references 8 and 9. In particular, in reference 9, it was desired to find a small reactor with long

Fig. 10.1—Sodium reactor survey contour map.

Fig. 10.2—Endurance and safety as a function of core loading and diameter.

10.4 TWO-DIMENSIONAL GEOMETRIES

endurance at SIR power which would fit into the S1G container. In order to find possible design points satisfying these criteria an endurance survey was made of small core beryllium-moderated sodium-cooled reactors in which the control is provided by rotating cylinders. The same sodium velocity was used in all calculations and account was taken of the increasing volume fraction occupied by fuel and structural material with decreasing core volume. The results of this study are given in Fig. 10.2. (Reproduced from Fig. 10 of Report KAPL-Memo-MLS-5.⁹) It can be seen in Fig. 10.2 that cores with diameters between 23 and 22 in. and core loadings varying from about 54 to 82 kg were of possible interest. The endurance in this region varies between 2500 and 3500 hr and the safety margin extends from -\$4 to -\$8. Representative calculation times per reactor point were about 3 hr.

10.4 TWO-DIMENSIONAL GEOMETRIES

The two techniques described above have utilized one-dimensional spherical reactor geometries. In the case of the S1G the use of this geometry did not realistically account for the angular dependency introduced by the presence of the rotating control cylinders. The development of the $(r-\theta)$ method, which involves the two cylindrical space dimensions of radius and angle, received impetus from a desire to create a multigroup technique which would effectively treat the unique S1G geometry.

An early application of the $(r-\theta)$ technique was a survey of a single well-defined reactor, namely, the S1G, over its entire lifetime. The purpose of the survey was to obtain information concerning control characteristics, flux, and power distributions throughout the reactor history, and to make comparison with experiment whenever possible.¹⁰ A number of $(r-\theta)$ calculations were performed to trace the behavior of the reactor at discrete time intervals corresponding to the completely shutdown condition, the clean critical condition, an intermediate condition, and, finally, the depleted condition near the end of the reactor operational life cycle. In general, the results of this survey were in good agreement with measurements made in preliminary pile assemblies (PPA, PTR) representing various stages of reactor life. Representative calculation times on the UNIVAC are about 6 hr per source iteration, and several iterations are usually required.

Another two-dimensional scheme of great utility is the $(r-z)$ method, which employs the cylindrical coordinates radius and height. A suitable reactor model for an $(r-z)$ calculation is one in which angular variations are either nonexistent or relatively insignificant so that they can be ignored. The $(r-z)$ method has been employed in various surveys. For instance, in a study of helium-cooled reactors¹¹ this method was used to determine the effect on power distribution and critical mass brought about by changes in the L/D (length-to-diameter) ratio. Another study¹² considered the effect on excess reactivity and shutdown safety of lengthening the S1G core and increasing the fuel loading. Representative calculation times per source iteration are again of the order of 6 hr on the UNIVAC.

In conclusion it might be pointed out that while reactor surveys are costly, the survey procedure is nevertheless more economical than the building of numerous critical assemblies to investigate a broad area of interesting nuclear designs.

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Chapter 11

SELF-SHIELDING CALCULATIONS FOR SIR-TYPE REACTORS

P. L. HOFMANN and M. L. STORM

11.1 INTRODUCTION

Many criticality calculations for practical reactor geometries are performed by establishing an equivalent homogeneous reactor model whose nuclear properties are then analyzed. The transformation of the actual heterogeneous reactor geometry to the equivalent homogeneous model can be accomplished by the employment of effective absorption cross-sections in the homogeneous model. The factors used to modify the actual absorption cross-sections in order to obtain effective values are referred to as self-shielding factors.

The self-shielding scheme used at KAPL in conjunction with the multigroup formulation of the age-diffusion equation was developed by H. Hurwitz, Jr. and G. M. Roe. The mathematical development of this theory is described in detail in references 1 to 3.

The application of the Hurwitz-Roe self-shielding methods to specific SIR-type geometries is discussed in this chapter. Simplified analytical approximations for these self-shielding factors, which were used extensively in multigroup survey calculations, are also developed. Initial application of this theory to S1G/S2G was made by H. Hurwitz, Jr., G. M. Roe, and W. J. C. Bartels.

A brief outline of the Hurwitz-Roe scheme is given in the following paragraphs. The age-diffusion equation can be written as²

$$-\nabla \cdot \frac{1}{3\Sigma_{tr}} \nabla nv + \Sigma_a nv = -\frac{\partial q}{\partial u} + S(r,u) \quad (11.1)$$

where nv is the neutron flux per unit lethargy, $S(r,u)$ is the source of neutrons, and q is the slowing-down density defined as

$$q = nv \xi \Sigma_s \quad (11.2)$$

Equation 11.1 is obtained assuming Σ_a is much smaller than Σ_s , but this assumption is not always justified. One way of accounting for large absorptions consists of the following procedure. The space-independent solution of the age-diffusion equation in a source-free energy range is compared with the corresponding solution of the Boltzmann equation. This comparison then indicates what adjustments are necessary to establish equivalence between the slowing-down processes as calculated by the two methods. A description of this approach is given in references 2 and 3, where various solutions of the space-independent Boltzmann equation are presented.

One possible modification² is to change the definition of the slowing-down density, but this procedure has not been adopted at KAPL. Instead, the relation $q = nv \xi \Sigma_s$

11.1 INTRODUCTION

has been retained, and the absorption cross-sections have been adjusted by means of properly chosen (self-shielding) factors in order to give the desired variation of q with energy for the space-independent problem. This method accounts for both the presence of large absorption and the heterogeneity of the assembly.

The space-independent solution of the age-diffusion equation in a source-free energy region is

$$q = q_0 \exp \left(- \int^u \frac{\Sigma_a}{\xi \Sigma_s} du \right) \quad (11.3)$$

As discussed in references 2 and 3, an approximate solution of the Boltzmann equation is

$$q = q_0 \exp \left(- \int^u \lambda(u') du' \right) \quad (11.4)$$

where $\lambda(u)$ is related to $P_c(u)$, the probability that a neutron scattered into the lethargy u will be captured before being scattered again, by the relation

$$\frac{(1-\alpha)(1-\lambda)}{1-\alpha^{1-\lambda}} = 1 - \bar{P}_c(u) \quad (11.5)$$

In the above, $\alpha = (M-1)^2 / (M+1)^2$, where M is the mass of the scattering material relative to the neutron mass and $\bar{P}_c$ is the average value of $P_c(u)$ when the spacing of the absorption resonances is small compared to the average energy loss in a scattering collision. If the absorption resonances are broad, this averaging is not necessary and $P_c(u)$ may be used. The solution of Eq. 11.5 can be written as

$$\lambda = \frac{\bar{P}_c}{\xi} \beta(\bar{P}_c) \quad (11.6)$$

where β is a correction factor which is unity when $\bar{P}_c$ is small. In one approximation β is taken as

$$\beta = \frac{1}{1 - [1 - (\gamma/\xi)] \bar{P}_c} \quad (11.7)$$

where γ is a function of α . Comparing Eqs. 11.3 and 11.4, it is seen that, if the term Σ_a appearing in the age-diffusion equation were to be replaced by $(\Sigma_a)_{\text{eff}}$ chosen so that

$$\frac{(\Sigma_a)_{\text{eff}}}{\xi \Sigma_s} = \lambda \quad (11.8)$$

the desired correspondence between the two solutions of the space-independent problem would be obtained. Thus, using Eq. 11.6,

$$(\Sigma_a)_{\text{eff}} = \bar{P}_c \beta(\bar{P}_c) \Sigma_s = \frac{\bar{P}_c}{\Sigma_a} \Sigma_s \beta(\bar{P}_c) \Sigma_a = F \beta(\bar{P}_c) \Sigma_a \quad (11.9)$$

In Eq. 11.9 the self-shielding factor F is defined as

$$F = \frac{\bar{P}_c}{\Sigma_a} \Sigma_s \quad (11.10)$$

In summary, the following procedure applies in obtaining $(\Sigma_a)_{eff}$: (1) calculate $P_c(u)$; (2) average $P_c(u)$ when necessary; and (3) correct with the factor β when the average absorption probability is large. (In most practical cases it is sufficient to let $\beta = 1$.) The averaging of $P_c(u)$ over resonances has been done by G. M. Roe for various geometries.³ In the energy region of small resonance spacings where $P_c(u)$ is usually averaged, the self-shielding factors tend to be close to unity.

In practice the calculation of $P_c(u)$ is usually quite difficult; several simplifying assumptions must be made, and the over-all treatment is semiquantitative at best. However, the effects of self-shielding on criticality are not major since, in the energy range where self-shielding is important (low energy), leakage is unimportant. The main effect of self-shielding is to change the neutron spectrum in the core. Multigroup calculations by I. Bornstein on SIR-type reactors loaded with 35 and 40 kg of U^{235} and with clean reflectors show that the difference in k between calculations performed with and without self-shielding factors is about 2.5 per cent.

In the following sections a description of the application of these procedures to the SIR-type geometry will be presented. Only the low-energy region where the self-shielding factors are appreciably different from unity will be discussed.

11.2 SELF-SHIELDING FACTORS

The reactor core is depicted as consisting of two regions: (1) the fuel region - containing fuel and associated absorbers in contact with the fuel (e.g., fission products) - and (2) the moderator region - containing structural materials and coolant in addition to the moderator. It is assumed that these materials are uniformly mixed.

The probability of capture before scattering is then considered as being made up of two component probabilities: (1) P_o , the probability of capture in the fuel region before scattering or capture by the moderator region, and (2) P_m , the probability of capture before scattering in the moderator region. Hence

$$P_c = P_o + P_m \quad (11.11)$$

The probability of scattering before capture in the fuel region is $1 - P_o$, so that P_c can be approximated by

$$P_c = P_o + \frac{\Sigma_{am}}{\Sigma_{am} + \Sigma_s} (1 - P_o) \quad (11.12)$$

where Σ_{am} is the macroscopic absorption cross-section in the moderator region and Σ_s is the macroscopic scattering cross-section (scattering in fuel is neglected). The above formula would be correct if structural and moderator materials were mixed homogeneously.

P_o is approximated by

$$P_o = \frac{f\Sigma_{ao}}{\Sigma_s + \Sigma_{am} + Cf\Sigma_{ao}} \quad (11.13)$$

where Σ_{ao} is the macroscopic absorption cross-section for the fuel region, f is the geometrical fuel-element self-shielding factor^{4,5} and C is a parameter designed to take account of mutual shielding between fuel elements. When fuel is "black" ($f\Sigma_{ao}$ approaching a constant value), P_o is redefined as P_∞ , and C is obtained from Eq. 11.13. The calculation of P_∞ and hence C , depends on the specific reactor under consideration.

For a 50-kg core, the largest value β takes at isolated resonances is about 1.08. More typical values of β are in the neighborhood of 1.02. Since the subsequent self-shielding calculations are simpler when β is unity, it is convenient to let $\beta = 1$ in Eq. 11.9 and obtain

11.3 APPLICATION TO SIR-TYPE GEOMETRY

$$P_c = P_o + P_m = \frac{(\Sigma_s)_{eff}}{\Sigma_s} = \frac{(\Sigma_{ao})_{eff}}{\Sigma_s} + \frac{(\Sigma_{am})_{eff}}{\Sigma_s} \quad (11.14)$$

Since $(\Sigma_{ao})_{eff}/(\Sigma_{am})_{eff}$ is equal to the ratio of capture probabilities in the two regions,

$$P_o = \frac{(\Sigma_{ao})_{eff}}{\Sigma_s} = \frac{A\Sigma_{ao}}{\Sigma_s}$$

and

$$P_m = \frac{(\Sigma_{am})_{eff}}{\Sigma_s} = \frac{B\Sigma_{am}}{\Sigma_s} \quad (11.15)$$

where A and B are the self-shielding factors for the fuel region and moderator region, respectively.

Solving for A and B yields

$$A = \frac{\Sigma_s}{\Sigma_{ao}} P_o = \frac{f\Sigma_s}{\Sigma_s + \Sigma_{am} + fC\Sigma_{ao}}$$

$$B = \frac{\Sigma_s}{\Sigma_{am}} P_m = \frac{\Sigma_s}{\Sigma_{am} + \Sigma_s} (1 - P_o) = \frac{\Sigma_s}{\Sigma_{am} + \Sigma_s} \left(1 - A \frac{\Sigma_{ao}}{\Sigma_s}\right) \quad (11.16)$$

11.3 APPLICATION TO SIR-TYPE GEOMETRY

In applying the previously developed formulas for A and B, it is first necessary to obtain an expression for the coupling parameter C in terms of P_∞ . P_∞ is the limiting value of P_o when the fuel is "black." Under these conditions (see, for example, reference 5), the geometrical self-shielding factor approaches the value

$$\frac{1}{2(2V/S)\Sigma}$$

where V, S, and Σ are the volume, surface area, and macroscopic absorption cross-section, respectively, of the fuel element. Σ is related to Σ_{ao} by $\Sigma_{ao} = v\Sigma$, where v is the fuel-element core volume fraction. Hence, in the limit of black fuel, $f\Sigma_{ao}$ approaches $Sv/4V$, and Eq. 11.13 becomes

$$P_\infty = \frac{Sv/4V}{\Sigma_s + \Sigma_{am} + C(Sv/4V)} \quad (11.17)$$

$$C = \frac{1}{P_\infty} - (\Sigma_s + \Sigma_{am}) \frac{4V}{Sv} \quad (11.18)$$

The calculation of P_∞ depends on the specific reactor core under consideration. Usually $\Sigma_s > \Sigma_{am}$, and furthermore Σ_s is fairly constant in the energy range under consideration so that C is not a sensitive function of energy. This can be seen clearly in Fig. 11.1 where C is plotted as a function of lethargy.

In the following paragraphs the evaluation of P_∞ , and hence C for the SIR-type geometry will be described.

The birthplace of slow neutrons is the reactor beryllium, and it is assumed that the birth rate is uniform, so that the fraction of moderated neutrons born in the central beryllium (r_1) and the fraction born in the interstitial regions (r_2) are pro-

portional to the volumes involved. ("Central" and "interstitial" denote geometrical configurations in the reactor; an idealized model is shown in Fig. 11.2.)

Fig. 11.1—Variation of coupling parameter C (Eq. 11.18) with lethargy u .

First consider the moderated neutrons born in the centrally located beryllium. A uniform volume distribution of isotropic neutron sources within that region is assumed. The fractional escape of neutrons of a given lethargy, from the beryllium in that region, is calculated by first-collision theory using the self-shielding curve for an infinite cylinder.^{4,5} Since a scattering collision removes a neutron from the lethargy under consideration, the scatterings can be treated as absorptions for the purpose of this calculation, and the total cross-section is used in computing the self-shielding factor. In this case,⁶ the escape probability is equal to the self-shielding factor for neutrons isotropically incident upon the surface bounding the beryllium, so that the escape probability is read directly from the self-shielding curves.

The probability of passage through the fuel when the fuel is black must now be determined. It is assumed that the neutrons emanate from the centrally located beryllium with a cosine distribution and that the probability for passage through the fuel is a purely geometrical effect which could be determined graphically for representative positions on the surface. An average probability of 0.24 is obtained; in other words, 76 per cent of the neutrons leaving the central beryllium are absorbed in the adjoining fuel region when the fuel is black.

The disposition of the 24 per cent of the neutrons escaping from the central beryllium is then treated in the following approximate manner, which permits a closed expression to be obtained for P_{∞} .

The complicated geometry exterior to the central beryllium and fuel regions is replaced by equivalent concentric annuli representing the successive interstitial beryllium, fuel, and central beryllium regions which the neutron encounters (Fig. 11.2). The chance of passing through the equivalent beryllium annuli is calculated on the basis of exponential attenuation, whereas the chance of passage through the successive fuel regions is approximated by the previously mentioned value of 0.24. The following notation is introduced. As a neutron goes from the center to the outside of Fig. 11.2:

p_1 is the chance that a neutron born in the central beryllium reaches the surface

Fig. 11.2—Sketch of equivalent core geometry (not to scale).

ϕ is the chance that a neutron passes through a fuel region

q_1 is the chance that a neutron passes through the interstitial region

p_2 is the chance that a neutron passes through the equivalent central beryllium region.

Of the neutrons born in the central beryllium, the fraction that is absorbed in the fuel is

$$r_1 [p_1(1 - \phi) + p_1\phi q_1(1 - \phi) + p_1\phi q_1\phi p_2(1 - \phi) + p_1\phi q_1\phi p_2\phi q_1(1 - \phi) + p_1\phi q_1\phi p_2\phi q_1\phi p_2(1 - \phi) + \dots] = \frac{r_1 p_1(1 - \phi)(1 + \phi q_1)}{1 - \phi p_2 \phi q_1} \quad (11.19)$$

assuming an infinite array of concentric annuli.

Now consider the neutrons born in the first ring of interstitial beryllium exterior to the beryllium under consideration. Denoting by q_0 the fraction escaping from the equivalent interstitial region, one finds in analogy to Eq. 11.19 that the probability of capture in fuel of the neutrons born in this interstitial beryllium is given by

$$\frac{r_2 q_0(1 - \phi)(1 + \phi p_2)}{1 - \phi p_2 \phi q_1} \quad (11.20)$$

Finally, by adding Eqs. 11.19 and 11.20, the total probability of capture before scattering in fuel when the fuel is black is obtained as

$$P_\infty = \frac{r_1 p_1(1 - \phi)(1 + \phi q_1) + r_2 q_0(1 - \phi)(1 + \phi p_2)}{1 - p_2 q_1 \phi^2} \quad (11.21)$$

With the above expression for P_∞ , C can now be obtained from Eq. 11.18. It then is assumed that P_∞ is the same for each fuel assembly regardless of its location in the reactor.

As an illustrative example consider neutrons near lethargy $u = 17$ (~0.4 ev). The following values are obtained for the p 's and q 's:

$$p_1 = 0.33, p_2 = 0.27, q_0 = 0.74, q_1 = 0.82,$$

so that, with $\phi = 0.24$, $r_1 = 0.69$, and $r_2 = 0.31$, one finds $P_\infty = 0.397$. Hence, returning to Eq. 11.18 with $\Sigma_s + \Sigma_{am} = 0.49$ and $4V/Sv = 2.61$, a representative value of C is obtained as 1.24. Referring again to Fig. 11.1, it is seen that C is not a sensitive function of energy in the range of interest. With C determined, the self-shielding factors A and B are obtained from Eq. 11.16.

The results of calculations for A and B for the clean 50-kg core, in the lethargy range from $u = 13.8$ to $u = 18.7$, are shown in Fig. 11.3 as a function of u . In Fig. 11.4, A and B have been replotted as a function of $\Sigma_{ao}/\xi\Sigma_s$. It is seen that the functional relation in Fig. 11.4 is much simpler than that of Fig. 11.3. By use of the above data, average A 's and B 's for pertinent lethargy groups were calculated and plotted as a function of the group-averaged $\Sigma_{ao}/\xi\Sigma_s$. The resulting curves were then fitted by the following empirical expressions⁷:

$$\begin{aligned} \bar{A} &= \frac{1}{1 + 0.4(\Sigma_{ao}/\xi\Sigma_s)} \\ \bar{B} &= \frac{1 + \bar{A}}{2} \end{aligned} \quad (11.22)$$

These forms have been used extensively in multigroup surveys employing automatic computing machines. The above empirical expressions can be derived from Eq. 11.16 in the following manner:

With $\Sigma_g > \Sigma_{am}$, Eq. 11.16 reduces to

$$A = \frac{1}{1 + fC (\Sigma_{ao}/\Sigma_g)} \quad (11.23)$$

Furthermore, for most geometries of practical interest, f can be approximated by^a

$$f = \frac{1}{1 + (3V/S) \Sigma}$$

Fig. 11.3—SIG self-shielding factors A and B as a function of u.

11.3 APPLICATION TO SIR-TYPE GEOMETRY

Fig. 11.4—Self-shielding factors A and B as a function of $\frac{\Sigma_{a0}}{\xi \Sigma_s}$.

where V , S , and Σ are the volume, surface area, and macroscopic absorption cross-section, respectively, of the fuel element. For SIR-type fuel elements, f can be re-expressed as

$$f = \frac{1}{1 + \delta (\Sigma_{a0}/\xi \Sigma_s)} \quad (11.24)$$

where

$$\delta = \frac{3R}{2} \frac{\xi \Sigma_s}{v}$$

R is the fuel element equivalent radius, and v is the volume fraction of the fuel elements in the core. Substitution of Eq. 11.24 into Eq. 11.23 yields

$$A = \frac{1}{1 + (\delta + C\xi)(\Sigma_{a0}/\xi \Sigma_s)} = \frac{1}{1 + \epsilon (\Sigma_{a0}/\xi \Sigma_s)} \quad (11.25)$$

where

$$\epsilon = \delta + C\xi$$

In the energy region of interest, C and $\xi \Sigma_s$ are slowly varying functions. With $C \cong 1.25$, and with typical values for the other constants ϵ is approximately 0.42, in satisfactory agreement with Eq. 11.22. Even if C were taken as unity, as a first approximation, a value of 0.37 would be obtained for ϵ .

Similarly, for $\Sigma_B > \Sigma_{am}$, B can be written as

$$B = 1 - A \xi \frac{\Sigma_{ao}}{\xi \Sigma_o} = 1 - A \xi \frac{1 - A}{A\epsilon} = 1 - \xi \frac{(1 - A)}{\epsilon} \quad (11.26)$$

after substituting for $\Sigma_{ao}/\xi \Sigma_s$ from Eq. 11.25. With $\xi \cong 0.2$ and $\epsilon \cong 0.4$, Eq. 11.25 reduces to the form given in Eq. 11.22; namely, $B = (1 + A)/2$.

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Chapter 12

STATISTICAL THEORY ESTIMATION OF FISSION-PRODUCT POISONING

P. GREEBLER and H. HURWITZ, Jr.

12.1 INTRODUCTION

Prediction of the endurance of a nuclear reactor requires an accurate computation of poisoning resulting from long-lived fission products. Recent theoretical and experimental studies of nuclear cross-sections have furnished sufficient information for such calculations.

Several estimates of fission-product poisoning in thermal reactors have been made from the considerable data available on thermal cross-sections.^{1,2} Poisoning of an intermediate-energy reactor was calculated by Clark³ and more recently by King,⁴ who computed reactivity coefficients for six of the most important fission-product nuclides. King's calculations were based primarily on a few prominent absorption resonances in the low-energy region. Thus far no calculation of poisoning has been reported which includes the effect of the absorption cross-sections of all the long-lived fission products at higher energies.

Businaro, Gallone, and Morgan made some theoretical calculations⁵ of nuclear cross-sections in the energy range 10^2 to 10^6 ev, based on recent studies of the statistical behavior of nuclear resonances. In particular, cross-sections were calculated for 11 nuclides varying in mass number from 82 to 153, this being the range of atomic mass that includes virtually all the fission products. From these cross-sections and curves of the yields of the stable fission products, the total cross-section of all the fission products was estimated at energies from 10^2 to 10^6 ev. (For convenience we shall designate the microscopic absorption cross-section of all the fission products, in barns per fission, as the "fission-product cross-section," and denote it by the symbol $\bar{\sigma}_p$. Since it is frequently convenient to include in this cross-section the contribution of U^{236} resulting from radiative capture of neutrons by U^{235} , we denote the fission-product cross-section including U^{236} by the symbol $\bar{\sigma}_p'$.)

Values of the fission-product cross-section $\bar{\sigma}_p$ estimated by Businaro, Gallone, and Morgan, were roughly five times higher than an earlier estimate made at KAPL, the KAPL estimate being that $\bar{\sigma}_p$ was of the order of the cross-section of a boron atom.⁶ All the 11 nuclides used by Businaro, Gallone, and Morgan to calculate the fission-product cross-section $\bar{\sigma}_p$ were even Z-odd N or odd Z-even N types (Z and N being the numbers of protons and neutrons, respectively), and none of the nuclides possessed a number of neutrons within two of the magic numbers 50 or 82. Even-even nuclei possess considerably smaller cross-sections, on the average, than odd-even, even-odd, or odd-odd nuclei. Furthermore, nuclei with magic N have very small cross-sections.⁷ Since one-half of the long-lived fission products are even-even nuclei and a large fraction have or are near magic N, it is highly probable that omission of these factors accounted for the large $\bar{\sigma}_p$ values obtained in reference 5.

Fission-product cross-sections are evaluated over an energy range 10^2 to 10^5 ev for the work described here, taking into consideration the magic N and even Z-even N effects. In fact, the contribution of each of the long-lived fission products to the fission-product cross-section $\bar{\sigma}_p'$ is considered separately. Values of $\bar{\sigma}_p'$ thus obtained are used in multigroup calculations to yield the poisoning in the higher energy groups (40 to 10^7 ev) contributed by all the fission products in an intermediate reactor.

Since application of recent nuclear theory to the computation of cross-sections at higher energies is the primary objective of this study, we consider first the underlying principles of this analysis.

12.2 METHOD OF ANALYSIS

Following the procedure of Weisskopf,⁸ we make a distinction between fine structure and gross structure of nuclear cross-sections. Knowledge of the fine structure of a given nuclide requires a list of the location and intensity of each resonance in the neutron-energy spectrum; in other words, E_0 , the resonance energy, Γ_γ , the radiation width, and Γ_n , the neutron width, must be specified for each resonance. Knowledge of the gross structure involves the average cross-section in a specified energy interval, the fluctuations caused by individual resonances being smoothed out. The distinction is important since the fine structure can be determined only by detailed experimental study, whereas the gross structure can be predicted by nuclear theory from a relatively small amount of experimental data. In practical applications the fine structure is more important at low energies; whereas the gross structure is more important at high energies. In computing the fission-product poisoning at thermal energies, for example, it is more useful to know that the peak of the strong resonance of Sm^{149} occurs at 0.096 ev than it is to know that this nuclide has typically six resonances of a certain average strength in each 50-ev energy range. For a similar computation in the neighborhood of 1000 ev, on the other hand, it is less useful to know the cross-section at precisely 1000 ev (which may or may not coincide with a resonance peak) than it is to know the average cross-section in the range between 975 and 1025 ev. At sufficiently high energies, depending upon the particular nuclide being considered, appreciable overlapping of the resonances is bound to occur; therefore only the gross structure can be readily observed in any case.

Absorption cross-sections of a nuclide may be identified with the gross structure, for practical applications, at energies that are high compared with the average energy spacing between resonances. For nuclei with nonzero spin, let D be the average spacing between the energy levels of each spin state of the compound nucleus formed during the nuclear interaction. This statement implies that for each nuclide there exists an average energy spacing D between resonance peaks, independent of energy, a fact that has been confirmed experimentally for many nuclei for neutron energies up to about 1 Mev.⁹ The average nuclear absorption cross-section of a nuclide near a given energy E can be computed by integrating over an individual resonance with respect to energy and multiplying the integral by D^{-1} , the number of resonances per unit energy interval. This average cross-section will be correct, however, only if the resonance chosen for integration possesses the average strength for the resonances that occur near energy E . This requires that the neutron width, Γ_n , be appropriately averaged. Values of the radiation width Γ_γ , for a given nuclide do not vary appreciably over the energy range of interest. Neutron widths, Γ_n , vary as $E^{1/2}$; in addition, the reduced neutron widths $\Gamma_n^0 = \Gamma_n E^{-1/2}$, vary from one resonance to another within a macroscopic energy interval. (The reduced neutron width Γ_n^0 does not have the $E^{1/2}$ dependence of Γ_n . This permits comparison of neutron widths with all resonances effectively reduced to 1 ev.) This variation is described most aptly as a Gaussian distribution with respect to $(\Gamma_n^0)^{1/2}$, as proposed by Porter and Thomas.^{10,11} Further details concerning these factors are discussed in Sec. 12.6, where the following equation for the average absorption cross-section at energy E of a given nuclide is derived:

$$\bar{\sigma}_a = \left[\frac{4.12 \times 10^6 \bar{\Gamma}_n^0}{E^{1/2} D} \phi(w) \right] \left(1 + \frac{A^{1/2} E^{1/2}}{3160} \right)^2 \quad (12.1)$$

where A = nuclide mass number (atomic weight)

$\bar{\sigma}_a$ = average absorption cross-section at energy E (barn)

E = energy (ev)

$\bar{\Gamma}_n^0$ = average reduced neutron width (ev)^{1/2}

D = average spacing between resonances of the same spin (ev). For even Z -odd N or odd Z -even N nuclides, D is actually twice the experimental resonance spacing, as discussed in Sec. 12.6.

TABLE 12.1— $\phi(w)$ FOR SMALL VALUES OF w
(PORTER-THOMAS DISTRIBUTION)

w	$\phi(w)$	w	$\phi(w)$	w	$\phi(w)$
0	0	0.10	0.069	0.25	0.140
0.02	0.017	0.12	0.080	0.30	0.160
0.04	0.032	0.14	0.090	0.35	0.179
0.06	0.045	0.16	0.100	0.40	0.196
0.08	0.057	0.18	0.110	0.50	0.227
		0.20	0.119		

$w = \Gamma_\gamma / [E^{1/2} (\bar{\Gamma}_n^0/D)D]$, where Γ_γ = radiation width (ev).

$\phi(w)$ is obtained from the solid curve of Fig. 12.3 and is defined by Eq. 12.12.

$\phi(w)$ is tabulated in Table 12.1 for small values of w .

The second term on the right, involving energy and mass number of the nuclide, does not differ appreciably from unity at energies below 10^4 ev. At 10^4 ev it contributes roughly an additional 30 per cent to cross-sections of nuclides in the atomic mass region of fission products, and above 10^5 ev this term is of considerable magnitude. An approximation to Eq. 12.1, valid at very low energies, where $\phi(w) = 1$, is

$$\bar{\sigma}_a = \frac{4.12 \times 10^6}{E^{1/2}} \bar{\Gamma}_n^0/D \quad (12.2)$$

At high energies $\phi(w) = w$, and the first term in brackets on the right side of Eq. 12.1 varies as E^{-1} , being directly proportional to $\bar{\Gamma}_\gamma/D$. At sufficiently high energies that the second term in Eq. 12.1 becomes large compared with unity, the cross-section becomes independent of energy. Thus the average (gross) cross-section has a large value at low energies, decreasing at first as $E^{-1/2}$ as the energy is increased, then decreasing more rapidly as E is increased, and finally approaching a constant value at very large E . At energies above 1 Mev the level spacing D may vary appreciably with energy,⁹ and this factor has not been taken into account in Eq. 12.1.

Fission-product cross-sections $\bar{\sigma}_p$ and $\bar{\sigma}_{p'}$ are evaluated as $\sum_i Y_i \bar{\sigma}_{ai}$, where Y_i is the fractional yield of the i th fission product and $\bar{\sigma}_{ai}$ is the absorption cross-section of the i th product, (computed from Eq. 12.1.) For the evaluation of $\bar{\sigma}_p$, $\sum_i Y_i = 2$; whereas, for the evaluation of $\bar{\sigma}_{p'}$, which includes U^{235} , $\sum_i Y_i = 2 + \alpha = 2.38$ for a typical intermediate reactor. The summation is taken over all the long-lived stable fission products, which are listed in Table 12.2. Also listed are Z and N numbers for each isotope, per cent fission yields taken directly from Table 5 of King's report,⁴ and values of D , Γ_γ , and the ratio $\bar{\Gamma}_n^0/D$. It should be noted from Eq. 12.1 that these latter three values specify $\bar{\sigma}_a$ as a function of energy for a given nuclide.

Feshbach, Porter, and Weisskopf¹² have made a theoretical evaluation of $\bar{\Gamma}_n^0/D$ as a function of nuclear mass number, using the "cloudy crystal ball" model of the nucleus. The theoretical evaluation, shown as the solid curve of Fig. 12.1, is compared with measured values of $\bar{\Gamma}_n^0/D$ obtained by Harvey et al.⁹ The experimental points confirm the general form of the theoretical curve, but the maximum is appreciably less than predicted. The $\bar{\Gamma}_n^0/D$ values listed in Table 12.2 were obtained from the dashed curve of Fig. 12.1. A large number of Γ_γ values have been compiled by Hughes and Harvey¹³ for the 1955 book of neutron cross-sections. Many of these values are plotted in Fig. 12.2, and the solid curve was used to obtain the Γ_γ values listed in Table 12.2.

Level spacings (D) have been measured for a large number of nuclides by Harvey et al.,⁹ and several of these are among the fission products listed in Table 12.2. (Those D values in Table 12.2 which are listed with specified limits of error were obtained from references 9 and 14). For many of the fission products, level spacings are estimated from the 1955 compilation of neutron cross-sections by counting resonance peaks in the low-energy region of good resolution.¹³ Cross-section plots of natural elements containing several isotopes are treated by assuming that each isotope of odd mass number accounted for six times the number of unidentified resonances as an even Z -even N isotope.

TABLE 12.2—DATA FOR FISSION-PRODUCT CROSS-SECTION

Nuclide	Z	N	Yield (Y) $\times 10^2$	Spacing (D), ev	Radiation width (Γ_γ), ev	$\frac{\Gamma_\gamma^0}{D} \times 10^4$	Fission-product absorption cross- section, barns per fission			
							At 10^2 ev	At 10^3 ev	At 10^4 ev	At 10^5 ev
Kr ⁸³	36	47	0.59	500	0.20	0.30	0.03	0.007	0.0012	0.00031
Kr ⁸⁴	36	48	1.11	2,000	0.20	0.31	0.03	0.004	0.0005	0.00009
Kr ⁸⁵	36	49	0.33	2,000	0.20	0.32	0.01	0.001	0.0001	0.00002
Rb ⁸⁵	37	48	1.20	2,000	0.20	0.32	0.03	0.004	0.0005	0.00009
Kr ⁸⁶	36	50	2.14	10,000	0.20	0.33	0.02	0.002	0.0003	0.00006
Rb ⁸⁷	37	50	2.82	6,000	0.20	0.33	0.03	0.003	0.0004	0.00007
Sr ⁸⁸	38	50	3.56	2,000	0.20	0.34	0.08	0.015	0.0019	0.00032
Y ⁸⁸	39	50	4.58	6,000	0.20	0.34	0.06	0.007	0.0008	0.00013
Sr ⁹⁰	38	52	5.62	6,000	0.20	0.35	0.07	0.008	0.0009	0.00016
Zr ⁹¹	40	51	6.10	2,000	0.20	0.35	0.13	0.024	0.0029	0.00049
Zr ⁹²	40	52	6.32	2,000	0.20	0.36	0.14	0.025	0.0030	0.00052
Zr ⁹³	40	53	6.47	2,000	0.20	0.36	0.14	0.026	0.0032	0.00054
Zr ⁹⁴	40	54	6.55	2,000	0.19	0.37	0.14	0.027	0.0033	0.00055
Mo ⁹⁵	42	53	6.61	370 $\pm$ 120	0.18	0.37	0.40	0.070	0.0132	0.00288
Zr ⁹⁶	40	56	6.61	2,000	0.17	0.38	0.14	0.025	0.0040	0.00052
Mo ⁹⁷	42	55	6.56	370 $\pm$ 120	0.16	0.38	0.38	0.064	0.0134	0.00207
Mo ⁹⁸	42	56	6.48	500 $\pm$ 150	0.15	0.40	0.32	0.052	0.0100	0.00169
Tc ⁹⁹	43	56	6.16	32	0.15	0.40	0.84	0.216	0.0504	0.01278
Mo ¹⁰⁰	42	58	5.56	500 $\pm$ 150	0.14	0.41	0.27	0.042	0.0075	0.00126
Ru ¹⁰¹	44	57	4.60	100	0.14	0.42	0.48	0.097	0.0181	0.00504
Ru ¹⁰²	44	58	3.56	200	0.13	0.43	0.28	0.048	0.0092	0.00214
Rh ¹⁰³	45	58	2.41	40	0.13	0.44	0.33	0.077	0.0170	0.00396
Ru ¹⁰⁴	44	60	1.50	400	0.12	0.45	0.08	0.012	0.0025	0.00041
Pd ¹⁰⁵	46	59	0.92	50	0.12	0.46	0.12	0.026	0.0056	0.00160
Pd ¹⁰⁶	46	60	0.53	100	0.11	0.47	0.05	0.010	0.0019	0.00052
Pd ¹⁰⁷	46	61	0.26	50	0.11	0.48	0.03	0.008	0.0015	0.00039
Pd ¹⁰⁸	46	62	0.13	100	0.10	0.50	0.01	0.002	0.0005	0.00011
Ag ¹⁰⁹	47	62	0.028	31 $\pm$ 6	0.10	0.51	0.004	0.0009	0.0003	0.00006
Cd ¹¹¹	48	63	0.018	50	0.09	0.52	0.002	0.0004	0.0001	0.00002
Cd ¹¹²	48	64	0.013	100	0.09	0.53	0.001	0.0002	0.0001	0.00001
Cd ¹¹³	48	65	0.012	50	0.08	0.54	0.002	0.0003	0.0001	0.00002
In ¹¹⁵	49	66	0.011	14 $\pm$ 2	0.08	0.55	0.002	0.0005	0.0001	0.00002
Te ¹²⁶	52	74	0.18	300	0.09	0.78	0.01	0.002	0.0003	0.00004
I ¹²⁷	53	74	0.38	15 $\pm$ 3	0.09	0.81	0.08	0.018	0.0036	0.00077
Te ¹²⁸	52	76	0.71	400	0.10	0.84	0.04	0.008	0.0009	0.00011
I ¹²⁹	53	76	1.20	50	0.10	0.87	0.23	0.042	0.0075	0.00216
Te ¹³⁰	52	78	1.97	800	0.11	0.90	0.09	0.012	0.0015	0.00025
Xe ¹³¹	54	77	2.85	100	0.11	0.94	0.42	0.073	0.0147	0.00270
Xe ¹³²	54	78	4.24	400	0.12	0.98	0.30	0.054	0.0066	0.00110
Cs ¹³³	55	78	6.70	42 $\pm$ 5	0.12	1.03	1.61	0.316	0.0571	0.01602
Xe ¹³⁴	54	80	7.53	2,000	0.13	1.08	0.21	0.022	0.0027	0.00045
Cs ¹³⁵	55	80	6.00	100	0.13	1.14	1.04	0.186	0.0378	0.00716
Xe ¹³⁶	54	82	6.24	10,000	0.14	1.20	0.03	0.003	0.0004	0.00007
Cs ¹³⁷	55	82	4.70	300	0.14	1.27	0.45	0.068	0.0121	0.00020
Ba ¹³⁸	56	82	6.61	10,000	0.15	1.35	0.04	0.004	0.0005	0.00009

TABLE 12.2—(Continued)

Nuclide	Z	N	Yield (Y) $\times 10^2$	Spacing (D), ev	Radiation width (Γ_γ), ev	$\frac{\Gamma_\gamma^0}{D} \times 10^4$	Fission-product absorption cross- section, barns per fission			
							At 10^2 ev	At 10^3 ev	At 10^4 ev	At 10^5 ev
La ¹³⁹	57	82	6.59	300	0.15	1.44	0.72	0.122	0.0184	0.00299
Ce ¹⁴⁰	58	82	6.51	10,000	0.15	1.53	0.04	0.004	0.0005	0.00009
Pr ¹⁴¹	59	82	6.41	200	0.14	1.62	0.93	0.164	0.0252	0.00414
Ce ¹⁴²	58	84	6.22	6,000	0.13	1.71	0.04	0.004	0.0005	0.00009
Nd ¹⁴³	60	83	5.90	200	0.12	1.80	0.76	0.107	0.0193	0.00335
Nd ¹⁴⁴	60	84	5.06	500	0.11	1.89	0.40	0.051	0.0062	0.00104
Nd ¹⁴⁵	60	85	3.95	100	0.10	1.97	0.90	0.145	0.0275	0.00455
Nd ¹⁴⁶	60	86	3.07	500	0.10	2.05	0.24	0.025	0.0031	0.00052
Pm ¹⁴⁷	61	86	2.26	30	0.09	2.13	0.81	0.145	0.0239	0.00419
Sm ¹⁴⁷	62	85	0.09	17	0.09	2.21	0.04	0.010	0.0015	0.00034
Nd ¹⁴⁸	60	88	1.79	1,000	0.08	2.29	0.07	0.008	0.0009	0.00014
Sm ¹⁴⁹	62	87	1.40	17	0.08	2.37	0.68	0.123	0.0220	0.00612
Nd ¹⁵⁰	60	90	0.178	1,000	0.08	2.44	0.02	0.002	0.0003	0.00005
Sm ¹⁵¹	62	89	0.486	17	0.08	2.51	0.25	0.044	0.0080	0.00178
Sm ¹⁵²	62	90	0.305	50	0.08	2.58	0.08	0.013	0.0016	0.00027
Eu ¹⁵³	63	90	0.122	2.2 $\pm$ 0.3	0.09	2.65	0.11	0.023	0.0075	0.00158
Sm ¹⁵⁴	62	92	0.100	100	0.09	2.72	0.02	0.003	0.0004	0.00007
Gd ¹⁵⁵	64	91	0.055	20	0.09	2.79	0.03	0.008	0.0009	0.00013
Gd ¹⁵⁶	64	92	0.028	50	0.09	2.79	0.01	0.001	0.0003	0.00004
Gd ¹⁵⁷	64	93	0.016	20	0.09	2.72	0.01	0.002	0.0004	0.00007
U ²³⁸	92	144	38.0	16	0.03	1.00	7.65	1.444	0.3111	0.06952

Fig. 12.1—Experimental and predicted values of Γ_γ^0/D . Solid curve is "cloudy crystal ball" theory prediction.

Fig. 12.2— Γ_γ versus atomic weight of target nucleus.

TABLE 12.3—FISSION-PRODUCT CROSS-SECTIONS

Cross-section	Symbol	Unit	10^2 ev	10^3 ev	10^4 ev	10^5 ev	10^6 ev
Fission-product absorption (including U^{238})	$\bar{\sigma}_p$	Barns per fission	23.0	4.2	0.80	0.17	0.07
U^{238} absorption	$Y\bar{\sigma}_a$	Barns per fission	7.6	1.4	0.31	0.07	0.03
Fission-product absorption	$\bar{\sigma}_p$	Barns per fission	15.4	2.8	0.49	0.10	0.04
σ_p (computed by Businaro et al.)		Barns per fission	47	9.9	1.7	0.30	0.24
U^{235} fission	$\bar{\sigma}_f$	Barns per atom	23	8.5	3.7	1.7	1.3

Insufficient cross-section data were available to estimate D for several of the nuclides listed in Table 12.2. Fortunately, most of these nuclei occur in the regions near magic N ; therefore their contribution to the total fission-product poisoning was very small. Spacings for these nuclei were obtained from plots of D versus neutron number N . These plots were made from the data of Harvey et al.,^{9,14} with separate curves for even-even, even-odd, and odd-even nuclei. Harvey's data indicate that, in the region between the magic N numbers 50 and 82, even-even nuclei possess D values from 3 to 10 times greater than those with odd atomic mass numbers. D values for all types of nuclei rise steeply as magic N numbers are approached, and extrapolation of the curves indicates a D value of the order of 10^4 ev for even Z -magic N nuclei. This figure was used as an upper limit for the D values of even Z -magic N nuclides listed in Table 12.2, being revised downward when sufficient data were available to suggest such

revision. All the even Z-magic N nuclei contribute so little to the fission-product cross-section that large errors in their estimated D values produce negligible effect on the calculated results. Another factor considered in estimating D values, in the absence of specific cross-section data, was that nuclei with large N numbers relative to the isotope of maximum stability would have larger D values than the isotopes with smaller N of identical Z and even-odd type.

12.3 DISCUSSION OF RESULTS

Values of $Y\bar{\sigma}_a$ for each fission product are listed in the four right-hand columns of Table 12.2 for energies from 10^2 to 10^5 ev. Table 12.3 shows the computed fission-product cross-sections at energies from 10^2 to 10^6 ev. (The value at 10^6 ev was computed using the asymptotic condition $\phi(w) = w$). The factor $A^{1/3}$ in Eq. 12.1 was taken as 5 for all fission products and 6.2 for U^{238} . Listed separately are fission-product cross-sections $\bar{\sigma}_p$, and $\bar{\sigma}_p$, and $\bar{\sigma}_a$ for U^{238} weighted by its fission yield Y. Fission-product cross-sections $\bar{\sigma}_p$ computed by Businaro et al.⁵ are also listed, and it is seen that these are about three times greater than our values. Fission cross-sections of U^{235} are also listed in Table 12.3 for reference purposes. Comparison of cross-sections obtained by the statistical approach with available experimental information is discussed in reference 21.

The importance of taking into consideration the large D values of even Z-even N nuclei and of magic N nuclei can be realized from the following results of the calculations made for 10^3 ev:

1. Even Z-even N nuclei (excluding U^{238}) constitute 50 per cent of the total fission yield (1.01 of 2.0), but they contribute only 15 per cent to the fission-product cross-section $\bar{\sigma}_p$.
2. Nuclei with N numbers from 48 to 52 (inclusive) constitute 17 per cent (0.34 of 2.0) of the total fission yield, but they contribute only 3 per cent to $\bar{\sigma}_p$.
3. Nuclei with N numbers from 80 to 84 (inclusive) constitute 34 per cent (0.68 of 2.0) of the total fission yield, and they contribute 25 per cent to $\bar{\sigma}_p$. If we separate the nuclides Cs^{135} , Cs^{137} , La^{139} , Pr^{141} , and Nd^{143} from this group, the remaining nuclides (all even Z-even N) constitute 19 per cent of the total fission yield and contribute only 3 per cent to $\bar{\sigma}_p$.

Analysis of the function $\phi(w)$, appearing in Eq. 12.1, for various distributions of reduced neutron width Γ_n^0 produced the set of curves shown in Fig. 12-3. The Porter-Thomas distribution used for our calculations (solid curve) yields $\phi(w)$ values lower than those predicted by an exponential distribution, but higher than an exponential distribution with respect to the square root of Γ_n^0 . $\phi(w)$ is maximum for a hypothetical distribution in which Γ_n^0 is equal for all resonances. $\phi(w)$ values for small w are tabulated in Table 12.1 for the Porter-Thomas distribution.

Fig. 12-3— $\phi(w)$ versus w .

To estimate fission-product poisoning at higher energies, multigroup calculations were made of the rate of decrease of reactivity in a model of a SIR-type reactor. Fifteen energy groups, from 10 Mev to 0.075 ev, were used for these calculations following the procedures described in references 6 and 15. Values of fission-product cross-section averaged over lethargy for each group and corrected for shielding in the fuel element are listed in Table 12.4. Actual numerical computation of fission-product poisoning was carried out by perturbation methods, using adjoints evaluated earlier by Hofmann¹⁶ for the appropriate model (clean and depleted reactor conditions are represented by separate models). Statistically evaluated cross-sections $\bar{\sigma}_p$ were used for the first eight groups. The energy spread of the eighth group (38 to 170 ev) is sufficiently large to justify use of the cross-sections $\bar{\sigma}_p$, but the higher groups (lower energies) are too narrow in energy spread to permit use of these statistically evaluated cross-sections. (Since most of the fission-product cross-section $\bar{\sigma}_p$ is contributed by nuclei with D values of 100 ev or lower, it is reasonable to assume that $\bar{\sigma}_p$, averaged over a lethargy group, is a good approximation of the actual averaged fission-product cross-section, provided the energy spread of the group exceeds 100 ev. A curve of $\bar{\sigma}_p$ versus energy was extrapolated below the 100-ev value listed in Table 12.3 by utilizing Eq. 12.2 to calculate $\bar{\sigma}_p$ at 0.01 ev.) It should be noted that, since $\bar{\sigma}_p$ is a "grand gross structure," its applicability extends to lower energies than that of the gross structure of most of the individual nuclides. Fluctuations at lower energies caused by fine structure of individual nuclei are smoothed out considerably as a result of the summation over all the fission products.

The rate of decrease of reactivity for a clean reactor resulting from fission-product absorption between 10 Mev and 38 ev was computed as 11¢ per mol of U^{235} fissions or 2.1¢ per 1000Mwh. This is 29 per cent of the total fission-product poisoning (34 per cent for the depleted condition), which is discussed in the next section.

TABLE 12.4—MULTIGROUP EVALUATION OF FISSION-PRODUCT POISONING IN A SIR-TYPE REACTOR

Group No.	Lethargy u	Energy (E), ev	Fission-product cross-section, barns			Rate of reactivity loss, ¢/1000Mwh	
			Uncorrected	Corrected for shielding in fuel	Corrected for burnout (depleted reactor)	Clean reactor	Depleted reactor
1	0-0.5	$10^7-6.2 \times 10^6$	0.029			0	0
2	0.5-1	$6.2 \times 10^6-3.8 \times 10^6$	0.032			0	0
3	1-2	$3.8 \times 10^6-1.4 \times 10^6$	0.047			0	0
4	2-3	$1.4 \times 10^6-5.0 \times 10^5$	0.073			0	0
5	3-4	$5.0 \times 10^5-1.8 \times 10^5$	0.11	0.11	0.11	0	0
6	4-8	$1.8 \times 10^5-3.4 \times 10^3$	0.56	0.55	0.55	0.13	0.11
7	8-11	$3.4 \times 10^3-1.7 \times 10^2$	6.1	5.9	5.9	0.74	0.69
8	11-12.5	170-38	28	26	26	1.18	1.14
9	12.5-13.75	38-10.8	19	17	17	.43	0.42
10	13.75-15	10.8-3.1	52	45	45	.78	0.72
11	15-16	3.1-1.13	26	24	23	.26	0.24
12	16-17	1.13-0.40	59	50	28	.45	0.22
13	17-17.75	0.40-0.195	25	17	15	.06	0.06
14	17.75-18.7	0.195-0.075	1495	900	320	2.37	0.93
15	18.7	0.075	1390	720	270	.82	1.09
Totals						7.22	5.62

12.4 MULTIGROUP CALCULATIONS OF FISSION-PRODUCT POISONING: COMPARISON WITH EXPERIMENTAL DATA

Fission-product cross-sections for the seven higher lethargy groups, also listed in Table 12.4, were calculated from the latest available cross-section data.¹³ The strength of each absorption resonance of a nuclide was averaged over the lethargy group

in which it occurred and was weighted by the fission yield of the nuclide. Unidentified resonances in cross-section plots of natural elements containing several isotopes are treated, as suggested by King,⁴ by assigning a 3:1 favorability ratio (ratio of reactivity coefficients) for isotopes with odd mass numbers over those with even Z -even N . Cross-sections of Sm^{151} in the subcadmium region are calculated by the method of King,⁴ using the ratio of thermal cross-sections of Sm^{149} to Sm^{151} for the entire region. This procedure is also followed for Nd^{143} and Nd^{145} . In energy groups 10 through 15 most of the fission-product cross-section is contributed by a few well-known resonances. In group 9, however, many unidentified and poorly resolved resonances contribute to the cross-section; therefore the value for this group is, at best, a rough estimate. For the two highest lethargy groups most of the fission-product cross-section is a result of the giant Sm^{149} resonance.

Table 12.4 lists fission-product cross-sections for both clean and depleted conditions and shows the contribution of each group to the rate of decrease of reactivity. Effective burnup factors of 0.32, 0.78, and 0.94 are applied to Sm^{149} , Sm^{151} , and Rh^{103} , respectively, for the depleted condition based on an endurance of 90,000 Mwh. Correction factors for U^{235} shielding of the fission products in the fuel elements have been calculated from the formula on page 18 of reference 6. Rates of decrease of reactivity shown in Table 12.4 are 7.2¢ and 5.6¢ per 1000 Mwh for clean and depleted conditions, respectively.

If we add to these values the reactivity loss caused by fuel burnup, 2.7¢ and 4.5¢ per 1000 Mwh for clean and depleted conditions¹⁷, the predicted long-term rate of decrease of reactivity is 9.9¢ and 10.1¢ per 1000 Mwh for clean and depleted conditions, respectively. In Chap. 20 are analyzed experimental data on the rate of reactivity loss in the intermediate reactor as a result of long-lived fission-product poisons and fuel burnup. The experimental value, 12.5¢ per 1000 Mwh, obtained by several different methods of analysis, is in reasonable agreement with our calculated results.

12.5 CONCLUSIONS

Absorption cross-sections of fission-product nuclei can be evaluated from statistical features of nuclear resonances. For energies that are high compared with the average level spacing, D , the computed cross-section should be a sufficiently good approximation for practical applications. For energies that are lower than D , the computed cross-section generally will be a poor approximation. Limitations on the accuracy of the statistical method at higher energies arise from inadequate data on nuclear resonances. The principal source of error is the lack of reliable D values for many nuclei, and a secondary cause for error is the uncertainty of the $\bar{\Gamma}_n^0/D$ values predicted from theoretical considerations.

Various distributions of reduced neutron width, Γ_n^0 , which have been proposed are analyzed for the effect of each on the computed cross-section. A hypothetical distribution in which Γ_n^0 is equal for all resonances yields a maximum computed cross-section, and values in decreasing order are computed for distributions of Γ_n^0 that are exponential, Gaussian with respect to the square root of Γ_n^0 , and exponential with respect to the square root of Γ_n^0 .

Fission-product cross-sections are evaluated at energies from 10^2 to 10^5 ev (and approximately at 10^6 ev), using the statistical method. The statistical approach is particularly applicable to fission products since the effective cross-section per fission is obtained by summing over a large number of cross-sections. Approximately one-half of all stable fission products are even Z -even N nuclei. Also one-half of the fission products have magic or near-magic neutron numbers (50 ± 2 or 82 ± 2). Level spacings for nuclei in these groupings are generally high. This accounts for the fact that our computed cross-sections are considerably lower than those calculated earlier by Businaro, Gallone, and Morgan. Multigroup calculation fission-product poisoning, from our statistical cross-section values for the higher energy groups, produced results in good agreement with experimental data on fission-product poisoning in the S1G.

Since fission-product cross-sections computed by the statistical method are accurate only at relatively high energies, their usefulness for reactor calculations increases with increasing energy spectrum of the reactor. For a thermal reactor the importance of the long-lived poison Sm^{149} overrides all other stable fission products. In an intermediate reactor a higher percentage of the fission-product absorption occurs

12.6 APPENDIX

at energies that are sufficiently high for application of the statistical method. For high-energy reactors fission-product poisoning may be calculated entirely by the statistical process.

12.6 APPENDIX

A. Derivation of Equation 12.1

The Breit-Wigner single-level equation for the absorption cross-section is (considering only nucleons of angular momentum $\ell = 0$)

$$\sigma_a = \frac{\lambda_0^2 g}{4\pi} \left(\frac{E_0}{E} \right)^{1/2} \frac{\Gamma_n \Gamma_\gamma}{(E - E_0)^2 + (\Gamma/2)^2} \quad (12.3)$$

where Γ_n , Γ_γ , and Γ = neutron, radiation, and total level widths
 E = energy

E_0 = energy at the resonance peak

$\lambda_0 = h(2mE_0)^{-1/2}$ = neutron wave length at resonance energy

g = statistical weight of the spin state of the compound nucleus formed during interaction

Integrating Eq. 12.3 over energy, neglecting the variation of energy except in the term $E - E_0$, and extending the lower limit of integration to $-\infty$ for convenience, yields, as the resonance strength,

$$\Omega = \int_{-\infty}^{\infty} \sigma_a dE = \frac{gh^2}{4mE_0^{1/2}} \frac{\Gamma_\gamma \Gamma_n^0}{\Gamma_\gamma + E_0^{1/2} \Gamma_n^0} \quad (12.4)$$

where the reduced neutron width $\Gamma_n^0 = E_0^{-1/2} \Gamma_n$.

For a specified nuclide, Γ_γ is approximately constant over the energy range of interest, but Γ_n^0 values vary according to a statistical distribution. It is convenient to introduce a variable x , defined by $x = \Gamma_n^0 / \bar{\Gamma}_n^0$, where $\bar{\Gamma}_n^0$ is the average neutron width for the nuclide. The statistical distribution of Γ_n^0 values is now expressed as a probability function $P(x)$. The quantity $P(x)$ expresses the probability that a randomly selected level of the compound nucleus will possess a neutron width ratio $\Gamma_n^0 / \bar{\Gamma}_n^0$ that is equal to x (within the limits x to $x + dx$). Introducing the variable $w = \Gamma_\gamma / E_0^{1/2} \bar{\Gamma}_n^0$, multiplying Ω by $P(x)$, and integrating over x yields as the average resonance strength,

$$\bar{\Omega} = \frac{gh^2 \bar{\Gamma}_n^0}{4mE_0^{1/2}} \phi(w) \quad (12.5)$$

where

$$\phi(w) = \int_0^{\infty} \frac{xP(x) dx}{1 + (x/w)} \quad (12.6)$$

If D is the average level spacing for the spin state of statistical weight g , there are D^{-1} resonances per unit energy interval. Thus the average cross-section at energy E due to the levels of the spin state being considered is

$$\bar{\sigma}_a = \frac{gh^2}{4mE^{1/2}} \frac{\bar{\Gamma}_n^0}{D} \phi(w) \quad (12.7)$$

For energies sufficiently high that the barrier penetration factor for $\ell = 1$ is not appreciably less than unity, we must correct Eq. 12.7 for the effect of the nucleons of

orbital angular momentum $\ell > 0$. This is accomplished approximately^{18,19} by multiplying Eq. 12.7 by the factor $[1 + (A^{1/3} E^{1/2} / 3160)]^2$. Evaluating the constants of Eq. 12.7 so that $\bar{\sigma}_a$ is expressed in barns and energies are expressed in electron volts yields

$$\bar{\sigma}_a = \frac{4.12 \times 10^6 g}{E^{1/2}} \frac{\bar{\Gamma}_n^0}{D} \phi(w) \left(1 + \frac{A^{1/3} E^{1/2}}{3160}\right)^2 \quad (12.8)$$

Excepting the g factor, Eq. 12.8 is identical with Eq. 12.1. For nuclei with zero spin (even Z -even N), g is unity. For our calculations we assume that $g = 1/2$ for each spin state of odd- A target nuclei. Since spin values of such nuclei in the fission-product region tend to be high, the assumption of $g = 1/2$ is a good approximation. Since there are two spin states for each target nucleus of nonzero spin, we sum over the two states; so the factor g in Eq. 12.8 is set equal to unity for all nuclides. It should be noted, however, that D is the average level spacing for a single spin state, which is twice the average spacing between experimental resonances for odd- A target nuclei.

B. Evaluation of $\phi(w)$

Calculations of $\phi(w)$ from Eq. 12.6 yield the following expressions for the various distributions $P(x)$ of neutron width that were analyzed:

Uniform distribution, $P(x) = \delta(x - 1)$ where δ denotes the Dirac δ function.

$$\phi(w) = w/(w + 1) \quad (12.9)$$

Exponential distribution [Hughes et al.]⁹, $P(x) = e^{-x}$

$$\phi(w) = w[1 + we^w \text{Ei}(-w)] \quad (12.10)$$

where

$$\text{Ei}(-x) = \int_x^\infty \frac{e^{-u} du}{u} \quad (12.11)$$

Gaussian distribution of $(\Gamma_n^0)^{1/2}$ [Porter-Thomas]¹¹, $P(x) = (2\pi x)^{-1/2} e^{-x/2}$

$$\phi(w) = w - 4\left(\frac{w}{2}\right)^{3/2} e^{w/2} \int_{(w/2)^{1/2}}^\infty e^{-u^2} du \quad (12.12)$$

Exponential distribution of $(\Gamma_n^0)^{1/2}$ [Bethe]²⁰, $P(x) = (2x)^{-1/2} e^{-(2x)^{1/2}}$

$$\phi(w) = w \left(1 - (2w)^{1/2} \left\{ \text{Ci}(2w)^{1/2} \sin(2w)^{1/2} + \left[\frac{\pi}{2} - \text{Si}(2w)^{1/2}\right] \cos(2w)^{1/2} \right\}\right) \quad (12.13)$$

where

$$\text{Ci}(x) = \int_x^\infty \frac{\cos u du}{u} \quad (12.14)$$

$$\text{Si}(x) = \int_0^x \frac{\sin u du}{u} \quad (12.15)$$

Eqs. 12.9, 12.10, 12.12, and 12.13 are plotted in Fig. 12.3. The Porter-Thomas distribution was used for our calculations since there is theoretical justification for this distribution, and it appears to give the best fit for nuclei in the atomic number region of fission products.¹¹ Businaro, Gallone, and Morgan used the exponential distribution for their calculations, which makes their $\phi(w)$ values about 10 per cent higher than those used for our computations.

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Chapter 13

A XENON CALCULATION PROCEDURE ADAPTABLE TO DIGITAL COMPUTERS

P. L. HOFMANN, H. HURWITZ, Jr., and E. WACHSPRESS

13.1 INTRODUCTION

This report summarizes a digital-computer procedure used at KAPL for the determination of the xenon concentration and its effect on reactivity during reactor operation and after shutdown. The techniques that are outlined were devised to be particularly applicable to intermediate-spectrum reactors.

In general, the approach has relied—as far as terminology and formalism are concerned—on multigroup calculation methods as developed by Ehrlich and Hurwitz.¹ Particular emphasis has been placed on (1) steady-state operation and (2) peak xenon override.

For both of these conditions methods are described for calculating the xenon concentration and its effect on reactivity.

It should be noted that the xenon behavior in SIR-type reactors differs markedly from that encountered in rod-controlled water-moderated thermal reactors. The xenon reactivity defects in the intermediate reactor are much smaller than comparable defects for the thermal reactor. Furthermore, the peak xenon defect following complete shutdown occurs much sooner in the former than in the latter and is not much larger than the steady-state defect.

13.2 STEADY-STATE XENON CONCENTRATION

In this section the build-up of the steady-state xenon concentration is reviewed by considering the formation, radioactive decay, and burnup of xenon.

Xenon-135 is the result of the radioactive chain

Tellurium-135 decays rapidly, and Cesium-135 is fairly stable. We are therefore primarily concerned with the chain

There is also evidence of direct formation of xenon as a fission product; the amount of xenon formed in this manner is small compared to that produced by the radioactive decay of iodine.

The rate of change of nuclei per cubic centimeter of I^{135} and Xe^{135} can be described by the following differential equations:

$$\frac{dN_I}{dt} = -\lambda_I N_I - \sigma_I \phi N_I + y_I \phi \Sigma f \quad (13.1)$$

13.3 PEAK XENON CONCENTRATION

$$\frac{dN_{Xe}}{dt} = \lambda_I N_I + y_{Xe} \phi \Sigma_f - \lambda_{Xe} N_{Xe} - \sigma_{Xe} \phi N_{Xe} \quad (13.2)$$

where N_I = nuclei per cubic centimeter of I^{135}
 N_{Xe} = nuclei per cubic centimeter of Xe^{135}
 λ_I = decay constant of I^{135}
 λ_{Xe} = decay constant of Xe^{135}
 y_I = I^{135} fission yield
 y_{Xe} = Xe^{135} direct fission yield
 ϕ = neutron flux
 σ_{Xe} = microscopic Xe^{135} capture cross-section
 σ_I = microscopic I^{135} capture cross-section
 Σ_f = macroscopic U^{235} fission cross-section

The cross-sections are averaged over the energy spectrum of the reactor, $\phi\sigma = \int \phi\sigma(E) dE$. This spectrum depends upon the condition of the reactor, and unless otherwise specified the σ 's refer to the steady-state condition.

Since σ_I and y_{Xe} are comparatively small, we can neglect them, obtaining from Eqs. 13.1 and 13.2

$$\frac{dN_I}{dt} = y_I \phi \Sigma_f - \lambda_I N_I \quad (13.3)$$

$$\frac{dN_{Xe}}{dt} = \lambda_I N_I - \lambda_{Xe} N_{Xe} - \phi \sigma_{Xe} N_{Xe} \quad (13.4)$$

For steady-state operation we obtain

$$y_I \phi \Sigma_f - \lambda_I N_I = 0 \quad (13.5)$$

$$\lambda_I N_I - \lambda_{Xe} N_{Xe} - \phi \sigma_{Xe} N_{Xe} = 0 \quad (13.6)$$

The xenon steady-state concentration, N_{Xe}^{ss} , is, from Eqs. 13.5 and 13.6,

$$N_{Xe}^{ss} = \frac{y_I \phi \Sigma_f}{\lambda_{Xe} + \phi \sigma_{Xe}} \quad (13.7)$$

where y is written, instead of y_I , because of the small amount of direct xenon formation. This value of y will be referred to as the "xenon yield."

13.3 PEAK XENON CONCENTRATION

When a reactor that has been operating for some time is shut down to zero power, the xenon concentration will build up to a maximum and then decay since the direct xenon yield is smaller than the xenon burnout at steady state; i.e.,

$$y_{Xe} \phi \Sigma_f < \sigma_{Xe} \phi N_{Xe}^{ss}$$

When the flux falls to zero, dN_{Xe}/dt , Eq. 13.2, becomes positive. After the iodine decays (N_I becomes small), dN_{Xe}/dt becomes negative. Hence N_{Xe} must reach a maximum.

With $\phi = 0$, corresponding to shutdown, Eqs. 13.3 and 13.4 become

$$\frac{dN_I}{dt} = -\lambda_I N_I \quad (13.8)$$

$$\frac{dN_{Xe}}{dt} = \lambda_I N_I - \lambda_{Xe} N_{Xe} \quad (13.9)$$

At $t = 0$, N_I and N_{Xe} are the steady-state values obtained from Eqs. 13.5 and 13.6, provided that the reactor has operated over a period long enough for a steady-state condition to be obtained before shutdown. The solution of Eqs. 13.8 and 13.9 for N_{Xe} is

$$N_{Xe} = y\phi\Sigma_f \left[\left(\frac{1}{\lambda_{Xe} + \phi\sigma_{Xe}} - \frac{1}{\lambda_{Xe} - \lambda_I} \right) e^{-\lambda_{Xe}t} + \frac{1}{\lambda_{Xe} - \lambda_I} e^{-\lambda_I t} \right] \quad (13.10)$$

where ϕ is the steady-state reactor flux before shutdown ($t = 0$). The xenon concentration will reach a maximum at a time t given by

$$t = \frac{1}{\lambda_{Xe} - \lambda_I} \ln \left(\frac{\lambda_I + \phi\sigma_{Xe}}{\lambda_{Xe} + \phi\sigma_{Xe}} \frac{\lambda_{Xe}}{\lambda_I} \right) \quad (13.11)$$

By substituting the value of t given by Eq. 13.11 into Eq. 13.10, we obtain

$$N_{Xe}^{peak} = y\phi\Sigma_f \left\{ \frac{1}{\lambda_{Xe} + \phi\sigma_{Xe}} \left(\frac{\lambda_{Xe}}{\lambda_I} \frac{\lambda_I + \phi\sigma_{Xe}}{\lambda_{Xe} + \phi\sigma_{Xe}} \right)^{\lambda_{Xe}/(\lambda_I - \lambda_{Xe})} + \frac{1}{\lambda_{Xe} - \lambda_I} \left[\left(\frac{\lambda_{Xe}}{\lambda_I} \frac{\lambda_I + \phi\sigma_{Xe}}{\lambda_{Xe} + \phi\sigma_{Xe}} \right)^{\lambda_I/(\lambda_I - \lambda_{Xe})} - \left(\frac{\lambda_{Xe}}{\lambda_I} \frac{\lambda_I + \phi\sigma_{Xe}}{\lambda_{Xe} + \phi\sigma_{Xe}} \right)^{\lambda_{Xe}/(\lambda_I - \lambda_{Xe})} \right] \right\} \quad (13.12)$$

We are particularly concerned with Eqs. 13.7 and 13.12. It is convenient to introduce at this point a quantity B , called the "peak reduction by burnup," defined by the equation

$$B = \frac{N_{Xe}^{peak}}{N_{Xe}^{ss-nb}} \quad (13.13)$$

N_{Xe}^{ss-nb} is the steady-state xenon concentration with no burnup and is the xenon concentration obtained from Eq. 13.7 by equating $\sigma_{Xe} = 0$.

$$N_{Xe}^{ss-nb} = \frac{y\phi\Sigma_f}{\lambda_{Xe}} = y\phi\Sigma_f \tau_{Xe} \quad (13.14)$$

where τ_{Xe} is the mean lifetime of xenon. When the values for N_{Xe}^{peak} in Eq. 13.12 and N_{Xe}^{ss-nb} in Eq. 13.14 are substituted into Eq. 13.13, we find that $y\phi\Sigma_f$ cancels out and that B is a function of $\phi\sigma_{Xe}$

$$B = \lambda_{Xe} \left\{ \frac{1}{\lambda_{Xe} + \phi\sigma_{Xe}} \left(\frac{\lambda_{Xe}}{\lambda_I} \frac{\lambda_I + \phi\sigma_{Xe}}{\lambda_{Xe} + \phi\sigma_{Xe}} \right)^{\lambda_{Xe}/(\lambda_I - \lambda_{Xe})} + \frac{1}{\lambda_{Xe} - \lambda_I} \left[\left(\frac{\lambda_{Xe}}{\lambda_I} \frac{\lambda_I + \phi\sigma_{Xe}}{\lambda_{Xe} + \phi\sigma_{Xe}} \right)^{\lambda_I/(\lambda_I - \lambda_{Xe})} - \left(\frac{\lambda_{Xe}}{\lambda_I} \frac{\lambda_I + \phi\sigma_{Xe}}{\lambda_{Xe} + \phi\sigma_{Xe}} \right)^{\lambda_{Xe}/(\lambda_I - \lambda_{Xe})} \right] \right\} \quad (13.15)$$

A plot of B as a function of $\phi\sigma_{Xe}$ is given in Fig. 13.1. This quantity B is very convenient for calculation purposes.

13.4 CALCULATION OF REACTIVITY EFFECTS

The purpose of the calculations is to evaluate the loss of reactivity brought about by (1) steady-state xenon concentration and (2) peak xenon concentration after shutdown.

The loss of reactivity with peak xenon gives an indication of the excess reactivity that must be available to start the reactor when the xenon concentration is at a maximum.

Although the xenon formation in power reactors is space-dependent, many machine codes currently in use do not allow for sufficient space-dependent composition variations to permit a proper evaluation of the nonuniform xenon distribution. The xenon effects

Fig. 13.1—Peak burnout reduction factor. Note: the asymptotic value of B_m from Eq. 13.15 is ~ 0.313 .

are often approximated by assuming that the distribution of xenon is uniform in a specified region. However, machine codes that automatically calculate pointwise xenon concentrations are in preparation. The effect of the xenon space dependency on reactivity and power distribution can be obtained from the procedure described here without resorting to pointwise composition specifications. At the time when the majority of the S1G/S2G calculations were performed, available machine codes were restricted to three-region (core, reflector, and blanket) spherical reactor models.

It is assumed that the presence of xenon does not change the position in the reactor at which a low-energy neutron is absorbed. (Low energy here refers to the energy region in which the xenon absorption cross-section is large.)

* * *

A multigroup calculation is performed, and the fission distribution in the reactor is divided into two parts, F^H and F^L ; F^H fissions per second per cubic centimeter are those fissions in the higher energy ranges where the xenon cross-section is negligible, and F^L are those fissions in the energy range where the xenon absorptions compete with fission. In this low range an average σ_{Xe}/σ_f is used. For S1G/S2G calculations F^L is taken as the fissions in the thermal group (lethargy $u = 18.7$) and in the first nonthermal group ($u = 18.7$ to $u = 17.75$). These groups have been chosen so that the average ratio of σ_{Xe} to σ_f for each group is the same. Thus the average σ_{Xe}/σ_f is independent of neutron energy in this low-energy region. The ratio of fissions to total absorptions in the core is also essentially constant in these two groups.

Multigroup methods are used to calculate F^H and F^L , assuming no xenon is present. Then the effect of xenon on F^L is calculated, giving a new distribution, $F^{L'}$, and a total fission rate of $F' = F^H + F^{L'}$. The calculation is iterated until F' and the assumed normalized source S for the multigroup calculation agree in shape. In practice few iterations are required. As derived in Vol. I, Chap. 9, this publication, the reactivity is given by

$$k = \frac{\int F'(\nu F_H) d(\text{Vol})}{\int S(\nu F_H) d(\text{Vol})}$$

where ν is the neutrons produced per fission and F_H is the iterated fission probability.² Care should be taken to distinguish between the symbol F^H , which represents high-energy fissions, and F_H , which represents the iterated fission probability for fission neutrons.

As an example, in spherical co-ordinates the factor νF_H may be approximated by

$$1 - \frac{1}{2} \left(\frac{r}{r_{\text{core}}} \right)^2$$

in the absence of an adjoint multigroup calculation. The evaluation of k is correct independent of the choice of S if νF_H is correct. If νF_H is approximately correct, the error in k will be small, particularly because S will also have substantially the correct shape. The effect of xenon on reactivity is obtained by a comparison of F (no xenon) with the F' calculated for a particular xenon concentration and distribution.

13.5 STEADY-STATE XENON REACTIVITY

In this section the calculations are described in greater detail. First, consider the steady-state xenon concentration. Xenon is produced at point m at a rate

$$R_m = f_m N_u y \quad (13.16)$$

where y is the xenon yield, N_u is the number of U^{235} atoms per cubic centimeter, and f_m is the fission rate per uranium atom at position m in the core and at specified reactor power. The latter can be written as

$$f_m = f_m^L + f_m^H$$

To account for the reduction in f_m^L due to steady-state xenon, we introduce the term C_m :

$$C_m = 1 - \epsilon_m = \frac{\phi_m^L}{\phi_0^L} \Bigg]_m = \frac{\text{low-energy flux with equilibrium xenon}}{\text{low-energy flux, no xenon}} \Bigg]_m \quad (13.17)$$

The term ϵ_m has been introduced as a convenience.

The steady-state xenon concentration $N_{Xe_m}^{ss}$ at point m can now be obtained. From Eq. 13.7,

$$N_{Xe}^{ss} = \frac{y \phi \Sigma_f}{\lambda_{Xe} + \phi \sigma_{Xe}} = \frac{y \phi \Sigma_f \tau}{1 + \phi \sigma_{Xe} \tau}$$

where τ is the mean life of Xe^{135} .

By introducing C_m , Eq. 13.7 can be rewritten and applied to point m :

$$N_{Xe_m}^{ss} = \frac{(f_m^H N_u y + C_m f_m^L N_u y) \tau}{1 + C_m \phi_{0m}^L \sigma_{Xe} \tau} \quad (13.18)$$

In the denominator, ϕ_0^L has been written for ϕ_0 because it is the only flux that reacts with xenon.

By substituting Eq. 13.16 into Eq. 13.18, we obtain

$$N_{Xe_m}^{ss} = \frac{(f_m^H N_u y + C_m f_m^L N_u y) \tau}{1 + C_m \phi_{0m}^L \sigma_{Xe} \tau} = \frac{R_m \tau - (1 - C_m) f_m^L N_u y \tau}{1 + C_m \phi_{0m}^L \sigma_{Xe} \tau} \quad (13.19)$$

By assuming the same total absorptions in the last two groups with or without xenon, we have (ignoring the change in self-shielding caused by the presence of xenon)

$$C_m = \frac{\Sigma_a}{\Sigma_a + N_{Xe_m}^{ss} \sigma_{Xe}} \quad (13.20)$$

where $\Sigma_a = (A^L/F^L) N_u \sigma_T$ is the macroscopic absorption cross-section of the low-energy group without xenon. A^L and F^L are the total core absorptions and fissions, respectively, in the low-energy region, obtained from the multigroup calculation.

If Eq. 13.20 were substituted into Eq. 13.19, an equation in $N_{Xe_m}^{ss}$ would result which

could be solved for $N_{Xe_m}^{ss}$. For the purposes of machine calculations, however, this step is bypassed, and the effect of the xenon concentration on reactivity is calculated directly. A number of constants are now defined which are of use in the UNIVAC procedure.

From Eqs. 13.17 and 13.20, we note

$$\epsilon_m = 1 - C_m = 1 - \frac{\Sigma_a}{\Sigma_a + N_{Xe_m}^{ss} \sigma_{Xe}} = \frac{N_{Xe_m}^{ss} (\sigma_{Xe}/\Sigma_a)}{1 + N_{Xe_m}^{ss} (\sigma_{Xe}/\Sigma_a)} \quad (13.21)$$

Substituting Eq. 13.19 in Eq. 13.21,

$$\epsilon_m = \frac{(\sigma_{Xe}/\Sigma_a) (R_m \tau - \epsilon_m f_m^L N_u y \tau)}{1 + (1 - \epsilon_m) \phi_{0m}^L \sigma_{Xe} \tau + (\sigma_{Xe}/\Sigma_a) (R_m \tau - \epsilon_m f_m^L N_u y \tau)} \quad (13.22)$$

Now let

$$\alpha_m \equiv \frac{R_m \tau \sigma_{Xe}}{\Sigma_a} \quad (13.23)$$

$$\beta_m \equiv \phi_{0m}^L \sigma_{Xe} \tau = f_m^L \tau (\sigma_{Xe}/\sigma_f) \quad (13.24)$$

In terms of these constants Eq. 13.22 becomes

$$\begin{aligned} \epsilon_m &= \frac{\alpha_m - \epsilon_m f_m^L N_u y \tau (\sigma_{Xe}/\Sigma_a)}{1 + \alpha_m + \beta_m - \epsilon_m \beta_m - \epsilon_m f_m^L N_u y \tau (\sigma_{Xe}/\Sigma_a)} \\ &= \frac{\alpha_m - \epsilon_m \beta_m (\Sigma_f/\Sigma_a) y}{1 + \alpha_m + \beta_m - \epsilon_m \beta_m [1 + (\Sigma_f/\Sigma_a) y]} \end{aligned} \quad (13.25)$$

Now let

$$\delta_m = \beta_m \left(1 + \frac{\Sigma_f}{\Sigma_a} y \right) = \beta_m \left(1 + \frac{F^L}{A^L} y \right) \quad (13.26)$$

$$\epsilon_{0m} = \frac{\alpha_m}{1 + \alpha_m + \delta_m} \quad (13.27)$$

$$\gamma_m = \frac{\delta_m \alpha_m}{(1 + \alpha_m + \delta_m)^2} = \frac{\delta_m \epsilon_{0m}}{1 + \alpha_m + \delta_m} \quad (13.28)$$

Note that α_m , β_m , δ_m , ϵ_{0m} , and γ_m can all be calculated from known quantities obtainable from the multigroup calculation. If Eqs. 13.26, 13.27, and 13.28 are substituted in Eq. 13.25, we obtain

$$-\epsilon_m^2 \beta_m \left(1 + \frac{\Sigma_f}{\Sigma_a} y \right) + \left(1 + \alpha_m + \beta_m + \beta_m \frac{\Sigma_f}{\Sigma_a} y \right) \epsilon_m - \alpha_m = 0 \quad (13.29)$$

By dividing by $-(1 + \alpha_m + \delta_m)\epsilon_{0m}$ and simplifying further, we obtain

$$\gamma_m \left(\frac{\epsilon_m}{\epsilon_{0m}} \right)^2 - \frac{\epsilon_m}{\epsilon_{0m}} + 1 = 0 \quad (13.30)$$

Let

$$\chi_m = \frac{\epsilon_m}{\epsilon_{0m}} \quad (13.31)$$

By solving the quadratic Eq. 13.30 in χ_m , we get

$$\chi_m = \frac{1}{2\gamma_m} (1 \pm \sqrt{1 - 4\gamma_m}); \quad 0 < \gamma_m < \frac{1}{4} \quad (13.32)$$

(γ_m is positive by definition; χ_m is real.)

It should be noted here that an extraneous root has been introduced in Eq. 13.32. The physical impossibility of the sign before the radical being positive can be shown by examining the limiting value of χ_m as γ_m approaches zero.

In order to simplify the UNIVAC procedure by eliminating the square root, it is convenient to use the following approximation:

$$\sqrt{1 - 4\gamma_m} = \frac{1}{2} \left(1 - 2\gamma_m + \frac{1 - 4\gamma_m}{1 - 2\gamma_m} \right) \quad (13.33)$$

We then obtain

$$\chi_m \approx \frac{1 - \gamma_m}{1 - 2\gamma_m} \quad (13.34)$$

This approximation is in error by only 1 per cent for $\gamma_m = 0.16$ and is adequate for the range of γ_m under consideration. In practice γ_m is rarely close to its maximum value of 0.25.

From known quantities γ_m can be calculated, and hence χ_m and therefore ϵ_m can be obtained. It is then a simple matter to determine the low-energy fissions $F_m^{L'}$ in the presence of steady-state xenon. For steady-state conditions

$$F_m^{L'} = F_m^L (1 - \epsilon_{0m} \chi_m) = F_m^L (1 - \epsilon_m) \quad (13.35)$$

13.6 PEAK XENON REACTIVITY

For the case of the peak xenon condition, a similar procedure can be followed. Reference should be made to Eq. 13.13, which relates the peak xenon concentration N_{Xe}^{peak} and the quantity N_{Xe}^{ss-nb} . The latter quantity is obtainable from Eq. 13.19 by setting the flux equal to zero. We then obtain

$$N_{Xe_m}^{ss-nb} = \tau (R_m - \epsilon_{0m} \chi_m f_m^L N_u y) \quad (13.36)$$

By using Eq. 13.13, the peak xenon concentration is

$$N_{Xe_m}^{peak} = B_m \tau (R_m - \epsilon_{0m} \chi_m f_m^L N_u y) \quad (13.37)$$

The quantity B (or B_m) given in Fig. 13.1 is a function of $\phi_m \sigma_{Xe}$.

$$\phi_m^L \sigma_{Xe} = (1 - \epsilon_{0m} \chi_m) \frac{\beta_m}{\tau} \quad (13.38)$$

It should be emphasized that the unperturbed flux was corrected for the presence of steady-state and not peak xenon. Further, for UNIVAC calculations, B_m given in Fig. 13.1 may be approximated by

$$B_m = \frac{1 + 2.11 \times 10^4 (\phi \sigma_{Xe})_m}{1 + 7.01 \times 10^4 (\phi \sigma_{Xe})_m}$$

$$= \frac{1 + 0.44\beta_m(1 - \epsilon_m)}{1 + 1.47\beta_m(1 - \epsilon_m)} \quad (13.39)$$

(Note $\tau = 4.78 \times 10^4$ sec)

If a peak xenon concentration is present, the low-energy fissions F_m^L are reduced, resulting in

$$F_m^L \Big|_{\text{peak}} = F_m^L \Big|_{\text{peak}} \left(\frac{\Sigma_a}{\Sigma_a + N_{\text{Xe}}^{\text{peak}} \sigma_{\text{Xe}}} \right)_m \quad (13.40)$$

On substituting Eq. 13.37 in Eq. 13.40 and simplifying, we find

$$F_m^L \Big|_{\text{peak}} = F_m^L \Big|_{\text{peak}} \frac{1}{1 + B_m[\alpha_m - \epsilon_m \beta_m (F_m^L/A_m^L) y]} \quad (13.41)$$

13.7 RECAPITULATION

The entire procedure is now summarized.
The multiplication constant k is given by

$$k = \frac{\int P(\nu F_H) d(\text{Vol})}{\int S(\nu F_H) d(\text{Vol})} \quad (13.42)$$

If no xenon is present, we substitute $F_m = F_m^L + F_m^H$ for P and solve for k .

With steady-state xenon present we solve Eq. 13.34 to obtain X_m and hence ϵ_m . Then from Eq. 13.35 we obtain the reduced fissions $F_m^{L'}$ and hence we substitute $F_m^{L'} + F_m^H$ for P in Eq. 13.42 and solve for k . (This must be iterated until $F_m^{L'} + F_m^H$ is proportional to the assumed source.)

In the case for peak xenon, $F_m^{L'} \Big|_{\text{peak}}$ is obtained from Eq. 13.41, and the appropriate k is calculated from Eq. 13.42.

The change in reactivity, Δk , for the presence of steady-state and peak xenon can then be easily obtained.

* * *

The above procedure is strictly correct for obtaining the conditions for criticality with steady-state xenon. In this case, as indicated above, the fission rates F_m^L and F_m^H are calculated by the multigroup methods with no xenon present. The problem is iterated until the corrected distribution $F' = F^H + F^{L'}$ is $1/\nu$ times the assumed fission neutron source. The method automatically corrects for the fact that it is $F_m^{L'}$ rather than F_m^L which determines the rate of xenon production. (This follows from the form of Eq. 13.19. Actually, the correction in the power at any point due to the presence of steady-state xenon is only a few per cent.) Once the steady-state problem has been performed, the reduction factors $\{1 + B_m[\alpha_m - \epsilon_m \beta_m (F_m^L/A_m^L) y]\}^{-1}$ as indicated in Eq. 13.41 can be used directly for the peak xenon case, the terms in the reduction factor being evaluated for the steady-state xenon case, but the high-energy fissions and low-energy absorptions being evaluated for the peak xenon case.

The procedures outlined in this chapter have been applied extensively to S1G calculations. In general, good agreement with S1G test data has been obtained. (See Secs. 14.4 and 20.12.)

REFERENCES

1. R. Ehrlich and H. Hurwitz, Jr., Multigroup Methods for Neutron Diffusion Problems, Nucleonics, 12(2): 23 (February 1954). See also Vol. I, Chap. 2, this publication.	13.1
2. H. Hurwitz, Jr., Note on the Theory of Danger Coefficients, Report KAPL-98. See Vol. I, Chap. 9, this publication and also Chap. 7, this volume.	13.4

Referred to
in section

Chapter 14

PREDICTION OF FISSION-PRODUCT POISONING

J. S. KING

14.1 INTRODUCTION

The importance of a reliable prediction of fission-product poisoning of a reactor design is well established. Without this information no real estimate of lifetime or endurance is possible. Predictions for a SIR-type reactor were particularly desirable since over one-third of the total mechanical control is allocated to long-lived fission-product poisoning. At the same time, theoretical predictions were particularly difficult since so little was known of the complicated cross-sections in the intermediate-energy region. Consequently certain critical-assembly data were gathered together by M. E. Way and the writer. These data, though scattered and incomplete, served as the basis for an experimental determination.¹⁻³

14.2 LONG-LIVED FISSION-PRODUCT ESTIMATES

It should be stated at once that an accurate experimental study would be feasible only if individual reactivity measurements could be made (in the critical facility) on most of the important fission-product isotopes. It has been impossible to make such measurements because most of the isotopes are unstable and cannot be obtained in separated form suitable for reactivity determinations. Consequently many indirect results must be reverted to by observing how fission-product "stand-ins" behave. This shortcoming, together with approximations on the change in reactivity values with lifetime, make the critical-facility estimate of the endurance of marginal reliability; i.e., probably not better than ± 25 per cent.

A. Experimental Basis

The experimental data were collected from three areas of measurement:

1. Direct reactivity coefficient measurements of unshielded samples of as many separated and stable product isotopes as were available. The reactivity coefficient measurement technique followed the pattern described in Chap. 8. The data were obtained as a function of position in the core and degree (simulated) of core depletion. In some cases extrapolation to infinite dilution depended on comparison with self-shielding data on other similar isotopes. (It can be considered that fission-product deposition in the core is not great enough to require self-shielding corrections.)
2. Inferred reactivity coefficients for unavailable isotopes from interpolations of a general systematic survey of stable, separated isotopes as a function of atomic number, Z . Such a survey requires a fairly large program. It was actually undertaken in an early PPA mock-up (in an era when separated isotopes were difficult to obtain, and measurement techniques were unrefined). Figure 14.1 gives the data and shows actually two curves, one for even A and one for odd A nuclei ($A \equiv$ mass number). Further details are given in Sec. 14.5. (It should be

said that with the rapid and accurate pile oscillator technique developed at KAPL and the availability of many enriched isotopic samples, a much more complete and reliable survey could be made today.)

3. Measurements of the power distribution, $f(r,t)$, and favorableness function, $F_L(r,t)$, as functions of core position, r , for several mocked-up lifetime conditions, t , of the reactor design. The power data provided knowledge of proper weighting factors for the reactivity-coefficient data, which were taken at a single central core position. The study of F_L provided the connection between absorption rates and reactivity coefficients.

