**Evolution of Flow Disturbances
in
Cocurrent Gas-Liquid Flows**

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Abstract

Long wave stability curves for two-layer laminar flow, laminar liquid and a turbulent gas (simulated with a polynomial profile) using the full differential equations and boundary conditions are compared to the standard methods of predicting flow regime transitions -- one dimensional equations. The differential models predict instability at significantly less severe conditions than the integral equations. This difference is as much as an order of magnitude when plotted on friction velocity -- liquid depth coordinates. Because this plot removes most of the effect of the base state, it is clear that the one-dimensional models are not predicting linear stability of anything. This means either that agreement between one-dimensional models with observed transitions is fortuitous because the flow is not sufficiently long or because convenient choices have been made for adjustable parameter values. Otherwise, flow regime transitions are not linked with linear stability.

A polynomial profile gas, laminar liquid model, developed to provide a first approximation to turbulent flow, gives growth curves that are close to the two-layer laminar exact solutions if the interfacial friction velocity and liquid depth are matched. The primary differences are that the wavelength is predicted to be somewhat shorter for the turbulent model and the growth rate is slightly larger for a laminar gas. These linear stability studies point to the need to determine first the base state before any significant results can be obtained.

Analytical and numerical solutions for laminar flow in a rectangular channel over a solid wavy surface demonstrate that the ratio of wavelength to channel height has a profound effect on the behavior of the pressure and shear stress variations. If the wavelength is long compared to channel height, the pressure is completely in phase with the wave *slope*, not the wave height, as occurs for high Re in an infinitely high channel. Nonlinear effects reduce the relative magnitude of the shear stress variation and phase angle which could explain saturation in the growth of waves (e.g. Craik's "slow waves") that are formed by shear variation for very thin liquid layers. However the pressure variation and phase are increased so that this is not a likely explanation for the saturation of waves on thicker layers.

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Description of Research

Comparison of linear theories based on differential and one-dimensional equations

We examined (see attached paper by Kuru) the problem of linear stability of stratified channel flow with the goal of comparing different stability models and determining the role of turbulence. Figure 1 shows a comparison of different models that are predicting long wave stability. Lin and Hanratty (1986) used a macroscopic momentum model (i.e., one dimensional momentum equation). Our calculations employ the full two-layer Orr-Sommerfeld equations and boundary conditions. We show both the laminar gas-laminar liquid result and a polynomial gas (Pai, 1953) - laminar liquid that is intended to predict the *qualitative* effect of turbulence. The differences between the macroscopic and differential models are probably important when predicting flow regimes. The differential model, particularly when a turbulence model is added, predicts instability at U_{SG} substantially below the one-dimensional model if the interface is assumed smooth. Using a roughness factor, $f_i = 2 f_s$, improves the agreement somewhat. While the interface does become rough, this comparison suggests that f_i is acting more like an adjustable constant than fundamental part of the model. The difference between the laminar and polynomial models occurs because there is considerably more interfacial shear for the polynomial model at equal flow rates. Consequently, instability is predicted at lower U_{SG} .

Figure 2. shows the comparison plotted as liquid friction velocity versus liquid depth. In this plot we have added the predictions of another one-dimensional model by Crowley et al. (1993). This plot was done in an attempt to remove the effect of the base state (i.e. fluid depths and velocities) which varies with the models. Friction velocity and liquid depth seem to be the two factors that contribute most to the instability mechanism (for fixed fluid properties). First note that short waves are unstable at much lower interfacial shear than long waves. When compared in

Figure 1. Comparison of long wave linear stability models on Mandhane coordinates.

Figure 2. Comparison of different stability models in terms of interfacial stress and liquid height. This plot is intended to remove the effect of different base states.

terms of the mechanism of instability (i.e., interfacial shear) the two differential models differ slightly. The polynomial model (see attached paper by Kuru et al. for a description) has a slightly higher transition. This difference is due entirely to the different shape of the polynomial velocity profile. Note that the difference is small enough that it would be difficult to verify experimentally. The one-dimensional model of Crowley et al. is a factor of 2-3 above the two differential models for thin liquid films and converges somewhat for thicker layers. Lin and Hanratty's model is another factor of 2 higher at small h and crosses at higher liquid depth.

