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Cross Flow Induced Vibrations in Staggered Arrays of Cylindrical Structures

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy
in Mechanical Engineering

by

Jure Marn

1991

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I dedicate this work to Zdenka who stood by me and helped me fight.

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List of Symbols

Roman letters:

A	...	area [m^2]
C	...	terms in equation of tube motion, associated with fluid damping
C, D_{1,2}	...	constants in solution of equations (4.20, 4.45)
F	...	fluid force, subscripts x and y denote the force components
F_{TL}	...	force in transverse, longitudinal direction, also lift, drag [N/m]
H	...	constant arising from boundary conditions
K	...	terms in equation of tube motion, associated with fluid stiffness
M	...	terms in equation of tube motion, associated with acceleration
P	...	pressure, function of x only, ~denotes dimensionless P
Re, Re_s	...	ordinary and streaming Reynolds numbers
U_r	...	dimensionless "reduced velocity", used as a mean velocity
V	...	volume [m^3]
V	...	mean velocity vector with components V _θ and V _r
U, V	...	components of mean velocity vector in local x and y directions, from equation (4.14) on used as a function of x only
X, Y	...	cylinder displacements in global Cartesian coordinates directions, from equation (4.14) on used as a function of x only, (in eq. 4.23 hat denotes dimensional X)
a, b, c⁺, r_{lf}	...	various lengths used in determination of boundary conditions
c	...	damping parameter of the cylinder
c	...	shear coefficient
d	...	diameter of the tube
e	...	energy
f	...	liquid component
f	...	a function of ...
f	...	body forces vector [N]
f[*]	...	dimensionless decay function
g	...	gravity, gaseous component
i	...	imaginary unit
i	...	numbering of adjacent cylinders
k	...	wavenumber
k_s	...	stiffness of the tube
l₀	...	characteristic decay length, usually 2 d [m]
m	...	tube mass per unit depth
p	...	pressure
q	...	heat flux
r	...	radial coordinate
r_{inn,out}	...	inner and outer radiuses of computational domain
s	...	exponent in void fraction distribution
s	...	coordinate along streamline
s_{a,s}	...	position of the attachment, separation point
s_{e,i}	...	position of exit, inlet from control volume calculational domain

t	...	time
u, v	...	components of perturbed velocity vector in local x and y directions
$\vec{v}$	...	velocity vector with components $\vec{u}$ and $\vec{v}$ in local x and y coordinate directions
w_f	...	liquid shear on the wall
w_g	...	gaseous shear on the wall
x, y	...	local coordinate system axes

Greek letters:

Γ	...	vapor generation
Σ	...	sum
α	...	void fraction
α	...	angle between attachment point on the tube and midplane [rad]
δ	...	logarithmic decrement
η	...	z component of vorticity
θ	...	angular coordinate
θ_0	...	side borders of computational domain
λ	..	exponents of the solution of eqs. (4.20, 4.45)
π	...	Ludolf number
ρ	...	density
η, ξ	...	dummy variables in integral involving s^*
λ_m	...	tube mass [kg]
ω	...	frequency, subscript n denotes natural frequency of the tube
ν	...	kinematic viscosity of the fluid

Other symbols:

$*$	...	dimensionless variables
$\dot{}$	...	second and first derivatives with respect to time in eq. (4.11 and 4.13)
$\ddot{}, \dot{}$	...	third, second and first derivatives wrt local x -coordinate in equations (4.15) and beyond
∇	...	gradient operator
∂	...	partial derivative
$\times$	...	cross product
$\Im$	...	imaginary part of complex number
$\Re$	...	real part of complex number

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ABSTRACT OF THE DISSERTATION

Cross Flow Induced Vibrations in Staggered Arrays of Cylindrical Structures

by

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Flow induced vibrations caused instability is the subject of this investigation. The bulk of the work performed is theoretical in nature, the comparison with some of existing experimental data is given for each of four models described.

First model encompasses the effects of prescribed motion on the cylinder. Such circumstances occur in the case of vortex shedding initiated instability. The reduced velocity within the cylinder array is low and there is no coupling between the adjacent cylinders.

Second model assumes certain form of vibration and corresponding behavior of the perturbed velocity field in temporal and one of spatial coordinates thus transforming partial differential equations into ordinary differential equations and

takes into account the motion of the neighboring cylinder. This corresponds to fluid elastic controlled instabilities. The resulting equations are solved analytically. The model is used for better understanding of the equations of cylinder motion as well as for quick estimates of threshold of instability.

Third model relaxes an assumption about the form of vibration in spatial direction and uses the vorticity formulation of equation of fluid motion to account for fluid-solid interaction. Due to complicated forms of governing equations the analytical approach could no longer be used. Alternate Direction Implicit (ADI) scheme was used to solve partial differential equations, integrals were evaluated using numerical techniques as well. This model shows excellent agreement with experiments and offers an empirical formula connecting reduced velocity, streaming Reynolds number and pitch to diameter ratio as design tool.

Fourth model is very similar to the third one. The subject of the analysis is two phase (air-water mixture) flow. The void fraction distribution is found to be the single most decisive factor to determine the onset of instability for such a domain.

In conclusion two distinct mechanism were found to be responsible for flow induced vibration caused instabilities, 1) outside source controlled periodic excitation (such as vortex shedding) - described by the first model, and 2) fluid elastic forces - described by second, third and fourth models. For the values of reduced velocity below 0.7 first model is proposed, for the values above 0.7, the rest.

1 OBJECTIVE

Flow induced vibrations are a very important phenomena taking place in many engineering applications, especially in those connected to the flow of fluid through tube bundles. An example is flow induced vibrations in heat exchangers which can lead to fatigue of the tubes and then to leakage. To design systems so that flow vibrations are not a problem, one needs to understand the phenomena so as to predict and avoid dangerous flow conditions.

The objective of this dissertation is to develop a robust tool which will enable the engineer to predict the onset of instability in a arbitrary staggered cylinder array subject to single and partial two phase flow. One of the applications for this tool is a nuclear power station (PWR) steam generator. A schematic of PWR is shown in figure 1. These stations are under much more stringent safety requirements than other heat exchangers and the problem of fatigue is very closely monitored. Each year a certain amount of tubes is inspected. If the cracks are found, the tube is plugged. After a sufficient number of tubes is made inoperable the power generation station has either to reduce power (and earnings) or replace the steam generator. The latter is an extremely expensive procedure and utilities try to avoid it at all costs. Therefore the prediction of circumstances leading to flow instabilities will be welcomed.

Figure 1. Schematic of PWR primary circuit.