B. Calculational Approximations

The reactivity coefficient data were combined with measurements of $F_L(r,t)$ and $f(r,t)$ to calculate the reactivity loss produced by the accumulation of each fission product in the manner now to be described. In principle, one should use Eq. 14.3, which is now to be derived, to do this. In practice, however, several approximations had to be made, as in Eqs. 14.6 and 14.7.

For a particular core position denoted by the position vector r and the volume element dr , a single fission product at time t after startup poisons the core by an amount

$$\Delta k(r,t) dr = RC(r,t) n(r,t) dr \quad (14.1)$$

Here $\Delta k(r,t)$ = the reactivity loss in dollars per unit volume

$RC(r,t)$ = the reactivity coefficient in dollars per mole for the fission product at the given position r at the time t after startup

$n(r,t)$ = the number of moles of the fission product per unit volume at r and t

Let L = Avogadro's number (6.02×10^{23} atoms/mole)

Y = the fission yield of the particular fission product in question

λ = the radioactive decay constant ($\lambda = 0$ if the fission product is stable)

$f(r,t)$ = the fission rate per unit volume at r and t

$\Delta a(r,t) \equiv \int_0^\infty dE \sigma_a(E) \phi(E,r,t)$, = the rate of absorptions per atom of fission product (assumed dilute enough that self-shielding may be neglected)

Then the production, decay, and depletion of the particular fission product follow the equation

$$\frac{dn(r,t)}{dt} = [Y f(r,t)/L] - [\lambda + \Delta a(r,t)] n(r,t) \quad (14.2)$$

By substituting the solution of this equation for n into Eq. 14.1, one obtains the exact, though cumbersome, relation

$$\begin{aligned} \Delta k(r,t) = & \left\{ RC(r,t)(Y/L) \exp \left(- \int_0^t dt' [\lambda + \Delta a(r,t')] \right) \right\} \\ & \times \left\{ \int_0^t dt'' f(r,t'') \exp \left(+ \int_0^{t''} dt' [\lambda + \Delta a(r,t')] \right) \right\} \end{aligned} \quad (14.3)$$

This expression, integrated over the volume of the core, gives the reactivity loss produced by that particular fission product in the reactor.

* * *

More usable expressions can be obtained if advantage is taken of the fact that in SIR-type reactors the quantities $f(r,t)$ and $\Delta a(r,t)$ in the coefficients on the right-hand side of Eq. 14.2 vary only slightly with time. Accordingly it is a reasonable approximation to use, in place of Eq. 14.2, an equation in which these quantities are replaced by constants $\bar{f}$ and $\bar{\Delta a}$ which may be near the time-average values of those quantities from startup ($t = 0$) to final shutdown ($t = t_0$).

$$\frac{dn}{dt} = [Y \bar{f}(r)/L] - [\lambda + \bar{\Delta a}(r)]n \quad (14.4)$$

where

$$\bar{f}(r) \cong \frac{1}{t_0} \int_0^{t_0} dt' \bar{f}(r, t')$$

$$\bar{\Delta a}(r) \cong \frac{1}{t_0} \int_0^{t_0} dt' \Delta a(r, t')$$

The solution to Eq. 14.4 is

$$n(r, t) = (Y/L) \bar{f}(r) (1 - \exp \{-[\lambda + \bar{\Delta a}(r)]t\}) / [\lambda + \bar{\Delta a}(r)] \quad (14.5)$$

which allows the reactivity to be written in the form

$$\Delta k(r, t) = RC(r, t) (Y/L) \bar{f}(r) t C(r, t) \quad (14.6)$$

Here

$$C(r, t) \equiv [1 - e^{-(\lambda + \bar{\Delta a})t}] / (\lambda + \bar{\Delta a})t$$

is a decay factor.

When $(\lambda + \bar{\Delta a})t$ is small, $C(r, t)$ is close to unity. Furthermore, if $\lambda + \Delta a(r, t)$ is small enough during the life of the reactor to ensure that $\int_0^{t_0} dt' (\lambda + \Delta a) \ll 1$, then the value of $\bar{f}(r) dr$ is indeed close to the time-average fission rate in dr . (This can be seen by expanding Eq. 14.3 and comparing with Eq. 14.6.) If P is the total reactor power in watts,

$$\int dr \bar{f} \cong \int dr f(r, t) = 3.12 \times 10^{10} \times P \quad (14.7)$$

We can integrate Eq. 14.6 in this case so that

$$\bar{\Delta k}(t) \equiv \int dr \Delta k(r, t) = \frac{3.12 \times 10^{10}}{6.02 \times 10^{23}} YPt \frac{\int dr RC(r, t) \bar{f}(r)}{\int dr \bar{f}(r)} \quad (14.8)$$

or

$$\bar{\Delta k}(t) = 5.18 \times 10^{-14} YPt (RC)_{\text{eff}}(t)$$

It is clear that $(RC)_{\text{eff}}$ is a core-distributed reactivity value that has been weighted by a time-average power distribution. In practice, the time-average power distribution $\bar{f}(r)$ is taken as a mean between mock-up power distribution measurements for $t = 0$ and similar measurements for a time corresponding to the end point in question.

* * *

When $\int_0^{t_0} dt' (\lambda + \Delta a)$ is of the order of unity, the depletion factor $C(r, t)$ in Eq. 14.6 must be measured, at least approximately. This is done in the critical-assembly mock-ups by measurements to determine $\bar{\Delta a}(r)$, which necessitates knowledge of the absorption integral at every point in the core and of its variation with burnup. A fair approximation is possible by assuming $\bar{\Delta a}(r)$ equal to the absorption integral for a core mock-up corresponding to mid-life condition; i.e.,

$$\bar{\Delta a}(r, t_0) \cong \Delta a(r, t_0/2)$$

The quantity $\overline{\Delta a}(r, t_0/2)$ is found from simultaneous measurements of $RC(t_0/2)$ and $F_L(t_0/2)$ combined according to the Hurwitz prescription:⁴

$$\overline{\Delta a}(r, t_0) \cong \Delta a(r, t_0/2) = \frac{RC(r, t_0/2)}{F_L(r, t_0/2)} \frac{3.12 \times 10^{10} P}{6.02 \times 10^{23}} \quad (14.9)$$

From this value of $\overline{\Delta a}(r, t_0)$, the value of $C(r, t_0)$ is computed. Then, as in Eq. 14.8, integration of Eq. 14.6 over the volume of the core leads to

$$\begin{aligned} \overline{\Delta k}(t) &= t(Y/L) \int dr RC(r, t) C(r, t) \overline{f}(r) \\ &= 5.18 \times 10^{-14} YPt \int dr RC(r, t) C(r, t) \overline{f}(r) / \int dr \overline{f}(r) \\ &= 5.18 \times 10^{-14} YPt (RC \times C)_{\text{eff}} \end{aligned} \quad (14.10)$$

Use of the quantity $(RC)_{\text{eff}}(t)$ that was defined in Eq. 14.8 leads to a new quantity $C_{\text{eff}}(t)$ defined by

$$\overline{\Delta k}(t) = 5.18 \times 10^{-14} YPt (RC)_{\text{eff}}(t) C_{\text{eff}}(t) \quad (14.11)$$

Although in Eq. 14.10, $RC(r, t)$ explicitly appears only for the final time t at which $\overline{\Delta k}(t)$ is desired, it must not be forgotten that implicit in the approximate expression for $C(r, t)$ is a knowledge of both $RC(r, t/2)$ and $F_L(r, t/2)$ (by Eq. 14.9). Even if $\Delta a(r, t)$ could be obtained for all times by applying Eq. 14.9 to a complete knowledge of RC and F_L and if thus $\overline{\Delta a}(r)$ could be computed and used in Eq. 14.5, the $\overline{\Delta k}(t)$ based upon Eq. 14.6 would still not be so accurate as a $\overline{\Delta k}(t)$ based upon Eq. 14.3. In the critical-assembly mock-ups, data for $t = 0$, $t = t_0$, and perhaps one intermediate value of t were obtained, and rough averages were found. Since C can be quite small for the most potent absorbers, the errors made in these approximations may have been fairly severe.

* * *

When $\lambda + \Delta a$ is very large, as in the case of very short-lived fission products or highly poisonous ones (Xe^{135}), it is evident that the reactivity effect must take a saturation value early in any period of operation of the reactor and that the effective reactivity coefficient is not one obtained by a time-average fission rate $\overline{f}(r)$ as was used in Eqs. 14.8 and 14.10. In this case it is possible to make a direct approximation to the exact expression Eq. 14.3. Since $\lambda + \Delta a$ is large, $\int_0^t dt' (\lambda + \Delta a)$ is a rapidly increasing function of t ; indeed, it is $(\lambda + \Delta a)t$ if the flux ϕ is constant with time. The exponential factor in the integral over t'' in Eq. 14.3 is an even more rapidly increasing function, so that $f(r, t'')$ can be replaced by its value at t , the upper limit of integration, with little effect on the result, to give

$$\Delta k(r, t) dr \cong dr RC(r, t) (Y/L) \exp \left[- \int_0^t dt' (\lambda + \Delta a) \right] f(r, t) \int_0^t dt'' \exp \left[+ \int_0^{t''} dt' (\lambda + \Delta a) \right] \quad (14.12)$$

The integral over dt'' is then approximated, for large $(\lambda + \Delta a)t$, by

$$\int_0^t dt'' \exp \left[+ \int_0^{t''} dt' (\lambda + \Delta a) \right] \cong \exp \left[+ \int_0^t dt' (\lambda + \Delta a) \right] / (\lambda + \Delta a)$$

so that

$$\Delta k(r, t) dr \cong dr (Y/L) RC(r, t) f(r, t) / [\lambda + \Delta a(r, t)] \quad (14.13)$$

By using the Hurwitz formula (Eq. 14.9) for $\Delta a(r, t)$, one finally obtains the xenon reactivity loss as the volume integral over the expression.

$$\Delta k(r, t) dr = dr (Y/L) RC(r, t) f(r, t) / [\lambda + 5.18 \times 10^{-14} P RC(r, t) / F_L(r, t)] \quad (14.14)$$

Here Y is the cumulative fission yield of the mass-135 chain through Xe^{135} , λ is the Xe^{135} decay constant, and $f(r,t)$ is normalized as in Eq. 14.7.

14.3 ESTIMATES OF CORE POISONING IN A DEPLETED REACTOR

Altogether there are about 65 important fission-product nuclides. These are listed in Table 14.1 with their approximate yields. For all but the five isotopes noted in Table 14.2, only the indirect reactivity coefficient measurements of Fig. 14.1 were available. These were made in an early PPA core (PPA-5) of a different composition. Reactivity coefficients for these 60 nuclides were crudely estimated by normalizing the curves by the average of the reactivity coefficient ratios from PPA-12 and PPA-20, for several test samples measured in both mock-ups.

Experimental data on F_L 's, reactivity coefficients, and power distributions were obtained in PPA-19 and PPA-20, which mocked cores for a clean reactor ($t = 0$) and for a depleted reactor ($t \sim 100,000$ Mwh) if the two can be considered as two stages in the life of a single reactor. Unfortunately only a very few fission-product samples were available: namely, Rh^{103} , Cs^{133} , and natural samarium. However, data on these, together with

TABLE 14.1—STABLE AND LONG-LIVED
FISSION-PRODUCT YIELDS

Nuclide	Fractional yield $\times 100$, %	Nuclide	Fractional yield $\times 100$, %
Kr^{83}	0.59	Te^{128}	0.18
Kr^{84}	1.11	I^{127}	0.38
Kr^{85}	0.33	Te^{128}	0.71
Rb^{85}	1.20	I^{129}	1.20
Kr^{86}	2.14	Te^{130}	1.97
Rb^{87}	2.82	Xe^{131}	2.85
Sr^{88}	3.56	Xe^{132}	4.24
Y^{89}	4.58	Cs^{133}	6.70
Sr^{90}	5.62	Xe^{134}	7.53
Zr^{91}	6.10	Cs^{135}	6.00
Zr^{92}	6.32	Xe^{136}	6.24
Zr^{93}	6.47	Cs^{137}	4.70
Zr^{94}	6.55	Ba^{138}	6.61
Mo^{95}	6.61	La^{139}	6.59
Zr^{96}	6.61	Ce^{140}	6.51
Mo^{97}	6.56	Pr^{141}	6.41
Mo^{98}	6.48	Ce^{142}	6.22
Tc^{99}	6.16	Nd^{143}	5.90
Mo^{100}	5.56	Nd^{144}	5.06
Ru^{101}	4.60	Nd^{145}	3.95
Ru^{102}	3.56	Nd^{146}	3.07
Rh^{103}	2.41	Pm^{147}	2.26
Ru^{104}	1.50	Sm^{147}	0.09
Pd^{105}	0.92	Nd^{148}	1.79
Pd^{106}	0.53	Sm^{149}	1.40
Pd^{107}	0.26	Nd^{150}	0.718
Pd^{108}	0.13	Sm^{151}	0.486
Ag^{109}	0.028	Sm^{152}	0.305
Cd^{111}	0.018	Eu^{153}	0.122
Cd^{112}	0.013	Sm^{154}	0.100
Cd^{113}	0.012	Gd^{155}	0.055
In^{115}	0.011	Gd^{156}	0.028
		Gd^{157}	0.016

TABLE 14.2—POISONING INFERRED FROM DIRECT REACTIVITY-COEFFICIENT MEASUREMENTS

Nuclide	Fractional yield $\times 100$, %	(RC) _{eff} (t_0), \$/mole	(C) _{eff} (t_0)	$\Delta k(t_0)$, \$
Rh ¹⁰³	2.41	0.81	0.93	0.37
Cs ¹³³	6.70	0.42	0.97	0.54
Sm ¹⁴⁷	0.09	2.03	0.80	0.03
Sm ¹⁴⁹	1.40	17.10	0.27	1.30
Sm ¹⁵¹	0.486	4.23	0.64	0.27
U ²³⁸	38.0	0.082	1.00	0.60
Total				3.11

reasonable assumptions about the reactivity coefficient split among the samarium isotopes, provided at least a crude experimental basis for a major fraction of the fission poisoning at the end of roughly 100,000 Mwh life. Table 14.2 summarizes the data for six major isotopes. The reactivity coefficient for U²³⁸ was estimated¹ from Rh¹⁰³ data. It is included in the tables as an effective "fission product." Data obtained from Fig. 14.1 lead to an addition of \$1.86 to $\Delta k(t_0)$ for the 60 isotopes not listed in Table 14.2, the weaker absorbers among the fission products. Adding the two gives a total poison balance after 100,000 Mwh (6 kg fuel depletion) of \$4.97 with an estimated uncertainty of ± 25 per cent. This number has yet to be completely verified. It agrees reasonably well with the result of a statistical calculation (Chap. 12). See the discussion of these in Sec. 20.12.

It should be observed that the crudity and uncertainty of this fission-product study is due primarily to lack of separated isotope samples. The approximations of the method should not introduce an error any greater than the uncertainty of the yields themselves, i.e., ± 10 per cent. Use of extremely small samples of many separated isotopes in pile oscillator measurements could have led to a long-lived fission-poison estimate of about ± 10 per cent uncertainty.

14.4 XENON MOCK-UP MEASUREMENTS

Xenon poisoning in an intermediate-energy spectrum is less important relative to long-lived poisons than in thermal reactors. Nevertheless it was expected to contribute some 30 per cent to the total burned-out core fission-product poisoning in S1G. Hence some experimental estimate was desired to complete the total poison prediction. This was made by using samarium to simulate xenon in the PPA critical assembly. The strong resonance in Sm¹⁴⁹ at 0.0975 ev is close enough to the Xe¹³⁵ resonance at 0.0863 ev that the following two assumptions are valid in the intermediate spectrum: (1) The neutron spectrum seen by sub-cadmium samarium is the same as that seen by Xe¹³⁵; (2) the iterated fission probability, F_L , is the same for the two resonances. By use of these assumptions, together with the assumption that the ratio of absorption integrals for the two resonances is equal to the ratio of their known level strengths, a knowledge of xenon poisoning was inferred from detailed reactivity measurements of thin samarium samples placed at representative core positions. The difference between the samarium reactivity coefficients for bare and for cadmium-covered samples defines the reactivity coefficient for absorption in the Sm¹⁴⁹ resonance at 0.0975 ev. Multiplying this result by the ratio of level strengths (3.6×10^5 barn-ev for Xe¹³⁵ and 7250 barn-ev for Sm¹⁴⁹) gives a reactivity coefficient for Xe¹³⁵ as a function of core position.

As a check on the above assumptions, numerical integrations of absorption rates for Sm¹⁴⁹ and Xe¹³⁵ were also made with rather crude spectrum measurements from PPA data and with cross-sections as given in Report BNL-170.⁵ The numerical results were 15 per cent lower than those from the simple level-strength ratios, and in actual application a mean value between the two results was used.

Once the xenon reactivity coefficient was obtained, the calculation of steady-state xenon poisoning was made with the same general approach as for long-lived fission

products described above. As before, experimental knowledge is necessary, for all points in the core, of the reactivity coefficient, RC, the iterated fission probability, F_L , and the ratio of the power at the given position relative to total core power. Unlike the long-lived poison case, however, these need only be known at the time, t , of interest since this strong absorber quickly reaches a saturated condition. The steady-state xenon poisoning for both the clean ($t \approx 10,000$ Mwh) and depleted ($t \approx 100,000$ Mwh) cores was calculated by Eq. 14.14. The results were

$$\Delta k (t \approx 10,000 \text{ Mwh}) = \$2.76 \pm 15 \text{ per cent}$$

$$\Delta k (t \approx 100,000 \text{ Mwh}) = \$3.02 \pm 15 \text{ percent}$$

The initial data from a power reactor gave a clean-core steady-state xenon poisoning of \$2.30, in reasonable agreement with the first of these values. However, in the light of recent xenon cross-section data and further measurements on samarium in PTR mock-ups of the clean and dirty S1G, the original 1954 estimates¹ have been reworked. The comparison obtained with the S1G power facility show excellent agreement both at steady-state and peak xenon. As reported by Way,² these comparisons appear in Table 14.3.

TABLE 14.3—XENON POISONING EFFECTS ON REACTIVITY

	Steady state, \$	Peak, \$	Hours until peak
Measured clean Δk	2.4	2.5	2-4
Estimated clean Δk	$2.4 \pm 20\%$	$2.6 \pm 20\%$	4.0
Estimated dirty Δk	$2.2 \pm 20\%$	$2.4 \pm 20\%$	4.3

14.5 REACTIVITY COEFFICIENTS AS A FUNCTION OF ATOMIC NUMBER

The work reported in this section was done under the general supervision of G. Dessauer. It was briefly reported by T. M. Snyder at the Geneva Conference in 1955.⁶

Figure 14.1 combines the experimentally measured central reactivity coefficient values in PPA-5 in a manner guided by the following assumptions about effective nuclear absorption cross-sections in the intermediate reactor neutron spectrum (these assumptions are really about the average nuclear level spacings, as discussed by Hurwitz and Bethe⁷):

1. Odd-mass nuclides (both odd Z -even N and even Z -odd N) have effective absorption cross-sections that are generally slowly varying with Z and N . Odd-even nuclides in a given region of Z have cross-sections roughly equal to those of the even-odd nuclides in that region.
2. Even-even nuclides (practically all the even-mass ones) also generally have effective absorption cross-sections slowly varying with Z . Their cross-sections are considerably smaller (by a factor of about $1/3$) than those of neighboring odd-mass nuclides.
3. Cross-sections of nuclides that possess closed shells, or are within one or two nucleons of a closed shell, are exceptions from the above two assumptions and are much smaller, in accordance with the observations of Hughes et al.⁸

In Fig 14.1, solid circles show the reactivity coefficients measured for the odd- Z elements. These elements are comprised almost exclusively of odd-even nuclides.

There are enough values available to allow fairly complete interpolation to values for the neighboring even-odd nuclides, which appear as open circles in Fig 14.1--an interpolation that is, however, restricted by the measurements of the reactivity coefficients for even- Z elements, as will now be explained. The odd- A (odd-mass) curve, the highest one on the figure, is made up of these two kinds of points, plus some (with a question mark) that had to be almost sheer guesses because of a lack of pure rare-earth samples for measurement.

The reactivity coefficient values observed for even- Z elements are considered as being made up of two contributions: first, that of the even-odd nuclides and, second, that of the even-even nuclides. The contribution of the even-odd nuclides was computed as the product of the open-circle value on the odd- A curve and the fractional abundance of such nuclides in the element. After that the contribution of the even-even nuclides could be computed by subtracting the contribution of the former from the measured reactivity

Fig. 14.1—Reactivity changes at the center of PPA-5 due to capture alone.

coefficient value. This, divided by the fractional abundance of even-even nuclides in the element, gave an estimated reactivity coefficient for even-A nuclides shown as an open triangle on the even-A curve.

A measured reactivity coefficient for an even-Z element thus actually set upper and lower limits for the reactivity coefficients both of the even-odd and the even-even nuclides; and the smoothness of interpolation on the odd-A curve was a compromise to give simultaneously a reasonable smoothness to the resulting even-A curve.

Table 14.4 gives the reactivity coefficient measurements and the calculations on which Fig. 14.1 was based.

TABLE 14.4—MEASURED AND ESTIMATED REACTIVITY COEFFICIENTS FOR CENTER OF PPA-5

Z	Element	Odd-even nuclides		Even-odd nuclides		Even-even nuclides	
		RC, ¢/mole	Abundance, %	RC, ¢/mole	Abundance, %	RC, ¢/mole	Abundance, %
17	Cl	0.84	100	0.84	---	---	---
19	K	0.62	99.99	0.62	---	---	---
20	Ca	0.30	---	---	0.14	0.51	99.86
22	Ti	0.15	---	---	12.9	0.33	87.1
23	V	0.26	99.75	0.26	---	---	---
24	Cr	0.13	---	---	9.5	0.76	90.5
25	Mn	3.2	100	3.2	---	---	---
26	Fe	0.30	---	---	2.2	2.5	97.8
27	Co	6.4	100	6.4	---	---	---
28	Ni	0.38	---	---	1.1	2.75	98.9
29	Cu	1.25	100	1.25	---	---	---
30	Zn	0.48	---	---	4.1	1.65	95.9
31	Ga	2.15	100	2.15	---	---	---
32	Ge	0.94	---	---	7.8	3.7	92.2
33	As	6.3	100	6.3	---	---	---
34	Se	1.45	---	---	7.5	8.9	92.5
35	Br	12.3	100	12.3	---	---	---
36	Kr	---	---	---	10.6*	---	0.90*
37	Rb	2.15	100	2.9*	---	---	---
38	Sr	---	---	---	1.6*	---	0.40*
40	Zr	0.16	---	---	11.2	0.5*	88.8
41	Nb	4.0	100	4.0	---	---	---
42	Mo	3.2	---	---	25.2	8.0*	74.8
44	Ru	11.2	---	---	29.7	29	70.3
45	Rh	19	100	19	---	---	---
46	Pd	8	---	---	22.2	22	77.8
47	Ag	25.3	100	25.3	---	---	---
48	Cd	10	---	---	25.1	29.5	74.9
49	In	34	100	34	---	---	---

*Deduced by making special assumptions about the nuclides with closed shells of neutrons ($N=50, 82,$ and 126) which are consistent with the data of Hughes et al.,⁸ on fission neutron cross-sections.

TABLE 14.4—(Continued)

Z	Element	Odd-even nuclides			Even-odd nuclides		Even-even nuclides	
		RC, ¢/mole	Abundance, %	RC, ¢/mole	Abundance, %	RC, ¢/mole	Abundance, %	RC, ¢/mole
50	Sn	1.73	---	---	83.5	7.0	16.5	0.7
---	---	---	---	---	---	---	---	---
52	Te	2.2	---	---	7.9	13.0	92.1	1.3
53	I	22	100	22	---	---	---	---
54	Xe	---	---	---	---	36.8*	---	3.1*
55	Cs	11.6	100	11.6	---	---	---	---
56	Ba	0.64	---	---	17.9	3.1*	82.1	0.3*
---	---	---	---	---	---	---	---	---
58	Ce	0.23	---	---	---	---	100	0.74*
---	---	---	---	---	---	---	---	---
60	Nd	0.93	---	---	20.5	1.5*	79.5	1.0*
---	---	---	---	---	---	---	---	---
62	Sm	---	---	---	28.8	110*	71.2	11*
63	Eu	137	100	137	---	---	---	---
---	---	---	---	---	---	---	---	---
---	---	---	---	---	---	---	---	---
---	---	---	---	---	---	---	---	---
---	---	---	---	---	---	---	---	---
---	---	---	---	---	---	---	---	---
70	Yb	15.4	---	---	30.5	30	69.5	8.6
---	---	---	---	---	---	---	---	---
72	Hf	17	---	---	32.3	26	67.7	12.7
73	Ta	33.8	100	33.8	---	---	---	---
74	W	15.6	---	---	14.3	41	85.7	11.5
75	Re	50.7	100	50.7	---	---	---	---
76	Os	18.4	---	---	17.6	61	82.4	9.2
77	Ir	75.4	100	75.4	---	---	---	---
78	Pt	12.5	---	---	33.7	31	66.3	3.2
79	Au	30	100	30	---	---	---	---
80	Hg	6.4	---	---	30.1	17	69.9	1.9
81	Tl	2.3	100	2.3	---	---	---	---
82	Pb	0.2	---	---	21	0.4*	79	0.2*
83	Bi	0.2	100	0.2	---	---	---	---

*Deduced by making special assumptions about the nuclides with closed shells of neutrons ($N = 50, 82,$ and 126) which are consistent with the data of Hughes et al.,⁸ on fission neutron cross-sections.

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Chapter 15

EVALUATION OF SIR-TYPE REFLECTOR CONTROL IN CRITICAL ASSEMBLIES

J. A. BISTLINE, Jr.

15.1 INTRODUCTION

Methods of control for intermediate reactors were investigated almost from the start of the critical assembly program at the Knolls Atomic Power Laboratory (see Chap. 2). PPA-12 was built to investigate the use of a translating side reflector as a control element. A different control method, involving a rotating beryllium cylinder faced on one side with B_4C , was subsequently tried, and the results were encouraging. PPA-15 incorporated six regions of B_4C in the reactor to simulate six rotating cylinders. A detailed mock-up of a design for these cylinders was constructed for use in the PPA (see Sec. 15.5).

Techniques for measuring the control value of the poison cylinders were worked out and various control poisons were evaluated.¹⁻⁵ The Proof Test Reactor (PTR) facility was used to provide additional design information on cylindrical reflector control elements.

This chapter discusses the techniques developed for measuring the control value of the cylinder mock-ups and gives a comparison of the effectiveness of various control poisons in an intermediate spectrum. The critical assembly studies were carried out mainly in PPA-18 and in the PTR. These facilities are also briefly described in this chapter.

The application of these experimental data to the evaluation of endurance characteristics and safety margins are discussed in Chap. 19 of this volume. Two-dimensional multigroup calculations of these quantities are discussed in Chap. 18.

15.2 EXPERIMENTAL FACILITIES

A. Description of PPA-18

The PPA-18 was a low-power flexible nuclear mock-up of a SIR-type design. The control cylinders were mocked-up in PPA-18 by six hexagonal prisms consisting of 37 slugs per prism located at the corners of the reactor. Five of the 37 slugs contained B_4C powder in steel cans and 29 of the slugs were beryllium reflector slugs. The arrangement of the mocked-up control cylinders is shown in Fig. 15.1.

The location of the control cylinders was chosen to make the PPA core area approximately equal to the reactor core area and to make the distance between the control cylinders and the core center as close as possible to that in the reactor. Rotation of the PPA control cylinders was accomplished by exchanging the B_4C -filled rods with the beryllium rods around the periphery of the control-cylinder hexagon. Because of the geometry of the PPA tube matrix and the location of the control cylinder, the PPA control-cylinder mock-up could only simulate the control-cylinder angles of 6, 26, 46, . . . 146 deg.

Control and safety of the reactor was provided by the control cylinders. In the PPA the control cylinders were fixed as described, and safety was provided by (1)

Fig. 15.1—Cross-section view of equatorial plane of the PPA. The following regions are outlined: (1) the core in the center; (2) the six control cylinders, (a) the five B_4C tubes and (b) the three-tube void region; (3) the beryllium reflector region between the cylinders; and the stainless-steel primary shield.

Fig. 15.2—PPA-18. Cross-sectional view of movable half. B₄C at 66-deg position.

splitting the assembly into two halves and (2) removing several fuel and reflector rods at high speed. Figure 15.2 indicates the location of the safety and control rods. Control was provided by the accurate positioning of calibrated control rods.

B. Description of the Proof Test Reactor (PTR)

The PTR was a critical assembly built to the S1G dimensions. (A more detailed description is given in Chap. 3. See Fig. 3.5) The PTR was mounted vertically on a 25-ton hydraulic elevator except for one-sixth of the fuel core materials which were suspended from a tower above the assembly. As the elevator was raised the suspended fuel rods entered the core to make a complete assembly.

Although all parts were manufactured to the size of the cold S1G dimensions, the PTR matrix tubes were mounted in an expanded array corresponding to the size of the S1G core at operating temperatures.

The PTR fuel rods consisted of flat disks of fuel, beryllium, stainless steel, and magnesium (to mock sodium) stacked in thin-walled stainless-steel tubes.

The six S1G control cylinders were duplicated in the PTR but with the following differences:

1. They did not control the PTR since their position could be changed only by manual adjustment with the reactor shutdown.
2. The PTR cylinders were made with simplified welding, beryllium machining, and support structures. Coolant channels were present in the beryllium slab of only one PTR cylinder.
3. The PTR cylinders were not immersed in sodium.

PTR control was provided by manual adjustment of the six control elements in the shutdown position until the reactor was near critical when the elevator was raised to complete the assembly. Fuel control rods were then driven into the core to make the assembly critical.

15.3 EVALUATION OF CONTROL MARGINS

Since the control cylinders rotate from their least reactive position at 0 deg to their most reactive one at 140 deg and since the PPA-18 was critical with the control cylinders at 66 deg, two types of experiments were indicated:

1. Rotation of the cylinders from 66 to 6 deg with a subcritical reactor to measure the reactivity loss (safety margin) in the clean reactor.
2. Rotation of the cylinders from 66 to 146 deg to measure the excess reactivity over clean critical. This is a measure of the reactivity available to compensate for the effects of fuel burnup and fission-product poisoning during operation.

Several techniques were used to evaluate each of these control values. For rotation from 66 to 6 deg, the following techniques were used:

- A. The single-cylinder method.
- B. The reciprocal multiplication method (1/M method).
- C. The reactivity-coefficient method.

For rotation from 66 to 146 deg, these techniques were used:

- A. The single-cylinder method.
- D. The reactivity-coefficient method.
- E. In the PTR the rotation from clean critical (60 deg) to depleted (140 deg) was compensated by removing fuel and by adding cadmium to simulate fission-product poisons.

The results of these methods are shown in the different control-cylinder calibration curves for the PPA-18 in Fig. 15.3. The probable value for the B_4C poison safety margin between 66 and 6 deg was $\$(8.5 \pm 1.5)$. The probable value for the excess reactivity between 66 and 146 deg was $\$(12.7 \pm 1.3)$.

The results of the PTR calibration are shown in Fig. 15.4. Integration of this curve indicates an excess reactivity of $\$13.0$ for control-element rotation between 66 and 140 deg.

Since B_4C was considered the most probable control poison, all rotation experiments were made with B_4C rods. The effectiveness of alternate control poison was then compared with the B_4C data. A description of each of the techniques listed above follows.

A. Single-cylinder Method

The single-cylinder method of control evaluations consists of alternate small-step rotations of two of the control cylinders in opposite directions from the standard loading critical position. With each step the reactivity change is measured on control rods calibrated by the rod-drop technique as the rotation progresses. By this method a critical reactor control curve for one cylinder is obtained over its entire range. When the PPA control cylinders were at the 6-, 66-, 126-, and 186-deg positions, it was replaced by the detailed mock-up, and the change in reactivity was measured. Appropriate corrections were applied to the safety margins at 46, 26, and 66 deg on the basis of the reactivity difference between the detailed mock-up and the PPA control cylinders at the 6-deg position.

This method does not simulate the simultaneous motion of all six cylinders which characterizes S1G operation; it therefore does not record the accompanying interaction of the control cylinders. It predicts a somewhat larger safety margin than is found in the actual case when the six cylinders are at 6 deg. Similarly, the excess reactivity indicated at 146 deg on the basis of single-cylinder data is larger than the actual value with all six cylinders. Table 15.1 is a summary of the data that were obtained from the PPA-18 single-cylinder experiment. The value for six cylinders was obtained simply as six times the corrected value for the single cylinder. A curve of the results is given in Fig. 15.3:

This method is the fastest one to use and is particularly useful in comparing different poisons at various cylinder positions.

Fig. 15.3—PPA-18. Detailed mock-up calibrations of poison control cylinders.

TABLE 15.1— B_{1C} SINGLE-CYLINDER EVALUATION OF THE SAFETY-MARGIN AND EXCESS REACTIVITY

Position, deg	One cylinder, \$	Values using detailed mock-up, \$	Six cylinders, \$
6	-1.83	-1.47	-8.82
26	-1.63	-1.31*	-7.86
46	-1.11	-0.89*	-5.34
66	0	0	0
86	+1.04		+6.24
106	+1.96		+11.76
126	+2.30		+13.80
186	+2.71		+16.26

*Calculated by ratios from the PPA cylinder measurements.

B. Reciprocal Multiplication Method

The reciprocal multiplication method was devised to allow a measurement of the reactivity change produced by rotating all six control cylinders from 66 to 6 deg without making any changes in the reactor core. This method consists in counting leakage neutrons that escape from the subcritical assembly; it is subject to large error because of the low counting rates when the control cylinders are at 6 deg. The multiplication, M , is measured by observing the counting rate of neutron chambers relative to the counting rate for source neutrons before loading the reactor. The ratio of counting rate for a given load to the counting rate for zero load is defined as the multiplication, and the reciprocal multiplication is a measure of the reactivity difference between the given load and criticality. Chapter 4 discusses these matters in more detail.

The position of the B_4C during the rotation of the control cylinders not only affects the pile reactivity or the multiplication but also affects the neutron leakage from the core. Consequently the apparent multiplication as measured by the external neutron detectors differs from the true multiplication of the reactor. An attempt was made to overcome this difficulty by calibrating incremental reciprocal multiplications at different control-cylinder positions in terms of known and relatively constant reactivity changes.

The three safety rods in the center of the reactor (Fig. 15.2) were used to introduce the known reactivity change in the reactor for this experiment. These were calibrated by a rod-drop measurement in the critical reactor before the start of the experiment. Since these rods were at the center of the core, their reactivity value should have been least affected by the control-cylinder rotations on the outside edge of the core, and it was assumed throughout the experiment that their reactivity value remained constant. These three rods were dropped out of the reactor after each change in the control-cylinder position to obtain a calibration of the apparent multiplication at that counting rate.

In order to improve the accuracy with the detailed mock-up, two experiments were necessary. In the first experiment the shape of the curve of reactivity versus control-cylinder position was found by rotating the six PPA cylinders in 20-deg steps from 66 to 6 deg. Since the early experiments had shown that the stainless steel had its greatest effect in the 6-deg position, it was important to obtain the best possible measurement relating the PPA mock-up to the detailed mock-up in this position. The precision of the experiment is a function of the counting rate and is poor when the reactor is \$10.00 below critical, which occurs when all six PPA cylinders are at 6 deg. For this reason, substitution of the detailed mock-up for that of the PPA when the control cylinders were at 6 deg would have given a poor measurement.

This difficulty was overcome in a second experiment. In this second experiment the total loss in reactivity due to the rotation of the detailed mock-up from 66 to 6 deg in one step was found. Starting with the detailed mock-up and the PPA cylinders at 66 deg, the detailed mock-up alone was rotated from 66 to 6 deg, and the counts were taken. In this step the counting rate was high enough that a large error was not inherent in the experiment. This one measurement was not sufficient, however, since the interaction effect was not yet known. The detailed mock-up was then removed, and the 2-in.-diameter boron tubes were inserted to determine whether the poison value of the detailed mock-up (as given by the counting rate) could be matched by some configuration of these boron tubes. It was found possible to match the counting rate by removing the two outside rods of the five-tube configuration. This pattern of three 2-in.-diameter B_4C tubes was then used to simulate the control cylinders at 6 deg. The experiment was continued by rotating the other five cylinders from 66 to 6 deg and using this modification at 6 deg. The reduction in the safety margin produced by this refinement was applied to the 46- and 26-deg steps, where the detailed mock-up could not be used, on the basis of the reactivity difference between this measurement and the five-tube configurations at 6 deg.

The results are given in Table 15.2 and appear on the control curve in Fig. 15.3. The estimated probable error for this measurement is $\pm \$1.00$ for the total value at 6 deg.

TABLE 15. 2-RECIPROCAL-MULTIPLICATION
EVALUATION OF THE SAFETY MARGIN

Rotation, deg	Reactivity change for six cylinders, \$	
66-46		4.79
46-26		1.65
26-6		0.71
66-6	Total	7.15

C. Reactivity-Coefficient Method (Safety Margin Measurement)

The reactivity-coefficient method likewise can measure the reactivity change produced by moving all six control cylinders from 66 to 6 deg. The rationale of this method for safety-margin tests is to increase the reactivity of the pile by adding a reactive material at the center of the core. It is necessary to assume that the neutron spectrum is not changed appreciably at the core-control-cylinder interface region; for a different neutron spectrum would undoubtedly cause a change in the competition of neutron absorptions between boron and fuel, and in neutron leakage.

Since enough extra fuel was not available, a hydrogenous material, polyethylene, was chosen to increase the reactivity by lowering the neutron-energy spectrum at the core center. The reactivity coefficient of the polyethylene was measured after each step by calibrated control rods, and the value of the control cylinders was determined from the dollar value of the polyethylene which was added to restore criticality after each rotation of the control cylinders. Twelve polyethylene disks, 2 in. in diameter and 0.1 in. thick, were distributed in the first 5 in. of each fuel rod in the three central rings when the control cylinders were at 6 deg.

Appropriate corrections were applied to the safety margins at 46, 26, and 6 deg on the basis of the reactivity difference between the detailed mock-up and the PPA control cylinder at the 6-deg position. The results are shown in Table 15.3 and

TABLE 15. 3-REACTIVITY-COEFFICIENT METHOD FOR
EVALUATING SAFETY MARGIN

Rotation for all control cylinders, deg	Effectiveness of PPA control cylinders, \$		Effectiveness of detailed mock-up (six cylinders), \$
66-46		6.81	5.47
46-26		3.47	2.79
26-6		0.73	0.59
66-6	Total	11.01	8.85

appear on the control curve shown in Fig. 15.3. The probable error assigned to the safety-margin evaluation by this method is $\pm \$1.00$ for the total value at 6 deg.

D. Reactivity-Coefficient Method (Excess-Reactivity Measurement)

This application of the method consists in using cadmium slivers in the core as a means of simulating the fission-product poisoning and burnup. These slivers were used in each of the three corners of the core interstices. Sufficient cadmium was added during each rotational step to allow all the control cylinders to be rotated out 20 deg. The reactivity coefficient of the cadmium was measured at each step by calibrated control rods. The results are given in Table 15.4 and appear on the control curve shown in Figure 15.3.

The largest source of error in this experiment was the difficulty of obtaining consistent reactivity coefficients for the cadmium slivers. The probable error as-

TABLE 15.4—REACTIVITY COEFFICIENT
EVALUATION OF EXCESS REACTIVITY

Position of control cylinders, deg	Excess reactivity, β
66	0
86	5.7
106	9.7
126	11.7
146	12.7

signed to the total excess reactivity obtained by this method is $\pm \$1.3$ for the total value at 146 deg.

E. Simulated Fission-Product Poison Fuel Removal Method

This method consists in measuring the control-element sensitivity, $dk/d\theta$, at critical in steps from clean to spent critical. These steps are established by (1) uniformly removing fuel to simulate burnup, (2) uniformly adding the appropriate amount of poison (in the form of cadmium sleeves) to simulate the fission products resulting from the fuel burnup, and then (3) adjusting the control elements in phase to restore criticality.

This measurement was made on the PTR. At each step the PTR control rods were calibrated by the rod-drop method and the cents per degree per control element was measured by moving the element a fixed amount and compensating with the calibrated control rod. The differential control-element curve, Fig. 15.4, was obtained. The total excess reactivity over clean critical, $\$14.4$, was obtained as six times the integral of this curve from 60 to 140 deg.

Fig. 15.4—PTR control-cylinder differential calibration.

15.4 COMPARISON OF ALTERNATE CONTROL MATERIALS FOR S_1G

During the latter part of the program, the control cylinder was investigated for possible improvements in nuclear properties, engineering design, and cost. This study involved the investigation of alternate control poisons, rearrangement of the beryllium, and changes of the structural supports. Comparison of a number of the poisons was made in the PPA. The PTR experiments, on the other hand, were concerned both with alternate poisons and with the rearrangement of beryllium in the cylinder as a result of engineering changes. The reactivity contribution of the beryl-

lium in the cylinder is important since in the depleted reactor the beryllium face of the cylinder is toward the core and directly affects the endurance of the reactor.

A. PPA-18 and PPA-20 Control Poison Comparison

The PPA-18 work consisted in substituting another set of 50 poison tubes into the detailed mock-up in place of the B_4C at the 6-deg position. The results are expressed in Table 15.5 as total control value for six cylinders. Since all poisons have the least effect on core reactivity at the 140-deg position of the cylinders, a change in control value is a consequence of the change in the safety margin.

In PPA-20, a critical assembly similar to PPA-18 in construction but containing ~50 kg of U^{235} and a $Be:U^{235}$ ratio near 140:1, a comparison was made on the basis of the detailed mock-up for B , B^{10} , and B_4C . Hafnium metal strips were also compared with the B_4C detailed mock-up. In addition, attempts were made to increase

TABLE 15. 5—CONTROL POISON EVALUATIONS IN PPA-18

Material	Density, g/cc	Total control for all six cylinders, %
B_4C	1.9	20.8
Samarium-gadolinium oxide mixture	3.88	17.9
Samarium-gadolinium oxide mixture	4.98	18.7
Hafnium oxide	2.38	17.1

the control possible in a cylinder by increasing the area of the poison face and by backing the poison with NaOH to obtain greater thermalization of the neutrons passing through the poison. The results are shown in Table 15.6. The data used to compile this table were taken with one cylinder at 6 deg. The single-cylinder safety margin was multiplied by six and reduced by 5 per cent to obtain the total shutdown safety margin. (This 5 per cent is the estimated reduction in the safety margin due to coupling at 6 deg.)

B. PTR Control Poison Comparison

Some detailed experiments were made by D. J. Anthony with the PTR for a modified control cylinder (see Fig. 15:5). This cylinder had

TABLE 15. 6—SAFETY MARGINS AND CLEAN-CRITICAL POSITIONS FOR SIX CONTROL CYLINDERS WITH SEVERAL POISONS IN PPA-20 USING THE DETAILED MOCK-UP

Control cylinder element	Density	Angle of cylinder face covered with poison, deg	Safety margin, %
B_4C ($\frac{3}{8}$ -in.-dia. tubes)	1.9 g/cm ³	130	7.9 ± 0.5
	1.9 g/cm ³	110	7.1 ± 0.4
	1.9 g/cm ³	100	6.9 ± 0.4
B_4C (backed by NaOH)	1.9 g/cm ³	110	7.9 ± 0.5
B^{10} ($\frac{3}{8}$ -in.-dia. tubes)	1.4 g/cm ³	110	10.6 ± 0.9
	1.4 g/cm ³	100	10.4 ± 0.9
Boron ($\frac{3}{8}$ -in.-dia. tubes)	0.6 g/cm ³	110	5.8 ± 0.4
	0.6 g/cm ³	100	5.6 ± 0.3
Boron (back row of tubes removed)	0.6 g/cm ³	110	4.5 ± 0.3
Hafnium (sheet form)	0.45-in. thick	100	5.1 ± 0.3
Hafnium (backed by NaOH)	0.45-in. thick	100	6.7 ± 0.4
Hafnium	0.20-in. thick	100	3.7 ± 0.3

Fig. 15.5—Modified control cylinder in the 140-deg position.

TABLE 15.7—POISON COMPARISON ON PTR TEST CYLINDER
0-DEG POSITION

Run	Outer row	Middle row	Inner row	Δk ,*
0	B ₄ C	B ₄ C	Aluminum	0
1	B ₄ C	Aluminum	Aluminum	+17
2	Hafnium	Hafnium	Hafnium	+26
3	Hafnium†	Hafnium†	Hafnium†	+36
4	Hafnium	Hafnium†	Hafnium†	+28
5	Hafnium	Hafnium	Aluminum	+34
6	Hafnium	Aluminum	Aluminum	+54
7	Rare earth‡	Hafnium	Rare earth‡	+35
8	Rare earth‡	B ₄ C	Rare earth‡	+22
9	Rare earth‡	Aluminum	Aluminum	+65

* (+) means less poisonous than the 0 run and therefore a reduction in safety margin.

† Hafnium rods clad with stainless steel.

‡ Rare-earth compounds (60.4 per cent samarium; 27 per cent gadolinium; 1 per cent praseodymium; 0.7 per cent neodymium; and 10 per cent other).

1. Three rows of 1/2-in.-diameter poison tubes covering an angle of 116 deg on the cylinder face; the outer row held 23 poison tubes, the middle row, 19 tubes, and the inner row, 15 tubes.
2. Two rows of beryllium tubes; the outer row contained eighteen 1.25-in.-diameter tubes, and the inner row, nine 1.75-in.-diameter tubes.

In this cylinder, sodium in the cylinder voids could be mocked up with aluminum plates cut to fit around the rods. The experimental results obtained from comparing various combinations of poisons in this cylinder are given in Table 15.7. Since these measurements were made with the test cylinder at 0 deg and the opposite cylinder moved out to compensate for the test cylinder, the results are subject to the errors introduced by distortion of the flux. The probable experimental error associated with these results due to control-element calibration, position, etc., is about $\pm 0.5\%$.

The modified cylinder was also used to measure the reactivity value of beryllium in the cylinder at 140 deg in the PTR. The distributed reactivity coefficient measured in positions R-1 through R-6 (see Fig. 15.5) along the entire length of the cylinder was $+0.048\%$ /mole.

15.5 APPENDIX: DESCRIPTION OF THE DETAILED MOCK-UP OF THE CONTROL CYLINDER

The poison section of a PPA control cylinder consisted of five 2-in.-diameter B_4C tubes. The prism shape of the PPA cylinder dictated a design of the detailed mock-up such that it could be used only at the corner prism angles of 6, 66, and 126 deg. An example of a detailed mock-up is seen in Fig. 15.6. In addition to the

Fig. 15.6—Detailed mock-up of a control cylinder at the 6-deg position. The number of rods at the left are core rods. The slugs at top and bottom lettered with R are stationary reflector rods and the ones on the right with MD indicate control-cylinder beryllium rods. The smaller cylinders are tubes containing boron carbide and rods of aluminum, magnesium, and stainless steel. The triangular-ended pieces are beryllium triflutes used in this case to mock-up a straight line edge of a beryllium region. The flat strip crossing the core slugs was in place for determining a power distribution along a traverse across the reactor. (See Chaps. 6 and 24.)

stainless-steel tubes containing the poison, some aluminum and magnesium rods were used to simulate the S1G sodium. The mock-up also included some stainless-steel and beryllium rods used to simulate the large concentration of stainless steel in the cylinder. The poison tubes were removable, and sets of 50 containing boron carbide, natural boron, B^{10} , a rare-earth mixture (oxides of samarium and gadolinium), and hafnium oxide were available for testing in the mock-up. A hafnium metal-slab mock-up was also available for use in place of the tubes. The densities of the poisons tested are listed in Table 15.6.

Reactivity Comparison of the S1G Control Cylinder with the PPA Detailed Mock-up

Although much effort was made to duplicate accurately the S1G control cylinder design, some discrepancies between the detailed mock-up and the S1G control cylinders still existed and had to be evaluated by reactivity-coefficient experiments. These discrepancies included (1) absence of sodium, (2) differences in stainless-steel content throughout the mock-up, (3) differences in the amount of sodium, stainless steel, and beryllium between control-cylinder face and core.

An analysis of the B_4C and stainless-steel regions of the S1G cylinder and the PPA detailed mock-up showed that the following differences in reactivity existed per cylinder:

1. At 6-deg position: S1G excess k over PPA mock-up = $(3 \pm 10)\%$.
2. At 66-deg position: S1G excess k over PPA mock-up = $(-2 \pm 10)\%$.
3. At 126-deg position: difference was insignificant.

Since these measurements showed such small differences, no correction was made to the control calibration curves.

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Chapter 16

BURNABLE POISON CALCULATIONS FOR INTERMEDIATE REACTORS

P. L. HOFMANN and M. L. STORM

16.1 INTRODUCTION

In power reactors designed to operate for lengthy periods without refueling, an excess amount of fuel over that required for initial criticality must be available to compensate for the reactivity loss brought about by fuel depletion and fission-product accumulation. Furthermore, a control scheme must be provided to maintain criticality at all times. The more conventional control schemes usually consist in the mechanical introduction and removal of neutron-absorbing materials (e.g., translating core control rods or control cylinders). To reduce the use of mechanical control devices to a minimum and still provide for high endurance at large powers, one alternate procedure is to build into the reactor core a suitably chosen neutron absorber (burnable poison) which compensates for the initial excess fuel investment. Ideally this absorber would be destroyed in such a manner that the reactivity loss caused by fuel depletion and fission-product accumulation is exactly compensated by the reactivity gain associated with poison depletion. Since in practice this compensation is never perfect, some mechanical control devices are needed to take account of the reactivity mismatch, as well as to provide the necessary control for power maneuvers and shutdown.

Two obvious requirements for suitable burnable poisons exist. The first requirement is that the neutron-absorption cross-section of the poison be large compared to that of fuel in order that the poison residue at the end of the core life be small. However, too large a neutron-absorption cross-section may be undesirable because the initial poison depletion and consequent associated reactivity increase might then be too rapid. A proper choice of burnable poison elements with regard to geometry and composition, and hence self-shielding, can provide a remedy for this defect. A second requirement of such a poison is that the isotope resulting from neutron capture be a weak absorber.

The present chapter describes burnable poison calculations performed for submarine intermediate reactors. A convenient measure of the suitability of a specific burnable poison is its effect on reactor endurance. Accordingly, all the calculations described here are concerned with the changes in endurance that can be brought about by the addition of burnable poisons.

The applications of burnable poisons considered here resulted from efforts to modify SIR-type reactors that were already designed. They did not attempt to design new reactors incorporating burnable poisons from the outset. Hence, when any specific scheme for the introduction of burnable poisons is evaluated, it should be remembered that there are competing means of achieving endurance gains in SIR-type reactors which do not involve the use of burnable poisons. For example, if the U^{235} loading were increased and sufficient beryllium removed to maintain the original initial safety margin, an endurance gain would result of about 4500 Mwh for each kilogram of U^{235} added. On the other hand, if the U^{235} loading were increased without the removal of beryllium, the gain would be about 7500 Mwh/kg added. This last approach could be only used if initially there were sufficient safety to permit the subsequent reduction in safety margin.

16.2 ONE-GROUP REACTOR MODEL

Two types of burnable poison modifications of SIR-type reactors have been considered. They can be categorized as compensated and uncompensated additions of burnable poisons. Compensated addition means the simultaneous addition of fuel and burnable poison in such a manner as to maintain the initial safety margin of the unpoisoned reactor. One is then interested in the endurance increase brought about by the change. In uncompensated additions burnable poison alone is added to the core to increase the initial safety margin.

The initial investigation of the addition of burnable poisons to SIR-type reactors was done by Hurwitz and Nims in 1952^{2,3} and the program was continued for three years.

A simplified one-energy-group approach is described in Secs. 16.2-16.4 and in Sec. 16.5 a more elaborate multigroup formulation of this problem is described.

16.2 ONE-GROUP REACTOR MODEL

In this section a one-group semianalytical method for calculating the effects on endurance of the addition of burnable poisons to SIR-type reactors is considered. The change in reactivity with time of the reactor under consideration is formulated in terms of available experimental data, and use is made of the one-group poison depletion model described in Report KAPL-847². The reactivity requirements allotted to peak xenon and fine control are assumed constant throughout the reactor history, and the endurance is calculated as that time when the remaining excess reactivity can no longer compensate the loss resulting from depletion, fission-product formation, and the burnable poison residue. Machine methods utilizing a multigroup formulation, to be discussed in Sec. 16.5, can take a more realistic account of the many factors involved in a burnable poison study. Nevertheless, it is helpful to utilize simple approaches to the problem in order to obtain qualitative ideas of effects and trends before embarking on a detailed program of machine calculations.

Consider the SIR-type reactor. An observable when the reactor is in operation is the setting of the control cylinders as a function of the megawatt hours of operation. In order to make endurance estimates it is necessary to correlate cylinder positions with fuel depletion.

The reactivity of the assembly is a function of its composition and configuration. During critical operation the reactivity variations brought about by changes in composition and configuration must occur in a compensating manner. The major compositional changes are the depletion of fuel, the accumulation of fission products, and the depletion of the burnable poison. The changes in configuration are caused by the motion of the control cylinders.

For small changes in the state of the assembly, the reactivity change can be expressed in terms of the first order partial derivatives of reactivity with respect to the reactor parameters.⁴ The physical interpretation of these derivatives is in terms of the so-called "reactivity coefficients" (RC's). The complexity of the problem arises from the unknown dependence of each coefficient on all the reactor variables.

The purpose of this section is to present a qualitative picture, the assumption is made that the reactivity coefficients are constant over the range of parameters under consideration. To this end, effective values of reactivity coefficients are selected. For example, in the case of fuel, values of about 40 and 70¢/kg of U²³⁵ have been measured in the clean and depleted SIR-PPA cores, respectively, and for simplicity an average value of 55¢/kg of U²³⁵ has been adopted.

Now consider in more detail a SIR-type reactor that does not utilize burnable poisons and is critical at time zero with the following characteristics: (1) control range, ΔK_C ; (2) initial safety margin, $\Delta K_S(0)$; and (3) initial excess reactivity, $\Delta K_E(0) = \Delta K_C - \Delta K_S(0)$. Let the peak xenon defect, assumed constant with time, be ΔK_{Xe} , and let the reactivity requirement for fine control be ΔK_{FC} . Then the initial potential reactivity available for shim control, $\Delta K_{SH}(0)$, to compensate for fuel burnup and fission-product formation is taken as

$$\Delta K_{SH}(0) = \Delta K_E(0) - |\Delta K_{Xe}| - |\Delta K_{FC}| \quad (16.1)$$

Let $M(t)$ be the U^{235} content in kilograms at time t . The reactivity loss, ΔK_U , due to fuel burnout will be expressed as

$$\Delta K_U(t) = -d_U[M(0) - M(t)] \quad (16.2)$$

where d_U is the effective worth of the fuel in dollars per kilogram. $M(t)$ and $M(0)$ are related by

$$M(t) = M(0) - gt \quad (16.3)$$

where g is the rate of fuel depletion in kilograms per hour or, more generally, in kg/Mwh. Thus Eq. 16.2 becomes

$$\Delta K_U(t) = -d_U gt \quad (16.4)$$

For SIR-type reactors, g can be taken as 60×10^{-6} kg/Mwh.⁵ The reactivity loss due to non-xenon fission-product formation, ΔK_{FP} , is taken as

$$\Delta K_{FP}(t) = -ft \quad (16.5)$$

where f , the rate of fission-product poisoning, is taken as 4.5¢/1000 Mwh from estimations by the method of Chap. 14. (Measurements in S1G (Chap. 20) indicate that this represents an underestimate of the fission-product poisoning.)

The endurance of the reference reactor is now taken as that time, t_f , when

$$\Delta K_{SH}(0) = (f + d_U g)t_f \quad (16.6)$$

Now consider the addition of burnable poison and fuel, or burnable poison alone, to the reactor. In the following, primed quantities will refer to the reactor with burnable poisons, whereas unprimed quantities denote the reference system.

Let $D(0)$ be the initial dollar value of poisons added. The new potential reactivity available for shim control, $\Delta K'_{SH}(0)$, is taken as

$$\Delta K'_{SH}(0) = \Delta K_{SH}(0) - D(0) + d_U[M'(0) - M(0)] \quad (16.7)$$

whereas the new initial safety margin, $\Delta K'_S(0)$, is

$$\Delta K'_S(0) = \Delta K_S(0) + D(0) - d_U[M'(0) - M(0)] \quad (16.8)$$

It is assumed in the above that the total control range and reactivity requirements for peak xenon and fine control are unchanged, so that

$$\Delta K'_{SH}(0) + \Delta K'_S(0) = \Delta K_{SH}(0) + \Delta K_S(0) \quad (16.9)$$

When fuel and burnable poison are added in a compensating fashion

$$D(0) = d_U[M'(0) - M(0)] \quad (16.10)$$

and hence

$$\Delta K'_{SH}(0) = \Delta K_{SH}(0), \Delta K'_S(0) = \Delta K_S(0) \quad (16.11)$$

16.2 ONE-GROUP REACTOR MODEL

When burnable poisons alone are added, $M'(0) = M(0)$, and

$$\begin{aligned}\Delta K'_{SH}(0) &= \Delta K_{SH}(0) - D(0) \\ \Delta K'_S(0) &= \Delta K_S(0) + D(0)\end{aligned}\quad (16.12)$$

Consider the addition of several poisons of typical initial dollar value $D_j(0)$ where

$$\sum_j D_j(0) = D(0) \quad (16.13)$$

The reactivity change due to poison burnup, ΔK_{BP} , is taken as

$$\Delta K_{BP}(t) = \sum_j D_j(0)[1 - P_j(t)] \quad (16.14)$$

where $P_j(t)$ is the fraction of the initial amount of the j th poison remaining at time t . It is assumed as before that effective constant burnable poison reactivity coefficients can be chosen.

Thus the reactivity change, $\Delta K(t)$, due to fuel burnup, fission-product formation, and poison burnup is taken as

$$\Delta K(t) = \sum_j D_j(0)[1 - P_j(t)] - (f + d_U g)t \quad (16.15)$$

When each burnable poison consists of a single absorbing isotope, P_j is given by²

$$P_j = U^{\sigma_j/\sigma_U} \gamma_j (1 - P_j) \quad (16.16)$$

where $U = M(t)/M(0)$ is the fraction of the initial fuel loading remaining at time t ; σ_j and σ_U are effective microscopic poison and total fuel absorption cross-sections; and γ_j is related to the poison self-shielding factor, f_j , by the relation

$$f_j = \frac{1}{1 + \gamma_j P_j}$$

σ_U includes the fuel self-shielding factor, which is assumed constant in deriving Eq. 16.16.

To eliminate one parameter, γ_j , in these illustrative calculations, we restrict ourselves to the case of time-independent poison self-shielding factors ($\gamma_j = 0$). Then P_j is given by expressions of the form

$$P_j = U^{\alpha_j} \quad (16.17)$$

where α_j is the ratio of the effective poison to total fuel microscopic absorption cross-section. A discussion of the effects of varying the parameter γ_j is given in references 2 and 3, where consideration is given to the broad range of shapes of reactivity variation which are obtainable by adjustment of the poison self-shielding factor.

For burnable poisons that consist of two absorbing isotopes, where the second isotope is created by neutron capture in the first, Eq. 16.17 cannot be used for the second isotope. It is shown in Sec. 16.12 that if $P_1 = U^{\alpha_1}$ for the first isotope then for the second isotope

$$P_2 = \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} (U^{\alpha_2} - U^{\alpha_1}) + U^{\alpha_2} \quad (16.18)$$

where $P_j = N_j(t)/N_j(0)$, N_j being the concentration per cubic centimeter of the j th isotope.

In the following subsections some illustrative examples of the types of calculations that can be performed with this reactor model are given.

16.3 COMPENSATED POISON ADDITION (ONE-GROUP MODEL)

A. Single Burnable Poison

As a first illustrative example, the compensated addition of burnable poisons to a SIR-type reactor is considered. If we assign a value of \$10 to $\Delta K_{SH}(0)$ (an estimate based on experiments performed in low-power mock-ups of the S1G), the endurance of the reference reactor with no burnable poison is obtained from Eq. 16.6 as that time, t_f , when

$$t_f = \frac{\Delta K_{SH}(0)}{f + d_{UG}} \quad (16.19)$$

With the compensated addition of poison and fuel (Eq. 16.11), $\Delta K'_{SH}(0) = \Delta K_{SH}(0)$ and $\Delta K'_S(0) = \Delta K_S(0)$. If, in addition, it is postulated that the unpoisoned reactor has a minimum safety margin, then it is of interest to ascertain under what conditions the reactor with burnable poisons will always possess at least this minimum safety margin. One can then examine the reactivity change, $\Delta K(t)$, caused by fuel burnup, fission-product formation, and poison burnup in order to see when $\Delta K(t)$ has a maximum. Since the control range is assumed constant in this model, the presence of a maximum in $\Delta K(t)$ would indicate a reduction in the safety margin from its initial value. Restricting oneself for the present to a single burnable poison, one obtains from Eqs. 16.15 and 16.17

$$\Delta K(t) = D_1(0)[1 - P_1(t)] - (f + d_{UG})t = D_1(0)(1 - U^{\alpha_1}) - (f + d_{UG})t \quad (16.20)$$

$$\text{or} \quad \Delta K(t) = D_1(0) \left\{ 1 - \left[1 - \frac{gt}{M'(0)} \right]^{\alpha_1} \right\} - (f + d_{UG})t$$

$$\text{Then} \quad \frac{d(\Delta K)}{dt} = \frac{gD_1(0)\alpha_1}{M'(0)} \left[1 - \frac{gt}{M'(0)} \right]^{\alpha_1-1} - (f + d_{UG}) \quad (16.21)$$

and

$$\frac{d^2(\Delta K)}{dt^2} = - \left[\frac{g}{M'(0)} \right]^2 D_1(0)\alpha_1(\alpha_1 - 1) \left[1 - \frac{gt}{M'(0)} \right]^{\alpha_1-2} \quad (16.22)$$

Examining Eq. 16.22 one sees that since

$$1 - \frac{gt}{M'(0)} = \frac{M'(t)}{M'(0)} > 0$$

then for $\alpha_1 > 1$, $d^2(\Delta K)/dt^2$ is negative. Hence for $\alpha_1 > 1$, $\Delta K(t)$ is always concave downward, and, to ensure that $\Delta K(t)$ has no maximum during the reactor lifetime, it is sufficient to impose the condition that $\frac{d(\Delta K)}{dt} \Big|_{t=0}$ be zero or negative. It is then found

from Eq. 16.21 that when

$$\frac{f + d_{UG}}{gD_1(0)\alpha_1/M'(0)} \geq 1 \quad (16.23)$$

the reactor will always have an adequate safety margin.

When fuel and burnable poison are added in a compensating fashion,

16.3 COMPENSATED POISON ADDITION (ONE-GROUP MODEL)

$D_1(0) = d_U[M'(0) - M(0)]$; and Eq. 16.23 can be written as

$$\frac{f + d_U g}{d_U g \left[\frac{M'(0) - M(0)}{M'(0)} \right] \alpha_1} \geq 1 \quad (16.24)$$

For any given compensated fuel increase, $\Delta M = M'(0) - M(0)$, a range of α 's can be found for which Eq. 16.24 is satisfied. Denote the maximum value of α for each ΔM by $\alpha_{\max}$. In Fig. 16.1, $\alpha_{\max}$ is plotted as a function of ΔM . The loading, $M(0)$, of the reference reactor was taken as about 50 kg for the calculations. It is seen from Fig. 16.1 that, for small compensated additions, poisons that burn up rapidly (large values of α) can be used without a resultant loss of safety margin. However, the larger the poison addition, the smaller the permissible range of α 's that will satisfy the condition that no maximum occur in $\Delta K(t)$.

The endurance is obtained as that time, t_f , when

$$\Delta K(t_f) + \Delta K'_{SH}(0) = 0 \quad (16.25)$$

Substitution of Eq. 16.20 in Eq. 16.25 yields

$$\left\{ 1 - \left[1 - \frac{gt_f}{M'(0)} \right]^{\alpha_1} \right\} D_1(0) + \Delta K'_{SH}(0) = (f + d_U g)t_f \quad (16.26)$$

Fig. 16.1— $\alpha_{\max}$ as a function of ΔM for compensated burnable poison additions. For specified ΔM , α must be less than $\alpha_{\max}$ to ensure an adequate safety margin at all times.

In solving Eq. 16.26 for t_f , various values of ΔM were chosen, and $D_1(0)$ was then obtained from Eq. 16.10. For each value of ΔM a range of α values was selected (Fig. 16.1), so that in each case an adequate safety margin was maintained. The endurance t_f was obtained by graphical solution of Eq. 16.26. The results are shown in Fig. 16.2, where ΔE , the endurance gain above that of the reference reactor (Eq. 16.19), is plotted as a function of ΔM for various values of α . For $\alpha = 17$, $\Delta E/\Delta M = 6900$ Mwh/kg and for

Fig. 16.2—Endurance gain as a function of increase in fuel loading for compensated burnable poison additions.

16.3 COMPENSATED POISON ADDITION (ONE-GROUP MODEL)

this large value of α the endurance return does not differ greatly from the value of 7100 Mwh/kg obtained in this model (see Sec. 16.13) for infinite α .

The endurance results obtained from this simple model do not exhibit any leveling off or decrease in $\Delta E/\Delta M$ for sufficiently large mass additions has been obtained in the more detailed multigroup calculations discussed in Secs. 16.6 - 16.7.

B. Two Absorbing Isotopes

As a second illustrative example consider the compensated addition of natural hafnium and treat only the Hf^{177} and Hf^{178} isotopes. Then $\Delta K(t)$ is obtained from Eqs. 16.15, 16.17, and 16.18 as

$$\Delta K(t) = D_1(0)[1 - P_1(t)] + D_2(0)[1 - P_2(t)] - (f + d_{UG})t \quad (16.27)$$

or

$$\Delta K(t) = D_1(0) (1 - U^{\alpha_1}) + D_2(0) \left[1 - \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} (U^{\alpha_2} - U^{\alpha_1}) - U^{\alpha_2} \right] - (f + d_{UG})t$$

where subscripts 1 and 2 refer to Hf^{177} and Hf^{178} , respectively.

Proceeding as before:

$$\begin{aligned} \frac{d(\Delta K)}{dt} = & D_1(0)\alpha_1 \frac{g}{M'(0)} U^{\alpha_1-1} + D_2(0) \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} \frac{g}{M'(0)} (\alpha_2 U^{\alpha_2-1} - \alpha_1 U^{\alpha_1-1}) \\ & + D_2(0)\alpha_2 \frac{g}{M'(0)} U^{\alpha_2-1} - (f + d_{UG}) \quad (16.28) \end{aligned}$$

since $\frac{dU}{dt} = \frac{-g}{M'(0)}$. In addition

$$\begin{aligned} \frac{d^2(\Delta K)}{dt^2} = & - \left[\frac{g}{M'(0)} \right]^2 D_2(0)\alpha_2(\alpha_2 - 1)U^{\alpha_2-2} \left[1 + \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} \right] \\ & - \left[\frac{g}{M'(0)} \right]^2 D_1(0)\alpha_1(\alpha_1 - 1)U^{\alpha_1-2} \left\{ 1 - \frac{[D_2(0)/D_1(0)][N_1(0)/N_2(0)]}{1 - (\alpha_2/\alpha_1)} \right\} \quad (16.29) \end{aligned}$$

In the following numerical example we restrict ourselves to the case of unshielded hafnium. PTR oscillator measurements (Chap. 17) for zero-thickness hafnium isotopes yielded central reactivity coefficients (RC's) of -14.2 and -3.86\$/mol for Hf^{177} and Hf^{178} and values of $\alpha_1 = 8.2$ and $\alpha_2 = 2.2$ for the distributed isotopes. Using these values, we see that $\alpha_1 > 1$, $\alpha_2 > 1$, $(\alpha_2/\alpha_1) < 1$ and

$$\frac{[D_2(0)/D_1(0)][N_1(0)/N_2(0)]}{1 - (\alpha_2/\alpha_1)} = \frac{(RC)_2/(RC)_1}{1 - (\alpha_2/\alpha_1)} < 1$$

so that $d^2(\Delta K)/dt^2$ as given by Eq. 16.29 is negative. (As discussed in Chap. 17, a constant conversion factor was used to obtain distributed reactivity coefficients from the measured central values. This is in good agreement with perturbation theory calculations.) Thus, again postulating a minimum safety margin in the unpoisoned reactor, we

obtain the following criterion that $\frac{d(\Delta K)}{dt} \Big|_{t=0}$ be zero or negative:

$$\frac{(f + d_{UG}) + [g/M'(0)] \alpha_1 D_2(0) [N_1(0)/N_2(0)]}{[g/M'(0)] [D_1(0)\alpha_1 + D_2(0)\alpha_2]} \geq 1 \quad (16.30)$$

Equation 16.30 reduces to Eq. 16.23 when $D_2(0) = 0$.

Similarly the endurance is taken as that time, t_f , when

$$D_1(0)[1 - U(t_f)^{\alpha_1}] + D_2(0) \left\{ 1 - \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} [U(t_f)^{\alpha_2} - U(t_f)^{\alpha_1}] - U(t_f)^{\alpha_2} \right\} + \Delta K'_{SH}(0) = (f + d_U g)t_f \quad (16.31)$$

For compensated additions of natural hafnium

$$D_1(0) + D_2(0) = d_U[M'(0) - M(0)] \quad (16.32)$$

and

$$\frac{D_1(0)}{D_2(0)} = \frac{N_1(0)}{N_2(0)} \frac{(RC)_1}{(RC)_2} \quad (16.33)$$

permitting the evaluation of $D_1(0)$ and $D_2(0)$.

Endurances were obtained by graphical solution of Eq. 16.31 for a large range of values of the parameters satisfying Eq. 16.30. The results are shown in Fig. 16.3 where ΔE is plotted as a function of ΔM . In each case the ratio (α_1/α_2) was fixed on the basis of multigroup calculations for unshielded hafnium, as 4.1.

Fig. 16.3—Endurance gain as a function of increase in fuel loading for compensated additions of hafnium. (α_1/α_2 fixed at value 4.1).

Fig. 16.4—Variation of P_1 and P_2 with U .

The variation of P_1 and P_2 with U is shown in Fig. 16.4 for the same range of α 's. The maximum value of P_2 is given (see Sec. 16.5) by

$$P_2^{\max} = \left[\frac{\alpha_2}{\alpha_1} + \frac{\alpha_2}{\alpha_1} \left(1 - \frac{\alpha_2}{\alpha_1} \right) \frac{N_2(0)}{N_1(0)} \right]^{1/[\alpha_1/\alpha_2 - 1]} \left[1 + \frac{N_1(0)}{N_2(0)} - \frac{\alpha_2}{\alpha_1} \right] \quad (16.34)$$

and this maximum occurs at a value of $U = U_M$ given by

$$U_M = \left[\frac{\alpha_2}{\alpha_1} + \frac{\alpha_2}{\alpha_1} \left(1 - \frac{\alpha_2}{\alpha_1} \right) \frac{N_2(0)}{N_1(0)} \right]^{1/(\alpha_1 - \alpha_2)} \quad (16.35)$$

Thus $P_2^{\max}$ depends only on the ratios α_1/α_2 and $N_1(0)/N_2(0)$; whereas the value of U at which the maximum occurs depends in addition on the difference $(\alpha_1 - \alpha_2)$.

16.4 UNCOMPENSATED POISON ADDITION (ONE-GROUP MODEL)

Consider the addition of burnable poison alone to the reference reactor. Such an addition might be contemplated if the boron in the SIR-type control cylinder were replaced by material with poorer absorption properties so that the control cylinders did not provide adequate safety margin. One is then interested in restricting the resulting loss of endurance. As described in Eq. 16.12, the effect in the model is to decrease the available shim control and increase the safety margin, so that

$$\Delta K'_{SH}(0) = \Delta K_{SH}(0) - D_1(0)$$

$$\Delta K'_S(0) = \Delta K_S(0) + D_1(0)$$

In the following illustrative calculations the reference model is again taken as a 50-kg reactor with $\Delta K_{SH}(0) = \$10$, and we will consider the uncompensated addition of \$3 and \$5 worth of poison.

First, consider the addition of a single burnable poison. One finds from Eq. 16.23 that, for $D_1(0) = \$3$, $\Delta K(t)$ will have a maximum for $\alpha_1 > 22$ and that, for $D_1(0) = \$5$, $\Delta K(t)$ will have a maximum for $\alpha_1 > 13$. The endurance calculations are performed, as was described previously, and the results are shown in Fig. 16.5 for the range of α 's where an adequate safety margin is always maintained.

For the case of natural hafnium the uncompensated addition of \$3 and \$5 worth of poison is also considered. By using the same parameters as before ($\alpha_1/\alpha_2 = 4.1$), one finds from Eq. 16.30 that, for $D(0) = \$3$, $\Delta K(t)$ has a maximum for $\alpha_1 > 38$ and that, for $D(0) = \$5$, $\Delta K(t)$ has a maximum for $\alpha_1 > 22$. The results of the endurance calculations are shown in Fig. 16.6 where the endurance loss is plotted as a function of α_1 for each of the poison additions considered.

16.5 MULTIGROUP APPROACH

The approach discussed in Sec. 16.2 has the advantage of great simplicity and is valuable in providing qualitative information regarding the effects on reactor endurance of the nature, quantity, and location of the poison elements. For example, for any given burnable poison a change in the location of the poison elements would be reflected by a change in the corresponding value of α to be used. However, a major drawback of the method lies in the difficulty of selecting effective parameters valid over the intermediate spectrum as it changes with reactor operation.

In order to cope with the disadvantages of the one-group model, the more elaborate method of performing multigroup calculations at discrete time intervals during the reactor life has been employed. In this approach a number of criticality calculations are performed at various stages of fuel burnup, and the endurance is determined as that time when the reactor can no longer override peak xenon. The details of the method to be used in this type of calculation were evolved in 1953 in discussions with

Fig. 16.5—Endurance loss as a function of α for uncompensated burnable poison additions.

G. J. Habetler, H. Hurwitz, Jr., and J. B. Nims, Jr., and the first description of the method is given in reference 6. The programming of the method for UNIVAC calculation was done by G. J. Habetler and M. A. Ketchum.

The calculations that form the basis of this method are of the multigroup type developed by Hurwitz and Ehrlich.⁷ All calculations were performed for three-media (homogeneous composition) spherical models consisting of a core, reflector, and blanket region. The composition and dimensions of the model are given in Table 16.1. When the cylindrical core is converted to an equivalent spherical model, it is assumed that a sphere with 8 per cent less core volume and the same percentage composition has the same reactivity as the cylindrical core. Control-drum rotations were represented by introducing varying uniformly distributed amounts of boron into the reflector region. To render this procedure more realistic, the boron concentration corresponding to the 0-deg drum position was chosen to give agreement with the safety margin predicted on the basis of PPA experiments.⁸ From the results of these experiments it was estimated that the reactor would be $-\$7.70$ subcritical with the drums in the 0-deg position (boron face turned all the way toward the core).

Fig. 16.6—Endurance loss as a function of α_1 for uncompensated additions of hafnium.

Hence the 0-deg position was represented by a reflector boron concentration such that the multigroup calculation for the model yields a safety margin of $-\$7.70$. The most reactive position, that is the position where the drum beryllium face is turned in toward the reactor core, was represented by a reflector that contained no boron.

The effects of fission-product buildup were treated by replacing the non-xenon fission products by boron distributed uniformly in the core at the rate of 0.8 boron atoms per fission. This follows the procedure outlined in Report KAPL-975⁹ in which the "boron yields" per fission was chosen to give agreement with the estimated reactivity value of the fission products after 60,000 Mwh of operation. (Measurements in the S1G (Chap. 20) indicate now that this represents an underestimate of the fission-product poisoning.) It is assumed that the non-xenon fission products build up linearly with the megawatt hours of operation. The mathematical treatment of the xenon buildup during reactor operation and after shutdown follows the methods described in Report KAPL-993.¹⁰

The calculations are now described in greater detail. The basic purpose of all calculations is to obtain the endurance in hours at full-power operation for various SIR-type reactors. A limitation on the endurance is the requirement that at all times sufficient control must be available to override peak xenon. The following procedure is used.

It is postulated that the reactor operates at full power over its entire life span, which is arbitrarily divided into equal time increments. (An increment of 25,000 Mwh has been used for all calculations.) The assumption is made that xenon builds up to its steady-state value instantaneously at $t = 0$, and a critical calculation is obtained at this instant by adjustment of the reflector boron concentration. From the results of this first calculation, a rate of U^{235} and poison burnout is obtained. Assuming that this rate remains unchanged over the first time interval $\Delta t_1 = t_1 - t_0$, one

can then calculate the U^{235} and burnable poison concentration at t_1 . On the assumption of a linear buildup of fission products, the amount of fission products (or rather their boron equivalent) that has accumulated during Δt_1 can be calculated. We are now in a position to perform another multigroup calculation with a new fuel and burnable poison loading and a core boron concentration characteristic of the fission-product accumulation at t_1 . Criticality is again achieved by adjustment of the reflector boron, which mocks the compensating control-drum rotation. Before proceeding any further, however, we must test whether sufficient excess reactivity is available to override the peak xenon concentration at this point. For this test all the boron is removed from the reflector (i.e., the drums are turned to the most reactive position), and the reactivity of the assembly is determined. This excess reactivity is compared with the peak xenon defect. As long as the excess reactivity is greater than the peak xenon defect, we have an indication that peak xenon can be overridden and can, therefore, proceed with the calculation for the next time interval. This procedure is repeated until the peak xenon test fails.

The concentration of reflector boron corresponding to the 0-deg drum position is so chosen that the calculated safety margin for the clean reactor is in agreement with that predicted on the basis of PPA experiments. When burnable poisons are added in a compensating manner, the first step in the calculation is to adjust the initial fuel loading to maintain the desired safety margin with the reflector boron concentration characteristic of the 0-deg drum setting. Once the initial fuel and burnable poison loadings are adjusted, the reflector boron is changed to yield a critical assembly at time zero, and the calculation proceeds as was described.

The following items of interest are obtained from this burnable poison multigroup coding:

1. Results of the initial loading iteration
2. Endurance
3. Absorption integrals at 25,000 Mwh time intervals for fuel and burnable poisons, from which one can calculate the parameter α
4. Reactor composition at 25,000 Mwh time intervals

In the next subsection, pertinent results obtained from the application of this burnable poison code are presented.

16.6 DESCRIPTION OF MULTIGROUP CALCULATIONS

As was mentioned previously, the concepts of the burnable poison coding were evolved in 1953. The first detailed description of results obtained from this approach appeared¹¹ in early 1954, and, from this date until September 1955, various applications of the multigroup approach were made. In describing these calculations, we shall adhere fairly closely to the chronological order of their appearance.

In the application of this multigroup coding to the investigation of the effects of burnable poisons, a reference endurance calculation is performed for a reactor model with no burnable poison; then endurance calculations are carried out for modified reactors that contain the specific poisons under consideration. Thus, in applying this coding to burnable poison calculations, one is assessing various poisons from the point of view of the possible endurance changes brought about by their use.

An important factor in all burnable poison calculations is the self-shielding of the poison, and hence the manner in which the poison is introduced into the core. Since one objective of the burnable poison investigations was to consider possible modifications of SIR-type reactors already designed, the question of poison location was initially resolved by choosing the modification which would result in the least design change. Changes within the fuel element were considered, but no changes in the over-all fuel element dimensions were contemplated. Thus the initial investigations considered adding burnable poison and more fuel to the originally designed fuel elements. This approach offered advantages in design and manufacturing schedules.

A. Boron

The first burnable poison multigroup calculations considered the compensated addition of boron and fuel to the fuel elements. In these calculations the fuel, burnable poison, boron equivalent fission products, and xenon were self-shielded with

the factor A usually applied in multigroup calculations for SIR-type reactors to the absorption cross-sections of fuel and associated absorbers in contact with fuel. The self-shielding factor A_i for the i th group was obtained^{9,12} from the equation

$$A_i = \frac{1}{1 + 0.4 (\Sigma_{a1}/\xi\Sigma_{s1})} \quad (16.36)$$

where Σ_{a1} is the macroscopic absorption cross-section of fuel and associated absorbers and $\xi\Sigma_{s1}$ is the macroscopic slowing-down power for the i th group. The self-shielding factor B applied to the absorption cross-section of moderator, structural material, and coolant is taken in this scheme^{9,12} as

$$B_i = \frac{1 + A_i}{2} \quad (16.37)$$

The main result of the series of boron calculations, which will be discussed in more detail later, was that the endurance is increased by approximately 3000 Mwh/kg of U^{235} added.¹

* * *

For comparison purposes the coding was then applied to the following non-burnable-poison modifications of the reactor:

1. The U^{235} loading of the fuel elements was increased and sufficient beryllium was removed to maintain the original initial safety margin.
2. The U^{235} loading was increased without the removal of beryllium, resulting in a reduced initial safety margin.

The results of these calculations¹ were that the endurance gain was 4500 Mwh/kg of U^{235} added by the first method and 7500 Mwh/kg of U^{235} added by the second method.

These non-burnable-poison modifications of SIR-type reactors provided competitive processes for achieving endurance gains and could be used as yardsticks in evaluating the merit of any proposed scheme that involved the use of burnable poisons. The non-burnable-poison modifications were also checked calculations for the multigroup coding in the sense that the values of 4500 and 7500 Mwh/kg were in reasonable agreement with estimates of endurance changes that were based on the then current values for fission-product poisoning and the reactivity coefficients of fuel and beryllium (a description of this method for analyzing endurance changes is given in Chap. 19).

B. Cadmium

The next series of calculations considered the compensated addition of cadmium and fuel to the reactor fuel elements. The self-shielding scheme employed was the same as was used in the boron calculations, and the endurance return was found¹³ to be about 4500 Mwh/kg of U^{235} added for fuel additions up to about 13 kg. At some loading increase in the vicinity of 15 kg, the endurance of the reactor began to decrease with the further compensated addition of fuel and cadmium. This effect (to be discussed in more detail in Sec. 16.9) is attributed to the decrease in the effectiveness of the reflector control system (in the spherical model) for sufficiently large additions of fuel and cadmium.

C. Hafnium

Calculations were also performed for the compensated addition of fuel and natural hafnium to the fuel elements. (Since these hafnium calculations were performed, revised group average cross sections for hafnium isotopes have been obtained.¹⁴ The use of these revised data results in better agreement with experimental reactivity coefficients than was obtainable with previously available cross-section information.) Only the Hf^{177} and the Hf^{178} isotopes were considered, and provisions

were made in the coding for the burnup of Hf^{177} and Hf^{178} as well as the formation of Hf^{178} from Hf^{177} . It is of interest to mention briefly the self-shielding factors calculated by J. B. Nims, Jr., which were applied to the hafnium isotopes. Self-shielding factors were calculated for individual resonances and various concentrations of hafnium atoms by means of the methods described by G. M. Roe.¹⁵ An average weighted self-shielding factor for each energy group in the hafnium resonance region was then found for each hafnium concentration. In obtaining the group self-shielding factor, the values for individual resonances were weighted by the contribution of the corresponding resonances to the total group absorption cross-section. The resulting values for the group average self-shielding factors as a function of hafnium concentration were then fitted with empirical expressions for use in UNIVAC calculations. The above procedure assumes that there is no interference between the resonances of hafnium and the materials in contact with it. In the energy region below the resonances, the hafnium was self-shielded with the factor A described in Eq. 16.36. The other materials in contact with hafnium in the fuel element were self-shielded with the factor A in all energy groups.

D. Effects of Self-Shielding Changes

The next series of calculations was selected to see how sensitive the results of the multigroup approach were to extreme variations of the burnable poison self-shielding factors. Accordingly calculations were performed for boron, cadmium, and hafnium when the poisons were unshielded and not in intimate contact with fuel. This is an idealized limiting case and corresponds most nearly to uniform dispersal

Fig. 16.7—Moles of unshielded burnable poison as a function of fuel increase required to maintain a constant initial safety margin.

of moderate amounts of poison in the moderator and structure. In all calculations the poisons were not self-shielded, and the remaining materials were self-shielded with the factors A and B mentioned previously. These self-shielding factors were, of course, approximate, and more exact treatment would require recourse to the detailed procedures described in references 12 and 15. However, this approach permitted determination of the effects of an extreme choice of self-shielding, and, in addition, certain results of the calculations, such as the effective α 's, could be compared with information obtained from experimental measurements for zero thickness isotopes.

We now consider in detail the results of the shielded and unshielded burnable poison multigroup calculations. In particular, the initial loading iterations, ratios of effective poison cross-section to total fuel cross-section, endurance, and burnup characteristics will be presented and compared wherever possible with available data.

16.7 INITIAL LOADING

The initial burnable poison loadings (Δp , moles) and associated fuel increases (ΔM , kg) required to maintain a constant initial safety margin are shown in Fig. 16-7 for the case of unshielded boron, cadmium, and hafnium and in Figs. 16.8 to 16.10 for the case of poison in intimate contact with fuel. (The endurance returns reported in Report KAPL-1171⁶ for the shielded hafnium calculations were afterward found to be incorrect as a result of errors discovered in the UNIVAC tapes. How-

Fig. 16.8—Variation of moles of natural boron as a function of fuel increase required to maintain a constant initial safety margin.

ever, the errors did not affect the results of the initial loading iterations, which are presented in Fig. 16.10.) The results shown in Fig. 16.7 have been replotted in Figs. 16.8 to 16.10 to facilitate comparison between the two sets of calculations. All fuel and burnable poison loadings reported are equivalent reactor quantities as obtained from the spherical model (see Table 16.1) of 8 per cent less core volume. In adjusting the fuel loading for each specified amount of burnable poison to yield an initial safety margin of \$7.70, the reflector boron concentration corresponding to the 0-deg drum position was fixed at 0.001234×10^{24} nuclei/cm³ in all calculations.

TABLE 16.1 - SPHERICAL REACTOR MODEL

	Core radius, cm	40	
	Reflector radius, cm	58	
	Blanket radius, cm	83	
Composition (nuclei/cm ³ $\times 10^{-24}$) of moderator, coolant, and structure			
Material	Core	Reflector	Blanket
Beryllium	0.0632	0.0931	
Sodium	0.00743	0.00398	0.00450
Stainless steel	0.00702	0.00380	0.0679

Fig. 16.9—Variation of moles of natural cadmium as a function of fuel increase required to maintain a constant initial safety margin.

Fig. 16.10—Variation of moles of natural hafnium as a function of fuel increase required to maintain a constant initial safety margin.

It is seen in Figs. 16.7 and 16.9 that the cadmium loading curve increases rapidly for fuel increases greater than 8 kg. Since cadmium is a thermal poison, this behavior reflects the decreasing importance of the low-energy groups in the shutdown condition for sufficiently large compensated additions of cadmium and fuel.

For moderate simultaneous additions of fuel and unshielded poison the ratios $\Delta p/\Delta M$ are in accord with the ratios $(RC)_{\text{fuel}}/(RC)_{\text{poison}}$ evaluated using calculated distributed reactivity coefficients for the clean shutdown condition.¹⁶ [These reactivity coefficients were:

- for fuel (~90 per cent U^{235}), 44.2¢/kg of U^{235} ;
- for unshielded boron, - 30.8¢/mole;
- for unshielded cadmium, - 93.2¢/mole;
- for unshielded hafnium, - 89.2¢/mole.]

This comparison is given in Table 16.2. In each case the values used in obtaining the ratio $\Delta p/\Delta M$ were taken from the first calculated points shown in Fig. 16.7 for the respective poisons.

The greatest difference occurs for the case of boron where the amount of poison added in the burnable poison calculations is quite large. It is seen that, for moderate compensated additions of fuel and burnable poisons, extrapolation of the calculated reactivity-coefficient data seems reasonable.