There are clearly differences in the predictions between differential and one-dimensional models. It is likely that as conditions are varied, they could be much larger than those shown for the present air-water flow. Consequently, use of one-dimensional models to predict flow regimes is suspect. Efforts are needed to determine if the reported agreement between one-dimensional models is fortuitous, either because of limited length experiments or convenient choices for parameter values, or if regime transitions are not directly linked to linear instability.

Comparison of growth rates from polynomial gas and laminar gas.

Figure 3 shows a set of growth curves for an air-water system just above the neutral stability conditions for short waves. The polynomial and laminar gas models are compared. This is done by adjusting the laminar gas Reynolds number to match the liquid height and friction velocity of the polynomial model. The laminar-laminar flow peak is at lower wavenumber and its growth rate is consistently higher by a factor of about 2. These differences in the growth predictions can be explained by examining the stress components shown in figure 4 which are calculated for an air-water system at some nominal conditions. In this figure τ_{SR} and τ_{SI} are the real (in phase with wave height) and imaginary (in phase with the wave slope) components of the perturbation shear stress at the air-water interface. The corresponding pressure components are P_{SR} and P_{SI} . It is

Figure 3. Growth rates for laminar and polynomial gas models.

Figure 4. Calculated stress components at the wavy interface for the parabolic and polynomial profiles.

known that PS_I is the component that contributes most to growth of moderate wavelength waves close to neutral stability. Figure 4 shows that PS_I is quite different for the two models with the secondary peak for a parabolic gas, which occurs at a nondimensional wavenumber $k d = 0.8$, being higher and at a lower wavenumber than the polynomial gas. This is in exact qualitative agreement with the growth rates. It must be noted that in figure 6 of the Kuru et al. paper, a similar comparison is done and real data for turbulent flow over a solid wavy surface are included. In that figure the measured PS_I is similarly peaked at a wavenumber higher than for a parabolic profile but the magnitudes of the two cases are much closer.

It will be difficult to verify the growth rate predictions experimentally. The factor of 2 difference in growth might be measurable for sufficiently high R_G ; but if R_G is too large, then there is no useful linear growth range. It is possible that the differences in wavelength are accessible by experiments. However while there is a significant difference in the maxima, it is not sharp. The wide band of unstable modes is enough to make growth rate experiments difficult.

Laminar flow over a wavy surface in a channel

The wave phenomena that occur in gas liquid flow are strongly influenced by the pressure and shear stress variations of the gas flow above the waves. Almost all the numerous previous experimental and theoretical studies have focused on the short wave situation where the wavelength is short compared to the channel height. In the attached paper by Montalbano, we describe our analytical and numerical results for flow over waves where the wavelength is long or order 1 compared to channel height.

The linear and weakly nonlinear problems were solved analytically. The linear results are:

$$\frac{\hat{|\tau_w|}}{\langle \tau_w \rangle} = 4 + \frac{1}{35} \text{Re } \alpha + \frac{2}{8085} (\text{Re } \alpha)^2 + \frac{4}{15} \alpha^2 + \frac{1}{105} \text{Re } \alpha^3 -$$

$$\frac{17 I}{5659500}(\text{Re } \alpha)^3 - \frac{1123}{15765750} \alpha^4 \text{Re}^2 - \frac{949801}{24397953580000} (\alpha \text{Re})^4 + \frac{4}{1575} \alpha^4 + \dots, \quad (1)$$

and

$$\frac{\hat{p}_w}{\langle p_w \rangle} = -\frac{36 I}{\alpha \text{Re}} - \frac{54}{35} + \frac{36 I}{5 \text{Re}} \alpha + \frac{117}{13475} I \alpha \text{Re} - \frac{57}{175} \alpha^2 + \frac{52 I \alpha^3}{175 \text{Re}} + \frac{1233}{12262250} (\alpha \text{Re})^2 + \frac{31}{175175} I \alpha^3 \text{Re} - \frac{7861641}{6099488395000} I (\alpha \text{Re})^3 + \dots \quad (2)$$