The final capability of a model will consider single phase flow and air water two phase flow. In a PWR, the secondary circuit water enters a steam generator as a liquid water, see figure 2. As it flows through the bundle of tubes it progressively vaporizes until it reaches a quality of about 90 % and leaves the steam generator at the top of the structure. Above the structure there are separators and driers. The problem is three dimensional, however this will be very hard to achieve. By using reasonable assumptions the steam generator could be transformed into three two dimensional regions. At the bottom of the steam generator we encounter the cross-flow of liquid water in the bundle. This flow is turned upward and follows the

direction parallel to tubes. This is a second two dimensional region. At the top the tubes form an inverted U shape, and this could be viewed as cross-flow again and approximated as a third two dimensional region. This dissertation will address flow induced vibrations due to cross-flow over arrays of the cylindrical structures in the lower region of tube bundle and will formulate the starting point for the research leading to steam-water mixture induced flow vibrations in the upper portion of the steam generator.

Figure 2. Typical U-tube steam generator.

At the top of steam generator the steam and water are not flowing in the same direction. The steam exits at the top through the moisture separators, the liquid returns through the downcomer and re-enters the process (Figure 3 shows the simplified schematic of the process). This can affect the process by two mechanisms.

Figure 3. Actual and Analyzed Problem.

First is increased interfacial drag. As it is shown in chapter 7 increased interfacial drag in fact stabilizes the cylinder bundle at lower reduced velocities while it does not show any sizeable impact on stability threshold at higher reduced velocities. Second mechanism is the actual flow direction difference. Flow direction will affect the boundary condition by reducing the velocity at the solid-liquid interface. Decrease in perturbed velocity will again reduce the danger of instable flow at lower mass damping parameters and will essentially fail to affect the upper stability boundary at higher reduced velocity. In the limit (when flow becomes parallel to the cylinders the experiments found that the instability occurs at much higher reduced velocities.

This shows that the most dangerous conditions require cross flow over the cylinder bundle. Conservative analysis will thus address the aligned flow of both phases over the bundle especially since exact direction of liquid/vapor cross flow cannot be determined neither experimentally nor analytically.

1.1 SIMILARITY/DIFFERENCE BETWEEN VARIOUS MODELS DESCRIBED

A number of phenomena are known to be important contributors to vibration of tubes in a bundle with cross flow. Experimental investigations have often simplified the problem to a single flexible tube in a channel or in a bundle of rigid tubes. A series of four models were developed that correspond to experimental efforts. The first model assumes one flexible tube in an array of rigid cylinders. It is

anticipated that the tube will vibrate due to some forced motion (e.g. vortex shedding) and the fluid response is evaluated. The results will show that the multiple frequencies of a single vibrating cylinder are not important for the threshold of dynamic instabilities and that the calculations may be carried out in quasi stationary fashion (Fourier transform type of approach). This scales the value of real frequency against the natural frequency of the tube and hints that reduced velocity (reversed Strouhal number based on natural frequency) does not depend on the frequency of the vibration but on other parameters such as mass per unit length, density of incoming fluid, diameter of the tube, pitch to diameter ratio and other parameters. The validity of the results is limited to low reduced velocity/low mass damping parameter region which is in line with vortex shedding influence.

The second model assumes certain (so called normal modes) forms of vibration based on the suggestions of the first model. In particular, the phenomena can be treated as quasi stationary and is most likely to be periodic in both time and one of the spatial coordinates. A wave-equation like transformation is accomplished by assuming the normal modes of the perturbed velocity field in the temporal and one of the spatial coordinates transforms partial differential into ordinary differential equations. The concept of a basic computational unit is introduced by taking into consideration the motion of the neighboring cylinder. The resulting equations are solved analytically. While the model cannot be applied to a real steam generator geometry without reservation it demonstrates that the displacement theory proposed by Chen (1987) is valid. It also provides the reader with insight on how the boundary

conditions effect the solution to the equations.

Analysis into normal modes leads to analysis based on the vorticity transport equation (the third model). This model relaxes the assumption about the form of the vibration in spatial coordinates and retains only temporal ansatz. This complicates the governing equations to the point that the analytical approach can no longer be used. The governing equations are discretized and a finite difference scheme (Alternate Direction Implicit - ADI) is used to solve them. Simpson's rule is used to evaluate integrals. The results and their good agreement with experimental data suggest that the phenomena is inertia controlled. Further, the good agreement with experiment enables us to develop a correlation that can be used as a design tool.

The fourth model is very similar to the third one as far as the mathematical approach to the solution of the equations is concerned. A two-fluid representation using the TRAC governing equations is the basis for the fourth model. Two phase (air-water) flow in a tube bundle is studied. This model results again will show fluid elastic instability to be inertia controlled in the upper reduced flow velocity region while the interfacial drag reduces the impact of momentum at lower reduced velocities.

2 LITERATURE SURVEY

Review of past research led to the conclusion that most theoretical investigations of the flow induced vibrations can be generalized within two categories: the first, with emphasis on the response of the structure (solid mechanics dominate), and second, which deals extensively with the fluid mechanics of the problem. I believe that a more balanced approach dealing with both the fluid and the structural response is needed. There have been a large number of structural response dominated investigations. In this work the reader will find a critique of the main ideas behind each of the above categories and supplemented by author's understanding and solutions to the problem.

2.1 MECHANISMS OF VIBRATIONS

Paidoussis (1988) divides vibrations into two types:

- a) Strouhal type periodicity in the interstitial flow which leads the resonance and subsequent vibration of the cylinder. This type of mechanism is often associated with expressions such as "jet switch", "wake swing" and "jet instabilities".
- b) Fluidelastic instability which occurs when the flow velocity exceeds a threshold value.

The first type of vibration can be explained by considering the periodic behavior of vortices created by flow passing through a bundle of cylinders. When the frequency of the vortices approaches the natural frequency of the cylinders in the bundle, the motion is amplified and is observed as a resonance. This phenomenon is known as "vortex shedding".

The second type of vibration can be described as flutter and is connected to the dynamic response of the cylinder and fluid-solid interactions. This effect is diminished by damping of the system making fluidelastic instabilities less likely to occur in water flow than in air flow.

There is a third type of vibration possible: it is due to turbulent motion of fluid flow past certain flow velocity. This work will not address this problem as the mechanisms behind turbulence are not known well enough.

Chen (1987) views vibrational behavior as a static or dynamic type of behavior. He claims that the structure responds to the flow induced force in different manners. The structure may deflect statically, become unstable by divergence in flow, resonate with periodic excitation of the flow, respond to random fluid excitation or be subjected to dynamic instability by flutter. He further suggests that the types of response can be classified according to excitation mechanisms.