Comparison of $\Delta p/\Delta M$ with experiment can only be made in an approximate manner. Measurements have been made of central reactivity coefficients for un-

shielded poisons for the clean-critical condition. In addition, PTR measurements of distributed core reactivity coefficients for boron, cadmium, and hafnium¹⁷ show that the change in poison value from clean-critical to the clean-shutdown condition is small compared to experimental uncertainties of 10 to 15 per cent. Thus, if one estimates the distributed reactivity coefficients from the central unshielded values and, in addition, assumes that they are unchanged in going to the shutdown condition, comparison can then be made with the ratios listed in Table 16.2. The ratios obtained in this manner are about 40 per cent less than the ratios $(RC)_{\text{fuel}}/(RC)_{\text{poison}}$

TABLE 16.2—COMPARISON OF $(RC)_{\text{fuel}}/(RC)_{\text{poison}}$ WITH $\Delta p/\Delta M$ OBTAINED FROM THE UNSHIELDED BURNABLE POISON CALCULATIONS

Material	$(RC)_{\text{fuel}}/(RC)_{\text{poison}}$	$\Delta p/\Delta M$, moles/kg
Unshielded boron	1.44	1.80 ($\Delta p = 20.9$, $\Delta M = 11.6$)
Unshielded cadmium	0.47	0.56 ($\Delta p = 1.92$, $\Delta M = 3.43$)
Unshielded hafnium	0.50	0.52 ($\Delta p = 1.05$, $\Delta M = 2.01$)

obtained on the basis of the calculated reactivity coefficients. In addition to the approximations inherent in the above procedure, a partial cause for the discrepancy is that the fuel reactivity coefficient estimated from experiment is less than the calculated one, whereas the poison reactivity coefficients estimated from experiment are greater than the calculated reactivity coefficients. Therefore, in forming the ratio $(RC)_{\text{fuel}}/(RC)_{\text{poison}}$, the difference between calculated and experimentally derived quantities accentuates the difference between their ratios.

In Figs. 16.8 to 16.10 comparison is made of the results of the initial loading iterations obtained for unshielded poison and poison in intimate contact with fuel. Both sets of calculations are consistent in the sense that for a given fuel loading more shielded poison is required to maintain the initial safety margin than in the unshielded cases.

16.8 EFFECTIVE POISON-TO-FUEL CROSS-SECTION RATIO

A parameter of interest in the one-group treatment of poison burnup discussed in Sec. 16.3 is the ratio, α , of the effective microscopic poison absorption cross-section to the effective microscopic total fuel cross-section. The multigroup coding employed in the burnable poison calculations records the absorption integrals for fuel and burnable poison at 25,000 Mwh time intervals, thus permitting the determination of α . Values of α so obtained for B^{10} and Cd^{113} when the poisons are either unshielded or in intimate contact with fuel are plotted as a function of time in Figs. 16.11 and 16.12 for various calculations. Corresponding values for unshielded hafnium appear in Fig. 16.13. Each curve is labeled by the value of ΔM required to maintain the initial safety margin when burnable poisons are added. (The corresponding value of Δp can be obtained from Figs. 16.8 to 16.10.)

On examination of Fig. 16.11, it is seen that α_{p10} for any given initial fuel loading is fairly insensitive to time and hence drum position for both the shielded and unshielded calculations. However, the values for α_{Cd113} shown in Fig. 16.12 vary appreciably over the life of the reactor, making it more difficult to select a single constant ratio of effective cross-sections for use in a simple theory.

At time zero the calculations can be compared with the ratios of effective zero-thickness poison cross-section to total fuel cross-section as obtained from experimental reactivity coefficient data. Values of 3.58 and 22.5 were obtained for B^{10} and Cd^{113} and values of 8.2 and 2.2 were obtained (chap. 17) for Hf^{177} and Hf^{178} , respectively. Extrapolation to $\Delta M = 0$ of the zero-time ratios given in Figs. 16.11 to

Fig. 16.11—Variation of the ratio (α_{B-10}) of the effective B^{10} and total fuel absorption cross-sections.

16.13 for the unshielded calculations yields values of 4.25 and 21 for B^{10} and Cd^{113} and 7.80 and 1.85 for Hf^{177} and Hf^{178} . These values are in good agreement with the experimental information.

16.9 ENDURANCE GAINS

The endurance gain in Mwh, ΔE , the initial number of kilograms of fuel added, ΔM , and the quotient $\Delta E / \Delta M$ obtained from the multigroup calculations for unshielded poison and poison in intimate contact with fuel are summarized in Table 16.3.

The first item of interest in Table 16.3 is the marked drop in endurance of both the shielded and unshielded cadmium cases for compensated fuel increases greater

Fig. 16.12—Variation of the ratio (α_{Cd-113}) of the effective Cd^{113} and total fuel absorption cross-sections.

Fig. 16.13—Variations of the quantities α_{177} , α_{178} and $\frac{\alpha_{177}}{\alpha_{178}}$ for unshielded hafnium.

than about 14 kg. In some of these calculations as noted in Table 16.3, the reactor did not go critical at time zero in the presence of steady-state xenon with no boron in the reflector. This effect can be explained as follows.

Although fuel and cadmium are added in a compensating manner, the compensation is carried out in the shutdown condition to maintain a constant initial safety margin. According to spherical model calculations,¹⁸ the distributed reactivity coefficient of unshielded cadmium for the clean-critical condition is 23 per cent greater than the corresponding reactivity coefficient for the clean-shutdown condition. This fact results in decreased reflector control in the spherical model, and, for sufficiently large compensated additions of cadmium and fuel, the decrease in reflector control is sufficient to prevent the reactor from going critical in the presence of steady-state xenon.

The endurance gain, ΔE , is plotted as a function of ΔM for the unshielded calculations in Fig. 16.14, and comparison between the two sets of calculations is made in

TABLE 16.3 - ENDURANCE RESULTS

Burnable poison	Unshielded poison			Poison in intimate contact with fuel		
	Fuel increase, ΔM , kg	Endurance gain ΔE , Mwh	$\Delta E/\Delta M$, Mwh/kg	Fuel increase, ΔM , kg	Endurance gain ΔE , Mwh	$\Delta E/\Delta M$, Mwh/kg
Natural boron	11.6	39,000	3,300	9.90	31,000	3,200
	22.4	63,000	2,800	19.7	57,000	2,900
	29.2	70,000	2,400	25.3	73,000	2,900
Natural cadmium	3.43	24,000	6,900	6.27	36,000	5,700
	5.94	42,000	7,100	13.7	64,000	4,700
	9.03	65,000	7,200	18.9	Negative*	
	13.6	100,000	7,400			
	17.7	Negative†				
	18.7	Negative‡				
	19.7	Negative‡				
Natural hafnium	2.01	6,900	3,400			
	10.4	34,000	3,300			
	19.3	58,000	3,000			

* The endurance of this reactor was about 25,000 Mwh

† The endurance of this reactor was less than 25,000 Mwh

‡ The reactor did not go critical at time zero in the presence of steady-state xenon with no boron in the reflector.

Figs. 16.15 and 16.16. The sharp decrease at large ΔM in the ΔE curve for cadmium is not shown in Figs. 16.14 and 16.16.

The addition of poison in intimate contact with fuel which is self-shielded in the same manner as the fuel and the idealized case of the addition of unshielded poison are similar from the viewpoint of the one-group model of poison burnup discussed in Sec. 16.3. In each case the fraction of the initial amount of poison remaining in the reactor, P , is related to the fraction of the initial amount of fuel remaining, U , by the relation $P = U^\alpha$, where α is the ratio of effective microscopic poison cross-section to effective microscopic total fuel cross-section. For boron and cadmium in intimate contact with fuel, the poison and fuel cross-sections are self-shielded with the same factor; whereas in the unshielded calculations only the fuel cross-section has been self-shielded. In both cases the self-shielding parameter, γ , discussed in reference 2, is zero.

In view of this similarity it is of interest to examine in more detail the results of the multigroup endurance calculations and to compare them with the predictions of the one-group model discussed in Sec. 16.3.

It is seen from Fig. 16.14 and Table 16.3 that the endurance gains obtained from the shielded and unshielded boron calculations are quite similar until fuel loadings are reached for which saturation effects in the unshielded calculations become appreciable. The values of α for the shielded boron calculations are about 20 per cent less than the corresponding values of α (see Fig. 16.11) for the unshielded calculations. These values of α are qualitatively consistent with the fact that the endurance returns from the unshielded calculations are somewhat greater than those of the shielded calculations for moderate fuel additions where saturation effects have not appeared in the unshielded calculations. Finally, referring to Fig. 16.2 one sees that for $\alpha = 4$, by use of the simple one-group theory, a value of $\Delta E/\Delta M = 3700$ Mwh/kg is predicted and that for $\alpha = 3$ a value of $\Delta E/\Delta M = 2800$ Mwh/kg is predicted. These results are in reasonable agreement with the multigroup calculations for a corresponding range of α 's.

Referring to Fig. 16.16 and Table 16.3 and restricting our attention to moderate fuel additions, where decrease in reflector control is not apparent, we see that the unshielded cadmium calculations yield larger endurance returns than the shielded cases. This result is in qualitative agreement with the relative magnitudes of the values of α for corresponding calculations (see Fig. 16.12). Owing to the large variation in the values of α for the cadmium calculations, it is more difficult to select effective values

Fig. 16.14—Variation of endurance increase as a function of fuel increase when unshielded boron, cadmium, and hafnium are used as burnable poisons.

for comparison with the results of the one-group approach. However, it is seen that, in the unshielded calculations where the values of α range from about 14 to 32, the endurance returns are in reasonable agreement with the value of $\Delta E/\Delta M = 7000$ Mwh/kg obtained in the one-group model for infinite α (Sec. 16.5). In addition, for comparison with the shielded calculations, we find in Fig. 16.2 that as α ranges from 6 to 9, $\Delta E/\Delta M$ varies from 5000 to 6000 Mwh/kg, in general agreement with the multigroup results. Finally, in regard to the unshielded cadmium calculations, it should be mentioned that the rapid burnup of poison associated with the large values of α resulted in the occurrence of maximums in the excess reactivity as calculated at 25,000 Mwh time intervals. This peaking is probably associated with a corresponding reduction in the safety margin. The effect did not occur in the shielded cadmium calculations or in any of the boron or hafnium cases.

Finally, let us consider the results of the unshielded hafnium calculations shown in Fig. 16.14 and Table 16.3. By choosing $\alpha_{177} = 7$ and $\alpha_{177}/\alpha_{178} = 4.1$ (see Fig. 16.13), one finds that the corresponding value of $\Delta E/\Delta M = 3200$ Mwh/kg, obtained from the one-group model (see Fig. 16.3), is in agreement with the multigroup results. One point that should be mentioned in connection with the unshielded hafnium calculations is that the self-shielding factor for a resonance absorber is less than 1 even for a homogeneous medium.¹⁵ Thus the self-shielding factors of unity, applied to Hf¹⁷⁷ and

Fig. 16.15—Endurance increase as a function of fuel increase when boron is used as a burnable poison.

Hf^{178} in the cases considered, are greater than what would occur if the hafnium were mixed uniformly with the moderator. The departure of these factors from unity can be calculated by using the methods and graphs presented in reference 15. For the three hafnium loadings shown in Fig. 16.7, the self-shielding factors of unity applied to all energy groups are high by at most 10 per cent for Hf^{177} . Although the self-shielding factor for Hf^{178} is about 0.93 for the first calculation, it decreases to about 0.7 and 0.55 for the last two cases. Thus, in the last two hafnium calculations, the poisoning effect of the Hf^{178} residue is overemphasized.

16.10 POISON AND FUEL BURNUPS

The fraction of the initial amount of burnable poison remaining in the reactor at the end of the reactor lifetime, P_p , and the fraction of the initial fuel loading remaining at these times, U_p , are summarized in Table 16.4. (Since multigroup calculations are performed at 25,000 Mwh time intervals, P_p and U_p are obtained by interpolation of the given data.) In addition, the value α_p that will relate P_p and U_p by the expression $P_p = U_p^{\alpha_p}$ is listed for each case. Each multigroup calculation is identified by the corresponding value of ΔM obtained from the initial loading iterations.

No value of α_p is listed for Hf^{178} where the relation between P and U is a more complicated function of U than U^α . If the finite time intervals used in the multigroup burnup

Fig. 16.16—Endurance increase as a function of fuel increase when cadmium is used as a burnable poison.

calculations are considered, the values of α_p listed in Table 16.4 are in reasonable agreement with corresponding values (shown in Fig. 16.11 to 16.13) obtained from the multigroup absorption integrals.

In all cases the value of P_p for Hf^{178} is greater than 1. It is of interest to compare these results with those shown in Fig. 16.4, as obtained from the one-group model. Choosing $\alpha_1 = 7$, one finds that, for $U = 0.86$, $P_1 = 0.35$ and $P_2 = 1.155$. This is in fair agreement with the results listed in Table 16.4.

16.11 COMMENTS

The previous sections have been concerned mainly with the detailed discussion of two calculational schemes (i.e., one-group and multigroup) for determining the effects on endurance of burnable poison additions to SIR-type reactors. In this section we comment briefly on several broad areas that merit consideration in the application of burnable poisons to any reactor. In particular, three areas may be considered: (1) non-uniform distribution of burnable poisons, (2) choice of suitable burnable poison elements, and (3) effect of burnable poisons on reactor temperature coefficients.

Nonuniform poison distributions offer the possibilities of placing the poisons in a region of high α , thus increasing the endurance return. Although such poison distributions have not been discussed explicitly in the previous sections, the one-group method

TABLE 16.4 - BURNUP RESULTS

Burnable poison	Unshielded poison				Poison in intimate contact with fuel			
	ΔM , kg	P_f	U_f	α_f^*	ΔM , kg	P_f	U_f	α_f^*
B^{10}	11.6	0.500	0.857	4.5	9.90	0.581	0.857	3.5
	22.4	0.535	0.858	4.1	19.7	0.598	0.854	3.3
	29.2	0.565	0.863	3.9	25.3	0.610	0.853	3.1
Cd^{113}	3.43	0.0010	0.853	43	6.27	0.053	0.847	18
	5.94	0.0011	0.840	39	13.7	0.124	0.837	12
	9.03	0.0028	0.825	32				
	13.6	0.0028	0.804	27				
Hf^{177} Hf^{178}	2.01	0.300 1.188	0.869	$\alpha_{177} = 8.6$				
Hf^{177} Hf^{178}	10.4	0.310 1.180	0.859	$\alpha_{177} = 7.7$				
Hf^{177} Hf^{178}	19.3	0.338 1.179	0.856	$\alpha_{177} = 7.0$				

* α_f equals $\ln P_f / \ln U_f$.

can be used through choice of a suitable α characteristic of the specified nonuniform distribution. However, the multigroup coding described in Sec. 16.5 was limited to a single homogeneous spherical core region. Since these multigroup calculations were performed, a new machine code has been developed by G. J. Habetler and M. Ketchum which permits the division of the core into concentric regions, each possessing a different (homogeneous) composition. The new coding has the additional advantage of permitting calculations to be performed for one-dimensional cylinders and slabs as well as for spheres. It also makes it possible to investigate problems in power distribution associated with the poison localization.

Certainly one of the questions to be answered in the application of burnable poisons to any reactor is the choice of a suitable burnable poison specimen. This choice affects not only the metallurgical area, but it affects also the physics of the situation by way of the associated self-shielding considerations. In most practical situations metallurgical considerations are paramount, and the physics consequences must be accepted.

The effect of the use of burnable poisons on the temperature coefficients of the reactor is important in control and safeguard considerations. In this respect two effects must be given careful consideration. If the burnable poison is a resonance absorber in intimate contact with fuel, a significant negative Doppler temperature coefficient may result. If on the other hand a poison like cadmium with a single low-lying broad resonance is used, the thermal base effects predominate. If the neutron temperature is sufficiently high, the possibility of positive thermal base effects should be recognized.

16.12 APPENDIX: BURNUP EXPRESSIONS FOR TWO ISOTOPES

In the following discussion we assume a monoenergetic neutron spectrum. All cross-sections are effective microscopic cross-sections which are assumed to be corrected by constant self-shielding factors. The final results can be expressed in terms of ratios, α , of the effective poison to total fuel microscopic absorption cross-sections. α can be obtained from experiment or from multigroup calculations as

$$\frac{\iint \phi \sigma_j dE dV}{\iint \phi \sigma_U dE dV}$$

where j denotes the j th isotope.

The burnup of each isotope is described by the expressions

$$\frac{dN_1}{dt} = -N_1\phi\sigma_1 \quad (16.38)$$

$$\frac{dN_2}{dt} = N_1\phi\sigma_1 - N_2\phi\sigma_2 \quad (16.39)$$

where N is the concentration in atoms per cubic centimeter.

The burnup of fuel is described by the relation

$$\frac{dN_U}{dt} = -\phi N_U \sigma_U \quad (16.40)$$

and, if the reactor is operating at a constant power level,

$$\phi N_U \sigma_f = F \text{ (fissions/cm}^3 \text{ sec)} \quad (16.41)$$

Then, using Eqs. 16.40 and 16.41,

$$\frac{dN_U}{dt} = -\phi N_U \sigma_U = -F \frac{\sigma_U}{\sigma_f} \quad (16.42)$$

or

$$N_U(t) = N_U(0) - F \frac{\sigma_U}{\sigma_f} t$$

and

$$U = \frac{N_U(t)}{N_U(0)} = 1 - ht \quad (16.43)$$

where

$$h = \frac{F (\sigma_U/\sigma_f)}{N_U(0)}$$

One obtains, from Eqs. 16.38, 16.41, 16.42, and 16.43,

$$\frac{1}{N_1} \frac{dN_1}{dt} = -\phi\sigma_1 = -\frac{\sigma_1 F}{N_U \sigma_f} = -\frac{\sigma_1}{\sigma_U} \frac{F(\sigma_U/\sigma_f)}{N_U} = \frac{\sigma_1}{\sigma_U} \frac{1}{N_U} \frac{dN_U}{dt} \quad (16.44)$$

or

$$P_1 = \frac{N_1(t)}{N_1(0)} = U^{\sigma_1/\sigma_U} = U^{\alpha_1}$$

where

$$\alpha_j \equiv \frac{\sigma_j}{\sigma_U}$$

Equation 16.39 can be written as

$$\begin{aligned}\frac{dN_2}{dt} &= \frac{N_1\sigma_1 F}{N_U\sigma_f} - \frac{N_2\sigma_2 F}{N_U\sigma_f} = \frac{\sigma_1}{\sigma_U} \frac{N_1}{N_U} F \frac{\sigma_U}{\sigma_f} - \frac{\sigma_2}{\sigma_U} \frac{F(\sigma_U/\sigma_f)}{N_U} N_2 \\ &= \alpha_1 N_1(0) \frac{N_1(t)/N_1(0)}{N_U(t)/N_U(0)} \frac{F(\sigma_U/\sigma_f)}{N_U(0)} - \alpha_2 \frac{F(\sigma_U/\sigma_f)}{N_U/N_U(0)} N_2(t)\end{aligned}\quad (16.45)$$

or

$$\frac{dN_2}{dt} + \frac{\alpha_2 h}{U(t)} N_2(t) = \alpha_1 N_1(0) \frac{P_1(t)}{U(t)} h$$

Substitution of Eq. 16.43 and 16.44 in 16.45 yields

$$\frac{dN_2(t)}{dt} + \frac{\alpha_2 h}{1 - ht} N_2(t) = \alpha_1 N_1(0) h (1 - ht)^{\alpha_1 - 1} \quad (16.46)$$

Equation 16.46 is readily integrated by rewriting it as

$$\frac{d}{dt} [(1 - ht)^{-\alpha_2} N_2(t)] = \alpha_1 N_1(0) h (1 - ht)^{\alpha_1 - 1 - \alpha_2} \quad (16.47)$$

Integrating Eq. 16.47 and using the notation $N_2(t)|_{t=0} = N_2(0)$ yields

$$N_2(t) = N_2(0)(1 - ht)^{\alpha_2} + \frac{N_1(0)}{1 - (\alpha_2/\alpha_1)} [(1 - ht)^{\alpha_2} - (1 - ht)^{\alpha_1}] \quad (16.48)$$

or

$$P_2(t) = \frac{N_2(t)}{N_2(0)} = U^{\alpha_2} + \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} (U^{\alpha_2} - U^{\alpha_1}) \quad (16.49)$$

The maximum value of P_2 occurs when

$$\begin{aligned}\frac{dP_2}{dU} &= \alpha_2 U^{\alpha_2 - 1} + \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} (\alpha_2 U^{\alpha_2 - 1} - \alpha_1 U^{\alpha_1 - 1}) \\ &= U^{\alpha_2 - 1} \left[\alpha_2 + \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} (\alpha_2 - \alpha_1 U^{\alpha_1 - \alpha_2}) \right] = 0\end{aligned}$$

or when

$$U = \left[\frac{\alpha_2}{\alpha_1} + \frac{\alpha_2}{\alpha_1} \left(1 - \frac{\alpha_2}{\alpha_1} \right) \frac{N_2(0)}{N_1(0)} \right]^{1/(\alpha_1 - \alpha_2)} = U_M \quad (16.50)$$

The maximum value of P_2 is then given by

$$\begin{aligned}
P_2^{\max} &= U_M^{\alpha_2} \left[1 + \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} (1 - U_M^{\alpha_1 - \alpha_2}) \right] \\
&= U_M^{\alpha_2} \left\{ 1 + \frac{N_1(0)/N_2(0)}{1 - (\alpha_2/\alpha_1)} \left[1 - \frac{\alpha_2}{\alpha_1} - \frac{\alpha_2}{\alpha_1} \left(1 - \frac{\alpha_2}{\alpha_1} \right) \frac{N_2(0)}{N_1(0)} \right] \right\} \\
&= U_M^{\alpha_2} \left\{ 1 + \frac{N_1(0)}{N_2(0)} \left[1 - \frac{\alpha_2}{\alpha_1} \frac{N_2(0)}{N_1(0)} \right] \right\}
\end{aligned}$$

or

$$P_2^{\max} = \left[\frac{\alpha_2}{\alpha_1} + \frac{\alpha_2}{\alpha_1} \left(1 - \frac{\alpha_2}{\alpha_1} \right) \frac{N_2(0)}{N_1(0)} \right]^{\frac{1}{(\alpha_1/\alpha_2) - 1}} \left[1 + \frac{N_1(0)}{N_2(0)} - \frac{\alpha_2}{\alpha_1} \right] \quad (16.51)$$

16.13 APPENDIX: INCREASE IN FUEL LOAD FOR COMPLETE POISON BURNUP

The endurance of the reference reactor (no burnable poisons) is obtained from Eq. 16.10 as that time, t_f^0 , when

$$(f + d_U g) t_f^0 = \Delta K_{SH}(0) \quad (16.52)$$

For the compensated addition of burnable poison, $\Delta K_{SH}'(0) = \Delta K_{SH}(0)$. Since the poison burns out completely, the endurance is that time, t_f' , when

$$(f + d_U g) t_f' = \Delta K_{SH}(0) + d_U \Delta M \quad (16.53)$$

By subtracting Eq. 16.52 from Eq. 16.53

$$(f + d_U g)(t_f' - t_f^0) = (f + d_U g)\Delta E = d_U \Delta M$$

Hence, using the numerical values given in Sec. 16.2, one obtains

$$\frac{\Delta E}{\Delta M} = \frac{d_U}{f + d_U g} = \frac{55 \text{ ¢/kg}}{(4.5 \times 10^{-3} + 55 \times 60 \times 10^{-6}) \text{ ¢/Mwh}} = 7000 \text{ Mwh/kg}$$

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Chapter 17

CRITICAL-ASSEMBLY INVESTIGATION OF BURNABLE POISONS

D. J. ANTHONY and R. T. FROST

17.1 INTRODUCTION

Burnable poisons can be used to reduce the range of mechanical control required for a proposed reactor design. They can also be employed in the modification of existing designs by permitting increased loadings and thus gaining increased endurance without changing the mechanical control features of the reactor.

The ideal burnable poison is one which burns out at such a rate that the resulting gain in reactivity is equal to the loss in reactivity associated with fuel depletion and the buildup of fission-product poisons and which leaves no burnable poison residue at the end of core life. If such a poison can be found, then the increase in endurance is a function only of the initial fuel increment. In practice, however, a perfect reactivity match is not attainable. Furthermore, the gain in endurance may be limited by the amount of burnable poison residue. This residue depends upon the absorption cross-section of the burnable poison material, its geometrical self-shielding, the amount of poison used, and the fuel burnup. Metallurgical feasibility and suitability, as well as the effect on the over-all reactor temperature coefficient, must also be considered in the selection of a practical burnable poison.

In this chapter the analytical and experimental techniques that were used in the investigation of burnable poisons in the PTR critical assembly will be discussed from the point of view of modifying an existing reactor design.

17.2 CRITERIA FOR THE SELECTION OF A BURNABLE POISON

The following criteria can be established to determine the relative merits of various isotopes as burnable poisons in an intermediate-energy neutron spectrum:

1. A high isotopic absorption cross-section is required. The question of whether this cross-section is too high does not usually arise in practice since the effective poison cross-section can be modified by suitable adjustment of the self-shielding parameter. Self-shielding provides a degree of freedom which makes possible favorable burnup characteristics. The relative values of averaged isotopic cross-sections can be obtained by examining the zero-thickness reactivity coefficients, for the reactivity coefficient is proportional to the ratio of absorptions in poison to the fission rate in the reactor.¹
2. If an isotope of mass number A is to be considered as a burnable poison, the isotopic cross-section must be very large compared to that for the isotope of mass number $A + 1$, unless the half-life of the $A + 1$ isotope is short enough to prevent its building up appreciably.
3. An important criterion in selecting a burnable poison for a reactor is its effect on the over-all reactor temperature coefficient. In this regard both thermal base and Doppler broadening effects must be assessed. The possibility of tolerating positive thermal base effects depends on the specific reactor under consideration.

4. It might be desirable that the Doppler effect due to absorption resonances be as large as possible, so that some prompt negative temperature effects could be obtained by placing the isotope in intimate thermal contact with the fuel. Since, however, this effect would diminish with poison burnup, the comfort achieved by adding the poison to the clean reactor might be but a temporary one. Such an expedient might only serve to conceal a latent over-all prompt positive temperature coefficient in the reactor, which would remain undiscovered until the reactor had been in operation for some time. Hence this procedure might be unwise except in a reactor whose temperature coefficient was already thoroughly understood in the absence of the burnable poison.
5. The incorporation of the burnable poison must be feasible metallurgically. Any suitable burnable poison will have a very high cross-section, so that even when it is appreciably self-shielded the specimen will be physically thin. This fact offers relatively great freedom in finding a suitable location for the poison.

17.3 EXPERIMENTAL MEASUREMENTS

A. Pile Oscillator Measurements of Reactivity Coefficients

Since hafnium was a neutron-absorbing material extensively investigated in the PTR burnable poison program, a discussion of several measurements on hafnium will serve to illustrate the experimental techniques that can be used to evaluate the relative merits of various isotopes as possible burnable poisons in accordance with some of the previously described criteria.

The reactivity-coefficient values for unshielded Hf^{176} , Hf^{177} , Hf^{178} , Hf^{179} , and Hf^{180} were calculated from the results of pile oscillator measurements (details of the pneumatic type of pile oscillator technique are described in Chap. 9) of isotopically enriched samples of hafnium obtained from the Oak Ridge National Laboratory. In order to determine unshielded reactivity coefficients, self-shielding measurements were made with natural hafnium as well as with the enriched samples of Hf^{177} and Hf^{178} , the high absorption cross-section isotopes; the capsulated oscillator samples were diluted with aluminum powder and the experimental results were corrected for the reactivity effect introduced by the aluminum content.

It should be emphasized that only milligram quantities of these samples were measured with the pile oscillator. Gram quantities would have been necessary for conventional reactivity-coefficient measurements in which the reactivity change resulting from the introduction of the sample or samples must be compensated by displacement of the calibrated PTR control rods (as in Chap. 7). Since the supply of many enriched isotopes that might be suitable for use as burnable poisons is still somewhat limited, the necessity of using the pile oscillator technique becomes obvious when one is searching for a suitable burnable poison.

The self-shielding curves for natural hafnium and the enriched Hf^{177} and Hf^{178} samples, measured in the center of the clean PTR core, are shown in Figs. 17.1 and 17.2.

Fig. 17.1—Central PTR oscillator measurements on natural hafnium.

Fig. 17.2—Central PTR oscillator measurements on enriched hafnium samples.

In order to establish the accuracy of the pile oscillator method, the calculated central unshielded reactivity coefficients for the hafnium isotopes, listed in Table 17.1, were used to estimate the reactivity coefficient for natural hafnium. The results indicated a central unshielded value of -410 ¢/mol, which is in good agreement with the oscillator-measured value of -393 ¢/mol of natural hafnium.

Because of the lack of sufficient hafnium isotope cross-section data, it was necessary to utilize experimental methods to determine accurately the parameter σ_p/σ_t , the ratio of unshielded poison cross-section to total fuel cross-section. This ratio can be used to calculate the reactivity effect of the burnout of Hf^{177} and the consequent formation of Hf^{178} , as was done by the method of Sec. 16.2.

The distributed σ_p/σ_t ratios, listed in Table 17.1, were calculated from distributed reactivity coefficients according to the method of Hurwitz, described in Report KAPL-98.¹ A factor of 0.24, which is the ratio of the measured unshielded distributed reactivity coefficient of natural hafnium (-95 ¢/mol) to the measured central value of -393 ¢/mol, was used to convert the central reactivity coefficients to distributed values. The fact that the ratio σ_p/σ_t for Hf^{178} is appreciable makes natural hafnium less desirable as a burnable poison, particularly if the amounts of burnable poisons to be used are fairly large.

TABLE 17.1--CENTRAL REACTIVITY COEFFICIENTS OF UNSHIELDED HAFNIUM ISOTOPES

Isotope	Abundance, %	Central RC, ¢/mol	RC contribution to natural Hf, ¢/mol	σ_p/σ_t , distributed values
Hf^{176}	5.2	-197	-10	1.1
Hf^{177}	18.5	-1420	-263	8.2
Hf^{178}	27.1	-386	-105	2.2
Hf^{179}	13.8	-152	-21	0.86
Hf^{180}	35.2	-32	-11	0.18
			Total	
			-410	
Natural hafnium		-393		2.2

B. Measurement of Doppler Coefficient by Oscillatory Heating

Oscillatory heating measurements of the Doppler effect for natural hafnium in the central region of the PTR showed a small increase of poisoning with increasing tem-

perature. The value for $T(dk/dT)$, given in Table 33.1 of Chap. 33, shows that a mole of centrally located hafnium (which, at the thickness used in the experiments, should give approximately \$1 of poisoning) would increase its poisoning value by only 2.5¢ if its absolute temperature were increased by a factor of e . Since reasonable poison loadings for a SIR-type of reactor would have to be of the order of several dollars of reactivity, the associated reactivity change would be of the order of 10¢ for an e -fold increase in temperature. This small value of the Doppler coefficient for hafnium is the result of the low energy of the resonances in which the principal absorption occurs.

C. Other Measurements

Most of the experimental physics work was concerned with the evaluation of criteria 1, 2, and 4 given in Sec. 17.2. Several experiments were performed to determine whether the reactivity effect of large amounts of poison loaded into the central core region (hafnium, cadmium, and boron samples were used) varied as the control cylinder boron was moved in from the clean-critical position (66 deg) to the shutdown position (0 deg). Inverse-multiplication measurements indicated that the boron and cadmium were about 30 per cent more poisonous, while the hafnium was only 15 per cent more poisonous when the control cylinders were rotated to the 0-deg position. The increase in value of the centrally located poisons is apparently due to flux peaking at the center of the core which results from the location of control cylinder B_4C at the core-reflector interface in the shutdown condition.

The same experiments were repeated with the three poisons uniformly distributed throughout the core; no appreciable change in poison value with control cylinder position was seen. More extensive experimental investigations would be required to evaluate fully the use of burnable poisons in SIR-type reactors. In particular, it would be necessary to

1. Determine the contribution of the poison to the over-all reactor temperature coefficient,
2. Investigate possible coupling effects that might exist between neighboring fuel elements containing the burnable poison or between burnable poison elements and reactor control element poison,
3. Evaluate self-shielding effects arising from design requirements.

17.4 ENDURANCE CALCULATIONS

In order to illustrate the relative merits of B^{10} , Cd^{113} , Sm^{149} , and natural hafnium (assumed to consist of Hf^{177} and Hf^{178} only) when used as burnable poisons to extend reactor endurance, consider the following example. Suppose that \$3.0 of unshielded burnable poison is to be uniformly distributed throughout the core of a SIR-type reactor. What additional hours of endurance can be obtained by adding additional fuel to offset the loss in reactivity that this poison produces, for each of the poisons listed above? Since the fuel has a reactivity coefficient of 44 ¢/kg of U^{235} in the clean reactor, the amount of fuel added is 6.82 kg of U^{235} .

Experimentally, 1 kg of U^{235} is worth 78¢ in the depleted reactor (Chap. 19), and fission products (aside from xenon and samarium) plus fuel depletion build up a reactivity defect at the rate of 9.3¢/1000 Mwh (Sec. 20.12). Hence each kilogram of U^{235} added to the fuel can compensate in reactivity for $78¢/(9.3 \times 10^{-3} \text{ ¢/Mwh}) = 8400 \text{ Mwh}$ of additional operation.

If, therefore, the poison burned out completely before the termination of the life cycle of the reactor, then the resulting gain in endurance would be equivalent to 8400 Mwh/kg of U^{235} added.

However, when burnout is incomplete, the gain in endurance, T , is equal to

$$\frac{Y - XP}{F}$$

where Y = total worth of fuel added ($78¢/\text{kg of } U^{235} \times 6.82 \text{ kg}$)

X = total worth of poison added (300¢)

P = relative amount of poison in the core, at end of core life

F = reactivity defect rate ($9.3 \times 10^{-3} \text{ ¢/Mwh}$)

The second term in the numerator represents a reactivity loss due to the poison residue and can be calculated by the method of Hurwitz and Nims (Sec. 16.2) which describes burnup of the poison in terms of geometrical self-shielding, the ratio of σ_p/σ_t , and the relative amount of fuel burnup. When self-shielding is neglected, the equation for the burnup of a burnable poison consisting of a single isotope A reduces to Eq. 16.17.

$$P_A = U^{\sigma_A/\sigma_t}$$

where P_A = relative amount of poison remaining in the reactor

U = relative amount of fuel remaining

σ_A = effective cross-section of isotope A

σ_t = total absorption cross-section for fuel

The formation and burnup of isotope A + 1 (e.g., Hf^{178} resulting from the burnup of Hf^{177}) is represented, when self-shielding is neglected, by Eq. 16.18:

$$P_{A+1} = -\frac{\rho\sigma_A}{\sigma_A - \sigma_{A+1}} U^{\sigma_A/\sigma_t} + \left[1 + \frac{\rho\sigma_A}{\sigma_A - \sigma_{A+1}} \right] U^{(\sigma_{A+1}/\sigma_t)}$$

where ρ is the ratio of the initial amounts in the reactor of isotopes A and A + 1, respectively.

The calculations of the endurance gained per kilogram of U^{235} for the various poisons which are summarized in Table 17.2 are based on reactivity-coefficient experiments in the PTR. However, since the experiments involved the reactivity measurements of small samples (<50 g), the applications of the results are limited to relatively small additions of fuel, probably up to 5 kg.

Although in the case of Cd^{113} and Sm^{149} the gain in endurance is equivalent to the gain resulting from the addition of fuel, the burnup rate of these two isotopes is so high that the gain in reactivity exceeds the loss resulting from the depletion of fuel and accumulation of fission products and thus requires the use of some shim control. By proper self-shielding of the poison, however, it is possible to obtain more nearly compatible burnup rates.

TABLE 17.2--RESULTS OF ENDURANCE CALCULATIONS

Method	Megawatt hours gained per kg of U^{235} added	σ_p/σ_t	Poison residue, %
Uncompensated addition of \$3 of fuel	8400		
Addition of \$3 of fuel compensated by the use of burnable poison			
B^{10}	5700	3.58	-1.7
Cd^{113}	8400	22.5	-
Sm^{149}	8400	26.7	-
Natural hafnium*	6500	2.1	-1.2

* Assumed to consist of Hf^{177} and Hf^{178} only.

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in section

17.2, 17.3A

Chapter 18

TWO-DIMENSIONAL (r - θ) MULTIGROUP ANALYSIS OF A SIR-TYPE REACTOR

P. L. HOFMANN and E. L. WACHSPRESS

18.1 INTRODUCTION

This chapter presents the results of an r - θ multigroup analysis of an intermediate spectrum power reactor over its entire lifetime. The term r - θ multigroup signifies that multigroup calculations were performed for reactor configurations with material compositions varying in both the radial, r , and angular, θ , directions. References 1 to 4 are pertinent to multigroup calculational techniques.

In this survey 13 calculations were performed on a UNIVAC. These calculations were made for an r - θ model representing the reactor in

1. The completely shutdown condition
2. The clean-critical condition
3. An intermediate stage during its operational life cycle
4. The depleted condition near the end of its operational life cycle

For each reactor configuration, determinations were made of

1. The reactivity variation with control-drum position near critical
2. The flux distributions for 15 energy groups
3. The power distribution
4. The neutron balance in the various reactor regions

The results of these determinations are described in some detail in this chapter.

In the past, several one-dimensional multigroup calculations (spherical model) for reactors have been performed. The one-dimensional limitation makes the results of these calculations somewhat unrealistic when applied to SIR-type reactors. This is particularly true for the treatment of the control-drum rotation. The development of the r - θ multigroup method received impetus from a desire to create a multigroup calculation that would effectively treat the unique geometry of a rotating control cylinder. The results of the r - θ analysis display a greater degree of realism than previously achieved by means of one-dimensional multigroup calculations.

Results of this r - θ analysis have been compared with applicable PPA experiments. These comparisons have substantiated the usefulness of this technique in treating the azimuthal variation introduced by the presence of control drums.

18.2 REACTOR MODEL

A. Simplified Reactor Model

A simplified reactor model was analyzed in order to limit the UNIVAC calculation time to a reasonable order of magnitude and to be consistent with the accuracy of the calculational technique.

The first simplification was to reduce the number of different regions in the reactor model. Only five regions were specified: core, reflector, B_4C , void, and shield. The core is defined as that region which contains U^{235} .

The core is surrounded by the reflector, which consists of six stationary reflectors and six control drums. The control drums in turn are divided into a B_4C region, a void region, and a non- B_4C region. The composition of the stationary reflectors and of the non- B_4C region of the control drums is similar enough to permit these two regions to be lumped together in the model and designated as the reflector.

In the reactor a sodium-filled gap lies between the core proper and the reflector. The sodium gap is a region in itself. However, since this region is comparatively small, it is arbitrarily divided equally between the reflector and the core. The B_4C region is a mock-up of the B_4C region of the control drum. The shield region mocks part of the immediate steel-sodium layer surrounding the reactor.

B. Mesh Mock-up

The simplified model was modified still further in the configurations actually used in the calculations. An even number of increments in the θ direction was chosen to provide for 30-deg representation of mock-ups having symmetry about the 30-deg line. The size of the increments in the radial direction, Δr , was chosen to give the proper core area in the median plane ($r \approx 2.2$ cm). The angular increment $\Delta\theta$ was approximately 2.3 deg, i.e., 26 increments in a 60-deg sector.

The UNIVAC coding limited the mesh mock-up to a maximum number of interfaces in the r direction of seven plus the outer boundary. The maximum number of interfaces in the θ direction is limited to seven plus the boundary edges.

Sketches of the mesh mock-ups for the various drum positions are shown in Fig. 18.1. That these drums positions are given to the nearest half degree is no indication of the accuracy of the mock-ups. An attempt was made in deciding upon a mock-up to reproduce a specific drum position (e.g., 60-deg). Then the angle between the leading edge of the poison sector and the line between the center of the core and the center of the rotating drum plus half the nominal 110-deg (or 55-deg) arc of the poison sector was taken as the drum rotation actually mocked. In the 60.5-deg case the leading edge of the poison was 5.5 deg. Therefore $55 + 5.5 = 60.5$ deg as shown in Fig. 18.1.

C. Compositions

The per cent compositions of the various regions, exclusive of U^{235} and fission products, that were used in the computations are listed in Table 18.1.

TABLE 18.1--COMPOSITION OF REACTOR MODEL

Exclusive of U^{235} and fission products					
Material	Grams/ cubic centimeter	Volume fractions			
		Core	Reflector	B_4C region	Shield
Beryllium	1.81	0.524	0.843		
Sodium	0.86	0.334	0.106		0.200
Stainless steel	7.85	0.081	0.051		0.800
Magnesium	1.74	0.018			
Oxygen		1.77×10^{21} nuclei/cm ³			
Boron				0.0222×10^{24} nuclei/cm ³	

The B_4C region consists of stainless-steel tubes filled with B_4C and cooled by sodium. In the model only boron was introduced in this region in a concentration equivalent to that in the actual region.

The void region was treated by the method described in Report KAPL-1172.⁴ The diffusion length in this region, to allow properly for axial streaming of neutrons, was estimated as 5.75 cm.

Five different cores appear in this series of calculations:

1. Fresh clean reactor, no xenon
2. Fresh reactor, steady-state xenon

Fig. 18.1—Mesh mock-ups for various control drum positions.

3. Partially depleted reactor, fission products, steady-state xenon
4. Depleted reactor, fission products, steady-state xenon
5. Depleted reactor, fission products, peak xenon

With the components chosen, the partially depleted reactor had 75,000 Mwh of operation and the depleted reactor had produced 115,000 Mwh. Fission products other than xenon were simulated by natural boron distributed uniformly in the core. It was assumed that 0.8 boron atom per fission would represent an adequate fission-product equivalent. (This procedure follows the method outlined on page 17 of Report KAPL-975.⁵) The xenon concentrations, steady-state and peak xenon after shutdown from full power, were calculated for a clean reactor using a spherical model. The xenon concentration change with fuel burnup was ignored in this series of calculations.

In Table 18.2, the U^{235} , boron, and xenon concentrations for the various reactor cores are listed. Uniform spatial fuel depletion and fission-product buildup have been assumed.

TABLE 18.2-- U^{235} , BORON, AND XENON CONCENTRATIONS

Reactor Core	U^{235} , volume fraction	Boron, nuclei/cm ³	Xenon, nuclei/cm ³
Fresh, clean	0.00970	0	0
Fresh, steady-state xenon	0.00970	0	1.34×10^{18}
Partly depleted, fission products, steady-state xenon	0.00885	2.31×10^{19}	1.34×10^{18}
Depleted, fission products, steady-state xenon	0.00840	3.46×10^{19}	1.34×10^{18}
Depleted, fission products, peak xenon	0.00840	3.46×10^{19}	1.44×10^{18}

D. Input Data

All input data used in these calculations are tabulated in Table 18.3, considerably rounded off. A self-shielding scheme as described in Report KAPL-1418⁶ was used to account for core heterogeneities.

TABLE 18.3--GROUP DEPENDENT INPUT DATA

Group	Δu	$\frac{1}{\xi \Sigma_g}$	$c = D\Delta u$	$\Sigma_a \Delta u$	$\Sigma_f \Delta u$	$1/\omega_2$
Calc. 1. Core: Fresh, clean						
1	0.5	51.6	1.60	0.00032	0.00030	3.33
2	0.5	49.5	1.53	0.00032	0.00030	1.67
3	1.0	41.4	2.61	0.00064	0.00059	1.67
4	1.0	22.5	1.45	0.00068	0.00062	1.15
5	1.0	17.3	1.09	0.00080	0.00071	1.03
6	4.0	12.9	3.33	0.0066	0.00513	0.887
7	3.0	12.0	2.20	0.0209	0.0140	0.870
8	1.5	11.9	1.05	0.0270	0.0177	0.840
9	1.25	11.9	0.858	0.033	0.0220	0.833
10	1.25	11.9	0.852	0.0378	0.0196	0.847
11	1.0	11.9	0.702	0.0157	0.0084	0.935
12	1.0	11.9	0.613	0.0361	0.0262	0.833
13	0.75	11.9	0.474	0.0528	0.0387	0.735
14	0.95	13.2	0.626	0.0793	0.0591	0.654
15	1.0	14.4	0.682	0.0929	0.0691	0
Calc. 2. Core: Fresh, steady-state Xe						
1	0.5	51.6	1.60	0.00032	0.00030	*
2	0.5	49.5	1.53	0.00032	0.00030	
3	1.0	41.4	2.61	0.00064	0.00059	
4	1.0	22.5	1.45	0.00068	0.00062	
5	1.0	17.3	1.09	0.00080	0.00071	
6	4.0	12.9	3.33	0.0066	0.00513	
7	3.0	12.0	2.20	0.021	0.0140	
8	1.5	11.9	1.05	0.027	0.0177	
9	1.25	11.9	0.86	0.033	0.0220	
10	1.25	11.9	0.852	0.038	0.0196	
11	1.0	11.9	0.702	0.016	0.0084	
12	1.0	11.9	0.673	0.036	0.0262	
13	0.75	11.9	0.473	0.054	0.0384	
14	0.95	13.2	0.615	0.087	0.0547	
15	1.0	14.4	0.672	0.100	0.0633	

* $1/\omega_2$ was the same for all cores.

TABLE 18.3--(Continued)

Group	Δu	$\frac{1}{\xi \Sigma_s}$	$c = D\Delta u$	$\Sigma_a \Delta u$	$\Sigma_f \Delta u$	$1/\omega_2$
Calc. 3. Core: Partly depleted, fission products and steady-state Xe						
1	0.5	51.6	1.60	0.00029	0.00027	*
2	0.5	49.5	1.53	0.00029	0.00027	
3	1.0	41.4	2.61	0.00059	0.00054	
4	1.0	22.5	1.45	0.00063	0.00057	
5	1.0	17.3	1.09	0.00074	0.00065	
6	4.0	12.9	3.33	0.00112	0.00469	
7	3.0	12.0	2.20	0.00195	0.0128	
8	1.5	11.9	1.05	0.00253	0.0163	
9	1.25	11.9	0.86	0.0314	0.0202	
10	1.25	11.9	0.85	0.0361	0.0181	
11	1.0	11.9	0.70	0.0164	0.0076	
12	1.0	11.9	0.67	0.0361	0.0239	
13	0.75	11.9	0.47	0.0524	0.0354	
14	0.95	13.2	0.62	0.0864	0.0503	
15	1.0	14.4	0.67	0.0994	0.0583	
Calc. 4. Core: Depleted, fission products and steady-state Xe						
1	0-0.5	51.6	1.60	0.00028	0.00026	*
2	0.5-1.0	49.5	1.53	0.00028	0.00026	
3	1-2	41.4	2.61	0.00057	0.00051	
4	2-3	22.5	1.45	0.00060	0.00054	
5	3-4	17.3	1.09	0.00071	0.00062	
6	4-8	12.9	3.33	0.00587	0.00445	
7	8-11	12.0	2.20	0.0187	0.0122	
8	11-12.5	11.9	1.05	0.0244	0.0155	
9	12.5-13.75	11.9	0.86	0.0308	0.0193	
10	13.75-15	11.9	0.855	0.0353	0.0172	
11	15-16	11.9	0.701	0.0168	0.0072	
12	16-17	11.9	0.673	0.0361	0.0227	
13	17-17.75	11.9	0.475	0.0519	0.0337	
14	17.75-18.7	13.2	0.617	0.0859	0.0480	
15	18.7	14.4	0.673	0.0990	0.0557	
Calc. 5. Core: Depleted, fission products and peak Xe						
1	0-0.5	51.6	1.60	0.00028	0.00026	*
2	0.5-1	49.5	1.53	0.00028	0.00026	
3	1-2	41.4	2.61	0.00057	0.00051	
4	2-3	22.5	1.45	0.00060	0.00054	
5	3-4	17.3	1.09	0.00071	0.00062	
6	4-8	12.9	3.33	0.00587	0.00445	
7	8-11	12.0	2.20	0.0188	0.0122	
8	11-12.5	11.9	1.05	0.0244	0.0155	
9	12.5-13.75	11.9	0.862	0.0304	0.0193	
10	13.75-15	11.9	0.855	0.0353	0.0172	
11	15-16	11.9	0.701	0.0168	0.00724	
12	16-17	11.9	0.673	0.0361	0.0227	
13	17-17.75	11.9	0.475	0.0519	0.0337	
14	17.75-18.7	13.2	0.616	0.0865	0.0477	
15	18.7	14.4	0.673	0.0995	0.0553	

* $1/\omega_2$ was the same for all cores.

TABLE 18.3-- (Continued)

Group	Δu	$\frac{1}{\xi \Sigma_s}$	$c = D \Delta u$	$\Sigma_a \Delta u$	$\Sigma_f \Delta u$	$1/\omega_2$
Reflector						
1	0.5	35.4	1.24	0.000014		2.50
2	0.5	34.2	1.19	0.000014		2.08
3	1.0	28.20	1.99	0.000029		1.67
4	1.0	15.1	1.08	0.000029		1.19
5	1.0	11.5	0.816	0.000029		1.10
6	4.0	8.45	2.43	0.000156		1.08
7	3.0	7.89	1.66	0.000377		1.00
8	1.5	7.77	0.807	0.000392		0.99
9	1.25	7.76	0.671	0.000655		0.98
10	1.25	7.76	0.671	0.00124		0.962
11	1.0	7.76	0.536	0.00175		0.971
12	1.0	7.76	0.535	0.00289		0.962
13	0.75	7.82	0.403	0.00319		0.952
14	0.95	8.69	0.561	0.00655		0.926
15	1.0	9.54	0.640	0.00855		0
Group	Δu	$\frac{1}{\xi \Sigma_s}$	$c = D \Delta u$	$\Sigma_a \Delta u$	$\frac{1}{\omega_2}$ Calc. 4,5	$\frac{1}{\omega_2}$ Calc. 1,2,3
B ₄ C Region						
1	0.5	137.	5.08	0.0017	2.77	3.80
2	0.5	145.	5.34	0.0014	2.38	2.60
3	1.0	139.	9.92	0.0025	1.37	1.21
4	1.0	115.	7.30	0.0032	0.862	1.17
5	1.0	80.8	4.58	0.0070	1.18	1.34
6	4.0	61.0	12.2	0.0907	0.813	0.79
7	3.0	60.1	5.49	0.307	0.980	0.517
8	1.5	60.1	1.33	0.447	0.709	0.694
9	1.25	60.1	0.62	0.751	0.725	0.670
10	1.25	60.1	0.365	1.377	0.633	0.674
11	1.0	60.1	0.167	1.941	0.599	0.688
12	1.0	60.1	0.104	3.173	0.394	0.679
13	0.75	60.1	0.051	3.63	0.521	0.955
14	0.95	60.1	0.0428	7.10	0.461	0.952
15	1.0	60.1	0.035	9.33	0	
Shield Region						
1	0.5	129.	1.13	0.00021	2.50	2.64
2	0.5	127.	1.11	0.00021	1.92	1.70
3	1.0	134.	2.23	0.00041	2.08	1.34
4	1.0	127.	1.95	0.00041	0.82	1.07
5	1.0	104.	1.28	0.00041	1.27	1.05
6	4.0	92.0	6.65	0.00175	1.04	1.30
7	3.0	47.8	1.74	0.00386	1.47	1.20
8	1.5	36.6	0.659	0.00538	1.03	0.972
9	1.25	36.6	0.548	0.0090	0.97	0.930
10	1.25	36.6	0.542	0.0170	0.92	0.870
11	1.0	36.6	0.427	0.0241	0.94	0.874
12	1.0	36.6	0.420	0.0397	0.69	0.819
13	0.75	36.5	0.309	0.0455	0.962	0.901
14	0.95	36.5	0.378	0.0900	0.524	0.859
15	1.0	36.4	0.389	0.117	0	

TABLE 18.3-- (Continued)

Group	Δu	$\frac{1}{\xi \Sigma_s}$	$c = D\Delta u$
Void Region			
1	0.5	100	2.88
2	0.5	100	2.88
3	1.0	100	5.75
4	1.0	100	5.75
5	1.0	100	5.75
6	4.0	100	23.0
7	3.0	100	17.3
8	1.5	100	8.62
9	1.25	100	7.19
10	1.25	100	7.19
11	1.0	100	5.75
12	1.0	100	5.75
13	0.75	100	4.31
14	0.95	100	5.46
15	1.0	100	5.75

The average neutron degradations from one lethargy group to the next were treated by a method outlined in Report TID-2009.¹ (The average degradation in the i th lethargy group $\bar{q}$ is related to the degradation out of the i th group q_1^0 by $\bar{q} = \omega_2 q_1^0$, where ω_2 is determined by trial and error.)

18.3 REACTIVITY RESULTS

In Table 18.4 the excess Δk for the 13 multigroup calculations are listed. (The dollar reactivity values are based on 0.0085 in $\Delta k = \$1$.)

One quantity that is of particular interest with regard to the control characteristics of the reactor is the control sensitivity (cents per degree of rotation) of the control cylinders. This sensitivity at various control-cylinder positions was determined (by a graphical method) from the values listed in Table 18.4. Actually the sensitivity curve (not shown) is a composite of two curves. Up to about 70 deg the clean reactor is in

TABLE 18.4--SUMMARY OF RESULTS

Reactor Core	Drum position, deg	Excess Δk , \$
Fresh, clean	0	-9.62
Fresh, clean	31	-7.38
Fresh, clean	60.5	-3.21
Fresh, clean	70.5	0.51
Fresh, steady-state xenon	60.5	-4.71
Fresh, steady-state xenon	70.5	-1.50
Fresh, steady-state xenon	79	1.50
Partly depleted, fission products, steady-state xenon	79	-4.44
Partly depleted, fission products, steady-state xenon	93	0.44
Partly depleted, fission products, steady-state xenon	101.5	1.18
Depleted, fission products, steady-state xenon	0	-19.70
Depleted, fission products, peak xenon	121	0.59
Depleted, fission products, peak xenon	139	2.35

Fig. 18.2—Reactor control characteristics.

various stages of subcriticality. The clean reactor becomes critical in the neighborhood of 70 deg. It is maintained critical by the continuous supply of positive reactivity from the control drums as the fuel is depleted. Therefore the second portion of the curve for rotations greater than 70 deg always applies to a critical reactor. The point of maximum sensitivity ($\sim 41\text{¢/deg}$) occurs in the neighborhood of clean critical (near 70 deg).

Another representation of the control characteristics can be obtained by integrating under the control sensitivity curve. The resulting S curve is shown in Fig. 18.2. Examination of this curve indicates an over-all control range of \$24.5. This value is in good agreement with a \$22 estimate obtained from PPA experiments.

It would be possible to plot an S curve for any specific core composition (i.e., point in the reactor lifetime). This curve would differ slightly from Fig. 18.2. However, it is possible to use Fig. 18.2 to estimate the safety available in the shutdown condition. For example, Fig. 18.2 shows a reactivity difference of \$21.1 from the 117-deg drum rotation to the 0-deg position. From the calculation for the 0-deg drum position of a core that was critical at 117 deg (the last calculation summarized in Table 18.4) the safety was determined as \$19.7, compared with \$21.1 obtained from Fig. 18.2. In this comparison the difference between steady-state and peak xenon (which is about 50¢) has been ignored. This fact indicates that control curves in future surveys of comparable reactors may be determined for one particular core condition and the resulting curves may be interpreted in the same manner as the dynamic curve of Fig. 18.2.

Knowledge of the over-all control range makes it possible to estimate the endurance of the reactor. In Fig. 18.3, the critical control-drum position has been plotted as a function of full-power hours of endurance. The calculated values have been extrapolated to the 140-deg drum position, which was taken to be the maximum mechanical range of the control drum. On this graph we have ignored the differences between steady-state

Fig. 18.3—Critical drum position throughout life.

and peak xenon concentrations. The full-power endurance estimated from the graph is ~140,000 Mwh. Although this endurance estimate is, of course, subject to uncertainties it should be pointed out that the peak xenon concentration of 1.44×10^{16} nuclei/cm³ is probably a high value for depleted reactors since it was actually calculated for a hard spectrum, clean reactor model.

18.4 FLUX DISTRIBUTIONS

One of the results of the multigroup calculations is a complete set of spatial flux distributions for each energy group. Contour maps of pertinent flux distributions are shown in Fig. 18.4. On the left side of the figure, from top to bottom, are the contour maps of the total, subcadmium, and thermal fluxes for the fresh clean reactor with control drums set at 70 deg. On the right-hand side of the figure are the comparable maps for the depleted reactor with peak xenon and with drums at 139 deg.

The total flux is the summation over the entire flux spectrum comprising all 15 lethargy groups. The subcadmium flux is the summation over only the lowest three energy groups (lethargy u between 17.0 and 18.7). The thermal flux is simply the flux of the lowest energy group ($u = 18.7$). The flux distributions shown refer to the reactor median plane. As normalization in an $r - \theta$ calculation is somewhat arbitrary, an attempt was made to normalize in such a manner as to give numerical values of fluxes and powers comparable with those obtainable from the plane $z = 0$ of a similar $r - z$ calculation.

At the time these multigroup calculations were performed, the iteration techniques had not yet been fully perfected. Since convergence is slowest in the central reactor region, the flux values for this region are somewhat inaccurate. Schemes are now available which speed up the convergence rate in the central region; consequently better results can now be obtained.

An examination of Fig. 18.4 shows that the total flux patterns are reasonably circular in the central part of the core region. The effect of the reflector nonuniformities becomes apparent as the reflector is approached. The patterns for subcadmium and thermal flux show very definitely the effect of the reflector. High flux peaks are built up in the beryllium portion of the reflector with deep minima occurring in the boron region. This thermal-flux peakage is particularly pronounced in the clean reactor. The thermal-flux peak occurs near the boundary of the stationary reflector and the non-B₄C portion of the control cylinder.

Fig. 18.4—Flux distribution for reactor in initial and final stages. (Contours in median plane; relative units).

18.5 POWER DISTRIBUTIONS

The power at the point (r, θ) is obtained from the summation

$$\sum_1 \phi_1(r, \theta, u_1) \Sigma_{f1} \Delta u_1$$

where ϕ_1 is the relative average flux and Σ_{f1} is the average macroscopic fission cross-section for lethargy group i .

Fig. 18.5—Power distribution for reactor in initial, intermediate, and final stages.
(Contours in median plane; average reactor power = 1.00)

Figure 18.5 shows power-distribution contour maps for five pertinent reactor configurations. Since the power distributions are derived from the flux distributions, the inaccuracies in the flux distributions near the center of the reactor appear also in the power distributions. However, the power distributions shown in Fig. 18.5 have been corrected by assuming a Bessel function fit near the reactor core center.

The power distributions pertain to the median plane of the reactor. However, because of the normalization procedure mentioned in Sec. 18.4, one may consider the power values shown on the contour maps to be relative to an average reactor power of unity. With this normalization the average power in the median plane is 1.15.

The power distributions calculated here may be compared with the results of Chap. 21 in a pair of critical assemblies, PPA-20 (66 deg) and PPA-19 (106 deg), that respectively mocked clean and depleted reactors of a type similar to those discussed here. Figure 18.6 is a composite cross-section of three critical assemblies, and it illustrates the positioning of the boron slugs that represent the control drums at three different settings, 66 deg, 106 deg, and 126 deg. The arrows on Fig. 18.6 indicate the directions along which traverses of the power distributions in the critical assemblies have been measured.

Figure 18.7 shows comparisons of the calculated and the experimental power variations along four traverses in the midplane of each pair of reactors. The difference in control drum setting between PPA-19 and the depleted reactor (106 deg vs 121 deg) makes the comparisons less valid for the traverse on the right of Fig. 18.7. A more direct comparison between calculations and PPA measurements is given in Report KAPL-1172.⁴ Nevertheless the reasonable agreement found here indicates the usefulness of applying the two-dimensional (r, θ) multigroup techniques to the detailed analysis of intermediate spectrum reactors with complicated geometry.

Fig. 18.6—Composite cross-section of three control cylinder settings in the PPA.

18.6 NEUTRON BALANCES

Detailed neutron balances for two reactor conditions are presented in Tables 18.5 and 18.6. The neutron balance of Table 18.5 refers to the fresh clean reactor with the drums rotated to 70 deg. Table 18.6 shows the neutron balance for the depleted reactor with peak xenon and with the drums rotated to 139 deg.

A great deal of useful information can be extracted from these neutron balances. One interesting aspect, for instance, is the energy-dependent fission distribution (i.e., fraction of fissions which occur in each energy group). Figure 18.8 shows a plot of cumulative fissions vs lethargy u for the two reactor configurations under discussion. For the clean reactor one obtains a median fission energy of ~43 ev; whereas a median fission energy of ~29 ev is obtained for the depleted reactor. These fission energies are, of course, characteristic of the reactor median plane and may, therefore, be somewhat higher than the average median fission energies for the entire reactor. As fuel depletion proceeds and the control drums rotate the B_4C region away from the reactor core, the spectrum becomes softer. The hardening effect due to the fission-product accumulation appears to be too small to change this trend.

The median fission energies for five successive reactor configurations are listed in Table 18.7.

Fig. 18.7—Comparison of power distributions, calculated and observed.

TABLE 18.5--NEUTRON BALANCE FOR FRESH CLEAN REACTOR AT 70.5 DEG

Group	u	X_F	Core					Reflector					Blanket				
			D^I	A	D^O	L_Z	L	D^I	A	D^O	L_Z	L	D^I	A	D^O	L_Z	L
1	0-0.5	0.023		0.0001	0.0197	0.0004	0.0027		0.0000	0.0025	0.0000	-0.0025		0.0000	0.0000	0.0000	0.0000
2	0.5-1	0.1020	0.0197	0.0009	0.0965	0.0039	0.0204	0.0025	0.0000	0.0211	0.0004	-0.0190	0.0000	0.0000	0.0001	0.0000	-0.0001
3	1-2	0.4450	0.0965	0.0066	0.4112	0.0238	0.0999	0.0211	0.0001	0.1106	0.0033	-0.0928	0.0001	0.0000	0.0011	0.0002	-0.0012
4	2-3	0.300	0.4112	0.0082	0.6182	0.0157	0.0691	0.1106	0.0001	0.1717	0.0021	-0.0633	0.0011	0.0001	0.0011	0.0002	-0.0004
5	3-4	0.1300	0.6182	0.0091	0.6835	0.0112	0.0443	0.1717	0.0006	0.2075	0.0016	-0.0381	0.0011	0.0001	0.0014	0.0002	-0.0005
6	4-8		0.6835	0.0520	0.5379	0.0236	0.0700	0.2075	0.0003	0.2452	0.0042	-0.0421	0.0014	0.0002	0.0020	0.0008	-0.0017
7	8-11		0.5379	0.1130	0.3901	0.0106	0.0242	0.2452	0.0007	0.2255	0.0026	0.0164	0.0020	0.0005	0.0033	0.0002	-0.0021
8	11-12.5		0.3901	0.1051	0.2750	0.0036	0.0064	0.2255	0.0006	0.2056	0.0012	0.0181	0.0033	0.0007	0.0032	0.0001	-0.0006
9	12.5-13.75		0.2750	0.0875	0.1846	0.0020	0.0009	0.2056	0.0010	0.1864	0.0009	0.0173	0.0032	0.0010	0.0028	0.0001	-0.0006
10	13.75-15		0.1846	0.0640	0.1209	0.0013	-0.0015	0.1864	0.0017	0.1666	0.0008	0.0173	0.0028	0.0016	0.0022	0.0000	-0.0010
11	15-16		0.1209	0.0200	0.1008	0.0008	-0.0008	0.1666	0.0021	0.1517	0.0006	0.0122	0.0022	0.0017	0.0017	0.0000	-0.0013
12	16-17		0.1008	0.0345	0.0670	0.0006	-0.0012	0.1517	0.0032	0.1367	0.0005	0.0113	0.0017	0.0022	0.0012	0.0000	-0.0017
13	17-17.75		0.0670	0.0318	0.0372	0.0003	-0.0023	0.1367	0.0032	0.1238	0.0004	0.0092	0.0012	0.0019	0.0010	0.0000	-0.0017
14	17.75-18.7		0.0372	0.0252	0.0158	0.0002	-0.0039	0.1238	0.0064	0.1045	0.0005	0.0124	0.0010	0.0029	0.0008	0.0000	-0.0027
15	18.7		0.0157	0.0335	0.	0.0002	-0.0180	0.1045	0.0418	0.	0.0005	0.0622	0.0008	0.0187	0.	0.0001	-0.0180
Totals		1.00		0.5914		0.0983	0.3103		0.0618		0.0196	-0.0814		0.0315		0.0020	-0.0335

TABLE 18.5--(Continued)

Group	u	Absorber					Void		Fissions		Totals		
		D ^I	A	D ^O	L _Z	L	L _Z	F	S	A	D ^O	L _Z	L ^O
1	0-0.5		0.0000	0.0002	0.0000	-0.0002	0.0000	0.0001	0.0230	0.0001	0.0224	0.0005	0.0000
2	0.5-1	0.0002	0.0001	0.0010	0.0001	-0.0012	0.0001	0.0009	0.1244	0.0010	0.1187	0.0046	0.0000
3	1-2	0.0010	0.0007	0.0024	0.0024	-0.0046	0.0006	0.0060	0.5637	0.0074	0.5252	0.0305	0.0007
4	2-3	0.0024	0.0010	0.0032	0.0021	-0.0039	0.0007	0.0075	0.8252	0.0093	0.7942	0.02080	0.0008
5	3-4	0.0032	0.0021	0.0050	0.0012	-0.0050	0.0007	0.0082	0.9242	0.0119	0.8973	0.0149	0.0001
6	4-8	0.0049	0.0203	0.0029	0.0024	-0.0207	0.0021	0.0406	0.8973	0.0728	0.7880	0.0332	0.0034
7	8-11	0.0029	0.0367	0.0010	0.0006	-0.0354	0.0009	0.0758	0.7880	0.1509	0.6200	0.0149	0.0022
8	11-12.5	0.0010	0.02260	0.0006	0.0000	-0.0222	0.0000	0.0689	0.6200	0.1290	0.4845	0.0050	0.0015
9	12.5-13.75	0.0006	0.0165	0.0002	0.0000	-0.0162	0.0000	0.0577	0.4844	0.1060	0.3741	0.0030	0.0014
10	13.75-15	0.0002	0.0134	0.0001	0.0000	-0.0133	0.0001	0.0333	0.3741	0.0807	0.2898	0.0022	0.0014
11	15-16	0.0001	0.0092	0.0001	0.0000	-0.0091	0.0000	0.0108	0.2898	0.0331	0.2543	0.0014	0.0010
12	16-17	0.0001	0.0076	0.0000	0.0000	-0.0076	0.0000	0.0250	0.2543	0.0474	0.2049	0.0011	0.0008
13	17-17.75	0.0000	0.0047	0.0000	0.0000	-0.0047	0.0000	0.0234	0.2049	0.0417	0.1620	0.0006	0.0006
14	17.75-18.7	0.0000	0.0053	0.0000	0.0000	-0.0053	0.0000	0.0188	0.1620	0.03987	0.1211	0.0007	0.0004
15	18.7	0.0000	0.0167	0	0.0000	-0.0167	0.0001	0.0249	0.1211	0.1107	0	0.0008	0.0096
Total			0.1570		0.0092	-0.1661	0.0053	0.4017		0.8417		0.1343	0.02040

D^I = Degradation in
 L^O = Leakage out of reactor at outer radial boundary
 A = Absorption
 D^O = Degradation out

L_Z = End leakage based on axial buckling = $0.00089515 = (\pi/H)^2$
 S = Neutron source (X + D^I)
 L = Leakage out of region in r- θ plane
 X_F = Fission spectrum

TABLE 18.6--NEUTRON BALANCE FOR DEPLETED REACTOR AT 139 DEG

Group	u	X_F	Core					Reflector					Blanket				
			D^I	A	D^O	L_Z	L	D^I	A	D^O	L_Z	L	D^I	A	D^O	L_Z	L
1	0-0.5	0.023		0.0001	0.0194	0.0004	0.0031		0.0000	0.0030	0.0000	-0.0030		0.0000	0.0000	0.0000	-0.0000
2	0.5-1	0.1020	0.0194	0.0008	0.0947	0.0039	0.0220	0.0030	0.0000	0.0243	0.0004	-0.0217	0.0000	0.0000	0.0002	0.0000	-0.0002
3	1-2	0.4450	0.0947	0.0057	0.4062	0.0236	0.1046	0.0243	0.0001	0.1206	0.0036	-0.1001	0.0002	0.0001	0.0018	0.0002	-0.0020
4	2-3	0.300	0.4062	0.0072	0.6130	0.0155	0.0705	0.1206	0.0001	0.1871	0.0023	-0.0689	0.0018	0.0001	0.0011	0.0003	0.0003
5	3-4	0.130	0.6130	0.0081	0.6800	0.0112	0.0438	0.1871	0.0001	0.2259	0.0017	-0.0405	0.0011	0.0001	0.0018	0.0002	-0.0009
6	4-8		0.6800	0.0474	0.5477	0.0240	0.0610	0.2259	0.0003	0.2696	0.0046	-0.0486	0.0018	0.0003	0.0019	0.0010	-0.0013
7	8-11		0.5477	0.1077	0.4150	0.0051	0.0198	0.2696	0.0008	0.2628	0.0031	+0.0030	0.0019	0.0005	0.0037	0.0002	-0.0025
8	11-12.5		0.4150	0.1054	0.3060	0.0041	-0.0004	0.2628	0.0008	0.2506	0.0014	0.0099	0.0037	0.0006	0.0032	0.0001	-0.0001
9	12.5-13.75		0.3059	0.0933	0.2157	0.0024	-0.0054	0.2506	0.0012	0.2340	0.0011	0.0143	0.0032	0.0009	0.0028	0.0000	-0.0004
10	13.75-15		0.2157	0.0734	0.1486	0.0016	-0.0080	0.2340	0.0021	0.2134	0.0010	0.0175	0.0026	0.0014	0.0021	0.0000	-0.0009
11	15-16		0.1486	0.0269	0.1263	0.0010	-0.0055	0.2134	0.0028	0.1965	0.0006	0.0133	0.0021	0.0016	0.0017	0.0000	-0.0012
12	16-17		0.1263	0.0446	0.0866	0.0008	-0.0056	0.1965	0.0042	0.1785	0.0007	0.0131	0.0017	0.0021	0.0010	0.0000	-0.0014
13	17-17.75		0.0866	0.0422	0.0500	0.0003	-0.0060	0.1785	0.0042	0.1618	0.0004	0.0121	0.0010	0.0016	0.0009	0.0000	-0.0016
14	17.75-18.7		0.0500	0.0375	0.0215	0.0002	-0.0093	0.1618	0.0083	0.1357	0.0006	0.0171	0.0009	0.0027	0.0004	0.0000	-0.0022
15	18.7		0.0215	0.0596	0	0.0004	-0.0384	0.1357	0.0492	0	0.0033	0.0832	0.0004	0.0163	0	0.0000	-0.0159
Totals		1.00		0.6597		0.0945	0.2458		0.0741		0.0251	-0.0993		0.0282		0.0022	-0.0303

TABLE 18.8--(Continued)

Group	u	Absorber				Void		F	S	Totals			
		D ^I	A	D ^O	L _Z	L	L _Z			A	D ^O	L _Z	L ^O
1	0-0.5		0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.023	0.0001	0.0224	0.0005	+0.0000
2	0.5-1	0.0000	0.0000	0.0001	0.0000	-0.0001	0.0000	0.0007	0.1244	0.0008	0.1192	0.0043	0.0000
3	1-2	0.0001	0.0001	0.0006	0.0005	-0.0011	0.0003	0.0052	0.5642	0.0060	0.5293	0.0283	0.0007
4	2-3	0.0003	0.0002	0.0006	0.0005	-0.0007	0.0004	0.0064	0.8293	0.0076	0.8018	0.0190	0.0009
5	3-4	0.0006	0.0005	0.0010	0.0001	-0.0010	0.0004	0.0070	0.9318	0.0087	0.9087	0.0135	0.0009
6	4-8	0.0010	0.0058	0.0008	0.0007	-0.0063	0.0014	0.0359	0.9087	0.0537	0.820	0.0317	0.0033
7	8-11	0.0008	0.0118	0.0006	0.0002	-0.0118	0.0008	0.0700	0.8200	0.1208	0.6821	0.0094	0.0077
8	11-12.5	0.0006	0.0086	0.0002	0.0000	-0.0082	0.0003	0.0670	0.6821	0.1154	0.5600	0.0059	0.0009
9	12.5-13.75	0.0002	0.0076	0.0001	0.0000	-0.0075	0.0002	0.0591	0.5600	0.1030	0.4524	0.0037	0.0008
10	13.75-15	0.0001	0.0077	0.0001	0.0000	-0.0076	0.0002	0.0358	0.4524	0.0847	0.3642	0.0028	0.0008
11	15-16	0.0001	0.0060	0.0000	0.0000	-0.0060	0.0001	0.0116	0.3642	0.0372	0.3245	0.0019	0.0005
12	16-17	0.0000	0.0057	0.0000	0.0000	-0.0057	0.0001	0.0280	0.3245	0.0565	0.2661	0.0016	0.0004
13	17-17.75	0.0000	0.0040	0.0000	0.0000	-0.0040	0.0001	0.0274	0.2661	0.0521	0.2127	0.0008	0.0005
14	17.75-18.7	0.0000	0.0054	0.0000	0	-0.0054	0.0001	0.0207	0.2128	0.0539	0.1577	0.0010	0.0002
15	18.7	0.0000	0.0279	0	0.0000	-0.0279	0.0003	0.0331	0.1577	0.1529	0	0.0041	0.0007
Totals			0.0913		0.0020	-0.0933	0.0046	0.4080		0.8532		0.1284	0.0184

D^I = Degradation in
 L^O = Leakage out of reactor at outer radial boundary
 A = Absorption
 D^O = Degradation out

L_Z = End leakage based on axial buckling = $0.00089515 = (\pi/H)^2$
 S = Neutron source (X + D^I)
 L = Leakage out of region in r- θ plane
 X_F = Fission spectrum

Fig. 18.8—Cumulative fission versus lethargy distribution.

The general softening of the spectrum as the drums rotate can be observed. The addition of steady-state xenon to the fresh clean reactor brings about a hardening of about 2 ev in the median fission energy.

TABLE 18.7--MEDIAN FISSION ENERGIES

Reactor Core	Drum position, deg	Median fission energy, ev
Fresh, clean	70.5	43.3
Fresh, steady-state xenon	70.5	45.5
Partly depleted, fission products, steady-state xenon	93	37.2
Depleted, fission products, peak xenon	121	32.1
Depleted, fission products, peak xenon	139	29.0

The authors wish to acknowledge the contribution made by D. V. Steward toward the success of the machine computations.

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Chapter 19

ESTIMATION OF REACTOR ENDURANCE

CHARACTERISTICS BY CRITICAL EXPERIMENTS

D. J. ANTHONY and H. B. STEWART

19.1 INTRODUCTION

The total energy available from the complete life cycle of a reactor power plant can be evaluated approximately by virtually zero-power experiments on a critical experiment mock-up assembly. The low power operation and the flexibility of the critical assembly make it possible to conduct experimental programs on a variety of reactor designs efficiently and economically.

During the course of the S1G design program, experimental and analytical techniques developed at KAPL were used at the PPA for endurance evaluations of S1G. Some aspects of these techniques were presented in laboratory Reports KAPL-837¹ and KAPL-988². This chapter is a review of these principles and an outline of their application to the performance evaluation of a power reactor.

The problem is to evaluate

1. The static differences in reactivity between the critical assembly and the power reactor resulting from detailed differences in composition and geometry;
2. The changes in reactivity associated with the burnup of fuel and the consequent reactor control compensations during power operation.

In general, the static reactivity difference between the critical assembly and the power reactor is different at the various phases of the reactor life because the reactivity values of the reactor components vary with the changes in control-element geometries, fuel burnups, and fission-product poison accumulations. However, from the standpoint of total reactor endurance, these reactivity comparisons are important only in the maximum and minimum reactivity conditions, or the freshly loaded (clean) and depleted (dirty) conditions.

Since the total energy delivered by a power reactor is directly proportional to the amount of fuel burned (the capture-to-fission ratio is virtually constant throughout life), the endurance evaluation consists in determining the difference in fuel load between the fresh reactor and the spent reactor. The fresh fuel load is established in the critical experiment by loading fuel until the reactor is critical with the control elements in a minimum reactivity position; at this position a predetermined increment of the total available control (i.e., that between shutdown and clean critical) is retained as a fast safety mechanism. The magnitude of this control increment or safety margin, as it is usually referred to, is dictated by the speed of shutdown from full-power operation that is required to prevent reactor core damage following a loss of coolant flow.

The spent-reactor fuel inventory is established by the stepwise conversion of the fresh reactor to the spent reactor by

1. Uniformly removing fuel to simulate burnup
2. Adding an appropriate amount of poison (e.g., cadmium) to mock fission-product poisons resulting from fuel burnup
3. Maintaining criticality by means of the control elements until the control elements are in the maximum reactivity position.

19.2 THE PPA AS A NUCLEAR REPLICA OF A SIR-TYPE REACTOR

However, since the fuel burns up at different rates in different parts of the reactor core and at different times, a small correction must be applied to the experimentally measured reactivity value of fuel in the spent reactor. Details of this correction are discussed in Sec. 19.6.

19.2 THE PPA AS A NUCLEAR REPLICA OF A SIR-TYPE REACTOR

Since it is important to maintain a certain degree of flexibility in the critical experiments, it is not possible to mock a power reactor in detail. Consequently, the Preliminary Pile Assemblies differed in several respects from the SIR-type reactors which they were intended to simulate.

The PPA is a hexagonal array of stainless-steel tubes each having an 0.02-in. thick wall and a 2.04-in. outside diameter, with 2.04-in. between centers. The assembly is divided into a stationary half and a movable half, as shown in Fig. 19.1; each half contains 817 tubes 27-in. long. When the halves are together, control of the assembly is achieved by the use of 10 calibrated movable fuel and reflector rods arranged to move in four banks of 2 or 3 rods each. Similarly, 10 fuel rods arranged to move in five banks of 2 or 3 rods each are used as a fast safety device.

When typical SIR-type cores were mocked, 181 tubes were used. The stationary reflector in each core was simulated by beryllium slugs of 1.96-in. diameter, whereas the control cylinders were mocked by hexagonal prisms consisting of 37 PPA tubes, four tubes on a side, of a total cross-sectional area approximating that of a control

Fig. 19.1—A Preliminary Pile Assembly, PPA-16. This assembly had a longer core than most other assemblies.

cylinder. Beryllium slugs made up the reflector portion of the cylinder, and five 1.93-in. diameter cans of boron carbide along the perimeter of each hexagon approximated the poison portion of the control cylinder. The six control cylinders mocked in this way could be "rotated" by interchanging B_4C cores and beryllium slugs manually when the halves of the assembly were separated.

Both the power reactors and the PPA have symmetry in the longitudinal direction, but the radial asymmetries and geometric boundaries in SIR-type reactors are difficult to mock exactly in the PPA. In the typical core the PPA mock-up could be arranged to have a core area within 2 per cent, a (core center)-to-(control cylinder center) distance within 1 per cent, and the control cylinder area within 20 per cent, of the corresponding areas and distances in the power reactor. The PPA dimensions were chosen, whenever possible, to approximate the dimensions of the power reactor at operating temperature.

Another difference between the PPA and the power reactors was the use of aluminum and magnesium in the PPA to take the place of sodium in the reactor. The fuel elements were also different; the PPA fuel disks, 0.04 in. thick and 1.96 in. in diameter, were made of uranium-aluminum alloy, about 35 per cent uranium and 65 per cent aluminum by weight.

19.3 THEORY

With the approach described in Report KAPL-988 by Tonks,² the reactivity of a power reactor at any time may, in general, be written as a function of reactor materials (including fuel remaining), amount of poison resulting from the accumulation of fission products, available control, and effects due to geometry. Thus

$$K = K(\theta, M_1, F_1, G_1)$$

where θ = control position

M_1 = reactor materials including fuel

F_1 = fission-product poisons including xenon

G_1 = effects due to geometry

By means of a Taylor expansion then it is possible to go from a point in "K space" defined by the critical-assembly (PPA) variables to a point characterized by the power reactor variables. If only the first-order terms are retained, the following equation results:

$$\delta K = K_2 - K_1 = K_\theta(\theta_2 - \theta_1) + \sum_i K_{M_i}(M_{i2} - M_{i1}) + \sum_i K_{F_i}(F_{i2} - F_{i1}) + \sum_i K_{G_i}(G_{i2} - G_{i1}) \quad (19.1)$$

where $K_\theta = \partial K / \partial \theta$, the control sensitivity

$K_{M_i} = \partial K / \partial M_i$, the reactivity coefficient of a particular reactor material

K_{F_i} = the reactivity coefficient of the fission products

K_{G_i} = the "reactivity coefficient" of a particular geometrical change

Subscripts 1 and 2 refer to the PPA and power reactor variables, respectively.

The reactivity coefficient of a material, defined as $\partial K / \partial M$, is the reactivity change per unit mass of material. It is customary to express the reactivity coefficient in units of cents per mole (or gram-atom) since this places the reactivity coefficients of all elements on an atomic basis, similar to cross-sections. The reactivity coefficient not only differs from one material to another, it is also a function of the position in the reactor and a function of the control-cylinder positions and composition of the reactor. However, it is independent of the reactor power level.

As discussed in Sec. 19.1, from a standpoint of total reactor endurance, the reactivity comparisons are important only in the freshly loaded (clean) and the depleted (dirty) conditions. Therefore the following cases will be considered:

Case 1: Shutdown Condition ($\theta = 0$ deg)

By assuming that no fission-product poison is present ($F_1 = 0$) and that $\theta_2 = \theta_1 = 0$ deg, Eq. 19.1 becomes

$$K_2 - K_1 = \sum_i K_{M_i} (M_{i_2} - M_{i_1}) + \sum_i K_{G_i} (G_{i_2} - G_{i_1}) \quad (19.2)$$

where the summation terms represent the reactivity comparison of the power reactor and the critical assembly both in the shutdown condition. From the measured safety margin in the critical experiment, $1 - K_1$, the safety margin for the power reactor, $1 - K_2$, can be estimated.

Case 2: Clean-critical Start-up

If both the critical assembly and the power reactor are assumed to be clean critical, i.e., $K_1 = K_2 = 1$, then Eq. 19.1 becomes

$$\delta K = 0 = K_\theta (\theta_2 - \theta_1) + \sum_i K_{M_i} (M_{i_2} - M_{i_1}) + \sum_i K_{G_i} (G_{i_2} - G_{i_1}) \quad (19.3)$$

because $F_1 = 0$, i.e., no fission-product poisons are present in the clean-critical reactor.

Solving for θ_2 , the power reactor control-cylinder position, Eq. 19.3 becomes

$$\theta_2 = \theta_1 - \frac{1}{K_\theta} \left[\sum_i K_{M_i} (M_{i_2} - M_{i_1}) + \sum_i K_{G_i} (G_{i_2} - G_{i_1}) \right] \quad (19.4)$$

which indicates that the difference in clean-critical position depends on the deviations in composition and geometry between the reactor and the PPA.

Case 3: Depleted Condition ($\theta = 140$ deg)

In this condition, Eq. 19.1 reduces to

$$\delta K = 0 = \sum_i K_{M_i} (M_{i_2} - M_{i_1}) + \sum_i K_{F_i} (F_{i_2} - F_{i_1}) + \sum_i K_{G_i} (G_{i_2} - G_{i_1}) \quad (19.5)$$

for $K_2 = K_1 = 1$ and $\theta_2 = \theta_1 = 140$ deg.

Since the energy developed by a power reactor is directly proportional to the amount of fuel burned, it becomes necessary to investigate a reactor in the poisoned condition in order to estimate endurance. Experimentally this situation is realized by converting the clean-critical mock-up to the depleted condition by the stepwise removal of fuel to simulate burnup and by the addition of cadmium to simulate the accumulation of fission-product poisons, along with the appropriate rotation of the control elements.

The problem then is to estimate the corresponding fuel burnup in the power reactor for the same control setting as is mocked by the cadmium-poisoned critical assembly. By starting with Eq. 19.5, in order to solve for the fuel burnup in the power reactor, and with

m = initial fuel load

m_2 = fuel burnup in the power reactor

m_1 = fuel burnup mocked in the critical assembly

Y = moles of cadmium ($Y_Z + Y_X$) in the critical assembly

Z = moles of fission products (other than xenon) in the power reactor

X = moles of xenon in the power reactor

N_j = moles of reactor materials (other than fuel)

G_i = effects due to geometry

the following equation results:

$$\delta K = 0 = K_m(m_1 - m_2) + (ZK_Z - Y_ZK_Y) + (XK_X - Y_XK_Y) + \sum_i K_{G_i}(G_{i2} - G_{i1}) + \sum_j K_{N_j}(N_{j2} - N_{j1}) \quad (19.6)$$

The total amount of cadmium, Y , has been written as $Y = Y_Z + Y_X$ since cadmium is used to mock both the xenon poisoning and the fission-product poisoning.

The number of moles of fission products resulting from the burnup of m_2 moles of U^{235} is

$$Z_1 = 2m_2/(1 + \alpha)$$

where α is the capture-to-fission ratio for U^{235} . Since

$$Y = Y_Z + Y_X$$

the amount of burnup in the reactor may be written as

$$m_2 = \frac{m_1 K_m + XK_X - YK_Y + [\sum_i K_{G_i}(G_{i2} - G_{i1}) + \sum_j K_{N_j}(N_{j2} - N_{j1})]}{K_m - [2K_Z/(1 + \alpha)]} \quad (19.7)$$

The two summation terms in brackets in Eq. 19.7 represent the effect of the reactivity deviation of the critical assembly from the power reactor in the poisoned condition as a result of inaccuracies in reactor geometry and material choices.

19.4 EVALUATION OF THE REACTIVITY DIFFERENCES BETWEEN S1G AND PPA

As was discussed in Sec 19.2, it is not possible to mock a power reactor in detail and still maintain the required degree of flexibility in the critical experiment. On this account, analytical evaluation of the geometrical differences had to be used along with the reactivity-coefficient results to estimate completely the differences between the S1G and the PPA. Furthermore, from the standpoint of total reactor endurance, the reactivity comparisons are important only in the clean and dirty conditions.

The results of the reactivity comparisons or analyses, which are used to evaluate the geometrical and compositional differences between the power reactor and the critical-assembly mock-up on the basis of experimentally determined reactivity coefficients of the various reactor materials, are represented by the summation terms in Eqs. 19.2, 19.3, and 19.5 for the different phases of the life cycle of the reactor.

For example, one reactivity analysis, the details of which are summarized in Table 19.1, indicated that the S1G was more reactive than the PPA by $\$(1.7 \pm 1.7)$ in the clean shutdown condition (all six control cylinders in the 0-deg position), and $\$(0.79 \pm 1.7)$ more reactive in the dirty condition (the control cylinders each at 140 deg). Actually the reactivity coefficients used in the reactivity analysis of the shutdown (0 deg) phase were determined in the clean condition for which the control cylinders were in the 62-deg position. The reactivity coefficients were assumed to be the same for the 0-deg position as for the 62-deg position. On the other hand, the analysis of the dirty condition was properly based on reactivity coefficients evaluated in a cadmium-poisoned and fuel-depleted PPA. However, the assumption above has been justified by subcritical measurements.

Since the reactivity analysis established by means of Eq. 19.2 that the S1G was more reactive than the PPA by $\$(1.7 \pm 1.7)$ in the shutdown condition, then the S1G safety margin, the amount of reactivity below clean critical when the control cylinders are in the 0-deg or shutdown position, was estimated to be $-\$(5.7 \pm 1.8)$. These numbers are based on a PPA calibration of the safety margin (Fig. 19.2), which was measured as $-\$(7.4 \pm 0.7)$ by inverse-multiplication techniques.