The weakly - nonlinear results are

$$\frac{\bar{\tau}_w}{\langle \tau_w \rangle} = 1 + a \left(4(\cos(\alpha x) - \frac{1}{35} \alpha \text{Re} \sin(\alpha x)) + a^2 (3 + 3 \cos(2\alpha x) + \frac{9}{70} \alpha \text{Re} \sin(2\alpha x)) + \dots \right), \quad (3)$$

and

$$\frac{\bar{p}_w}{\langle p_w \rangle} = -\frac{12}{\text{Re}} x + a \left(36 \frac{\sin(\alpha x)}{\alpha \text{Re}} - \frac{54}{35} \cos(\alpha x) \right) + a^2 \left(\frac{54}{2} \frac{\sin(2\alpha x)}{\alpha \text{Re}} - \frac{54}{\text{Re}} x \right) + \dots \quad (4)$$

Figure 5 displays the total shear variation from Eq. 3, as well as the contribution from linear and nonlinear terms of the expansion for $\text{Re}=500$, $a/H = 0.05$ and $a = 0.02, 0.05, 0.08$. The nonlinear contribution, which consists of the overtone of the wave and a constant term, is positive over most of the wave; its largest contributions occur a little downstream of the crest and the trough. The effect is more pronounced as the wavenumber increases. Nonlinear effects increase the maximum in stress, which is upstream of the crest for the linear case, and move it back toward the wave crest. The nonlinear correction reduces the drag in the trough and moves the maximum negative value upstream. There is an overall increase in drag because of the presence of the $3a^2$ term. The major nonlinear correction to the pressure variation is the additional pressure drop term, $-54/\text{Re } x$, which

Figure 5. Shear stress deviation over a wave comparing linear and nonlinear contributions. $Re=500$, $\alpha=.02, .05, .08$

Figure 6. Variation of phase shift with Reynolds number for $\alpha = .125, .25, .375, .5$ for $\lambda = 16.666$

increases the pressure drop substantially. The other nonlinear term contributes only slightly to the result.

An interesting result for a moderate value of the wavenumber is shown in figure 6. The shear stress phase angle (degrees upstream of the crest) as a function of Reynolds number is shown for all four amplitudes with $\lambda=16.666$ ($\alpha = .377$). At all amplitudes there is a sharp increase in phase at low Re which then levels off at around $Re=300$. Also as amplitude increases there is a decrease in phase in shift that is close to linear. At low Reynolds numbers, both Re and amplitude are important for determining phase shift; at higher Reynolds numbers amplitude is the determining factor. Equation 1 shows that the phase shift of the shear increases with increasing wave number for long waves. Numerical results of Montalbano (1994) (not shown here) agree with the long wave trends up to dimensionless wavenumbers of 0.75. For wave numbers larger than .75, the phase shift decreases with increasing wave number in agreement with Benjamin (1959) who analytically demonstrated this decrease with increasing α .

The phase angle for the pressure minimum varies from -102° to -140° as the Reynolds number increases from 10 to 500. The inviscid value is -180° . For fixed Reynolds number the phase angle moves away from the inviscid value as the amplitude is increased. For $Re = 100$, the phase angle is -137° , -131° and -101° for amplitudes of 0.125, 0.25 and 0.5. Similar behavior occurs at other Reynolds numbers. The effect is even larger if separation occurs.

Other work

In another project we have compared measured wave amplitudes in a two-layer laminar *rotating* flow with weakly-nonlinear theory. The results agree well. We are currently preparing a publication for similar measurements in the air-liquid channel. This was partially supported on this grant and it will be forwarded when it is completed.

We are preparing a publication documenting the transition to three-dimensional waves in gas-liquid flow. This work was partially supported on this grant and it will be forwarded when it is completed.

We are preparing a publication on the how speeds of solitary waves scale with wave shape and amplitude. It is interesting that the speed scales with the total volume of water within the wave, not the amplitude as classic theories predict. This work was partially supported on this grant and it will be forwarded when it is completed.