Static displacements are caused by steady drag and lift forces and with

oscillatory forces that have frequencies much lower than the structural resonant frequencies of the cylinders (tubes). Static instability (divergence) is caused by a fluid stiffness force. Buckling of an elastic tube while transporting steady flow is an example of divergence.

Dynamic response is a structural response to various excitation mechanisms such as vortex shedding or turbulence. Once the excitation is known, the structural response can be calculated in a straightforward manner. Dynamic instability is typically caused by high velocity flow. Examples are flutter and whirling instability of cantilevered pipe conveying fluid. These modes of instability could be classified as a single mode and coupled mode flutter.

Both of the above descriptions rely on the assumption that the fluid force is something foreign to the system or is being controlled by an outside source, whether a pressure gradient or something else. Most of the flows are driven by a pressure gradient yet the local behavior is dependent only on local geometry and flow conditions. It is our opinion that the structure affects the local fluid motion as much as the fluid flow affects the structural response. Further the equations for fluid motion are valid and uniquely describe the stresses acting within the bundle. Since the fluid forces result mainly from the pressure field, the equations of the fluid motion could be easily coupled with the equations of the solid motion. Treating the fluid forces as strictly a function of Reynolds number, which is in effect what most of the people investigating the structural response were doing, is incomplete and

possible unnecessary. Flow induced vibrations are described by coupled fluid and solid equations of the motion and should be modeled accordingly.

Until the beginning of 1980 the mechanism of fluid elastic instability was not a consideration when flow induced vibrations in the bundles were mentioned. Rather, the problems were attributed to vortex shedding. During the last decade quite the opposite was found to be true. Vortex shedding is not important relative to fluid elastic instability for velocity and mass damping parameter ranges existing in steam generators. One can construct the experiment where the fluid elastic instability alone is taking place and compare that with similar experiment known for both vortex shedding and fluid elastic forces induced instabilities. Basis for such comparison is in the work of Kiya et al. (1980) who found that vortex shedding occurs for a pair of cylinders (behind first cylinder) if the pitch to diameter ratio exceeds 1.4. Below that value the two cylinders and pair of standing Foepl vortices act as a single elastic body. Since turbulent buffeting is not an important excitation mechanism there remains only self induced fluid elastic instability as a source of vibrations. If one wants to show that vortex shedding importance is in fact a marginal contributor in arrays of cylinders one should proceed as follows:

- 1) observe the dynamic instability of an array of cylinders with pitch to diameter ratios less than 1.4
- 2) observe the dynamic instability for a similar array as under 1) but with a pitch

to diameter ratio larger than 1.4. The difference could be attributed to vortex shedding.

Data by many researchers (Gibert et al. 1978, 1981, Chen and Jendrzejcyk 1981, 1987) for single phase flow and by Pettigrew et al (1988) for two phase flow show little difference between the cases of pitch to diameter ratio below and above 1.4 for variety of mass damping parameters. For two phase flow (Pettigrew), the threshold of instability (i.e. the flow velocity at which the instability occurs) even increases as the pitch to diameter ratio is increased. This would not be the case if vortex shedding were to have a significant impact on the threshold of instability, but is in line with fluid elastic mechanism in which the effect of a neighboring body decreases with increasing pitch to diameter ratio.

The conclusion one reaches from the above discussion is the following: although there is no clear difference in the cylinder feedback whether caused by forced motion (vortex shedding) or self induced motion (fluid elastic instability) while both mechanism are present (that is for pitch to diameter ratios larger than 1.4) the former mechanism does not generate forces sufficient to cause instability in the cylinder arrays observed in experiments.

An important questions is how the physical and geometric properties of the tubes influence the instabilities? Table 1 shows some of the common characteristics of several experiments carried out during last five years. In all cases fluid elastic

forces were identified as main mechanism behind the unstable behavior of the tubes.

Source	Size	Type of Support	Difference in x/y Freq.	Flow	Angle of Attac.	Remarks
1	4x5	linear iso-viscoelast.	around 1%	1 ϕ	60	found "second stability" bound.
2	60x10	cantilever+ damper	none mentioned	1 ϕ	90,60, 30	compared 10 and 30 Hz tubes
3	4x8	cantilever+ damper	none mentioned	1 ϕ	30	
4	8x7	piano-wire	none observed	1 ϕ	45	observed divergence
5	7x10	cantilever	none observed	2 ϕ	60,30	the most comprehensive
6	12x8	piano-wire	some observed at high velocit. (over 10 m/s)	1 ϕ	45	instability for $P/d > 2.12$ (single flex. tube)
7	25 cyl.	multispan	some observed at low velocit.	2 ϕ	30	real-life SG tubes

Table 1. Comparison of Experiments.

- [1] Andjelic, M., Popp, K., "Stability Effects in a Normal Triangular Cylinder Array", 1988 Int. Symposium on Flow-Induced Vibration and Noise, 1988 ASME Winter Annual Meeting, Chicago, Illinois, Vol.3
- [2] Teh, C.E., Goyder, H.G.D., "Data for the Fluidelastic Instability of Heat Exchanger Tube Bundles", 1988 Int. Symposium on Flow-Induced Vibration and Noise, 1988 ASME Winter Annual Meeting, Chicago, Illinois, Vol.3
- [3] Lever, J.H., Rzentkowski, G., "An Investigation in Post-Stable Behavior of a Tube Array in Cross Flow", 1988 Int. Symposium on Flow-Induced Vibration and Noise, 1988 ASME Winter Annual Meeting, Chicago, Illinois, Vol.3

- [4] Paidoussis, M.P., Price S.J., Nakamura, T., Mark, B., Njuki, W., "Flow-Induced Vibrations and Instabilities in a Rotated-Square Cylinder Array in Cross Flow", 1988 Int. Symposium on Flow-Induced Vibration and Noise, 1988 ASME Winter Annual Meeting, Chicago, Illinois, Vol.3
- [5] Pettigrew, M.J., Tromp, J.H., Taylor, C.E., Kim, B.S., "Vibration of Tube Bundles in Two-phase Cross-Flow: Part 2 - Fluidelastic Instability", 1988 Int. Symposium on Flow-Induced Vibration and Noise, 1988 ASME Winter Annual Meeting, Chicago, Illinois, Vol.3
- [6] Price, S.J., Kuran, S., "Fluidelastic Stability of a Rotated Square Array with Multiple Flexible Cylinders Subject to Cross-Flow", 1990 Pressure Vessels and Piping Conference, PVP Vol. 189, Nashville, Tennessee
- [7] Yeh Y.S., Chen, S.S., "Flow-Induced Vibration of Component Cooling Water Heat Exchanger", 1990 Pressure Vessels and Piping Conference, PVP Vol. 189, Nashville, Tennessee

Angle of attachment is the angle between two adjacent tubes. 60 is the angle found in normal triangular arrays, 45 refers to rotated square array, 30 is parallel triangular array and 90 stands for normal square array.