TABLE 19.1—REACTIVITY COMPARISON OF S1G AND PPA*

	Clean shutdown, \$ ($\theta = 0$ deg)	Dirty condition, \$ ($\theta = 140$ deg)	Estimated uncertainty, \$
Step 1: Differences in core beryllium composition	+0.64	-0.38	± 0.3
Step 2: Since the area of the PPA core is about 2% larger than that of S1G, material was conceptually removed from the edge of the PPA core and redistributed throughout the PPA core by means of the appropriate reactivity coefficients	+0.54	+0.40	± 0.03
Step 3: Differences in core-material composition other than beryllium	-1.34	-1.62	± 0.3
Step 4: Differences in geometry and composition of reflector regions	+1.45	+1.94	± 0.3
Step 5: Several other differences in addition to those listed above; each is assumed to be independent of control-cylinder positions			
(a) Neutron streaming	0.0	0.0	± 1.0
(b) S1G quality control of reactor materials	+0.75	+0.75	± 0.5
(c) Neutron temperature effects	†	†	± 1.0
(d) Doppler effect for fuel	-0.3	-0.3	± 0.2
(e) Iron lumping in S1G reflectors	†	†	± 0.5
Totals	+1.74	+0.79	± 1.7

* Positive contribution means S1G more reactive than PPA.

† No data.

19.5 REACTOR ENDURANCE EVALUATION

Completion of the necessary clean-critical experiments (e.g., reactivity-coefficient measurements, power-distribution determinations, self-shielding studies, and control evaluations) is followed by the conversion of the critical assembly to the mock-up of the depleted power reactor. This is accomplished by uniformly decreasing the fuel loading to simulate a particular burnup and by adding a uniform amount of cadmium, compatible with the simulated fuel burnup, to mock the poison effect of xenon and the other fission products. The amount of cadmium added can be determined approximately from estimated values for xenon and the long-lived fission-product poisons, on the assumption that cadmium adequately mocks the poisons. Criticality is then restored by rotation of the six control cylinders.

This stepwise procedure is repeated until the control cylinders are in their most reactive position (140 deg). This establishes the fuel burnup that is to be mocked and the poison value of the cadmium in the PPA.

Reactivity coefficients of the reactor materials are measured in this 140-deg critical assembly to allow a reactivity comparison of the PPA and the reactor in the depleted condition. Other critical experiments are performed to enable the calculation of the fuel burnup in the reactor by means of Eq. 19.7.

Finally, reestimations of the xenon and fission-product poison effects in the reactor can be made in view of experimental results obtained from the PPA mock-up.

19.6 REACTIVITY EFFECT OF NONUNIFORM FUEL BURNUP

Thus far only uniform fuel burnup has been considered since m_1 moles of fuel were removed uniformly from the PPA for experimental convenience in order to

Fig. 19.2—Estimated SIG safety margin dependence upon control cylinder position, based on measurements in PPA mock-ups.

mock the depleted reactor. The distribution of the fuel burnup in the power reactor depends on the history of the reactor, and, in general, the reactivity effect of the fuel actually burned out differs from that which would arise from the uniform removal of the same quantity of fuel. The total apparent reactivity loss caused by the uniform removal of m moles of fuel is $m\bar{K}_m$. The total apparent reactivity loss resulting from a nonuniform burnup of fuel is

$$\sum_i m_i K_{m_i} = \frac{m}{pt} \sum_i K_{m_i} \int_0^t p_i dt = m\bar{K}_m \quad (19.8)$$

where m_i is the burnup of fuel in the i th volume element and p_i is the power generated in the fuel of that volume element. The total reactor power is p (assumed constant). Furthermore, p_i is a function of the control-cylinder positions and consequently the time, t , since reactor startup. Therefore, $\bar{K}_m$ is a burnup-weighted distributed reactivity coefficient that includes the effect of nonuniform fuel burnup.

The value of $\bar{K}_m$ can be obtained from the PPA fuel reactivity coefficients and the time-integrated power associated with each fuel element in the reactor.

The total reactivity difference between the reactor and PPA due to fuel depletion, which is represented by the first term in Eq. 19.6, should be written

$$m_1 K_m - m_2 \bar{K}_m = (m_1 - f m_2) K_m$$

where $f = \bar{K}_m / K_m$ is the ratio of the burnup-weighted distributed reactivity coefficient of fuel to the uniformly distributed reactivity coefficient measured in the PPA.

Estimation of the S1G fuel burnup, based on a calculated value of 1.08 ± 0.05 for the ratio of the burnup-weighted and the uniformly distributed reactivity coefficient for fuel, results in an estimated endurance of 125,000 Mwh $\pm$ 25 per cent.

A similar situation arises from the uniform loading of cadmium in the PPA core to mock the accumulation of fission products in the power reactor since the deposition of fission-product poisons, resulting from fuel burnup, has the same distribution as the fuel burnup, provided the burnup rate of the poison is small. (The xenon poison is treated separately from the other fission products because of its large cross-section and relatively short half-life.)

The calculated values of peak xenon and fission-product poisons which were used in the final evaluation of the endurance of the power reactor were obtained as described in Chap. 14 and in Report KAPL-1151.³ They were based on the power-weighted reactivity coefficients and a consideration of the burnup rates of the significant fission-product poisons.

19.7 CONCLUSION

Three items of available S1G data are presented here to indicate the degree of effectiveness of the experimental and associated analytical techniques discussed in this chapter.

1. An estimate of the expected endurance of S1G, based on the first 25,000 Mwh of operation, indicated an endurance which was not far from the 125,000 Mwh value predicted above. (Note that this endurance prediction was based only on experiments with the flexible PPA. Subsequent experiments with the more accurate PTR indicated an endurance very close to the endurance presently expected of S1G.)
2. The S1G safety margin was measured as $-(6.5 \pm 0.75)$, which compares favorably with the predicted value of $-(5.7 \pm 1.8)$.
3. The most recent estimated xenon poison values (based on PTR measurements) are compared in Table 19.2 with the values measured for the clean S1G in full-power operation.

TABLE 19.2—XENON POISON IN THE CLEAN S1G

	Steady-state value, %	Peak value, %
Estimated (see Chap. 14)	$-2.4 \pm 20\%$	$-2.6 \pm 20\%$
Measured	-2.4	-2.5

These data indicate a quite satisfactory degree of effectiveness for the methods of comparison and extrapolation described in this chapter.

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Chapter 20

LOADING, NUCLEAR TESTING, AND POWER OPERATION OF THE S1G

R. G. STATER, L. G. BARRETT, J. A. BISTLINE, Jr., and R. C. DAHLBERG

20.1 INTRODUCTION

The first nuclear testing of the S1G, the land-based prototype of the Submarine Intermediate Reactor¹, was begun on March 10, 1955. Although extensive preliminary testing had been performed in the KAPL Proof Test Reactor (PTR)², it was necessary to make certain tests during the initial startup of S1G for safeguards reasons and additional tests throughout the reactor life to measure fundamental nuclear characteristics not available from critical experiments.

This chapter describes nuclear test procedures and results obtained during the first 45,000 megawatt hours (Mwh) of operation. The first test involved the loading of fuel, measurement of the nuclear safety margin, and determination of the critical control-element positions. The reactor was subsequently filled with sodium coolant, and the control elements were calibrated by the "rod-drop" technique. The effect of sodium flow and pressure on reactivity was measured. Detailed testing was then performed to evaluate temperature and power coefficients in S1G in reference to naval requirements for maneuvering stability. Endurance predictions were made for S1G on the basis of the rate of reactivity decrease in the clean reactor caused by the accumulation of fission products and depletion of fuel.

One significant result of S1G testing has been the degree to which nuclear characteristics have been predicted successfully by PTR measurements and reactor physics calculations. This gives added confidence to present critical-assembly and analytical work.

The scope of nuclear testing included items on initial loading, power calibration, control-element calibration, sodium filling of the core, sodium-flow and sodium-pressure effects, temperature- and power-coefficient measurements, fission-product buildup and fuel-depletion reactivity effects, and power-distribution measurements. Subjects such as shielding measurements, reactor transient behavior and maneuvering characteristics, and decay heat determinations are not included in this category.

20.2 INITIAL LOADING³

The objective of the first test performed was to load the fuel slugs into the dry (no sodium) S1G core while ensuring an adequate safety margin.

By loading the reactor dry and operating at very low power levels, it was possible to obtain shielding data that could be acquired at no other time during the reactor life.

To obtain access to the top of the reactor for fuel loading, it is necessary to remove the six control drive mechanisms and lock the cylinders in the least reactive position, i.e., with the boron poison face toward the core. In addition, it is necessary to remove the secondary-safety-rod drive and container thimble. Thus the S1G employs no cocked safety devices during fueling operations, and consequently it was imperative that one rely heavily on obtaining interpretable multiplication data. The multiplication of neutrons in a subcritical assembly is, as in Chap. 4,

$$M = \frac{1}{1 - k_{\text{eff}}} \quad (20.1)$$

where M is the actual reactor multiplication and k_{eff} is the effective multiplication factor.

In the case of S1G, where a neutron source is present in the core and where geometrical factors such as instrument location and neutron attenuation must be considered, this equation can be modified to the following form:

$$M_{\text{app}} = \frac{G}{1 - k_{\text{eff}}} = \frac{C}{C_0} \quad (20.2)$$

where M_{app} = apparent reactor multiplication as measured by nuclear instrumentation

G = geometric factor

C_0 = counting rate with source rod alone in core

C = subsequent counting rate after loading some fuel slugs or, if all slugs are loaded, after rotating control cylinders through an interval on approach to criticality

In general, it is more convenient to use the reciprocal of the multiplication of source neutrons or inverse multiplication.

$$IM = \frac{1 - k_{\text{eff}}}{G} = \frac{C_0}{C} \quad (20.3)$$

where IM = inverse multiplication = $1/M_{\text{app}}$.

Thus, as the reactor approaches criticality, k_{eff} approaches unity and the inverse multiplication approaches zero.

Extensive preliminary investigation of various loading schemes was made on the PTR to develop a slug loading sequence that would supply the best multiplication data to ensure safe loading of S1G.⁴ During the actual loading, inverse-multiplication data were used as a continual monitor of the nuclear status of the reactor by comparison with corresponding PTR results. As a result of the close nuclear similarity between the S1G and the PTR, the S1G safety margin could be assumed to be at least as large as that of the PTR as long as the extrapolation of S1G loading curves to zero inverse multiplication did not fall at a lower loading than for the PTR.

During loading the primary-coolant system contained helium, and electrical heating units were used to maintain the average reactor temperature between 275 and 300°F. The first fuel that was loaded contained a neutron source. After it was loaded, counting rates were determined on four proportional counters, and these rates were taken as the source count. Fuel slugs were then loaded in a designated pattern, and the inverse multiplication was calculated at specified intervals.

The S1G loading curves as obtained from the four proportional counters are shown in Fig. 20.1. For comparative purposes the PTR curves are shown in Fig. 20.2. The inverse multiplication decreased almost linearly for both reactors after about one-fourth of the fuel had been loaded. The initial increase in inverse multiplication can be attributed to the fact that the first fuel loaded around the source tends to decrease the neutron leakage more than to increase reactor multiplication; i.e., the magnitude of the geometric factor, G , was decreasing. The differences between the S1G and PTR loading curves were caused by the inexact mock-up of shielding and fuel in the PTR. In addition, the PTR was built to simulate reactor conditions at full operating temperature, whereas the S1G was at 275 to 300°F.

On completion of fuel loading, the first estimate of the S1G safety margin was obtained by insertion of the secondary safety rod into the central matrix position. By converting the counting rates before and after insertion to inverse multiplication and assuming the secondary safety rod to be worth $\$(2.70 \pm 0.20)$ as measured in the PTR, the safety margin was estimated as

$$SM = \frac{IM}{\Delta IM} \Delta k(\$) = \frac{0.328}{0.068} \$2.70 \approx \$13.00 \quad (20.4)$$

Fig. 20.1—SIG loading curve. The four curves are the records of four proportional counters in different locations.

Fig. 20.2—PTR loading curve. The three curves are the records of three counters in three different locations. The dashed curves indicate the shapes of the curves that would have been expected had there been no attempt to mock up sodium in the PTR.

where SM = safety margin of S1G without sodium in core

IM = inverse multiplication with control cylinders least reactive and secondary safety rod cocked

ΔIM = absolute magnitude of change in inverse multiplication on insertion of secondary safety rod into core

$\Delta k(\$)$ = absolute magnitude of reactivity change causing ΔIM (i.e., \$2.70).

After installation of the control-cylinder drive mechanisms, the approach to criticality was begun. The secondary safety rod was first cocked, and the control cylinders were then rotated in phase toward their most reactive position. Rotation was stopped and inverse-multiplication measurements were made at specified rotational increments (Fig. 20.3). (During this phase of the test, six thermocouples attached to selected parts of one special fuel assembly were carefully monitored to ensure against excessive core temperature because of nuclear-power generation.) Before counting-rate losses in the proportional counters became appreciable, current readings were

Fig. 20.3—Cylinder rotation to criticality in the dry S1G

obtained from ion chambers feeding d-c zero-stabilized amplifiers in the low-level circuit.⁵

Within about \$3.00 of critical the inverse multiplication became a linear function of in-phase control-cylinder position and could therefore be extrapolated quite accurately to the critical position. As criticality was approached, cylinder rotational increments were reduced, and finally one cylinder at a time was positioned forward a few degrees. The clean-critical in-phase control-cylinder position was found to be 78.9 deg. This value was in very good agreement with PTR predictions. A two-dimensional multigroup calculation had estimated the dry clean-critical in-phase cylinder position to be 82 deg. This compares favorably with the actual position and should give added confidence to the analytical methods being used.

After criticality was reached, the power level was increased to a steady-state level of about 50 watts. (At 1¢ below critical the power was approximately 10 watts. Initial power-level estimates prior to any power calibrations in S1G were made with the reactor a few cents subcritical. If the S1G and the PTR were identical nuclear-wise, the multiplication of source strength should be the same in the two reactors at the same degree of subcriticality so that the relative power levels should have been directly proportional to source strengths. Power levels for the PTR were known accurately. Then, by associating the subcritical power level with instrument readings, subsequent estimates of S1G power levels at critical were available.) The secondary safety rod was calibrated

by the standard rod-drop technique. The reactivity value of the rod was calculated as follows:

$$\frac{n}{n_0} = \frac{1}{1 + \Delta k} \quad (20.5)$$

or

$$\Delta k = \frac{n_0 - n}{n} = \frac{1.17 - 0.33}{0.33} = \$(2.55 \pm 0.10)$$

where Δk = negative reactivity value of rod (\$)

n_0 = steady-state neutron level at 50 watts as measured by proportional counter immediately prior to rod drop = 1.17×10^6 cpm

n = neutron level immediately after rod drop as obtained by extrapolation of delayed-neutron decay curve back to time of initiation of drop = 0.33×10^6 cpm

The fact that the actual calibrated rod value of \$2.55 in S1G was less than the value of \$2.70 measured in the PTR is probably attributable to composition differences: the S1G contained no sodium during dry startup, whereas the PTR contained magnesium as a sodium mock-up. By use of the calibrated value of the secondary safety rod, the actual dry safety margin at 275°F was then

$$SM = \frac{0.328}{0.068} \times \$2.55 = \$(12.30 \pm 2.50)$$

The probable error of the safety-margin measurement is set at ± 20 percent of its value because of a 3 per cent probable error in each inverse-multiplication measurement and a 4 per cent error on the secondary-safety-rod value. It should also be pointed out that the rod calibration was performed at criticality, whereas the safety-margin calculation was based upon inverse-multiplication changes at about \$12.00 subcritical. It is implicitly assumed that the safety-rod value remains constant for the two cases. Experience with the PTR has shown that reactivity values change as the control cylinders are moved to their least reactive position. Although the magnitude of this change is somewhat uncertain, it is probably not very large. The estimated dry safety margin from PTR measurements had been \$12.00.

It is possible to obtain a measure of the control-cylinder sensitivity at criticality from the slope of the inverse-multiplication data plotted during the first rotation to critical if it is assumed that the safety margin is directly related to inverse multiplication. Bottom-counter data indicated the control sensitivity to be about 4.2¢/deg/cylinder. Side counters indicated less sensitivity, but it is believed that the position of the adjacent control-cylinder poison influenced leakage to these instruments, i.e., caused the geometric factor, G , to vary with cylinder position. The predicted control sensitivity from PTR measurements with a sodium mock-up was 4.5¢/deg/cylinder. Analytical methods⁶ had indicated a sensitivity of 5.3¢/deg per cylinder.

20.3 POWER-LEVEL CALIBRATION⁷

It was necessary to calibrate the S1G neutron-flux instrumentation readings in terms of reactor power level for two reasons:

1. To prevent any reactor operation at power levels appreciably greater than 1 kw which might damage the reactor structure during certain testing performed without the benefit of the primary coolant

2. To allow evaluation of shielding test results.

The technique employed for calibration consisted basically in comparing fuel activations in S1G with similar activations in the PTR at a known power level.

Two removable fuel elements were inserted into an accessible matrix position. Subsequently the reactor was brought to criticality, and the power level was quickly adjusted to a steady-state value that was estimated to be 20 watts by a method devised F. G. LaViolette and H. B. Stewart. This power level was maintained for exactly 3 hr; then a manual scram was initiated and the two fuel elements were removed for gamma-radiation scanning⁸ (Chap. 21). The power-level calibration was then simply a matter of comparing the activity of the S1G elements with that of two identical elements irradiated for 3 hr in

the PTR at a power level known to be 19 ± 2 watts.⁹ The actual S1G power level was calculated as follows:

$$P_{S1G} = \frac{C_{S1G}}{C_{PTR}} P \quad (20.6)$$

where P_{S1G} = S1G power level (watts)

P_{PTR} = PTR power level = 19 watts

C_{S1G} = integral of counts obtained on automatic scanner using two fuel elements irradiated in S1G

C_{PTR} = integral of counts obtained on automatic scanner using two fuel elements irradiated in PTR

For an average of five scans the S1G power level was calculated to be 18.4 ± 2 watts (versus the 20-watt estimate). No corrections for decay of radiation were required since both the S1G and PTR scans were made at exactly the same time after reactor shutdown. The S1G nuclear instrument readings during the 3-hr run were taken as equivalent to an 18.4-watt power level, and subsequent dry operation was based upon this calibration.

The accuracy of the fuel activation technique for power calibration as applied to this test is, of course, limited by the similarity of the S1G and PTR power distributions. The automatic scanner provided a longitudinal power distribution (Fig. 20.4), which compared favorably with the PTR distribution.¹⁰ However, no data were obtained for radial power distributions in the clean S1G. It was not expected that the radial distributions would have compared quite so well since the PTR contained magnesium as a sodium mock-up; this caused a difference of about 15 deg between the in-phase wet critical PTR control-cylinder positions and the in-phase dry critical S1G positions.

It might be noted that, after filling the primary-coolant system with sodium, a recalibration between instrument readings and power level was necessary because of sodium attenuation between the core and instruments. However, for the same degree of subcriticality, the equilibrium reactor power level caused by source multiplication should be identical with or without sodium. After the wet critical in-phase control-cylinder position and control-cylinder sensitivity were determined, it was possible to establish a known degree of subcriticality and recalibrate the instruments.

Fig. 20.4—Longitudinal power distributions in PTR and S1G

20.4 FILLING WITH SODIUM ¹¹

After completion of dry testing the primary-coolant system was prepared for filling with sodium. It was necessary, of course, to ensure nuclear safety during the filling of the core and to measure the reactivity changes that occurred. It was particularly important from a safeguards point of view to know the speed and magnitude of the reactivity changes that might occur when sodium entered the core. Hence a procedure for filling the core and reflector was adopted which minimized the possibility of a nuclear incident. In an intermediate reactor the danger exists of introducing into the core hydrogenous material which has a relatively large positive reactivity coefficient.

Before the primary-coolant system was charged with sodium, the following three steps were performed:

1. The average reactor temperature was adjusted to about 275°F by use of electrical heating units.
2. The primary-coolant system was evacuated to below 100 μ and was constantly monitored until it was evident that there were no volatile substances such as oil and water in the system.
3. The secondary safety rod was cocked, one control drum was positioned at 70 deg, and the other five drums remained at 0 deg. By this procedure the dry reactor was somewhat nearer criticality and thus became more sensitive to small reactivity changes while additional safety was immediately available. During the entire filling operation the zero-stabilized amplifiers were set on their most sensitive range scale; the cocked safeties would then scram if the power reached a level of 2 or 3 watts.

When all reactor and liquid-metal instrumentation had been checked and all reactor and liquid-metal components found to be in working order, the charging operation was begun. The initial charging rate of the sodium into the system was 10 gpm until the sodium level was up to, but not in, the lower plenum chamber. At this point filling was halted and the counting rate was determined.

The procedure then called for a filling rate of 4 gpm, stopping at 4-in. height increments for counting-rate determinations. This technique was designed to safeguard the reactor, even if large amounts of oil had remained undetected in the primary-coolant system. However, the sodium level in the core rose at a somewhat faster rate than was expected, and it was not possible to determine the reactor multiplication after each 4-in. rise in the level. The level in the core was estimated with the following instrumentation:

1. Proportional counters. The sodium level, as well as the subcriticality of the reactor, could be estimated by observing changes in the reactor multiplication as the sodium entered the system. The side counters gave the most reliable indication since they were relatively unaffected by any neutron attenuation from the sodium.
2. Thermocouples on the reactor container. Initially, the thermocouples indicated various temperatures, depending on their proximity to electrical heating units. However, as the level of the sodium was raised, all thermocouples successively indicated the same temperature. Temperature changes were gradual, however, and it was only possible to specify the sodium level to within a few inches.
3. Sodium level instrumentation in the hold tank of the sodium service cell. The level of sodium was also approximated by measuring the volume of sodium pumped and comparing this with calculated sodium volumes in the coolant system.

At no time during the filling was the sodium observed to have a negative reactivity coefficient. This substantiated experimental work of Anthony and Schulz¹² which indicated that the reactivity coefficient was always positive, even in the reflector.

After sodium filling had been completed, the approach to wet critical was made. Inverse multiplication was plotted as a function of cylinder position as for the dry rotation to criticality. It was found that, for an average reactor temperature of 450°F, the wet critical in-phase cylinder position was 58.5 deg (Fig. 20.5), a shift of 20 deg from the dry critical position.

The safety margin was determined in two ways: (1) the degree of subcriticality was calculated by using Eq. 20.4 for changes in count rate as the cylinders were rotated through a definite increment; and (2) the curve of control-cylinder sensitivity was integrated up to the critical position. The average safety margin from these two determinations was (6.30 ± 1.00) at 450°F.

Fig. 20.5—Cylinder rotation to criticality in the wet. S1G

The reactivity value of the sodium was determined from the difference between the dry and wet safety margins. The safety margin of the dry S1G at 275°F had been found to be \$12.30. Adjusting this value for the temperature coefficient of the reactor without sodium (from Table 20.3), one obtains \$12.60 for the dry S1G at 450°F. Hence the nominal value of all the sodium (with the control cylinders at 0 deg) was \$12.60 - \$6.30 = \$6.30. Estimates based on reactivity-coefficient measurements in KAPL critical assemblies had indicated that the core sodium alone should be worth about \$4.00.

20.5 SECONDARY-SAFETY-ROD AND CONTROL-CYLINDER CALIBRATION^{13,14}

Immediately after the primary-coolant system had been filled with sodium, a check was made on the secondary-safety-rod reactivity value and on control-cylinder sensitivities. Two methods of calibration were used. Both involved the rod-drop technique (Eq. 20.5).

The first method required dropping the secondary safety rod into the critical reactor to determine its worth. Then with the secondary safety rod remaining completely inserted, criticality was re-established and the power level was restored to the original pre-rod-drop level by adjusting the control cylinders in phase toward their most reactive position. In this manner the cylinders were calibrated by relating the cylinder rotation

TABLE 20.1--CONTROL-CYLINDER SENSITIVITY BY SECONDARY-SAFETY-ROD-DROP METHOD

	Run drop No.		
	1	2	3
Power level, watts	100	100	100
Average reactor temperature, °F	370	370	650
In-phase cylinder position before rod drop, deg	57.6	57.5	60.0
In-phase cylinder position after rod drop, deg	67.0	67.3	70.0
Rotation interval	9.4	9.8	10.0
Value of rod drop, \$	2.23 ± 0.15	2.33 ± 0.20	2.45 ± 0.12
Average control sensitivity over interval, ¢/deg/cylinder	3.95 ± 0.25	4.01 ± 0.3	4.12 ± 0.2

necessary to restore power and the calibrated rod value. This method provided an average control-cylinder sensitivity over a 10-deg range. The results obtained by this method are shown in Table 20.1.

Some difficulty was experienced in making the safety-rod drop to the fully inserted position during this experiment because of the safety circuit modification required to scram the secondary safety rod independently of the control cylinders (primary safeties). Therefore, the total safety-rod value was probably closer to the PTR-predicted value of \$2.70 than the measured values listed in Table 20.1 would suggest. The measured values were well within the design specification requiring a minimum value of \$2.00.

The control sensitivity calculations were based on these measured safety-rod values. The accuracy of this technique of measuring control sensitivity is limited because the neutron flux at the core-control-cylinder interface tends to increase relative to the average flux when the safety rod is inserted. Since the relative effectiveness of a neutron absorber is roughly proportional to the square of the neutron flux, the control sensitivity values obtained by this method would be expected to be slightly higher than the actual values during normal operation.

The second method utilized for measuring control-cylinder sensitivities made use of the equipment developed for power-coefficient measurements (see Chap. 35). It proved possible to move a control cylinder so quickly through the limited 2-deg fine-control interval that the rod-drop technique could be applied for calibration. The time required for the cylinder to "drop" through the 2-deg range was between 120 and 180 msec. During initial reactor operation sensitivity measurements were performed in a range around the in-phase critical position by positioning the cylinders slightly out of phase so that the one being calibrated was at the desired location. Since during normal operation the cylinders remain within ± 5 deg of the in-phase position and since sensitivity values are affected by deviations from the in-phase location of cylinders, this method of obtaining a range of calibration was generally limited to within 5 deg of in-phase. With extended power operation and the associated build-up of xenon and fission products, it has been possible to extend the range of calibrations from the initial wet critical position of 58.5 deg up to 81 deg.

Neutron response to a typical 2-deg drop is shown in Fig. 20.6. Table 20.2 lists some of the data associated with the test results shown in Fig. 20.7.

Although the control-cylinder motion was not recorded for these particular runs, some windup and backlash apparently existed in the system so that the cylinder bounced on reaching the lower 2-deg limit. This can be noted in the neutron-level behavior at the end of the drop shown on Fig. 20.6.

Fig. 20.6—Neutron level response to a rapid 2-deg control-cylinder rotation. Some control-cylinder "bounce" is evident.

In Fig. 20.7 there is an apparent drop in sensitivity values by an amount between 0.15 and 0.25¢/deg per cylinder for the runs made at 73 deg and higher. Such a shift could result from a change of as little as 4 min in the 2-deg stroke. Since these runs were made using the No. 5 rather than the No. 1 control cylinder as for the previous runs, it is conceivable that the differences in the two drive systems caused the apparent sensitivity shift. For comparison purposes Fig. 20.7 also shows sensitivities predicted from PTR testing by Anthony, Schemel, and MacKinney.¹⁵

The S1G control curve (Fig. 20.8) was obtained by an integration of the sensitivity values in Fig. 20.7. The broken portion of the curve is an estimate made from PTR measurements.

TABLE 20.2--CONTROL-CYLINDER CALIBRATION

Control cylinder No.	Approx. power level, kw	Mwh operation	Av. control-element position, deg	Av. temp., °F	Control sensitivity, ¢/deg
1	50	0	57.8	353	3.66
1	16	0	57.5	410	3.79
1	500	8500	71.0	560	4.05
1	600	8500	71.0	539	4.16
1	600	8500	68.0	540	4.18
1	600	8500	68.0	540	4.06
1	600	8500	65.0	540	4.07
1	600	8500	62.0	540	3.86
5	300	30,000	73.0	407	4.00
5	300	30,000	73.0	408	3.93
5	300	30,000	75.1	408	3.97
5	300	30,000	75.1	408	4.00
5	300	30,000	77.0	408	4.04
5	300	30,000	77.0	408	3.93
5	300	30,000	79.1	408	3.98
5	300	30,000	79.1	408	4.04
5	300	30,000	81.1	408	3.94
5	300	30,000	81.1	408	3.95

Fig. 20.7—Control-cylinder sensitivity as a function of cylinder position. The squares represent data obtained by compensating for the secondary-safety-rod drop (Table 20.1). The circles and triangles are data obtained by 2-deg rapid rotations of control cylinders (No. 1 and 5, respectively). See text, Sec. 20.5.

Fig. 20.8—SIG control curve.

20.6 SODIUM FLOW AND PRESSURE EFFECTS ON REACTIVITY^{16, 17}

Although a flow effect on reactivity was not expected, maneuverability considerations made it necessary to investigate thoroughly all possible sources of reactivity change during normal operation.

In experiments performed at low power (1 to 25 watts), a negative effect up to about 23¢ was observed as the flow rate was varied between zero and rated flow (Fig. 20.9). The time constant of this effect appeared to be less than 4 sec.

An analysis of the test results and subsequent experiments indicated that these reactivity variations were probably the result of slight movements of the core matrix structure. In gas-flow tests, pressure gradients were found between the side reflector and the core matrix regions; these gradients caused the outer part of the matrix to bow toward the reflector. The maximum deflection of the matrix into the gap between the core and reflector was about 0.08 in. With the use of PTR reactivity coefficients, a reactivity decrease of about 30¢ due to bowing was calculated to be possible. After studying (1) the effect of the flow-induced reactivity changes on the control system and (2) the possibility of contact between the control cylinders and the core matrix at higher power levels, the existence of the flow effect was judged not to be serious.

Other experiments were made at low power to measure the effect of primary-coolant-system pressure on the reactivity of the clean critical SIG.

Fig. 20.9—Effect of coolant flow rate on reactivity.

The results obtained at both high and low flow rates indicated the existence of a slight positive pressure effect on reactivity. A positive pressure effect of 1.5¢ to 2.0¢ was found in going from 35 to 100 psig. It was concluded that the operation of the reactor would not be influenced greatly by pressure changes in the sodium.

20.7 TEMPERATURE- AND POWER-COEFFICIENT MEASUREMENTS

(Similar material on temperature and power coefficients appears in Chaps. 27 through 37. The emphasis here is on the over-all nuclear testing; whereas in these other chapters the subject is examined in its more specialized aspects.)

A nuclear characteristic of primary importance to the entire power plant from the standpoint of maneuvering stability and safety is the power coefficient of reactivity. To ensure satisfactory maneuvering stability in S1G, it was necessary for the over-all power coefficient to be negative. From the standpoint of safety during rapid maneuvers, it was very desirable to have a fast coefficient component which was negative or at least not strongly positive. Reactor kinetics studies using the pile simulator have shown that

the plant can be maneuvered satisfactorily providing the fast coefficient does not exceed a positive value of about $0.02\text{¢}/^{\circ}\text{F}$ increase in fuel temperature.

Prior to any high-power operation of S1G, a measurement of the so-called "isothermal temperature coefficient" was made. As used in this chapter, the temperature coefficient is associated with changes in reactor temperature induced by changes in sodium temperature made externally to the core as contrasted to the power coefficient which is associated with temperature changes initiated by heat generation in the fuel. Although the temperature coefficient is not strictly applicable to the normal situation during a power maneuver, it is a convenient criterion since it can be measured at low power or even when the reactor is subcritical if desired. The isothermal-temperature-coefficient measurement has provided an easy method for continual low-power monitoring during the reactor life to detect any drift in the temperature coefficient and possibly to warn of unexpected changes in the magnitude of the over-all coefficient.

During the approach to and initial operation at rated power, a variety of techniques were used to measure the S1G power coefficient, and these tests were repeated periodically throughout the core life to observe any changes in the coefficient. Such tests involve basically a comparison of the reactor transfer function or the neutron-level response at low power with similar characteristics at higher powers.

Both the temperature and power coefficients consist of several contributing components, each having a particular algebraic sign, magnitude, and response time. They include

1. Structural expansion effects. The expansion of the core and reflector with increasing temperature has several effects on reactivity. (See Chap. 29 for quantitative calculations.) The various contributions are difficult to calculate because of their complicated dependence on the mechanical construction of the core.
The geometrical expansion causes the reactor buckling to decrease with a corresponding reduction in neutron leakage. This, of course, has a positive effect on reactivity. On the other hand, the expansion decreases the density of all materials in the reactor and results in increases in neutron age and diffusion length. This latter effect tends to increase the leakage or reduce reactivity and outweighs the positive buckling effect. Mechanical motion of the control cylinders relative to the core may cause additional reactivity changes. The time of response for various structural expansion effects to occur after a power-level change should vary between 10 sec in the core and 30 sec in the reflector.
2. Sodium-expansion effects. Since sodium has a positive reactivity effect in the S1G core and reflector of about $\$6.30$ (i. e., moderation properties outweigh absorption), expansion of sodium on heating reduces the number of sodium nuclei in the reactor and results in a decrease in reactivity. The response time for the major portion of the sodium in the core should be between 0.2 and 0.6 sec, a fairly fast coefficient.
3. Thermal-base effect. As the temperature of the beryllium moderator increases, the thermal-neutron temperature distribution shifts upward and results in temperature or power effects on reactivity. Since an appreciable fraction of the sub-cadmium fissions in the S1G core is due to neutrons thermalized in the reflector and scattered back to the core, a decrease in the reflector steel absorption cross-section with temperature may have a positive effect on reactivity. This effect, in the case of the reflector structure, can lag substantially behind the core temperature.
The most serious potential positive thermal-base effect is related to Xe^{135} . The peak of the xenon resonance is at such low energy, 0.08 ev, that a temperature increase of the core moderator causes the xenon-to-fission cross-section ratio to decrease and produce a positive reactivity contribution. Similar effects, perhaps negative, may arise from the variation in α for U^{235} with energy and also with the accumulation of other fission products. Nearly 25 per cent of the beryllium in the core is in direct contact with sodium; this results in a response time of about 1 sec for the thermal-base effect.
4. Doppler effect. (See Chap. 32.) The Doppler coefficient is important because of the large changes possible in fuel temperature in extremely short times. This coefficient results from the broadening of the U^{235} cross-section resonances with

increased fuel temperature. A positive contribution is made by the fission cross-section; whereas a negative contribution is obtained from the absorption cross-section. Experimental measurements in the PTR indicate the over-all Doppler effect to be negative in the clean reactor. (See Chap. 33.) The response to this effect is for all practical purposes nearly instantaneous with power.

Because of the importance of determining temperature- and power-coefficient information, particularly concerning fast-response components, the several techniques that have been employed on S1G are discussed individually.

20.8 ISOTHERMAL TEMPERATURE COEFFICIENT¹⁸

For this test the reactor power was adjusted to 1 kw with the average reactor temperature at 500°F and sodium flow at 63 per cent rated flow. Since at this low power essentially no heat was being generated in the core, temperature had to be maintained and varied, when required, by sources external to the reactor. One source of heat was the electrical heating units on the primary-coolant system. Heat was also obtained from the electromagnetic pumps by operating them at high flow rates. Under these conditions it was possible to increase the average reactor temperature at a rate of 10°F/hr. During this test water was present in the steam generators. The control-cylinder motion necessary to maintain a constant neutron level was recorded as a function of the reactor temperature. Because of the small changes in reactivity, one cylinder was adjusted at a time to minimize position uncertainty. The temperature coefficient was calculated from the measured over-all reactor temperature increase and the known control-cylinder sensitivity. The observed isothermal temperature coefficient of reactivity at low power was found to agree better with predicted coefficient than one might expect from the uncertainties in the calculations. A breakdown of the predicted result is given in Table 20.3.

TABLE 20.3--PREDICTED ISOTHERMAL TEMPERATURE COEFFICIENT (CLEAN CORE)

Contributor	Coefficient, ¢ /°F
1. Structural expansion ¹⁹ (Chap. 29)	-0.17
2. Sodium expansion ¹⁸ (Chap. 28)	-0.09
3. Thermal-base effect (cf. Chap. 28)	0.10
4. Doppler effect ²⁰ (Chap. 33)	-0.04
Over-all coefficient	-0.20

20.9 STEADY-STATE TEMPERATURE COEFFICIENT

The steady-state temperature coefficient differs in principle from the isothermal temperature coefficient in that the temperature distribution throughout the reactor core resembles the temperature distribution during normal-power operation. These somewhat more refined temperature coefficient measurements were made at 50 and 100 per cent rated power by varying the reactor inlet sodium temperature. All tests were performed with rated ΔT across the reactor, with constant sodium flow and with different xenon concentrations. The inlet sodium temperature was decreased about 50°F by lowering pressure in the steam generators. Control-cylinder motion was used to measure the associated reactivity change.

Figure 20.10 shows the experimental results as a function of xenon concentration. For a given measurement the analysis was complicated by the xenon buildup during the finite time required for the change in inlet temperature. This correction for xenon was of the same magnitude as the change in reactivity with temperature. The fact that the temperature coefficient for both the 50 and 100 per cent power runs decreased with increasing xenon concentration indicates the xenon contribution to be a positive coefficient. The apparent increase in temperature coefficient in going from 50 to 100

Fig. 20.10—Steady-state temperature coefficient as function of xenon concentration. Values are given relative to the zero power value taken as -1.0 .

per cent power has not been explained; possibly it was due to experimental errors. The temperature coefficient had been expected to become less negative with increasing power level because of the logarithmic relation of the Doppler effect with temperature.²⁰

20.10 POWER COEFFICIENT BY PILE-OSCILLATOR TECHNIQUE²¹

A series of power-coefficient measurements was made on S1G at six power levels before full power was reached. Power coefficients were measured by means of pile-oscillator techniques, with the reactivity variation being introduced through small oscillations of a control cylinder. (See Chaps. 34 and 35 and also reference 22.) In essence, this technique involves the measurement of the reactor transfer function at different power levels. The changes occurring in the reactor transfer function as power is increased result from the effects of the various reactor temperature coefficients.

The response of a reactor with negative temperature-coefficient feedback to a small sinusoidal variation in reactivity is given by

$$\frac{\delta n}{n_0} = \frac{A(j\omega)}{1 - A(j\omega)\Gamma(j\omega)n_0} \delta k \quad (20.7)$$

where $\delta n/n_0$ = fractional power change

δk = oscillating reactivity ($\$$)

$\Gamma(j\omega)$ = power coefficient of reactivity

$A(j\omega)$ = low-power reference transfer function

When the reference transfer function is obtained at low power, the thermal response of the reactor is negligible. The clean S1G reference function that was observed checked well with calculations and with the PTR reference function (Chap. 34 and reference 23.)

Solving Eq. 20.7 for the temperature effect $\Gamma(j\omega)n_0$, we have

$$-\Gamma(j\omega)n_0 = \frac{\delta k}{\delta n/n_0}(j\omega) - \frac{1}{A(j\omega)} \quad (20.8)$$

To obtain an average power coefficient in units of dollars per megawatt, $\Gamma(j\omega)n_0$ should be divided by the power change in megawatts between the low- and high-power runs.

The small reactivity oscillations (δk) of the control cylinder were recorded from the output of a selsyn transmitter in the control drive mechanism. To provide the best comparison of experimental results with calculated results, a maximum reactivity oscillation of $\pm 4\phi$ was employed. For safety reasons mechanical stops were used in the control drive mechanism which set 2 deg as the maximum possible full stroke obtainable. An ionization chamber was used as the sensing element for the neutron level or $\delta n/n_0$. A description of the apparatus employed and the results are given in Chap. 35.

20.11 POWER COEFFICIENT BY ROD-DROP TECHNIQUE

The rod-drop technique for measuring the power coefficient (Chap. 36 and reference 24) resembles the pile-oscillator method in that it is a comparison of low- and high-power characteristics, but in this case it is a measurement of neutron-level response to a "drop" of the control cylinder through a limited 2-deg stroke. Basically this method involves calculation of the $\Delta k(t)$ required to produce the observed neutron-level response during and immediately following the drop. The difference between the calculated Δk 's obtained from a low-power reference run and from a high-power run is then a measure of the reactivity effects of temperature changes during the high-power drop.

$$\Delta k_T(t) = \Delta k(t) - \Delta k_0(t) \text{ and } \frac{\Delta k_T(t)}{\Delta P(t)} = \text{power coefficient } (\phi/\text{Mw}) \quad (20.9)$$

where $\Delta k_T(t)$ = reactivity resulting from temperature changes in reactor components during a rod drop

$\Delta k(t)$ = total reactivity change (control element and temperature effects) as calculated from neutron-level response at a given power level

$\Delta k_0(t)$ = calculated reactivity change caused by control element during rod drop at low power with no temperature effects

$\Delta P(t)$ = power change resulting from $\Delta k_0(t)$

Figure 20.11 shows typical results in terms of power coefficients, i.e., cents per megawatt power change, for reactor operation at 40 per cent rated power, 500°F average reactor temperature, and 100°F ΔT . The power coefficient varies with time after drop because various core components respond to a change in power with different time constants. The initial buildup of xenon appears to make the coefficient somewhat less

Fig. 20.11—Power coefficients after 30,000 Mwh of operation. Results of rod-drop measurements made after 0, 30, and 237 hours of continuous operation.

20.12 . REACTIVITY EFFECT OF FUEL DEPLETION AND FISSION-PRODUCT ACCUMULATION

negative. As in the case of pile-oscillator tests, an increase in the prompt negative effect has been observed after periods of extended operation. The coefficients in Fig. 20.11 were obtained after about 30,000 Mwh of operation and are not appreciably different from results for similar reactor conditions during the initial extended full-power operation.

20.12 REACTIVITY EFFECT OF FUEL DEPLETION AND FISSION-PRODUCT ACCUMULATION²⁵

Measured reactivity defects (decrease in reactivity) and defect rates in the S1G during initial power operation were due to fission-product buildup, fuel depletion, and Li^6 accumulation in beryllium. The reactivity defect was followed as a function of time by observing the compensating motion required on the six control cylinders. Cylinder positions were converted to reactivity units by using the control curve (Fig. 20.8), and these values were subsequently corrected for the sodium-flow effect (Fig. 20.9) and for the temperature effect.

The defect rate is of primary importance in predicting reactor endurance. In order to determine the reactivity defect rate, it was necessary to log the S1G power history by making periodic sodium heat balances around the reactor. Power was calculated as

$$P = 2.348 \rho C_p F \Delta T \quad (20.10)$$

where P = power (watts)

ρ = sodium density at average reactor temperature (lb/cu ft)

C_p = specific heat of sodium at average reactor temperature (Btu/lb/°F)

F = sodium-flow rate (gpm)

ΔT = sodium temperature rise across reactor (°F)

Power and reactivity defect variations during a typical one-week period of operation are shown in Fig. 20.12.

Fig. 20.12—Power and reactivity defect variations during one week of S1G operation. Reactivity defects were calculated from control cylinder settings. The reactor was shut down for about three days during this period.

Fig. 20.13—Growth of reactivity defect with reactor operation. Xenon effects have been taken out; what is shown here is the result of fuel depletion and non-xenon fission-product buildup.

Determination of the defect rate was complicated by the existence of major fission-product poisons having transient characteristics, namely, Xe^{135} and Sm^{149} . The best method found for obtaining the non-xenon defect rate was to determine as a function of the accumulated megawatt hours of operation the reactivity defects at reactor startups after periods of extended shutdown where little or no xenon was present. Based upon the first 45,000 Mwh of operation, the non-xenon defect rate was 12.5¢/1000 Mwh as shown in Fig. 20.13. The non-xenon, non-samarium defect rate was calculated to be 9.3¢/1000 Mwh by correcting the measured rate of 12.5¢/1000 Mwh for the initial transient behavior of Sm^{149} , using PTR reactivity-coefficient data obtained by King (Chap. 14.) Since the Sm^{149} concentration will begin to approach saturation after about 1000 hr of full-power operation, the over-all non-xenon defect rate should decrease from 12.5¢/1000 Mwh in the clean reactor toward 9.3¢/1000 Mwh in the depleted reactor.

The measured non-xenon defect rate agrees reasonably with the sum of the various reactivity loss contributions expected in the clean reactor (Table 20.4). The loss due to fuel depletion has been evaluated quite accurately in the PTR mock-up of the clean S1G to be 2.6¢/1000 Mwh and should not be subject to any large uncertainty. The poisoning effect of the buildup of Li^6 in the beryllium has been calculated (Chap. 26) to be 1.1¢/1000 Mwh; again, this can add only slightly to the uncertainty of the total defect rate. There is, however, considerable uncertainty as to the poisoning effect of the fission products.

A straight theoretical computation (Chap. 12) of the fission-product poisoning, assuming that the nuclear level widths and spacings of fission products are distributed statistically much as in other materials, has led to an initial defect rate of 7.2¢/1000 Mwh. Of this 2.4¢/1000 Mwh is due to Sm^{149} , and 4.8¢/1000 Mwh is due to the other non-xenon fission products. The defect rate diminishes to 5.6¢/1000 Mwh as the reactor becomes depleted and the Sm^{149} approaches saturation. This treatment is truly statistical only for the high-energy resonances, and special effects have to be considered for the important low-energy resonances such as in Sm^{149} .

Reactivity coefficient measurements in the PTR (Chap. 14) on a few fission-product nuclides, combined with judicious extrapolation of such measurements on a number of elements in the fission-product region, have been used to estimate an average fission-product defect rate of 4.7¢/1000 Mwh over the core life. Of this, 4.7¢, 1.3¢ is due to Sm^{149} , and 3.4¢ is due to the remaining non-xenon fission products. As can be seen in

Table 20.4, the defect rates that are predicted by these two methods are somewhat lower than the measured value.

Reactivity observations carried on for more than 75,000 Mwh of operation, indicated that the defect rate was maintained at about the initial 12.5¢/1000 Mwh value. Inasmuch as the Sm^{149} is bound to approach saturation after long operation, the other sources of reactivity loss must have increased to offset the decreasing Sm^{149} contribution. Fuel depletion seems most likely to have done this.

TABLE 20.4--REACTIVITY DEFECT RATES: EXPECTED CONTRIBUTIONS VS VALUES MEASURED IN S1G

Source of defect	Clean value, ¢/1000 Mwh	Depleted value, ¢/1000 Mwh	Average value, ¢/1000 Mwh
Fuel depletion (PTR data)	2.6	4.3	3.5
Li^6 buildup (calculated)	1.1	1.1	1.1
Sm^{149}	2.4	0	1.3
Other fission products (except xenon)	4.8	5.6	3.4
Expected totals	10.9	11.0	9.3
S1G measured values, ¢/1000 Mwh	12.5	12.5	12.5

*Calculated (Chap. 12).

†PTR data (Chap. 14).

The clean full-power steady-state xenon defect was measured as \$2.30 by comparing cylinder positions after an extended run at full power with the critical position at the next startup. This value is in good agreement with the prediction from multigroup calculations by Hofmann and Nevins²⁶ of \$2.14. The peak of the xenon transient appeared to occur about 3 hr after shutdown and was only 10¢ greater than the steady-state defect.

Any prediction of reactor endurance, defined as the total energy output accumulated before the reactor loses the ability to override peak xenon following a shutdown from full power, has at this point the following inherent uncertainties:

1. Defect rate. The non-xenon, non-samarium defect rate of 9.3¢/1000 Mwh as measured in the clean reactor may not be constant during reactor life because of the changing neutron spectrum.
2. Control curve. The S1G excess reactivity above clean critical and that available for endurance must be based on PTR measurements where fission products are mocked up by cadmium.
3. Transient poison reactivity. The reactivity allowances in the depleted reactor for Xe^{135} and Sm^{149} are based upon PTR measurements only.

20.13 POWER DISTRIBUTION

As yet no complete set of power-distribution data has been obtained in the S1G. Other than the scanning of individual elements during dry power level calibration, only four assemblies have been removed and scanned. The burnup in these four was relatively low since they had been in the core an equivalent of slightly more than 25,000 Mwh. The longitudinal scans showed depressions in activity at the center of the fuel region and about halfway toward either end, as appear in Fig. 20.14. These depressions were caused by steel reinforcements in the fuel assemblies. The actual activity without the structural steel would presumably be a smooth curve as estimated by the dashed line. Unfortunately Fig. 20.14 is not a true power distribution; for it has been found that there was a high variable background count related to the position of the assembly being scanned, which tended to increase the values at the ends of the assembly more than it did at its center.

Fig. 20.14—Longitudinal scan of fuel assembly. See text, Sec. 20.13, for explanation of the dips in the curve.

20.14 SUBSEQUENT TESTING

Some testing was continued through the core life of S1G since critical experiments can provide only limited information on the effects of fission-product accumulation and fuel depletion on certain reactor characteristics. In particular, control-element calibrations were performed after every 10-deg in-phase motion owing to non-xenon fission products and to fuel depletion; the isothermal temperature coefficients and sodium-flow coefficients were monitored monthly; the reactivity defect as a function of integrated power operation was followed; and power coefficient measurements were taken at intervals of 30,000 Mwh. This continued testing served in general to substantiate the results described in this chapter.

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Part III

**HEAT GENERATION AND
NUCLEAR MATERIALS PROBLEMS**

J. R. STEHN

Chapter 21

POWER-DISTRIBUTION MEASUREMENTS ON CRITICAL ASSEMBLIES

G. B. GAVIN, E. B. FEHR, and R. T. FROST

21.1 INTRODUCTION

For reactor analysis and design purposes the power distribution as well as a variety of other reactor characteristics must be known as a function of control-element position and of fuel burnup. To study the significant aspects of these characteristics, both in the quasi steady-state condition and as a function of time, two critical assemblies were constructed. One assembly (PPA-20, 66-deg position) simulated a clean reactor, and the other (PPA-19, 106-deg position) simulated a depleted reactor.

The results of the fission-rate measurements in each assembly have direct application in (1) choosing the sizes of the coolant orifices, (2) analyzing differential stresses due to heat gradients, and (3) evaluating fuel burnup and reactor endurance.

Later work was done in the PTR. A sample of the results of such work is also included in this chapter. In general, the PTR results verified those obtained from the PPA's.

21.2 CRITICAL ASSEMBLIES USED

The PPA-20 was a mock-up of a clean reactor having about 50 kg of fuel, but it had approximately \$1 worth of cadmium to make it possible to locate all six of the control cylinders at a 66-deg position. (The clean reactor would be critical with all six control drums rotated 58 deg from their least reactive, or boron-toward-core, position. The hexagonal matrix of the PPA's allows them to mock up cylinder positions only at 6, 26, 46, 66, ... 126 deg. The cadmium added allowed use of the nearest possible angle to 58 deg.) The cadmium was in the form of slivers and was loaded in the interstitial region of the tube matrix. Because of the presence of the cadmium, PPA-20 simulates the reactor somewhere between clean start-up and the steady-state xenon condition. The PPA-19 contained approximately 45 kg of fuel and \$8 worth of cadmium, which corresponds to the same reactor with steady-state xenon after about 100,000 Mwh of operation. Since in this depleted condition the control cylinder would have been rotated to move the boron away from the reactor core, no attempt was made to mock the control cylinder in detail in any sector of the PPA-19. Three control cylinders shown in Fig. 21.1 indicate the 66-deg location of the control cylinders in the PPA-20; another three cylinders show the 106-deg position for the PPA-19 mock-up. (No actual critical assembly, of course, had such a mixed set of control-cylinder positions.)

Except for the fuel and cadmium in the reactor core, the compositions of the PPA-19 and PPA-20 were practically the same. The detailed compositions are given in Table 21.1, wherein the abbreviation SS is used for Stainless Steel.

The sample of PTR data was taken on a modification of PTR-3 that was intended to simulate the depleted condition of a reactor, with control cylinders set at 126 deg. The composition of this reactor is also included in Table 21.1. The cadmium in it was in the form of 0.6-mil-thick plating considerably thinner than the slivers used in the PPA's; hence it was more effective as a poison in the PTR.

Fig. 21.1—Two halves of equatorial plane.

TABLE 21.1--CORE COMPOSITIONS

Material	PPA-20, 66 deg	PPA-19, 106 deg	PTR-3, 126 deg
Be, moles	30,248	30,248	29,962
SS, moles	3,322	3,327	3,386
Mg, moles	3,173	3,638	2,256
Al, moles	3,842	3,354	4,049
U ²³⁸ , moles	14.6	12.62	14.8
U ²³⁵ , moles	220	192.16	202.5
Cd, moles	9.4	52.9	12.75
Poison value of Cd, \$	1	8	9

21.3 TECHNIQUES OF POWER-DISTRIBUTION MEASUREMENT

The methods used for comparing the local fission rates in these assemblies evolved from simple activation techniques. Basically, a measurement of the fission-product activity in PPA fuel disks was used to determine the power-distribution characteristics because it is the most direct and probably the most accurate type of measurement. The integral type disk-activation measurements were effected by placing standard PPA fuel disks at appropriate positions throughout the reactor core, removing these disks after an exposure, and counting the gamma activity induced in the disks. Such measurements integrate the power distribution over each 2-inch diameter fuel disk used and are accordingly too coarse-grained to represent steep power gradients accurately.

Two special techniques were developed to show small-scale power variations, at least on a relative basis. These techniques had the common feature that they measured the activity variations along U^{235} strips that had been irradiated in the assembly. The thickness of these strips was such that they had the same self-shielding as the fuel. Some strips were placed along radial lines, as in Fig. 15.6, on the mid-plane interface (between the two halves of the assembly). Other strips were inserted axially along the PPA fuel tubes at the edges of the fuel disks.

The first technique¹ was to compare the blackening of photographic film on which the U^{235} strips were laid after the reactor was shut down. This radioautographic technique, which was used in the PPA-19 measurements, suffered from the usual large errors of film density measurements.

The second technique²⁻⁴ was the use of an automatic "scanner" which measured the beta activity of successive narrow portions of the strip. This scanner was used in the PPA-20 measurements; it was designed to have high resolution, to reproduce results well, to be fast in operation, and to correct automatically for the time decay factor.

In all these measurements the data obtained by the different techniques have been intercompared, and the most reasonable results have been chosen for the power distribution given in subsequent figures. Table 21.2 summarizes the techniques used in this power distribution study.

TABLE 21.2--TECHNIQUES EMPLOYED FOR FISSION-RATE MEASUREMENTS

	Fuel-disk counting ⁵	Fuel-strip radioautograph ¹	Fuel-strip counting ²⁻⁴
Irradiated material	PPA fuel disk highly enriched in U^{235}	U^{235} Al strip, 31% U^{235} by weight	U^{235} strip, highly enriched
Dimensions of irradiated material	2-in.-diameter disk, 40-mil thickness	Rectangular strips $1/2 \times 8$ in., 7-mil thickness	Rectangular strips $3/4 \times 20$ in., 3-mil thickness
Effective U^{235} thickness	0.115 gm/cm ²	0.115 gm/cm ²	0.156 gm/cm ²
Radioactivity detected	Fission-product γ decay	Fission-product β decay	Fission-product β decay
Detecting instrument	End-window Geiger counter or Scintillation counter	Type K, x-ray film and densitometer	End-window Geiger counter
Geometric resolution	Area of PPA fuel disk, 2 in. in diameter	Densitometer aperture, usually 1/8-in. diameter	Adjustable slit usually $3/16 \times 3/4$ in.
Information obtained	Absolute fission rate per disk	Gradient details and relative power distribution	Gradient details and relative power distribution

A. Normalization of Results

In presenting the results of fission-rate measurements, it has been the practice at KAPL to establish the fission rate of the central PPA fuel disk as unity and to normalize the fission rates at all other disks to the central value. This is convenient for illustrative purposes and it also permits an absolute evaluation of the reactor power level in terms of a calibrated PPA fuel disk located at the center of the reactor.

The integral fission rates per fuel disk in the peripheral slugs of the core, however, do not represent the fission-rate gradients which are developed across the disk. To indicate the shape and magnitude of the radial gradients and to maintain general consistency, the results of the measurements along the last 2 in. of the radial traverses have been renormalized so that (1) the slope of the gradient is based on the differential

21.4 RELATIVE POWER DISTRIBUTIONS

fuel-strip measurements and (2) the area under the more finely resolved curve is equivalent to the result of the integral measurement over the 2-in. disk.

21.4 RELATIVE POWER DISTRIBUTIONS

A. Radial Power Gradients

A list of maximum power gradient indices for the peripheral fuel slugs in each assembly is given in Table 21.3. The indices indicate the ratio $\Delta f/\bar{f}$, where Δf is the maximum change in the fission rate across the PPA fuel slug and $\bar{f}$ is the average fission rate developed by the slug when the control cylinders are at the 106-deg position. The 106-deg position was chosen as a reasonable compromise between the initial 66-deg position and the 140-deg position of maximum reactivity.

TABLE 21.3--POWER GRADIENT INDICES

PPA-20 (66 deg)				PPA-19 (106 deg)			
Slug	Traverse	Δf	$\Delta f/\bar{f}$	Slug	Traverse	Δf	$\Delta f/\bar{f}$
2	B	0.06	0.10	151	G	0.23	0.37
3		0.31	0.48	163		0.20	0.30
4		0.51	0.61	172	H	0.38	0.46
9	C	0.38	0.43	171		0.47	0.53
18		0.26	0.34	97	E	0.32	0.41
13		-0.26	-0.38	110		0.25	0.36
5	D	-0.41	-0.64	124	F	0.25	0.39
1		-0.35	-0.59	138		0.23	0.40

The indices in Table 21.3 are given for successive adjacent fuel slugs around the periphery of the core, going from right to left on the power contour diagrams of Figs. 21.2 and 21.3.

Fig. 21.2—Power contours in equatorial plane, clean reactor (PPA-20).

Fig. 21.3—Power contours in equatorial plane, depleted reactor (PPA-19).

B. Longitudinal Power Variations

The longitudinal distributions of the relative power which was developed in several fuel slugs in PPA-20 (66-deg position) and PPA-19 (106-deg position) are shown in Figs. 21.4 and 21.5. These slugs were located in the half of the PPA's that corresponds

Fig. 21.4—Longitudinal power distribution, clean reactor (PPA-20), in slugs 4, 38, 51 and 91 of Fig. 21.1.

Fig. 21.5—Longitudinal power distribution, depleted reactor (PPA-1).

to the lower half of the reactor. They are identified by the numbers shown on Fig. 21.1. The relative power developed along a longitudinal plane in each assembly is shown by contour curves in Figs. 21.6 and 21.7. The planes for which the contour curves are representative intersect the equatorial plane along the lines C and H shown in Fig. 18.6.

Fig. 21.6—Power contours, clean reactor (PPA-20) along longitudinal plane, traverse C.

Fig. 21.7—Power contours, depleted reactor (PPA-19) along longitudinal plane, traverse H.

Fig. 21.8—Longitudinal power distribution in PTR-3.

21.4 RELATIVE POWER DISTRIBUTIONS

The relative power generation along the entire lengths of two PTR fuel elements is shown in Fig. 21.8. The power development was not symmetrical, for there was an appreciably smaller amount of reflecting material on top of the reactor (the location of the mocked-up sodium entrance and exit flow pipes) than there was on the bottom of the reactor.

C. Radial Power Variations

The distribution of the relative power developed along the equatorial plane in two radial directions in one assembly is shown in Fig. 21.9. These two and the other traverses studied for the two reactors are indicated by the lines marked A through H on Figs. 18.6, 21.2, and 21.3. The relative power developed throughout the equatorial plane in each assembly is illustrated by contour curves in Figs. 21.2 and 21.3.

The relative power developments along the four equatorial plane traverses of PTR-3, J, K, L, M of Fig. 18.6, are shown in Fig. 21.10.

Fig. 21.9—Radial power distribution in the equatorial plane, PPA-20.

Fig. 21.10—Radial power distribution in equatorial plane, PTR-3.

21.5 ABSOLUTE POWER LEVEL

The absolute reactor power level in each assembly is not of direct interest since the operating level is arbitrary and does not alter the general characteristics of the power distribution. In order to compare the absolute fission rate at any point in the reactor for different control-cylinder positions, however, it is necessary to normalize the total relative distribution in each assembly to a common operating level. The following ratios are therefore of interest.

	PPA-20 (66 deg)	PPA-19 (106 deg)
$\frac{\text{Total reactor fission rate}}{\text{Fission rate of the central fuel disk}}$	$(1.4 \pm 0.05) \times 10^4$	$(1.3 \pm 0.05) \times 10^4$
$\frac{\text{Fission rate of the central fuel disk}}{\text{Average fission rate per disk}}$	1.6 ± 0.05	1.5 ± 0.05

For typical measurements the PPA was operated at a power level such that the absolute fission rate induced in the central fuel disk was approximately 2×10^6 fissions per second, corresponding to a normal operating level slightly below 1 watt.

A. Estimating Total Power from Mid-plane Measurements

More emphasis should be placed on measuring the variations in power density across the equatorial plane of the reactor than on measuring its longitudinal variations. This is due to the following two circumstances:

1. Although the shape of the radial power distributions varies strikingly from one traverse to another, the shape of the longitudinal power distribution differs comparatively little from one slug to another and can readily be estimated from that of neighboring slugs.
2. Thermal stresses arising from radial gradients in the heat production, particularly in the integrated heat production of an entire fuel slug, require more engineering-design attention than those arising from longitudinal gradients.

Since it required less experimental time to measure the radial power distribution in the equatorial plane of a PPA than to measure the longitudinal one, it was of interest to correlate the integral power characteristic of each fuel slug with the power developed by each slug at the equatorial plane. The ratio of $\bar{f}$ to f_1 is given for several slugs in Table 21.4 where $\bar{f}$ is the average fission rate of the slug and f_1 is the fission rate of the disk 0.2 in. from the equatorial plane.

With the exception of the fuel slugs in the peripheral ring near the boron, it is apparent that the ratio is practically constant to within experimental accuracy. The assumption that this ratio is constant permits quick and reliable surveys of the integral power characteristics of various assemblies with a minimum of reactor operating time.

TABLE 21.4--COMPARISON OF EQUATORIAL POWER AND AVERAGE POWER

PPA-20 (66 deg)		PPA-19 (106 deg)	
Slug	$\bar{f}/f_1$	Slug	$\bar{f}/f_1$
1	0.84	138	0.80
2	0.82	151	0.81
3	0.81	163	0.85
4	0.83	172	0.83
5	0.87	124	0.80
6	0.92	137	0.86
9	0.89	171	0.85
13	0.87	110	0.86
16	0.87	149	0.87
18	0.88	97	0.88
24	0.88	122	0.87
27	0.85	96	0.86
49	0.89	94	0.87
50	0.90	107	0.88
91	0.89	91	0.87
92	0.88	92	0.87

A more thorough comparison of the relative power distributions along the equatorial plane appears on Fig. 21.11 for the PTR study. The analogous figures for the power values averaged over the whole lengths of the PTR fuel slugs appear on Fig. 21.12.

B. Experimental Uncertainties

A sufficient number of counting events were obtained for each activation measurement to reduce statistical uncertainties to less than 1 per cent. Instrument instability, particularly in high-voltage regulation, limited reproducibility to between 1 and 2 per cent. Small variations in the location of the samples in the counters as well as in the desired core position could introduce an additional error of about 1 per cent. Measurements are generally good to about 3 per cent, with a maximum uncertainty of about 5 percent.

Fig. 21.11—Radial power distribution, values in the equatorial plane, PTR-3.

Measurements of the radial gradients in PPA-20 were reproducible to about 1 per cent. The radial gradients in PPA-19 were not checked for reproducibility but on the basis of similar data are believed to be correct within 5 per cent. The difference in precision is attributed to the superiority of the strip-scanning technique. The normalization of the radial gradient data obtained from the strip measurements to the results of the disk measurements can be in error by a few per cent because a slightly different volume element is activated in each set of data. (A common volume is required to make the two sets of data completely consistent.)

Interpolations and a numerical integration of the experimental data give a value for the integrated power distribution which is uncertain by 3 or 4 per cent. The uncertainty in the fission rate per unit power throughout the core is therefore about 5 per cent. In the region along the core reflector boundary where the large gradients exist, the fission rate per unit power is subject to large uncertainties if applied to smaller elemental volumes than normally occupied by a fuel-disk element. The uncertainty associated with this smaller resolution based on the strip scanning measurements should not exceed 10 per cent.

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Fig. 21.12—Radial power distributions, average fuel slug values, PTR-3.

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Referred to in section

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Chapter 22

GAMMA-RAY AND NEUTRON HEAT GENERATION

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Chapter 22

GAMMA-RAY AND NEUTRON HEAT GENERATION

INTRODUCTION

H. HURWITZ, Jr., and M. L. STORM

22.1 INTRODUCTION

The determination of the magnitude and distribution of the heat generation within a reactor is obviously essential to the design of the reactor cooling system. During steady-state operation at full power, the major part of the energy released by fission is deposited within the fuel-containing regions and a large fraction of this core heat generation arises from the conversion of the kinetic energy of the fission products into heat. On the other hand, the heat generation in the non-fuel-containing regions (e.g., reflector and blanket) arises from gamma-ray absorption and neutron slowing-down and capture processes.

For example, in a SIR-type reactor during steady-state operation, approximately 94 per cent of the total heat generation occurs in the core, 4 per cent occurs in the reflector, and 2 per cent in the blanket. A summary of the various energy sources and their fractional contributions to the heat generation in each region is given in Table 22.1.

TABLE 22.1 - FRACTIONAL CONTRIBUTION OF VARIOUS ENERGY SOURCES
TO THE HEAT GENERATION IN EACH REGION OF AN INTERMEDIATE REACTOR
(FOR STEADY-STATE OPERATION)

Energy Source	Region		
	Core	Reflector	Blanket
Kinetic energy of fission products	0.88	---	---
Fission product betas	0.04	---	---
Neutron degradation and (n, α) reaction in beryllium	0.02	0.27	---
Prompt fission gammas and capture gammas	0.04	0.57	0.89
Fission-product gammas	0.02	0.16	0.11

For thermal design purposes, the heat generated by gamma-ray absorptions must be known, not only in the reflector and blanket during operation at power, but also over the entire reactor during postoperation shutdown when the fission-product gamma-rays are the principal source of heat.

The spatial distribution of heat generation due to the fission-product betas and the kinetic energy of the fission products is given essentially by the fission distribution and as such is obtained by the experimental techniques described in Chap. 21 of this volume and by the multigroup calculations described in Vol. I, Chap. 2. In this article we consider the heat generation in SIR-type reactors produced by gamma-ray absorption and by neutron degradation and capture. Specific application is made to the case of steady-state full-power operation, but the methods employed are also applicable to the determination of transient heating effects.

In A of this chapter (Secs. 22.2-22.9), a general discussion is given of the problem of calculating the absorptions resulting from various surface and volume sources of gamma radiation in plane, spherical, and cylindrical geometries. The gamma-ray absorption problem is treated by the "straight-ahead" approximation (Secs. 22.2, 22.10, 22.11) and the assumption of exponential attenuation. A justification of this approach for the intermediate reactor is given in B (Sec. 22.10-22.13), where a gamma-ray multigroup calculation is used to determine effective energy absorption cross-sections. Finally in C (particularly Sec. 22.14-22.18), both the gamma-ray absorption distributions obtained in A and the effective cross-sections calculated in B are applied to the determination of the steady-state heat generation in a power reactor.

The calculations described in this article were performed in 1952, prior to the extensive use of large-scale computing machines which facilitate the treatment of multidimensional geometries. Thus, a necessary approximation in the heat-generation calculations was the use of multigroup data on neutron capture and degradation which were obtained for spherical representations of the reactor. The procedure employed, in general, was to treat various alternative simple models and compare the results, since machine techniques were not available for treating the actual geometry rigorously. However, the calculations performed for the various models were in reasonable agreement with one another, and the theoretically obtained heat generation agreed within experimental error with subsequent measurements (in 1953) of the gamma-ray heating at most points within the reactor. A comparison of the theoretical and experimental gamma-ray heating determinations is given in Chap. 23 of this volume.

Although machine methods are now available to handle the complicated geometrical considerations which arise in the determination of reactor heat generation, the analytical approaches described in this chapter are still useful. They predict absolute magnitudes and spatial distributions of heating rates that agree with experiment; and for preliminary design considerations they are preferable in their direct approach to a detailed program of machine calculations.

Chapter 22

GAMMA-RAY AND NEUTRON HEAT GENERATION

A. GAMMA-RAY ABSORPTION DISTRIBUTIONS FOR SIMPLE GEOMETRIES

H. HURWITZ, Jr., G. M. ROE, and M. L. STORM

22.2 INTRODUCTION

The design of reactor cooling systems is aided by a knowledge of the magnitude and distribution of the absorption of the gammas produced in the core, and of the secondary gamma radiation produced by neutron capture and inelastic scattering. In this chapter, the geometries of different regions of a reactor are idealized by considering plane, spherical, and cylindrical bodies with various surface and volume sources of gamma radiation. This idealization permits the analytic evaluation of most of the integrals arising in the absorption calculations. In addition, the "straight-ahead" approximation is assumed at all times; this assumes that, in all scattering events, the gamma-rays keep their direction unchanged (see Secs. 22.10 and 22.11). The gamma-ray attenuation is assumed to be exponential. The validity of these assumptions is considered in B, where the selection of effective energy-absorption coefficients is discussed.

For plane-geometry situations, the results of the distributed source calculations may be applied even when the regions involved have different absorption properties. For other geometries, the examples considered which involve distributed sources apply directly only to reactors in which the gamma-ray attenuation properties of the core and reflector regions are quite similar. On the other hand, problems which involve a core and reflector with different attenuation coefficients may be treated by first calculating the gamma-ray energy which escapes from the core and then considering this energy as a surface source at the core-reflector interface. Results are presented for several alternative simple surface-source angular distributions. The accuracy of this approach has been tested by comparing the results obtained in this way with distributed volume source calculations for the case of equal attenuation length in core and reflector. Agreement is found to be good, provided sensible choices are made for the angular distribution of the surface source.

As a guide for selecting mathematical functions to represent surface source angular distributions which occur in practice, it is helpful to investigate, in various limiting cases, the distribution of gamma-rays emitted from a surface A of a volume V containing a uniform isotropic source of radiation. Let dA , Fig. 22.1, be an element of area on A ; let x be the distance from a volume element dV in the source region to dA ; and let ϕ be the angle that the emergent beam makes with the normal to the surface. The solid angle $d\Omega'$ subtended at dV by the element of surface area is $d\Omega' = (dA \cos \phi)/x^2$, and the volume of an element in the source region is $dV = x^2 dx d\Omega$. Then the energy $E(\phi)$ emerging at dA per unit time into solid angle $d\Omega$ is given by the relation:

Figure 22.1

$$\begin{aligned}
 E(\phi) d\Omega dA &= \int_{x=0}^{x=x_{\max}} S dV d\Omega' e^{-\sigma x} \\
 &= S \int_0^{x_{\max}} dx x^2 d\Omega (dA \cos \phi / x^2) e^{-\sigma x} \\
 &= S d\Omega dA \cos \phi \left(1 - e^{-\sigma x_{\max}} \right) / \sigma
 \end{aligned} \tag{22.1}$$

where σ is the probability of absorption per unit length for a photon in V , and S is the gamma-ray energy emitted per unit volume, solid angle, and time.

Now consider the limiting cases when $\sigma x_{\max} \gg 1$ and when $\sigma x_{\max} \ll 1$. One then obtains from Eq. 22.1:

$$E(\phi) \sim \cos \phi \quad \text{for } \sigma x_{\max} \gg 1 \quad (\text{Lambert's Law}) \tag{22.2}$$

$$E(\phi) \sim x_{\max} \cos \phi \quad \text{for } \sigma x_{\max} \ll 1 \tag{22.3}$$

When A is the surface of a sphere of radius R , it is seen that $x_{\max} = 2R \cos \phi$ and hence, in the limiting cases of large and small σR , $E(\phi)$ is proportional respectively to $\cos \phi$ and to $\cos^2 \phi$.

When the surface A consists of two infinite planes enclosing a source region of width d , $x_{\max} = d / \cos \phi$. In this case the limiting forms for E are $E \sim \cos \phi$ for σd large, and E independent of ϕ for σd small. The latter case corresponds to the distribution for an isotropic plane source.

In Secs. 22.3 and 22.4, the cases of an infinite plane and a sphere with surface intensities of emitted radiation proportional to $\cos^n \phi$ ($n = 0, 1, 2$) will be considered. In Secs. 22.5 and 22.6, a region bounded by two infinite planes will be considered, first with an exponentially distributed source and then with a space-independent source of gamma-rays per unit volume. Finally, in Secs. 22.7 and 22.8, the cases of a sphere and cylinder with a constant source strength per unit volume will be treated under the assumption of equal mean free paths of the gamma-rays in the source and external region. In the cylindrical cases, the distribution is obtained along a radius of an infinite cylinder and along the axis of a finite cylinder with height equal to diameter.

The gamma-ray escape fractions are calculated for a uniform source in a sphere and slab and for an exponential source in a semi-infinite slab. Since the escape fractions for the uniform source cases are equal to the self-shielding factors, a plot of the self-shielding factors for sphere, slab, and cylinder is included in this report as Fig. 22.20.

In Sec. 22.9, asymptotic formulas are obtained for the energy absorption at large and small distances for plane and spherical sources with various angular distributions and with general assumptions about the attenuation curves. These yield results which have been used to check the specific calculations made for the case of exponential attenuation.

In order to compare the results for spheres, slabs, and cylinders quantitatively, it is convenient to consider the fraction of the escaping energy which is absorbed in thin

shells of unit thickness as a function of distance from the surface. Comparative plots are given for these functions. These plots are conveniently normalized and provide a simple means for obtaining results for arbitrary ratios of attenuation length to radius for spheres and cylinders with the various angular distributions of emerging radiation.

22.3 INFINITE PLANE WITH SURFACE INTENSITY PROPORTIONAL TO $\cos^n \phi$

Consider an infinite plane with an emitted intensity per unit area per solid angle per second given by

$$E = \alpha_n \cos^n \phi, \quad (n = 0, 1, 2), \quad \alpha_n \text{ a constant, } 0 \leq \phi \leq \pi/2$$

Let y be the co-ordinate of a source point P ; Q is the point at which the intensity of the absorbed radiation is to be found; x is the perpendicular distance from Q to the plane;

$\rho = \sqrt{y^2 + x^2}$ is the distance from P to Q ; and $\rho^2 d\Omega d\rho$ is the size of the volume element at Q . As before, ϕ is the angle between the normal to the surface and the beam of radiation emitted into the solid angle $d\Omega$.

If σ is the probability of absorption per unit length for a photon in the infinite region exterior to the plane, then the intensity of absorbed radiation per unit volume per unit time at Q is given by:

$$I_n = \int_{\rho=x}^{\infty} \frac{[\alpha_n \cos^n \phi d\Omega \cdot 2\pi y dy] \sigma e^{-\sigma \rho} d\rho}{\rho^2 d\Omega d\rho} = \int_x^{\infty} 2\pi \sigma \alpha_n \cos^n \phi \frac{e^{-\sigma \rho}}{\rho} d\rho \quad (22.4)$$

where $\cos \phi = x/\rho$.

The results of integrating Eq. 22.4 for $n = 0, 1, 2$ are as follows:

$$I_0 = 2\pi \sigma \alpha_0 \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du \quad (22.5)$$

$$I_1 = 2\pi \sigma \alpha_1 \left(e^{-\sigma x} - \sigma x \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du \right) \quad (22.6)$$

$$I_2 = \pi \sigma \alpha_2 \left((\sigma x)^2 \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du - (\sigma x - 1) e^{-\sigma x} \right) \quad (22.7)$$

The integral $\int_x^{\infty} (du/u) e^{-u}$ is tabulated as a function of x . It should be noted that Eqs. 22.5, 22.6 and 22.7, as well as the majority of the other equations in this section, can be expressed more compactly in terms of the functions² $E_n(x) = \int_0^{\infty} e^{-xu} u^{-n} du$. Most of the results in this section are expressed in terms of $E_1(x)$ as this function is widely tabulated.

Let the physically known quantity on the surface of the plane be g_n , the total intensity of radiation emitted per unit area per unit time. The constants g_n and α_n are related by

$$g_n = \int_0^{\pi/2} \alpha_n \cos^n \phi \cdot 2\pi \sin \phi d\phi$$

from which one easily obtains

$$g_0 = 2\pi \alpha_0 \quad (22.8)$$

$$g_1 = \pi \alpha_1 \quad (22.9)$$

$$g_2 = \frac{2}{3} \pi \alpha_2 \quad (22.10)$$

Before substituting Eqs. 22.8 through 22.10 into the expressions for I_n , it is convenient to define a normalized intensity I'_n in such a way that

$$\int_0^\infty I'_n d[\sigma x] = 1 \quad (22.11)$$

Since $\int_0^\infty I_n dx = g_n$, it is seen that

$$I_n = \sigma g_n I'_n \quad (22.12)$$

Fig. 22.2—Normalized absorption intensities, plane source with three possible angular distributions.

Using Eq. 22.12 and Eqs. 22.5 through 22.10,

$$I_0' = \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du \quad (22.13)$$

$$I_1' = 2 \left(e^{-\sigma x} - \sigma x \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du \right) \quad (22.14)$$

$$I_2' = \frac{3}{2} \left((\sigma x)^2 \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du - (\sigma x - 1)e^{-\sigma x} \right) \quad (22.15)$$

Equations 22.13, 22.14 and 22.15 are plotted in Fig. 22.2. The asymptotic behavior of the functions for large values of σx is given by the following:

$$I_0' \approx \frac{e^{-\sigma x}}{\sigma x} \quad (22.16)$$

$$I_1' \approx 2 \frac{e^{-\sigma x}}{\sigma x} \quad (22.17)$$

$$I_2' \approx 3 \frac{e^{-\sigma x}}{\sigma x} \quad (22.18)$$

(A direct derivation of these formulas is given in Sec. 22.9.)

The fraction of the radiation emitted from the plane that is absorbed within a distance x of the plane is given by

$$P_n = \int_0^x \frac{I_n dx}{E_n} = \int_0^x I_n' d[\sigma x] \quad (22.19)$$

Substitution of Eqs. 22.13, 22.14 and 22.15 into the above yields

$$P_0 = 1 - e^{-\sigma x} + \sigma x \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du \quad (22.20)$$

$$P_1 = 1 + e^{-\sigma x} [\sigma x - 1] - (\sigma x)^2 \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du \quad (22.21)$$

$$P_2 = 1 + \frac{(\sigma x)^3}{2} \int_{\sigma x}^{\infty} \frac{e^{-u}}{u} du - \frac{e^{-\sigma x}}{2} [(\sigma x)^2 - (\sigma x) + 2] \quad (22.22)$$

The fraction of the radiation emitted from a unit area of the plane that escapes into the region beyond x is, of course, given by $(1 - P_n)$. Equations 22.20-22.22 are plotted in Fig. 22.3

For application in Secs. 22.5 and 22.6, Eq. 22.5 ($n=0$) for the thin plane will be extended to the case in which the radiation traverses two media of different absorption probabilities per unit length. Suppose that the regions with cross-sections σ_0 and σ_1 are of width x and width h and the path lengths in the two regions are designated by ρ_0 and ρ_1 . The total path length $\rho = \rho_0 + \rho_1$ is related to the individual paths by:

$$\frac{x+h}{\rho} = \frac{x}{\rho_0} = \frac{h}{\rho_1} \quad (22.23)$$

Fig. 22.3—Fraction of radiation absorbed within distance x of a plane source with various angular distributions (Eqs. 22.20, 22.21, and 22.22).

The intensity of absorbed radiation per unit volume per sec at $x + h$ is given by:

$$I_0 = \int_{x+h}^{\infty} 2\pi \alpha_0 \frac{\sigma_1 e^{-(\sigma_0 \rho_0 + \sigma_1 \rho_1)}}{\rho} d\rho = \int_{x+h}^{\infty} \frac{2\pi \alpha_0 \sigma_1 e^{-\left(\frac{\sigma_0 x}{x+h} + \frac{\sigma_1 h}{x+h}\right)\rho}}{\rho} d\rho$$

or

$$I_0 = 2\pi \alpha_0 \sigma_1 \int_{\sigma_0 x + \sigma_1 h}^{\infty} \frac{e^{-u} du}{u} \quad (22.24)$$

22.4 SPHERE WITH SURFACE INTENSITY PROPORTIONAL TO $\cos^n \phi$

Consider a sphere with an emitted intensity per unit area per solid angle per second given by:

$$E = \alpha_n \cos^n \phi, \quad (n=0,1,2) \quad 0 \leq \phi \leq \frac{\pi}{2}$$

In Fig. 22.4, r is the distance from the center of the sphere to the point Q at which the intensity of absorbed radiation is to be found, R is the radius of the sphere, θ is the angle between r and the radius vector to an element of area S on the surface of the sphere, ρ is the distance from S to Q , and ϕ is the angle between the normal to the surface of the sphere and the beam of radiation emitted into the solid angle $d\Omega$.

The ρ , θ , and ϕ are connected by the relations:

$$\rho \cos \phi = r \cos \theta - R \quad (22.25)$$

$$\rho^2 = r^2 + R^2 - 2rR \cos \theta \quad (22.26)$$

Figure 22.4

or

$$\cos \phi = \frac{(r^2 - R^2) - \rho^2}{2R\rho} \quad (22.27)$$

If σ is the probability of absorption per unit path length for a photon in the infinite region exterior to the sphere, then the intensity of absorbed radiation per unit volume per second at Q is given by:

$$\begin{aligned} \mathcal{I}_n &= \int_{\theta=0}^{\theta_{\max}} \frac{[\alpha_n \cos^n \phi \, d\Omega \, 2\pi R^2 \sin \theta \, d\theta] \, \sigma e^{-\sigma \rho} \, d\rho}{\rho^2 \, d\Omega \, d\rho} \\ &= \int_{(r-R)}^{\sqrt{r^2 - R^2}} \frac{2\pi \sigma R}{r} \alpha_n \cos^n \phi \, \frac{e^{-\sigma \rho}}{\rho} \, d\rho \end{aligned} \quad (22.28)$$

where $\cos \phi$ is given by Eq. 22.27. The results of integrating Eq. 22.28 for $n = 0, 1, 2$ are as follows:

$$\mathcal{I}_0 = 2\pi \sigma \frac{R}{r} \alpha_0 \int_{\sigma(r-R)}^{\sigma \sqrt{r^2 - R^2}} \frac{e^{-u}}{u} \, du \quad (22.29)$$

$$\begin{aligned} \mathcal{I}_1 &= \frac{\alpha_1 \pi}{r} [\sigma(r+R)-1] e^{-\sigma(r-R)} - \frac{\alpha_1 \pi}{r} [\sigma \sqrt{r^2 - R^2} - 1] e^{-\sigma \sqrt{r^2 - R^2}} \\ &\quad - \frac{\alpha_1 \pi \sigma^2 (r^2 - R^2)}{r} \int_{\sigma(r-R)}^{\sigma \sqrt{r^2 - R^2}} \frac{e^{-u}}{u} \, du \end{aligned} \quad (22.30)$$

$$\begin{aligned} \mathcal{I}_2 &= \frac{\alpha_2 \pi \sigma^2 (r^2 - R^2)}{4r} \left[\sigma^2 (r^2 - R^2) - 4 \right] \int_{\sigma(r-R)}^{\sigma \sqrt{r^2 - R^2}} \frac{e^{-u}}{u} \, du \\ &\quad + \frac{\alpha_2 \pi}{4r} \frac{1}{\sigma R} \left\{ \sigma^2 (r^2 - R^2) [\sigma \sqrt{r^2 - R^2} - 1] - 2 [\sigma \sqrt{r^2 - R^2} + 1] \right\} e^{-\sigma \sqrt{r^2 - R^2}} \\ &\quad + \frac{\alpha_2 \pi}{4r} \frac{1}{\sigma R} \left\{ 2 [\sigma(r-R)+1] - \sigma^2 (r+R)^2 [\sigma(r-R)-1] \right\} e^{-\sigma(r-R)} \end{aligned} \quad (22.31)$$

22.4 SPHERE WITH SURFACE INTENSITY PROPORTIONAL TO $\cos^n \phi$

Let G_n be the total radiation emitted from the surface of the sphere per second. The constants G_n and α_n are related by

$$G_n = 4\pi R^2 \int_0^{\pi/2} \alpha_n \cos^n \phi \cdot 2\pi \sin \phi \, d\phi$$

from which the following are obtained

$$G_0 = 8\pi^2 R^2 \alpha_0 \quad (22.32)$$

$$G_1 = 4\pi^2 R^2 \alpha_1 \quad (22.33)$$

$$G_2 = \frac{8}{3}\pi^2 R^2 \alpha_2 \quad (22.34)$$

It is convenient to introduce a variable r' defined by

$$r' = r/R \quad (22.35)$$

Since the total radiation absorbed in the region external to the sphere is

$$\int_R^\infty \mathcal{I}_n 4\pi r^2 \, dr = \int_1^\infty \mathcal{I}_n \left(\frac{4}{3}\pi R^3 \right) 3r'^2 \, dr' = G_n$$

it is convenient to define a normalized intensity $\mathcal{I}'_n$ such that

$$\mathcal{I}'_n = \frac{G_n \mathcal{I}_n}{V_{\text{sphere}}}; \quad V_{\text{sphere}} = \frac{4\pi R^3}{3} \quad (22.36)$$

Substitution of Eqs. 22.32 through 22.36 into 22.29, 22.30 and 22.31 for $\mathcal{I}'_n$:

$$\mathcal{I}'_0 = \frac{\sigma R}{3r'} \int_{\sigma R(r'-1)}^{\sigma R \sqrt{r'^2-1}} \frac{e^{-u}}{u} \, du \quad (22.37)$$

$$\mathcal{I}'_1 = \frac{1}{3r'} \left\{ [\sigma R(r'+1)-1] e^{-\sigma R(r'-1)} - [\sigma R \sqrt{r'^2-1}-1] e^{-\sigma R \sqrt{r'^2-1}} \right. \\ \left. - (\sigma R)^2 (r'^2-1) \int_{\sigma R(r'-1)}^{\sigma R \sqrt{r'^2-1}} \frac{e^{-u}}{u} \, du \right\} \quad (22.38)$$

$$\mathcal{I}'_2 = \frac{1}{8\sigma R r'} \left\{ (\sigma R)^2 (r'^2-1) [(\sigma R)^2 (r'^2-1)-4] \int_{\sigma R(r'-1)}^{\sigma R \sqrt{r'^2-1}} \frac{e^{-u}}{u} \, du \right. \\ + \left[(\sigma R)^2 (r'^2-1) [\sigma R \sqrt{r'^2-1}-1] - 2(\sigma R \sqrt{r'^2-1}+1) \right] e^{-\sigma R \sqrt{r'^2-1}} \\ \left. + [2(\sigma R [r'-1]+1) - (\sigma R)^2 [r'+1]^2 (\sigma R [r'-1]-1)] e^{-\sigma R(r'-1)} \right\} \quad (22.39)$$

The equations for the infinite plane calculated in Sec. 22.3 could, of course, have been obtained from those derived in this section for a sphere by taking the limit as $R \rightarrow \infty$, $r \rightarrow \infty$, and $(r-R) \rightarrow x$. The correspondence between the absorption obtained for

22.4 SPHERE WITH SURFACE INTENSITY PROPORTIONAL TO $\cos^2\phi$

Fig. 22.6—Radiation absorption rates as function of distance from spherical sources or plane source.

Fig. 22.7—Radiation absorption as function of distance from spherical sources or plane source.