References

- Crowley, C. J., G. B. Wallis & Barry, J. J. 1992 Validation of a one-dimensional wave model for the stratified to slug flow regime transition, with consequences for wave growth and slug frequency. *Int. J. Mult. Flow* **18** 249-271.
- Crowley, C. J., G. B. Wallis & Barry, J. J. 1993 Dimensionless form of a one-dimensional wave model for the stratified flow regime transition. *Int. J. Mult. Flow* **19** 369-376.
- Lin, P. Y. & Hanratty, T. J. 1986 Prediction of the initiation of slugs with linear stability theory. *Int. J. Multiphase Flow* **12**, 79-98.
- Montalbano, E. D. 1994 Laminar flow over solid waves, Ph.D. Thesis, University of Notre Dame.
- Pai, S. I. 1953 On turbulent flow between parallel plates. *J. App. Mech.* **20**, 109-114.

Publications resulting from this grant

- L. A. Jurman and M. J. McCready, "Study of waves on thin liquid films sheared by turbulent gas flows", *Physics of Fluids A* **1**, pp 522-536, 1989.
- L. A. Jurman, K. Bruno and M. J. McCready, "Periodic and solitary waves on thin, horizontal, gas-sheared liquid films", *International Journal of Multiphase Flow* **15**, pp 371-384, 1989.
- K. Bruno and M. J. McCready, "Study of the processes which control the interfacial wave spectrum in separated gas-liquid flows", *International Journal of Multiphase Flow* **15**, pp 531-552, 1989.

M. J. McCready, L. A. Jurman, C. -A. Peng and S. E. Deutsch, "Evolution of interfacial disturbances in horizontal gas-liquid flows", pp 129-136, in Proceedings of the Eight Symposium on Energy Engineering Sciences. (1990).

M. J. McCready, M. Gupta, L. A. Jurman and K. Bruno, " Properties of small-scale waves in sheared gas-liquid flows", in *Air-Water Mass Transfer* ed. Wilhelms & Gulliver pp 116-125. ACSE 1991.

C. -A. Peng, L. A. Jurman, and M. J. McCready "Formation of solitary waves on gas-sheared liquid layers", *International Journal of Multiphase Flow*, 17, pp 767-782, 1991.

L. A. Jurman, S. E. Deutsch, and M. J. McCready "Interfacial mode interactions in horizontal gas-liquid flows", *J. Fluid Mech.* 238, pp 187-219, 1992.

M. Sangalli, Th. Prokopiou, M. J. McCready and H. -C. Chang, "Observed transitions in two-phase stratified gas-liquid flow", *Chem. Eng. Sci.*, 47, pp 3289-3296, 1992.

W. C. Kuru, E. D. Montalbano, J. F. Brennecke and M. J. McCready, " Evolution of long wave disturbances in horizontal gas-liquid flows", pp 32-39, in Proceedings of the Eleventh Symposium on Energy Engineering Sciences. (1993).

accepted:

T. J. Hanratty and M. J. McCready, "Phenomenological understanding of gas-liquid stratified flows", *Multiphase Science and Technology*, 10/92.

submitted:

W. C. Kuru, M. Sangalli and M. J. McCready, "Linear stability of stratified channel flow", *Int. J. Mult. Flow* on 4/15/94.

M. J. McCready and H. -C. Chang, "Formation of large disturbances on sheared and falling liquid films" submitted to: *Chem. Eng. Comm.* on 10/27/94.

E. D. Montalbano and M. J. McCready, "Laminar channel flow over long and moderate waves", submitted to *J. Fluids Engineering*, on 11/30/94.

Presentations resulting from this grant

M. J. McCready, S. E. Deutsch and L. A. Jurman, "Modal wave interactions on air-sheared liquid layers", annual meeting of the Fluid Mechanics division of the American Physical Society, Palo Alto, CA, November 1989.

M. Cheng, D. Enright, L. A. Jurman, H. -C. Chang and M. J. McCready, Nonlinear wave instabilities on sheared liquid layers: experiments and theory, AIChE Annual meeting, Fall, 1989.

M. Gupta and M. J. McCready, "Numerical and analytical study of interfacial mass transfer", AIChE Annual meeting, Fall, 1989.

M. J. McCready, L. A. Jurman, C. -A. Peng, and S. E. Deutsch, "Evolution of interfacial disturbances in horizontal gas-liquid flows", Eighth symposium on energy engineering sciences, Argonne, IL May, 1990.