The purpose of presenting this table is to show that the frequency remains to be the same in both, lift and drag, directions regardless for the type of the support one is using as long as two degrees of freedom movement is preserved by the clamping mechanism. This fact simplifies the analysis performed. The table encompasses just few of the experiments performed. While almost all observers found profound differences in the amplitudes of lift and drag, with the lift direction dominating, there were no observations of instabilities with different frequencies in direction x and y. This is true for both single and two phase flows. The reason for this behavior is probably in mechanism controlling the feedback of the structure. The

amplitude is the property of the system influenced by both, solid and fluid, parts while the frequency seems to be controlled solely by solid part of the structure. The frequency is dependent on the shape of the body provided that two degrees of freedom have been preserved. Since the cylinders are axisymmetric there is no reason why the frequencies would differ. On the other hand, there is no reason they should not as the lift and drag forces could have different sensitivities to disturbance. We will use experimental observations as basis for the assumption that the vibration frequencies in the two directions are the same. Before venturing into the arena of finite amplitude behavior further investigation should be carried out. The amplitudes do, however, depend on local flow conditions.

Based on experiments one can conclude that the mounting of the test section does not cause the results, in their scaled form, to change significantly. Most of the flexible tubes were cantilevered with some having other types of support (ranging from iso-viscous to piano wire suspension). The observed instabilities occurred under closely similar flow conditions allowing us to conclude that the proper scaling was used for the analysis to correctly account for different types of mounting. The most important parameters which need to be related in the analysis of steam generator design are critical gap velocity (velocity of the fluid in the gap between the tubes constituting the basis for Re and streaming Re number calculations), critical mass damping parameter, pitch to diameter ratio, angle of attachment (which represents the geometry of the bundle) and Reynolds (or streaming Reynolds) number. It is customary to declare one of the parameters as dependent on all of the others

especially if the dependent one can easily be controlled. Reduced velocity of the fluid can be regulated quite easily through various means (valves, turbine load etc.) thus its role as a dependent parameter.

If one were to single out the most important structural parameter, the mass damping parameter would most likely be the best candidate. The mass damping parameter is, in this study, regarded as the independent parameter and (by definition) is not a function of frequency, pitch to diameter ratio or angle of attachment; it is defined as a product of the logarithmic decrement (sometimes also denoted as the damping parameter) and the ratio between the true and substitute (equivalent) fluid (which flows over the structure) mass per unit length. For multiple tubes, there is no clear way of defining the parameter. Experiments used to determine the threshold of instability are performed using only one tube (which means that mass damping parameter of the array equals the mass damping parameter of one tube). If several tubes are used then the mass damping parameter could be based on a geometric average of all the tubes in the domain.

The question of multiple tubes and the mass damping parameter can also be viewed from the perspective of its impact on the critical reduced velocity. Rephrased, the problem would focus on the following question: given the flow what is a stable geometry? If one knows the desired pitch to diameter ratio and the angle of attachment, the only free parameter remaining is the mass damping parameter. If a designer chooses the liquid used and the material of the tubes, the tube diameter and

wall thickness can be uniquely determined from the its definition. The whole process is described (and a sample calculation presented) in Appendix B .

2.2 EQUATIONS OF THE STRUCTURAL MOTION

In a vacuum, the equations of motion of a cylinder taken from Chen (1987) are:

$$EI \frac{\partial^4 x_c}{\partial z^4} + C_s \frac{\partial x_c}{\partial t} + m_p \frac{\partial^2 x_c}{\partial t^2} - g_s(x_c, t) \quad (2.1)$$

and

$$EI \frac{\partial^4 y_c}{\partial z^4} + C_s \frac{\partial y_c}{\partial t} + m_p \frac{\partial^2 y_c}{\partial t^2} - g_s(y_c, t) \quad (2.2)$$

where x_c and y_c are cylinder displacements in the x and y direction, EI is the product of Young's modulus and the moment of inertia, C_s is the structural damping coefficient, m_p is the mass of the cylinder and g_s is the external force. If the external force is caused by fluid, there is an additional term added to equations (2.1) and (2.2), namely the added mass term. Added mass accounts for fluid next to the body that has to be accelerated with the body and thus poses an additional drag. The external force term is due solely to the effect of the surrounding fluid. It can be approximated after assuming certain fluid properties, such as inviscidness or

incompressibility. Equations (2.1) and (2.2) will then read:

$$EI \frac{\partial^4 x_c}{\partial z^4} + C_s \frac{\partial x_c}{\partial t} + m_p \frac{\partial^2 x_c}{\partial t^2} + \sum_{q=1}^{2N} \gamma_q \frac{\partial^2 x_{cA}}{\partial t^2} - g_s(x_c, t) \quad (2.3)$$

and

$$EI \frac{\partial^4 y_c}{\partial z^4} + C_s \frac{\partial y_c}{\partial t} + m_p \frac{\partial^2 y_c}{\partial t^2} + \sum_{q=1}^{2N} \gamma_q \frac{\partial^2 y_{cA}}{\partial t^2} - g_s(y_c, t) \quad (2.4)$$

Most authors differentiate between the component of the force in the x direction (Cartesian coordinate system), by naming it drag and denoting it F_D and in the y direction by naming it lift and denoting it F_L . There have only been a few papers published (none of them in the field of two phase flow) that theoretically treat drag and lift from first principles. Most of the authors (see Price (1988), Rajaona (1988) and others) use correlated data in the above equations. Price used an interesting approach where instead of using raw data he used measured partial derivatives of lift and drag *coefficients* with respect to the flow and crossflow directions. He analyzed the 8 cylinders in a 3 x 3 array. Assuming a constant flow velocity (which is probably a pretty bold assumption), the fluid force on the cylinder in the x direction (streamwise, drag) is expressed as

$$F_x = \frac{1}{2} \rho d U^2 \left\{ C_{DC} - 2 C_{DC} \dot{\xi}_i \frac{d}{U_G} + C_{LC} \dot{\eta}_i \frac{d}{U_G} \right\} \quad (2.5)$$

and in the y direction (normal, lift) is

$$F_y = \frac{1}{2} \rho d U^2 \left\{ C_{LC} - 2 C_{LC} \dot{\xi}_i \frac{d}{U_G} + C_{DC} \dot{\eta}_i \frac{d}{U_G} \right\} \quad (2.6)$$