Substitution of Eqs. 22.37, 22.38 and 22.39 into Eq. 22.43 yields

$$S_0 = 1 + \frac{\sigma R}{2} (r'^2 - 1) \int_{\sigma R (r' - 1)}^{\sigma R \sqrt{r'^2 - 1}} \frac{e^{-u}}{u} du - e^{-\sigma R (r' - 1)} + \frac{e^{-\sigma R \sqrt{r'^2 - 1}}}{2 \sigma R} [\sigma R \sqrt{r'^2 - 1} + 1] - \frac{e^{-\sigma R (r' - 1)}}{2 \sigma R} [\sigma R (r' - 1) + 1] \quad (22.44)$$

$$S_1 = 1 - \frac{(\sigma R)^2}{4} (r'^2 - 1)^2 \int_{\sigma R (r' - 1)}^{\sigma R \sqrt{r'^2 - 1}} \frac{e^{-u}}{u} du + \frac{e^{-\sigma R \sqrt{r'^2 - 1}}}{4(\sigma R)^2} \left\{ -(\sigma R)^3 (r'^2 - 1)^{3/2} + (\sigma R)^2 (r'^2 - 1) - 2\sigma R \sqrt{r'^2 - 1} - 2 \right\} + \frac{e^{-\sigma R (r' - 1)}}{4(\sigma R)^2} \left\{ (\sigma R)^3 (r' - 1)^3 + (4\sigma R - 1)(\sigma R)^2 (r' - 1)^2 + [4(\sigma R)^2 - 4(\sigma R) + 2] \sigma R (r' - 1) + 2 - 4(\sigma R)^2 \right\} \quad (22.45)$$

$$S_2 = 1 + \frac{\sigma R (r'^2 - 1)^2}{16} [(\sigma R)^2 (r'^2 - 1) - 6] \int_{\sigma R (r' - 1)}^{\sigma R \sqrt{r'^2 - 1}} \frac{e^{-u}}{u} du + \frac{e^{-\sigma R \sqrt{r'^2 - 1}}}{16(\sigma R)^3} \left\{ (\sigma R)^5 (r'^2 - 1)^{5/2} - (\sigma R)^4 (r'^2 - 1)^2 - 4(\sigma R)^3 (r'^2 - 1)^{3/2} + 12\sigma R \sqrt{r'^2 - 1} + 12 \right\} + \frac{e^{-\sigma R (r' - 1)}}{16(\sigma R)^3} \left\{ (\sigma R)^5 (r' - 1)^5 + (1 - 6\sigma R) (\sigma R)^4 (r' - 1)^4 + [4 + 6\sigma R - 12(\sigma R)^2] (\sigma R)^3 (r' - 1)^3 + [12\sigma R + 12(\sigma R)^2 - 8(\sigma R)^3] (\sigma R)^2 (r' - 1)^2 + [-12 + 12\sigma R + 8(\sigma R)^3] \sigma R (r' - 1) - [12 + 16(\sigma R)^3] \right\} \quad (22.46)$$

These functions are plotted versus $\sigma R (r' - 1)$ in Figs. 22.8A, 22.8B and 22.8C for $\sigma R = 1.0, 1.5, 2.0$, on the same graphs are plotted the corresponding curves for a plane that are given by Eqs. 22.20, 22.21 and 22.22.

22.5 EXPONENTIAL SOURCE IN SEMI-INFINITE REGION

Consider two semi-infinite regions R_0 and R_1 which have different gamma-ray absorption cross-sections per unit length, σ_0 and σ_1 . Assume a distributed isotropic source of gamma radiation of the form $\sigma_N e^{-\sigma_N q}$. This type of distribution would correspond to capture gammas arising from a flux of neutrons entering R_0 from R_1 and having a diffusion length $(1/\sigma_N)$ in R_0 . For convenience, distances measured outward from the interface into each region will be considered positive.

To find the absorption of gamma-rays in R_1 consider the contribution in R_1 from one thin plane located in R_0 (using Eq. 22.24) with $\alpha_0 = (\sigma_N e^{-\sigma_N q}) dq / 4\pi$, and $\sigma = \sigma_1$ and then

Fig. 22.8A—Fraction of radiation absorbed within distance r of spherical sources or within distance x of plane source.

Fig. 22.8B—Fraction of radiation absorbed within distance r of spherical sources or within distance x of plane source.

Fig. 22.8C—Fraction of radiation absorbed within distance r of spherical sources or within distance x of plane source.

sum over all q . Upon doing this, it is found that I_1 , the intensity per unit volume per second of radiation absorbed in R_1 is:

$$\begin{aligned}
 I_1 &= \int_0^\infty \frac{\sigma_1 \sigma_N}{2} e^{-\sigma_N q} dq \left[\int_{(\sigma_0 q + \sigma_1 h)}^\infty \frac{e^{-u}}{u} du \right] \\
 &= \frac{\sigma_1}{2} \left\{ \int_{\sigma_1 h}^\infty \frac{e^{-u}}{u} du - e^{-\left(\frac{\sigma_N}{\sigma_0}\right) \sigma_1 h} \int_{\left(\frac{\sigma_N}{\sigma_0} + 1\right) \sigma_1 h}^\infty \frac{e^{-u}}{u} du \right\} \quad (22.47)
 \end{aligned}$$

To find the absorption at any point q in R_0 , consider contributions from thin planes at q_1 and q_2 (see Fig. 22.9 and use Eq. 22.5 with $(\alpha_0 = \sigma_N e^{-\sigma_N q} dy/4\pi$ and $\sigma = \sigma_0$), sum over q_1 as q_2 varies from $q+\epsilon$ to ∞ , q_2 as q_2 varies from 0 to $q-\epsilon$, and then take

Figure 22.9

22.5 EXPONENTIAL SOURCE IN SEMI-INFINITE REGION

the limit as $\epsilon \rightarrow 0$ (the introduction of ϵ is necessary as the integrands are unbounded). Upon doing this, I_0 the intensity per unit volume per second of radiation absorbed in R_0 is:

$$\begin{aligned}
 I_0 &= \lim_{\epsilon \rightarrow 0} \frac{\sigma_0 \sigma_N}{2} \left\{ \int_{q+\epsilon}^{\infty} \left[\int_0^{\infty} \frac{e^{-u}}{u} du \right] e^{-\sigma_N q_1} dq_1 \right. \\
 &\quad \left. + \int_0^{q-\epsilon} \left[\int_0^{\infty} \frac{e^{-u}}{u} du \right] e^{-\sigma_N q_2} dq_2 \right\} \\
 &= \frac{\sigma_0}{2} \left\{ e^{-\sigma_N q} \ln \left| \frac{\sigma_0 + \sigma_N}{\sigma_0 - \sigma_N} \right| + \int_{\sigma_0 q}^{\infty} \frac{e^{-u}}{u} du - e^{-\sigma_N q} \int_{(\sigma_0 - \sigma_N)q}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.48)
 \end{aligned}$$

Although the gamma-ray flux is continuous everywhere, the absorption intensity may be discontinuous. In particular, at $h = 0$, the intensity I_1 is equal to $(\sigma_1/2) \ln [(\sigma_N + \sigma_0)/\sigma_0]$, and at $q = 0$, I_0 is equal to $(\sigma_0/2) \ln [(\sigma_N + \sigma_0)/\sigma_0]$. This discontinuity at the interface can be removed by introducing normalized intensities defined by

$$I'_1 = I_1/\sigma_1, \quad I'_0 = I_0/\sigma_0 \quad (22.49)$$

If, in addition, the following quantities are defined

$$\alpha = \frac{\sigma_0}{\sigma_N}, \quad y = \sigma_0 q \quad x = \sigma_1 h \quad (22.50)$$

and Eqs. 22.49 and 22.50 are introduced into Eqs. 22.47 and 22.48, one obtains for the normalized intensities:

$$I'_1 = \frac{1}{2} \left\{ \int_x^{\infty} \frac{e^{-u}}{u} du - e^{-x/\alpha} \int_{(1+1/\alpha)x}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.51)$$

$$I'_0 = \frac{1}{2} \left\{ e^{-y/\alpha} \ln \left| \frac{\alpha+1}{\alpha-1} \right| + \int_y^{\infty} \frac{e^{-u}}{u} du - e^{-y/\alpha} \int_{(1+1/\alpha)x}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.52)$$

At the interface

$$I'_0 = I'_1 = \frac{1}{2} \ln \left[1 + \frac{1}{\alpha} \right] \text{ at } x=y=0 \quad (22.53)$$

Eqs. 22.51 and 22.52 are plotted in Fig. 22.10 for values of the parameter α equal to 0, 1/2, 1, 2, and ∞ .

When $\alpha = 0$, Eqs. 22.51 and 22.52 reduce to

$$I'_1 = \frac{1}{2} \int_x^{\infty} \frac{e^{-u}}{u} du; \quad I'_0 = \frac{1}{2} \int_y^{\infty} \frac{e^{-u}}{u} du$$

Fig. 22.10—Normalized absorption intensities near interface between semi-infinite regions, one of which has an exponential source.

which represent the absorptions due to a thin plane (except for the different normalization factor).

When $\alpha = 1$, it is found that

$$I_1' = \frac{1}{2} \left\{ \int_x^\infty \frac{e^{-u}}{u} du - e^x \int_{2x}^\infty \frac{e^{-u}}{u} du \right\} \quad (22.51a)$$

$$I_0' = \frac{1}{2} \left\{ e^{-y} [\gamma + \ln 2y] + \int_y^\infty \frac{e^{-u}}{u} du \right\} \quad (22.52a)$$

where $\gamma = 0.5772$ is Euler's constant; and when $\alpha = \infty$, both I_0' and I_1' are equal to zero.

Since the total source strength is unity, the fraction of the radiation that enters R_1 from R_0 can be obtained by calculating the total absorption in R_1 . When this is done, it is found that the fraction F escaping into R_1 is:

$$F = \frac{1}{2} \left\{ 1 + \alpha \ln \frac{\alpha}{1+\alpha} \right\} \quad (22.54)$$

This function is plotted in Fig. 22.11.

In the normalization defined by Eq. 22.49

$$\int_0^\infty I_1' d(\sigma_1 h) + \int_0^\infty I_0' d(\sigma_0 q) = 1$$

It is often convenient to normalize the intensity of absorbed radiation in R_1 alone. Since $\int_0^\infty I_1 dh = F$, we can introduce a normalized intensity $\mathcal{I}_1'$ defined by

$$\mathcal{I}_1' = \frac{I_1}{\sigma_1 F}$$

Fig. 22.11—Fraction of radiation escaping from semi-infinite region containing exponential source.

so that

$$\int_0^\infty \mathcal{L}'_1 d(\sigma_1 h) = 1 \quad (22.55)$$

Substituting Eqs. 22.55, 22.54 and 22.50 into 22.47,

$$\mathcal{L}'_1 = \frac{\int_0^\infty \frac{e^{-u}}{u} du - e^{x/\alpha} \int_{[1+\frac{1}{\alpha}]x}^\infty \frac{e^{-u}}{u} du}{1 + \alpha \ln \left[\frac{\alpha}{1+\alpha} \right]} \quad (22.56)$$

and at $x=0$,

$$\mathcal{L}'_1 = \frac{\ln \left[1 + \frac{1}{\alpha} \right]}{1 + \alpha \ln \left[\frac{\alpha}{1+\alpha} \right]}$$

Fig. 22.12—Radiation intensity distributions from exponential source in semi-infinite region, normalized to non-source region total of unity (Eq. 22.56).

Eq. 22.56 is plotted in Fig. 22.12 for $\alpha = 0, 1, \infty$. when $\alpha = 0$, Eq. 22.56 reduced to the equation for a thin plane:

$$J'_1 = \int_x^\infty \frac{e^{-u}}{u} du$$

and when $\alpha = \infty$, Eq. 22.56 reduces to the case of a plane source with a cosine angular distribution

$$J'_1 = 2 \left[e^{-x} - x \int_x^\infty \frac{e^{-u}}{u} du \right]$$

22.6 UNIFORM SOURCE IN FINITE REGION

The case of an exponential source distribution in a region of finite width d can be obtained by considering this distribution to be the sum of two exponential distributions, S_1 and S_2 , of opposite sign in semi-infinite regions, as indicated in Fig. 22.13.

Figure 22.13

The results for the total distribution can then be obtained by combining the results for the two component sources.

22.6 UNIFORM SOURCE IN FINITE REGION

Consider a finite region R_0 of width s with unit source strength of gamma-rays per unit volume. Let the absorption cross section per unit length be σ_0 in R_0 , and let it be σ_1 and σ_2 in the semi-infinite regions R_1 and R_2 on both sides of R_0 . For convenience, distances measured outward in both directions from the interface separating R_0 and R_1 will be considered positive.

To find the absorption of gamma-rays in R_1 , consider the contribution in R_1 from one thin plane in R_0 (using Eq. 22.24 with $\alpha_0 = (dq/4\pi)$) and then sum over all q from $q = 0$ to $q = s$. Upon doing this it is found that the intensity per unit volume per second of absorbed radiation is:

$$I_1 = \int_0^s \frac{\sigma_1}{2} \left[\int_{(\sigma_0 q + \sigma_1 h)}^{\infty} \frac{e^{-u}}{u} du \right] dq$$

$$= \frac{\sigma_1}{2\sigma_0} \left\{ e^{-\sigma_1 h} - e^{-(\sigma_0 s + \sigma_1 h)} - \sigma_1 h \int_{\sigma_1 h}^{\infty} \frac{e^{-u}}{u} du + (\sigma_0 s + \sigma_1 h) \int_{(\sigma_0 s + \sigma_1 h)}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.57)$$

Figure 22.14

To find the absorption at any point q in R_0 , consider contributions from thin planes at q_1 and q_2 (see Fig. 22.14; use Eq. 22.5 with $\alpha_0 = dq/4\pi$), sum over q_1 as q_1 varies from $q + \epsilon$ to s , sum over q_2 as q_2 varies from 0 to $q - \epsilon$, and then take the limit as $\epsilon \rightarrow 0$ (the introduction of ϵ is necessary as the integrands are unbounded). Upon doing this, the intensity per unit volume per second of radiation absorbed in R_0 is found to be:

$$I_0 = \lim_{\epsilon \rightarrow 0} \left\{ \frac{\sigma_0}{2} \int_{q+\epsilon}^s \left[\int_{\sigma_0(q_1-q)}^{\infty} \frac{e^{-u}}{u} du \right] dq_1 + \frac{\sigma_0}{2} \int_0^{q-\epsilon} \left[\int_{\sigma_0(q-q_2)}^{\infty} \frac{e^{-u}}{u} du \right] dq_2 \right\} \quad (22.58)$$

$$= \frac{1}{2} \left\{ 2 - e^{-\sigma_0 q} - e^{-\sigma_0(s-q)} + \sigma_0 q \int_{\sigma_0 q}^{\infty} \frac{e^{-u}}{u} du + \sigma_0(s-q) \int_{\sigma_0(s-q)}^{\infty} \frac{e^{-u}}{u} du \right\}$$

The intensity of absorbed radiation I_2 in R_2 is obtained from Eq. 22.57 by substituting σ_2 for σ_1 and $(q-s)$ for h . Thus

$$I_2 = \frac{\sigma_2}{2\sigma_0} \left\{ e^{-\sigma_2(q-s)} - e^{-[\sigma_0 s + \sigma_2(q-s)]} - \sigma_2(q-s) \int_{\sigma_2(q-s)}^{\infty} \frac{e^{-u}}{u} du \right. \\ \left. + [\sigma_0 s + \sigma_2(q-s)] \int_{[\sigma_0 s + \sigma_2(q-s)]}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.59)$$

At $h = q = 0$, Eq. 22.57 and 22.58 reduce to

$$I_1 = \frac{\sigma_1}{2\sigma_0} \left\{ 1 - e^{-\sigma_0 s} + \sigma_0 s \int_{\sigma_0 s}^{\infty} \frac{e^{-u}}{u} du \right\}$$

$$I_0 = \frac{1}{2} \left\{ 1 - e^{-\sigma_0 s} + \sigma_0 s \int_{\sigma_0 s}^{\infty} \frac{e^{-u}}{u} du \right\}$$

while Eq. 22.59 becomes, at $q = s$,

$$I_2 = \frac{\sigma_2}{2\sigma_0} \left\{ 1 - e^{-\sigma_0 s} + \sigma_0 s \int_{\sigma_0 s}^{\infty} \frac{e^{-u}}{u} du \right\}$$

It is convenient to remove these discontinuities by introducing normalized intensities defined by

$$I_1' = \frac{I_1}{\sigma_1 s}, \quad I_0' = \frac{I_0}{\sigma_0 s}, \quad \text{and} \quad I_2' = \frac{I_2}{\sigma_2 s} \quad (22.60)$$

In addition, the following quantities are defined:

$$\alpha = \sigma_0 s, \quad x = \sigma_1 h, \quad y = \sigma_0 q, \quad \text{and} \quad y' = \sigma_2(q-s) \quad (22.61)$$

Substitution of Eq. 22.60 and 22.61 into 22.57, 22.58 and 22.59 yields:

$$I_1' = \frac{1}{2\alpha} \left\{ e^{-x} - e^{-(\alpha+x)} - x \int_x^{\infty} \frac{e^{-u}}{u} du + (x+\alpha) \int_{x+\alpha}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.62)$$

22.6 UNIFORM SOURCE IN FINITE REGION

$$I'_0 = \frac{1}{2\alpha} \left\{ 2e^{-y} - e^{-(\alpha-y)} + y \int_y^\infty \frac{e^{-u}}{u} du + (\alpha-y) \int_{\alpha-y}^\infty \frac{e^{-u}}{u} du \right\} \quad (22.63)$$

$$I'_2 = \frac{1}{2\alpha} \left\{ e^{-y'} - e^{-(\alpha+y')} - y' \int_{y'}^\infty \frac{e^{-u}}{u} du + (y'+\alpha) \int_{y'+\alpha}^\infty \frac{e^{-u}}{u} du \right\} \quad (22.64)$$

Note that at $x = y = y' = 0$:

$$I'_0 = I'_1 = I'_2 = \frac{1}{2\alpha} \left\{ 1 - e^{-\alpha} + \alpha \int_\alpha^\infty \frac{e^{-u}}{u} du \right\}$$

Eqs. 22.62, 22.63 and 22.64 are plotted in Fig. 22.15 for values of the parameter equal to 0.5, 1, 2 and ∞ . The curve for each α is, of course symmetrical about $y = (1/2)\alpha$ and the interface between R_0 and R_2 lies at $y = \alpha$ for each value of the parameter. The normalized intensities defined by Eq. 22.60 are such as to make the area under each curve in Fig. 22.15 equal to unity.

Fig. 22.15—Normalized absorption intensities for uniform source in slabs of normalized thickness, $\alpha = \sigma_0 s$ (Eqs. 22.62, 22.63, and 22.64).

The total absorption in R_1 is given by

$$\int_0^\infty I_1 dh = \frac{1}{4\sigma_0} \left\{ 1 - (\sigma_0 s)^2 \int_{\sigma_0 s}^\infty \frac{e^{-u}}{u} du + e^{-\sigma_0 s} [\sigma_0 s - 1] \right\} \quad (22.65)$$

hence, the fraction of the radiation in R_0 that escapes into R_1 is

$$f_{SL} = \frac{1}{s} \int_0^\infty I_1 dh = \frac{1}{4\alpha} \left[1 - \alpha^2 \int_\alpha^\infty \frac{e^{-u}}{u} du + e^{-\alpha} (\alpha - 1) \right] \quad (22.66)$$

This function is plotted in Fig. 22.16. (Note that f_{SL} is one-half of the self-shielding factor given in Fig. 22.20 for a slab.) The fraction of the emitted radiation that escapes into R_2 is given by the same expression.

Fig. 22.16—Fraction of radiation from slab with uniform source that escapes from one surface (Eq. 22.66).

It is seen from Eq. 22.66 that

$$\int_0^\infty I_1 dh = f_{SL} S$$

Hence, the intensity of absorbed radiation in R_1 alone can be normalized by defining

$$\mathcal{J}'_1 = \frac{I_1}{\sigma_1 f_{SL}} \quad (22.67)$$

so that

$$\int_0^\infty \mathcal{J}'_1 d(\sigma_1 h) = 1$$

Substitution of Eqs. 22.67, 22.67 and 22.61 into 22.57 yields

$$\mathcal{J}'_1 = 2 \frac{\left\{ e^{-x} - e^{-(x+\alpha)} - x \int_x^\infty \frac{e^{-u}}{u} du + (x+\alpha) \int_{x+\alpha}^\infty \frac{e^{-u}}{u} du \right\}}{1 - \alpha^2 \int_\alpha^\infty \frac{e^{-u}}{u} du + e^{-\alpha} (\alpha - 1)} \quad (22.68)$$

Equation 22.68 is plotted in Fig. 22.17 for values of the parameter α equal to 0, 1 and ∞ . When $\alpha=0$ the equation reduces to the equation for a thin plane:

$$\mathcal{J}'_1 = \int_x^\infty \frac{e^{-u}}{u} du$$

and when $\alpha=\infty$, the equation reduces to the equation for a plane with a cosine distribution of sources:

$$\mathcal{J}'_1 = 2 \left[e^{-x} - x \int_x^\infty \frac{e^{-u}}{u} du \right]$$

Fig. 22.17—Normalized absorption intensity near slab with uniform source, for normalized slab thickness, $\alpha = \sigma_0 s$ (Eq. 22.68).

An alternative normalization to that defined by Eq. 22.60 is to introduce the normalized intensities:

$$I_1'' = \frac{\sigma_0}{\sigma_1} I_1; \quad I_0'' = I_0, \quad I_2'' = I_2 \frac{\sigma_0}{\sigma_2} \quad (22.69)$$

and use the following variables

$$\alpha = \sigma_0 s, \quad \bar{X} = \frac{\sigma_1 h}{\sigma_0 s}, \quad \bar{Y} = \frac{q}{s}, \quad \bar{Y}' = \frac{\sigma_2 (q-s)}{\sigma_0 s} \quad (22.70)$$

This normalization is sometimes more convenient for the application of the results to specific numerical cases. Substitution of Eqs. 22.69, 22.70, into 22.57, 22.58 and 22.59 yields:

$$I_1'' = \frac{1}{2} \left\{ e^{-\alpha \bar{X}} - e^{-\alpha(1+\bar{X})} - \alpha \bar{X} \int_{\alpha \bar{X}}^{\infty} \frac{e^{-u}}{u} du + \alpha(1+\bar{X}) \int_{\alpha(1+\bar{X})}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.71)$$

$$I_0'' = \frac{1}{2} \left\{ 2 - e^{-\alpha \bar{Y}} - e^{-\alpha(1-\bar{Y})} + \alpha \bar{Y} \int_{\alpha \bar{Y}}^{\infty} \frac{e^{-u}}{u} du + \alpha(1-\bar{Y}) \int_{\alpha(1-\bar{Y})}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.72)$$

$$I_2'' = \frac{1}{2} \left\{ e^{-\alpha \bar{Y}'} - e^{-\alpha(1+\bar{Y}')} - \alpha \bar{Y}' \int_{\alpha \bar{Y}'}^{\infty} \frac{e^{-u}}{u} du + \alpha(1+\bar{Y}') \int_{\alpha(1+\bar{Y}')}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.73)$$

Equations 22.71, 22.72 and 22.73 are plotted in Fig. 22.18 for values of the parameter equal to 0, 0.1, 0.5, 1, 2, and ∞ . The source region is more clearly defined in this representation, and the interface between R_0 and R_2 is now at the fixed vertical line $Y = 1$. The area under all curves in Fig. 22.18 is unity. For the case where all σ 's are equal, these graphs have the direct physical significance of being the ratio of the gamma-ray absorption at a point to the absorption that would exist if the source were a unit source throughout all space.

Fig. 22.18—Normalized absorption intensities for uniform source in slab (Eqs. 22.71, 22.72, and 22.73).

22.7 UNIFORM SOURCE IN SPHERICAL REGION

Consider a sphere of radius R with a unit isotropic source of gamma-rays per unit volume per second. It will be assumed that the absorption cross-sections for gamma-rays inside and outside the source region are equal. The intensity of absorbed radiation per unit volume per second at any point is given by

$$I = \int \frac{\sigma e^{-\sigma|r-r'|}}{4\pi|r-r'|^2} dV \quad (22.74)$$

where r' denotes the distance from the center of the sphere to the volume element where the absorption is being calculated, r denotes the variable of integration in the source region, and dV is the element of volume in the source region. Equation 22.74 can be put in a more convenient form by dividing both sides by σ and then taking the partial derivative with respect to σ , yielding.

$$-\Psi = \frac{\partial \left(\frac{I}{\sigma} \right)}{\partial \sigma} = - \int \frac{e^{-\sigma|r-r'|}}{4\pi|r-r'|} dV \quad (22.75)$$

Then Ψ satisfies the following equations inside and outside of the source region:

$$-\nabla^2 \Psi + \sigma^2 \Psi = 1 \quad \text{in } V \quad (22.76)$$

$$-\nabla^2 \Psi + \sigma^2 \Psi = 0 \quad \text{outside of } V \quad (22.77)$$

Thus the solution for Ψ is

$$\Psi = \frac{1}{\sigma^2} - \frac{\alpha \sinh \sigma r}{r} \quad \text{in } V \quad (22.78)$$

$$\Psi = \beta \frac{e^{-\sigma r}}{r} \quad \text{outside of } V \quad (22.79)$$

where r now denotes the distance from the center of the sphere of the volume element where the absorption is being calculated. The constants of integration α and β are obtained as follows:

At the center of the sphere, Ψ is found from Eq. 22.75 to be

$$\Psi = \int_0^R \frac{e^{-\sigma r}}{4\pi r} 4\pi r^2 dr = \frac{1}{\sigma^2} \left[1 - (1+\sigma R)e^{-\sigma R} \right]$$

Comparison of this result with Eq. 22.78 evaluated at $r = 0$ yields

$$\alpha = \frac{(1+\sigma R)e^{-\sigma R}}{\sigma^3}$$

Imposing the condition that Ψ be continuous at $r = R$ yields

$$\beta = \frac{R}{\sigma^2} \left[e^{\sigma R} - \left(1 + \frac{1}{\sigma R}\right) \sinh \sigma R \right]$$

Hence, it may be seen that

$$\Psi = \frac{1}{\sigma^2} \left[1 - (1+\sigma R)e^{-\sigma R} \frac{\sinh \sigma r}{\sigma r} \right] \quad \text{in } V \quad (22.80)$$

$$\Psi = \frac{Re^{-\sigma r}}{\sigma^2 r} \left[e^{\sigma R} - \left(1 + \frac{1}{\sigma R}\right) \sinh \sigma R \right] \quad \text{outside of } V \quad (22.81)$$

The intensity of absorbed radiation, I , is obtained from the relation

$$\frac{I}{\sigma} = \int_0^\sigma \Psi d\sigma = \int_\sigma^\infty \Psi d\sigma \quad (22.82)$$

Substitution of Eqs. 22.80, 22.81 into 22.82 yields:

$$I = 1 - \frac{1}{4} \left(1 + \frac{1}{r'} + \frac{1}{\sigma R r'} \right) e^{-\sigma R(1-r')} - \frac{1}{4} \left(1 - \frac{1}{r'} - \frac{1}{\sigma R r'} \right) e^{-\sigma R(1+r')} \\ + \frac{\sigma R(1-r'^2)}{4 r'} \int_{\sigma R(1-r')}^{\sigma R(1+r')} \frac{e^{-u}}{u} du \quad \text{for } r < R \quad (22.83)$$

$$I = \frac{1}{4} \left(1 + \frac{1}{r'} - \frac{1}{\sigma R r'} \right) e^{-\sigma R(r'-1)} - \frac{1}{4} \left(1 - \frac{1}{r'} - \frac{1}{\sigma R r'} \right) e^{-\sigma R(r'+1)} \\ - \frac{\sigma R}{4} \frac{(r'^2-1)}{r'} \int_{\sigma R(r'-1)}^{\sigma R(r'+1)} \frac{e^{-u}}{u} du \quad \text{for } r > R \quad (22.84)$$

where $r' = r/R$.

Equations 22.83 and 22.84 are plotted in Fig. 22.19 for σR equal to 1.0, 1.5, 2.0, and ∞ .

It was seen in Sec. 22.2 that spheres with surface intensities of emitted radiation proportional to $\cos \phi$ and $\cos^2 \phi$, respectively, are extreme limits corresponding to the

cases when σR inside the sphere is respectively very large and very small. The case of equal mean free paths considered in this section lies between these extreme limits. Hence one would expect the intensity of absorbed radiation given by Eq. 22.84 for the region external to the sphere to be bracketed by the results of Sec. 22.4 for a sphere with a cosine and cosine-squared surface distribution.

Fig. 22.19—Absorption intensity distributions for uniform source in sphere (Eqs. 22.83 and 22.84).

In order to make the comparison, the absorption given by Eq. 22.84 must be normalized to correspond to the normalization used in Sec. 22.4 Eq. 22.36. This is done in the following manner:

The total radiation G absorbed in the region external to the source region is calculated (by use of Eq. 22.84) as

$$\begin{aligned} G &= \int_{r=R}^{\infty} 4\pi r^2 I \, dr = 4\pi R^3 \int_1^{\infty} r'^2 I \, dr' \\ &= \frac{\pi R^3}{(\sigma R)^3} \left\{ (\sigma R)^2 - \frac{1}{2} + \left[\sigma R + \frac{1}{2} \right] e^{-2\sigma R} \right\} \end{aligned} \quad (22.85)$$

Define a normalized intensity, J , by

$$J = \frac{4\pi R^3 I}{3G} = \frac{I}{f_s} \quad (22.86)$$

where

$$f_s = \frac{3G}{4\pi R^3} = \frac{3}{4(\sigma R)^3} \left\{ (\sigma R)^2 - \frac{1}{2} + \left[\sigma R + \frac{1}{2} \right] e^{-2\sigma R} \right\} \quad (22.87)$$

is the fraction of the total radiation which escapes from the surface of the sphere. This function is plotted on Fig. 22.20 as the self-shielding factor for a sphere.

Fig. 22.20—Self-shielding factors (f) for slabs, cylinders, and spheres. The self-shielding factor is equal to the fraction of radiation escaping from the volume containing a uniform isotropic source. The absorption cross-section is σ .

If $(3J/\sigma R)r'^2$ is now plotted versus $\sigma R(r'-1)$, the results can be compared with $(3J'/\sigma R)r'^2$ for the sphere with a cosine and cosine squared surface distribution which were plotted versus $\sigma R(r'-1)$ in Figs. 22.6 and 22.7. This comparison is made in Figs. 22.21, 22.22 and 22.23 for σR equal to 1.0, 1.5 and 2.0. It is seen that the results of this section for equal mean free paths are closely bracketed by the results of Sec. 22.4 for a sphere with a cosine and cosine squared surface distribution.

22.8 UNIFORM SOURCE IN CYLINDRICAL REGION

Consider a cylinder of radius R and length z_0 , which contains a unit isotropic source of gamma-rays per unit volume per second. It will be assumed that the absorption cross-sections for gamma-rays inside and outside the source region are equal. The integrals arising in the calculation of the absorption can no longer be evaluated by purely analytic methods. However there are two special cases for which the integrations are relatively simple.

A. Case 1: Absorption in End Plane of Semi-Infinite Cylinder

Let I_p be the intensity of absorbed radiation per unit volume per second at a distance r from the axis of the cylinder and in a plane coincident with the end plane of the cylinder. Let ρ , θ and z be the cylindrical co-ordinates of the typical point in the cylinder, where z is measured from the end plane and the plane $\theta = 0$ includes the point at distance r . If the radius of the cylinder is R , I_p is given by:

$$I_p = \int_0^{\infty} dz \int_0^{2\pi} d\theta \int_0^R \frac{\sigma e^{-\sigma D}}{4\pi D^2} \rho d\rho \quad (22.88)$$

Fig. 22.21—Normalized radiation absorption intensities near spherical source. Radiation mean free path the same inside and outside of sphere. Uniform volume source is compared with surface sources angularly distributed as they were in Figs. 22.6 and 22.7.

where the distance D to the typical point (ρ, θ, z) is given by:

$$D^2 = z^2 + r^2 + \rho^2 - 2rp \cos \theta \quad (22.89)$$

Equation 22.88 can be put into a more convenient form by dividing both sides by σ and then taking the partial derivative with respect to σ , which yields:

$$\frac{\partial}{\partial \sigma} \left(\frac{I_p}{\sigma} \right) = -W$$

or

$$I_p = -\sigma \int_{\infty}^{\sigma} W d\sigma = \sigma \int_{\sigma}^{\infty} W d\sigma \quad (22.90)$$

Fig. 22.22—Normalized absorption intensities near spherical source. Radiation mean free path the same outside and inside of sphere. Uniform volume source is compared with surface sources angularly distributed as they were in Figs. 22.6 and 22.7.

where W is given by

$$W = \int_0^\infty dz \int_0^{2\pi} d\theta \int_0^R \frac{e^{-\sigma D}}{4\pi D} \rho d\rho \quad (22.91)$$

Upon substituting for D from Eqs. 22.89 and 22.91, W becomes:

$$W = \int_0^\infty dz \int_0^{2\pi} d\theta \int_0^R \frac{e^{-\sigma \sqrt{z^2 + r^2 + \rho^2 - 2r\rho \cos \theta}}}{4\pi \sqrt{z^2 + r^2 + \rho^2 - 2r\rho \cos \theta}} \rho d\rho \quad (22.92)$$

Fig. 22.23—Normalized radiation absorption intensities near spherical source. Radiation mean free path the same inside and outside of sphere. Uniform volume source is compared with surface sources angularly distributed as they were in Figs. 22.6 and 22.7.

The three integrations arising in Eq. 22.92 can be carried out successively if the following formulas are applied:

$$(a) \int_0^{\infty} \frac{e^{-\mu \sqrt{z^2 + v^2}}}{\sqrt{z^2 + v^2}} dz = K_0(\mu v)$$

$$(b) \int_0^{2\pi} K_0 \left[x \sqrt{1 - 2\gamma \cos \theta + \gamma^2} \right] d\theta = 2\pi I_0(\gamma x) K_0(x) \quad \text{if } \gamma \leq 1 \quad (22.93)$$

$$(c) \int x I_0(x) dx = x I_1(x)$$

$$(d) \int x K_0(x) dx = -x K_1(x)$$

$$(e) I_0(z) K_1(z) + I_1(z) K_0(z) = \frac{1}{z}$$

where I and K are the Bessel functions of imaginary argument. One then obtains:

$$W = \frac{R}{2\sigma} K_0(\sigma r) I_1(\sigma R) \quad \text{if } r > R \quad (22.94)$$

$$W = \frac{1}{2\sigma^2} [1 - \sigma R K_1(\sigma R) I_0(\sigma r)] \quad \text{if } r < R$$

Substitution of Eq. 22.94 into 22.90 yields:

$$I_p = \frac{1}{2} - \frac{\sigma R}{2} \int_{\sigma R}^{\infty} K_1(z) I_0\left(\frac{zr}{R}\right) \frac{dz}{z} \quad \text{for } r/R < 1 \quad (22.95a)$$

$$I_p = \frac{\sigma R}{2} \int_{\sigma R}^{\infty} K_0(z) I_1\left(\frac{zr}{R}\right) \frac{dz}{z} \quad \text{for } r/R > 1 \quad (22.95b)$$

When $r > R$, the asymptotic expressions for the Bessel functions³ may be inserted in Eq. 22.95b to give the following expansion for I_p :

$$\begin{aligned} I_p \approx & \frac{1}{4} \sqrt{\frac{R}{r}} \left\{ E_2[\sigma(r-R)] - \frac{(3r+R)}{8\sigma R r} E_3[\sigma(r-R)] \right. \\ & - \frac{3}{128} \frac{(r-R)(5r+3R)}{(\sigma R r)^2} E_4[\sigma(r-R)] \\ & \left. - \frac{3}{1024} \frac{(35r^3 - 5r^2 R + 9rR^2 + 25R^3)}{(\sigma R r)^3} E_5[\sigma(r-R)] - \dots \right\} \end{aligned} \quad (22.96)$$

where $E_n(x) = \int_1^{\infty} e^{-xu} u^{-n} du$ are tabulated functions.^{1,2}

Values of I_p in Table 22.1 were computed by means of Eq. 22.96 for $r > R$ and the numerical integration of Eq. 22.95a for $r < R$.

TABLE 22.1--ABSORPTION IN END PLANE OF SEMI-INFINITE CYLINDER

Absorption Intensity I_p as a function of $r' = r/R$ for σR equal to 1.0, 1.5 and 2.0

r'	I_p		
	$\sigma R = 1.0$	$\sigma R = 1.5$	$\sigma R = 2.0$
0.0	0.364	0.424	0.458
0.2	0.360	0.421	0.455
0.4	0.347	0.408	0.443
0.6	0.325	0.384	0.420
0.8	0.283	0.335	0.370
1.0	0.181	0.204	0.215
1.2	0.084	0.083	0.074
1.4	0.051	0.0441	0.0346
1.7	0.027	0.0197	0.0130
2.0	0.015	0.0095	0.0053

Equations 22.95 have been evaluated numerically for a much wider range of parameters by J. P. Simpelaar.⁴ The values in Table 22.1 for an end plane of a semi-infinite cylinder are just half as great as the values for any plane normal to an infinite cylinder.

B. Case 2: Distribution on the Axis of a Finite Cylinder with Equal Height and Diameter

Let I_A be the amount of radiation absorbed per unit volume per second on the extended axis of a cylinder of length z_0 and radius R , at a distance z from the end of the cylinder. I_A is given by:

$$I_A = \int_0^{z_0} dz'' \int_0^R 2\pi\rho \, d\rho \frac{\sigma e^{-\sigma\sqrt{(z+z'')^2 + \rho^2}}}{4\pi [(z+z'')^2 + \rho^2]} \quad (22.97)$$

Now $\rho d\rho$ can be replaced by $L dL$, where $L^2 = (z+z'')^2 + \rho^2$, so that one of the integrations in Eq. 22.97 can be carried out directly to yield:

$$I_A(z) = \frac{\sigma}{2} \int_0^{z_0} \left[E_1(\sigma z + \sigma z'') - E_1\left(\sigma \sqrt{(z+z'')^2 + R^2}\right) \right] dz'' \quad (22.98)$$

where $E_n(x) = \int_1^\infty e^{-xu} u^{-n} du$. Equation 22.98 was integrated numerically with the length z_0 equal to twice the radius; the results are given in Table 22.2.

The close agreement between the values of I_A at $z' = 0$ and I_p at $r' = 0$ shows that the flux at the end of a cylinder with equal height and diameter is almost as large as the flux at the end of a semi-infinite cylindrical source for the values of σR considered here.

Twice the values of I_p given in Table 22.1 yield the radial distribution in any normal plane of an infinite cylinder. It is of interest to compare the results for an infinite cylinder with those obtained in Sec. 22.5 for a sphere under the same assumption of equal mean free path. It is most significant to compare the fraction of emergent energy

TABLE 22.2--ABSORPTION ON AXIS OF FINITE CYLINDER

Absorption intensity I_A as a function of $z' = z/R$ for σR equal to 1.0, 1.5, and 2.0. The length of the cylinder was taken to be $2R$.

$z' = z/R$	I_A		
	$\sigma R = 1.0$	$\sigma R = 1.5$	$\sigma R = 2.0$
0.0	0.359	0.422	0.457
0.2	0.1700	0.1733	0.1623
0.4	0.0980	0.0900	0.0759
0.7	0.0475	0.0376	0.0274
1.0	0.0247	0.0169	0.0106

absorbed in a cylindrical shell dr with that absorbed in a spherical shell dr . In particular, we must normalize $2I_p$ and compare it with $(3J/\sigma R)r'^2$, which is plotted in Figs. 22.21, 22.22, and 22.23.

The normalization of $2I_p$, corresponding to that used in Eq. 22.86 to define f , is:

$$I_p' = \frac{2I_p}{f_c} \quad (22.99)$$

where f_c is the fraction of the total radiation which escapes from the surface of the cylinder. In addition, by considering the total absorption in the region exterior to a cylinder, it is seen that $(2I_p'/\sigma R)r'$ is the function which must be compared with $(3J/\sigma R)r'^2$ for given values of $\sigma R(r'-1)$. The results of this comparison are given in Table 22.3.

TABLE 22.3--COMPARISON OF $(3J/\sigma R)r'^2$ (SPHERE) WITH $(2I_p'/\sigma R)r'$ (IN ANY NORMAL PLANE OF INFINITE CYLINDER) FOR σR EQUAL TO 1.0, 1.5, AND 2.0

$\sigma R = 1.0$	$\sigma R(r' - 1)$	$\frac{2I_p' r'}{\sigma R}$	$\frac{3r'^2 J}{\sigma R}$	$\frac{2I_p' r'}{\sigma R}$
				$\frac{3r'^2 J}{\sigma R}$
	0.0	1.774	1.614	1.099.
	0.2	0.988	0.940	1.051
	0.4	0.700	0.685	1.022
	0.7	0.450	0.470	0.957
	1.0	0.294	0.330	0.891

TABLE 22.3--(Continued)

$\sigma R = 1.5$	$\sigma R(r' - 1)$	$\frac{2I_p' r'}{\sigma R}$	$\frac{3r'^2 J}{\sigma R}$	$\frac{2I_p' r'}{\sigma R}$
				$\frac{3r'^2 J}{\sigma R}$
	0.0	1.784	1.629	1.095
	0.3	0.870	0.800	1.088
	0.6	0.540	0.520	1.038
	1.05	0.292	0.300	0.973
	1.50	0.166	0.180	0.922
$\sigma R = 2.0$	0.0	1.792	1.717	1.044
	0.4	0.740	0.710	1.042
	0.8	0.404	0.415	0.973
	1.4	0.184	0.205	0.898
	2.0	0.088	0.105	0.838

The values for I_p were taken from Table 22.1, and the values for $(3J/\sigma R)r'^2$ were taken from Figs. 22.21, 22.22 and 22.23. The values of the fraction of the total radiation which escapes from the surface of the cylinder were taken from Fig. 22.20 which was originally obtained in connection with self-shielding problems.^{2,5}

Thus, for a given value of σR , it is seen that $2I_p' r' / \sigma R$ close to the surface is slightly greater than, but farther away grows less than, $3r'^2 J / \sigma R$.

Although calculations were not made for the cases of an infinite cylinder with a cosine and cosine-squared surface intensity of emitted radiation, estimates of the intensity of absorbed radiation can be made. A plot of $(2I_p' / \sigma R)r'$ versus $\sigma R(r' - 1)$ for an infinite cylinder with a cosine or cosine-squared surface intensity of emitted radiation would lie between the plots of $(3J/\sigma R)r'^2$ (sphere) and I' (infinite plane) in Figs. 22.6 and 22.7. Since there is not too great a spread between the above-mentioned graphs for sphere and plane, the estimate of $(2I_p' / \sigma R)r'$ can be made with reasonable accuracy.

For example, examination of Figs. 22.6 and 22.7 shows that no interpolation is required at the surface of the cylinder, and we find that for an infinite cylinder:

$$\frac{2I_p'}{\sigma R} r' \text{ (cosine distribution)} = 2$$

$$\frac{2I_p'}{\sigma R} r' \text{ (cosine-squared distribution)} = 1.50$$

As in the spherical case discussed in Sec. 22.7, these values bracket the average value of $(2I_p' / \sigma R)r' = 1.78$ at the surface of the cylinder, which was obtained under the assumption of equal mean free paths (see Table 22.3).

22.9 PLANE AND SPHERE WITH A MORE GENERAL KERNEL

In this section, expressions will be derived for the energy absorption at very large and very small distances from a plane and from a spherical surface source of radiation with various angular distributions. The straight-ahead approximation is used, but a more general kernel is assumed than in the previous sections. Let $G(\rho) d\rho$ represent the fraction of energy in a monodirectional beam which is absorbed in the path length $d\rho$ at a distance ρ from the source. Let the angular distribution of radiation from the surface under consideration be characterized by the function $f(\mu)$, where $f(\mu) d\mu$ gives the fraction of radiation emerging at angles ϕ from the normal to the surface for which $\cos \phi$ lies between μ and $\mu + d\mu$. The following forms for $f(\mu)$ will be considered:

Form for $f(\mu)$	Range of μ	
$f(\mu) = 1$	$0 \leq \mu \leq 1$	Isotropic (one-sided)
$f(\mu) = 2\mu$	$0 \leq \mu \leq 1$	Cosine distribution (Lambert's law)
$f(\mu) = 3\mu^2$	$0 \leq \mu \leq 1$	Cosine-squared distribution
$f(\mu) = \delta(\mu - 1)$		Monodirectional
$f(\mu) = 1/2$	$-1 < \mu < 1$	Fully isotropic

Let $K_p(x) dx$ and $K_s(x) dx$ be the fraction of radiation absorbed between the distances x and $x + dx$ from a plane (for the plane source, $K_p(x) dx$ refers to the radiation absorbed on only one side of the source) and spherical source, respectively. The expression $K_p(x) dx$ thus represents the radiation absorbed in a slab of thickness dx at distance x from the source, while $K_s(x) dx$ represents the radiation absorbed in a spherical shell of thickness dx and radius $r = (R + x)$ where R is the radius of the surface on which the source is located. The terms $K(x) dx$, $f(\mu) d\mu$, and $G(\rho) d\rho$ are related by

$$K(x) dx = dx \int d\mu f(\mu) G(\rho) (\partial \rho / \partial x) \quad (22.100)$$

where ρ as a function of μ and x is determined by the geometry. For the plane case, $K_p(x)$ is proportional to $I(x)$ where $I(x)$ represents the energy absorbed per unit volume at distance x from the source, whereas for a spherical source, $K_s(x)$ is proportional to $r^2 I(r)$ where $I(r)$ is the radiation absorbed per unit volume at radius r . From the definition it follows that the integral of $K_s(x)$ and $K_p(x)$ from $x = 0$ to infinity is unity. Furthermore, for any angular distribution for which $f(\mu) = 0, \mu < 0$, it will be seen that $K_s(0) = K_p(0)$. (For $f(\mu) = 1, \mu > 0$, both of these quantities are infinite.) In general, $K_s(x)$ falls below $K_p(x)$ for intermediate values of x and is greater than $K_p(x)$ for large x . In addition, for cases where $f(\mu) = 0$ for $\mu < 0$, $K_p(x)$ and $K_s(x)$ are larger than $G(x)$ for small values of x and smaller than $G(x)$ for large values of x .

For a plane source where $\rho = \frac{x}{\mu}$, Eq. 22.100 becomes

$$K_p(x) = \int_0^1 f(\mu) G\left(\frac{x}{\mu}\right) \frac{du}{\mu} \quad (22.101)$$

At a small distance from the source ($x \sim 0$), Eq. 22.101 yields

$$K_p(0) = \alpha G(0) \quad (22.102)$$

where

$$\alpha = \int_0^1 \frac{f(\mu) d\mu}{\mu}$$

and

$\alpha = \infty$	for $f(\mu) = 1$	$(0 \leq \mu \leq 1)$
$\alpha = 2$	$f(\mu) = 2\mu$	$(0 \leq \mu \leq 1)$
$\alpha = 3/2$	$f(\mu) = 3\mu^2$	$(0 \leq \mu \leq 1)$
$\alpha = 1$	$f(\mu) = \delta(\mu - 1)$	$(0 \leq \mu \leq 1)$
$\alpha = 1/2$	$f(\mu) = 1/2$	$(-1 < \mu < 1)$

For large x , Eq. 22.101 can be evaluated if it is assumed that at large distances $G(\rho)$ decreases roughly exponentially. In this case, the main contribution to Eq. 22.101 comes from $\mu \approx 1$. It is therefore legitimate to use the following approximate expression for $G(x/\mu)$:

$$G\left(\frac{x}{\mu}\right) \approx G(x) e^{-\nu x \left(\frac{1}{\mu} - 1\right)} \approx G(x) e^{-\nu x (1 - \mu)} \quad (22.103)$$

where

$$\nu = [-d(\ln G)/d\rho]_{\rho=x}$$

Eq. 22.101 then yields for large x :

$$K_p(x) \approx \beta \frac{G(x)}{\nu x} \quad (22.104)$$

where

$\beta = 1$	for $f(\mu) = 1$	$(0 \leq \mu \leq 1)$
$\beta = 2$	$f(\mu) = 2\mu$	$(0 \leq \mu \leq 1)$
$\beta = 3$	$f(\mu) = 3\mu^2$	$(0 \leq \mu \leq 1)$
$\beta = \infty$	$f(\mu) = \delta(\mu - 1)$	$(0 \leq \mu \leq 1)$

Note that for the monodirectional source $K_p(x) = G(x)$, and thus the infinite value quoted for β really means that the form given in Eq. 22.104 for $K_p(x)$ does not apply in this case. In addition, for the isotropic case (one-sided)

$$K_p(x) = \int_x^\infty \frac{G(\rho) d\rho}{\rho} \quad (22.105)$$

This reduces directly to the result in Eq. 22.104 for large x with the above approximation for $G(\rho)$.

For a spherical source of radius R , $K_s(x)$ is obtained from Eq. 22.100 as the following expression (note that $f(\mu) = 0, \mu < 0$, for most cases of interest):

$$K_s(x) = r \int_{-1}^{+1} \frac{f(\mu) G(\rho) d\mu}{\rho + \mu R} \quad (22.106)$$

where

$$r = x + R$$

$$\rho = \sqrt{x^2 - R^2 + \mu^2 R^2} - \mu R$$

Since $\rho = 0$ for $x = 0$ and $\mu > 0$, the value of $K_s(0)$ is the same for a spherical or plane source for all angular distributions having $f(\mu) = 0$ for $\mu < 0$. For large x , the relation between x and ρ becomes

$$\rho = r - \mu R = x + (1 - \mu)R \quad (22.107)$$

Hence Eq. 22.106 becomes for large x :

$$K_s(x) \approx \int_{-1}^{+1} f(\mu) G[x + (1 - \mu)R] d\mu \quad (22.108)$$

By means of the same approximation used for large x in the plane case, $K_s(x)$ can be written

$$K_s(x) \approx G(x) \int_{-1}^{+1} f(\mu) e^{-\omega R (1 - \mu)} d\mu \quad (22.109)$$

where

$$\omega = -d(\ln G)/dx$$

This leads immediately to the following results:

$$\begin{aligned} K_s(x) &\approx \frac{G(x)}{\omega R} \left(1 - e^{-\omega R} \right) && \text{for } f(\mu) = 1, \mu > 0 \\ K_s(x) &\approx \frac{2G(x)}{\omega R} \left\{ 1 - \left(\frac{1 - e^{-\omega R}}{\omega R} \right) \right\} && \text{for } f(\mu) = 2\mu, \mu > 0 \\ K_s(x) &\approx \frac{3G(x)}{\omega R} \left\{ 1 - \frac{2}{\omega R} \left[1 - \left(\frac{1 - e^{-\omega R}}{\omega R} \right) \right] \right\} && \text{for } f(\mu) = 3\mu^2, \mu > 0 \\ K_s(x) &\approx G(x) && \text{for } f(\mu) = \delta(\mu - 1) \\ K_s(x) &\approx \frac{1}{2} \frac{G(x)}{\omega R} \left(1 - e^{-2\omega R} \right) && \text{for } f(\mu) = \frac{1}{2}, -1 < \mu < 1 \end{aligned} \quad (22.110)$$

Note that for $f(\mu) = 1, \mu > 0$,

$$K_s(x) = \frac{r}{R} \int_{(x-R)}^{\sqrt{x^2 - R^2}} \frac{G(\rho) d\rho}{\rho} \quad (22.111)$$

while for $f(\mu) = \frac{1}{2}, -1 < \mu < 1$,

$$K_s(x) = \frac{1}{2} \frac{r}{R} \int_{x-R}^{x+R} \frac{G(\rho) d\rho}{\rho} \quad (22.112)$$

These relationships follow directly from Eq. 22.106 by means of the substitution

$$\phi_\mu = \frac{(\rho + \mu R) d\rho}{R\rho} \quad (22.113)$$

Note that Eqs. 22.111 and 22.112 are similar in form to Eq. 22.105 so that $K_s(x)$ for an isotropic source can be expressed simply in terms of $K_p(x)$ for a plane source. Thus for $f(\mu) = 1$, $\mu > 0$

$$K_s(x) = \frac{r}{R} \left\{ K_p(r - R) - K_p\left(\sqrt{r^2 - R^2}\right) \right\} \quad (22.114)$$

Similarly, for $f(\mu) = \frac{1}{2}$, $-1 < \mu < 1$,

$$K_s(x) = \frac{1}{2} \frac{r}{R} \left\{ K_p(x) - K_p(x + 2R) \right\} \quad (22.115)$$

Chapter 22

GAMMA-RAY AND NEUTRON HEAT GENERATION

B. MULTIGROUP TREATMENTS OF GAMMA-RAY ABSORPTION

H. HURWITZ, Jr., J. B. NIMS, Jr., and M. L. STORM

22.10 INTRODUCTION

In applying the results derived in Part A for the distribution of gamma-ray absorptions, the problem arises of selecting a single effective energy absorption cross-section for which the assumption of exponential attenuation is valid within a reasonable degree of accuracy. An energy absorption cross-section representative of the initial core gamma-ray spectrum can be obtained once the shape of the initial spectrum is known. This cross-section is not rigorously valid at all distances because of the change of the spectrum with penetration. The present study investigates the accuracy with which the absorption may be represented by an exponential approximation and determines the effective cross-section to be used in this representation.

To consider this question for SIR-type reactors, a multigroup calculation was performed to determine the spectral energy distribution and energy deposition rate as a function of distance from a point source along a straight-line path starting in the core and continuing into the reflector. The "straight-ahead" approximation (wherein the gamma-ray photons lose the proper amounts of energy on collisions with electrons but are assumed to keep their direction of travel unchanged) was used, and various representative path lengths in core and reflector material were chosen. Although the treatment of gamma-ray attenuation by the straight-ahead approximation and the multigroup method would not be entirely satisfactory in the case of shielding studies, this approach is considered to be useful for gamma-ray heating calculations, where the distances involved are comparatively small.

Although low-energy gamma radiation is scattered more than gamma radiation of higher energy, it is also absorbed more rapidly. Thus the absorption of low-energy radiation should not occur too far away from where it is assumed to take place in the straight-ahead approximation. Use of the straight-ahead approximation yields an underestimate of the energy absorbed near the source and an overestimate of the energy absorbed far away from the source. (It is to be noted that ample cooling is provided in the core where the gamma radiation is a minor contributor to the steady-state heat generation.) To counterbalance this effect, the low end of the spectrum is cut off at an energy of 0.5 mc^2 . This means that gamma radiation of energy less than 0.5 mc^2 is assumed to be absorbed immediately in this calculation.

The multigroup equations were formulated using two alternative methods:

1. The spectral energy density was assumed constant within each group.
2. The spectral energy density was assumed to vary linearly with energy within each group.

The results obtained by the two methods are in good agreement.

The effective energy absorption cross-section was found to be 0.067 cm^{-1} in the core and 0.056 cm^{-1} in the reflector for the path lengths considered. The inaccuracy in the absorption resulting from the use of these effective values does not exceed 4 per cent

in the core for the greatest path length used. In the reflector the inaccuracy is initially not greater than 9 per cent, and it decreases to 5 per cent at greater distances.

22.11 MATHEMATICAL FORMULATION

The penetration of gamma radiation through matter involves absorption processes due to the photoelectric effect and pair production, and energy degradation and scattering resulting from Compton collisions. Of primary interest in reactor heat generation problems is the energy deposition rate in material located at distances from core gamma-ray sources that are small compared to the distances of interest in shielding problems. In view of these relatively small distances (that is, within a few attenuation lengths of the source), it is reasonable to assume straight-ahead propagation. In this approximation the energy degradation in a Compton collision is considered, but Compton scattering is neglected. The equation governing the one-dimensional penetration and energy degradation of the gamma radiation is then given by⁶

$$\frac{\partial P(E, x)}{\partial x} + \mu(E)P(E, x) = \int_E^{E_m} k(E' \rightarrow E)P(E', x)dE' \quad (22.116)$$

where

- E = the photon energy in units of mc^2
- E_m = the maximum energy in the beam
- $P(E, x)$ = the number of photons in the beam passing point x per unit time in the energy range E to $E + dE$
- $\mu(E)$ = the total narrow-beam absorption and scattering cross-section
- $k(E' \rightarrow E)$ = the probability per unit length of path for a photon of energy E' to be Compton-scattered into the energy range E to $E + dE$. $k(E' \rightarrow E)$ is obtained from the Klein-Nishina formula, and is given as

$$k(E' \rightarrow E) = \frac{3}{8} N \sigma_{Th} \frac{E}{E'} \left\{ \frac{1}{E^2} + \frac{1}{E'^2} + \frac{1}{EE'} \left[\left(\frac{1}{E} - \frac{1}{E'} \right)^2 + 2 \left(\frac{1}{E'} - \frac{1}{E} \right) \right] \right\} \quad (22.117)$$

where σ_{Th} is the Thomson cross-section ($\sigma_{Th} = 8\pi r_0^2$, r_0 being the classical electron radius), and N is the number of electrons per unit volume.

In Eq. 22.116, the term $\mu(E)P(E, x)$ represents the loss of photons at x by pure absorption and scattering to lower energies. The term on the right represents the increase in density of photons of energy E brought about by the degradation of photons of energy E' , where E' varies from E to the maximum energy E_m .

It is convenient in this formulation to introduce the spectral energy density $Y(E, x)$ which is related to $P(E, x)$ by

$$Y(E, x) = EP(E, x) \quad (22.118)$$

Substitution of Eq. 22.118 into Eq. 22.116 yields

$$\frac{\partial Y(E, x)}{\partial x} + \mu(E)Y(E, x) = \int_E^{E_m} K(E' \rightarrow E)Y(E', x)dE' \quad (22.119)$$

where

$$K(E' \rightarrow E) = \frac{E}{E'} k(E' \rightarrow E) = \frac{3}{8} N \sigma_{Th} \frac{1}{E'^2} \left\{ 1 + \frac{E^2}{E'^2} + \frac{E}{E'} \left[\left(\frac{1}{E} - \frac{1}{E'} \right)^2 + 2 \left(\frac{1}{E'} - \frac{1}{E} \right) \right] \right\} \quad (22.120)$$

In the multigroup formulation the spectral energy range is divided into finite regions of arbitrary energy widths ΔE_1 . The magnitude of each group width ΔE_1 is given by $(E_{1+1} - E_1)$; and Y_1 is the spectral energy density corresponding to the energy E_1 . The multigroup equations are formed from Eq. 22.119 by first rewriting the right-hand side

as a summation of integrals whose limits of integration correspond to the group limits, and then integrating both sides over the i th group. One then obtains

$$\int_{E_1}^{E_{i+1}} \frac{\partial Y(E, x)}{\partial x} dE + \int_{E_1}^{E_{i+1}} \mu(E) Y(E, x) dE = \int_{E_1}^{E_{i+1}} dE \sum_{j \geq 1} \int_{E_j}^{E_{j+1}} K(E' - E) Y(E', x) dE' \quad (22.121)$$

In order to proceed further, assumptions are made as to the dependence of $Y(E, x)$ on energy. In particular, the following two approaches lend themselves to simple formulations of the multigroup equations. In the first approach the spectral energy density is assumed independent of energy within each group, and in the second approach the spectral energy density is assumed to be a linear function of energy within each group. These two methods will now be considered in turn.

Method A. $Y(E, x)$ independent of E within each group.

Eq. 22.121 then becomes:

$$\left[E_{i+1} - E_1 \right] \frac{\partial Y_i}{\partial x} + Y_i \int_{E_1}^{E_{i+1}} \mu(E) dE = \sum_{j \geq 1} Y_j \int_{E_j}^{E_{j+1}} f_{E' - \Delta E_1} dE' \quad (22.122)$$

where

$$f_{E' - \Delta E_1} = \int_{E_1}^{E_{i+1}} K(E' - E) dE \quad (22.123)$$

It is now convenient to introduce the total group energy N_i defined by

$$N_i = \int_{E_1}^{E_{i+1}} Y dE \quad (22.124)$$

Since in this formulation Y is independent of energy in each group, Eq. 22.124 becomes

$$N_i = Y_i (E_{i+1} - E_1) \quad (22.125)$$

Substitution of Eq. 22.125 into Eq. 22.122 yields

$$\frac{\partial N_i}{\partial x} + \bar{\mu}_i N_i = \sum_{j \geq 1} \bar{f}_{j,i} N_j \quad (i = 0, 1, \dots, n-1) \quad (22.126)$$

where n is the number of groups chosen,

$$\bar{\mu}_i = \frac{1}{E_{i+1} - E_1} \int_{E_1}^{E_{i+1}} \mu(E) dE \quad (22.127)$$

and

$$\bar{f}_{j,i} = \frac{1}{E_{j+1} - E_j} \int_{E_j}^{E_{j+1}} f_{E' - \Delta E_1} dE' \quad (22.128)$$

It should be noted that the series of differential equations represented by Eq. 22.126 are of the form

$$\frac{\partial N_s}{\partial x} + \bar{\mu}_s N_s = \bar{f}_{s,s} N_s$$

$$\frac{\partial N_{s-1}}{\partial x} + \bar{\mu}_{s-1} N_{s-1} = \bar{f}_{s,s-1} N_s + \bar{f}_{s-1,s-1} N_{s-1}$$

$$\frac{\partial N_{s-2}}{\partial x} + \bar{\mu}_{s-2} N_{s-2} = \bar{f}_{s,s-2} N_s + \bar{f}_{s-1,s-2} N_{s-1} + \bar{f}_{s-2,s-2} N_{s-2} \quad (\text{etc.})$$

which enables one to obtain analytic solutions for each N_s in succession without undue amounts of labor. This simplifying feature is due to the fact that the absorption of gamma radiation does not give rise to gamma radiation of higher energy, unlike the

fission absorption of neutrons. The initial conditions to be used are determined by the shape of the initial spectrum.

Method B. $Y(E, x)$ is a linear function of E in each group.

Let

$$Y = \frac{Y_1 + Y_{1+1}}{2} + \left[E - \frac{E_1 + E_{1+1}}{2} \right] \frac{Y_1 - Y_{1+1}}{E_1 - E_{1+1}} \quad \text{for } E_1 \leq E \leq E_{1+1} \quad (22.129)$$

Substitution of Eq. 22.129 into Eq. 22.121 yields, after some elementary operations

$$\frac{(E_{1+1} - E_1)}{2} \frac{\partial}{\partial x} (Y_1 + Y_{1+1}) + \bar{\mu}_1 \left[Y_1 E_{1+1} - Y_{1+1} E_1 \right] + (Y_{1+1} - Y_1) \bar{U}_1 = \sum_{j=1}^{\infty} \left\{ (Y_j E_{j+1} - Y_{j+1} E_j) \bar{f}_{j,1} + (Y_{j+1} - Y_j) \bar{g}_{j,1} \right\} \quad (22.130)$$

where $\bar{\mu}_1$ and $\bar{f}_{j,1}$ are again defined by Eq. 22.127 and 22.128 and where

$$\bar{U}_1 = \frac{1}{E_{1+1} - E_1} \int_{E_1}^{E_{1+1}} E \mu(E) dE \quad (22.131)$$

$$\bar{g}_{j,1} = \frac{1}{E_{j+1} - E_j} \int_{E_j}^{E_{j+1}} E' f_{E' - \Delta E_1} dE' \quad (22.132)$$

When Y_{1+1} equals Y_1 , Eq. 22.130 reduces to Eq. 22.126, as is to be expected.

The series of differential equations represented by Eq. 22.130 for n groups comprise n equations involving $n+1$ unknowns. To solve these equations, it is necessary to make an additional assumption. The customary procedure of setting the spectral energy density equal to zero at the upper end of the spectrum is followed.

Before proceeding further it is necessary to choose group widths ΔE_1 and to evaluate the coefficients appearing in Eq. 22.126 and 22.130. The group widths are chosen so that there is a reasonable degree of uniformity of source strength, absorption, and scattering cross-sections within a given group. The coefficients $\bar{\mu}_1$, $\bar{U}_1$, $\bar{f}_{j,1}$ and $\bar{g}_{j,1}$ depend on the composition of the region through which the gamma radiation is traveling.

22.12 APPLICATIONS TO SIR-TYPE REACTORS

The initial conditions to be used in obtaining solutions of Eqs. 22.126 and 22.130 are determined by the gamma-ray birth spectrum in the core. This spectrum was constructed in the following manner:

1. Measurements made on the Los Alamos Water Boiler and Fast Reactor⁷ show that the number of photons emitted per unit energy interval decreases exponentially with photon energy in an energy range of approximately 0.5 to 8 Mev. The gamma-rays comprising this spectrum consist of prompt fission gammas, delayed fission-product gammas, and capture gamma-rays primarily from the product nucleus U^{236} . From Fig. 3 of reference 7 an average representation of the variation of the photon density spectrum was found to be $\exp(-0.61 E)$, where, as before, E is in units of mc^2 . On this basis the spectral energy density was taken to vary as $E \exp(-0.61 E)$. This relation was used to distribute a total energy of 14 Mev/fission in the energy range 0.5 to 10 mc^2 . The total of 14 Mev/fission includes the following:
 - a. Prompt fission gamma-rays: 5 Mev/fission⁸
 - b. Delayed fission-product gamma-rays: 6 Mev/fission⁸
 - c. Capture gamma-rays from product nucleus U^{236} : 3 Mev/fission
(Calculated from absorptions obtained from critical mass calculations for the reactor)

This contribution to the total birth spectrum is indicated as the dashed curve in Fig. 22.24.

2. To the above total of 14 Mev/fission must be added 0.1 Mev/fission resulting from capture processes in sodium in the core, and 0.9 Mev/fission resulting from

Fig. 22.24—Gamma-ray birth spectrum.

capture processes in iron in the core. These numbers were deduced from the absorptions obtained in critical mass calculations. Since Na^{24} emits a 1.38- and a 2.76-Mev gamma, the total of 0.1 Mev/fission was distributed with a flat distribution in the range 2 to 6 mc^2 . Of the total of 0.9 Mev/fission resulting from neutron capture processes in iron,⁹ 0.3 Mev/fission was distributed with a flat distribution in the range of 2 to 6 mc^2 , and 0.6 Mev/fission was distributed in the same manner in the range 6 to 10 mc^2 . Finally, as an estimate of the gammas resulting from induced radioactivity in the core, a total of 1 Mev/fission was distributed with a flat distribution in the range 0.5 to 4 mc^2 .

The above contributions to the birth spectrum are depicted in Fig. 22.24. The components of the spectrum discussed in 1. and 2. were used to synthesize a smooth spectrum having a total energy of 16 Mev/fission which is represented by the solid line in Fig. 22.24.

With the birth spectrum thus obtained, it was necessary to divide the energy range 0.5 to 10 mc^2 into a finite number of groups and to obtain representations of the initial spectrum for either Method A or Method B, where the spectral energy density is assumed independent of energy or a linear function of energy within each group.

Five energy groups were chosen in order to keep the numerical work within the scope of hand calculations. These groups covered the following energy ranges

- Group 0: $0.5 \leq E < 1 \text{ mc}^2$
- Group 1: $1 \leq E < 2 \text{ mc}^2$
- Group 2: $2 \leq E < 4 \text{ mc}^2$
- Group 3: $4 \leq E < 6 \text{ mc}^2$
- Group 4: $6 \leq E < 10 \text{ mc}^2$

The choice of group widths, while arbitrary, was such that the narrowest groups lay in the energy region where the initial spectrum and the total absorption cross-section varied most rapidly with energy. (See Figs. 22.24 and 22.25.)

The initial spectrum and the representations of this spectrum that are to be used for Methods A and B are shown in Fig. 22.26. In each representation the total energy in each group is the same as the total energy in the continuous spectrum between the energy limits defining each group. In accordance with the discussion at the end of Sec. 22.11,

the spectral energy density was set equal to zero at the upper end of the spectrum in the representation used for Method B.

The relation between the total energy in a group N_i and the spectral energy density at the ends of each group is given by Eq. 22.125 for Method A. For Method B it is obtained by substituting Eq. 22.129 into Eq. 22.124, yielding:

$$N_i = \left(\frac{Y_i + Y_{i+1}}{2} \right) (E_{i+1} - E_i) \quad (22.133)$$

By integration over the continuous spectrum shown in Fig. 22.26 the following values were obtained for the total energy in each group

Group 0:	1.4 Mev/fission	0.5	$\leq E < 1 \text{ mc}^2$
Group 1:	3.5 Mev/fission	1	$\leq E < 2 \text{ mc}^2$
Group 2:	5.9 Mev/fission	2	$\leq E < 4 \text{ mc}^2$
Group 3:	3.1 Mev/fission	4	$\leq E < 6 \text{ mc}^2$
Group 4:	2.1 Mev/fission	6	$\leq E < 10 \text{ mc}^2$

The insertion of these values into Eqs. 22.125 and 22.133 yielded the representations of the spectrum shown in Fig. 22.26 that were used for Methods A and B.

In Sec. 22.13, where solutions are obtained for each method, results will be determined for an initial spectrum normalized to unity.

Finally, the coefficients $\bar{\mu}_1$, $\bar{U}_1$, $\bar{f}_{j,1}$, and $\bar{g}_{j,1}$, which appear in the multigroup Eqs. 22.126 and 22.130 must be determined.

A plot of the total absorption curve $\mu(E)$ for the reactor core and reflector material is given in Fig. 22.25. Knowledge of this function permits the determination of $\bar{\mu}_1$ and $\bar{U}_1$ from Eqs. 22.127 and 22.131 by means of numerical integration. The results are listed in Table 22.4.

TABLE 22.4--VALUES OF $\bar{\mu}_1$ AND $\bar{U}_1$ IN CORE AND REFLECTOR

Group Index	$\bar{\mu}_1$		$\bar{U}_1$	
	Core	Reflector	Core	Reflector
$i = 0$	0.2380	0.1726	0.1786	0.1264
$i = 1$	0.1536	0.1280	0.2243	0.1870
$i = 2$	0.1039	0.0920	0.3037	0.2637
$i = 3$	0.0791	0.0684	0.3935	0.3379
$i = 4$	0.0660	0.0542	0.5262	0.4311

It is seen from Eqs. 22.128 and 22.132 that $\bar{f}_{j,1}$ and $\bar{g}_{j,1}$ depend on the function $f_{E' \rightarrow \Delta E_1}$ which is defined by Eq. 22.123. Substitution of Eqs. 22.120 and 22.123 yields

$$\begin{aligned} \frac{f_{E' \rightarrow \Delta E_1}}{(3/8)N_{\text{Th}}} &= \left[\frac{1}{E'^2} \left(E + \frac{E^3}{3E'^2} + \frac{1}{E'} \left\{ \ln E + \left[\frac{1}{2E'^2} + \frac{1}{E'} \right] E^2 - \left[2 + \frac{2}{E'} \right] E \right\} \right) \right]_{E=E_1}^{E=E'} \\ &= \left[\frac{1}{E'^2} \left(E + \frac{E^3}{3E'^2} + \frac{1}{E'} \left\{ \ln E + \left[\frac{1}{2E'^2} + \frac{1}{E'} \right] E^2 - \left[2 + \frac{2}{E'} \right] E \right\} \right) \right]_{E=E_{i+1}}^{E=E_{i+1}} \quad (22.134) \\ &\quad \text{for } E_1 \leq E' \leq E_{i+1} \\ &\quad \text{for } E_{i+1} \leq E' \leq E_m \end{aligned}$$

Fig. 22.25—Total photon absorption cross-sections in reactor core and reflector.

The function $f_{E' \rightarrow \Delta E_1} / (3/8) N \sigma_{Th}$ is plotted in Fig. 22.27 for the group widths chosen. Knowledge of this function permits the determination of $\bar{f}_{j,i} / (3/8) N \sigma_{Th}$ and $\bar{g}_{j,i} / (3/8) N \sigma_{Th}$ from Eqs. 22.128 and 22.132. When these values are multiplied by $(3/8) \sigma_{Th}$ and by the number of electrons per unit volume (N being a different number in core and reflector material), the following in Table 22.5 are obtained for $\bar{f}_{j,i}$ and $\bar{g}_{j,i}$ in core and reflector material.

Substitution of the coefficients listed in Tables 22.4 and 22.5 into Eqs. 22.126 and 22.130 yields the desired sets of equations for Methods A and B in the core and reflector.

Fig. 22.26—Gamma-ray birth spectra.

TABLE 22.5—VALUES OF $\bar{f}_{j,1}$ AND $\bar{g}_{j,1}$ IN THE CORE AND REFLECTOR

	Core	Reflector		Core	Reflector
$\bar{f}_{4,4}$	0.0072	0.0063	$\bar{g}_{4,4}$	0.0594	0.0524
$\bar{f}_{4,3}$	0.0068	0.0060	$\bar{g}_{4,3}$	0.0512	0.0451
$\bar{f}_{4,2}$	0.0050	0.0044	$\bar{g}_{4,2}$	0.0385	0.0340
$\bar{f}_{4,1}$	0.0022	0.0019	$\bar{g}_{4,1}$	0.0167	0.0147
$\bar{f}_{4,0}$	0.0011	0.0010	$\bar{g}_{4,0}$	0.0086	0.0075
$\bar{f}_{3,3}$	0.0092	0.0082	$\bar{g}_{3,3}$	0.0516	0.0455
$\bar{f}_{3,2}$	0.0155	0.0137	$\bar{g}_{3,2}$	0.0754	0.0665
$\bar{f}_{3,1}$	0.0055	0.0049	$\bar{g}_{3,1}$	0.0266	0.0234
$\bar{f}_{3,0}$	0.0027	0.0024	$\bar{g}_{3,0}$	0.0131	0.0116
$\bar{f}_{2,2}$	0.0225	0.0198	$\bar{g}_{2,2}$	0.0715	0.0631
$\bar{f}_{2,1}$	0.0196	0.0173	$\bar{g}_{2,1}$	0.0541	0.0477
$\bar{f}_{2,0}$	0.0076	0.0067	$\bar{g}_{2,0}$	0.0211	0.0186
$\bar{f}_{1,1}$	0.0421	0.0372	$\bar{g}_{1,1}$	0.0670	0.0591
$\bar{f}_{1,0}$	0.0379	0.0334	$\bar{g}_{1,0}$	0.0465	0.0410
$\bar{f}_{0,0}$	0.0630	0.0556	$\bar{g}_{0,0}$	0.0593	0.0523

22.13 SOLUTIONS OF THE MULTIGROUP EQUATIONS

In this section, pertinent results of the solutions of the multigroup equations, subject to the initial representations of the birth spectrum depicted in Fig. 22.26 for Methods A and B, are presented in graphical form.

As mentioned before, calculations of gamma-ray absorption for various geometrical configurations are simplified by the use of an energy absorption kernel of the form

Fig. 22.27—Relative Compton scattering into each group.

$\sigma_a \exp(-\sigma_a x)$, where σ_a is an effective energy absorption cross-section to be obtained from the multigroup calculations.

The procedure followed is to calculate $N = \sum_1 N_i$, the total energy in all groups, as a function of distance along a straight-line path starting at a point source located in the core. Various representative distances of travel in core and reflector material are chosen. The following straight-line paths were considered

- Path 1 - 60 cm in core material only
- Path 2 - 20 cm in core and then 40 cm in reflector
- Path 3 - 40 cm in core and then 20 cm in reflector
- Path 4 - 60 cm in reflector material only (the source point being located at the core-reflector interface)

Fig. 22.28—Spectral energy distribution after various path lengths in core and reflector material.

In order to perform the calculations for Paths 2 and 3, the spectral energy distribution in the core must first be determined as a function of distance. Then, for example, in Path 2, the spectral energy distribution in the core after 20 cm of travel is used as the initial spectrum for the subsequent 40 cm of travel in reflector material. In addition, the spectral distributions for various distances of travel in both media are normalized to unity for purposes of comparison.

Finally, the solution for $N = \sum_1 N_i$, the total energy in the beam, is normalized to correspond to a total initial energy of unity. Then $(-\partial N/\partial x)$ is the fractional energy absorption per centimeter of path, and the straightness of a plot of this function on semi-logarithmic paper indicates how accurately the absorption can be represented by a single exponential.

It is seen that the solutions of Eq. 21.126 are of the form

$$N_i = \sum_{k \geq 1} \alpha_{ki} e^{-\beta_k x} \quad (k, i = 0, 1, 2, 3, 4)$$

for the energy in each group. $N = \sum_1 N_i$ and $(-\partial N/\partial x)$ are then calculated. In plotting the spectrum, Eq. 22.125 is used to obtain Y_i from N_i .

The solutions of Eq. 22.130 are also of the form

$$Y_i = \sum_{k \geq 1} \gamma_{ki} e^{-\delta_k x} \quad (k, i = 0, 1, 2, 3, 4)$$

The energy in each group, N_i , is obtained from Y_i by use of Eq. 22.133; $N = \sum_1 N_i$ and $(-\partial N/\partial x)$ are calculated as before.

The equilibrium spectrum is obtained by considering the dominant exponential term in each expression for Y_i at very large distances. (In the actual physical problem a true equilibrium is not reached, for the shape of the spectrum at high energies is continually changing. The equilibrium spectrum mentioned here is a mathematical equilibrium which arises from the multigroup treatment. This mathematical equilibrium may represent the true situation fairly well at intermediate distances but must not be considered to be valid for the high-energy part of the spectrum at extremely large distances.)

Figure 22.28 compares the normalized spectral energy distributions in core and reflector material at $x = 20$ cm, 40 cm, and at equilibrium as obtained by Methods A and B for Paths 1 and 4. Figure 22.29 compares the fractional energy absorption per centimeter of path as a function of distance of travel in each region. The curves starting at $x = 20$ and 40 cm correspond to Paths 2 and 3 where the radiation travels 20 and 40 cm, respectively, in core material before entering the reflector. The reflector curve starting at $x = 0$ (Path 4) corresponds to gamma radiation born at the core-reflector interface which then travels 60 cm in reflector material.

These illustrations show that essentially the same results are obtained by assuming the spectral energy density to be constant or a linear function of energy in each group. The most marked deviation in the results of the two methods occurs in the equilibrium cases (bottom of Fig. 22.28), where the spectral energy density in the linear case (Method B) is too high at 6 mc^2 . This is directly attributable to the assumption that $Y(E, x)$ is zero at the high end of the spectrum. At equilibrium the spectral energy distribution is fairly flat, hard to fit by a linear function which is constrained to zero at 10 mc^2 . This deviation is not manifested in the other spectral comparisons where $Y(E, x)$ decreases with increasing energy.

Detailed comparison of the spectral distributions in core and reflector shows that there is less energy in the lowest energy group in core material than in reflector material at corresponding distances of travel. This is due to the relatively greater absorption properties of core material at these energies (Fig. 22.25). In conjunction with this effect it is interesting to note (Figure 22.29, reflector curves starting at $x = 20$ and 40 cm) that a ray leaving the core is initially less strongly absorbed than it is after it has travelled farther into the reflector. This is due to the fact that the lower energy groups of gamma-rays leaving the core contain less energy than the corresponding groups of rays which travelled the same distance in all reflector material.

Fig. 22.29—Fractional energy absorption rates in reflector and core materials. Total energy in initial spectrum normalized to unity.

There is a slight tendency for the fractional energy absorption curves to be S-shaped as a function of distance as compared to a purely exponential absorption. This is illustrated by the exaggerated curve on the left in Fig. 22.30 where $\ln(-dN/dx)$ is plotted as a function of x . The true curve for $\ln N$ (on the right in Fig. 22.30) first falls below the exponential due to the softening of the spectrum by Compton collisions. At intermediate distances the $\ln N$ curve becomes less steep as there is less energy left in the beam and the spectrum is gradually rehardening. Finally, at large distances, the hardening effect predominates. Hence, the curve for $\ln(-dN/dx)$ first rises above the exponential absorption curve, falls below it at intermediate distances, and then rises again at very large distances. For example, the initial slope of the all-reflector curve shown in Fig. 22.29

Figure 22.30

is 0.052, at intermediate distances it is 0.056, while at still larger distances it approaches 0.048 which is the attenuation coefficient for the multigroup equilibrium spectrum.

It is to be noted that the fractional energy absorption per centimeter of path, which depends on the total energy in all groups, is less sensitive than the shape of the spectral energy density curve to minor variations in the initial conditions and compositions of the regions.

From consideration of Fig. 22.29 it is found that the effective energy-absorption cross-sections in the core and reflector are 0.067 and 0.056 cm⁻¹, respectively. In Fig. 22.31 the fractional absorption per centimeter of path in the core as obtained by the multigroup methods is compared with the function

$$-\frac{dN}{dx} = 0.067 e^{-0.067x} \quad (22.135)$$

The results given by this equation are in good agreement with multigroup calculations, and the error is only 4 per cent at a distance of 60 cm. At smaller distances it is considerably less. In Fig. 22.32 the fractional energy absorption per centimeter of path in the reflector as calculated by the multigroup method for radiation originally emitted from a unit source in the core is compared with the function

$$e^{-0.067d} \left\{ 0.056 e^{-0.056x} \right\} \quad (22.136)$$

where d is the distance travelled in the core and x is the distance travelled in the reflector. Equation 22.136 can be interpreted in the following manner:

The factor $e^{-0.067d}$ represents the strength at the core-reflector interface of radiation emitted from a unit source in the core d centimeters away from the interface. The factor $0.056 e^{-0.056x}$ represents the fractional energy absorption per centimeter of path in reflector material due to a unit source located at the core-reflector interface.

The results given by Eq. 22.136 are accurate within 9 per cent at small distances from the core-reflector interface, and the error decreases to less than 5 per cent at greater distances.

Since the effective energy-absorption coefficients of the core and reflector do not differ greatly, the possibility arises of choosing a single effective energy-absorption cross-section to represent both regions. This, of course, is not so accurate as treating each region separately but the assumption of equal mean free path in all regions simplifies certain absorption calculations for non-planar geometry. It is seen from Fig. 22.35 that no single exponential can represent exactly the absorption in both regions. Hence, the accuracy of such a choice depends on which path lengths occur most frequently. The values of 0.067 and 0.056 cm⁻¹ for the core and reflector suggest that a value of 0.06 cm⁻¹ be chosen to represent both regions. The function $0.06 e^{-0.06x}$ represents the absorption within 30 per cent in both regions for total path lengths in core and reflector not exceeding 40 cm.

It should be remembered that the multigroup calculation is not valid at large distances because of the neglect of angular scattering and the incorrect representation of

the top group which is gradually changing shape. However, the relative magnitudes of the lower end of the spectrum are reasonably good as given by the multigroup calculation.

As mentioned in Sec. 22.10, use of the straight-ahead approximation underestimates the energy absorbed near the source and overestimates the energy absorbed far away from the source. To counterbalance this effect, the multigroup spectrum was cut off at

Fig. 22.31—Fractional energy absorption rates in core material. Total energy in initial spectrum is normalized to unity.

Fig. 22.32—Fractional energy absorption rates in reflector material. Total energy in initial spectrum normalized to unity.

an energy of 0.5 mc^2 , and radiation of lower energy was assumed to be absorbed immediately.

One way of checking the multigroup calculation is to average an appropriate energy-absorption cross-section for core and reflector material over the initial spectrum and compare these average values with the initial fractional absorption per centimeter of path as given by the multigroup calculation (0.066 and 0.053 cm^{-1} are initial values in

core and reflector; see Fig. 22.35). An "appropriate" energy-absorption cross-section in this calculation is one that is consistent with the assumption that all radiation scattered below 0.5 mc^2 is absorbed immediately. This cross-section is obtained by subtracting from the total cross-section (μ_T) the cross-section (μ_S) for the fraction of the energy scattered to energies greater than 0.5 mc^2 . This scattering cross-section is given by

$$\mu_S(E') = \int_{0.5}^{E'} \frac{E}{E'} k(E' - E) dE = \int_{0.5}^{E'} K(E' - E) dE \quad (22.137)$$

where $k(E' - E)$ and $K(E' - E)$ are given by Eqs. 22.113 and 22.116, respectively. When this calculation is carried out and $\mu_a = \mu_T - \mu_S$ is formed (see Fig. 22.25 for μ_T), the results obtained from averaging μ_a over the initial spectrum agree exactly with the values of 0.066 and 0.053 cm^{-1} obtained from the multigroup calculation. (Differences between core and reflector material enter into the factor $(3/8)N\sigma_{Th}$ which appears in Eq. 22.117. This factor $(3/8)N\sigma_{Th}$ equals 0.1301 cm^{-1} for the reflector and 0.1475 cm^{-1} for the core material.)

Note that the results for the fractional absorption per centimeter of path do not differ greatly from straight lines (see Fig. 22.29). Thus the effective attenuation coefficients as obtained from the multigroup results at moderate distances are, for the case considered, very close to the initial attenuation coefficients (i.e., 0.067 and 0.056 as compared with 0.066 and 0.053 cm^{-1}). This would indicate that in reactor calculations it is a satisfactory approximation to use the appropriate energy absorption coefficient evaluated for the initial spectrum as discussed above.

Chapter 22

GAMMA-RAY AND NEUTRON HEAT GENERATION

C. HEAT GENERATION IN AN INTERMEDIATE REACTOR

H. HURWITZ, Jr., J. B. NIMS, Jr., and M. L. STORM

22.14 INTRODUCTION

In this final part of the chapter, we obtain the magnitude and distribution of the heat generated in an intermediate reactor by the absorption of gamma radiation, by neutron degradation, and by (n, α) reactions in beryllium. Account is not taken of the kinetic energy of fission products and fission betas in the core. All values are obtained for steady-state operation at 1 Mw.

As was mentioned previously in A, the procedure has been to treat various alternative simple models and compare the results, because techniques were not available for treating the actual geometry rigorously. The results obtained for the various models are in reasonable agreement with one another and with the experimental results described in Chap. 23.

Information on neutron degradation and gamma-source strengths resulting from neutron capture is obtained from neutron multigroup calculations. At the time of this study (1952) such calculations had been performed only for spherical representations of the reactor. Hence a necessary approximation was the employment of neutron multigroup results obtained for a spherical geometry. After the sources of gamma radiation have been determined, the magnitude and distribution of the absorption of the gamma radiation can be obtained approximately by various integrations of the type discussed in Sec. 22.2. The gamma-ray absorption problem is treated by the straight-ahead approximation and the assumption of exponential attenuation. A justification of this approach is given in B, where a gamma-ray multigroup calculation is carried out to determine the effective energy absorption cross-sections for the SIR-type core and reflector regions.

For convenience, the following schematic drawings of the regions of interest in this calculation are presented in Figs. 22.33 and 22.34.

It is evident from Fig. 22.33 that the setting of the movable control drum should be an important parameter in the heat-generation calculations. Finally, it should be pointed out that the power distribution in the core is nonuniform.

A first approach is to calculate the heat generation in the core, reflector, and thermal blanket on the basis of spherical geometry in which the core, reflector, and blanket are represented by concentric spherical regions. In this approximation it is possible to utilize fully the results of the critical mass calculations on the absorption and scattering of neutrons. The control drums are assumed to be in the 180-deg position, where the movable reflector material is in the position closest to the core. Account is taken of neutron degradation and (n, α) reactions in beryllium, as well as the absorption of gamma radiation born in the core, reflector, and thermal blanket.

A second approach, which takes account of the non-uniform gamma-ray source in the core and the plane geometry at the interface between the core and the stationary

Fig. 22.33—Schematic cross-sectional view through reactor mid-plane. Control drum angle, θ , is indicated.

reflector, is to represent the core as a slab with a source distribution that rises exponentially near the core-reflector interface. This model yields an overestimate of the heat generation due to the core gammas, but it will be seen that this effect is of importance only near the core-reflector interface.

With the information obtained from the above calculations as a background, a more realistic representation of the reactor as given by a cylindrical model is considered (Fig. 22.34). It is then possible to take account of the differences in composition of the top and side reflector and the top and side blanket. The distributions obtained from spherical calculations will be used to represent the heat generation in the equatorial plane of the cylinder. The total heat generations calculated via both models are in good agreement.

Fig. 22.34—Schematic vertical cross-sectional view of reactor.

In the above calculations the control drums were assumed to be in the 180-deg position. Heat generation in the reactor equatorial plane will also be calculated as a function of the control drum position, measured by angle θ on Fig. 22.33. In particular, the 0-deg, 90-deg, and 180-deg positions are considered.