M. J. McCready, M. Gupta, L. A. Jurman and K. Bruno, " Properties of small-scale waves in sheared gas-liquid flows", 2nd International Symposium on Gas Transfer at Water Surfaces, Minneapolis, MN, September, 1990.

S. E. Deutsch, L. A. Jurman and M. J. McCready, "Characterization of chaos in interfacial waves in an open flow system" AIChE Annual meeting, Fall, 1990.

C. -A. Peng, L. A. Jurman and M. J. McCready, "Characterization of solitary waves on thin, gas-sheared liquid layers" AIChE Annual meeting, Fall, 1990.

M. J. McCready, S. E. Deutsch, and L. A. Jurman, "Investigation of chaotic behavior of interfacial waves in an open flow system" annual meeting of the Fluid Mechanics division of the American Physical Society, Ithaca, NY, November 1990.

Th. Prokopiou, M. J. McCready & H. -C. Chang, "Transitions to three dimensional waves on horizontal gas-sheared liquid films", AIChE Annual meeting, Fall, 1991.

M. Sangalli, M. J. McCready & H. -C. Chang, "Spatial instability of two-phase stratified flow: theory and experiment", AIChE Annual meeting, Fall, 1991.

M. J. McCready "Disturbance origination and propagation in open flow systems", AIChE Annual meeting, Fall, 1991.

M. Sangalli, Th. Prokopiou, M. J. McCready & H. -C. Chang, "Transitions in stratified two-phase flows: theory and experiments", Conference: *Instabilities in multiphase flow*, Rouen France, May 11-13, 1992.

M. Sangalli, Th. Prokopiou, M. J. McCready and H. -C. Chang, "Observed transitions in two-phase stratified gas-liquid flow", 1st International conference on gas-liquid and gas-liquid-solid reactor engineering, Columbus OH, September 13-16, 1992.

M. Sangalli, M. Cheng, M. J. McCready and H. -C. Chang, "Interfacial turbulence in two-phase channel flow", AIChE Annual meeting, Fall, 1992.

M. J. McCready, M. Sangalli and H. -C. Chang, "Prediction of stratified flow stability by linear and non linear theories", AIChE Annual meeting, Fall, 1992.

M. J. McCready and G. Su, "Frequency selection of roll waves in a gas-liquid flow", annual meeting of the Fluid Dynamics division of the American Physical Society, Tallahassee, FL, November 1992.

W. C. Kuru, E. D. Montalbano, J. F. Brennecke and M. J. McCready, " Evolution of long wave disturbances in horizontal gas-liquid flows", Eleventh symposium on energy engineering sciences, Argonne, IL May, 1993.

M. Sangalli, M. J. McCready and H. -C. Chang, "Defect-mediated transition to interfacial turbulence in two-phase channel flow", AIChE Annual meeting, Fall, 1993.

W. C. Kuru, E. D. Montalbano, J. F. Brennecke and M. J. McCready, "Frequency selection of roll wave precursors in gas-liquid flows", AIChE Annual meeting, Fall, 1993.

M. J. McCready, "Issues, Questions and Accomplishments in Gas-Liquid Flows", presented at gas-liquid flow symposium, Urbana, IL November, 1994.

Graduate theses resulting from this grant (some are partially supported)

Ching-An Peng, "The formation of solitary waves in horizontal, gas-liquid concurrent flows" -- M. S., 1990

Lorraine A. Jurman, "Interfacial waves on sheared, thin liquid films" -- Ph. D., 1990.

Malini Gupta, "Fundamental analyses of heat and mass transfer in two-phase flows" -- Ph. D., 1991.

Theologos Prokopiou, "Nonlinear waves on liquid interfaces" -- Ph. D., 1992.
(with H. -C. Chang)

Chee K. Cheng, "Study of wave evolution in gas-sheared films and falling liquid films using optical imaging" -- M. S. 1994.

Mahesh Lakshkiarayanan, "The characterization of speeds of solitary waves" -- M. S. 1994.

Edward Montalbano, "Laminar flow over a solid waves" -- Ph. D. 1994.
(with J. F. Brennecke)

in progress

William Kuru, "Two-phase instabilities" -- Ph. D.

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