Both lift and drag are evaluated from the experimental data in terms of partial derivatives of the above coefficients with respect to streamwise and crossflow directions:

$$C_{DC} = C_{D0} + \sum_{i=1}^8 \left\{ \xi_i \frac{\partial C_L}{\partial \xi_i} + \eta_i \frac{\partial C_D}{\partial \eta_i} \right\} \quad (2.7)$$

and

$$C_{LC} = C_{L0} + \sum_{i=1}^8 \left\{ \xi_i \frac{\partial C_L}{\partial \xi_i} + \eta_i \frac{\partial C_D}{\partial \eta_i} \right\} \quad (2.8)$$

Note that

$$\frac{\partial C_L}{\partial \eta_i} - \frac{\partial C_D}{\partial \xi_i} = 0 \quad (2.9)$$

due to the orthogonal nature of the lift and drag forces.

Although this appears to be a complete analytical solution, the fact remains, that steady flow throughout the flow field was assumed. This in turn means, that the lift and drag forces are linear functions of the cylinder movement independent of the effect on the flow field of cylinders up and downstream. This effectively ignores that perturbations in the flow field, such as vortex shedding, that are functions of the Reynolds number. This leads us to conclude that just measuring, then correlating lift and drag might not be enough to lead to good solutions, and might not yield reliable predictions of forces exerted on the cylinders. Instead of simply denoting the function g_i in equations (2.3) and (2.4) as drag and lift forces, one should approximate the driving force on the cylinder as an integral of pressure as a function of position and time around the cylinders.

2.3 EQUATIONS OF FLUID MOTION

When evaluating the fluid motion, one must employ several different simplifying assumptions to obtain analytical or closed form solution. Probably the most restrictive is to assume potential flow (inviscid, irrotational). This type of approach was used by Kim (1988), who analyzed a pair of tandem cylinders subjected to vortex buffeting. The fluid mechanics part of the constitutive equations is formulated as a complex potential representing uniform flow past an arbitrary number of cylinders modeled by doublets. This flow is disturbed by introducing vortices. An acoustic field, calculated by obtaining the vortex path, is used to predict the pressure variations on the cylinders. The complex potential used is

$$W(z) = z - \sum_{m=1}^{NV} \frac{ik_m}{2\pi} \ln(z - z_m) + \sum_{m=1}^{NV} \sum_{j=1}^{\infty} W_{jm}^v + \sum_{j=1}^{\infty} (W_j^f + W_j^m) \quad (2.10)$$

where z stands for complex variable, NV is the number of vortices, and W^v , W^f and W^m are dimensionless complex potential, dimensionless complex potential due to the image vortices, dimensionless complex potential due to the image doublets and dimensionless image potential due to the cylinder motion respectively. The position of the m^{th} vortex is z_m and its strength is k_m . The vortex path is obtained from

$$\frac{d\vec{x}_i}{dt} = \vec{u}_i \quad (2.11)$$

Although the above approach appears to be an improvement, it still lacks some important features. First, it is limited to 2-D because the complex potential is only in the 2-D plane. This by itself is not a severe drawback, especially since this work will focus on one and two dimensional approximations as well. The problem is inability of ideas expansion to real three dimensional simulation should new solution techniques enable a researcher to deal with this kind of a problem. Further, potential flow is irrotational, which means that there is no vorticity generation. This is much more severe limit which constrains the usefulness of the solution and doesn't leave any room for future improvements of the analytical or numerical tools.

A better model was introduced by Lever and Weaver (1984) and further developed by Yetisir and Weaver (1988). The essence of the model is the idea of a non-steady control volume. The conservation equations derived in Appendix A are the continuity equation,

$$\frac{\partial}{\partial t}[\rho V(t)] + \nabla \cdot [\rho V(t)\vec{u}] = 0 \quad (2.12)$$

and the momentum equation,

$$\rho \frac{\partial}{\partial t}[\vec{u} V(t)] + \rho \nabla \cdot [\vec{u} \vec{u} V(t)] = \sum \vec{f} \quad (2.13)$$

Lever and Yetisir (1988) derived a slightly different form of momentum equation. For this reason it is derived in Appendix A. Further, the derivation shows

that neither author started with the same equation, yet both arrive after some algebra at the same result. Since we decided to adopt the Lever and Weaver idea of a flexible control volume, as it can be adjusted easily to fit modern numerical techniques, the derivation of the final forms of equations will be shown later in this text. Both above mentioned papers have assumed one-dimensional unsteady incompressible flow. There is only one flexible cylinder surrounded by rigid cylinders. Since the model is one dimensional, viscosity effects cannot be accounted for other than in the form of some kind of friction factor, as employed in both papers. The important conclusion, drawn from the papers is that there is a single complex transcendental equation, whose roots yield, in different combinations, different thresholds to different forms of the amplified vibrations, depending on flow Reynolds number.

3 ONE DIMENSIONAL APPROACH (FIRST MODEL)

In this section the one dimensional unsteady integral model is based on the idea of non-steady control volume is derived. All the assumptions used are stated where appropriate. The derivation is followed by the results for dynamic and static thresholds and compared to available experimental results valid for a rotated square array by Weaver and Fitzpatrick (1988) and Paidoussis (1988). This model will describe the cylinder response to the outside controlled vibration (vortex shedding).

As already mentioned the basis for this model is the concept of an unsteady control volume. The model is represented with a flexible streamtube within the cylindrical bundle. This is presented in figure 4. By choosing a flexible streamtube we are in effect assuming that all of the fluid is transported through the bundle though them, neglecting the flow parallel to the streamtubes. One of the cylinders in the bundle is flexible, the others are held fixed and that could be described as a main limit of the above model. In the real situation that would hardly be a case, especially in the steam generator application. Most of the tubes used in heat exchangers have length/diameter ratio as high as possible to get as much area where heat transfer takes places as possible, and are thus sensitive to any outside applied force.

The infinitesimally short segment of the streamtube represents the unsteady control volume as shown in figure 5.