22.15 SPHERICAL MODEL

In order to take full advantage of available data on the absorption and degradation of neutrons, the first heat-generation calculation is done for a spherical model whose dimensions are similar to those used in critical mass calculations. In particular, this model has a core radius of 39 cm, a reflector thickness of 18 cm, and a blanket thickness of 14 cm. This core has a volume of $2.5 \times 10^5 \text{ cm}^3$. (A cylindrical representation of the reactor is discussed in Sec. 22.17.) Results for a 1-Mw reactor power level are expressed in watts/cm³ of homogeneous material in the regions of interest. The contributions to the heat generation by gamma-rays born in the core, reflector, and blanket as well as by neutron degradation and (n, α) reactions in beryllium are considered.

The following gamma-ray source strengths and energy-absorption cross-sections are used in the core, reflector, and blanket:

1. Core

Total gamma-ray source strength of 16 Mev/fission which is equivalent to 80 kw at a reactor power level of 1 Mw. The effective energy absorption coefficient is 0.067 cm^{-1} .

2. Reflector

Total gamma-ray source strength of 13 kw. This is obtained from neutron-absorption data given in critical-mass calculations and the assumption that 8 Mev in gamma energy is released per neutron absorption. The effective energy-absorption cross-section of reflector material for gamma rays escaping from the core is 0.056 cm^{-1} . (The shape of and total energy in the core birth spectrum, as well as the values to be used for the effective energy absorption coefficients in core and reflector material, were discussed in Sec. 22.12.) Although the spectrum of the reflector-born gamma-rays, which arise from neutron capture in iron, contains less low energy than that of the gamma-rays which escape from the core, the contribution of these gammas to the heat generation is much less due to the weaker source strength. Hence, it is only a slight overestimate to assume that the energy-absorption cross-section for reflector-born gammas is also 0.056 cm^{-1} . Similarly, it is assumed that the energy-absorption cross-section of reflector material for the spectral energy distribution of blanket gammas which escape into the reflector is 0.056 cm^{-1} .

3. Blanket

Neutron multigroup calculations show that the neutrons can be considered to be exponentially absorbed within the first 14 cm of the blanket, the absorption coefficient being 0.46 cm^{-1} . In addition, on the assumption that 8 Mev in gamma energy is released for each neutron capture in the blanket iron, a total of 16 kw of gammas is found to be born in the blanket. In selecting an energy-absorption cross-section for blanket material which is considered to be 100 per cent iron, consideration was given to the spectrum of gamma rays entering the blanket from the reflector as well as to the gammas born in the blanket. Concerning the former, a mean energy-absorption coefficient¹⁰ for blanket material, averaged over gamma-ray spectra leaving the reflector, yielded an average energy-absorption cross-section of 0.21 cm^{-1} . Concerning the latter, calculations done by Nuclear Development Associates¹¹ show that the spectral energy density due to a plane monodirectional source in iron is essentially flat for initial source energies of 1/2, 1, 2, and 3 Mev, at penetrations of 4, 7, and 10 mean free paths. Although most of the energy contained in the capture gamma spectrum of iron is released as 7.6-Mev gammas,¹² a flat distribution was assumed valid and an average of the energy absorption coefficients for iron over a flat spectrum extending down to 0.3 Mev yielded an energy absorption coefficient of 0.207 cm^{-1} . Since the two values are approximately the same, an effective energy-absorption cross-section of 0.21 cm^{-1} is used for the blanket.

Using the above source strengths and energy-absorption coefficients in conjunction with the normalized absorption distributions given in Sec. 22.13, the heat generation due to

Fig. 22.35—Gamma-ray heat generation in a spherical model reactor. Heat generation rates integrated over the three regions are as follows [The notation (n,n') refers to neutron degradation. All figures are in kilowatts.]:

Heat Source	Core	Reflector	Blanket
Gammas born in core	58.4	15.7	9.4
Gammas born in reflector	1.6	7.7	
Gammas born in blanket	—	3.0	11.7
(n,α) and (n,n') reactions	18.6	10.1	—
Total of all sources	78.6	36.5	21.1

gamma-rays is obtained in the following manner. (The results are shown in Fig. 22.35.)

The total source of 80 kw (16 Mev/fission) in the 39-cm core was assumed to be uniformly distributed with a strength of 0.320 watt/cm^3 . (In reality there is a non-uniform power distribution in the core and this effect will be considered in Sec. 22.16 and in Sec. 22.21. However, except for heat generation near the core-reflector interface, the assumption of uniform source distribution is reasonably accurate.) The heat generation in the core was then obtained from Eq. 22.83 with the parameter $\sigma R = 2.61$.

One method for calculating the absorption of gamma-rays in the region external to a source region is to assume that the gamma-rays emanate from a surface source with an appropriate surface angular distribution and then calculate the absorption due to this surface distribution of sources. It was shown in Sec. 22.2 that a reasonable surface angular distribution for a sphere lies between a cosine and cosine-squared surface angular distribution, and the average of the absorptions obtained for both surface sources was used in this calculation. The fractional escape from the 39-cm core is 0.27 (Fig. 22.20), and the absorption in the reflector of the 21.6 kw of gammas leaving the core was calculated by using an average of the absorptions obtained from cosine and cosine-squared surface angular distributions (Eqs. 22.38 and 22.39) for the parameter $\sigma R = (0.056)(39) = 2.18$. (See Sec. 22.21 for the fraction escape from a spherical core with a non-uniform source distribution.) In calculating the heat generation in the blanket due to the core-born gammas, it was found that of the 21.6 kw that left the core, 5.9 kw escaped through the reflector into the blanket (Figs. 22.8B and 22.8C); and this energy was treated as emanating from a surface source on the reflector-blanket interface with a surface angular distribution that was the average of a cosine and cosine-squared distribution.

The total of 13.0 kw born in the reflector was assumed to be uniformly distributed with a source strength of $0.0245 \text{ watts/cm}^3$. (This over estimates the heating in the outer parts of the reflector and the blanket due to the reflector-born gammas and underestimates the heating in the center and inner parts of the reflector. However, the reflector-born gammas are only a small part of the total heating so that the under estimate is not serious.) The heat generation in core and reflector due to this source was obtained by considering two concentric spheres of radii 39 and 57 cm, each having a source strength of $0.0245 \text{ watts/cm}^3$ and an energy-absorption cross-section of 0.056 cm^{-1} . Subtraction of the heat generation (Eqs. 22.83 and 22.84) due to the smaller sphere from that due to the larger sphere yields the heat generation due to the spherical shell bounded by inner and outer radii of 39 and 57 cm. It is necessary to assume equal mean free paths to use the formulas referred to, and while the results obtained are correct for the reflector region, they are not valid for the core, whose cross-section is 0.067 cm^{-1} . However, this contribution to heat generation in the core is small, and the results in the core were modified to conform approximately to the true cross-section by multiplying the heat generation by $0.067/0.056$ and simultaneously changing the scale as measured from the core-reflector interface by the ratio $0.056/0.067$. The contribution of the reflector-born gammas to heat generation in the blanket was calculated by finding the escape from the reflector into the blanket, assuming the uniform source, and treating the 3.7 kw entering the blanket by the same method that was used in calculating the heat generation due to the 5.9 kw of core gammas that entered the blanket. In Fig. 22.35 the combined heat generation in the blanket region due to the total 9.6 kw of reflector and core gammas entering this region is shown.

The exponential source in the blanket can be treated by the analysis presented in Sec. 22.5, for values of the parameters σ_N , σ_0 , and σ_1 equal to 0.46, 0.21, and 0.056, respectively. The analysis referred to above was normalized to correspond to a unit source strength which is distributed according to the relation $\sigma_N \exp(-\sigma_N y)$. The conversion factor to the reactor at 1 Mw is $16,000/4\pi(57)^2 = 0.392$ in order to express the resulting heat generation in watts/cm^3 . Although the mathematics referred to is for plane geometry, this is a reasonable approximation since the reflector-blanket interface is 57 cm from the center of the core. However, to make allowance for the spherical geometry, the calculated heat generation due to the blanket-born gammas was multiplied by the factor $(57/r)^2$, r being the radial distance from the center of the core, in order to take account of the difference in heat generation for spherical and plane shells of thickness dr .

The heat generation due to neutron degradation and (n, α) reactions in beryllium was calculated by the method of M. H. Shackelford and D. Keaveney¹³ on the basis of information available from multigroup critical mass calculations for a 39-cm spherical core.

The results of all calculations based on the 39-cm core are plotted in Fig. 22.35 and the heat generation in each region due to the individual sources is tabulated. On the basis of this model the heat generation rates in the core, reflector, and blanket are 78.6, 36.5, and 21.1 kw, respectively, yielding a total heat generation in all three regions of 136.2 kw. Since the source energy available consisted of 137.6 kw (80.0 kw of core gammas, 13.0 kw of reflector gammas, 16.0 kw of blanket gammas, and 28.6 kw generated by neutron degradation and (n, α) reactions), it is seen that only 1.4 kw remains to be absorbed beyond 14 cm of blanket material. The total source of 137.6 kw (27.5 Mev/fission at a power of 1 Mw) is obtained as follows:

Source	Strength in Mev/fission
Prompt Fission Gammas	5.0
Delayed Fission Gammas	6.0
Capture Gammas: Blanket	3.2
Reflector	2.6
Core	4.0
Gammas Due to Induced Radioactivity in Core	1.0
Neutron Kinetic Energy	5.0
Kinetic Energy of α 's Released in (n, α) Reaction in Be	0.7
Total	27.5

22.16 HEAT GENERATION DUE TO A NON-UNIFORM CORE GAMMA-RAY SOURCE (PLANE MODEL)

Measurements of the power distribution in critical assemblies (Chap. 21, Table 21.3) show that the largest radial power gradients occur near the interface between the core and side reflector when measured in the direction of the beryllium reflector region as distinguished from the B_4C poison region. However, the power distribution in the radial direction was found to be fairly flat until a region which lies within 6 cm of the core-reflector interface was reached.

On the basis of these measurements an upper limit to the radial effect of non-uniform core gamma-ray sources was obtained by representing the core as a slab of 78-cm thickness. This is a reasonable approximation since the spatial orientation of the core and stationary reflector are well represented in plane geometry. The following source distributions were used (see Fig. 22.36):

Fig. 22.36—Source distribution in core as represented by a 78-cm slab.

1. An exponential source of $5.55 (0.1 e^{-0.1x})$ watts/cm³ in the 6 cm adjacent to the core-reflector interfaces where x (cm) is measured from the interface.
2. A uniform source of 0.305 watts/cm³ in the remaining 66 cm.

The above source strengths were chosen so that the average source per unit volume in the slab was 0.322 watts/cm³ in conformance with the source strength used in the last section for the 39-cm spherical model. In this plane model the gamma-ray source density at the core-reflector interface is 0.555 watts/cm³ which is a liberal upper bound for the source density.

With these source distributions the heat generation in core and reflector material was calculated by the methods and formulas presented in Secs. 22.5 and 22.6. The results of the calculation are shown in Fig. 22.37 as the SLAB curve. For comparison the heat generation due to a uniform distribution of core gamma-ray sources in a 39-cm-radius sphere is replotted on the same graph, as the SPHERE curve.

It is evident that the core gamma-heat generation in the reflector and last 6 cm of the core is greater when the core is represented by a slab with the above source distribution than it is when represented by a sphere with a uniform source distribution, the average source per unit volume being the same in both models. This is brought about by the additional transmission of gammas arising from the plane geometry and the exponential behavior of the source that was assumed in the plane model. It is seen in Fig. 22.37 that the excess heat generation of the plane model over the 39-cm spherical model is about 0.1 watts/cm³ at the core-reflector interface. This effect is appropriate for regions around the core periphery which are not close to boron.

The SPHERE, MODIFIED curve in Fig. 22.37 represents an approximate modification of the heat generation in the spherical model which takes into consideration the non-uniform core gamma source. It was obtained by reducing the results of the plane model in the reflector by the ratio $(39/r)^2$ at large distances, and by using the criterion that the total heat generation in the modified spherical model should not differ greatly from the unmodified spherical model. Although this calculation yields information concerning the peak heating to be expected in the equatorial plane of the reactor, it is desirable to use a systematic approach which will have more connection with the cylindrical reactor model. This will be done in the next section where the heat generation will be recalculated for a spherical model of smaller core radius (36 cm) but with the same total source strengths that were used in the last section for the 39-cm sphere. It is

Fig. 22.37—Effect of nonuniform core-gamma-source distribution on heat generation in core and reflector. "SLAB" refers to the 78-cm slab model of Fig. 22.36, with a uniform source out to 33 cm, and an exponentially increasing source from 33 to 39 cm. "SPHERE" refers to the 78-cm-diameter spherical model, with a uniform source out to 39 cm. Average source strength is same as in the "SLAB." By this model, the integrated core heating is 58.4 kw, and that of the reflector is 15.7 kw. "SPHERE, MODIFIED" refers to a modification which takes account of the nonuniform gamma-ray source in the sphere (See Sec. 22.16). By this model, the integrated core heating is 57.8 kw, and that of the reflector is 19.2 kw.

obvious that the heat generation obtained from this model will be greater than that obtained from the 39-cm model. In addition, it will be seen that the core gamma heating in the reflector region of the 36-cm spherical model is in agreement with the heat generation due to core gammas in the reflector equatorial plane of the cylindrical representation.

22.17 CYLINDRICAL MODEL

A SIR-type reactor core is taken to have a volume of $3 \times 10^5 \text{ cm}^3$ and a height of 70 cm. A cylinder of radius 36 cm and height 72 cm has a volume of $2.93 \times 10^5 \text{ cm}^3$, and these dimensions are used for a cylindrical representation of the reactor core. (An appropriate radius for a cylinder equal in length to the reactor core could be found to give the correct core volume, but the model with equal height and diameter was chosen for convenience. The slight difference in heat-generation values arising from the difference in dimensions is within the accuracy of the calculation.)

Although a cylindrical model is a more realistic representation of the reactor than is a spherical model, data on the absorption and degradation of neutrons were available only for the spherical model. In order to utilize these neutron data in a cylindrical model, it is assumed as a first approximation that the relationship of the heat generation in the spherical and cylindrical models arising from neutron degradation and capture gamma sources is the same as the relationship of the heat generation in the two models arising from a uniformly distributed core gamma source of 80 kw. The core-born gammas are a major contributor to heat generation throughout the reactor (exclusive of fission fragment kinetic energy which is deposited in the fuel elements), and a spherical model can be chosen that will yield core gamma heat generation equivalent to that obtained in a cylinder. The above procedure yields values for the total heat generation in the cylinder model that agree with the totals obtained in a spherical model.

The criterion used in choosing such a spherical model is that the ratio of volume to surface area of both sphere and cylinder be equal in order to ensure that the fractional escape of gamma-rays from the core be similar in the two models. The radius of the equivalent sphere is found to be 36 cm.

The heat generation obtained with a spherical model of core radius 36 cm, reflector thickness 18 cm, and blanket thickness 14 cm is shown in Fig. 22.38. The gamma-ray

Fig. 22.38—Heat generation in the Spherical model, 36-cm radius, power level 1 mw. Heat generation rates integrated over the three regions are in kilowatts as follows:

Heat Source	Core	Reflector	Blanket
Gammas born in core	57.2	16.2	10.2
Gammas born in reflector	1.6	7.8	
Gammas born in blanket	—	3.0	11.7
(n, α) and (n, n') reactions	18.6	10.1	—
Total heat generation	77.4	37.1	21.9

source strengths and energy-absorption cross-sections used in the core, reflector, and blanket are the same as those used in Sec. 22.15 for the 39-cm spherical model, and the heat generation due to gamma-rays is calculated in a like manner. As was discussed at the end of Sec. 22.16, the use of these data with the smaller core geometry yields larger heat-generation values, and this effect can be considered as taking into account a non-uniform core gamma source in the larger model. However, it is to be noted that the total heat generation of 136.5 kw in the 36-cm model is in good agreement with the total of 136.2 kw obtained with the 39-cm spherical model. In obtaining the heat generation due to neutron degradation and (n, α) reactions in the 36-cm sphere, the criterion used was that this distribution should yield the same total heat generation as that obtained in the 39-cm sphere. Thus, if the heat generation in one sphere is given by $f(r/R)$, where r is the radial coordinate and R is the radius of the sphere,

the same total heat generation in another sphere of radius T is obtained by using $(R/T)^3 f(r/T)$ as the new distribution function.

In calculating the heat generation in the cylindrical model of the reactor, it is convenient to consider separately the heat generation in the core, side reflector, and blanket, and the top and bottom reflector and blanket.

1. Core

Heat generation due to core-born gammas along the cylinder axis and in the radial equatorial plane can be obtained from Eqs. 22.139 and 22.95a, respectively. The results of performing these calculations for a total uniformly distributed core gamma source of 80 kw and an effective energy-absorption cross-section of 0.067 cm^{-1} are shown as the solid curves in Fig. 22.39.

In order to compare the above results with the heat generation in the 36-cm sphere due to core gammas, use is made of the transformation from spherical to cylindrical geometry that is given in Eq. 22.144. This transformation produces a heat-

Fig. 22.39—Heating of cylindrical core by core-born gammas. Reactor power level, 1 mw. Heating distributions along the axis of the cylindrical core (top curves) and radially in the equatorial plane (bottom curves). The solid curves are the cylindrical model distributions; the dashed curves are the transformed cylindrical distributions (See Sec. 22.17-1).

generation distribution in a cylinder such that the total heat generation in the sphere and cylinder are equal. In Fig. 22.39 a comparison is made of the heat generation along the cylinder axis and in the radial plane as obtained from the above-mentioned cylinder calculations and from the application of the transformation to the 36-cm spherical core gamma heat-generation distribution. It is seen that the comparison is good; and even the slightly greater values of the transformed spherical function near the core edge can be accepted as tending to compensate for the error due to neglecting the nonuniform core-gamma-source distribution in the cylinder.

On the basis of the above comparison the heat-generation profiles in the cylindrical core, including the heat generated by neutron degradation, and (n, α) reactions as well as by core gamma radiation, are obtained by applying the transformation given in Eq. 22.144 to the total heat-generation distribution in the 36-cm spherical core.

2. Side Reflector and Side Blanket

The heat generation in the reflector equatorial plane of the cylinder due to core-born gammas was calculated by means of Eq. 22.95b. Since this equation was derived on the assumption that the energy absorption coefficients of all regions are equal, a value of 0.06 cm^{-1} was used for this coefficient. (See Sec. 22.13 for the selection of this cross-section value.) In Fig. 22.40 a comparison is made of reflector heat generation due to core-born gammas as calculated for the 36-cm spherical model and as calculated in the equatorial plane of the cylinder. There is good agreement between the two distributions. The total heat generation in the equatorial plane of the cylinder, in both the side reflector and side blanket, will be represented by the total heat generation calculated in the reflector and blanket of the 36-cm spherical model. The variation

Fig. 22.40—Heating in reflector equatorial plane by core-born gammas. Reactor power level, 1 mw. The cylindrical model (radius of cylinder, 36 cm), assumed equal mean free paths for gammas in core and reflector. The spherical model (radius, 36 cm) treated the core gammas as a surface source with absorptions as the average of a cosine and a cosine-squared distribution.

with height of the total heat generation in the side reflector and blanket will be approximated by the function $\cos \pi z/108$. This was chosen because the longitudinal variation of the heat generation along the cylinder axis due to core-born gammas can, to a first approximation, be represented by a cosine variation which decreased from 1 to $1/2$ as z varies from 0 to 36 (see Fig. 22.41).

Fig. 22.41—Spatial variations of absorption of core-born gammas in side and end reflectors and blankets.

3. Top and Bottom Reflector and Blanket

Although the top and bottom reflector differ slightly in composition, it is not within the accuracy of the calculation to take this difference into account; the average of the compositions of both regions was used in the heat-generation calculations. Similarly, an average of the compositions of the top and bottom blanket regions was used in determining the heat generation in those regions.

Core-gamma heat generation along the cylinder axis in the reflector was calculated by means of Eq. 22.98. Since this equation was derived on the assumption that the energy-absorption coefficients of all regions are equal, a value of 0.06 cm^{-1} was used for this coefficient. The axial heat generation due to core-born gammas entering the blanket was obtained by treating the entering energy of 0.198 watts/cm^2 as energy emanating from a source on the reflector-blanket interface with a surface angular distribution that is an average of a cosine and cosine-squared distribution, Eqs. 22.14 and 22.15.

The remaining heat generated along the cylinder axis by reflector- and blanket-born gammas was obtained by representing the top and bottom reflector regions and the top and bottom blanket regions by plane slabs. The effective energy-absorption cross-sections of reflector and blanket regions for reflector- and blanket-born gammas were found, by the methods described in Sec. 22.15, to be 0.052 and 0.14 cm^{-1} , respectively. In determining the axial heat generation in the reflector and blanket due to reflector-born gammas, the results presented in Sec. 22.6 were used with a source strength (Sec. 22.15) of 0.024 watts/cm^3 . The heat generation due to blanket-born gammas was found by means of formulas in Sec. 22.5 for values of the parameters σ_N , σ_O , and σ_1 to be equal to 0.28 , 0.14 , and 0.052 , respectively. The value of $\sigma_N = 0.28$ is 0.60 of the neutron attenuation obtained for the side blanket where 0.60 is the ratio of the average volume fraction of steel in the top and bottom blankets to the volume fraction of steel in the side blanket. The conversion factor to express the heat generation by blanket-born gammas in watts/cm^3 for the 1 Mw reactor power level is again found to be 0.392 .

It should be noted that all results have been obtained in watts/cm³ of material of average regional composition. For design purposes it is necessary to obtain heat-generation values in beryllium or iron in the core and reflector regions. To do this it is necessary to consider separately gamma heat generation and neutron heat generation.

Thus, to obtain the heat generation in beryllium due to neutron degradation and (n, α) reactions, the values obtained for average regional composition should be divided by the volume fraction of beryllium in the region of interest. This is done because neutron scattering occurs mainly in beryllium, and (n, α) reactions occur only in beryllium.

The gamma heat generation in beryllium or iron is obtained by multiplying the results given for average regional composition by the ratio of the effective energy absorption cross-section of the individual material to that for the average composition. The effective energy absorption cross-section of iron in the core and reflector is 0.21 cm⁻¹, and the effective energy-absorption cross-section for beryllium in the core and reflector is 0.045 cm⁻¹. These values are obtained by averaging energy-absorption cross-sections for beryllium and iron over representative spectra in core and reflectors. The effective energy-absorption cross-sections for homogeneous material are 0.067 and 0.056 cm⁻¹ for core and reflector regions, respectively.

22.18 HEAT GENERATION IN THE SIDE REFLECTOR AS A FUNCTION OF CONTROL DRUM POSITION

In the cylindrical model considered in the last section, it was assumed that all control drums were in the 180-deg setting in which the poison region is farthest away from the core. However, a control drum setting of 90-deg more nearly represents the average operating position and this setting, as well as the 0 and 180-deg position will be considered when heat-generation profiles in the reflector equatorial plane are obtained in this section. Since the importance of control-drum setting in heat-generation calculations arises from the influence of the poison region on neutron flux, it is of interest to review available information on this effect.

Neutron multigroup calculations have been performed for a spherical core completely surrounded by a layer of boron whose thickness is determined by the amount of boron in the control drums and for a spherical model with no boron present. The latter case corresponds to the 180-deg control drum setting and the neutron data obtained from this calculation were utilized in previous sections. The former case corresponds more nearly to the 0-deg control drum setting (Fig. 22.42) although in this setting the side core - reflector interface is only 58 per cent covered with boron. Since multigroup information on the effect of the poison region on neutron flux is available for a spherical model with and without a boron shell around the core, it is seen that numerous approximations are required to apply these data to the geometry for different control-drum settings. The numbers obtained must therefore be regarded as a highly qualitative indication of heat-generation variations which may occur.

The effects to be considered with a change in control-drum position are:

1. The effect of the change in power distribution on gammas escaping from the core.
2. The effect of the change in neutron flux on neutron degradation and neutron absorption in the reflector, and neutron absorption in the control drum. This affects neutron heating and heat generation due to capture gammas.
3. The effect of control-drum iron on the attenuation of gamma-rays.

In calculating the heat-generation profiles for each control drum setting, various radial paths from the center of the core are considered. A rough guide for estimating the heat generation along each radial line in the equatorial plane is obtained by considering a spherical model corresponding to that radial line. The spherical model for each line is defined to have a core, control drum, and reflector

LINES OF EQUAL HEAT GENERATION IN REFLECTOR EQUATORIAL PLANE
CONTROL DRUM POSITION 180 DEGREES

LINES OF EQUAL HEAT GENERATION IN REFLECTOR EQUATORIAL PLANE
CONTROL DRUM POSITION 90 DEGREES

LINES OF EQUAL HEAT GENERATION IN REFLECTOR EQUATORIAL PLANE, ARBITRARY UNIT
CONTROL DRUM POSITION 0 DEGREES

Fig. 22.42—Heat generation contours in reflector equatorial plane. Relative values only.

Top: 180-deg. position of control cylinder

Middle: 90-deg. position of control cylinder

Bottom: 0-deg. position of control cylinder

region of thicknesses which are equal to the corresponding thicknesses along the radial line. The control drum region in the spherical model is in turn divided into two spherical shells, one containing the average composition of the poison tube region of the control drums and the other containing iron corresponding to the steel backbone of the control drum.

For each radial line, the values for neutron heating and core gamma escape obtained previously were reduced according to the control-drum setting and the proximity of the radial line to the poison region. This reduction was estimated by making use of multigroup results for the escape of fast neutrons from the core in the limiting cases of the spherical models with and without a layer of boron. The ratio of the escapes in these two models is 0.65. Hence, values for neutron heating and core gamma escape were reduced by factors varying from 0.65 to 1.0, depending on control-drum position and proximity of the radial line to the poison region. This approach is not unreasonable in view of the somewhat similar diffusion properties of fast neutrons and gammas. (For an independent calculation of the ratio of the fractional escapes for the extreme power distributions, see Sec. 22.21.)

Shielding of core gammas by the iron backbone in the control drum was estimated by use of the equivalent spherical shell in the 0-deg setting. In the 90-deg setting, the shielding effect was estimated by the shadow area subtended by the iron.

The heat generation resulting from capture gammas in the iron backbone is small except in the iron itself. The captures in iron were obtained by use of data for the limiting multigroup cases of a spherical model with and without poison. In the 0-deg position the absorption was obtained by considering the neutron current into the poison region and estimating the fraction absorbed in the iron backbone. In the 90-deg setting the absorption was obtained by appropriate interpolation of the limiting multigroup cases. Total absorptions calculated by these methods were in agreement with total absorption calculated by the use of reactivity coefficients (See Chap. 7).

Neutron capture in distributed reflector iron was estimated for each radial path by again using neutron absorption data from the neutron multigroup calculations with and without boron.

The results of performing these calculations in the reflector equatorial plane for control drum settings of 0, 90, and 180-deg are shown in Fig. 22.42. It is clear that the above procedure is highly qualitative. As discussed in the preceding section, the variation with height of these values can be approximated by the factor $\cos \pi z/108$.

22.19 APPENDIX: GAMMA-RAY ABSORPTION ON THE AXIS OF A CYLINDRICAL SOURCE REGION

Consider a cylindrical source region of radius R and height L with unit source strength per unit volume. Assuming straight-ahead propagation and exponential attenuation (σ being the effective energy absorption cross section), the intensity of absorbed radiation at any point z on the axis ($0 \leq z \leq L$) is given by

$$I(z) = \lim_{\epsilon \rightarrow 0} \left\{ \int_0^{z-\epsilon} dz_1 \int_0^R 2\pi r_1 dr_1 \frac{\sigma e^{-\sigma \sqrt{(z-z_1)^2 + r_1^2}}}{4\pi [(z-z_1)^2 + r_1^2]} + \int_{z+\epsilon}^L dz_2 \int_0^R 2\pi r_2 dr_2 \frac{\sigma e^{-\sigma \sqrt{(z_2-z)^2 + r_2^2}}}{4\pi [(z_2-z)^2 + r_2^2]} \right\} \quad (22.138)$$

where (z_1, r_1) ($z_1 \leq z$, $0 \leq r_1 \leq R$)

and (z_2, r_2) , ($z_2 \geq z$, $0 \leq r_2 \leq R$) are source points in the cylinder.

Equation 22.138 can be rewritten as:

$$I(z) = \lim_{\epsilon \rightarrow 0} \frac{\sigma}{2} \left\{ \int_0^{z-\epsilon} dz_1 \int_{\sigma(z-z_1)}^{\sigma\sqrt{(z-z_1)^2 + R^2}} \frac{e^{-u}}{u} du \right. \quad (22.139)$$

$$\left. + \int_{z+\epsilon}^L dz_2 \int_{\sigma(z_2-z)}^{\sigma\sqrt{(z_2-z)^2 + R^2}} \frac{e^{-u}}{u} du \right\}$$

$$= \lim_{\epsilon \rightarrow 0} \frac{\sigma}{2} \left\{ \int_0^{z-\epsilon} dz_1 \int_{\sigma(z-z_1)}^{\infty} \frac{e^{-u}}{u} du \right.$$

$$\left. + \int_{z+\epsilon}^L dz_2 \int_{\sigma(z_2-z)}^{\infty} \frac{e^{-u}}{u} du \right\} - \frac{\sigma}{2} \int_0^L d\lambda \int_{\sigma\sqrt{(\lambda-z)^2 + R^2}}^{\infty} \frac{e^{-u}}{u} du$$

Integrating and taking the limit as $\epsilon \rightarrow 0$ yields:

$$I(z) = \frac{1}{2} \left\{ 2 - e^{-\sigma z} - e^{-\sigma(L-z)} + \sigma z \int_{\sigma}^{\infty} \frac{e^{-u}}{u} du + \sigma(L-z) \int_{\sigma(L-z)}^{\infty} \frac{e^{-u}}{u} du \right\} \quad (22.139)$$

$$- \frac{\sigma}{2} \int_0^L d\lambda \left[\int_{\sigma\sqrt{(\lambda-z)^2 + R^2}}^{\infty} \frac{e^{-u}}{u} du \right]$$

Since $\int_{\lambda}^{\infty} \frac{e^{-u}}{u} du$ is a tabulated function, only the last term in Eq. 22.139 requires numerical integration. In particular, for $L = 2R$ the following results are obtained for the cases $\sigma R = 2$ and $\sigma R = 3$.

z/R	$I(z)$ for $\sigma R = 2$	$I(z)$ for $\sigma R = 3$
0	0.457	0.486
0.25	0.781	0.873
0.50	0.859	0.941
0.75	0.887	0.960
1.00	0.895	0.965

22.20 APPENDIX: A TRANSFORMATION FROM SPHERICAL TO CYLINDRICAL GEOMETRY

Let t be the distance from the center of a sphere of radius T ($0 \leq t \leq T$), and let $f(t/T) = f(t')$ be the heat-generation distribution in this sphere. The total heat generation in the sphere is given by

$$Q = 4\pi T^3 \int_0^1 f(t') t'^2 dt' \quad \text{for } t' = \frac{t}{T} \quad (22.140)$$

The heat-generation distribution as a function of position in a cylinder is, of course, not determined uniquely by a knowledge of $f(t')$. It is, however, frequently necessary for practical purposes to make a reasonable guess as to how the distribution might look in a cylinder. To achieve this it is convenient to make use of an ad hoc transformation which generates a function in the cylinder which has the desired volume integral of heat generation and behaves reasonably well along the axis and equatorial plane of the cylinder. This transformation may be obtained as follows:

Let the cylinder be one of radius R and height $2R$ so that it corresponds to the cylindrical representation of the reactor and, assuming cylindrical symmetry, let z and r be the usual cylindrical coordinates measured from the center of the cylinder. Then it is readily seen that the following heat-generation distributions in the cylinder yield a total heat generation which is equal to that given for the sphere in Eq. 22.140.

$$\begin{aligned} \frac{2}{3} \frac{T^3}{R^3} f\left(\frac{r}{R}\right) \frac{\alpha \frac{r}{R}}{\sin \frac{\alpha r}{R}} \cos \frac{\alpha z}{R}, \text{ for } |z| \leq r \\ \frac{2}{3} \frac{T^3}{R^3} f\left(\frac{z}{R}\right) \frac{\beta \frac{z}{R}}{2 J_1\left(\frac{\beta z}{R}\right)} J_0\left(\frac{\beta r}{R}\right), \text{ for } |z| \geq r \end{aligned} \quad (22.141)$$

where $\alpha = \alpha(r/R)$, $\beta = \beta(z/R)$, and the function $f(\lambda)$ is the same in both geometries.

One relation between α and β is obtained by imposing the condition of continuity on Eq. 22.141 at $z = r$. This yields the transcendental equation

$$\gamma y \cot \gamma y = \frac{y}{2} \frac{J_0(y)}{J_1(y)}, \quad 0 \leq y \leq \beta \quad (22.142)$$

where $\gamma = \frac{\alpha}{\beta}$ and $y = \beta \frac{z}{R} = \beta \frac{r}{R}$

The solution of Eq. 22.142 is

$$\alpha\left(\frac{r}{R}\right) \cong \sqrt{\frac{3}{8}} \beta \left\{ 1 + \frac{\left(\beta \frac{r}{R}\right)^2}{120} \right\} \quad (22.143)$$

Since heat-generation calculations along the axis of the cylinder and in the equatorial plane of the cylinder (see Fig. 22.39) show that the heat generation falls off by a factor of approximately one-half in proceeding from the center to the edge of the core, the parameter β will be chosen so $J_0(\beta) = 1/2$ or $\beta = 1.521$.

Using the above values for α and β , and considering the case of $T = R$, it is seen that the functions

$$\begin{aligned} \frac{2}{3} f\left(\frac{r}{R}\right) \frac{\alpha \frac{r}{R}}{\sin \frac{\alpha r}{R}} \cos \frac{\alpha z}{R}, \text{ for } |z| \leq r \\ \text{where } \alpha = 0.9314 \left\{ 1 + 0.0193 \left(\frac{r}{R}\right)^2 \right\} \\ \text{and } \frac{2}{3} f\left(\frac{z}{R}\right) \frac{1.521 \frac{r}{R}}{2 J_1\left(1.521 \frac{z}{R}\right)} J_0\left(1.521 \frac{r}{R}\right), \text{ for } |z| \geq r \end{aligned}$$

yield the same total heat generation in a cylinder as does the distribution $f(t/R)$ in a sphere.

It is to be noted that $\frac{\alpha \frac{r}{R}}{\sin \frac{\alpha r}{R}}$ increases from 1 to 1.168 as r/R varies from 0 to 1 and that $\frac{\beta \frac{z}{R}}{2 J_1\left[\beta \frac{z}{R}\right]}$ increases from 1 to 1.356 as z/R varies from 0 to 1.

Thus in the cylinder equatorial plane, the transformed function falls off less rapidly with radius than the original function. The falling-off along the cylinder axis is still less rapid. These effects are qualitatively similar to what may happen in actual cases, but there is no quantitative significance to the results. At $z = r = R$, the transformed function (Eq. 22.144) is 0.45 times the value of the spherical function at $r = R$. Since the derivatives of the transformation function are not continuous across $z = r$, some smoothing-out of the profiles is necessary. This smoothing-out process can be accomplished without appreciably changing the volume integral.

22.21 APPENDIX: FRACTIONAL ESCAPE FROM A SPHERICAL SOURCE REGION HAVING A NONUNIFORM SOURCE DISTRIBUTION

The purpose of this section is to determine the fractional escape from a sphere with a nonuniform source distribution. The results obtained are used to investigate the accuracy of the assumption of a uniform core-gamma-source distribution used in Sec. 22.15 and 22.17, and to investigate the effect of a change in power distribution on gamma-ray escape from the core.

In particular, two sets of extreme source distributions corresponding to experimental measurements of the power distribution in the PPA and theoretical values obtained from SIR neutron multigroup calculations are considered. These source distributions, $S_1(r)$, normalized to a total source of unity in a spherical core region, are shown in Fig. 22.43. The distribution S_1 corresponds to measurements along a radial line through the center of the boron region when the control drum is in the 66-deg position, and the distribution S_2 corresponds to measurements along a radial line through the center of the reflector region when the drum is in the 126-deg position. These distributions are normalized to unity in a 40-cm sphere. The distributions S_3 and S_4 are theoretical distributions obtained from multigroup calculations for a 39-cm spherical core with and without a B_4C shell around the sphere. They are normalized to unity in a 39-cm sphere.

To evaluate the fractional escape, an escape probability $f(r)$ for a unit isotropic point source at distance r from the center of a sphere is obtained, and then the

Fig. 22.43—Normalized nonuniform core gamma source distributions. S_1 —experimental distribution along radial line through center of boron region when drum is in 66-deg. position. S_2 —experimental distribution along radial line through center of reflector region when drum is in 126-deg. position. S_3 —theoretical distribution obtained from multigroup calculations for 39-cm spherical core with boron shell. S_4 —theoretical distribution obtained from multigroup calculations for 39-cm sphere with no boron present.

fractional escape from the sphere with source distribution $s_1(r)$ is given by $4\pi \int S_1(r) f(r) r^2 dr$. The function $f(r)$ is obtained in the following manner:

Consider a point, isotropic, unit source located a distance r from the center of a sphere of radius R . The effective energy absorption coefficient of the source region is σ and ρ is the radius vector from the source point to any point on the surface of the sphere. Then $f(r)$ is given by

$$f(r) = \frac{2\pi \int_{-1}^1 e^{-\sigma \rho} d(\cos \theta)}{4\pi}$$

where $R^2 = r^2 + \rho^2 + 2\rho r \cos \theta$

and $d(\cos \theta) = \frac{-\rho^2 + R^2 - r^2}{2\rho^2 r} d\rho$

Thus, changing variables,¹⁴

$$\begin{aligned} f(r) &= \frac{1}{2} \int_{R-r}^{R+r} \frac{\rho^2 + R^2 - r^2}{2\rho^2 r} e^{-\sigma \rho} d\rho \\ &= \frac{1}{4\sigma r} \left\{ [1 + \sigma(R+r)] e^{-\sigma(R-r)} - [1 + \sigma(R-r)] e^{-\sigma(R+r)} \right. \\ &\quad \left. - \sigma^2(R^2 - r^2) \int_{\sigma(R-r)}^{\sigma(R+r)} \frac{e^{-u}}{u} du \right\} \end{aligned} \quad (22.145)$$

Note that at $r = 0$, $f(0) = e^{-\sigma R}$ and at $r = R$, $f(R) = \frac{1}{2} + \frac{1 - e^{-2\sigma R}}{4\sigma R}$.

Using the above expression for $f(r)$, the fractional escapes F_1 obtained with source distributions $S_1(r)$ were calculated from $F_1 = 4\pi \int S_1(r) f(r) r^2 dr$ and were found to be:

$$\begin{array}{ll} F_1 = 0.22 & F_3 = 0.20 \\ F_2 = 0.29 & F_4 = 0.29 \end{array}$$

In comparison it is of interest to note that the fractional escape from a sphere with a uniform source distribution (see Sec. 22.7, Fig. 22.20) is 0.26 when the parameter $\sigma R = (0.067)(40) = 2.68$ and it is 0.27 when $\sigma R = (0.067)(39) = 2.61$. Thus it is seen that the assumption of uniform source distribution, as used in Secs. 22.15 and 22.17 for the 180-deg control drum position, is a reasonable one.

Using the above results it is also possible to check the assumption made in Sec. 22.18 that the effect of a change in power distribution is the same for both fast neutrons and gammas. It is seen that the ratio F_3/F_4 of the fractional escapes corresponding to the extreme source distributions obtained from neutron multigroup calculations is 0.69. This is in adequate agreement with the ratio of 0.65 used in Sec. 22.18. Using the escapes F_1 and F_2 based on the extreme experimental source distributions, it is seen that the ratio of the escapes is 0.76, which is in agreement with the ratio based on multigroup data.

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Chapter 23

GAMMA-RAY HEATING MEASUREMENTS IN THE PPA

J. A. RICH and R. E. SLOVACEK

23.1 INTRODUCTION

The distribution and magnitude of the heating that arises from gamma-ray and neutron interactions in the reflector and blanket of an intermediate reactor are of importance in the proper design of the cooling system for those regions. Calculations of the neutron and gamma-ray heating in such a reactor are described in Chap. 22, this volume. The measurements described here were made in PPA-20 (an assembly similar to the reactor) to determine experimentally the distribution and magnitude of the gamma-ray heating.

The gamma-ray field in the reactor originates from prompt fission gamma rays, delayed gamma rays associated with fission-product decay, capture gamma rays in structural materials (iron) and in fuel, and, finally, gamma rays resulting from the induced radioactivity in sodium and other materials in the core. The absence of sodium in the PPA in which the measurements were carried out did not make the PPA gamma-ray birth spectrum appreciably different from that of the reactor in question since the sodium gamma contribution is only 0.6 per cent of the total gamma-ray energy produced in the reactor. The sodium reactor gamma-ray birth spectrum (Fig. 22.24) has a peak at about 1 Mev and a long tail at higher energies. At 5 Mev, the spectral energy density is 8 per cent of maximum.

The beryllium and iron of the reflector and blanket absorb gamma-ray energy primarily by means of the Compton effect; however, the interpretation of the measurements is independent of the mechanism for producing the electron recoils. The recoil electron energy is transferred to the atoms of the medium by ionization and excitation and appears ultimately as heat energy in the medium.

Two methods of measuring the gamma heating are possible--a calorimetric measurement and an ionization type of measurement. These two techniques were compared recently at Oak Ridge¹ by using CO₂ in a graphite-walled cavity type ionization chamber and in a calorimeter which measured the rate of vaporization of liquid nitrogen. The results of the two methods agreed to within 3 per cent. The ionization method with a capacitive type ion chamber was used for the present measurements. This method was used because the temperature rise when the PPA operated at ~1 watt would be extremely small and would rule out a calorimetric measurement, an ion chamber without connecting leads could be located conveniently in almost any position in the assembly, and a large number of capacitive type chambers could be constructed at a relatively low cost and exposed simultaneously in the assembly for rapid accumulation of data.

The ionization produced in the detector gas can be related, with certain restrictions, to the ionization produced in the medium in the neighborhood of the detector. This relation has been studied theoretically and experimentally by Gray.²⁻⁵ This relation is known as Gray's principle. (See Eq. 23.1.)

23.2 GRAY'S PRINCIPLE

To understand the conditions for which this relation is valid, consider a small gas cavity in a solid medium surrounded on all sides by a thickness of medium which exceeds the maximum secondary electron range in the medium for the gamma rays in question. If the cavity dimensions are small compared to this electron range, the introduction of the gas cavity does not seriously perturb the velocity distribution of the electrons that cross the surface enclosing the gas cavity. The electron energy lost per unit volume had the electrons traversed the solid medium instead of the gas cavity is in the ratio ρ_v to the electron energy lost per unit volume in the gas cavity, where

$$\rho_v = \frac{\left(-\frac{dE}{dX}\right)_{\text{solid}}}{\left(-\frac{dE}{dX}\right)_{\text{gas}}} = \frac{E_v}{j_v W} \quad (23.1)$$

and E_v = the energy loss per unit volume in the solid medium
 j_v = the number of ion pairs per unit volume formed in the gas
 W = the average energy lost by an electron in forming an ion pair in the gas
 $\left(-\frac{dE}{dX}\right)$ = the stopping power or the electron energy loss per unit path length in a given medium

In the Gray relation, Eq. 23.1, W is assumed to be independent of energy⁵ and equal to 32.5 ev per ion pair or 52.1×10^{-19} watt-sec per ion pair to within 3 per cent for all electron velocities. Actually W increases for electron energies below 8 kev, but since the mean recoil electron energy for a reactor's gamma-ray spectrum is well above this value, the assumption of the constancy of W is justified.

The stopping power of a medium for electrons with a kinetic energy E is given by a slight modification of Heitler's expression⁶ for the loss of energy due to ionizing collisions,

$$\left(-\frac{dE}{dX}\right) = \frac{2\pi e^4 N_Z}{mc^2 \beta^2} \left[\ln \frac{E(E + mc^2)^2 \beta^2}{2(I_Z)^2 mc^2} + \left(\frac{mc^2}{E + mc^2}\right)^2 \right] \quad (23.2)$$

where e = the electronic charge
 Z = the atomic number of the medium
 N_Z = the number of electrons per cubic centimeter in the medium
 mc^2 = electron rest energy
 β = the ratio of the electron velocity to the velocity of light
 I_Z = the average ionization potential of an atom in the medium

This expression may also be written as

$$\left(-\frac{dE}{dX}\right) = N_Z S_Z \quad (23.3)$$

where S_Z is the stopping power per electron of the medium (of atomic number Z). Equation 23.1 can then be expressed as

$$E_v = \frac{N_Z S_Z}{N_{\text{gas}} S_{\text{gas}}} j_v W \quad (23.4)$$

or

$$E_v = \frac{N_Z}{N_{\text{gas}}} f_Z j_v W$$

The ratio $f_Z = S_Z / S_{\text{gas}}$ of the stopping power per electron in the solid medium relative to that in the gas is a slowly varying function of Z and is practically independent of the electron velocity. The quantity f_Z has been determined experimentally by Gray⁵ for

recoil electrons from Ra (B+C) gamma rays with air as the gas in the cavity. His results are in good agreement with Eq. 23.2 for the electron stopping power as a function of electron energy and Z of the medium. The theoretical values of f_Z for iron, aluminum, and beryllium are 0.87, 0.94, and 1.04, respectively, based on $f_Z = 1.0$ for air (mean $Z = 7.3$).

23.3 THE ION CHAMBER DETECTORS

Gray⁵ has shown that the absolute energy absorbed per unit volume may be determined with an accuracy of 5 per cent. The accuracy attainable with this technique is limited by uncertainties in the average energy required to form an ion pair on the gas, in the stopping power ratio, and in the ion collection efficiency. Another difficulty is inherent in the size of the gas cavity (ionization chamber) that is used: the spatial resolution of the details of the gamma-ray flux density can be no finer than this size though, of course, a kind of space average value is obtained, even in a steep flux gradient.

As stated in Sec. 23.2, the cavity in the ion chamber must have a wall thickness greater than the maximum range of the secondary electrons in the solid medium. It is also necessary that the production of ion pairs in the gas be due predominantly to electrons from the solid medium, that all ion pairs in the gas be collected, and that no multiplication occur as a result of collisions in the gas.

Figure 23.1 is a drawing of the capacitive-type ion chamber (developed by T. A. Rich, of the General Electric Company) for the heating measurements. Six cold-rolled

Fig. 23.1—Cross-sectional view of the ion chamber.

steel, ten aluminum, and ten beryllium chambers were constructed. The steel chambers were intended to be used in the steel reactor blanket, the beryllium ones in the beryllium reflector, and the aluminum ones in the core. Delays in obtaining the beryllium chambers, however, forced the use of aluminum chambers in some of the reflector measurements. For each of these chambers, the center electrode was made 0.81 cm in diameter and 1.53 cm in length. With a 0.025-cm gap spacing and a 0.12-cm³ cavity volume, the chamber sensitivity was such that a measurable voltage change was obtained across a chamber located anywhere in the reflector or in the blanket positions adjacent to the reflector. Simultaneous measurements in regions of relatively low gamma-ray flux were

made possible with four cold-rolled steel ion chambers which had a 0.076-cm gap spacing and a 0.33 cm^3 cavity volume; their larger cavity volume made them more sensitive.

Polystyrene was used as the insulating material because of its low leakage rate. Teflon was found to be unsatisfactory as an insulator. Spurious ionization produced in the cavity by proton recoils from the polystyrene should be small because of the small solid angle subtended by the effective cavity volume at the insulator. Extreme care had to be exercised in keeping the insulator surfaces clean to maintain a low leakage rate.

The chambers were tested for the presence of gas multiplication. Gamma rays from a 25-mg radium source were used to discharge the chambers, and the discharge rate dV/dt volts/sec was measured for initial charging voltages of 93 and 300 volts. The measured discharge rates were the same to within 5 per cent for the two initial voltages. Had gas multiplication been present in this range of charging voltages, the discharge rate would have been greater at the higher initial voltage.

A pocket chamber electrometer was modified to charge these chambers to 150 volts and to read the potential change ΔV across the chamber and reader capacitances after each run. The potential change was indicated by a voltmeter with two ranges (0-20 and 0-100 volts).

The quantity j_v (ion pairs per cubic centimeter-second-watts) used in the Gray relation (Eq. 23.4) is determined from the following expression in terms of the measured quantities

$$j_v = \frac{\Delta V(C_c + C_R)}{etPv_c} \quad (23.5)$$

where ΔV = potential change in volts

C_c = chamber capacitance in farads

C_R = reading instrument capacitance in farads

e = electronic charge in coulombs

v_c = effective cavity volume in cubic centimeters

P = reactor power level in watts

t = exposure time in seconds

Each run was made at a PPA power level of 4 watts for a time of 1800 sec. The power level was determined with previously calibrated ion chambers (see Chap. 6, Secs. 6.3 and 6.5, for the bases of calibration). The error in tP , namely the exposure that resulted from the periods of low-power operation while the assembly was being brought up to power, was measured and found to be negligible.

Chamber capacitances were measured with a capacitance bridge by the substitution method. The average capacitance of the more sensitive cold-rolled steel chambers was about $5.2 \mu\text{mf}$, whereas that of all the other chambers was $17 \mu\text{mf}$. A value of $C_R = 5.7 \mu\text{mf}$ was obtained for the modified reader capacitance as measured with a Q meter.

The effective cavity volume was taken to consist of the space around the center electrode adjacent to the bottom face and side of the center electrode. The space between the insulator and center electrode and between the cap and chamber added little to the effective cavity volume, for the small electric field in these regions resulted in poor ion-collection efficiency.

Jackets of the same material as the ion chamber were made to fit around the ion chambers to fill the remaining space inside a 2-in.-diameter PPA tube. The effective thickness of the ion-chamber walls was thereby increased to 2.1 cm. This thickness corresponds to the range of 7.3-Mev electrons in beryllium and of higher energy electrons in aluminum and iron. Since the gamma-ray spectrum in the PPA has relatively few quanta over 7 Mev, the Gray relation condition on chamber wall thickness is satisfied; the electron spectrum seen by the cavity is characteristic of the medium for which the measurement is being made.

It was convenient to use air at atmospheric pressure in the gas cavity even though this introduces a possible uncertainty in the measurement, namely, that protons from the (n,p) reaction on N^{14} give rise to an excess of measured ionization. This contribution to the gas ionization by the protons was determined as follows: four beryllium chambers and two of the more sensitive steel chambers were filled with CO_2 at atmospheric pressure. One beryllium chamber was located in each of the four rings in the stationary beryllium, and the two steel chambers were located in the first ring of the blanket region.

Beryllium and steel chambers filled with air at atmospheric pressure were placed in symmetrical positions in another sector of the assembly. Each CO_2 -filled chamber was compared to its air-filled counterpart as to ionization during reactor operation.

The proton ionization correction factor f_p is plotted in Fig. 23.2 as a function of radial distance from the core center. This correction factor is the ratio of the ionization in a given position as determined with a CO_2 -filled chamber to that determined with

Fig. 23.2—Air correction factor, f_p , as a function of radial position.

an air-filled chamber. The variation of f_p with the neutron flux is in the right direction. The value of f_p for a given ring obtained from the experimental curve in Fig. 23.2 was used to correct the quantity j_v , Eq. 23.4, as determined from measurements for the air-filled chamber.

23.4 DISCUSSION AND RESULTS

The experimental results do not represent the level of gamma-ray heating to be expected in a reactor that has been operated for a long period of time. The PPA was operated only 30 min, during which time the delayed fission-product gamma-ray generation rate did not reach its saturation value of 7 Mev/fission. Assuming a startup from a cold pile, the average delayed gamma-ray flux would be 5.5 Mev/fission during a 30-min run (see Chap. 24). Adding this to the 5 Mev/fission prompt gamma rays, one obtains 10.5 Mev/fission or about 88 per cent of the saturation fission gamma-ray flux (5 Mev/fission prompt plus 7 Mev/fission delayed). Since an unknown quantity of long-lived fission products had accumulated from previous PPA operations and since the capture gamma-ray flux from the pile materials was present through the 30-min run, it is estimated that an average of at least 90 per cent of the total saturated gamma-ray flux was available during a run. No correction was made to the measurements for this effect.

The gamma-ray heating distribution was determined in the equatorial plane of the PPA as well as along longitudinal traverses. Equatorial plane measurements were made with the small cavity (less sensitive) steel chambers in every position of the first blanket ring (adjacent to the reflector). The large cavity (more sensitive) steel chambers were used to obtain the heating in every position of the second blanket ring and in every second position of the third ring. Although the gamma-ray heating was obtained in every reflector position in the equatorial plane with the aluminum chambers, the beryllium

chamber measurements were limited to the stationary beryllium reflector region because the beryllium chambers were completed only near the end of the experiment. Simultaneous measurements were made in symmetrical positions in three sectors of the PPA to check the consistency of the measurements. At least two determinations of the heating were made in each blanket position and in each stationary beryllium position. The heating measurements at certain points in the control drum were made only once with the aluminum chambers. The heating was obtained at a total of 75 points in the equatorial plane.

The equatorial plane gamma-ray heating measurements made with the beryllium and steel chambers are presented in Fig. 23.3. The mean value of all measurements at a given position is presented.

Fig. 23.3—Gamma-ray heat generation in the reflector and blanket of PPA-20 measured with beryllium and steel chambers. Numbers at the center of each position give the local heating in units of 10^{-8} watts/cm³ per watt of power level.

The aluminum chamber measurements can be used to determine the gamma-ray heating in those beryllium regions in which measurements were not made with the beryllium chambers. The correction factor to be applied to the aluminum chamber measurements, determined from the heating measurements made in those positions in which both beryllium and aluminum chambers were used, was found to be 0.56 averaged for all positions. Theoretically, in regions of equal gamma-ray flux, this ratio should be equal to the ratio of stopping powers, $(\rho_{Be}/\rho_{Al}) = 0.68$. The perturbing effect of the aluminum jackets and chambers on the neutron and gamma-ray spectra in various parts of the reflector makes the interpretation of the aluminum measurements difficult. A

larger error is therefore assigned to the aluminum measurements. The equatorial plane gamma-ray heating as measured with the aluminum chambers and corrected to give the heating in beryllium is presented in Fig. 23.4.

NUMBERS AT THE CENTER OF EACH POSITION GIVE THE LOCAL HEATING IN UNITS OF 10^{-8} WATTS/CM³ PER WATT OF POWER LEVEL

Fig. 23.4—Gamma-ray heat generation in the reflector of PPA-20 measured with aluminum chambers. Numbers at the center of each position give the local heating in units of 10^{-8} watts/cm³ per watt-hour of power level.

The longitudinal heating distribution was obtained for the three stationary beryllium positions adjacent to the core and for three blanket positions adjacent to the reflector. Chambers were placed on either side of the center plane to verify the heat distribution symmetry about the equatorial plane. Time allowed only one determination of the longitudinal variation of the heating at most of the slug points.

The gamma-ray heating measured with the steel chambers along three longitudinal traverses in the blanket is given in Table 23.1.

TABLE 23.1--LONGITUDINAL HEATING IN BLANKET POSITIONS A, B, AND C (Fig. 23.4)

Distance from mid-plane cm	Ev (10^{-8} watt/cm per watt of power level)		
	Blanket position A	Blanket position B	Blanket position C
1.8	4.7	5.9	6.0
9.4	4.4	5.2	—
17.0	3.8	4.8	5.1
32.3	2.5	3.1	3.1
43.9	1.2	1.4	—
47.5	0.93	1.1	1.4
59.2	0.12	0.11	—

Table 23.2 gives the measured gamma-ray heating along three longitudinal traverses obtained with the beryllium ion chambers in the reflector. The heating in the last position along each traverse was obtained with steel chambers since these positions were located in the blanket at the ends of the PPA.

TABLE 23.2--LONGITUDINAL HEATING IN BLANKET
POSITIONS D, E, AND F (Fig. 23.4)

Distance from mid-plane cm	Ev (10^{-8} watt/cm ³ per watt of power level)		
	Reflector position D	Reflector position E	Reflector position F
1.8	4.6	5.0	5.1
18.5	4.1	—	4.5
27.2	3.4	3.7	3.8
35.6	2.2	2.5	2.9
43.7	2.3	1.8	1.4
51.3	0.95	1.0	0.88
55.1*	3.0	3.4	4.2

* Measurements were made with steel chambers.

Since the heating data were obtained from about 35 PPA runs with 30 different ion chambers, the uncertainties in all of the measured quantities were treated as random errors in estimating the experimental error. The largest uncertainty was the ± 15 per cent error in the pile power level. Other estimated probable errors are ± 2 per cent in the voltmeter reading ΔV , ± 3 per cent in the measured capacitances ($C_c + C_r$), ± 3 per cent in the mean energy W transferred in forming an ion pair, ± 7 percent in the air correction factor f_p , and the ± 10 per cent uncertainty in the effective cavity volume. An experimental error of ± 20 per cent, based on the propagation of random errors, is assigned to the beryllium and steel chamber measurements. A further uncertainty of 20 per cent in the Be/Al heating ratio leads to an estimated experimental error of 40 per cent for the aluminum chamber measurements.

A comparison of the experimental results can be made with the calculations of Hurwitz, Nims, and Storm (Chap. 22, Sec. 22.17). Figure 23.5 is a plot of the gamma-ray heating, in

Fig. 23.5—Gamma-ray heat generation along the radial traverse R_T in the equatorial plane of the intermediate reactor.

units of 10^{-8} watts/cm³ per watt of reactor power, in the equatorial plane along the radial traverse R_T (as indicated in Fig. 23.3) through the center of the stationary beryllium and blanket regions. The experimental points were obtained with the control drums at the 66 deg position. The solid curve in Fig. 23.5 represents the calculated gamma-ray heating for the 66 deg control drum position obtained by taking 75 percent of the calculated gamma-ray heating for a control drum position of 180 deg. (The fraction 75 per cent was obtained by interpolating the average heating between 0 and 180 deg, an admittedly crude procedure.) Since these results are for an average regional composition, an 80 per cent correction to obtain the gamma heating in the beryllium also has to be applied. This correction is just the ratio of the beryllium gamma-ray absorption coefficient (0.045 cm^{-1}) to the average absorption coefficient (0.056 cm^{-1}) for the reflector materials.

Table 23.3 gives the ratio of the calculated gamma heating to the measured gamma heating for the points along the radial traverse R_T (Fig. 23.3) in the equatorial plane.

TABLE 23.3--RATIO OF CALCULATED HEATING TO MEASURED HEATING ALONG RADIAL TRAVERSE R_T

Ring	Distance from core center cm	Ratio (watts/cm ³) calc (watts/cm ³) exp
10	38.3	1.2
11	42.8	1.0
12	47.4	1.1
13	51.9	1.4
14	56.4	1.0
15	61.0	0.7
16	65.5	0.5

The longitudinal heating distribution in the reflector adjacent to the core is plotted in Fig. 23.6. Since the longitudinal heating for the three positions was the same within the experimental error, the plotted experimental points represent the average value for

Fig. 23.6—Longitudinal distribution of heating in the reflector.

the three positions. The theoretical distribution is that for the center position. Figure 23.7 is a similar plot for the blanket region adjacent to the reflector. The longitudinal

Fig. 23.7—Longitudinal distribution of heating in the blanket.

distribution for both the reflector and blanket was approximated in the heating calculations by a function proportional to $(\cos \pi z/108)$, where z is the distance in centimeters from the center plane. This distribution is plotted as the solid curve in Figs. 23.6 and 23.7.

The ratio of the calculated gamma-ray heating to the experimentally determined heating for points along longitudinal traversals in the reflector and blanket is given in Table 23.4.

TABLE 23.4--RATIO OF CALCULATED HEATING TO MEASURED HEATING ALONG LONGITUDINAL TRAVERSES

Distance from center plane (z), cm	Reflector	Blanket
	$\frac{(\text{watts/cm}^3) \text{ calc}}{(\text{watts/cm}^3) \text{ exp}}$	$\frac{(\text{watts/cm}^3) \text{ calc}}{(\text{watts/cm}^3) \text{ exp}}$
1.78	1.2	1.1
9.40	—	1.2
17.0	—	1.2
18.5	1.2	—
27.2	1.2	—
32.3	—	1.3
35.6	1.2	—
43.8	1.1	1.4
47.5	—	1.1
51.3	0.5	—

As indicated in Tables 23.3 and 23.4, the agreement between the measured and calculated results is poor only at large distances from the core center. The approximations used in the heating calculations may not be sufficiently good to give the proper heating distribution near the reactor extremities.

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Chapter 24

DECAY HEAT AND ACTIVITY

J. R. STEHN and E. F. CLANCY

24.1 INTRODUCTION

In this chapter a means is developed for estimating the rate of heat generation accompanying the radioactive decay of the fission products subsequent to the shutdown of an enriched reactor. An expression is found for the decay power at an arbitrary cooling time following operation at a constant power level for an arbitrary length of time. The levels of beta and gamma activity of the fission products are also expressible for cooling times exceeding one day.

Not included in the results given in the subsequent sections of this chapter are three other sources of heat and radioactivity in the post-shutdown period:

1. Reactor constituents other than the fission products become radioactive. The principal activity of this type in sodium-cooled reactors, the 15-hr Na^{24} , can generate initially up to 0.03 per cent of the operating power level, but it never develops over a tenth as much power as the fission products. Several activities are generated in the stainless steel, but their contributions are negligible. The 23-min U^{239} and 2.3-day Np^{239} are important in reactors fueled by natural or low-enrichment uranium¹ (in natural uranium, for example, the Np^{239} alone initially generates 0.24 per cent of the operating power level, and considerably later it develops a third as much power as the fission products) but the contributions of these isotopes are negligible in reactors using fuel highly enriched in U^{235} .
2. The fission products continue to generate delayed neutrons after shutdown. The delayed neutrons maintain the neutron flux at a level of the order of 1 per cent of the pre-shutdown flux for times up to a minute or so, and this keeps the fission rate at a similar fraction of the operating power level. The power development from the delayed neutrons, roughly as important for the first few seconds as the decay power considered in this chapter, is properly a part of reactor neutron kinetics.
3. Photoneutrons generated by hard gamma radiation in beryllium tend to maintain the neutron flux and the fission rate. The gammas, whose energy must exceed the 1.67-Mev threshold for the $\text{Be}(\gamma, n)$ reaction, arise both from the Na^{24} and from the fission products.

Neutron-generating sources 2 and 3 above, while quite important in the first few seconds of shutdown, die down considerably more rapidly than does the effect of the fission products. Measurements by F. G. LaViolette (similar to those described in Chap. 25) of neutron flux levels in the S1G after shutdown from a 50-hr run at constant power enables us to make the comparison of Table 24.1 with fission-product decay powers calculated as described later in this chapter.

TABLE 24.1--DELAYED NEUTRON AND PHOTONEUTRON POWER COMPARED WITH FISSION-PRODUCT DECAY POWER

Time after shutdown, min	0.5	1	2	5	10	50
Delayed neutron and photoneutron power, % of operating power	0.58	0.27	0.106	0.040	0.026	0.016
Fission-product decay power, % of operating power	2.23	1.88	1.60	1.25	1.04	0.85
Ratio of the two powers	0.26	0.14	0.066	0.032	0.025	0.024

The remainder of this chapter considers solely the heating and radioactivity due to fission-product decay, which predominates over the other sources after the first few seconds following shutdown.

If one knew the fission yields, the half-lives, and the radiations of all the radioactive fission products, one could compute the sum of the exponentially decaying radiations accompanying fission-product decay in any given amount of fuel after any given length of exposure in a reactor at a constant power level. Calculations of this type have been made by a number of authors.²⁻⁹ Since little if anything is known about the shorter-lived of the fission products, such a summation computation for short cooling times is subject to considerable uncertainty. The work presented here consists in part of a summation study restricted to those fission products whose half-lives exceed 6 hr. This restricted summation study is adequate for cooling times of one day and longer.

For cooling times of less than a day, the estimations depend to a great extent upon experimental observation. Five different types of information are involved:

- Gamma-ray spectra and photon emission rates of irradiated fissile material after short times of cooling (10 msec to 1600 sec).
- Beta and gamma disintegration rates of foils of enriched uranium for cooling times between 20 and 300 min.
- Beta and gamma disintegration rates of irradiated natural uranium for cooling times ranging from 10 min to a year.
- Gamma and beta emission rates of enriched uranium measured immediately upon cessation of an extended irradiation.
- Heat generation rates measured calorimetrically on natural uranium fuel slugs, for cooling times of an hour and longer.

The complete description of the fission-product decay that is finally obtained has a time-dependence that is made a reasonable fit to the data of types A, B, and C, above. The absolute or numerical values are adjusted to match data of types D and E, above. These adjustments and fittings are done in the course of preparing a description, or a graphical representation, of the radioactive decay that follows a very short period of reactor operation (called here a "burst of fissions"). Another representation, the description of the decay following a very long period of operation ("infinite irradiation"), has a ready use in the treatment of the general case of a finite period of exposure.

24.2 EXPERIMENTAL INFORMATION AVAILABLE

A. Cooling Times up to 1600 Sec

(a) First Three Sec. of Cooling. Work by Brolley et al.¹⁰ has resolved discrepancies in the earlier measurements of the rate of decay of the fission-product radiations during the first few seconds of cooling. These authors conclude that neither the beta nor the gamma radiations decay appreciably during the first few milliseconds after fissioning, and that the gamma decay during the first 2.7 sec of cooling after a burst of fissions can be represented solely as a combination of the gammas emitted by two of the three shortest-lived nuclides that are known also to yield delayed neutrons in fission, namely, the ones with half-lives of 1.52 and 0.43 sec. (The 0.11-sec delayed neutron group more recently reported by Keepin and Wimett¹¹⁻¹³ was discovered since the work of Brolley et al.; its neutron yield is low enough that it might reasonably be expected to have negligible effect on the gamma emission.)

Brolley did not follow the decay of the beta radiation long enough to assign half-lives to it. We assume here that the beta decay has exactly the same time behavior as

the gamma decay, so that the contribution of the 1.52-sec activity to both gamma and beta radiation at zero cooling time is 40 per cent of the contribution of the 0.43-sec activity -- a ratio that can be calculated from Brolley's gamma-decay curve. We follow the time-dependence of the Brolley curve in the present work for the first 2.7 sec.

(b) Cooling through 17 Sec. Measurements made by Leipunsky et al.¹⁴ on fissile samples subjected to short irradiations indicate that the gamma photon emission rate varies with cooling time as $t^{-0.8}$ for cooling times between 1 and 17 sec.

This behavior agrees with that reported by Brolley in the cooling times common to the two experiments. We assume as before that both the beta- and gamma-energy emission rates have the time-dependence quoted by Leipunsky. We follow this time-dependence, however, only to 8-sec cooling time. This is a way of compromising between the behavior found by Leipunsky and the rather different behavior found by Zobel, now to be discussed; the two agree as to the proportional rates of decrease at about 8 sec.

(c) Cooling through 1600 Sec. Well-instrumented measurements by Zobel and Love¹⁵ give a great deal of information about the rate of gamma emission by U^{235} fission products in various energy ranges for times from roughly 1 sec to 1600 sec after relatively short irradiations. We extrapolate these results to lower energies and add a contribution (about 10 percent of the energy) for the 0 to 0.28-Mev photon range that this study did not cover, in order to obtain the rate of gamma energy emission. These data show a slower proportional rate of decrease for the first few seconds than do those of either Brolley or Leipunsky. We follow the time-dependence of Zobel's data from 8- to 1600-sec cooling time. We take the absolute values of the gamma power to be 80 percent of those reported by Zobel (which is within the reported possible error) for reasons to be discussed later with the integral measurements in Subsection D. We assume the beta power to be equal to the gamma power at each time of cooling.

B. Cooling Times between 20 and 300 Min

Although no such information appears to have been published, the manner of decay of the radioactivity of foils of fissile material after exposure in a low-power reactor is fairly well known. Beta and gamma counting rates on such foils were obtained at KAPL by G. B. Gavin and H. B. Stewart; the exposure period was 30 min and the cooling times were between 20 and 300 min. Allowing for the relatively long exposure period, we fit the data over a 30-min interval of cooling time by an assumed time-dependence of the form $t^{-\alpha}$. For beta counting rates, the value α appears to be roughly 1.2 over the whole time of cooling from 20 to 300 min.

For gamma counting rates, α increased from 1.2 at the shorter cooling times up to 1.6 at times between 200 and 300 min. The more rapid $t^{-1.6}$ decay is used significantly in the present work, as will soon be discussed in Subsection C.

C. Longer Times of Cooling

Many early studies of the beta and gamma activities of natural uranium after neutron irradiation were confined to measuring the counting rates for a particular sample under particular conditions as a function of time of cooling. Such measurements were not readily reduced to an absolute basis; furthermore, the effects of the U^{239} and Np^{239} produced by neutron capture in the U^{238} had to be estimated separately and subtracted in order to give the decay of the fission products alone. Way and Wigner summarized this work in their celebrated paper on the statistical nuclear theory approach to fission-product decay.¹⁶ The Way-Wigner expression for the decay rate, which involved terms in $t^{-1.2}$ and $t^{-1.4}$ after relatively long cooling times, fitted the data, at least roughly, over an astonishingly wide range of cooling times.

We assume a $t^{-1.6}$ dependence for both beta and gamma radiations in the range of cooling times from 3 hr to 1 day. This represents a more rapid rate of decay than the Way-Wigner expression. The experimental data summarized by Way and Wigner are not seriously inconsistent with our presentation. It is necessary to assume so rapid a decay rate in order to make a smooth transition between the two portions of the curve for which absolute energy emission rates are given; namely, Zobel's gamma values for times up to 1600 sec, and our own summation computation for times longer than 1 day.

Our summation study, as mentioned in Sec. 24.1, totals the radiation contributions of the individual fission-product nuclides whose half-lives exceed 6 hr and of their shorter-lived descendants. These nuclides occur in 32 of the known fission-product decay chains. The fission yields, half-lives, and radiations assumed in the calculations were those available in 1956.¹⁷⁻¹⁹ Neglect of the nuclides whose half-lives are less than 6 hr is estimated to cause less than 10 per cent error in the results for a cooling time of 1 day and less than 1 per cent error for a cooling time of 2 days. Radiochemical knowledge of the nuclides considered in the calculations is sufficiently precise that this summation study should represent the fission-product radiation quite accurately after more than 2 days' cooling.

D. Integral of Radiations over All Times of Cooling

A measurement of the intensity of the fission-product radiations immediately after the end of an exposure to a constant neutron flux constitutes an experimental determination of the time integral of the radiations following a burst of fissions. Since the radiations decay quite rapidly, an exposure can readily be made long enough to yield a good approximation to the integral over an infinitely long time -- in other words, to the amount of delayed energy emitted per fission.

According to a selection made by Way²⁰ from early measurements, the delayed beta energy is 7 Mev/fission and the delayed gamma energy is 5 Mev/fission. Peelle and co-workers²¹ recently reported a value of 4.8 Mev/fission for the portion of the delayed gamma energy that is emitted by photons in the energy range from 0.36 Mev to 5.8 Mev. Extrapolating the spectrum given by Zobel,²² we would estimate a total delayed gamma energy of 8.4 Mev/fission. With such conflicting values, it seems not unreasonable to take 7.0 Mev/fission as the delayed gamma energy.

In this work we assume that the delayed beta emission rate equals the delayed gamma rate at every instant of time, and that the total delayed gamma and beta energy is 14 Mev. Furthermore, we assume that the typical energy release per fission in an operating reactor is exactly 200 Mev, so that the delayed energy emission is 7 per cent of the total effective energy developed per fission.

The integral over-all times of the total heat emission rate following a burst of fissions should accordingly be 7 per cent of the energy developed in the burst. It was found impossible to meet this requirement if Zobel's data were taken as reported, for the integral turned out to be too large. With Zobel's data reduced by 20 per cent, however, the curve gave the desired 7 per cent integral. This does not seem too drastic a reduction, since Zobel quotes a possible error of 20 per cent in his results; nevertheless, it should be remarked that we are somewhat arbitrary both in assuming beta and gamma powers to be equal and in the 7 per cent choice for the total decay energy.

E. Calorimetric Measurements

The rate of heat generation in slugs of irradiated natural uranium after cooling times ranging from hours to days was determined calorimetrically by Day and Cannon²³ in a basic experiment. Substantial corrections had to be made for the energy carried away by gamma radiation escaping from the calorimeter²⁴ as well as for the heat generated by U^{239} and Np^{239} ; furthermore, the rate of fissioning in the slugs during exposure was not too well known. Nevertheless, the results of Day and Cannon constitute an independent test of the description for fission-product decay power that was found as described above. The points labelled "Experimental" on Figs. 24.1 and 24.2 are Day and Cannon's original values corrected for gamma escape and for U^{239} and Np^{239} heating. The curves on these figures are calculated from the results of the present work. The calculated curves tend to be low, but they reproduce the trend with time fairly well.

24.3 PRESENTATION OF RESULTS

Figures 24.3 and 24.4 show the time behavior of the total decay power, both beta and gamma, after a burst of fissions. These figures give six overlapping portions of the single basic curve which was developed in the manner discussed in the preceding section. The unit chosen for the ordinates of these figures is the watt per megawatt day. A megawatt day (Mwd) corresponds to a "burst" of 2.7×10^{21} fissions, or to the fissioning of

Fig. 24.1—Comparison of calculated curves with calorimetric data (short exposures).

Fig. 24.2—Comparison of calculated curves with calorimetric data (long exposures).

Fig: 24.3—Decay power following burst (short cooling times).

Fig. 24.4—Decay power following burst (long cooling times).

Fig. 24.5—Decay power after infinite irradiation.

1.05 grams of U^{235} . Although the curve is far from linear on the log-log plot used in Figs. 24.3 and 24.4, some portions of it can be reasonably well approximated by straight lines of slope corresponding to the usually assumed $t^{-1.2}$ behavior. From 10 sec to 3 hr, an approximation to the curve is $4.10 t^{-1.2}$ Mev/sec per fission; from 1 day to 100 days, it is very nearly $2.46 t^{-1.2}$ Mev/sec per fission. These can be compared with an early simple expression given by Way and Wigner²⁵: $2.66 t^{-1.2}$ Mev/sec per fission.

Figure 24.5 shows the decay power as a function of cooling time after an infinitely long irradiation. This curve is the integral of the "burst" curve of Figs. 24.3 and 24.4. For zero cooling time the curve of Fig. 24.5 has the 7 per cent value that was assumed for the infinite integral.

According to a simple theorem in radioactive decay, this "infinite irradiation" curve can be used to compute the decay power following any finite period of exposure to a constant neutron flux. The theorem states that the intensity of the fission-product

Fig. 24.6—Fission product power generation (beta and gamma combined). Example shown: for 8-sec irradiation followed by 2-sec cooling ($t+t_0=2+8=10$ sec) $P/P_0=0.0124$ (i.e., fission product power is 1.24 per cent of operating power).

radiations following a finite period of operation is simply the difference between the values on the infinite irradiation curve of Fig. 24.5 for two particular times of cooling:

1. the time elapsed from the end of the exposure period to the moment in question (the time normally considered to be the "cooling time" following a run), and
2. the time elapsed from the beginning of the exposure period to the moment in question (the sum of the time of exposure and the cooling time).

With this theorem, two nomograms have been constructed for rapid estimation of the decay-heat evolution following various times of operation. Figure 24.6 is the

Fig. 24.7—Fission product power generation (beta and gamma combined). Example shown: for 15-hr irradiation followed by 5-hr cooling ($t+t_0=5+15=20$ hr) $P/P_0=0.0022$ (i.e., fission product power is 0.22 per cent of the operating power).

nomogram designed for cooling times in the range of seconds and minutes, whereas Fig. 24.7 is the one designed for cooling times of hours or days.

If the cooling time is longer than the period of operation, the two particular times just defined are relatively close together, and it is difficult to read accurately the difference in the corresponding ordinates of Fig. 24.5, or to use the nomograms of Figs. 24.6 and 24.7. It is just such cases that invite the use of the "burst" curve of Figs. 24.3 and 24.4. In these cases, the approximation can be made that the decay heat following a period T (days) of operation at a constant power level P (Mw) is nearly the

Fig. 24.8—Beta and gamma powers following burst. The rates of energy emission in gamma-ray photons of various energy ranges are shown. The labelling of each curve indicates the photon energy range.

same as that following a burst of fissions that released PT (Mwd) of energy at the midpoint of the period of operation. Even if the cooling time is only twice the operating time, the error involved in treating the extended exposure as a burst is only 2 per cent if the reactor operating power does not change during the exposure, and the error cannot exceed 25 per cent no matter how the operating power level varies during the exposure. (These percentages apply, strictly speaking, only to the $t^{-1.2}$ portions of the curve.)

With a reactor whose power level is frequently changed, the burst curves appear to be as useful as the infinite irradiation curves. The operating history of such a reactor is complicated. It would require immense labor to break up the entire operating history into periods of operation at constant power, to calculate the contribution that

each period makes to the heat-generation rate at any given time after shutdown, and to total these contributions. A relatively simple way of obtaining the desired result is to break up the operating history into definite but not equal time intervals preceding the shutdown, regardless of whether or not the reactor was operating at a constant-power level during an interval. If these time intervals are so chosen that each is small compared to the subsequent time elapsing to the shutdown in question, one may use the "burst" formulation of Figs. 24.3 and 24.4 to calculate the contribution of each interval to the total decay power. It is helpful in this scheme to have a cumulative hourly record of the megawatt days of energy put forth by the reactor. Even though the contribution of any one interval may be in error by as much as 25 per cent of itself (in the case that the interval is half as long as the subsequent time to the shutdown), the sum of contributions from J intervals that contribute about equally is statistically liable to a relative error of $25/\sqrt{J}$ per cent.

Figures 24.8-24.11 give the results of the detailed summation study for cooling times exceeding a day. On each of these figures the beta and gamma contributions are shown separately; the contributions of various photon energy groups are also shown. These figures are useful in estimating the needs for shielding, as for a spent fuel element.

Figures 24.8 and 24.9 give the burst and infinite irradiation curves for the beta and gamma power developments. Figures 24.10 and 24.11 give similar curves for the beta and gamma activities; that is, for the rates of emission of beta particles and of gamma photons. These figures can be used for finite periods of exposure in exactly the same way as was described for Figs. 24.3, 24.4, and 24.5.

Fig. 24.9—Beta and gamma powers after infinite irradiation. The rates of energy emission in gamma-ray photons of various energy ranges are shown. The labelling of each curve indicates the photon energy range.

Fig. 24.10—Beta and gamma activities following burst. The rates of energy emission in gamma-ray photons of various energy ranges are shown. The labelling of each curve indicates the photon energy range.

Fig. 24.11—Beta and gamma activities after infinite irradiation. The rates of energy emission in gamma-ray photons of various energy ranges are shown. The labelling of each curve indicates the photon energy range.

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Chapter 25

PHOTONEUTRONS IN BERYLLIUM-MODERATED REACTORS

F. G. LaVIOLETTE

25.1 INTRODUCTION

Beryllium has a lower threshold for the production of photoneutrons than has any other element, so that beryllium-moderated reactors should show, more strongly than other reactors, transient source effects that result from the production of photoneutrons by the fission-product gamma-rays. In sodium-cooled reactors there is an additional effect caused by the sodium gamma-rays. After shutdown, the production of gamma-rays and of the corresponding photoneutrons decays at a relatively slow rate. These photoneutrons constitute in effect a neutron source.

Safety considerations dictate that a fairly intense source of neutrons be present in a reactor core whenever its reactivity is being increased from a subcritical condition, in order to provide means for indicating the approach to criticality. Sensitive proportional counters are required to measure the resulting neutron flux. It is a distinct advantage of a beryllium-moderated reactor that the long-lived photoneutrons appearing after reactor operation provide a sufficiently strong source to activate less sensitive detectors, such as fission counters and current chambers, for a considerable period of time after reactor shutdown. These photoneutrons, furthermore, can keep a beryllium-containing reactor virtually unaffected by any decay of the initially installed neutron source.

In the operation of such a reactor, it is desirable to predict how long the plant can be shut down before the photoneutron source becomes too weak to activate startup instruments with the control elements in their least reactive position. In addition it is desirable to distinguish the short-term photoneutron effects when estimating the xenon reactivity defect and to separate the photoneutron-induced fission heating from the decay-heat generation after shutdown.

Different methods were used for evaluating the short-period and the long-period photoneutron effects. Since the mathematical treatment of the technique for short-period measurements best illustrates the behavior of a reactor with photoneutrons, this will be discussed first. The short-term photoneutron decay characteristics were studied in "power-drop" experiments by D. J. Anthony and R. J. Schemel who used the PTR operating at low power levels. This experiment involved the observation of the reactivity variations required to keep the power level constant after a rapid reactivity change has produced a sudden decrease in power level. The reactor behavior in this case is illustrated in Fig. 25.1. Assume that the power level N_0 has existed for a long period of time prior to $t = 0$. At time zero the reactivity is suddenly decreased by an amount sufficient to cause a prompt decrease in power level to a value N_p . If no further reactivity change were made the power level would continue to decrease below N_p as the concentration of the delayed neutron precursors decayed from the initial value which was in equilibrium at the power level N_0 . However, if the reactivity is gradually restored the power level may be held constant at level N_p . In this case the reactivity defect (as measured by the displacement of the control element from the original critical position) varies with time in the same manner as the function describing the decay of the concentration of delayed-neutron

Fig. 25.1—Reactivity transient following a step change in power level.

precursors from its initial equilibrium value for level N_0 to the new equilibrium value for the level N_f .

The presence of photoneutron groups having decay periods longer than the periods of ordinary delayed neutrons causes a persistence of this reactivity defect. Furthermore, the strength of the photoneutron source relative to that of the delayed-neutron source may be evaluated in terms of this reactivity defect, as will now be shown.

25.2 POWER DROP MEASUREMENTS

Consider a reactor with several groups of different delayed-neutron and photoneutron periods. The reactor kinetics equations connecting the power level and the concentrations of the precursors of the various groups with the time are¹:

$$dN/dt = (k - 1 - \beta) N/\tau + \sum_i \lambda_i C_i \quad (25.1)$$

$$dC_i/dt = \beta_i N/\tau - \lambda_i C_i \quad (25.2)$$

No fixed source term is included in Eqs. 25.1 and 25.2, since the installed source strength may be considered negligible in comparison with the photoneutron source. The photoneutron source terms appear, together with the delayed-neutron groups, in the summation

$\sum_i \lambda_i C_i$. In these two equations:

N is proportional to the power level and is a measure of the fissions per unit volume per unit time,

C_i is a measure of the concentration of the i th group of delayed neutron or photoneutron precursors,

λ_i is the decay constant for the i th group,

β_i is the effective yield of the i th group, as a fraction of all the neutrons,

β is defined as $\sum_i \beta_i$,

τ is the prompt generation time for neutrons,

$(k-1)$ is the reactivity. For delayed critical, $k = 1$.

Experimentally, by appropriate manipulation of the reactor controls, the power level is made to drop suddenly from an original constant value of N_0 to a final constant value of N_f . The time at which this occurs is $t = 0$. The controls are thereafter varied in whatever manner is necessary to hold the power level fixed at N_f . (See Fig. 25.1.)

The concentration of the i th group of precursors, initially in equilibrium at a value $C_{i0} = \beta_i N_0 / \tau \lambda_i$, drops toward a final equilibrium value $C_{if} = \beta_i N_f / \tau \lambda_i$. According to Eq. 25.2, since N is constant when $t > 0$, the time dependence is

$$C_i(t) = C_{if} + (C_{i0} - C_{if}) e^{-\lambda_i t} \quad (25.3)$$

Since the experimental manipulations ensure that $dN/dt = 0$, Eq. 25.1 then gives an expression for the reactivity. Substituting for the C_1 in Eq. 25.1, and eliminating τ by means of expressions above, this equation is obtained:

$$\frac{1-k(t)}{\beta} = \frac{(N_o - N_f)}{N_f} \sum_i \left[\frac{\lambda_i (C_{1o} - C_{1f}) e^{-\lambda_i t}}{\sum_n \lambda_n (C_{no} - C_{nf})} \right] \quad (25.4)$$

The term $\sum \lambda_n (C_{no} - C_{nf})$ in the denominator on the right-hand side of Eq. 25.4 represents the total change in the delayed neutron and photoneutron source, i.e., the difference between the neutron source due to all precursors in equilibrium with the power level N_o and that due to all precursors in equilibrium with the new power level N_f . The summation in Eq. 25.4 is composed of terms

$$f_i(t) = \frac{\lambda_i (C_{1o} - C_{1f}) e^{-\lambda_i t}}{\sum_n \lambda_n (C_{no} - C_{nf})} \quad (25.5)$$

each of which represents the fraction of this total source change due to the i th group of precursors after decay for time t . It is to be noted that the decay period for the function $f_i(t)$ is identical to that of the generating precursor.

The delayed and photoneutron source due to all groups at time t is then

$$f(t) = \sum_i f_i(t) = \sum_i \left[\frac{\lambda_i (C_{1o} - C_{1f}) e^{-\lambda_i t}}{\sum_n \lambda_n (C_{no} - C_{nf})} \right] \quad (25.6)$$

Equation 25.4 can then be written as

$$f(t) = \frac{1-k(t)}{\beta} \frac{N_f}{N_o - N_f} \quad (25.7)$$

which presents directly, in terms of the observed reactivity, the fraction of delayed and photoneutrons remaining at time t . Note that the components of $f(t)$ due to photoneutron precursors having periods greater than the delayed-neutron periods will after a sufficient time dominate the value of $f(t)$. This is true even though the concentration of such precursors at initial equilibrium is much smaller than the concentration of ordinary delayed-neutron precursors.

This "power-drop" method is applicable to the determination of photoneutron contributions due to precursors with periods from a few minutes to several hours. Precursors with shorter periods are masked by the ordinary delayed neutrons. Precursors with longer periods do not build up to saturation value in the operating time usually convenient for such low-power experiments.

The sensitivity of the above technique for indicating the photoneutron effects is determined by the ratio of the initial sustained power level N_o to the final power level N_f . It is not desirable to operate critical assemblies at power levels N_o higher than a few watts for any appreciable length of time. Also, instrumental sensitivity establishes the minimum readable level N_f to be in the milliwatt range. Therefore the ratio N_o/N_f is limited to values of the order of 10^3 .

To sum up, the "power-drop" method consists of recording as a function of time the control-rod position, and hence the reactivity $k(t)$, which is necessary to maintain the reactor power level constant at the value N_f . The source function $f(t)$, calculated by means of Eq. 25.7, is then decomposed into its several exponential decaying components (Eq. 25.6) by well-known methods.

25.3 STARTUP MEASUREMENTS

A method developed by J. A. Bistline and the author may be used with a power reactor to obtain information on the weak long-period photoneutron groups which can not be determined conveniently by the power-drop method of Sec. 25.2. This method involves

measurements of neutron levels and control-element positions taken during the approach to criticality after the reactor has been shut down from a period of sustained high-power operation.

Since the long-period photoneutrons act as a nearly constant source, the straightforward way of measuring the strength of this source would be to find the neutron power level at some convenient degree of subcriticality. Unfortunately, the degree of subcriticality after high-power operation is not determined solely by the control-element setting. Temperature changes and xenon poisoning both strongly influence the value of the reactivity after shutdown. Therefore it is necessary to make an auxiliary measurement which in effect calibrates the control elements in terms of subcriticality.

The method for doing this is illustrated by Fig. 25.2. This shows the general time dependence of the transient power level and reactivity for the case where the reactor is shut down at time zero and returned to a near critical condition after a long time interval T , which may be in the range 1 to 1000 hr.

Fig. 25.2—Reactivity and power transients during shutdown and subsequent approach to criticality in startup method.

The approach to criticality is performed in a stepwise fashion as shown on the right of Fig. 25.2. Two power levels, N'_S and N_S , are selected, low enough to produce no significant core temperature changes but nevertheless associated with control element positions relatively near criticality. (This latter condition is imposed to minimize errors in neutron detector readings due to changes in control element geometry.) The control element is manipulated to hold the power constant at N'_S for a few minutes, and then to raise it and hold it constant at N_S .