Computational Domain

Figure 4. Flexible Streamtube Within the Bundle

Control Volume

Figure 5. The Unsteady Control Volume

3.1 DERIVATION OF ONE DIMENSIONAL UNSTEADY INTEGRAL MODEL

The continuity equation (see Appendix A for derivation),

$$\frac{\partial}{\partial t}[\rho V(t)] + \nabla \cdot [\rho V(t) \vec{u}] = 0 \quad (3.1)$$

scaled with

$$\begin{aligned} A &= \tilde{A}^* d ; u = \tilde{u}^* U_r \omega_n l_0 ; t = \frac{t^*}{\omega} \\ x &= s^* d ; p = \tilde{p}^* \rho d l_0 \omega_0^2 U_r \end{aligned} \quad (3.2)$$

yields the dimensionless form of the continuity equation,

$$\frac{1}{U_r} \frac{\omega}{\omega_n} \frac{\partial \tilde{A}^*}{\partial t^*} + \frac{l_0}{d} \frac{\partial}{\partial s^*} (\tilde{u}^* \tilde{A}^*) \quad (3.3)$$

The area, pressure and velocity are decomposed into mean and perturbed quantities. As long as the perturbed quantities remain much smaller than their mean flow counterparts one can superimpose the perturbed flow field on mean flow field which simplifies the governing equations while using linear perturbation theory.

$$\tilde{u}^* = U_r + u^* ; \tilde{A}^* = A_0^* + a^* ; \tilde{p}^* = p_0^* + p^* \quad (3.4)$$

These expressions are substituted into the dimensionless continuity equation (3.5) with the following simplifying assumptions:

- a) neglect terms of order (perturbation²)
- b) assume A_0^* and U_0^* are constant along a streamtube
- c) assume the velocity has a uniform profile in the direction perpendicular to the flow direction s^* (which effectively means using a 1-D approximation)

The basis for the above assumptions is that we are dealing with flow whose fluctuating components are of small amplitude (discard high order terms) and obey the continuity equation in the mean flow (moderate Re number and constant mean flow velocity and streamtube crosssection area), and that there is negligible cross flow (1-D approximation). With the above assumptions, the continuity equation becomes

$$\frac{1}{U} \frac{\omega}{\omega_n} \frac{\partial a^*}{\partial t^*} + \frac{l_0}{d} U' \frac{\partial a^*}{\partial s^*} + \frac{l_0}{d} A_0^* \frac{\partial u^*}{\partial s^*} = 0 \quad (3.5)$$

Integrating it along the stream tube from some arbitrary inlet position s_i^* to some arbitrary exit position s_e^* yields

$$\frac{1}{U} \frac{\omega}{\omega_n} \int_{s_i^*}^{s_e^*} \frac{\partial a^*}{\partial t^*} ds^* - \frac{l_0}{d} \left\{ U \left[a^*(s_i^*) - a^*(s_e^*) \right] + A_0^* \left[u^*(s_i^*) - u^*(s_e^*) \right] \right\} \quad (3.6)$$

The velocity can be calculated anywhere along the streamline with respect to some starting boundary conditions and an assumed form of area perturbation. The latter assumption can be defended by noting that the motion of the cylinder does not

become random unless we are dealing with turbulent flows. The final form of the continuity equation used in this model is

$$u^*(s^*) - u^*(s_i^*) + \frac{U_r}{A_0^*} [a^*(s_i^*) - a^*(s^*)] - \frac{d}{A_0^* U_r l_0} \frac{\omega}{\omega_{s_i^*}} \int_{s_i^*}^{s^*} \frac{\partial a^*(\xi^*)}{\partial \tau^*} d\xi^* \quad (3.7)$$

The pressure field is evaluated using the momentum equation, see Appendix A for derivation,

$$\rho \frac{\partial}{\partial t} [\bar{u} V(t)] + \rho \nabla [|\bar{u}| \bar{u} V(t)] - \sum \bar{f} \quad (3.8)$$

Using the assumptions listed above, one finds

$$\rho \frac{\partial}{\partial t} [Au] - \frac{\partial}{\partial x} [\rho u u A] - \frac{\partial}{\partial x} [pA] \quad (3.9)$$

and after scaling becomes

$$\frac{1}{U_r} \frac{\omega}{\omega_{s_i^*}} \frac{\partial}{\partial \tau^*} [\bar{u}^* \bar{A}^*] - \frac{l_0}{d} \frac{\partial}{\partial s^*} [\bar{u}^* \bar{u}^* \bar{A}^*] - \frac{1}{U_r} \frac{\partial}{\partial s^*} [\bar{p}^* \bar{A}^*] \quad (3.10)$$

Decomposing the flow field as before, neglecting high order terms (terms order of perturbation²) and noting that

$$\frac{\partial}{\partial \tau^*} (A_0^* U_r) - \frac{\partial}{\partial s^*} (A_0^* U_r l_0) - \frac{\partial}{\partial s^*} (A_0^* p_0^*) = 0 \quad (3.11)$$

yields

$$\begin{aligned} \frac{1}{U_r} \frac{\omega}{\omega_n} \frac{\partial}{\partial t^*} [A_0^* u^* + a^* U_r] - \frac{l_0}{d} \frac{\partial}{\partial s^*} [A_0^* U_r U_r + a^* U_r U_r + A_0^* U_r u^*] \\ - \frac{1}{U_r} \frac{\partial}{\partial s^*} [A_0^* p^* + a^* p_0^*] \end{aligned} \quad (3.12)$$

Integrating the momentum equation along the length of the control volume from some arbitrary inlet point s_i^* to some arbitrary exit point s_e^* yields

$$\begin{aligned} \frac{1}{U_r} \frac{\omega}{\omega_n} \int_{s_i^*}^{s_e^*} A_0^* \frac{\partial u^*}{\partial t^*} ds^* + \frac{1}{U_r} \frac{\omega}{\omega_n} \int_{s_i^*}^{s_e^*} U_r \frac{\partial a^*}{\partial t^*} ds^* - \\ \frac{l_0}{d} \{ 2U_r A_0^* [u^*(s_i^*) - u^*(s_e^*)] + U_r U_r [a^*(s_i^*) - a^*(s_e^*)] \} + \\ \frac{1}{U_r} \{ A_0^* [p^*(s_i^*) - p^*(s_e^*) + p_h^*] + p_0^* [a^*(s_i^*) - a^*(s_e^*)] \} \end{aligned} \quad (3.13)$$

where p_h^* is a surrogate to account for the hydraulic pressure loss (to account for error made by inviscid flow assumption). Also, the mean flow and perturbed flow velocities are assumed to have the same character. Up to this point our approach is essentially the same as that of Yetisir and Weaver. In equation (3.15) the partial derivative of the perturbation velocity with respect to time is an unknown function which can be related to the assumed perturbed area through the continuity equation,

The pressure at some arbitrary location s^* along the streamtube is then found

$$\begin{aligned} & \frac{\partial u^*(s^*)}{\partial \tau^*} - \frac{\partial u^*(s_i^*)}{\partial \tau^*} + \frac{U_r}{A_0^*} \left[\frac{\partial a^*(s_i^*)}{\partial \tau^*} - \frac{\partial a^*(s^*)}{\partial \tau^*} \right] \\ & - \frac{d}{A_0^* U_r l_0} \frac{\omega}{\omega_n} \int_{s_i^*}^{s^*} \frac{\partial^2 a^*(\xi^*)}{\partial \tau^{*2}} d\xi^* \end{aligned} \quad (3.14)$$

from

$$\begin{aligned} & p^*(s^*) - p^*(s_i^*) - \frac{\omega}{\omega_n} \int_{s_i^*}^{s^*} \left[\frac{\partial u^*(s_i^*)}{\partial \tau^*} + \frac{U_r}{A_0^*} \frac{\partial a^*(s_i^*)}{\partial \tau^*} \right] d\xi^* \\ & - \frac{d}{A_0^* U_r l_0} \left(\frac{\omega}{\omega_n} \right)^2 \int_{s_i^*}^{s^*} \int_{s_i^*}^{\xi^*} \frac{\partial^2 a^*(\eta^*)}{\partial \tau^{*2}} d\eta^* d\xi^* \\ & + \frac{2}{A_0^*} \frac{\omega}{\omega_n} |U_r| \int_{s_i^*}^{s^*} \frac{\partial a^*(\xi^*)}{\partial \tau^*} d\xi^* \\ & + \left(\frac{U_r l_0}{d} |U_r| U_r - p_0^* \right) \frac{a^*(s^*) - a^*(s_i^*)}{A_0^*} + p_h^* \end{aligned} \quad (3.15)$$

3.2 FORCES ON THE CYLINDER

The lift and drag are found by integrating the pressure around the cylinder as a function of time. As shown in figure 6, the resultant force, which balances the pressure on the cylinder, can be separated into two components, one in the x and the other in the y direction. The first one is sometimes called the drag force, whereas the

second is denoted as lift. Some researchers would call these the longitudinal and transverse components for the x and y directions, respectively. We will use subscripts L for flow (streamwise) and T for crossflow directions (to be consistent with other authors).

Figure 6. Forces on Cylinder.

The force on an infinitesimal area of the cylinder is

$$dF_T = dF \cos \beta = dF \cos \left(\frac{2s}{P} \right) = dF \cos \left(\frac{2s^*}{P/d} \right) \quad (3.16)$$

and the transverse force is

$$F_T = \int_{-s_0}^{s_0} dF_T = \int_{-s_0}^{s_0} p \cos \left(\frac{2s^*}{P/d} \right) ds^* \\ - \int_{-s_0}^{s_0} \rho d^2 l_0 \omega^2 U_r p^* \cos \left(\frac{2s^*}{P/d} \right) ds^* \quad (3.17)$$

which yields a "natural" scaling for the force in the transverse direction,

$$F_T = F_T^* \rho l_0 d^2 \omega_n^2 U, \quad (3.18)$$

The final form of the dimensionless transverse component of the fluid force on the cylinder is

$$F_T^* = \int_{-s_i^*}^{s_i^*} p^*(s^*, t^*) \cos\left(\frac{2s^*}{P/d}\right) ds^* \quad (3.19)$$

and similarly the streamwise component of the fluid force is

$$F_L^* = \int_{-s_i^*}^{s_i^*} p^*(s^*, t^*) \sin\left(\frac{2s^*}{P/d}\right) ds^* \quad (3.20)$$

or in the closed form

$$\begin{aligned} F_T^* = \int_{-s_i^*}^{s_i^*} & \left\{ p^*(s_i^*) - \frac{\omega}{\omega_n} \int_{s_i^*}^{s^*} \left[\frac{\partial u^*(s_i^*)}{\partial t^*} + \frac{U_r}{A_0^*} \frac{\partial a^*(s_i^*)}{\partial t^*} \right] d\xi^* \right. \\ & - \frac{d}{A_0^* U_r l_0} \left(\frac{\omega}{\omega_n} \right)^2 \int_{s_i^*}^{s^*} \int_{s_i^*}^{\xi^*} \frac{\partial^2 a^*(\eta^*)}{\partial t^{*2}} d\eta^* d\xi^* \\ & + \frac{2}{A_0^*} \frac{\omega}{\omega_n} |U_r| \int_{s_i^*}^{s^*} \frac{\partial a^*(\xi^*)}{\partial t^*} d\xi^* \\ & \left. + \left(\frac{U_r l_0}{d} |U_r| U_r - p_0^* \right) \frac{a^*(s^*) - a^*(s_i^*)}{A_0^*} + p_h^* \right\} \cos\left(\frac{2s^*}{P/d}\right) ds^* \end{aligned} \quad (3.21)$$

The remaining task is to determine the numerical or functional value of the variables inside the integral. Lever and Weaver (1984) and Yetisir and Weaver (1988) have determined that the effect of a flexible cylinder in an array of the rigid cylinders dies out within 2 diameter lengths upstream and downstream. That, in effect, means that all perturbation variables, such as pressure p^* , velocity v^* and area a^* as well as their derivatives with respect to both space and time are approximately zero more than two diameters away from their source. This simplifies equation (3.21) to

$$\begin{aligned}
 F_T^* = \int_{-s_i^*}^{s_i^*} \left\{ -\frac{d}{A_0^* U r_0} \left(\frac{\omega}{\omega_n} \right)^2 \int_{s_i^*}^{s^*} \int_{s_i^*}^{\xi^*} \frac{\partial^2 a^*(\eta^*)}{\partial t^{*2}} d\eta^* d\xi^* \right. \\
 + \frac{2}{A_0^*} \frac{\omega}{\omega_n} |U| \int_{s_i^*}^{s^*} \frac{\partial a^*(\xi^*)}{\partial t^*} d\xi^* \\
 \left. + \left(\frac{U r_0}{d} |U| U_r - p_0^* \right) \frac{a^*(s^*)}{A_0^*} + p_h^* \right\} \cos\left(\frac{2s^*}{P/d}\right) ds^*
 \end{aligned} \quad (3.22)$$

and similarly for

$$\begin{aligned}
 F_L^* = \int_{-s_i^*}^{s_i^*} \left\{ -\frac{d}{A_0^* U r_0} \left(\frac{\omega}{\omega_n} \right)^2 \int_{s_i^*}^{s^*} \int_{s_i^*}^{\xi^*} \frac{\partial^2 a^*(\eta^*)}{\partial t^{*2}} d\eta^* d\xi^* \right. \\
 + \frac{2}{A_0^*} \frac{\omega}{\omega_n} |U| \int_{s_i^*}^{s^*} \frac{\partial a^*(\xi^*)}{\partial t^*} d\xi^* \\
 \left. + \left(\frac{U r_0}{d} |U| U_r - p_0^* \right) \frac{a^*(s^*)}{A_0^*} + p_h^* \right\} \sin\left(\frac{2s^*}{P/d}\right) ds^*
 \end{aligned} \quad (3.23)$$