The amounts by which the reactor is subcritical at the levels N'_S and N_S , respectively, are $1-k'_S$ and $1-k_S$. If the neutron detector geometry is essentially unchanged between levels N'_S and N_S , and if there is no appreciable decay of the neutron source (the photoneutrons associated with operation at the original power level), the multiplication relation

$$N'_S(1-k'_S) = N_S(1-k_S) \quad (25.9)$$

may be used to express the difference in the two reactivities as measured by a calibrated control element:

$$\delta k_m = k_S - k'_S = (1-k'_S) \times (N_S - N'_S)/N'_S \quad (25.10)$$

By appropriate manipulation of the control element, the power level is raised to N'_S and held constant at that value for a few minutes; then it is raised further to N_S and held constant at that value for a similar length of time. The equilibrium settings of the control element of these two power levels will now be expressed in terms of the neutron source contributed by the photoneutron precursor concentrations $C_j(T)$ remaining from the previous power operation. Consider, for simplification, that the startup consists of

a simple one-step increase in power level from 0 to N_s . The delayed neutron and photoneutron precursor concentration associated therewith will rise from 0 to C_{1s} (see Eq. 25.3) obeying the relation

$$C_1(t) = C_{1s}(1 - e^{-\lambda_1 t}) \quad (25.11)$$

Here time is measured from the new startup. The time $t = 0$ is then a time interval T after shutdown from power operation. The longer-lived photoneutron precursors remaining from power operation, indicated by subscript j , must be taken into account explicitly as a constant source term in the kinetics equation analogous to Eq. 25.1:

$$\frac{dN}{dt} = (k-1-\beta)N_s/\tau + \sum_i \lambda_i C_{1i}(t) + \sum_j \lambda_j C_j(T) = 0 \quad (25.12)$$

By reasoning similar to that which led to Eq. 25.4, the reactivity necessary to maintain the power level at N_s is

$$\frac{1-k(t)}{\beta} = \frac{\sum_j \lambda_j C_j(T) - \sum_i \lambda_i C_{1is} e^{-\lambda_i t}}{\sum_i \lambda_i C_{1is}} \quad (25.13)$$

When the concentration of delayed-neutron precursors reaches equilibrium, which takes about 10 min, the exponential terms in the equation above have diminished to zero, so that

$$\frac{1-k(t)}{\beta} - \frac{1-k_s}{\beta} = \frac{\sum_j \lambda_j C_j(T)}{\sum_i \lambda_i C_{1is}} \quad (25.14)$$

Using the definition of $f(t)$ in Eq. 25.6, and the equilibrium values of C_{10} and C_{1s} from Eq. 25.2, we reduce the expression above to

$$\frac{1-k_s}{\beta} = \frac{f(T) \sum_i \lambda_i C_{10}}{\sum_i \lambda_i C_{1s}} = f(T) \left(\frac{N_o}{N_s} \right) \quad (25.15)$$

Finally, combining Eqs. 25.10 and 25.15, an expression for the photoneutrons remaining at time T after shutdown from power operation, expressed as a fraction of the total delayed and photoneutrons initially present, is:

$$f(T) = \delta k_m \left(\frac{N'_s}{N_s - N'_s} \right) \left(\frac{N_s}{N_o} \right) \quad (25.16)$$

In the application of this equation the measurement of the reactivity change δk_m and the ratio N'_s/N_s must be made under constant temperature conditions. Furthermore the time between the measurement of N'_s and N_s must be sufficiently short that changes in the xenon reactivity defect from the prior power operation will not produce a significant error in δk_m . Since the ratio N_o/N_s may be very large, in the range from 10^3 to 10^8 , neutron-level instrumentation must be provided which can cover this range.

The derivations of Eqs. 25.9 and 25.15 above assume that the equilibrium concentration of long-lived photoneutron precursors is small compared to the total equilibrium concentration of all delayed and photoneutron precursors. As a justification of this assumption, it is found in practice that the total concentration is within 1 per cent of the equilibrium value 10 min after a step change in power level. An additional assumption is that the spatial distribution and effectiveness of the photoneutrons in the subcritical reactor are the same as for delayed neutrons in the critical reactor. The error introduced by this assumption is considered acceptably small for the SIR-type reactors since significantly large errors occur in estimating the effective power-level history during which the longer-lived gamma-ray precursors are built up. However,

this assumption may not be valid for other reactor systems where the core geometry can change by large amounts as the control elements are moved to compensate for temperature changes.

25.4 EXPERIMENTAL RESULTS

Data on photoneutron decay in beryllium-moderated reactors have been obtained from power-drop tests on the PTR and from startup tests on the S1G and S2G reactors. These have been used to construct the curves of Fig. 25.3 for operating times from 1 hr to 100 hr.

The uppermost curve in Fig. 25.3 (for an operating time of 100 hr) was analyzed graphically to determine the relative values of the photoneutron source due to individual gamma emitters. The values shown in Table 25.1 indicate the approximate fraction of the total delayed neutrons contributed by each emitter after the reactor has been in operation for 100 hr. The analysis leading to Table 25.1 was based on half-life data taken

TABLE 25.1--PHOTONEUTRON YIELDS

Gamma-emitter Precursor	Half-life, hr	Photoneutron yield (fraction of the delayed neutrons), β_3 / β
Ba ¹⁴⁰ → La ¹⁴⁰	308	1×10^{-5}
Te ¹³²	77	2×10^{-5}
Te ¹³¹	30	6×10^{-5}
Na ²⁴	15	20×10^{-5}
I ¹³⁵	6.7	$20-50 \times 10^{-5}$
Kr ⁸⁸	2.8	$50-200 \times 10^{-5}$

Fig. 25.3—Decay of photoneutrons in beryllium-moderated reactors.

from the literature. The data provided by the curve of Fig. 25.3 are not considered sufficiently accurate to warrant any attempt to determine both the half-lives and the yields of the photoneutron groups. The values of β_1/β in Table 25.1 have been corrected to apply to an infinite thickness of beryllium.

25.5 REACTOR FISSION POWER-LEVEL AFTER SHUTDOWN

The fission power at time t after a reactor shutdown may be calculated from Eq. 25.7. Taking advantage of the fact that $N_f \ll N_o$ in this case, and writing $(1-k_d)$ for the shutdown reactivity, we find

$$N_f(t) = N_o f(t) \beta / (1-k_d) \quad (25.17)$$

The function $f(t)$ is given in Fig. 25.3, and $(1-k_d)/\beta$ is simply the shutdown margin in dollars. Applied to a \$5 shutdown, this equation indicates that the fission power falls below the gamma-decay power within 1 min after shutdown and is less than 10 per cent of the gamma-decay heat after 3 min.

If the neutron detector instrumentation indicates over the range from full power to about 10^{-8} of full power, Eq. 25.17 indicates that with a \$5 shutdown from extended operation at 10 per cent rated power, the neutron level is readable for over 100 days. This suggests the longevity of the photoneutron source in a beryllium-moderated reactor.

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25.2

Chapter 26

POISONING BY Li^6 GROWTH IN THE MODERATOR

L. G. BARRETT

26.1 INTRODUCTION

In a beryllium-moderated reactor the nuclear poison Li^6 is produced through the reactions

The He^6 decays into Li^6 with a half-life of 0.82 sec. The threshold for the (n, α) reaction on Be^9 is 0.72 Mev, and the cross-section goes up to a peak value of 100 millibarns. The cross-section for this reaction is shown in Fig. 26.1, which is based on recent measurements.¹ It should be noted that this cross-section curve differs radically from the curve given in Report BNL-325², which was based on earlier data.³ The cross-section curve shown in Fig. 26.1 is here extrapolated above the highest energy measurement at 4.4 Mev, but, since most fission neutrons are born with energies less than this value, the final results are insensitive to the method of extrapolation.

Fig. 26.1— $\text{Be}(n, \alpha)$ cross-section versus neutron energy (Mev.)

The calculations reported in this chapter were done to estimate the reactivity loss caused by the presence of Li^6 in a typical beryllium-moderated reactor and thus aid in analyzing endurance and long-term reactivity effects.

26.2 ANALYSIS

The differential equation for the concentration of Li^6 atoms as a function of time is

$$V_c \frac{dN_{\text{Li}}}{dt} = \int dr \int_0^\infty dE \left[N_{\text{Be}} \sigma(\text{Be}) - N_{\text{Li}} \sigma(\text{Li}) nv(E, r) \right] \quad (26.1)$$

Here N_{Li} and N_{Be} represent the numbers of atoms of Li^6 and Be^9 , respectively, per unit volume; $\sigma(\text{Li})$ is the absorption cross-section of Li^6 , and $\sigma(\text{Be})$ is the (n, α) cross-section. The volume of the core is V_c and $nv(E, r)$ is the neutron flux.

Dividing the range of integration of energy into subranges corresponding to the lethargy intervals Δu_i of the multigroup analysis of the neutron flux in the reactor, one obtains

$$\frac{d\bar{N}_{\text{Li}}}{dt} = \bar{N}_{\text{Be}} \sum_i \Delta u_i \bar{\sigma}_i(\text{Be}) \bar{nv}_i P - \bar{N}_{\text{Li}} \sum_i \Delta u_i \bar{\sigma}_i(\text{Li}) \bar{nv}_i P \quad (26.2)$$

The solution of Eq. 26.2, assuming that N_{Be} is constant, is

$$\frac{\bar{N}_{\text{Li}}}{\bar{N}_{\text{Be}}} = \frac{\sum_i \Delta u_i \bar{\sigma}_i(\text{Be}) \bar{nv}_i}{\sum_i \Delta u_i \bar{\sigma}_i(\text{Li}) \bar{nv}_i} \left[1 - e^{-tP \sum_i \Delta u_i \bar{\sigma}_i(\text{Li}) \bar{nv}_i} \right] \quad (26.3)$$

where i = the group index

$\bar{nv}_i$ = the neutron flux averaged over lethargy in the group and averaged over the core. It has been normalized to a fission-source strength of 1 neutron per cubic centimeter per second

$\bar{\sigma}_i$ = an average group microscopic cross-section

Δu_i = the lethargy group width

$\bar{N}_{\text{Li}}$ and $\bar{N}_{\text{Be}}$ = the average values of the Li^6 and Be^9 concentration over the core

$$P = \nu_c \frac{3.1 \times 10^{10} (\text{fissions per watt-sec}) \times (\text{power in watts})}{V_c}$$

P is the conversion factor between the multigroup power normalization of 1 fission neutron per cubic centimeter per second and the assumed power level of the core; and ν_c is the number of neutrons emitted per fission that would make the assembly studied by the multigroup method just critical.

For the purpose of calculating the Li^6 buildup in the core, the value for $\bar{nv}_i$ was taken from a clean ~50-kg one-dimensional multigroup case at the 30-cm-radius position. A similar calculation was done for the reflector; here the $\bar{nv}_i$ values were taken from the same case but at the radial position of 49 cm. The fluxes at these two particular radii are average flux values for the core and reflector, respectively. They are shown in Table 26.1.

The absorption cross-section of Li^6 for neutron energy E below 1200 ev was obtained by assuming the $1/v$ law to hold with a thermal value of 953 barns at $E = 0.025$ ev. This assumption appears to be justified upon examination of the total cross-section curve. In the range of u from 1.5 up to 9 (E from 2.2 Mev down to 1200 ev), the absorption cross-section for Li^6 was taken from the (n, α) curve in Report BNL-325.² Both the Li^6 (n, α) and the Be^9 (n, α) cross-sections were taken as unshielded. The average group cross-sections used and the $\bar{nv}_i$ values are tabulated in Table 26.1. The value of P was taken as 27×10^{10} for a 1-Mw reactor.

The reactivity coefficients for Li^6 were calculated according to the methods described by Hofmann,⁴ which are outlined in Sec. 32.2.

TABLE 26.1--MULTIGROUP QUANTITIES

Group	Range of u	$\bar{\sigma}_1(\text{Li})$, barns	$\bar{\sigma}_1(\text{Be})$, barns	$\bar{n}\bar{v}_1 \Delta u_1$ (at $r = 30 \text{ cm}$)
1	0-0.5	0.054	0.0089	0.25
2	0.5-1	0.069	0.0574	1.47
3	1-2	0.106	0.0605	10.67
4	2-3	0.321	0.0011	11.83
5	3-4	1.98	0	11.56
6	4-8	1.67	0	31.88
7	8-11	6.18	0	16.66
8	11-12.5	17.5	0	5.84
9	12.5-13.75	34.4	0	3.34
10	13.75-15	64.6	0	2.17
11	15-16	113	0	1.24
12	16-17	186	0	0.94
13	17-17.75	283	0	0.43
14	17.75-18.7	436	0	0.28
15	18.7	488	0	0.12
Weighted totals	0-18.7	1159	0.745	-

26.3 RESULTS

After 1000 Mwh of operation the amount of Li^6 produced is calculated to be 0.0203 mol. After ten times as long a period of operation, the amount of Li^6 is 1.92 mol - less than ten times as much because some of the lithium is burned up. These are the amounts of Li^6 developed in the roughly 28,000 mol of beryllium in the core.

Using a calculated distributed reactivity coefficient⁴ of -53¢/mol for unshielded Li^6 , one obtains reactivity losses after 10,000 and 100,000 Mwh equal to 1.08¢ and 102¢. Since the depletion of fuel and the buildup of fission products reduce reactivity by 12.5¢/1000 Mwh (Sec. 20.12) the reactor endurance after about 100,000 Mwh of operation should be diminished by about 8000 Mwh because of the growth of Li^6 in the core. The reactivity loss resulting from the growth of Li^6 in the reflector is negligible (~0.1¢ in 100,000 Mwh). There is no direct experimental evidence as to the magnitude of this lithium poisoning effect, for in the measurements made to date it has not been separated from the effects of fission-product buildup and fuel depletion.

If fuel assemblies are replaced by fresh ones after a life of something like 100,000 Mwh, and if 30 per cent of the core beryllium remains in the reactor, the new core will begin life with 33¢ less reactivity than the first core did. A third core, similarly, would have 66¢ less, a fourth one about \$1.00 less, and so on. Saturation effects would of course reduce these incremental reactivity losses after a number of reloadings, so that the handicap to infinitely aged core beryllium in this example would be \$2.86.

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26.3

Part IV

**REACTOR KINETICS AND
TEMPERATURE COEFFICIENTS**

R. T. FROST

Chapter 27

REACTOR KINETICS AND TEMPERATURE COEFFICIENTS

R. T. FROST

27.1 INTEREST FOR INTERMEDIATE REACTORS

The temperature coefficient of an intermediate-spectrum, sodium-cooled reactor is an order of magnitude smaller than that of a typical water-cooled plant. This is due to the relatively small coolant expansion effect and the relative unimportance of thermal spectrum shifts. In addition, the reactor kinetic behavior is distinguished from that of a typical water-cooled plant by the presence of a significant photoneutron effect arising from the beryllium moderator.

Considerable effort was expended in the intermediate reactor program in an attempt to predict, before the reactor was built, the temperature dependence and the kinetic characteristics of the power reactor. Information obtainable from reactor experiments and from calculations was applied to two broad problems. One was to predict the reactivity changes which would result from various power transients expected in the course of normal maneuvering, and particularly to insure that unusual maneuvers would not lead to self-sustained oscillations in power level. The results of these predictions were checked by actual tests in the reactor after it was built. The other problem was to estimate the course of reactor incidents (nuclear runaways). The second problem required a knowledge of those core characteristics which might affect the initiation and termination of a reactor incident: neutron generation time, prompt temperature coefficient, and the possible magnitude of reactivity increases resulting from such accidents as fuel element vaporization. Secs. 27.2 and 27.3 and Chaps. 28 through 37 cover the first problem, while Sec. 27.4 and Chap. 38 deal with the second.

27.2 TEMPERATURE COEFFICIENTS

The changes in reactivity with temperature are due primarily to four different phenomena: (a) thermal expansion of the reactor structure, (b) sodium expansion, (c) thermal base effects, and (d) the Doppler effect.

(a) The geometrical expansion causes the reactor buckling to decrease and thus tends to reduce leakage as the temperature is raised. On the other hand, the reduced density of core materials at the higher temperature increases the neutron migration area and thus tends to increase the leakage. This latter effect dominates and produces a net negative temperature coefficient as a result of thermal expansion of the core (Chap. 29).

(b) Sodium has a positive reactivity effect in the core because, in terms of reactivity, its neutron moderation is a stronger effect than its neutron absorption. However, thermal expansion of the sodium reduces the number of sodium nuclei in the core and thus results in a negative contribution to the reactor temperature coefficient. The experimental determination of reactivity coefficients for sodium as a function of position throughout the core and reflector, together with the known coefficient of thermal expansion, allows a simple determination of this reactivity effect (Chap. 28).

(c) As the temperature of the beryllium moderator increases, the mean energy of the neutrons near thermal is increased, both in the core and in the reflector. The phenomena associated with this are commonly lumped together and referred to as the "thermal bage effect."

In the core there are changes in reactivity accompanying the change in competition between absorption and fission. Because the xenon resonance is at 0.085 ev, the ratio of average xenon to average fuel cross-section should decrease as the thermal neutron distribution shifts upward beyond 0.085 ev, in energy with increased beryllium temperature. This produces a positive contribution to the temperature coefficient. Another effect of the thermal spectrum shift in the core is the change in fuel capture-to-fission cross-section ratio. Since this change is estimated to be small compared to the other effects, it was not considered further in the work which follows. In the reflector, the decrease in the ratio of absorption to scattering cross-section for steel means an increasing thermal diffusion length with increasing temperature; this increases the thermal current returned by the reflector and hence makes a positive contribution to the temperature coefficient.

An appreciable contribution to the temperature coefficient due to thermal spectrum changes with temperature was anticipated and observed. The resultant decrease in magnitude of the over-all temperature coefficient with the buildup of xenon in the S1G was in fact the most obvious systematic effect observed during reactor operations.

Later physics experiments and calculations (Chap. 31) have shown that the change in thermal spectrum with temperature in the central core region is negligible. These experiments were done with a partial core mock-up, a thermal column neutron source and a chopper and time-of-flight detector; the spectrum obtained was representative only of the central core region. Theoretical considerations, although carried through in detail only for the central core region, indicate that the thermal spectrum shifts are appreciable near the core-reflector interface and in the reflector itself. It is the outer regions of the reactor core, therefore, that produce the positive contribution to the temperature coefficient by xenon.

(d) The Doppler effect, which causes temperature broadening of the fuel resonances and decreases the self-shielding of both fission and capture resonances, is associated with fuel element temperature and hence contributes to the most rapid temperature coefficient. Although this effect is smaller in magnitude than the other effects for a given temperature excursion, its importance is enhanced by the fact that the fuel element temperature excursions are ordinarily much higher than the temperature variations in other core components.

Calculations on the fuel Doppler coefficient (Chap. 32) showed that the effect was a small difference between large positive and large negative effects. The positive effect results from the unshielding of the fission component of the resonances, while the negative effect results from the increase with temperature of parasitic captures accompanying a similar change in resonance self-shielding. A summary of the effect of the uncertainties in the experimental values of the capture-to-fission ratio (usually denoted as α) in the known U^{235} resonances has been given by H. Hurwitz.¹ The result of the calculation is fairly sensitive to α and its variation within each resonance. Calculations which were made in 1954, with the then-available values of α , gave the Doppler coefficient as a positive difference between positive and negative contributions that were approximately seven times as large as the difference between them. It seemed unlikely at that time, however, that the uncertainties in α were great enough that further refinements in neutron spectroscopy could bring the calculated results into agreement with the experimentally measured U^{235} Doppler effect. More recent information on α is such as to reduce this discrepancy somewhat, but the agreement is still not satisfactory.

Another possible error in the Doppler calculations lies in the fact that the net result is quite sensitive to the difference between the neutron "iterated fission probabilities" for resonance and for thermal neutrons. However, since the analyses of reactivity coefficients that involved calculated iterated fission probabilities (reported in Chap. 8) showed that reasonable agreement was obtained between calculation and experiment in most cases, this error is probably not large. It must be admitted, however, that the experiments were not analyzed for a possible consistent error in the calculated resonance values of iterated fission probability.

The experimental measurements of the Doppler change in the reactivity coefficient of fuel were supplemented by measurements for pure resonance absorbers for which good resonance parameter data were available. The experimental technique, described in

Chap. 33, consisted of a cyclic heating procedure and detection with a tuned circuit. The measurements indicated that the U^{238} portion of the fuel contributed about half of the over-all negative fuel coefficient, and that this over-all coefficient was appreciably negative over at least most of the core volume. It is worth noting that the measurements of the Doppler effect for the pure resonance absorbers were in fair agreement with the calculations and that the calculations in this case did not involve any cancellation between opposing effects such as mentioned in the case of fuel.

The so-called "isothermal" temperature coefficient gives the rate of reactivity change resulting from the effects listed above when all reactor components are at the same temperature. Measurements and calculations on this quantity appear in Chaps. 28 and 29. The isothermal temperature coefficient was measured near room temperature in a critical assembly. However, since the coolant was mocked with magnesium and aluminum in the critical assemblies, the reactivity change due to sodium expansion had to be measured by a separate experiment. Another difference between the critical assembly and the power reactor isothermal temperature coefficients arises from differences in mechanical construction of the cores which make the amounts of thermal expansion different in the two reactors (Chap. 28).

Although in concept the isothermal temperature coefficient is simple to deal with in terms of its components, it is not directly applicable in actual operation of the power reactor. There are always temperature gradients in the reactor operating at power. Hence there is practical interest in the power coefficient, the rate of change of reactivity with power level, which is discussed in Chap. 30.

27.3 RESPONSE TO VARIATIONS IN POWER LEVEL

In order to ensure that power transients would not give rise to power-level oscillations, a pile oscillator was used to measure the response of a reactor (Chap. 35) to sinusoidal variations of multiplication factor over a range of frequencies up to and including those at which only the fuel-temperature oscillations would be important. Preparatory experiments in the PTR critical assembly (Chap. 34) helped establish the feasibility of the technique. These experiments demonstrated that the effects of wave propagation are unimportant at the highest frequencies which were used in the experiments on the power reactor. The wave-propagation effect is simply the nonseparability of the spatial and time dependence of the neutron density during fast transients, and it is connected with the phenomenon of the different decay times for the various spatial modes in kinetic experiments of the transient type. Because of the relative ease of doing the experiments in the critical assembly, the experiments could be carried out therein at frequencies sufficiently high to obtain a measure of the prompt generation time divided by the effective delayed neutron fraction. Small suspected wave-propagation effects at the highest frequency, however, prevented this measurement from being more than a crude one.

A given set of temperature coefficients for the various reactor components (such as fuel, beryllium, and coolant) should lead to a unique response to reactivity oscillations as a function of frequency. Attempts were therefore made to correlate the experimental results with the thermal models; calculated or experimental values were used for the temperature coefficients, and an analog model for the solution of the heat transfer problem (Chap. 38). The reactivity effect of the xenon temperature coefficient which had been anticipated agreed qualitatively with the pile-oscillator results that were found in the presence of xenon, but only crude quantitative comparisons were possible. This was due in part to the large number of variables involved in the comparison, such as time constants and temperature coefficients of various other core components. The oscillator experiments at power were, however, extremely successful in their principal objective of demonstrating the stability of the reactor under a variety of operating conditions.

Semi-independent information regarding the kinetic response of the power reactor to sinusoidal inputs was obtained from transient experiments in which the reactivity was changed suddenly. Ideally, the transient analysis would be made with step-function changes in multiplication constant, but the mechanical inertia of the control elements forced the reactivity changes to be approximately of a "ramp" character (Chap. 36). Laplace transformation of the variations of reactivity and power during a transient gave values for the sinusoidal response which could be compared with the pile-oscillator re-

sults over a limited frequency range (Chap. 37). The information obtained with the ramp experiments is not entirely independent of the pile-oscillator results in that both types of experiment utilized some equipment in common (such as the ion chamber). Both types of experiment showed the same relative change in the temperature coefficient due to xenon, although an apparent normalization error in one (or both) of the experiments resulted in a factor-of-two difference in this coefficient as derived from the two experiments.

Although the "ramp" type of experiment is quicker to perform than the oscillator experiment, it does not give so directly the fundamental information regarding stability against power-level oscillations. Furthermore, since the transient response is measured over a limited time interval only, and the transformation procedure requires the response over an infinite interval, approximations are involved which render the results meaningless for all but the highest frequencies (above 1 radian per second). The results of the transient experiments are of some interest in their own right, however, since they can give a value for the prompt temperature coefficient, and since they show that oscillations are not induced by the types of power-level transients which are used in the experiments.

27.4 HAZARDS CONSIDERATIONS

The safeguards calculations for the reactor were based on experimental information on the reactivity effects occurring during an assumed reactor incident (Chap. 38). The most important of these reactivity effects result from: sodium ejection (the effect of which was derivable from the same reactivity coefficient measurements required for the isothermal temperature coefficient), fuel Doppler effect (the measured value was used), and the transport of fuel into the end reflector and out of the reactor vessel after fuel element rupture. Fairly accurate experimental information on this effect of fuel transport was available at the time the safeguards calculations were made.

Critical assembly measurements of hydrogen reactivity coefficients were also available and allowed study of incidents resulting from the introduction of hydrogen into the coolant stream. The safeguards problem also necessitated a prediction of the reactivity change resulting from fuel element heating in very fast assumed power excursions. The thermal time constant for the heating of the fuel element cladding, and hence for the reactivity change due to expansion and displacement of the element due to thermal strains, is considerably longer than the e-folding period assumed during some hypothetical accidents. This was the motivation for the work of Chap. 33 on the Doppler effect for the fuel, since this component of the temperature coefficient plays a role in even the fastest assumed accident. An example of feedback from physics to reactor design was a relaxation in the performance specifications for the reactor scram mechanisms after the demonstration of the negative value of the prompt Doppler coefficient. The experimental value of the ratio of prompt generation time to delayed-neutron fraction (which was larger than anticipated) was, unfortunately, not available at the time the hazard studies were made.

Although a complete quantitative understanding of all of the contributions to the temperature-induced reactivity effects was not achieved, a great deal of assurance regarding nuclear plant safety was obtained. Important results were obtained about the kinetic response of the plant without resorting to period measurements, which can be dangerous to make in the absence of detailed knowledge of the plant characteristics. The experimental verification by oscillator technique of calculations on the transient temperature effects was found to be rewarding despite the limited results obtained in its application.

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in Section

27.2

Chapter 28

ISOTHERMAL TEMPERATURE-COEFFICIENT MEASUREMENTS IN THE PTR

D. J. ANTHONY and A. L. MacKINNEY

28.1 INTRODUCTION

Prior to the construction of the Proof Test Reactor (PTR), it was impossible to obtain experimental information on the reactivity effects associated with temperature changes in a power reactor, other than the fuel Doppler effects. This was primarily because the Preliminary Pile Assembly (PPA) was not designed to simulate power reactor structural geometry in detail. (Descriptions of the PPA and the PTR appear in Chap. 3; detailed discussions of these reactivity effects appear in Chaps. 20 and 30.)

This chapter covers the results of measuring reactivity changes associated with isothermal temperature changes in the PTR. The general approach to the interpretation of these experiments is to subtract from the results obtained those contributions that can either be measured independently or calculated, in order to estimate the contributions that cannot otherwise be determined. Since differences exist between the PTR critical mock-up and the power reactor (a notable one is the absence of sodium in the PTR), care must be taken to predict the temperature coefficient for the power reactor prototype.

28.2 OBSERVATIONS

The isothermal temperature coefficient was measured for two reactor conditions. The reactor composition for each condition is summarized in Table 28.1. Cadmium was used to simulate both xenon and long-lived fission-product poisons in the depleted reactor mock-up.

TABLE 28.1--APPROXIMATE REACTOR CONDITIONS FOR ISOTHERMAL
TEMPERATURE-COEFFICIENT MEASUREMENTS

	Clean	Depleted
Fuel load (U^{235}), kg	~50	~45
Moderator-to-fuel ratio, moles Be/moles U^{235}	135	150
Control-cylinder position, deg	70	138
Cadmium load (distributed), %	0	9.5

The observations consisted of measurements of the positions of the control rods at criticality as a function of equilibrium temperature ranging from 56°F to 86°F. These measurements were made by the use of calibrated control rods. On the basis of previous PTR experiments the critical position was known to within 0.2°.

Temperatures were measured by calibrated resistance-type temperature bulbs located at representative positions throughout the reactor core and reflector. The reactor

temperature was controlled by appropriate adjustment of the air temperature in the cell. The over-all accuracy of the measurement of $\Delta k/\Delta T$ is estimated at ± 10 per cent. Table 28.2 lists the measured reactivity changes and associated temperature changes for each of the two reactor conditions.

TABLE 28.2--MEASURED COMBINED STRUCTURAL EXPANSION, THERMAL BASE, AND DOPPLER EFFECTS

Reactor condition	Temperature change, °F	Reactivity change, β	$\frac{\Delta k}{\Delta T}$, $\beta/^\circ\text{F}$
Clean	$+(16 \pm 1)$ (70 to 86)	-1.7 ± 0.2	-0.106
Depleted	$+(27 \pm 1)$ (56 to 83)	-2.8 ± 0.2	-0.103

In addition to the isothermal temperature-coefficient observations, measurements were made of the distributed reactivity coefficients (RC's) of sodium in the core and in the reflector of the PTR to allow an estimate of the temperature coefficient of the coolant in the power reactor. Similar RC measurements were made on the structural materials to allow an estimate of structural expansion effects. Measurements of Doppler effect were also carried out by the method described in Chap. 33.

28.3 RESULTS

In these PTR experiments the resultant of structural expansion, thermal base effect, and Doppler effect was measured directly by the techniques described above. The temperature coefficient due to sodium expansion was calculated as the sum of two quantities, one for the core, the other for the reflector, each quantity being the following product:

$$\text{Volume of region} \times \text{volume fraction of Na} \times \text{RC of Na} \times d\rho/dT.$$

Here the RC's are the distributed RC's of sodium for the two regions, and $d\rho/dT$ is the rate of change of the density of sodium with temperature, -9.8×10^{-5} mole/cu in./°F, which is equivalent to a volume expansion coefficient of $-1.61 \times 10^{-4}/^\circ\text{F}$.

The net effect of adding the sodium temperature coefficient to the over-all isothermal temperature coefficient is summarized in Table 28.3.

TABLE 28.3--PREDICTED OVER-ALL ISOTHERMAL TEMPERATURE COEFFICIENTS

	Clean	Depleted
Structural, thermal base, and Doppler effects (measured), $\beta/^\circ\text{F}$	-0.106	-0.103
Sodium contribution (calculated), $\beta/^\circ\text{F}$	-0.09	-0.05
Net coefficient predicted by PTR measurements, $\beta/^\circ\text{F}$	-0.196	-0.153

28.4 DEDUCTIONS

Using the Doppler effect evaluated experimentally and the structural expansion effect calculated as indicated below, one may estimate the thermal base effect from the measured isothermal temperature coefficient.

Structural and coolant expansion can be considered as the displacement of a fraction of the material in a region to the outer boundary of that region. The effect of this on the temperature coefficient is, then, for a given reactor material,

$$dk/dT = (1/V)(\Delta V/\Delta T)M(\text{RC}_{\text{distr}} - \text{RC}_x)$$

where $(1/V)(\Delta V/\Delta T) = 3\alpha$, where α is the linear coefficient of expansion of the combination of materials in the reactor

M = moles of the material
 RC_x = reactivity coefficient (ϕ /mole) of the material at surfaces of discontinuity (for the core, at the core-reflector interface, and for the reflector, the outside edge of the reflector)
 RC_{distr} = distributed reactivity coefficient (ϕ /mole) of the reactor material

A small difference in thermal expansion effects between the critical assembly and the power reactor arises because of the differences in details of mechanical construction of the core. In the power reactor, the core dimensions are determined primarily by steel structural components, so the thermal coefficient of expansion for the core can be taken to be equal to that of steel. The mechanical details of the PTR, however, are such that the radial expansion coefficient is that of steel, but the vertical expansion is a weighted mean of the expansion coefficients of the various core materials. This is true because the core was mocked by stacking disks of the various materials into vertical steel tubes, and the stack of disks was free to expand in the vertical direction. However, since there is little difference between the expansion coefficient in the radial and vertical direction, the volume expansion coefficient for the PTR is only slightly different from that for the power reactor.

Table 28.4 gives the various contributions to the isothermal temperature coefficients, including the thermal base effect determined by difference from the net predicted coefficient.

TABLE 28.4--CONTRIBUTIONS TO ISOTHERMAL TEMPERATURE COEFFICIENT (dk/dT)

	Clean	Depleted
Structural expansion (calculated), $\phi/^\circ F$	-0.13	-0.11
Coolant expansion (calculated), $\phi/^\circ F$	-0.09	-0.05
Thermal base effects (deduced), $\phi/^\circ F$	+0.09	+0.08
Doppler effect (calculated from Chap. 33), $\phi/^\circ F$	-0.07	-0.07
Predicted net coefficient (from Table 28.3), $\phi/^\circ F$	-0.20	-0.15

The contribution of $-0.13\phi/^\circ F$ for structural expansion for the clean reactor is to be compared with the value $-0.17\phi/^\circ F$ obtained in Chap. 29 by the multigroup calculation of the same quantity. The Doppler coefficient was calculated from the value 0.15×10^{-4} given for $T dk/dT$ in Table 33.1, on the assumption that this coefficient is inversely proportional to the absolute temperature. This is a kind of average value over the core.

28.5 DISCUSSION

There is reasonable agreement between the predicted clean isothermal temperature coefficient of $-0.20\phi/^\circ F$ based on PTR experiments and the value measured in the S1G, even though the following significant structural differences exist between the PTR and the power reactor: (1) Magnesium is used to mock up sodium in the PTR. (An estimate of the magnesium contribution in the expansion of the structure, however, shows it to be negligible.) (2) The PTR fuel rods consist of disk-type elements rather than S1G-type elements. (However, the calculation carried out above for the expansion effect of the structure, a calculation based on measured reactivity coefficients in the PTR, gives results agreeing reasonably well with the calculation of Chap. 29 on a reactor whose core expands like steel.)

The estimate for the depleted isothermal temperature coefficient, based on experiments in the cadmium-poisoned PTR, indicates a value which is 22 per cent less negative than the coefficient in the clean condition. An uncertainty arises in interpreting this

prediction, however, for only 60 per cent of the fission-product absorptions occur in the cadmium resonance region and are thus properly simulated by cadmium absorptions. Similarly, interpretation of the predicted decrease in thermal base effect in going from the clean to the depleted condition (Table 28.4) is difficult.

Since the temperature coefficient for the depleted poisoned power reactor has not been measured, it has not been possible to make a comparison with the critical-assembly measurements for this reactor condition as a check on the uncertainties just mentioned.

Chapter 29

CALCULATION OF TEMPERATURE COEFFICIENTS RESULTING FROM STRUCTURAL EXPANSIONS

P. L. HOFMANN and R. SIEGEL

29.1 CALCULATIONAL MODEL

The calculation of temperature coefficients resulting from dimensional changes in the reactor would, at first glance, appear to be a difficult task, since most practical reactor geometries are quite complex. However, experience has shown that reasonably accurate estimates can be obtained analytically by using rather simple calculational models to represent the actual reactor.

In the following paragraphs a brief description of some temperature coefficient calculations for an intermediate reactor is presented, and the results are compared with values based on experimental measurements.

To determine the structural expansion coefficients for the reactor, a series of nine 15-group multigroup calculations were performed which represented the reactor in various operational conditions and in various stages of expansion. The reactor was represented in all calculations by means of a three-region spherical model consisting of a core, reflector, and blanket. The dimensions and compositions used in these calculations are tabulated in Table 29.1.

Calculation No. 1 represents the reactor in the clean condition at operating temperature. A uniform amount of boron has been introduced into the reflector region to simulate the poisoning effect of the control cylinders. (A discussion of the method of choosing poison concentrations is given in Sec. 16.5.)

Calculation No. 2 represents the reactor in the depleted condition at high temperature. Since the poison faces of the control cylinders are turned away from the core in this condition, no boron concentration has been introduced into the reflector region. However, the core has been poisoned with xenon and boron, and the core fuel load has been reduced to account for fuel burnup. The core boron concentration is used to simulate the accumulation of fission products other than xenon.

Calculations No. 3 to 6 represent reactors with expanded cores; constant reflector and blanket thicknesses have been retained. In particular, calculation No. 3 and 4 respectively represent clean and depleted cores, 1 per cent expanded. Calculations No. 5 and 6 respectively represent clean and depleted cores, 2 per cent expanded.

In calculations No. 7 to 9 all regions (core, reflector, and blanket) have been expanded 1 or 2 per cent. Calculation No. 7 represents a 1 per cent expanded, clean reactor. Calculation No. 8 represents a 1 per cent expanded, depleted reactor and calculation No. 9 represents a 2 per cent expanded, depleted reactor.

A comparison of the calculated multiplication factors for these reactors makes it possible to deduce the temperature coefficient due to structural expansion.

29.2 RESULTS OF CALCULATIONS

The effect of expanding the core region alone and of expanding all regions including the core is shown graphically on Fig. 29.1. It is evident that the contribution from core expansion is generally about 80 per cent of the total. The results may be expressed in the form $\Delta k/k = \gamma \Delta V/V$; values of γ for a 1 per cent expansion are listed in Table 29.2.

TABLE 29.1--REACTOR SIZES AND COMPOSITIONS

	Calculation No.								
	1	2	3	4	5	6	7	8	9
Reactor size									
Core radius, cm	40.0	40.0	40.133	40.133	40.267	40.267	40.133	40.133	40.267
Reflector radius, cm	58.28	58.28	58.413	58.413	58.547	58.547	58.474	58.474	58.669
Blanket radius, cm	83.28	83.28	83.413	83.413	83.547	83.547	83.558	83.558	83.835
Reactor composition (nuclei/cc)									
Core									
U ²³⁵ ($\times 10^{24}$)	0.0004600	0.0003983	0.0004554	0.0003944	0.0004509	0.0003905	0.0004554	0.0003944	0.0003905
Be ($\times 10^{24}$)	0.0632	0.0632	0.06257	0.06257	0.06196	0.06196	0.06257	0.06257	0.06196
Na ($\times 10^{24}$)	0.00743	0.00743	0.007356	0.007356	0.007284	0.007284	0.007356	0.007356	0.007284
*SS ($\times 10^{24}$)	0.00702	0.00702	0.006950	0.006950	0.006882	0.006882	0.006950	0.006950	0.006882
B ^N ($\times 10^{19}$)	-	3.5512	-	3.5160	-	3.4811	-	3.5160	3.4811
Xe ($\times 10^{16}$)	-	1.0000	-	0.99009	-	0.98027	-	0.99009	0.98027
Reflector									
Be ($\times 10^{24}$)	0.0931	0.0931	0.0931	0.0931	0.0931	0.0931	0.09218	0.09218	0.09127
Na ($\times 10^{24}$)	0.00398	0.00398	0.00398	0.00398	0.00398	0.00398	0.00394	0.00394	0.003902
SS ($\times 10^{24}$)	0.00380	0.00380	0.00380	0.00380	0.00380	0.00380	0.003762	0.003762	0.003726
B ^N ($\times 10^{24}$)	0.000207	-	0.000207	-	0.000207	-	0.000207	-	0.000207
Blanket									
Na ($\times 10^{24}$)	0.00450	0.00450	0.00450	0.00450	0.00450	0.00450	0.004455	0.004455	0.004412
SS ($\times 10^{24}$)	0.0679	0.0679	0.0679	0.0679	0.0679	0.0679	0.06723	0.06723	0.06657

*SS = Stainless Steel

TABLE 29.2--STRUCTURAL EXPANSION TEMPERATURE COEFFICIENTS FOR A 1 PER CENT EXPANSION IN VOLUME

	$\gamma = \frac{\Delta k/k}{\Delta V/V}$	$\Delta k/k$ per $^{\circ}\text{C}$	$\phi/^{\circ}\text{F}$
Clean reactor			
Core expanded	-0.41	-2.2×10^{-5}	-0.14
All regions expanded	-0.50	-2.6×10^{-5}	-0.17
Depleted reactor			
Core expanded	-0.56	-3.0×10^{-5}	-0.20
All regions expanded	-0.66	-3.5×10^{-5}	-0.23

To obtain temperature coefficients it is necessary to relate expansion and temperature. This is not a simple relationship, since the structure is not homogeneous and various components have different thermal expansion coefficients. In order to make a simple estimate, it is assumed, as discussed in Sec. 28.4, that the expansion is governed by the steel structure with a coefficient of volume expansion equal to $dV/VdT = 52.9 \times 10^{-6}/^{\circ}\text{C}$.

Fig. 29.1—Reactivity effects of expansions.

REFERENCE

A 1 per cent volume expansion then requires a very large temperature increase. Since during most maneuvers the structural temperature change will not exceed this value, Table 29.2 includes only a 1 per cent expansion. Results for a 2 per cent expansion can be obtained from Fig. 29.1.

During testing of the reactor, an isothermal-temperature-coefficient determination was made. Since the isothermal temperature coefficient consists of various components, it is not easily possible to deduce what contribution was made by structural expansion alone. However, by estimating the contribution of each component it is possible to calculate a value comparable with the measured result. The results are reproduced in Table 29.3.

TABLE 29.3--REACTOR TEMPERATURE COEFFICIENTS
(CLEAN CONDITION)

Expansion effects	$\zeta/^{\circ}\text{F}$
Structure	-0.17
Coolant	-0.09
Thermal base effects	
Core	+0.01
Reflector	+0.09
Doppler effect	<u>-0.04</u>
Net effect	-0.20

Since the value of 0.20 $\zeta/^{\circ}\text{F}$ agrees reasonably well with the measured value, and since the expansion effect is one of the major contributors to the over-all isothermal coefficient, the methods used to estimate structural expansion temperature coefficients appear to have been reasonably sound.

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29.2

Chapter 30

TEMPERATURE AND POWER COEFFICIENTS IN THE POWER REACTOR

F. G. LaVIOLETTE

30.1 INTRODUCTION

Temperature changes in a reactor can alter its reactivity both through thermal expansion of the components and by alteration of conditions for interaction of neutrons with the material of each component. The temperature coefficient of a reactor is an expression of the reactivity change produced by a uniform incremental temperature change.

The temperature changes in a power reactor operating at high-power levels are not necessarily uniform; they depend upon the power level, the coolant properties and flow rates, and the thermal properties of the various reactor materials. Here an incremental change in power level may produce more significant temperature changes in some components than in others; therefore the reactivity coefficient of power level is not related to the temperature changes in any simple fashion.

The power coefficient of a reactor is an expression of the reactivity change produced by an incremental change in power level occurring under specified conditions of operation.

30.2 TEMPERATURE-COEFFICIENT MECHANISMS

There are several basic mechanisms through which temperature changes in a sodium-cooled reactor can alter the reactivity. Chief among these are (1) thermal expansion of the core structure, (2) thermal expansion of the sodium, (3) thermal base effect in the reflector and in xenon mechanisms, and (4) Doppler broadening of fuel resonances.

These effects have been already discussed to some extent in Chap. 27. The effects of thermal expansion of reactor structure and coolant are analyzed in Chaps. 28 and 29. Doppler broadening is treated in Chaps. 32 and 33. The thermal base effect is analyzed in Chap. 31. However, since the work of the present chapter throws particular light on the variation of the thermal base effect with xenon concentration, some further discussion is appropriate at this point.

The contribution to the thermal base effect which arises from temperature changes in the reflector results from changes in competition between scattering and absorption in the reflector. If the reflector temperature increases and thereby increases the average energy of the thermal neutrons, the cross-sections for absorption in the stainless steel and in the beryllium tend to decrease while the cross-section for scattering remains practically constant. The net effect is an increase in reactivity with an increase in temperature.

The other important thermal base effect results from competition between U^{235} and xenon neutron absorption. Since Xe^{135} has a strong resonance in the thermal energy region, its absorption cross-section variation with energy is quite different from that of U^{235} . In intermediate reactors a significant fraction of the neutrons have energies above the xenon resonance. In this region the xenon cross-section decreases with increasing neutron energy more rapidly than does the fuel cross-section. Therefore an increase in temperature tends to increase absorption (and hence fissions) in U^{235} relative to absorp-

tion in xenon. Thus an increase in temperature increases the reactivity, which means that this part of the thermal base effect also contributes positively toward the temperature coefficient.

Predicted values of the components of the temperature coefficient of the power reactor are given in Table 30.1. These values are based on calculation or on critical-assembly measurements, as indicated in each case.

TABLE 30.1--COMPONENTS OF TEMPERATURE COEFFICIENT

Temperature-coefficient component	Magnitude of component, $\zeta/^{\circ}\text{F}$			Source
	Fresh reactor, no xenon	Fresh reactor, peak xenon	Depleted reactor, peak xenon	
Structural expansion	-0.17	-0.17	-0.23	Calculation (Chap. 29)
Sodium expansion	-0.09	-0.09	-0.05	Measurement and Calculation (Chap. 28)
Reflector thermal base effect	+0.10	+0.10	+0.16	Calculation*
Xenon thermal base effect	0	+0.04	+0.04	Calculation*
Doppler effect	-0.04	-0.04	-0.04	Measurement (Chap. 33)
Resultant predicted temp. coefficient, $\zeta/^{\circ}\text{F}$	-0.20	-0.16	-0.12	

* The calculation of the thermal base effects was carried out in 1951 by H. Hurwitz, Jr., using methods discussed briefly in a series of Progress Reports.¹⁻⁴

30.3 TEMPERATURE-COEFFICIENT MEASUREMENTS

Measurements of the reactivity effects of temperature which have been made in the power reactor under steady-state conditions include (1) isothermal temperature coefficient, (2) steady-state temperature coefficient at power, and (3) steady-state power coefficient (Sec. 30.4).

The methods employed and some of the measured results are discussed below. The measurement of transient power coefficients in the reactor is discussed in Chaps. 35 and 36.

A. Isothermal Temperature Coefficient

An isothermal temperature coefficient refers to the reactivity change associated with a uniform increase in core temperature under conditions of zero temperature gradient. This temperature change can be accomplished in the reactor by electrical heating of the sodium coolant in a manner sufficiently gradual that the temperatures of all reactor components remain in equilibrium. A low-power level (below 500 kw) is maintained so that no significant temperature gradients are produced in the fuel elements or other components. Reactivity changes are determined from the change in position of previously calibrated control elements as the control-element position is adjusted to maintain a constant reactor-power level.

B. Over-all Temperature Coefficient at Operating Power Levels

When the temperature coefficient is measured with the reactor at power levels in the normal operating range, the temperature coefficient that is measured is a non-isothermal one. In this case, the power level and sodium flow rate are held constant while the temperature of the inlet sodium is altered. Although the reactor is not at the same temperature throughout, the spatial temperature distribution will remain approxi-

mately the same at different inlet temperatures, so the changes in temperature are nearly the same at all parts of the reactor core.

This measurement of the over-all temperature coefficient as a function of xenon poison concentration yields information on the thermal base effect. The results have verified reasonably well the prediction for the xenon portion of the thermal base effect given in Table 30.1.

C. Steady-state Power Coefficients

Steady-state power coefficients are obtained by observing the reactivity difference between two equilibrium conditions obtained with the reactor operating in the megawatt power range. Here again the reactivity change is measured by the change in position of a calibrated control element as that control element is adjusted to change the power level from one equilibrium value to another.

In the normal procedure for changing the power level the coolant flow rate is adjusted to maintain a constant temperature rise across the reactor (ΔT). The average reactor sodium temperature changes in this maneuver, for the temperature of the sodium leaving the steam generator rises somewhat with increases in the rate of heat transfer.

Reactivity changes accompany increases in power level as a result of (1) an over-all increase in reactor temperature and (2) an increase in the temperature difference between fuel and sodium. The reactivity change arising from the over-all temperature increase is

$$\delta k_T = \delta T_{Na} (C_T + C_{Xe} \$_{Xe})$$

where δT_{Na} = the change in the average sodium temperature
 C_T = the isothermal temperature coefficient
 C_{Xe} = the xenon thermal base effect per dollar of xenon
 $\$_{Xe}$ = the reactivity value of the xenon present in the reactor

The reactivity change connected with the excess of fuel temperature over coolant temperature is then, by difference

$$\delta k_F = \delta k_{\text{measured}} - \delta k_T$$

The fuel power coefficient can be expressed as the fuel reactivity change per unit change in power level, $\delta k_F / \delta P$, in cent per megawatt.

The data and results of steady-state power coefficient measurements on the plant are shown in Table 30.2.

The accuracy of the steady-state power coefficient measurements is limited by the following factors:

1. The reactivity changes are usually small (from 5 to 20¢), and short-term changes in the xenon concentration can cause serious errors.
2. Measurements of sodium temperature changes which may range between 20 and 40°F are subject to considerable error, partly because of reading and instrument errors and partly because of long-period transient fluctuations inherent in a large high-heat-capacity system.
3. Significant errors are introduced owing to the difficulty of obtaining steady steam conditions.

The temperature coefficients based on observed values of reactivity and temperature change may be uncertain by 10 to 20 per cent. The probable error to be associated with the power coefficients given in Table 30.2 ranges from 25 to 50 per cent. The relatively larger uncertainty in the power coefficients results from the fact that only 25 to 45 per cent of the observed reactivity change is attributable to the fuel temperature change (see last equation above).

TABLE 30.2--STEADY-STATE POWER COEFFICIENTS

Parameter held constant	Flow	ΔT	ΔT	ΔT	ΔT	ΔT	ΔT	ΔT
$\$_{Xe}$: xenon defect, \$	0.4	0.8	0.5	0.2	0.8	0.5	1.0	0.6
Power level range, %	14-25	13-28	21-37	20-40	28-54	37-51	54-70	53-82
Fuel power coefficient, $\phi/Mw (\delta k_p/\delta P)$	-0.33	-0.40	-0.38	-0.34	-0.41	-0.43	-0.28	-0.23

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Chapter 31

REACTOR THERMAL SPECTRUM

M. S. NELKIN and R. S. STONE

31.1 INTRODUCTION

The spectrum of an intermediate reactor is adequately described by the multigroup calculations (see Chap. 18) except for neutron energies near thermal. Since for these energies the spectrum is known to be position-dependent, a complete multigroup description should include several energy groups in the thermal energy region which are coupled together in a complicated manner due to the possibility of neutrons speeding up as well as slowing down in collisions with the moderator.

The situation is quite different in the case of a thermal reactor, where the neutrons in the energy range from zero to about 4 kT have approximately the same energy distribution independent of position in the reactor. This allows the use of a single thermal group with an almost Maxwellian distribution of neutron velocities arising from the near approach to thermal equilibrium permitted by the large number of scattering collisions before absorption.

Preliminary attempts were made to include a multigroup formulation of the thermal energy region for intermediate reactors, but the difficulty of the problem, combined with its lack of decisive importance, precluded the development of these techniques for use in actual reactor design calculations. It is appropriate to review here the procedure actually employed and to point out its inadequacies. In criticality calculations the procedure is probably adequate; for the key phenomenon, the competition between fuel absorptions and parasitic captures is not very sensitive to the shape of the thermal portion of the spectrum. There is some uncertainty as to the effective xenon cross-section, but the most important inadequacy is in the calculation for the thermal-diffusion length in the reflector, which is important in determining the control-cylinder effectiveness.

In the multigroup calculations, within the framework of a single thermal-group formulation, two approximations have been made which are inadequate. The first has been the assumption of a Maxwellian distribution of neutron energies for the thermal group, and the second has been the use of an imprecise definition of the "thermal-group" energy. In this definition a particular energy E_{th} has been used both as the lower boundary of the group just above thermal and as the energy at which cross-sections should be evaluated to give an approximation to the appropriate average cross-sections for the thermal group. The energy E_{th} was chosen so that the value of a $(1/v)$ cross-section at E_{th} would be the same as its average value over a Maxwellian distribution characteristic of the moderator temperature. A more serious question is the definition of the thermal-group boundary. To be consistent with neutron conservation, this should be chosen so that the thermal group contains the entire Maxwellian spectrum and not just the portion from zero to E_{th} . It should also be chosen so that scattering up into the lowest epithermal group from the thermal group can be neglected. This means that the thermal-group boundary, which represents the source energy for the thermal group, should properly be at some energy considerably greater than E_{th} , and that thermal cross-section averages should be taken over an energy region with this upper boundary.

Since a correct treatment of the thermal-energy region is so difficult, recent work at KAPL has been directed at a more fundamental understanding of the problem. Experimental and theoretical work has been carried out on the detailed thermal spectrum in the central region of various reactors. The results of this work are treated in this chapter as they relate to the PPA-20 reactor. The experimental results will be covered first, and then their interpretation. Finally the applications to intermediate reactors will be discussed qualitatively.

31.2 SPECTRUM MEASUREMENTS

The experimental approach used by Stone and Slovacek¹ was to measure the spectrum at the center of a subcritical assembly of the desired composition which had been placed in a thermal column of the KAPL Thermal Test Reactor.² A diagram of the equipment is given in Fig. 31.1.

Fig. 31.1—Experimental equipment arrangement for spectrum investigations.

The assembly was composed of 35 slugs, each 2 in. in diameter and 16 in. long. These slugs were composed of disks of fuel-aluminum alloy, beryllium, steel, and magnesium. The composition of this assembly, which had the same composition as PPA-20, is given in Table 31.1. The areas between the slugs were also filled with beryllium. The central slug in this array was only 8 in. long; the missing portion of this slug formed the re-entrant hole for the beam. The fission plate consisted of several enriched uranium disks placed on the internal end of each of the 35 slugs.

TABLE 31.1--COMPOSITION OF INTERMEDIATE ASSEMBLY
USED FOR SPECTRUM EXPERIMENTS

Material	Stainless steel	Mg	Al	Be	U ²³⁵
Atoms/cm ³ × (10 ⁻²⁴)	0.00689	0.00658	0.00797	0.0628	0.000457
Atoms/atom U ²³⁵	15	14	17	137	1

For this composition, $\sum_a^{\text{eff}}(kT)/\xi \sum_s = 2.45$

An observation hole (the re-entrant hole), penetrating to the center of the assembly from the outer face, served as a beam channel for neutrons emitted from the end of the channel. The end of the re-entrant hole was located more than three diffusion mean free paths away from any surface of the assembly. This hole was made small in order to

minimize the magnitude of its perturbation on the central flux, yet sufficiently large that the beam source was characteristic of the average core composition rather than that of individual core constituents.

The neutron beam was brought through the thermal column shield by a 1-meter steel collimator. The effect of inscattered neutrons from the sides of the collimator was less than 1 per cent. This was demonstrated experimentally by measuring the velocity spectra with and without a black boron absorber covering the neutron source area at the bottom of the re-entrant hole.

A flat plate neutron chopper was placed directly in front of the collimator exit. The function of this chopper was to divide the emergent beam into a series of short time bursts. To do this, not only did the chopper have to provide a large ratio of opened-to-closed transmission over the energy range to be investigated, but the transmission of higher energy neutrons had to be small in order to minimize background effects with hardened spectra. After a flight of 3 meters from the center line of the rotor, the time-chopped beam impinged on a BF_3 -filled proportional counter which served as a detector. The higher energy neutrons in a given time burst require a shorter time to traverse the distance from the shutter to the counter than the lower energy neutrons. It was therefore possible to deliver the output pulses from the detector to a time channel which would separate the pulses according to the corresponding neutron energies.

Background corrections to the observed counting rate data were determined. The resultant signal counting rate data were corrected for the energy dependence of shutter transmission and efficiency. A third energy-dependent correction relating the energy distribution of neutrons in the beam to the flux at the source region was also applied to these data. The corrected data could then be plotted in terms of relative flux per unit energy at the source region.

A series of activation experiments with bare and cadmium-covered fuel and resonance absorber strips was performed in this assembly. These experiments indicated that the spectrum was well stabilized over the central region of the assembly; they also provided a measure of the assembly buckling and the flux gradients in the beam source region. In addition, activation experiments were performed with a full central slug. The results of this series of experiments were compared to those with the half slug to show that the perturbation caused by the re-entrant hole was small.

The experimental result of the spectrum determination for an assembly temperature of 250°C is shown in Fig. 31.2. This spectrum is hardened sufficiently that there is no longer any semblance of a Maxwellian distribution of neutron energies. The dash-dot curve in this figure is the theoretically predicted spectrum calculated by Nelkin³ on the basis of the cross-sections of the assembly materials. The spectrum determination at 260°C is not shown since the observed spectrum was not significantly different from that obtained at 250°C .

31.3 INTERPRETATION OF EXPERIMENTS

No attempt will be made here to go into the details of the spectrum calculation, but a discussion will be given of the physical basis of the calculations. In the central portion of the assembly, the observed spectrum should be approximately independent of position since the removal of slow neutrons is primarily due to absorption rather than to leakage. The problem can therefore be reduced to a spectrum calculation for an equivalent infinite medium. The greatest uncertainty arises from the self-shielding factors needed to determine the effective absorption cross-section of the fuel elements. Once this cross-section has been estimated, the problem is reduced to the calculation of the spectrum in an infinite homogeneous mixture of absorber and heavy crystalline moderator. This calculation has been performed,⁴ and it yields the dash-dot curve of Fig. 31.2.

Consider now the comparison between theory and experiment. Two important qualitative features are correctly predicted. First, the flux per unit energy, $\phi(E)$, has no Maxwellian peak at 0.025 eV, but a broad maximum between 0.2 and 0.5 eV. Second, the spectrum is found to be approximately independent of temperature, both theoretically (see Fig. 31.3) and experimentally. There appears to be a discrepancy between the predicted and observed low-energy tail of the spectrum. The cause of this discrepancy has not been isolated; however, there are two possible explanations. The first is that experimentally there are very few neutrons in the low-energy region relative to those in the

Fig. 31.2—The intermediate reactor central spectrum at temperature $T = 25^\circ\text{C}$.

higher neutron-energy regions. Therefore the resultant value assigned to the flux at these low energies is much more dependent on the background counting rates than are those values which are assigned to higher neutron energies. A small error in the background determination would then be highly emphasized in the low-energy region. The second possible explanation is that the theoretical calculation is based on neutron conservation in each successively lower energy region. Therefore a small fractional error in the computation of neutron conservation at high energies would result in a large perturbation to the spectrum at low energies where the number of low-energy neutrons is small.

The importance of neutron conservation is not shown by the plot of $\phi(E)$ against $\log E$ in Fig. 31.2, wherein the area under the curve does not represent total integrated flux. This importance is shown, however, by the area in Fig. 31.3, wherein the theoretical curves for the flux per unit lethargy, $E\phi(E)$, are plotted against $\log E$ for $T = 25^\circ\text{C}$ and for $T = 260^\circ\text{C}$. This plot illustrates two important points. First, it is seen more directly that the Maxwellian peak has completely disappeared; for $E\phi(E)$ has become a monotonically increasing function of E . Second, it is seen that the total integrated flux below 0.03 eV is quite small, so that the disagreement between theory and experiment is of little significance in the determination of integral quantities such as average cross-sections.

31.4 APPLICATIONS TO SIR-TYPE REACTORS

If the thermal spectrum in the central part of the core were characteristic of the entire reactor, the problem of the thermal spectrum would be essentially solved, and the observed temperature independence of the spectrum would lead to the conclusion that "thermal base effects" on the temperature coefficient are negligible. However, as was pointed out in Chap. 30, this is not the case; for an important fraction of the thermal neutrons are thermalized in the reflector. The slow neutron spectrum will therefore be dependent on position, and will have an increasing temperature dependence as one goes away from the center of the core. Since experiments on the spectrum have been done only for the central region, and since space-dependent thermal spectra are not quanti-

Fig. 31.3—Theoretical calculations of PPA-20 central spectra.

tatively understood even in simple geometries, the remaining discussion of the thermal spectrum will be necessarily qualitative.

One important point is the dependence of the spectrum on the degree of fuel depletion. The clean reactor has its excess reactivity compensated by the turning in of the boron faces of the control cylinders. This reduces the number of thermal neutrons returned to the core. The fraction of sub-cadmium fissions in the PPA-20 is 10 per cent at the core center, increasing to 40 percent at the core-reflector interface, with a core average of 20 per cent. The central spectrum is temperature independent, and the thermal spectrum is everywhere considerably hardened. As the reactor becomes depleted, the effective boron content of the reflector is decreased, so that the thermalization of neutrons in the reflector and their return to the core is increased. The fraction of sub-cadmium fissions at the outside of the core will therefore increase. (Insofar as the variation of criticality with burnup is concerned, these effects can be calculated fairly reliably by the multigroup calculations.)

To understand the thermal base effects on the temperature coefficient discussed in Chap. 30, it is necessary to know the temperature dependence of the average spectrum in the reflector and in the core. It is clear that the reflector effect will increase as the reactor depletes; for more neutrons are thermalized in the reflector, and their spectrum becomes more sensitive to temperature, as the thermalization becomes more effective. Similarly, the xenon coefficient must increase since it is due primarily to the neutrons thermalized in the reflector; those in the central part of the core have a temperature-independent spectrum. Since both of these contributions to the thermal base temperature coefficient are positive, it can be concluded that this positive component of the temperature coefficient will increase during the course of the reactor lifetime. This, along with the rough estimates of Chap. 30, is all that can be said at present about the application of our knowledge concerning the spectrum of intermediate reactors.

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Chapter 32

THE CALCULATION OF DOPPLER TEMPERATURE COEFFICIENTS

L. G. BARRETT

32.1 GENERAL PRINCIPLES

The method that was used to attempt to calculate the Doppler coefficient for the intermediate reactor is described in this chapter. Since there was a fear that the Doppler coefficient for the fuel elements might be positive, considerable effort was exerted to predict this effect.

Because of the sensitivity of the results to the capture-to-fission ratio, α , in the resonances, there is an uncertainty, even as to sign, in the theoretical predictions for the Doppler effect in U^{235} . The sensitivity to α arises because the net effect is a relatively small difference between positive and negative effects due to unshielding of fission and capture cross-sections, respectively, as the resonance is broadened. Thus there exists for each resonance a critical value of α for which the net contribution of that resonance to the Doppler coefficient vanishes. For many of the important U^{235} resonances, there is uncertainty as to whether the measured α is greater or less than this critical value, and this leads to the ambiguity of the net contributions of that particular resonance to the over-all Doppler coefficient. Recent improvements in the resonance parameter data have tended to reduce the discrepancy between the calculated Doppler coefficient, summed over all resonances, and experiment.

The calculation of that portion of the temperature coefficient of reactivity that is due to Doppler broadening of resonances for an intermediate reactor can be performed by two methods. The first requires the application of perturbation theory and methods developed by Hurwitz¹ for the calculation of reactivity coefficients. This will be referred to as the adjoint method. For the second method, which is in principle a direct (i.e., not a perturbation) method,² one obtains the partial temperature derivatives of the parameters appearing in an integral equation for the multiplication factor of the reactor.

The direct method is exact only for a homogeneous bare reactor; to apply it to reactors of interest information obtained from a multigroup calculation is used. It is intrinsically incapable of indicating how the contribution of an individual fuel element depends upon the position of the fuel element in the reactor--a dependence that is quite important in order to obtain comparisons with experiments performed in local regions of the core. Therefore this chapter is for the most part devoted to the adjoint method.

The investigation of temperature coefficient is based on the premise that a reactivity change is produced by the change in resonance self-shielding when the resonance undergoes Doppler broadening. For any isolated resonance, the temperature coefficient will be proportional to $\bar{\sigma}^* \partial \bar{f} / \partial T$, where $\bar{\sigma}^*$ is the unshielded cross section and $\bar{f}$ is the self-shielding factor, both averaged over the Breit-Wigner expression for the resonance. A semi-analytic method for obtaining the self-shielding factor temperature derivative for slab geometry was developed by Roe.³⁻⁵ He also outlines a recipe to follow in obtaining the derivative for other geometries in terms of the derivative for the slab.

This recipe can be used in the perturbation expression for the reactivity coefficient, which will now be derived. This will then be applied to a spherical reactor model in order to obtain numerical results.

32.2 EXPRESSION FOR REACTIVITY COEFFICIENT

The reactivity change due to the insertion of a neutron absorber into a reactor¹ is given by

$$\frac{\Delta k}{k} = - \frac{\int_V \int_0^\infty \nu \delta \Sigma_A F(\underline{r}, E) dE dV}{\nu \int_V \int_0^\infty \Sigma_F F_H(\underline{r}) dE dV} \quad (32.1)$$

Here the integration is over the neutron energy E and the position variable $\underline{r}$;

dV = the element of volume

V = the volume of the reactor core

$\delta \Sigma_A$ = the change in macroscopic absorption cross-section, which is a function of position and is, of course, zero outside the volume of the inserted absorber

$F(\underline{r}, E)$ = the iterated fission probability for neutrons of energy E , at position $\underline{r}$

$F_H(\underline{r})$ = the iterated fission probability for fission neutrons

ν = the number of neutrons per fission

νF = the neutron flux, a function of both $\underline{r}$ and E

Also Σ_F = the macroscopic fission cross-section, is a function of position

$m(\underline{r}, E)$ = the adjoint function

$m_H(\underline{r}) \equiv \int_0^\infty m(\underline{r}, E) \chi(E) dE$ is the average of $m(\underline{r}, E)$ over the fission spectrum $\chi(E)$, where $\int_0^\infty \chi(E) dE = 1$

The iterated fission probability is related to the adjoint function by

$$F(\underline{r}, E) = m(\underline{r}, E) \left[\frac{\int_V \int_0^\infty \nu \Sigma_F dE dV}{\nu \int_V \int_0^\infty \nu m_H(\underline{r}) \Sigma_F dE dV} \right] \quad (32.2)$$

The proportionality factor in the bracket is just a normalization constant henceforth denoted as (NC). The multigroup calculations which give the adjoint function and neutron flux are normalized to a single source neutron per cubic centimeter.

The normalized number of fissions per cubic centimeter ($1/\nu$ or 0.40 at critical) is given by

$$F = \frac{\nu}{V} \int_V \int_0^\infty \nu \Sigma_F F_H(\underline{r}) dE dV \quad (32.3)$$

By taking the range of integration from $E = E_{th}$ (thermal energy) to $E = 10^7$ ev, converting to the lethargy variable $u = \ln(10/E)$, and substituting Eqs. 32.2 and 32.3 into Eq. 32.1, one finds the approximate relation

$$\frac{\Delta k}{k} = - \frac{(NC)}{FV} \int_V \int_0^{u_{th}} \nu(\underline{r}, u) m(\underline{r}, u) \delta \Sigma_A du dV \quad (32.4)$$

When the reactivity change is expressed in dollars per mole of an absorber which is distributed uniformly throughout the core, and Avogadro's constant as 0.6023×10^{24} atoms per mol is used, Eq. 32.4 becomes

$$\frac{\Delta k}{k} (\$) = - \frac{(NC)}{\beta FV} \left(\frac{0.6023}{V} \right) \int_V \int_0^{u_{th}} \nu(\underline{r}, u) m(\underline{r}, u) \sigma_A(u) du dV \quad (32.5)$$

Here β is the delayed neutron fraction, and σ_A is the effective microscopic absorption cross-section of the inserted absorber.

In spherical geometry, with the range of lethargy integration broken up into groups with index i , the reactivity becomes⁸

$$\frac{\Delta k}{k} (\$) = - \frac{(NC)}{\beta F V} \left(\frac{0.6023}{V} \right) 4\pi \sum_1 \bar{\sigma}_{A1} \int_0^{r_{\text{core}}} \bar{n v}_1(r) \bar{m}_1(r) r^2 dr \Delta u_1 \quad (32.6)$$

Here Δu_1 is the group width, and $\bar{\sigma}_{A1}$, $\bar{n v}_1(r)$, and $\bar{m}_1(r)$ are average values for the i th group.

It simplifies the analysis to suppose that the product $\bar{n v}_1 \bar{m}_1$, here denoted by ϕ_1 , depends linearly on $1/r$ between the two values of ϕ_1 (ϕ_{11} and ϕ_{12}) at two given values of r (r_1 and r_2 , the inner and outer radii of a spherical shell of thickness $\Delta r = r_2 - r_1$) and that ϕ_1 is a constant within the smallest shell. For sufficiently thin shells, no appreciable error will be introduced. Then one has the following interpolation formula for points in the shell:

$$\phi_1 = \frac{r_2 r_1}{r_2 - r_1} (\phi_{11} - \phi_{12}) \frac{1}{r} + \frac{r_2 \phi_{12} - r_1 \phi_{11}}{r_2 - r_1} \quad (32.7)$$

Using N concentric shells to fill the core (the typical shell having outer radius r_n):

$$\int_0^{r_{\text{core}}} \phi_1 r^2 dr = \Delta r \left[\sum_{n=0}^N \phi_{1n} r_n^2 - \phi_{1N} r_N \left(\frac{r_N}{2} + \frac{\Delta r}{6} \right) \right] \quad (32.8)$$

For a set of shells all of the same thickness Δr , $r_n = n\Delta r$ and

$$\Delta u_1 \int_0^{r_{\text{core}}} \bar{n v}_1(r) \bar{m}_1(r) r^2 dr = (\Delta r)^3 \left[\sum_{n=0}^N n^2 \bar{n v}_1(n) \bar{m}_1(n) \Delta u_1 - N \bar{n v}_1(N) \bar{m}_1(N) \left(\frac{N}{2} + \frac{1}{6} \right) \Delta u_1 \right] \quad (32.9)$$

The term in the brackets is denoted as D_{1A} .

Substituting Eq. 32.9 and $V = (4\pi/3) (\Delta r)^3 N^3$ one has, finally,

$$\frac{\Delta k}{k} (\$) = - \frac{(NC)}{\beta F} \left(\frac{3}{N^3} \right) \left(\frac{0.6023}{V} \right) \sum_1 \bar{\sigma}_{A1} D_{1A} \quad (32.10)$$

The reactivity change due to a change in the macroscopic fission cross-section¹ is analogous to Eq. 32.1:

$$\frac{\Delta k}{k} = \frac{\nu \int_V \int_0^\infty n v \delta \Sigma_F F_H(r) dE dV}{\nu \int_V \int_0^\infty n v F_H(r) dE dV} \quad (32.11)$$

Applying a treatment similar to that which led to Eq. 32.9, one obtains

$$\frac{\Delta k}{k} (\$) = \frac{(NC)}{\beta F} \cdot \left(\frac{3}{N^3} \right) \left(\frac{0.6023}{V} \right) \nu \sum_1 \bar{\sigma}_{F1} D_{1F} \quad (32.12)$$

where D_{1F} is defined as

$$D_{1F} \equiv \sum_{n=0}^N n^2 \bar{n v}_1(n) \bar{m}_H(n) \Delta u_1 - N \left(\frac{N}{2} + \frac{1}{6} \right) \bar{n v}_1(N) \bar{m}_H(N) \Delta u_1 \quad (32.13)$$

32.3 EXPRESSION FOR DOPPLER TEMPERATURE COEFFICIENT

To calculate the reactivity change per degree centigrade produced by Doppler broadening of a resonance, $\partial \bar{\sigma} / \partial T$ with the proper subscripts to denote absorption or

32.3 EXPRESSION FOR DOPPLER TEMPERATURE COEFFICIENT

fission is substituted for $\bar{\sigma}_{A1}$ or $\bar{\sigma}_{F1}$ in Eq. 32.10 or 32.14. Combining these equations, and summing both over the groups (i) and over the resonances (j) to obtain the total temperature coefficient, one finds

$$\frac{1}{k} \frac{dk}{dT} = \frac{1}{\beta} \left(\frac{3}{N^3} \right) \frac{(NC)}{F} \left(\frac{0.6023}{V} \right) \sum_i \sum_j \left(\bar{\sigma}_{Fj} D_{iF} - \bar{\sigma}_{Aj}^* D_{iA} \right) \frac{\partial \bar{f}_j}{\partial T} \quad (32.14)$$

The following relations now come into play:

$$\bar{\sigma}_{Fj}^* = \frac{\bar{\sigma}_{oj}^*}{1 + \alpha_j} \quad (32.15)$$

$$\bar{\sigma}_{oj}^* = \frac{\pi}{2} \frac{\sigma_{oj} \Gamma_j}{E_{oj}} \frac{1}{\Delta u_i} \quad (32.16)$$

In the above expressions, α_j , σ_{oj} , Γ_j , and E_{oj} are, respectively, the capture-to-fission ratio, peak total cross-section, total width, and energy, of the jth resonance.

Equation 32.14 becomes

$$\frac{1}{k} \frac{dk}{dT} = \frac{1}{\beta} \left(\frac{3}{N^3} \right) \frac{(NC)}{F} \frac{0.6023}{V} \sum_i \sum_j \left(\frac{\nu D_{iF}}{1 + \alpha_j} - D_{iA} \right) \left(\frac{\pi}{2 \Delta u_i} \frac{\sigma_{oj} \Gamma_j}{E_{oj}} \right) \frac{\partial \bar{f}_j}{\partial T} \quad (32.17)$$

From Eq. 32.17 it is seen that a resonance will contribute zero to the temperature coefficient if $\alpha_j = \left(\frac{\nu D_{iF}}{D_{iA}} - 1 \right)$. This value of α_j we denote by α_{crit} . It is the critical

value of α for a resonance. For $\alpha_j > \alpha_{crit}$, the resonance contributes in the direction of a negative coefficient; for $\alpha_j < \alpha_{crit}$, the resonance contributes toward a positive coefficient.

Equation 32.17 may be rewritten in the form

$$\frac{1}{k} \frac{dk}{dT} = \frac{1}{\beta} \left(\frac{3}{N^3} \right) \frac{(NC)}{F} \frac{0.6023}{V} \sum_i \frac{D_{iA}}{\Delta u_i} \sum_j \frac{\pi}{2} \left[\frac{\sigma_{oj} \Gamma_j}{E_{oj}} \frac{\partial \bar{f}_j}{\partial T} \left(\frac{1 + \alpha_{crit}}{1 + \alpha_j} - 1 \right) \right]_i \quad (32.18)$$

In practice, smooth functions of lethargy are formed from the group values of $\frac{D_{iA}}{\Delta u_i}$ and α_{crit} , and the group index is dropped to give

$$\frac{1}{k} \frac{dk}{dT} = \frac{1}{\beta} \left(\frac{3}{N^3} \right) \left(\frac{0.6023}{V} \right) \frac{(NC)}{F} \sum_j \left(\frac{D_A(u)}{\Delta u_j} \right) \frac{\pi}{2} \frac{\sigma_{oj} \Gamma_j}{E_{oj}} \frac{d\bar{f}_j}{dT} \left(\frac{1 + \alpha_{crit}}{1 + \alpha_j} - 1 \right) \quad (32.19)$$

To determine the temperature coefficient due to a resonance absorber at some position in the reactor, one replaces D_{iF} and D_{iA} with space-dependent functions. (Reference 6 uses R_{iF} and R_{iA} for these, in which case $\alpha_{crit} = \frac{\nu R_{iF}}{R_{iA}} - 1$). These functions have been

calculated for a one-dimensional model only, and when used they give the temperature coefficient per mole of absorber spread out uniformly over a spherical shell. For a pure absorber (the limiting case as $\alpha_j \rightarrow \infty$), the term $\frac{1 + \alpha_{crit}}{1 + \alpha_j}$ is zero.

32.4 SINGLE RESONANCE CONTRIBUTION TO DOPPLER TEMPERATURE COEFFICIENT

In order to determine the contribution of a single resonance to the over-all temperature coefficient, one substitutes for $\sigma_{oj} \Gamma_j$ the expression

$$\sigma_{oj} \Gamma_j = 1.306 \times 10^6 (2g\Gamma_n^0) / \sqrt{E_{oj}}$$

Here g is the statistical weight factor, and Γ_n^0 is the reduced neutron width; the term $(2g\Gamma_n^0)$ is a measure of the strength of a resonance. Then, for an isolated resonance at lethargy u_o ,

$$\frac{1}{k} \frac{dk}{dT} (u_o) = \frac{1}{\beta} \left(\frac{3}{N^3} \right) \frac{0.6023 (NC)}{V} \left(\frac{D_A (u_o)}{\Delta u} \right) \frac{4.1 \times 10^6 (2g\Gamma_n^0)}{2E_o^{3/2}} \left[\frac{(1 + \alpha_{crit})}{(1 + \alpha_o)} - 1 \right] \frac{df}{dT} (u_o) \quad (32.20)$$

Through $(df/dT) (u_o)$, this expression depends implicitly on $(2g\Gamma_n^0)$ and Γ . Figure 32.1 shows how changes in $(2g\Gamma_n^0)$ and Γ affect the contributions of a single resonance at various values of u_o . Here the factor $\left[\frac{(1 + \alpha_{crit})}{(1 + \alpha_o)} - 1 \right]$ was taken to be -1 for each type of

resonance, corresponding to the case of a pure absorption resonance ($\alpha = \infty$); and the values $D_A / \Delta u$ and α_{crit} were obtained from a multigroup calculation. Strong and weak resonances are represented by $(2g\Gamma_n^0)$ values of 10^{-3} ev and 10^{-4} ev, respectively. Broad and narrow resonances are represented by Γ values of 0.15 and 0.05 ev, respectively. Besides varying with $(2g\Gamma_n^0)$ and Γ , the value of $(df/dT) (u_o)$ depends upon the absorber mass number, concentration, geometry, and temperature. A power reactor fuel element was selected as an example with the temperature taken to be 1000°K . The U^{235} resolved resonances are shown in Fig. 32.1, so that one may see at a glance the approximate curves that would apply to the actual resonances. Figure 32.2 shows a plot of $(df/dT) (u_o)$ for each of the resonance types selected. In Roe's curves for the calculation of df/dT , a parameter $\mu \tau$ appears which describes the absorber geometry and the scattering properties of moderator and fuel. In the present calculations, the appropriate value of this parameter was taken as $3.77 \times 10^{-4} \sigma_o$ barns.

In Fig. 32.1 a dashed vertical line indicates the upper energy limit of resolved resonances in U^{235} . In order to calculate the temperature coefficient contribution from unresolved resonances one may predict the resonance parameters from average cross-sections and low lying resonance data.²

The sign of the contribution of a resonance to the over-all Doppler temperature coefficient is determined by the value of α in the resonance. Figure 32.3 shows curves of α_{crit} as a function of lethargy for different positions in the core. Superimposed on these curves are the available values of the resonance α 's and their experimental uncertainties.⁷ A resonance whose value of α lies below a given curve contributes toward a positive coefficient, whereas one whose α value is above the curve contributes toward a negative coefficient. The resonance spectrum is included to show the number of resolved resonances for which α has not yet been determined. Figure 32.4 shows the dependence on α and the lethargy of the temperature coefficient due to a "strong narrow" resonance. A more detailed treatment² has shown that the temperature dependence of the df_j/dT is such as to give a reactivity effect nearly proportional to the logarithm of the absolute temperature of the resonance material.

32.5 COMPARISON WITH EXPERIMENT

Calculations have been performed using both the adjoint and direct methods to determine the Doppler temperature coefficient of reactivity in an intermediate reactor for a few resonance absorbers.⁸ The agreement with experiment has been good for pure ab-

Fig. 32.1—Effect of energy, strength, and width of resonance on temperature coefficient.

sorbers. At present it is felt that the experimental uncertainty of the available resonance values and the lack of α values on other resolved resonances does not permit a good determination of the Doppler temperature coefficient for U^{235} . In fact, a positive Doppler coefficient has been calculated from the resonance parameters given in Report BNL-325⁹; yet Doppler temperature coefficient measurements in the PTR (Chap. 33) and pile oscillator tests on the power reactor (Chap. 35) indicate a negative Doppler coefficient.

Recently, cross-section measurements have been refined^{10,11} which enable the re-evaluation of the contribution of three resonances to the Doppler temperature coefficient. The calculation is so sensitive to the knowledge of α that the new data cause the previous positive Doppler coefficient to be roughly halved. Refinements in the data are therefore tending to change the calculated value in the proper direction to obtain agreement with experiment.

Fig. 32.2—Effect of resonance parameters on self-shielding temperature derivative.

Fig. 32.3—Capture-to-fission ratios for individual resonance, and critical values for different regions.

Fig. 32.4—Effect of capture-to-fission ratio (α) upon "strong narrow" resonance contribution to Doppler temperature coefficient.

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Chapter 33

MEASUREMENT OF DOPPLER COEFFICIENT BY OSCILLATORY HEATING

R. T. FROST and A. L. MacKINNEY

33.1 DOPPLER EFFECT MEASUREMENT BY OSCILLATORY HEATING

Because accurate calculations of the fuel Doppler effect are difficult and because this effect contributes significantly to the prompt temperature coefficient of the reactor, it was necessary to make experimental measurement of the Doppler effect. The smallness of the reactivity change obtainable under the practical constraints on the experiment led to some difficulties which will be mentioned under the subject of signal detection. In view of the ambiguity of the theoretical estimates of fuel Doppler effect, experiments were also done with the pure absorbers U^{238} and hafnium in order to obtain a comparison with the estimates based on the differential cross-section data. The agreement of these estimates with experiment for the pure absorbers was as good as could be expected, considering the uncertainties in the cross-section data and the estimated probable error (15 per cent) of the experiments.

Since there are other reactivity changes in addition to that produced by the Doppler effect when a small sample is heated in a critical assembly, these changes must be taken into careful account in an experiment designed to measure the effect due only to temperature broadening of resonances. These other temperature-induced reactivity changes, to be discussed in the following three sections, are:

1. Self-shielding changes in the sample due to the variation of density with temperature
2. Sample motion accompanying thermal strains, since the sample is ordinarily located in regions of the reactor where the reactivity coefficient varies with position
3. Temperature effects induced in reactor components other than the sample by heat transfer from the sample.

33.2 SELF-SHIELDING EFFECTS DUE TO DENSITY CHANGES

Self-shielding effects accompanying temperature-induced changes in density are not particularly serious in an intermediate energy neutron spectrum for most materials of the thicknesses required in the reactors. For a material having a relatively small Doppler coefficient, under conditions where self-shielding is a more sensitive function of thickness, however, this effect would be expected to compete seriously with the Doppler contribution to reactivity change. Experimentally determined self-shielding curves are available for the materials used in the Doppler measurements, and the effect due to density change can be evaluated by reference to them.

The self-shielding curves presented in Chap. 8 show the experimentally determined self-shielding factor f as a function of the product ξ of atomic density and slab thickness. For a cylindrical sample, the radius plays the role of slab half-thickness. The contribution of the sample to the reactor multiplication constant is proportional to the self-shielding factor. As the material is heated, both its atomic density and its radius will

change. The derivative of f with respect to the temperature thus determines the temperature coefficient resulting from this effect.

Although practical interest lies in the changes in self-shielding as they result from the combination of Doppler effect, fuel density changes, and thermal strains, the latter two effects should be calculable from other experimental information on: self-shielding as a function of thickness, variation of reactivity coefficient with position, and thermal strains. It is desirable to separate the reactor temperature coefficients, in so far as possible, into a sum of simple components.

Since the atomic density varies as $(1 + \alpha T)^{-3}$ and the radius as $(1 + \alpha T)$, their product ξ will vary approximately as $(1 - 2\alpha T)$, where α is the coefficient of linear thermal expansion, and the temperature rise T is low enough that $\alpha T \ll 1$. Hence the fractional change in the reactivity Δk of the sample per degree of heating due to expansion is

$$\frac{d \ln \Delta k}{dT} = \frac{d \ln f}{d \ln \xi} \frac{d \ln \xi}{dT} = -2\alpha \frac{d \ln f}{d \ln \xi} \quad (33.1)$$

Inspection of the fuel self-shielding curves shows that values of $-\frac{d \ln f}{d \ln \xi}$ range from 0.01 at the center to 0.1 at the core-reflector interface in the reactor. Since $2\alpha = 2 \times 10^{-5}$ for the stainless steel cladding, the values of $\frac{d \ln \Delta k}{dT}$ range from 2×10^{-7} to 2×10^{-6} .

The observed values of $\frac{d \ln \Delta k}{dT}$ are approximately -2.5×10^{-5} for the fuel. This value is obtained by combining data for enriched fuel from Table 33.1 with reactivity coefficient data for an average position similar to those given in Chap. 8. The average density effect can therefore be neglected. It is to be noted, however, that the size of the effect precludes an accurate Doppler measurement averaged over the core-reflector boundary region, where the density effect would become approximately 10 per cent of the measured effect.

33.3 THERMAL STRAINS OF SAMPLE

If the sample is located in a position where there is a gradient in the statistical weight function for the particular material, then the small changes in sample position with heating must be taken into account. To obtain an estimate of the order of magnitude of this effect, we consider a highly idealized situation that gives an upper bound for the resulting reactivity change.

Consider a long thin element that extends radially through the reactor and is constrained at the center of the core but is free to expand lengthwise into the reflector at one end. Its reactivity contribution is proportional to the average value of the reactivity coefficient function $W(x)$ over its length. At the initial temperature T , the length is L and the average value of $W(x)$ will be denoted by $\bar{W}$. At the temperature $T + \Delta T$, the length is increased to $L + \Delta L$ and the average value of the function W changes to

$$\bar{W} + \Delta \bar{W} = \frac{\bar{W} L + W_e \Delta L}{L + \Delta L} \quad (33.2)$$

where W_e is the value of the reactivity coefficient at the unconstrained end of the sample. If we expand this out to first order in small quantities, we find that the fractional change in W averaged over the sample volume is given by

$$\frac{\Delta \bar{W}}{\bar{W}} = \frac{\Delta L}{L} \left(\frac{W_e}{\bar{W}} - 1 \right) \quad (33.3)$$

Hence we have

$$\frac{d \ln \Delta k}{dT} = \frac{d \ln \bar{W}}{dT} = \left(\frac{W_e}{\bar{W}} - 1 \right) \frac{d \ln L}{dT} = \left(\frac{W_e}{\bar{W}} - 1 \right) \alpha \quad (33.4)$$

If this is not to compete with the Doppler effect in the measurement, the quantity in parentheses must be made very much smaller than unity. One might consider minimizing this effect by adjusting the length of the sample so that the quantity in parentheses vanishes. Since, however, one ordinarily wishes to measure a Doppler effect averaged over the reactor volume, such a constraint on sample position would be quite unwelcome. Furthermore, at points in the reactor where the statistical weight function equals its core average for fuel, the function W can have steep gradients and small positioning errors would be important.

To guarantee that the expansion effect be down by more than an order of magnitude from 2α in this case, it is apparent that the sample motion must be constrained at many points along its length. This consideration led to the helical constructions shown in Fig. 33.1 where the helix is wound from wire or fuel-containing steel tubing. These helices were wound onto quartz forms to which were affixed rows of glass beads that constrained the sample at each turn. The only important motions due to thermal cycling, as observed with a traveling microscope, were radial expansions and contractions perpendicular to the axis of the helix. The variation in statistical weight for fuel in the reactor perpendicular to the axis of the helix was small enough so as not to give significant effects on this account except in the $r = R$ position (R = cylindric radius of reactor), where the entire helix would lie in the strong radial flux gradient of the core-reflector interface. Measurements were therefore not made in this position.

Fig. 33.1—Special element for Doppler measurements.

33.4 TEMPERATURE CHANGES INDUCED IN SAMPLE ENVIRONMENT

The desire to eliminate temperature effects induced in reactor components other than the sample by radiation and conduction from the sample is the principal reason for resorting to a pile-modulation technique for the measurements, in which the modulation is accomplished by cyclic heating of the sample. By choosing a modulation cycle that is short compared to the time constant for thermal response of the adjacent slugs and matrix tubes, these other effects can be effectively eliminated. To check that the amplitude of the temperature oscillation of these other reactor parts, including the heating of the assembly itself, was sufficiently small with the modulation period chosen (15 sec), separate runs were made with a heater which was an exact duplicate of the fuel helix except that the steel tubing did not contain fuel. In the case of measurements with U^{238} and hafnium, the Doppler effects of the samples were large enough that separate runs of this sort did not have to be made.

33.5 APPARATUS AND PROCEDURE

A. Sample Heating and Temperature Monitoring

The helices were wound with a pitch large enough that mutual self-shielding due to adjacent turns was small compared with the self-shielding in a single turn, and the ex-

perimental results can therefore be conveniently compared with calculations for an infinite cylinder self-shielding geometry with the cross-section of the element from which the helix is wound. The total length of the helices when wound was such as to extend from the equatorial plane to the end of the reactor core.

The samples were heated by passing an electrical current through the helix winding. In the case of the uranium-oxide-filled helix, the current was carried by the steel tube, and heat transfer to the uranium oxide region was rapid enough that the fuel temperature lag between it and the tubing was unimportant. By automatic cycling of the heating current and the cooling air, it was possible to achieve a temperature-versus-time wave shape that was closely triangular with equal positive and negative slopes and an amplitude of several hundred degrees centigrade.

Peak temperatures could be measured with an optical pyrometer and corrected for non-black-body conditions while the temperature variation with time was recorded by means of thermocouples welded to the sample at several positions along its length. Typical thermocouple records are shown in Fig. 33.2. Small corrections were made to take account of the variation of the temperature wave shape from the ideal triangular pattern. For the hafnium measurements, it was necessary to use a blast of helium gas instead of air for cooling to prevent progressive oxidation of the sample.

Fig. 33.2—Typical thermocouple records.

B. Signal Detection

The detection of the oscillating component of the reactor flux in pile-modulation experiments was best accomplished by large ion chambers located inside the reactor. It was possible in this manner to obtain alternating currents of sufficient magnitude that electronic amplification was not necessary. An even more important consideration dictating the use of large ion chambers was that statistical fluctuations in the ion chamber current had to be minimized when performing experiments in which both the amplitude of oscillation and average reactor power were low.

An additional source of noise remains as a fundamental limitation to the sensitivity of experiments of this kind, even when amplifier and ion chamber noise are minimized in the manner mentioned above: the statistical fluctuations in the reactor power level. These fluctuations have been analyzed by O. R. Frisch.¹ Consideration of these fluctuations led to the result that, when a reactivity oscillation of amplitude δ is measured, the signal-to-noise ratio is approximately

$$\frac{S}{N} = \frac{\delta}{\sqrt{2}} \sqrt{N_f} \quad (33.5)$$

where N_f is the total number of fissions taking place during the experiment. (Noise is here defined as the standard deviation of the answers obtained in a series of similar experiments.) This expression is correct under the assumption that the ion chamber has a relatively high probability ν of detecting a fission in the reactor, the condition for its validity being $\nu A^2(\omega) \gg 1$, where $A(\omega)$ is the reactor transfer function. In typical experiments at KAPL, this product was of the order of 10.

C. Signal Measurement

Although Eq. 33.5 is independent of the band width over which alternating signals from the ion chamber are accepted in the signal measuring device, in practice the following considerations make it necessary to utilize some band-pass device selectively tuned to the pile-modulation frequency.

With a given noise level in which the measurement is made, the accuracy of measurement in principle will be independent of the "Q" or band width of the measuring device. As the Q of the device is increased, the detector becomes responsive to a narrower range of frequencies in the Fourier spectrum of the noise, and its root mean square displacements due to noise will decrease as $Q^{-1/2}$. The signal spectrum, on the other hand, consists of a single sharp line exactly at the center of the passband; hence the amplitude of the measured signal will not decrease as the passband is narrowed. The noise-to-signal ratio per cycle will therefore vary as $Q^{-1/2}$. However, as Q is increased, the correlation between successive cycles of response of the measuring device will be increased so that fewer independent data are obtained in a given time. In calculating the probable error to associate with a measurement of a number of cycles, the standard deviation in the series of readings cannot be used alone; the correlation which exists between successive cycles of response must also be considered.

As can be seen by considering that the envelope of the response to a pulse decays according to the law $e^{-\omega t/2Q}$, where ω is the angular frequency of oscillation, the "memory" of the device is proportional to Q. Hence, in a given running time the number of essentially independent readings varies as Q^{-1} . Dividing the noise-to-signal ratio per cycle by the square root of the number of independent measurements, one finds that the noise-to-signal ratio for the data obtainable in a fixed running time is independent of Q.

The necessity for some sort of tuned circuitry when experiments are performed in which either the reactor power or amplitude of the oscillating component of reactor flux are small is dictated by the practical requirement of reasonable signal-to-noise ratio per cycle. Although in principle the signal and the noise could be recorded directly and the resultant function Fourier-analyzed to obtain the signal, this becomes impossible in practice when the signal is small compared to the total noise amplitude. If the Fourier analysis is done graphically, the limitation arises from the limited accuracy of the plot where the small signal would become lost unless the data were recorded with considerable precision. If an electromechanical type of Fourier analysis is contemplated, the limitation arises in circuit nonlinearities and drifts under condi-

tions where a small signal superimposed on a large total noise amplitude tends to get lost in the nonlinearities and/or drifts.

The simple tuned circuit which was used consists of a high-sensitivity moving-coil galvanometer which serves as a resonant detector when the pile-modulation frequency is properly adjusted. Mounting of the galvanometer under an evacuated bell jar at a pressure such that the mean free path of molecules is much greater than the linear dimensions of the galvanometer eliminates air damping and enables one to obtain a Q in excess of 100. With this arrangement, galvanometer deflections of 1 cm are obtained for reactivity oscillations of 10^{-7} at 10 watts of reactor power. From Eq. 33.5 it can be seen that the signal required to give a signal-to-noise ratio of unity under the practical conditions of a 10-watt run which lasts for 10^3 sec ($N_F = 3 \times 10^4$) gives the result $\delta = 10^{-7}$. This, then, is the limitation to the sensitivity of pile-modulation experiments carried out in low-power critical assemblies, and not the lack of detector response to small signals.

The galvanometer deflection was read at discrete times during each modulation cycle. Timing signals were furnished by the time-base generator.

D. Calibration

A pile oscillator for calibration purposes was designed to give a saw-tooth oscillation, as shown in Fig. 33.2, with an amplitude of $1/2 \times 10^{-4}$, which is large enough to be measured on the control rods, but still small enough to ensure the validity of small oscillation theory. The detector is calibrated in terms of the pile oscillator by driving the latter in continuous rotation through selsyns from a time-base generator. The peak amplitude of response of the galvanometer is recorded after equilibrium had been reached. A calibrated attenuator ahead of the detector is used to decrease sensitivity when large reactivity oscillations are observed, as, for example, in the calibrating runs with the pile oscillator. The over-all arrangement of this experimental apparatus is depicted in Fig. 33.3.

Fig. 33.3—Doppler experiment instrumentation.

In order to eliminate the influence of pile drifts, a high-pass filter was interposed between the ion chamber and the galvanometer detector. At very low frequencies, the absolute value of its transfer function varies as the third power of the frequency. In practice, it was found that moderate reactor drifts had no appreciable effect on the galvanometer zero position. Since peak-to-peak oscillation amplitudes were recorded, such a zero drift would not have any influence on the measurements in any event. However, since the amplitude of the oscillation is proportional to the reactor flux level, it was convenient to eliminate reactor drift as much as possible before commencing measurements so as to eliminate large corrections for variation of reactor flux during the measurement. Control rods were never moved during the course of the oscillation measurements, to avoid induced transients in the detector.

33.6 RESULTS

Doppler effects were measured for enriched fuel, U^{238} , and hafnium in assemblies having different beryllium ratios. The Doppler effect measurement was repeated for enriched fuel in a cadmium-poisoned assembly which simulated a spent power reactor. The measurements in all cases were made on samples extending from the equatorial plane to the end of the reactor. Results are shown in Table 33.1. Each value of $T dk/dT$ is to be interpreted as the reactivity change per mol of material whose absolute temperature is increased by a factor e . The estimated probable error in all of the measurements is 15 per cent, except in the case of the oxide fuel measurements at $r/R = 0.92$, and the fuel-steel mixture, where thermal strains increased the probable error to approximately 30 per cent. The thicknesses of fuel, U^{238} , and hafnium were 1.1×10^{-3} , 1.0×10^{-3} , and 1.8×10^{-3} mol/cm², respectively. The hafnium sample was a 0.034-in. diameter wire. The fuel-steel mixture had a fuel thickness of 0.61×10^{-3} mol/cm².

TABLE 33.1--DOPPLER EFFECT MEASUREMENTS IN THREE ASSEMBLIES

Material	Radial Position in Reactor, r/R	Temperature Excursion, °C	$T dk/dT$ per mol T in °K
PPA-21A was an unpoisoned assembly having approximately 60 kg fuel load, a radius of 35 cm, a Be:U ²³⁵ atom ratio of 93:1, and a median fission energy of approximately 125 ev.			
Fuel	0 (central axis)	400 - 750	-0.26×10^{-4}
	0.5	400 - 750	-0.17×10^{-4}
U ²³⁸	0	400 - 750	-2.07×10^{-4}
Hf	0	400 - 750	-1.46×10^{-4}
PTR-2 was an unpoisoned assembly having fuel load of approximately 50 kg, a Be:U ²³⁵ atomic ratio of 137:1, and a mean fission energy of approximately 50 ev.			
Fuel	0	325 - 725	-0.23×10^{-4}
	0.5	325 - 725	-0.15×10^{-4}
U ²³⁸	0	320 - 700	-2.2×10^{-4}
	0.5	320 - 700	-1.1×10^{-4}
Hf	0	340 - 760	-2.1×10^{-4}
PTR-4 was a poisoned assembly, with a fuel inventory of 45 kg, a Be:U ²³⁵ atom ratio of 150:1 and \$9.5 of cadmium poisoning to simulate fission products.			
Fuel	0	450 - 650	-0.31×10^{-4}
	0.5	415 - 642	-0.20×10^{-4}
	0.92	453 - 653	-0.10×10^{-4}
Fuel-steel mixture	0	504 - 591	-0.31×10^{-4}
	0.5	520 - 606	-0.16×10^{-4}

It is of some interest to compare the reactivity change measured for a resonance absorber with the change predicted on the basis of the individual resonance level parameters for the two extremes of the temperature excursion studied. The self-shielding of U^{238} as a function of thickness has been evaluated and is shown in Fig. 33.4. The curves were derived on the basis of the self-shielding values for separate resonances taken from Roe's self-shielding graphs [Fig. 1.6.17-1.6.20 of the Reactor Handbook]². The level parameters were taken from BNL-297,³ which included data up to 195 ev.

The sample thickness used in the experimental U^{238} measurement is depicted by the arrow on the abscissa scale which corresponds to a change in reactivity coefficient of $0.76 \times 10^{-4}/\text{mol}$ for a temperature change equal to twice the amplitude of the fundamental Fourier component of the temperature oscillation. This is to be compared with the measured change of $0.86 \times 10^{-4}/\text{mol}$. This agreement can be taken as satisfactory, considering the experimental uncertainties.

Fig. 33.4—Self-shielding of reactivity coefficient for U^{238} near center of PPA-20.

The curves show that the temperature change in reactivity coefficient remains relatively constant for thicknesses equal to 7 per cent of the thickness used in the experiment. But this is just the thickness of the U^{238} impurity in the enriched fuel samples used to measure the fuel Doppler coefficient. Hence, to very good approximation, it can be assumed that the U^{238} contribution to the fuel Doppler measurement should be of the order of 1/14 of its value measured for the thicker geometry, or -0.15×10^{-4} , near the central axis of the reactor. Thus the contribution of U^{238} is approximately one-half of the total effect measured for the fuel.

From the fuel measurements it was deduced that the fuel Doppler effect averaged over the reactor volume would amount to $-0.074\text{¢}/^\circ\text{F}$ at 70°F . This component of the reactor temperature coefficient is expected to be approximately inversely proportional to the absolute temperature, and hence would be only $-0.022\text{¢}/^\circ\text{F}$ at a fuel element temperature of 1000°K which might conceivably be encountered in a sodium-cooled reactor at full power.

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33.5

33.6

33.6

Chapter 34

CRITICAL ASSEMBLY KINETICS STUDIES BY OSCILLATOR TECHNIQUE

R. T. FROST and R. J. SCHEMEL

34.1 EXPERIMENTAL DETERMINATION OF ZERO-POWER RESPONSE TO SINUSOIDAL CHANGES IN MULTIPLICATION FACTOR

Chapters 28 and 33 have shown the manner in which two special types of temperature coefficients for a power plant can be measured experimentally. However, in order to be sure that a nuclear power plant has no undesirable kinetic characteristics, it is necessary to study the auto-induced reactivity changes resulting from power transients in which temperature changes in reactor components having different time constants enter with varying degrees. The auto-induced reactivity change $\delta k'$ is assumed to be proportional to the oscillating component of reactor power δn . The factor of proportionality, Γ , which may be called the power coefficient, must in general be a complex number to account for the fact that the reactivity change $\delta k'$ will have a different phase from the power oscillation. It will of course depend upon the frequency of oscillation, ω . We thus write

$$\delta k' = \Gamma(\omega) \times \delta n$$

where the real reactivity and power level changes are both assumed to vary with time as the real part of the corresponding complex numbers $\delta k'$ and δn .

Experimental assurance must be obtained that at no power level can there exist unstable conditions which would lead to self-sustained power-level oscillations. In such a situation, finite oscillations would be possible with no oscillation in reactivity other than that furnished by the auto-induced reactivity. Let n_0 be the time average of the power level of the reactor. If the amplitude and phase of the relative power level oscillation $\delta n/n_0$ and of the total reactivity oscillation δk are combined in polar form into complex numbers, then the ratio of these complex numbers is defined as the zero-power reactor transfer function A_0 :

$$\delta n/n_0 = A_0 \delta k. \quad (34.1)$$

In general δk includes a contribution from the direct effects of a pile oscillator as well as the contribution $\delta k'$ resulting from the oscillation in power level through the power coefficient. The phenomenon of self-sustained power level oscillations can occur if a finite power oscillation δn is possible in the absence of an externally applied oscillation of the multiplication constant. In this case the total reactivity oscillation δk is composed solely of the auto-induced reactivity oscillation $\delta k'$. The fact that $\delta k = \delta k'$ implies that $(1/A_0)(\delta n/n_0) = \Gamma \delta n$, and the condition for instability with non-vanishing δn is

$$1/n_0 = A_0 \Gamma \quad (34.2)$$

Information regarding A_0 and Γ is of interest in connection with the possibility that relation (Eq. 34.2) might be satisfied at some particular power level and frequency.

In this chapter, we shall be concerned with the measurement of $1/A_0$. This is accomplished by studying the reactor kinetic response to sinusoidal changes of multiplication factor δk superimposed on an average multiplication of unity, done at such low reactor powers that the auto-induced reactivity oscillation from fuel element temperature changes is negligible.

34.2 FOURIER TRANSFORM OF DELAYED-NEUTRON KERNEL

In addition to its importance as a reference for the measurement of A at high-power levels where the auto-induced reactivity effects manifest themselves as an apparent change from A_0 , the "zero-power" reactor transfer function is important in its own right because of its close connection with the fundamentals of the reactor's kinetics. This connection can be seen in the following way.

Consider a typical fission at time $t = 0$. The probability of emission of delayed neutrons from the resultant fission fragments is called, following Hurwitz,¹ the "delayed-neutron kernel" $K(t)$. This kernel is normalized so that its time integral is equal to the total effective yield of delayed neutrons β from a single fission. Since the delayed neutron activity at any time t in a nuclear reactor can be considered to be the sum of separate contributions from fissions occurring at past times τ integrated over the power history of the assembly, the delayed neutron source can be written as

$$\int_{-\infty}^t d\tau \frac{n(\tau)}{\tau_0} K(t-\tau)$$

Here $n(\tau)$ is the neutron density at the time τ , and τ_0 is the prompt fission life-time, so that $n(\tau)/\tau_0$ is the fission rate at the time τ . The one-group, space-independent pile kinetic equations then become

$$\dot{n}(t) = \frac{k_p - 1}{\tau_0} n(t) + \int_{-\infty}^t d\tau \frac{n(\tau)}{\tau_0} K(t-\tau) \quad (34.3)$$

where k_p is the prompt multiplication. (More precise definitions of the symbols n and τ_0 appearing in the one-group space-independent kinetic model are to be found in Chap. 1.6 of the Reactor Handbook.²)

Let us consider an experiment in which the prompt multiplication varies with time as the real part of $\delta k e^{j\omega t}$ and seek a solution which is the real part of $\delta n e^{j\omega t}$. Substitution into Eq. 34.3 and a simple transformation of variable yields

$$j\omega \delta n e^{j\omega t} = \frac{k_0 - 1 + \delta k e^{j\omega t}}{\tau_0} (n_0 + \delta n e^{j\omega t}) + \frac{n_0}{\tau_0} \int_0^\infty dZ K(Z) + \frac{\delta n}{\tau_0} e^{j\omega t} \int_0^\infty dZ K(Z) e^{-j\omega Z} \quad (34.4)$$

The first integral is simply the total yield of delayed neutrons of all types, which we shall denote by β . In the beryllium-moderated power reactor, this fraction includes the delayed yield of photoneutrons from fission-product gamma rays. The second integral is simply the Fourier transform of the delayed-neutron kernel, which we shall denote by $K^*(\omega)$. Equating the constant terms and the time-dependent terms separately in Eq. 34.4 yields:

$$1 - k_p^0 = \beta \quad (34.5)$$

and

$$j\omega \delta n \approx -\frac{\beta \delta n}{\tau_0} + \frac{n_0 \delta k}{\tau_0} + \frac{\delta n}{\tau_0} K^*(\omega) \quad (34.6)$$

where we neglect a second-order term involving the product $\delta k \delta n / n_0$, on the assumption that $\delta k \ll 1$ and $\delta n / n_0 \ll 1$. (The effect of neglecting this term has been discussed by M. S. Nelkin in an appendix to KAPL-1138.³) We shall write Eq. 34.6 in the alternative form

$$\frac{\delta n}{n_0} = \frac{\delta k}{j\omega\tau_0 + \beta - K^*(\omega)} \quad (34.7)$$

Comparison with Eq. 34.1 shows that, in absolute units (i.e., not in dollars),

$$1/A_0(\omega) = j\omega\tau_0 + \beta - K^*(\omega) \quad (34.8)$$

A measurement of the phase of $1/A_0(\omega)$ at frequencies where it attains its high-frequency asymptotic form $(j\omega\tau_0 + \beta)$ yields a value for τ_0/β . Thus a value for the prompt generation time can be extracted in those cases where β is known.

In principle, a measurement of $1/A_0(\omega)$ over a sufficiently wide frequency range can enable one to obtain the shape of the delayed-neutron kernel by the Fourier inversion

$$K(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega [j\omega\tau_0 + \beta - 1/A_0(\omega)] e^{j\omega t} \quad (34.9)$$

The values of the integrand of Eq. 34.9 for negative values of ω are determined by the requirement that $K(t)$ be real. In other words, $K^*(-\omega)$ is the complex conjugate of $K^*(\omega)$. A "sufficiently wide" frequency range means that the experiments be carried into the high-frequency region where $K^*(\omega)$ effectively vanishes and, $1/A_0(\omega) \approx j\omega\tau_0 + \beta$, and to low enough frequencies that $1/A_0(\omega)$ approaches its low-frequency asymptotic form, which is a circular arc of radius

$$\left(\frac{\tau_0}{\beta} + \sum \frac{f_\mu}{\lambda_\mu} \right) / 2 \sum \frac{f_\mu}{\lambda_\mu}$$

in the right half-plane of a plot of $\text{Im}(1/A)$ vs $\text{Re}(1/A)$ tangent to the imaginary axis at the origin. Here f_μ and λ_μ^{-1} are the effective yields and life-times for the delayed neutron precursors. The properties of $K(t)$, that it is real and vanishes for negative t , imply that

$$K(t) = \frac{2}{\pi} \int_0^\infty d\omega \text{Re}[K^*(\omega)] \cos \omega t = - \frac{2}{\pi} \int_0^\infty d\omega \text{Im}[K^*(\omega)] \sin \omega t \quad (34.10)$$

so that in principle $K(t)$ can be determined from either the real or the imaginary part of K^* .

34.3 EXPERIMENTAL METHOD AND APPARATUS

The method of measurement of A which was originally developed at Harwell, and which was used in the earliest of the power-reactor experiments, consisted of performing an electromechanical Fourier analysis of the current from an ion chamber monitoring the reactor flux by means of sine and cosine integrators. However, in order to obtain experimental results as free as possible of electronic drifts and noise, a simpler system was developed for the critical assembly measurements, which reduced to a minimum the amount of experimental apparatus necessary and which rendered the results independent of electronic drifts during the measurements. The system has, in addition, the inherently great accuracy of a null-balance measurement, and the results are independent of drifts in reactor power during the measurements.

A uniformly rotating shaft generates simultaneously an oscillating multiplication constant in the reactor by means of a rotating absorber, and a sinusoidal voltage of known phase and amplitude by means of a rotating potentiometer. The amplitude and phase of the current from an ion-chamber located in the reactor can then be measured simply by adjusting the amplitude and phase of the electrical reference signal until it

produces a null when superimposed upon the sinusoidal component of the ion chamber current.

The apparatus embodying this null-balance system was installed in the Proof Test Reactor (PTR). Details of its parts and operation follow.

A. Motor Drive

The apparatus consists of a synchronous motor with mechanical coupling through a gear-shift mechanism to the continuously rotating pile oscillator located in the reactor core. The operator can select by remote control any one of 13 gear ratios, differing one from the next by a factor of two. The frequency range available is therefore $2^{12} = 4096$.

B. Pile Oscillator

The reactivity oscillator consists of a pair of coaxial cylindric shells, one of which is rotated inside the other by direct mechanical coupling to the gear-shift mechanism. Around half of one shell is bent a thin rectangular sheet of hafnium to form a half-cylinder; around half of the other shell is bent a sheet of hafnium cut in the shape of a sinusoid. Relative rotation of the two shells alternately covers and exposes the sinusoidally shaped sheet, and the resulting changes in self-shielding produce sine-wave oscillations of the reactor multiplication constant with an amplitude of the order of one cent. The wave form of the oscillations must be fairly accurately sinusoidal in order to allow determination of the phase of $1/A_0$.

C. Sine Potentiometer

In addition to driving the pile oscillator, the output of the gear-shift mechanism drives a continuously rotating potentiometer through a differential gear. This precision potentiometer is simply a voltage divider whose tap furnishes a voltage proportional to the sine function of the angular position of its rotating shaft. The third shaft of the differential gear is driven by a reversible slow-speed motor which can be started when it is desired to change the relative phase ψ between the angular positions of the pile oscillator and the arm of the potentiometer. This phase difference is recorded on a mechanical register which can be observed directly through a window in the reactor cell.

D. Null-balance Circuit

The over-all arrangement of the equipment is shown in Fig. 34.1. The ion chamber current $i_0 + \delta i \cos(\omega t - \phi)$ is passed to ground through the sine potentiometer (whose resistance amplitude is r) and a precision decade resistor box (of resistance R). The maximum resistance to ground that was used in these experiments was 0.3 megohms. This, together with an estimated upper limit for the capacitance of the ion chamber cable, leads to an extraneous phase lag of 0.2 deg at a frequency of 2 cps. At lower frequencies, the phase lag is negligible; at the highest frequency used (15 cps) the phase lag from this source was estimated to be $3 \text{ deg} \pm 1 \text{ deg}$.

A simple calculation shows that the alternating voltage appearing on the rotating potentiometer arm is

$$E = \left(R + \frac{r}{2} \right) \delta i \cos(\omega t - \phi) - \frac{r}{2} i_0 \cos(\omega t - \psi) - \frac{r}{4} \delta i \cos[2\omega t - (\phi + \psi)] \quad (34.11)$$

This voltage is observed by means of an electronic amplifier, the output of which passes through a variable-frequency band-pass filter to a recorder. The expression (Eq. 34.11) is based upon the assumption that the current drawn by the amplifier is small (which requires in practice that the impedance looking into the amplifier be over 10^8 ohms). The band-pass filter is adjusted to a narrow range of frequencies in the neighborhood of the modulating frequency. This is necessary to improve the signal-to-noise ratio in the low-power critical experiments, where pile fluctuations are important and would otherwise completely obscure the alternating voltage on the potentiometer arm as the conditions for balance (Eqs. 34.12 and 34.13) are being approached. Even at the highest reactor power which was used in these experiments (10 watts), the Fourier components of

Fig. 34.1 PTR pile oscillator instrumentation.

the pile fluctuations in the band-pass range of the filter were large enough to obscure the small second-harmonic term (the last term in Eq. 34.11) remaining when the fundamental term in the alternating voltage on the potentiometer arm was reduced to zero at balance.

E. Null-balance Conditions

The fundamental frequency component of alternating voltage appearing on the rotating potentiometer arm is reduced to zero only when

1. there is zero phase difference between the pile oscillator and potentiometer arm angular position:

$$\arg(1/A_0) = \arg(n_0/\delta n) = \arg(i_0/\delta i) = \phi = \psi, \quad (34.12)$$

and

2. the decade resistor R is adjusted to satisfy the conditions, evident from Eqs. 34.1 and 34.11, that

$$\left| \frac{1/A_0}{\delta k} \right| = \left| \frac{n_0}{\delta n} \right| = \left| \frac{i_0}{\delta i} \right| = \frac{2R}{r} + 1. \quad (34.13)$$

It is to be noted that conditions in Eqs. 34.12 and 34.13 are independent of reactor power level because the oscillating component of the reactor flux is proportional to the average value of flux.

F. Experimental Procedure

The transfer function of the PTR critical assembly was measured, by use of the equipment described, over a frequency range from 0.03 cps to 15 cps. The results of a measurement at a typical frequency are shown in Figs. 34.2 and 34.3, which are plots at a fixed frequency of the alternating voltage seen by the null-signal amplifier, plotted against ψ and R separately. The dashed curve of Fig. 34.2 was obtained with R set at a roughly estimated value, whereas the solid curve on this figure was obtained after Fig. 34.3 was obtained and utilized to set R to a more appropriate value. The sharpness of the null which can be obtained in either phase or amplitude depends, of course, upon

how close the other variable is set to its null value, but in practice it was found that an iteration consisting of just three plots, taken in the order of the ones shown, was sufficient. This was true when the measurements proceeded from one frequency to the next in a monotonic manner, so that rough preliminary values for R could be estimated.

Fig. 34.2—Error signal as function of phase for two different resistances.

Fig. 34.3—Error signal as function of resistance R with phase fixed at $\psi = 26.8$ deg, $\omega = 2.94 \text{ sec}^{-1}$.

34.4 RESULTS

The experimental data at the various frequencies are plotted (o) in Fig. 34.4 along with the theoretical curve (x) which is the inverse transfer function evaluated from Eq. 34.8 with the known delayed groups and effectiveness factors calculated for the reactor.⁴ The contribution of the photoneutrons to the total delayed-neutron yield has been neglected.

The experimental data in the high-frequency region are best fitted by a prompt generation time of approximately $37 \mu \text{ sec}$. It is to be borne in mind that at the highest frequencies used in these experiments the one-group, space-independent kinetic equations have only approximate validity because wave-propagation effects begin to set in. From the work of Wigner⁵ we can estimate that phase shifts of the order of several degrees could occur at the highest frequency, depending upon the relative location of oscillator and ion chamber in the reactor core. Because wave-propagation effects are so poorly understood, and because the transfer-function phase angle is known with only fair accuracy, Fourier inversion of the experimental data via Eq. 34.10 was not performed to obtain direct information about the delayed-neutron kernel.

Fig. 34.4—Reactor transfer function inverse $A_o^{-1}(\omega)$.

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Chapter 35

MEASUREMENT OF POWER COEFFICIENTS BY PILE OSCILLATOR

L. G. BARRETT

35.1 INTRODUCTION

Because it is important to know as accurately as possible the temperature and power coefficients of a reactor, particularly the fast response components, various techniques have been used on the power reactors to determine these coefficients. The technique to be described here is the pile oscillator method. In this method the reactor response to sinusoidal reactivity oscillations is measured at low power and then compared with the closed loop response measured at higher power. The difference between these responses is due to the combined effects of the temperature coefficients and their time responses. The result of the measurements then is the determination of the over-all power coefficient as a function of frequency. In principle, if the tests are performed under various reactor operating conditions, the analysis of the results can provide information both on the thermal model for the reactor as well as on the individual temperature coefficient components. The present chapter covers the basic theory of the pile oscillator technique, describes the equipment and procedures employed, and gives results obtained.

35.2 THEORY OF POWER COEFFICIENT MEASUREMENTS BY PILE OSCILLATOR TECHNIQUES

As outlined in Chapter 34, if the reactor is subjected to small sinusoidal oscillations in reactivity, $\delta k e^{j\omega t}$, there are power oscillations, $\delta n e^{j\omega t}$, about the average power level n of the reactor. On the basis of the expected proportionality between δk and $\delta n/n$, the reactor transfer function A is defined by

$$\delta n/n \equiv A \delta k, \text{ or } 1/A \equiv \delta k/(\delta n/n) \quad (35.1)$$

Here δk includes only the oscillating reactivity produced directly by the pile oscillator and does not include a contribution from the auto-induced reactivity oscillations resulting from the oscillation in reactor power. In the case of low power levels, where the total reactivity oscillation is due solely to this direct contribution from the pile oscillator, and the contributions from the power coefficients are negligible, A becomes the A_0 introduced in Chapter 34. The expression for $1/A_0$ can also be derived by a Fourier transform of the delayed neutron kernel

$$K(t) = \sum_i f_i \lambda_i e^{-\lambda_i t}$$

as mentioned in Chap. 34. Here f_i is the fraction of neutrons in the i th delayed group, and λ_i is the decay constant for the i th group.

Like δn , A is a complex amplitude; it is a function both of frequency ω and power level n . The inverse transfer function $1/A$ lends itself to more ready interpretation and will be used henceforth in this chapter.

The functional form of $1/A$ can be found in the case of a reactor operating at low

power and undergoing reactivity oscillations of a period which is long compared to the prompt neutron generation time. The one-group space-independent model of reactor kinetics has been used [by Siegel and Hurwitz, in KAPL-1138]¹ to develop the form

$$1/A_o = j\omega \left(\frac{\tau}{\beta} + \sum_i \frac{f_i}{\lambda_i + j\omega} \right) \quad (35.2)$$

Here $1/A_o$ is the reference (i.e., low power) transfer function inverse, in dollars of reactivity,

ω is the oscillation frequency, in radians per second,

τ is the prompt neutron generation time in seconds,

β is the delayed neutron fraction,

f_i is the fraction of delayed neutrons in the i th delayed neutron group,

λ_i is the decay constant of the precursor of the i th delayed neutron group.

This special case is one of low power level, so that there is no dependence of $1/A$ upon n . This special case is the basis for Fig. 35.2, which will be discussed subsequently in this section. (cf. Fig. 34.4.)

At higher power levels the power oscillations cause appreciable temperature oscillations, differing in different parts of the reactor, which produce reactivity oscillations by virtue of the temperature coefficients of reactivity. These changes modify the initial oscillating reactivity change in the same manner as feedback modifies the input signal of an amplifying system.

Such a system can be represented as shown in block diagram form in Fig. 35.1.

Figure 35.1

The response of this system is given by

$$\frac{\Delta n}{n} = \frac{A_o \Delta k}{1 - A_o n \Gamma} = \frac{\Delta k}{1/A_o - n \Gamma} \quad (35.3)$$

In this system, Γ is a power coefficient of reactivity; it is in general a complex quantity depending upon ω (and perhaps n as well). Using Eq. 35.1, one has the relation

$$1/A = 1/A_o - n \Gamma \quad (35.4)$$

This relation can be used to compute the power coefficient Γ in dollars per megawatt as the measured difference between a low-power transfer function inverse and a high-power transfer function inverse, divided by the difference in megawatts between the two power levels. On a polar plot of the imaginary part of $1/A$ vs its real part, $n \Gamma$ for any given frequency is the vector difference between two measured $1/A$ values. The sign of Γ has been chosen to make a positive power coefficient correspond to an increase in reactivity as the power is raised.

The various components of the temperature and power coefficients have been reviewed in previous chapters. For the power coefficients the thermal time constants of these various contributors are of basic importance. These are as follows:

1. The time of response for various structural expansion effects to occur after a power level change should range from 10 seconds in the core up to 30 seconds in the reflector.
2. The response time for the major portion of the sodium in the core should be between 0.2 and 0.6 seconds.
3. Since, as was pointed out in previous chapters, the thermal base effect is appreciable only in the outer portions of the core and in the reflector, this effect should depend to a considerable extent upon reflector temperature, which will lag substantially behind the core temperature.

4. The Doppler effect is for all practical purposes nearly instantaneous with power. The response time found by an electric circuit analogue calculation described by Siegel² was approximately 0.2 seconds.

The oscillating heat generation in the fuel causes temperature fluctuations in various components of the reactor. The magnitude and phase relations of the component temperature responses are dependent on the oscillation frequency. At high frequencies (ω greater than 10 radians/second) only components with very fast time responses, such as the fuel and sodium coolant, can respond thermally, while at low frequencies (ω about 0.1 radian/sec) components with slow responses, such as core structure, can also undergo appreciable temperature oscillation.

The most important information to be gained from the experiments is the nature of the temperature coefficients having short time responses, because these will appear during rapid power transients. The nature of these coefficients is determined by examining the high frequency portion of the experimental curve. In this range the fuel, and to a smaller extent the sodium, responds thermally to the fluctuating heat generation in the fuel. Thus any change in the transfer function at high frequencies in going from low to high power can be attributed to the fuel Doppler temperature coefficient and to the sodium temperature coefficient.

Figure 35.2 illustrates in (A) the shape of a typical reference transfer function (for the special low-power case described above) and in (B) the directions of the changes that

Fig. 35.2—Inverse transfer functions and power contributions.
A. Reference transfer function inverse.
B. Possible power (temperature) contributions ($-n\Gamma$) which, added to reference transfer inverse, yield net transfer function inverse.

power coefficients of various signs and phase can produce. A prompt negative coefficient with negligible phase will cause the $-n\Gamma$ vector to point to the right. The thermal capacity of the fuel and sodium will cause their response to lag somewhat behind the power changes, and the $-n\Gamma$ vector will point down toward the right if the temperature coefficients are predominantly negative. If the prompt temperature coefficient is positive, the $-n\Gamma$ vector is rotated 180 degrees from the vector produced by a prompt negative temperature coefficient. Thus, the nature of the very prompt coefficients can be directly observed at the high frequency end of the curve.

At low frequencies, it is more difficult to interpret the combined temperature effects. Here we can extrapolate back the information obtained at high frequencies and possibly, with some knowledge of the thermal responses of other reactor components, isolate the different contributions to the over-all temperature coefficient.

35.3 DESCRIPTION OF TEST APPARATUS

In the following description of the pile oscillator test equipment, reference is made to the functional block diagram of Figure 35.3. A variable speed motor drive was used to rotate the rotor of a sine potentiometer. An alternating voltage ($V \sin \omega t$) was then available at the rotating contact. This signal, when amplified in a magnetic amplifier, was used to actuate the solenoids in the fine control hydraulic drive on the control cylinder.

It was necessary to keep the δk variation small to provide the best comparison of experimental results with calculations. A maximum reactivity oscillation of ± 4 cents was employed, limiting power excursions to ± 5 per cent. In actual practice, the amplitude of control cylinder oscillation was kept at 0.85 deg or less. For safety reasons mechanical stops were used in the control drive mechanism which set a rotation of 2 deg as the maximum possible full stroke obtainable.

The control cylinder position was monitored by a selsyn transmitter in the control drive mechanism. This transmitter was electrically connected to the stators of two

Fig. 35.3—Pile oscillator functional block diagram.

selsyn control transformers. The rotors of both control transformers were locked with a null at the center of the range of oscillation after criticality was established at the desired power level. One control transformer was used to provide an error voltage to a discriminator which was coupled to the magnetic amplifier so as to maintain a closed feedback loop. In operation, this feedback loop prevented the center of the cylinder oscillation from drifting. The other control transformer signal, when fed through a phase detector, produced an output voltage proportional to cylinder position. This voltage was monitored to provide a meter indication and a recording of control cylinder position during oscillation. When properly calibrated this recording gave the amplitude of the oscillating reactivity δk in dollar units.

An ionization chamber was used to monitor the neutron level. This chamber fed into a Zero Stabilized DC Amplifier (ZSA) which had sufficient feedback to degenerate the input resistance of the amplifier to such a low level that phase shift was negligible in the combined circuit of chamber and amplifier. The steady-state neutron level n obtained from the output of the ZSA was recorded on a strip chart recorder. In order to obtain the neutron level oscillation, n was bucked out by a dc voltage in the input circuit of a functional amplifier (AS_1). An inverter stage (AS_1 2) with a gain of unity was used so that both plus and minus $D \sin(\omega t + \Psi_K)$, the oscillating neutron level, could be obtained. When both these voltages were applied to another sine potentiometer, which was ganged with the input signal sine potentiometer, the output of the rotating contacts provided voltages proportional to the products $D \sin \omega t \sin(\omega t + \Psi_K)$ and $D \cos \omega t \sin(\omega t + \Psi_K)$. These two voltages were integrated separately in functional electronic integrators. The integrators provided voltage outputs E_1 and E_2 proportional respectively to:

$$\int_0^T D \sin \omega t \sin(\omega t + \Psi_K) dt = DT/2 \cos \Psi_K + \text{oscillating terms},$$

and

$$\int_0^T D \cos \omega t \sin(\omega t + \Psi_K) dt = DT/2 \sin \Psi_K + \text{oscillating terms}.$$

By recording E_1 and E_2 simultaneously on an X-Y recorder, a vector was plotted whose amplitude was proportional to $DT/2$. In addition, the angle included between the vector and a reference axis was equal to the phase angle Ψ . The phase angle Ψ was composed of the phase shifts in the magnetic amplifier, fine control, control drive to cylinder, selsyn system, and the phase detector, in addition to the phase shift Ψ_K in the reactor itself. In order that the phase shift in the reactor might be determined separately, an additional phase measurement was made in which the output of the phase detector was compared with $\sin \omega t$. This phase angle was called Ψ_{MA} . The reactor transfer function phase angle was found from $\Psi = \Psi_K - \Psi_{MA} + \Psi_C$. Ψ_C was a small phase-angle correction which was based upon previously determined phase differences between indicated cylinder position and actual cylinder position.

The only remaining parameter needed to describe the reactor transfer function was the amplitude of the neutron level oscillation, δn . This could be obtained in two ways; either by examining the magnitude of the vector plotted on the X-Y recorder over a given time interval, or by the simpler method of recording the neutron level oscillation directly on a separate channel of the same recorder used for recording the δk signal. (The latter method assumes no distortion in the oscillation.) Finally, the reciprocal transfer function at a given power level and frequency was given by:

$$1/A = 1/A_0 - n \Gamma = \left| \delta k / (\delta n/n) \right| e^{j\Psi}$$

35.4 TEST PROCEDURE

Pile oscillator tests were performed at various power levels as shown in Table 35.1. These tests were started only after the power level, reactor ΔT , and sodium flow rate had reached equilibrium.

The control cylinder was set at the middle of its 2-deg range and the rotors of both selsyn control transformers were nulled and locked at this position. Before actual pile oscillation measurements were performed, the equipment was calibrated and linearity measurements were made on the various recorders and associated circuitry. The δk channel was calibrated by moving the cylinder through its 2-deg range, and the δn

channel was calibrated so that a change of ± 10 percent in steady-state neutron level corresponded to the maximum linear range of the recorder.

Linearity of the recording channels was measured by using a ramp generator, whose voltage changed linearly with time. In the case of the δk recording channel, the ramp generator was used to move the control cylinder at a constant rate through its 2-deg range. The δn recording channel linearity was measured by applying the output of the ramp generator to the AS₁ No. 1 amplifier input and making the normal recording on the δn channel.

To relate cylinder motion to reactivity units, control sensitivity measurements were made by the rod-drop method. This method and the results obtained are discussed in Section 20.5.

35.5 SUMMARY OF DATA

After all corrections were applied to the phase and magnitude measurements of the inverse transfer functions, the data were plotted as a function of frequency, and smooth curves were obtained.

Figure 35.4 is a polar plot of the inverse transfer functions found for the four tests listed in Table 35.1. (This figure can be compared with Fig. 34.4, which shows the theoretical curve and the PTR measurements.) The points on each curve are serially numbered to indicate the frequency corresponding to each point.

Fig. 35.4—Observed inverse transfer functions for sinusoidal reactivity oscillations.

The theoretical high frequency asymptote for the zero power reference transfer function is shown as the vertical line intersecting the real axis at 100 cents. The fact that the measured locus of $1/A$ for the 10-kw test crosses this asymptote is probably the fault of the experimental technique rather than being the result of temperature effects at this low power. A possible cause of systematic error in the measurements of the transfer function at various power levels, particularly at the higher frequencies, is the attenuation and phase shift of the neutron density waves as they are propagated from the core through the reflector to the ion chamber position. The ion chamber was located in the reactor at an appreciable distance from the core. Since any such systematic error in the transfer function measurements should be the same for tests at both high and low power, the differences between such tests, and hence the derived power coefficients, should still have significance.

In each case a strong negative prompt power coefficient is indicated at high frequencies. The vector $(-n\Gamma)$ generally points down and to the right for the 30 and 90 per cent power tests, to indicate an over-all negative coefficient lagging behind the neutron

TABLE 35.1--PILE OSCILLATOR TESTS

Power level, per cent of full power	Average control cylinder position deg	Range of xenon reactivity defects	Remarks
(10 kw)	57.2	~ 0	Reference test
30	60.7	\$0.31 - \$0.50	7 to 15 hours after start of 30% power run
90	67.8	\$0.56 - \$1.40	6 to 17 hours after start of a full power run
90	74.6	\$2.05 - \$2.07	82 to 95 hours after start of a full power run

level oscillation. The combination of thermal time constant due to heat conduction and another due to the coolant transport lag effect has been shown capable of producing such a loop in the reactor response as was observed in the 90 per cent power test. The xenon test at 90 percent power, that was made after about 90 hours at full power with nearly equilibrium xenon concentration, shows a large increase in the negative fast power coefficient as compared with the 90 per cent test that was made after about 10 hours at full power.

Polar plots of $(-n\Gamma)$, obtained from the inverse transfer functions plotted in Fig. 35.4, are shown in Fig. 35.5. The magnitude of the average power coefficient Γ is plotted in Fig. 35.6 as a function of frequency.

Fig. 35.5 -- Observed power contributions, $(-n\Gamma)$, using 10-kw transfer function.

Fig. 35.6—Power coefficient, $|\Gamma|$, versus oscillation frequency.

35.6 ANALYSIS OF DATA

In order that the results of the experiments may be compared with theory, a description should be made of the method employed by Siegel² for estimating the power coefficient for small sinusoidal oscillations in power. In this method the oscillation in temperatures of the fuel, coolant, beryllium, and the upper matrix plate were calculated for small sinusoidal oscillations in power. These temperatures were obtained both from a UNIVAC calculation and from a simple thermal model using an equivalent electric circuit. The latter model enabled hand calculations to be performed and could be used in making quick estimates of cyclic temperature responses for conditions differing from those covered by the UNIVAC calculations. The best available values for the individual temperature coefficients were then multiplied by the appropriate temperature changes to determine the various component reactivity changes. The individual effects were then added vectorially to account for the phase relations of the temperature responses. The combined effects gave the power coefficient as a function of frequency.

To compare the results of the pile oscillator tests with Siegel's theoretical predictions, Figs. 35.7 and 35.8 are polar plots of the $(-n\Gamma)$ vectors at frequencies of 0.0146 and 0.750 cps. (One vector was calculated for approximately \$1 xenon reactivity defect, and another was calculated for approximately \$2.5 xenon reactivity defect.)

Fig. 35.7—Power contribution $(-n\Gamma)$ at frequency 0.015 cps.

Fig. 35.8—Power contribution ($-n\Gamma$) at frequency 0.75 cps.

At the lower (0.0146 cps) frequency (Fig. 35.7), the calculated effect of the added xenon was to decrease the magnitude of the ($-n\Gamma$) vector from that of the clean condition and rotate it counter-clockwise about 11 deg. The measured effect was an increase in the magnitude of the ($-n\Gamma$) vector and a counter-clockwise rotation of about 19 deg. The increase in ($-n\Gamma$) is probably a result of the increase in the prompt coefficient (see the second following paragraph). The measured counter-clockwise rotation of the ($-n\Gamma$) vector at this low frequency suggests the existence of a slow positive xenon temperature coefficient with a phase lag greater than 90 deg in addition to the increase in fast negative coefficient observed in the presence of xenon. A slow xenon coefficient with a large phase lag is plausible if one assumes that a xenon thermal base effect occurs in the core regions adjacent to the reflector and that thermal transfer to the reflector occurs largely through convection.

At the higher (0.750 cps) frequency (Fig. 35.8), the ($-n\Gamma$) vector for the 90 per cent power test compares favorably with the calculated vector for the clean reactor. The calculated effect of the xenon temperature coefficient was to rotate the ($-n\Gamma$) vector counter-clockwise slightly with very little increase in magnitude. In actuality, however, the ($-n\Gamma$) vector was rotated counter-clockwise considerably and its magnitude was nearly doubled in the presence of the additional xenon. Such a large effect, apparently a result of fission product build up, is surprising. Assuming the measurements to be valid, the following possible reasons can be advanced to explain the results obtained.

As fission-product gases build up, the thermal conductivity of the fuel element may be expected to decrease; and therefore at high frequencies the temperature oscillation may have increased by as much as 20 per cent during the full-power continuous operation run. This could have caused the increased apparent prompt fuel temperature coefficient in the 90 per cent power xenon test. In addition, neutron spectrum changes during the long run may have had some effect on all temperature coefficients. Another possible explanation may be an increase in the prompt negative temperature coefficient resulting from Doppler broadening of fission-product resonances. The fission products, being intimately in contact with the fuel, would have the same rapid thermal time response as the fuel. There appears to be no way at present to choose from among these various speculations.

Other experimental evidence has been obtained which indicates the existence of an increase in fast temperature coefficient with buildup of fission products. The fast 2-deg rod-drop measurements obtained during the same full-power run were analyzed for temperature effects by Slovacek. (See Sec. 36.4.) He found that the prompt coefficient had increased by about the same factor (namely, 1.8) as was here observed in the two 90 per cent power pile oscillator tests.

Operational problems and other tests prevented a repetition of the full-power oscillator tests, and the question always remains as to the possibility of measurement errors indicating a false fission product sensitivity in the prompt temperature coefficient. Recent preliminary rod-drop and pile oscillator tests conducted at 40 per cent full power during prolonged operation indicate that the prompt coefficient did change quite strongly with reactor operation, and that it reached an equilibrium value when xenon reached equilibrium.

REFERENCES

In these later experiments, the experimental setup and procedure was partially converted to the null balance arrangement outlined in Chapter 34. Due, however, to a non-availability of a separate rotating pile oscillator, use was again made of one of the control cylinders, which was driven as described in this chapter through a servo system. This kept the measurements from being obtained in as simple a manner as described in Chap. 34.

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Chapter 36

TEMPERATURE EFFECTS FROM ROD-DROP EXPERIMENTS

R. E. SLOVACEK

36.1 INTRODUCTION

A series of pile-oscillator and rod-drop experiments were performed on the power reactor during initial runs approaching the full-power run. In this chapter the temperature effects obtained from the rod-drop experiments are discussed.

The equipment used in the rod-drop experiments was the same as part of that used in the pile-oscillator experiments (Chap. 35). The sinusoidal signal to the magnetic amplifier was replaced by an auxiliary battery supply to obtain a step-function driving potential for the rod "drops" and rod "jumps." The control cylinder was rotated the full 2 deg between stops in about 0.18 sec. A "rod drop" consisted of this 2-deg rotation made in the direction to reduce reactivity. A "rod jump" consisted of a 2-deg rotation in the opposite direction. As in the pile-oscillator experiments, recordings were obtained of cylinder position and change in neutron level for each run. In addition, time markers on each recording calibrated the chart speed.

36.2 ANALYSIS

The transient temperature effects on pile reactivity were obtained by analyzing the neutron level as a function of time during a fast power transient such as occurs in a rod-drop experiment.

If we neglect the effect of the source, the pile kinetic equations for reactivity changes encountered in the present rod-drop experiments may be written as¹

$$\frac{dn}{dt} = \frac{\beta(\Delta k - 1)}{\tau_o} n + \sum_{i=1}^6 \lambda_i C_i \quad (36.1)$$

and

$$\frac{dC_i}{dt} = \frac{\beta_i n}{\tau_o} - \lambda_i C_i \quad (36.2)$$

where n = instantaneous fission rate in the pile

β = delayed neutron fraction

β_i = delayed neutron fraction of group i

λ_i = decay constant of group i

Δk = dollars of reactivity above or below critical

τ_o = prompt generation time (seconds)

and C_i is proportional to the number of fission product delayed neutron precursors of group i

Equation 36.1 can be rewritten as

$$\frac{\tau_0}{\beta} \sum_{i=1}^6 \lambda_i C_i = (1 - \Delta k) n \left[1 + \tau_0 \frac{\frac{1}{n} \frac{dn}{dt}}{\beta(1 - \Delta k)} \right] \quad (36.3)$$

In a typical rod-drop experiment, a 10 per cent drop in neutron flux level accompanies a reactivity change of the order of -10 cents ($\sim 10^{-3}$ in absolute units) in 0.18 sec. Since $\beta \sim 10^{-2}$, then one obtains

$$\frac{\beta(1 - \Delta k)}{\frac{1}{n} \frac{dn}{dt}} \sim \frac{10^{-2}(1 - 0.001)}{0.10/0.18} \sim 2 \times 10^{-2} \text{ sec}$$

For this intermediate reactor, the value of τ_0 is equal to 37×10^{-6} (Sec. 34.4); therefore

$$\tau_0 \frac{\frac{1}{n} \frac{dn}{dt}}{\beta(1 - \Delta k)} \sim 37 \times 10^{-6} / 2 \times 10^{-2} \sim 2 \times 10^{-3}$$

This can be neglected when added to unity in Eq. 36.3, so that

$$(\tau_0 / \beta) \sum_{i=1}^6 \lambda_i C_i \cong (1 - \Delta k) n \quad (36.4)$$

or

$$\Delta k(t) \cong 1 - (\tau_0 / \beta) \sum_i \lambda_i C_i(t) / n(t) \quad (36.5)$$

The delayed-neutron production rate depends upon the past history of fissioning in the reactor:

$$\sum_i \lambda_i C_i(t) = \int_{-\infty}^t dt' n(t') \cdot \sum_i \lambda_i f_i e^{-\lambda_i(t-t')} \quad (36.6)$$

Hence a knowledge of $n(t)$ for all past times (obtainable from the neutron levels recorded during the experiment) leads to a knowledge of the change in pile reactivity.

The net change in reactivity $\Delta k(t)$ is the resultant of the change due to the motion of the control element $\Delta k_{CE}(t)$ and the change due to the effect $\Delta k_T(t)$ of the temperature changes in various reactor components during a power transient. A negative temperature coefficient makes these two changes oppose each other in sign for either a rod-drop or rod-jump experiment. That is, the net change in reactivity is less than the impressed change in reactivity from a control element if a net negative temperature coefficient exists in the reactor.

36.3 CALCULATIONS

The flux-level data were plotted to determine zero time by extrapolating the linear change in the power level or the control-element position back to the initial power level or control position. This was required since no zero time marker had been recorded and because the recorder trace usually started in a position on the chart where the deflection was nonlinear.

After the proper calibration has been applied to the corrected recorder deflections, the change in neutron level $\Delta n(t)$ in volts was obtained. The initial power level was indicated as a voltage (n_0) on a sensitive research voltmeter. The normalized neutron level $n(t)$ was then given by

$n(t) = 1 - \Delta n(t)/n_0$ for a rod drop, or

$n(t) = 1 + \Delta n(t)/n_0$ for a rod jump.

The delayed neutron level was calculated by numerical integration in a manner now to be indicated.

Previous to a rod drop or a rod jump at $t = 0$, the pile level was held constant for a large time compared to the longest delayed-neutron period, so that $n(t) = 1$ for $t < 0$. For this case, the normalized delayed-neutron level is expressed by modifying Eq. 36.6 to read:

$$(\tau_0/\beta) \sum_1 \lambda_1 C_1(t) = (\tau_0/\beta) \int_{-\infty}^0 dt' \sum_1 \lambda_1 f_1 e^{-\lambda_1(t-t')} + (\tau_0/\beta) \int_0^t dt' n(t') \sum_1 \lambda_1 f_1 e^{-\lambda_1(t-t')} \quad (36.7)$$

Numerical evaluation of the expression 36.7 is readily carried out if the power $n(t)$ is approximated as a linear function of the time during each of a number of equal time intervals of length h . (The typical interval ends at time $t_m = mh$ after the rod drop; the final one ends at time $t_M = Mh = t$.) Under this approximation, Eq. 36.7 becomes

$$(\tau_0/\beta) \sum_1 \lambda_1 C_1(t) = \sum_{m=0}^M D_m n(t_m) \quad (36.8)$$

where the coefficients D_m are given by

$$\begin{aligned} D_0 &= -K(t) - L(t) + L(t-h) \\ D_m &= L(t_m+h) - 2L(t_m) + L(t_m-h) \\ D_M &= K(0) - L(0) + L(h) \end{aligned} \quad (36.9)$$

and the functions K and L are defined as

$$\begin{aligned} K(t) &= (\tau_0/\beta) \sum_1 f_1 e^{-\lambda_1 t} \\ L(t) &= (\tau_0/\beta) \sum_1 (f_1/h\lambda_1) e^{-\lambda_1 t} \end{aligned} \quad (36.10)$$

The first numerical evaluation approach to Eq. 36.7 was in Report KAPL-706² which gives more of the details than appear here.

The following delayed-neutron group constants were used in the numerical evaluation of the coefficients:

$f_1 = 0.033$	$\lambda_1 = 0.0125 \text{ sec}^{-1}$
$f_2 = 0.220$	$\lambda_2 = 0.0315 \text{ sec}^{-1}$
$f_3 = 0.282$	$\lambda_3 = 0.153 \text{ sec}^{-1}$
$f_4 = 0.319$	$\lambda_4 = 0.455 \text{ sec}^{-1}$
$f_5 = 0.113$	$\lambda_5 = 1.61 \text{ sec}^{-1}$
$f_6 = 0.033$	$\lambda_6 = 13.9 \text{ sec}^{-1}$

Time intervals used were $h = 0.02, 0.04$, and 0.2 sec, respectively, for post-drop times up to $0.2, 0.28$, and 2 sec. The numerical values found for expression 36.8 were then substituted, together with the normalized neutron level $n(t)$, into Eq. 36.5 for the change in reactivity. Figure 36.1 is a typical curve showing the net change in pile reactivity $\Delta k(t)$ for a rod drop at full power.

Also plotted in Fig. 36.1 is the variation of the reactivity $\Delta k_{CE}(t)$ produced by the motion of the control element. The cylinder position was recorded as a function of time and the magnitude of the reactivity change was computed from control-cylinder calibration curves obtained in the PTR and in the power reactor at low burnup (Fig. 15.4). The value of $\partial k_{CE}/\partial \theta$ used was the value at the mean cylinder position during the rod drop.

Fig. 36.1—Reactivity transients following rod drop at full power. $\Delta k(t)$ is the total reactivity change deduced for the flux level; $\Delta k_{CE}(t)$ is the reactivity change produced directly by control element motion; and $\Delta k_T(t)$, the difference between the two, is the reactivity change resulting from the temperature variation.

The difference between the $\Delta k(t)$ and $\Delta k_{CE}(t)$ curves is the change in reactivity due to changes in the temperature of various reactor components during the power transient. In other words, the temperature effect is given by

$$\Delta k_T(t) = \Delta k(t) - \Delta k_{CE}(t) \quad (36.11)$$

where $\Delta k(t)$ and $\Delta k_{CE}(t)$ are both negative for rod drops (power decreases) and are both positive for rod jumps (power increases). If the temperature coefficient is negative as in this reactor, $\Delta k_T(t)$ is positive for rod drops and negative for rod jumps.

36.4 RESULTS

The results obtained from the rod drops and jumps from 5 to 100 per cent power are presented in Figs. 36.2-36.7. The normalized neutron level $n(t)$ and the temperature effect on reactivity $\Delta k_T(t)$ as given by Eq. 36.11 are shown for the rod drops and jumps at each power level.

Figure 36.2 is a plot of the normalized neutron levels and of the computed Δk_T values found at various times up to 2 sec after a rod drop at 5 per cent power. The probable errors are indicated on a few typical points. Figure 36.3 is a similar pair of plots for a rod jump. It is evident that the temperature effects are quite small, effectively zero, at this low power level.

The estimate of the error assigned to n is based on the errors in reading the chart for Δn , in reading n_0 , and in calibrating the chart. The error in $\Delta n/n_0$ is about 5 per cent near 0.04 sec but decreases to about 2 per cent after 0.15 sec. The estimated error

Fig. 36.2—Normalized power level and temperature-induced reactivity changes following rod drop at 5 per cent power.

Fig. 36.3—Normalized power level and temperature-induced reactivity changes following rod jumps at 5 per cent power.

Fig. 36.4—Power and temperature-induced reactivity changes following rod drop after 5 hr at full power.

Fig. 36.5—Power and temperature-induced reactivity changes following rod jump after 5 hr at full power.

Fig. 36.6—Power and temperature-induced reactivity changes following rod drops after 80 hr at full power.

Fig. 36.7—Power and temperature-induced reactivity changes following rod drops after 97 hr at full power.

assigned to $\Delta k_T(t)$ is based on the errors assigned to n , the coefficients D_m and to the 3 per cent error assigned to $\Delta k_{CE}(t)$. The $\Delta k_T(t)$ estimated error does not include the effect of the zero-time error (± 0.02 sec); this can affect the results significantly in the first 0.2 sec, as is evidenced in all the results by the large scatter of points in the first 0.2 sec when compared to the scatter in the next 1.8 sec. Since the estimated error of $\Delta k_T(t)$ is an absolute error, it is expected that the results for two successive rod drops should be internally consistent within a smaller error than the estimated error. When the mean cylinder position and the equipment calibration are the same for two successive rod drops or rod jumps, the results are indeed internally consistent (see, for example, Figs. 36.6 and 36.7). When the mean cylinder position and equipment calibration are different for various rod drops at a given power level, the results are consistent within the estimated error.

Figures 36.4-36.7 show the results of tests made at various times during a long high-power run. As noted in each of the n curves, the change in power level ($\Delta n/n_0$) varies approximately linearly from 0 to about 7.5 per cent in 0.18 sec and then varies approximately linearly from 7.5 to about 8.5 percent at 2 sec. At a given power level, of course, the detailed variation differs from this general variation because the magnitude of the temperature effect $\Delta k_T(t)$ differs for various power levels while the impressed reactivity $\Delta k_{CE}(t)$ is approximately the same at each power level. A detailed calculation on the transient reactor component temperatures would have to be done to see if a consistent set of temperature coefficients for the various reactor components would fit the present experimental results at the various power levels. Only once have rod drops at full power been so analyzed and compared with the experimental result.³ In that comparison it appeared that the fuel transient is actually faster than was expected from the analysis.

To obtain some comparison of the results at various power levels, the $\Delta k_T(t)$ values from rod drops and rod jumps at 20, 30, and 100 per cent power were normalized to a negative 1 per cent change in neutron level at 0.2 sec and plotted in Fig. 36.8. The estimated error is indicated on three points of each curve. Within the estimated error, the rod-drop and rod-jump results at a given power level are in good agreement.

Since the power transient occurs in about 0.2 sec, only the fuel and (to some extent) the sodium temperature coefficient can contribute an effect in this time. The other reactor components have small temperature coefficients or long time constants associated with them. There is an apparent change in the time constants of the prompt temperature effect when the results obtained at the beginning are compared with those found later on in the high-power run. The final 0.2-sec value of Δk_T is 1.8 times the earlier value, whereas the final 2-sec value of Δk_T is only 1.3 times the value at the beginning of the run.

36.5 CONCLUSIONS

The inherent stability of the reactor has been demonstrated in these rod-drop and rod-jump tests by the fact that a suddenly impressed reactivity change has invariably resulted in the development of opposing and smaller reactivity changes arising from temperature effects in the reactor. The calculation of the integrated temperature effect from the various reactor-component temperature coefficients and temperature transients during a power transient is admittedly somewhat uncertain; therefore, the method described in this chapter is a valuable tool for obtaining the integrated effect directly from the measurements obtained during the power transient. In addition, the results found by this method furnish an independent check on the pile-oscillator results (see Chap. 37). The uncertainty in the measurement of the temperature effect on reactivity during a power transient is estimated at about $\pm 0.35\%$ with the experimental techniques described in this chapter.

For a power transient developing within 0.2 sec at high power, the integrated negative temperature effect at 0.2 sec is 0.20% for each 1 per cent power change in the clean reactor and 0.35% for each 1 per cent power change after xenon and other effects have built up during about 90 hr of continuous reactor operation. Two seconds after initiation of the power transient at full power, the temperature effects have reached an approximate steady-state value of 0.38% for each 1 per cent power change in the clean reactor and 0.51% for each 1 per cent power change after about 90 hr of reactor operation.

Within the experimental error in the rod-drop experiments, all the results at the various power levels indicated that a net negative temperature effect exists with a response time of less than 2 sec.

Fig. 36.8—Temperature-induced negative reactivity normalized to unit (1 per cent) negative power change (rod drops or jumps occurred in 0.2 sec).

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Referred to in Section

36.2

36.3

36.4

Chapter 37

METHOD FOR COMPARISON OF TEMPERATURE COEFFICIENT FOR ROD-DROP AND PILE-OSCILLATOR EXPERIMENTS

R. SIEGEL

37.1 INTRODUCTION

The previous two chapters have shown how the power coefficient of a power reactor may be measured by two different methods. One method measures the reactor response to sinusoidal variations in multiplication, while the other method measures the time dependence of neutron level after a step-function change in multiplication constant. These two methods, the sinusoidal analysis and the transient analysis, are intimately connected through the formalism of the Laplace transform. It is the purpose of the present chapter to make comparison between these two types of data which were obtained from the power reactor.

In the rod-drop experiment (Chap. 36), the power variation with time, $\Delta n(t)$, is caused by the combined reactivity introduced by control-element motion, $\Delta k_C(t)$, and the reactivity change, $\Delta k_T(t)$, produced by temperature coefficients. The combined reactivity change, $\Delta k_C + \Delta k_T$, can be calculated from the experimental power curve by use of the pile kinetics equations, and the temperature effect $\Delta k_T(t)$ is then found by subtracting the known control-element reactivity variation from this result.

In the pile-oscillation experiment (Chap. 35), the control element is given a small sinusoidal oscillation, and the magnitude and phase relation of the resulting sinusoidal power oscillation δn is recorded. In the small signal limit, the sinusoidal oscillation in power level $\delta n/n$ about its average value n is proportional to the sinusoidal reactivity oscillation δk induced as a result of the control-element motion. We regard both oscillations as the real part of a corresponding complex exponential function, and write the constant of proportionality as A :

$$\delta n/n = A \delta k \quad (37.1)$$

The sinusoidal power oscillation gives rise to temperature oscillations which in turn produce oscillations in reactivity δk_T through the various temperature coefficient mechanisms discussed in Secs. 27.2, 30.2 and 35.2. On the assumption of linearity, these oscillations will also be sinusoidal and proportional to the power-level oscillation. We may, therefore, define a complex constant of proportionality between the power oscillations and the resultant temperature-induced reactivity oscillations:

$$\delta k_T = \Gamma \delta n \quad (37.2)$$

The reactivity oscillation, therefore, contains a direct contribution δk_C from the control-element motion, and an indirect contribution δk_T due to the power-level oscillation:

$$\delta k = \delta k_C + \delta k_T \quad (37.3)$$

Combining Eqs. 37.1, 37.2 and 37.3 and indicating explicitly the dependence of the complex quantities upon the oscillation frequency ω , we have:

$$\frac{\delta k_C(j\omega)}{\delta n(j\omega)/n} = \frac{1}{A(j\omega)} - n\Gamma(j\omega) \quad (37.4)$$

In order to see if the two types of experimental results are consistent, a basis for comparison of $n\Gamma(j\omega)$ and $\Delta k_T(t)$ is needed.

We can think of the reactor heat-flow system as having a complex impedance $\Gamma(p)$ which gives the temperature coefficient response resulting from a power transient,

$$\Delta k_T(p) = \Gamma(p)\Delta n(p)$$

For sinusoidal oscillations the complex number p becomes the pure imaginary $j\omega$, and the above expression becomes identical with Eq. 37.2. From the responses measured in the rod-drop experiments, the sinusoidal impedance $\Gamma(j\omega)$ is obtained and compared with the results of the pile oscillator experiment.

The comparison is here made for two different reactor conditions. For the "clean" condition the reactor had operated for about 5 hr at high power and had approximately \$1 xenon reactivity defect. For the "partially depleted" condition the reactor had operated for about 95 hr at high power and had a \$2.5 xenon reactivity defect.

37.2 ANALYSIS

Since the temperature effects are practically linear over the small power changes taking place during the experiments, the reactivity change caused by temperature coefficients during a power transient can be expressed as a displacement-type integral equation

$$\Delta k_T(t) = \int_0^t \Gamma(t-\tau)\Delta n(\tau)d\tau \quad (37.5)$$

The kernel $\Gamma(t-\tau)$ gives the reactivity change at time t due to a unit impulse in power at time τ and is equal to zero for $t < \tau$. Since the Laplace transform of a convolution of Γ and Δn is equal to the product of the individual transforms of $\Gamma(t)$ and $\Delta n(t)$, the transform of Eq. 37.5 gives

$$\Delta k_T(p) = \Gamma(p)\Delta n(p)$$

The desired value of $n\Gamma(j\omega)$ is then given by

$$n\Gamma(j\omega) = \frac{\Delta k_T(j\omega)}{\Delta n(j\omega)/n} \quad (37.6)$$

The procedure for comparing the rod-drop results with those of the pile oscillator can then be summarized as follows:

1. Approximate the rod-drop curves of $\Delta k_T(t)$ and $\Delta n(t)$ by algebraic functions of time.
2. Take Laplace transforms of these expressions to obtain

$$L[\Delta k_T(t)] = \Delta k_T(p)$$

$$L[\Delta n(t)] = \Delta n(p)$$

3. Replace p by $j\omega$ and form the ratio

$$\frac{\Delta k_T(j\omega)}{\Delta n(j\omega)/n}$$

This expression is then evaluated for several values of ω and compared with the oscillator results for $n\Gamma(j\omega)$ obtained from Eq. 37.4.

No precise rule exists for determining the range in frequency for which Eq. 37.6 will be valid for a given rod-drop transient. The experimental values as shown in Figs. 37.1-

Fig. 37.1—Average reactivity change due to temperature transients for two rod drops in clean reactor at full power.

37.4 extend only throughout the first 2 secs after the start of the rod drop. It is apparent that the temperature effects with long-time constants cannot respond appreciably in this time interval; thus, the results cannot be expected to yield values for low-oscillation frequencies. To make a rough estimate, we consider the 2-sec transient as being similar to a quarter of an oscillation cycle which corresponds to a frequency of 0.786 rad/sec. On this basis results have been calculated only for frequencies above $\omega = 1$ rad/sec. Details of the calculation are presented in the following sections.

37.3 RESULTS

A. Rod Drops for Clean Reactor

The curves of $\Delta k_T(t)$ and $\Delta n(t)$ which were obtained with the reactor after only 5 hr full-power operation are given in Figs. 37.1 and 37.2.

The experimental data for Δk_T were fitted with a sum of exponential functions

$$\pm \Delta k_T(t) = 2.87 - 2.71e^{-20t} - 2.27e^{-4.4t} - 0.604e^{-0.5t} \quad (37.7)$$

where a plus sign corresponds to a rod drop, a minus sign to a rod jump. This expression is shown in Fig. 37.1, along with the experimental curve. Since the experimental Δk_T curve represents an average of values for three runs, there is some scatter in the data (note the $\Delta n/n$ values on Fig. 37.2); hence, it was not felt worthwhile to obtain a more accurate curve approximation. The accuracy is further discussed in Sec. 37.4.

The $\Delta n(t)/n$ data were approximated by an average curve of the same type as Eq. 37.7:

$$\pm \Delta n(t)/n = 0.0932 - 1.45te^{-20t} - 0.0736e^{-25.8t} - 0.0196e^{-0.5t} \quad (37.8)$$

Fig. 37.2—Power changes for two rod drops and a rod jump in clean reactor at full power.

This relation has been plotted on Fig. 37.2 along with the experimental data for three runs.

Substituting into Eq. 37.6 the ratio of the Laplace transforms of the expressions in Eqs. 37.7 and 37.8, and noting that Δk_T and Δn are of opposite sign, we have

$$-n\Gamma(j\omega) = \frac{2.87}{j\omega} - \frac{2.7}{(j\omega + 20)^2} - \frac{2.27}{(j\omega + 4.4)} - \frac{0.604}{(j\omega + 0.5)} \quad (37.9)$$

$$= \frac{0.0932}{j\omega} - \frac{1.45}{(j\omega + 20)^2} - \frac{0.0736}{(j\omega + 25.6)} - \frac{0.0196}{(j\omega + 0.5)}$$

Strictly speaking, Eq. 37.9, and similar expressions which follow, are valid only as a limit as the real part of the transform variable p is made infinitesimally small. This is because of the nonconvergence of the Laplace transform of the constant terms for purely imaginary values of p . Physically, the limiting procedure can be visualized by considering that the function transformed contains a slowly decaying exponential factor with a relaxation time of several years, for example. In the limit, for very long decay times, the transform will approach the value in Eq. 37.9 as a limit.

Equation 37.9 has been evaluated numerically with the results:

ω , rad/sec	$-\Gamma(j\omega)n$ cents
1	30.3 - 3.99j
5	20.1 - 9.45j
10	14.8 - 6.39j
15	13.4 - 4.00j

These values of $-n\Gamma$ are plotted as solid circles on Fig. 37.5, where they have been vectorially added to the reference transfer function inverse $1/A_0$ computed by use of the pile kinetics equations¹ as indicated in Chap. 34 and shown in Fig. 34.4. The resultant locus represents values of $\delta k_c(j\omega)/(\delta n(j\omega)/n)$, deduced from rod-drop data, which, according to Eq. 37.4, can be compared directly to the results of the pile oscillator experiments.

B. Simplified Approximation

To gain some insight into how sensitive the values of $n\Gamma(j\omega)$ are to the accuracy of the curve-fitting, a simple exponential approximation was tried for both the Δk_T and Δn curves for the clean reactor:

$$\Delta k_T = 2.62 [1 - e^{-2.9t}]$$

$$\Delta n/n = 0.083 [1 - e^{-5.9t}]$$

These expressions have been plotted on Figs. 37.1 and 37.2 and do not approximate the data near the beginning of the rod drop ($0 < t < 0.2$ sec) quite as well as the relations used previously. Taking the ratio of transforms gives

$$-n\Gamma(j\omega) = \frac{2.62}{0.083} \left[\frac{\frac{j}{\omega} + \frac{1}{j\omega + 2.9}}{\frac{j}{\omega} + \frac{1}{j\omega + 5.9}} \right]$$

Evaluating this expression for a few values of ω gives the results

ω , rad/sec	$-n\Gamma(j\omega)$ cents
1	29.8 - 4.95j
5	19.6 - 6.97j
10	16.8 - 4.29j
15	16.1 - 2.99j

These values are plotted for the clean reactor as open circles on Fig. 37.5 and are close to those given by Eq. 37.9. Since the differences between each of the two approximations on Figs. 37.1 and 37.2 are about the same magnitude as the differences between the approximations and the actual data, it may be concluded that the results obtained are close to the values which would result from using more precise approximations to the experimental values.

Fig. 37.3—Average reactivity change due to temperature transients for two rod drops and a rod jump in partially depleted reactor at full power.

C. Rod Drops for Partially Depleted Reactor

The $\Delta k_T(t)$ curve for the reactor after 95 hr of high-power operation was fitted by the expression

$$\Delta k_T(t) = 3.42 - 33.8te^{-12t} - 2.80e^{-16t} - 0.62e^{-0.43t} \quad (37.10)$$

This is plotted on Fig. 37.3 where it is compared with the experimental curve.

As suggested by Fig. 37.4 the $\Delta n(t)/n$ data could be approximated quite well by two straight lines. This convenient approximation, however, produces an oscillation in the expression for $n\Gamma(j\omega)$, as is shown in Sec. 37.5. To avoid this oscillatory behavior, the curve was approximated as shown on Fig. 37.4 by a function with a continuous derivative:

$$\Delta n(t)/n = 0.081 - 1.23te^{-20t} - 0.0637e^{-25t} - 0.0173e^{-0.5t} \quad (37.11)$$

Fig. 37.4—Power change for two rod drops in partially depleted reactor at full power.

The ratio of the transforms in Eqs. 37.10 and 37.11 gives

$$-n\Gamma(j\omega) = \frac{3.42}{j\omega} - \frac{33.8}{(j\omega + 12)^2} - \frac{2.80}{j\omega + 16} - \frac{0.62}{j\omega + 0.43} \quad (37.12)$$

$$= \frac{0.081}{j\omega} - \frac{1.23}{(j\omega + 20)^2} - \frac{0.0637}{j\omega + 25} - \frac{0.0173}{j\omega + 0.5}$$

Numerical values of this expression are tabulated below. These points were plotted as solid circles on Fig. 37.5 and a smooth dashed curve drawn through them.

ω , rad/sec	$-n\Gamma(j\omega)$ cents
1	42.9 - 1.38j
5	38.6 - 9.36j
10	30.5 - 11.1j
15	25.7 - 8.88j

37.4 COMPARISON WITH PILE OSCILLATOR DATA

The dashed curves of Fig. 37.5 are plots in the complex plane of the values of $\delta k_c/(\delta n/n)$ from Eq. 37.4. It is to be remembered that both real and imaginary parts of the quantities plotted depend upon the frequency ω , and that the plots are loci with the frequency varying as one moves along a given locus. The solid curve gives the

Fig. 37.5—Reactor transfer function inverse for sinusoidal oscillations (as obtained from rod-drop experiments).

complex values, for various frequencies ω , of $n\delta k/\delta n$ for small values of n . This is simply the zero-power inverse transfer function discussed in Chap. 34; it was evaluated by means of Eq. 35.2. The values $n\delta k/\delta n$, evaluated experimentally from the rod-drop data by means of Eqs. 37.9 and 37.12, are plotted as the dashed curves for the reactor at power, both clean and partially depleted.

These data can be compared directly with the experimentally determined transfer functions from Chap. 35. The pile-oscillator data from the appropriate runs appear on Fig. 35.4, and it can be seen that both the rod-drop and the oscillator data show about a factor-of-two increase in the power coefficient $-n\Gamma$ in going from the clean reactor to the partially depleted reactor. (It is to be noted that the power contribution for a given oscillation frequency is the vector difference between the dashed and solid curve in Fig. 37.5.) Both sets of data also show a clockwise "looping" in the complex plane of the $-n\Gamma$ values. However, the magnitude of the power contribution $-n\Gamma$ as evaluated from the pile-oscillator data is approximately 50 per cent higher than the corresponding values derived from the rod-drop data. This probably reflects a normalization error in one or both of the experiments.

The counter-clockwise rotation of the power contribution $-n\Gamma$ observed in the pile-oscillator tests as the reactor changed from the clean to the partially depleted condition, which was explainable as a xenon thermal base effect in the outer core regions (Sec. 35.6), was not observed in the rod-drop experiments. This qualitative discrepancy is not understood.

37.5 APPENDIX: Approximation of $\Delta n(t)/n$ Curve by Straight Lines

The $\Delta n(t)/n$ curve in Fig. 37.4 could be approximated by two straight lines: the first passes through the origin with slope A and runs from $t=0$ to $t=t_1$, the second has a slope B and runs from $t=t_1$ to $t=\infty$. If we let $\delta(t)$ be a unit step function with these values:

$$\begin{aligned}\delta(t) &= 0 & t < 0 \\ \delta(t) &= 1 & t > 0,\end{aligned}$$

the curve of $\Delta n/n$ can be represented by $\Delta n/n = At - \delta(t - t_1)[A - B][t - t_1]$.

The Laplace transform of this function is:

$$L(\Delta n(t)/n) = \frac{A}{p^2} - e^{-t_1 p} \frac{(A - B)}{p^2}$$

When $j\omega$ is substituted for p , the term of the form $e^{-t_1 p}$ becomes

$$e^{j\omega t_1} = \cos \omega t_1 - j \sin \omega t_1$$

Thus, the straight line approximation leads to an oscillation in the transform which is proportional to the discontinuity in slope of the curve. To avoid this oscillation, the $(\Delta n/n)$ curve in Sec. 37.3 C was approximated by a sum of exponentials rather than by two straight lines.

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Referred to
in Section

37.3

Chapter 38

ANALOG CALCULATION OF A RUNAWAY

J. E. BARNES

38.1 INTRODUCTION

In order to translate into control design parameters the measurements and calculations of the various factors relative to the reactor kinetics, it is necessary to solve the time- and temperature-dependent reactor kinetic equations under various assumed operating conditions. The most stringent requirements upon the control system (rapidity of scram, control effectiveness, etc.) are imposed primarily by the behavior of the reactor under assumed accident conditions. Thus the emphasis of the kinetics studies is on the simulation of the time-dependent effects which can arise in a runaway. The control system must be so designed that it is capable of protecting the reactor from serious damage under accident conditions that might reasonably arise.

Under normal reactor operating conditions, the only large and rapid reactivity changes possible are those introduced by the reactor control system. However, if an abnormal power surge is taking place, there are other sources of reactivity (see Fig. 38.1). For example, the temperature of the sodium may increase, leading to an appreciable negative reactivity change. Large temperature excursions in the fuel may cause a reactivity change due to the fuel Doppler coefficient. A fuel element rupture may lead to a momentary increase and a subsequent decrease in reactivity (the "auto-catalytic effect"). Finally, and probably most important, there is the possibility of reactivity increase caused by the rapid entry of hydrogenous material into the reactor core; this might occur if there were a leak into the primary coolant system.

In order to determine the effect of incidents of the type described above, it is necessary to solve a time-dependent equation (the reactor kinetics equation) for the power level of the reactor. The solution then serves as a source term in the equation describing the thermal dependence of the reactivity effects in the reactor. The reactivity effects are then fed back into the reactor kinetics equations to find the (new) resultant reactivity and power level. This procedure is continued until the incident terminates, and the final desired result is the amount of energy stored in the reactor core during the incident.

The way in which the kinetics equations were solved in the work described here was by electrical simulation in which the coupling between the heat transfer and reactor kinetics mechanisms was represented by electrical interconnections between various components of the simulator.

One assumption which was made for simplification is that the spatial dependence of the flux within the reactor core does not vary with power level, time, or core temperature.

The several parts of the nuclear incident computer are indicated in Fig. 38.2. These various circuits simulate the reactor kinetics, the thermal behavior of the fuel elements and of the reactor coolant, and the coefficients of reactivity (which include the coolant temperature coefficient, the Doppler coefficient of the fuel, and the reactivity effects which might arise if the fuel elements should burst), the normal reactivity changes due to the reactor control system, the safety controls of reactivity, and, finally, the reactivity which may accidentally be introduced by plant casualties. Each of these simulations are discussed in turn in the following section.

NOTE:

T_W DETERMINES THE PRESSURE
AT WHICH THE FUEL ELEMENT
SPLITS OPEN

Fig. 38.1—Block diagram of the elements of nuclear incident calculations.

38.2 REACTOR KINETICS EQUATIONS

The reactor kinetics equations are:

$$\frac{dn}{dt} = \sum_{i=1}^6 \lambda_i C_i - (1 - \Delta k) n \frac{\beta}{\tau_0}$$

$$\frac{dC_i}{dt} = \beta f_i \frac{n}{\tau_0} - \lambda_i C_i$$

where τ_0 is the mean effective neutron lifetime,
 n is the reactor neutron density,
 Δk is the reactivity excess over delayed critical, in dollars,
 C_i is the density of the i th group of delayed neutron precursors,
 λ_i is the decay constant of the i th group of delayed neutron precursors,
 β is the effective delayed yield, taken as 0.0085. (The photoneutron contribution has been neglected and is believed to be less than 0.001.)
 f_i is the yield of the i th group of delayed neutron precursors.

Fig. 38.2—Incident computer diagram.

A precise definition of the quantities in this equation has been given by Goertzel in the Reactor Handbook.¹

Since most reactor incident calculations need to be accurate only in the vicinity of prompt critical (that is, $\Delta k = +\$1$), a one-group model of delayed neutron precursors was used in this computer setup. To derive an equivalent one-delayed-group model, one may take the Laplace transform of the reactor kinetics equations and force the one-group model to produce the same reactor period as the six-group model with the same assumed initial values of Δk and flux and the same initial contribution of delayed neutrons. It is to be noted that the equivalent decay constant of the one-group model is a function of the reactor period. The defect of this model is that the steady-state of the precursor is reached too quickly, but this is not a serious defect in nuclear incident calculations.

The computer setup for the reactor kinetics equations, shown in the upper part of Fig. 38.2, consists of the Δk servo-multiplier, amplifiers 13 and 15, integrators 1 and 2, and scale factor potentiometers 2, 3, 4, 5, and 14. The time is scaled 50 to 1 (i.e., 1 sec of machine time represents 20 milliseconds of real time) and 1 volt at integrator 1 is 100 per cent rated reactor power level.

Thermal Model of the Fuel Element

The analog of the fuel element is derived by direct consideration of the heat transfer characteristics in a unit length of one of the fuel elements (henceforth called the average fuel element). The analog must allow for the variations in the fuel temperatures during transients. It does this by setting up the equations of heat transfer involving the temperatures of the fuel and other materials which affect the heat flow. A typical computer setup for the thermal model of the fuel element is shown at the right in Fig. 38.2. This setup, which is rather general, may for example apply to the case of a double layer of fuel at temperature T_F sandwiching a backing layer at temperature T_M and sandwiched in turn by a wall at temperature T_W that conducts heat to a coolant layer which is at temperature T_{Na} .

The temperatures are scaled and normalized so that 1 volt on the computer represents 100°F. The thermal model of the fuel element appears in Fig. 38.2 in amplifiers 3, 4, 16, 5, 17, and 6.

Coolant Temperature Coefficient

A sodium temperature coefficient of -0.1 cent/°F is included in the model. It is introduced in the analog at the input of amplifier 12 as indicated by the symbol Δk_{Na} on Fig. 38.2.

Doppler Coefficient of Reactivity

The experimentally determined value of the fuel Doppler coefficient (from Chap. 33) was used in the analog calculations. Barrett has shown that the Doppler effect varies approximately logarithmically with the temperature of the fuel.² Consequently, the logarithmic function is set up on the function generator, FG#2-D in Fig. 38.2. The function generator is driven by the fuel temperature, and its output, Δk_D , feeds into the reactivity summator, amplifier 12.

Autocatalytic Effect

The autocatalytic effect may be as large as +\$3 if the entire fuel load bursts out of containment and redistributes itself in the worst possible way, and it may reverse sign in about 30 milliseconds.

The probability that a fuel element will burst open is assumed to depend primarily upon the fuel temperature T_F . Also, since the vapor pressure of the fuel is a sharply rising function of temperature, fuel element bursting should be very nearly a threshold phenomenon. On this account it is assumed that elements burst in a narrow range of fuel temperatures and, further, that the probability of bursting an element depends mainly on the power distribution throughout the active section of the reactor core. The instantaneous rate of bursting fuel elements $P_B(t)$ is, therefore,

$$P_B(t) = P(T_F) dT_F / dt$$

the product of the probability as a function of temperature and the time rate of change of fuel temperature.

To include the autocatalytic reactivity in the solution of the reactor kinetics equations, it is necessary to assume that each element bursting contributes its proportionate fraction of the effect produced by all the fuel, and furthermore, that the total reactivity effect of the bursting fuel elements follows the equation

$$\Delta k_{AC} = \int_{-\infty}^T P_B(t) k_{AC}(T-t) dt$$

where $P_B(t)$ is the instantaneous rate of bursting.

The function $k_{AC}(t)$ is called the autocatalytic kernel and must have the delta-function (impulsive) response of the following nature:

$$\text{At } t = 0, k_{AC}(0) = 0$$

$$\text{At } t = 0.015 \text{ sec, } k_{AC}(0.015) = + \$3/W \text{ (its maximum value),}$$

$$\text{At } t = 0.030 \text{ sec, } k_{AC}(0.030) = 0$$

After a long time, $k_{AC} (t > 0.1) = \$20/W$, corresponding to total removal of the fuel in the element from the end reflector. Here W is the total number of fuel elements in the reactor.

The equipments which compute the convolution integral are shown in Fig. 38.2 as FG#1- n_B , n_B SERVO, the Function Switch FN and amplifiers 10, 14, 11, 19S, 20S, and 7. Amplifier 18S computes the total percentage of fuel elements which fail, N_B . The purpose of the function switch is to prevent the computer setup from "repairing" the fuel elements. The time scale is slow enough that this function can be performed manually. On the first incidence of fuel element failure the function switch is closed when amplifier 14 first goes positive and reopened when amplifier 11 goes to zero. It is again closed when amplifier 4 exceeds its previous peak and reopened when amplifier 11 goes to zero.

Reactivity Introduced by the Control System

It was assumed that the maximum rate at which the controls can insert reactivity is 20¢/sec. In the computer setup, this rate of reactivity is set by adjusting scale factor potentiometer 19.

Reactivity Introduced by Hydrogenous Material

The method of studying reactivity casualties caused by the rapid introduction of hydrogenous materials into the reactor core is to reduce the casualty to an equivalent high rate of reactivity introduction, depending on the amount of hydrogen in the sodium and the rate of sodium flow. The reactivity rate is produced in the computer setup of Fig. 38.2 by adjusting the integrating rate of integrator 7 with scale factor potentiometer 23 in the same way as for the control system runaway problem. Since +\$2 of reactivity is available in the model, high rates of reactivity increase can be studied.

38.3 RESULTS OF CALCULATION

The results of a typical incident calculation may be seen in Fig. 38.3 which shows the flux, the total reactivity, the fuel temperature, and the percentage of fuel elements burst. The initial conditions assumed for the problem are: flux, full rated value; coolant flow, full rated value; initial exponential rise period, 30 milliseconds.

In a rough way, the course of this particular runaway may be described as follows. Soon after the reactivity rise above prompt critical (at $t = 0$ on Fig. 38.3), the flux rises up to a level about 30 times the flux at full power, and the fuel temperature increases rapidly, but not up to the bursting threshold, all in about 50 milliseconds. The Doppler coefficient reduces the reactivity faster than the control element increases it during this time, so that the reactivity falls below prompt critical and the flux drops down to about 5 times the full power level at $t = 0.1$ second. Fuel temperature rises more gradually thereafter, and the sodium temperature coefficient begins to reinforce the Doppler and other negative coefficients in keeping the reactivity below prompt critical, even though the flux level and temperature steadily mount.

When the central fuel temperature passes the fuel element bursting threshold, at about 0.9 sec, the elements begin to burst and continue to do so more and more rapidly, each adding momentarily to the reactivity and the ensuing flux for 30 milliseconds after it bursts, but subtracting from the reactivity thereafter. At about 1.0 sec the reactivity begins to drop precipitously, and the flux follows. Soon thereafter the last of the bursts occurs, the reactivity goes below delayed critical, and the 90 per cent of the fuel remaining in the reactor begins to cool down. Sustained by the delayed neutrons, the flux holds at a level several times full-power level, but the fuel continues to cool down for several seconds. There being no scram to stop the controls, the reactivity gradually builds up again; after a number of seconds it may again exceed prompt critical and begin another runaway.

From studies of this nature, control actuation and safety systems may be designed to prevent damage to the reactor assembly. In the calculation leading to Fig. 38.3, however, it was assumed that no scram was available. The incident was terminated only by the destruction of the fuel elements and their subsequent removal from the core.

Fig. 38.3—Results of fuel element bursting after control runaway.

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