The number of the unknowns (a^* , p^* , ω) exceeds number of the equations (eqs. 3.15 and 3.28) by one. Thus we have (similar to earlier authors) assumed the harmonic functional dependance of a^* on t^* and s^* to be

$$a^*(s^*, t^*) = a^*(s_m^*, t^*) f(s^*) e^{i\Phi^*(s^*)} \quad (3.24)$$

where the subscript m stands for the value unit area function at the minimum gap, f^* is the decay function for the perturbation and Φ^* is a phase function along the streamtube. f^* and Φ^* are given by

$$\begin{aligned} f^*(s^*) &= \frac{1}{1+b(s_a^*+s^*)^a} & s \leq -s_a^* \\ &= 1 & -s_a^* \leq s^* \leq s_s^* \\ \Phi^*(s^*) &= \frac{1}{U_r} \frac{s^*+s_a^*}{s_s^*-s_a^*} \end{aligned} \quad (3.25)$$

where a and b are constants chosen to fit the boundary conditions far up- and downstream. From the geometry of the problem it can be deduced that the minimum perturbation area function will have the following form

$$a^*(s_m^*, t^*) = x_0^* e^{i\omega t^*} \sin \alpha + y_0^* e^{i\omega t^* + \Psi} \cos \alpha \quad (3.26)$$

where Ψ stands for phase lag and x_0^* and y_0^* are amplitudes in the flow and crossflow directions, respectively. This formulation of the area function allows us to calculate derivatives of the area function with respect to time as

$$\frac{\partial a^*}{\partial t^*} - ia^* ; \quad \frac{\partial^2 a^*}{\partial t^{*2}} = -a^* \quad (3.27)$$

3.3 STABILITY ANALYSIS

The equations of motion for the cylinder are written (Yetisir and Weaver, 1988)

$$\begin{aligned} \lambda m \ddot{x} + c \dot{x} + kx &= \sum a_x F_x \\ \lambda m \ddot{y} + c \dot{y} + ky &= \sum a_y F_y \end{aligned} \quad (3.28)$$

where λm , c and k stand for mass of the cylinder, the damping coefficient and the stiffness, x and y are displacement of the cylinder in the flow and crossflow directions, respectively. Recognize also, that the RHS term in equation (3.28) contain the effects of two 1-D streamtubes touching the cylinder from the left and right sides. The only remaining connection to be made is between the displacement of the cylinder and the gap perturbations,

$$\begin{aligned} a_1^*(s^*, t^*) &= x^*(t^*) \sin\left(\frac{2s^*}{P/d}\right) + y^*(t^*) \cos\left(\frac{2s^*}{P/d}\right) \\ a_2^*(s^*, t^*) &= x^*(t^*) \sin\left(\frac{2s^*}{P/d}\right) - y^*(t^*) \cos\left(\frac{2s^*}{P/d}\right) \end{aligned} \quad (3.29)$$

By using equation (3.29) in equation (3.28) and noting that movement in the flow direction enhances effects of both sides of cylinder on drag, whereas movement in crossflow direction enhances lift only, one may conclude that cylinder movement in the flow and crossflow direction do not affect each other and the equations of motion are decoupled. Thus, the phase lag function Ψ can be neglected, at least at this order of approximation. To obtain a solution to the equations of cylinder motion, the damping parameter c and the stiffness k will be taken as

$$c = \lambda m \delta \frac{\omega_n}{\pi} \quad (3.30)$$

$$k = \lambda m \omega_n^2$$

Introducing equation (3.30) into (3.28), and scaling yields the characteristic equations,

$$\begin{aligned} & -\frac{m}{\rho d^2} \left(\frac{\omega}{\omega_n} \right)^2 + \frac{i}{\pi} \frac{m \delta}{\rho d^2} \left(\frac{\omega}{\omega_n} \right) + \frac{m}{\rho d^2} - 2F_L^* l_0 U_r \sin \alpha \\ & -\frac{m}{\rho d^2} \left(\frac{\omega}{\omega_n} \right)^2 + \frac{i}{\pi} \frac{m \delta}{\rho d^2} \left(\frac{\omega}{\omega_n} \right) + \frac{m}{\rho d^2} - 2F_T^* l_0 U_r \cos \alpha \end{aligned} \quad (3.31)$$

where δ represents logarithmic decrement of the tube damping. These equations will yield the ratio ω/ω_n and δ for any given flow conditions. Of course, since the ratio ω/ω_n is needed to evaluate the fluid forces, an iterative scheme must be employed. The results are found as fluid forces as a function of time, and as stability thresholds, associated with real and complex roots of the ratio ω/ω_n .

The stability thresholds need to be divided into two categories. One is connected to a damping matrix, and another one to the fluid stiffness. These modes are called dynamic and static (divergence) thresholds, respectively. Terms on the LHS of equation (3.31) are not complete fluid stiffness and damping parameter terms. They have to be associated with terms of the same order (in ω/ω_n) from expressions for streamwise (drag) forces. Denoting the terms, associated with acceleration (order ω/ω_n^2) with M , those associated with fluid damping with C , and the last associated with fluid stiffness with K , we arrive at

$$\begin{aligned}
 M_L &= \left[\frac{m}{\rho d^2} + \frac{2 \sin \alpha}{A_0^* U_r l_0^*} \int_{s_i^*}^{s_i^*} \sin \left(\frac{2s^*}{P/d} \right) \int_{s_i^*}^{s_i^*} a^*(\eta) d\eta d\xi ds^* \right] \\
 C_L &= i \left[\frac{\delta}{\pi} \frac{m}{\rho d^2} - 4 \sin \alpha \frac{U_r}{A_0^*} \int_{s_i^*}^{s_i^*} \sin \left(\frac{2s^*}{P/d} \right) \int_{s_i^*}^{s_i^*} a^*(\xi) d\xi ds^* \right] \\
 K_L &= \frac{m}{\rho d^2} - 2 \sin \alpha \frac{l_0^*}{A_0^*} U_r^3 \int_{s_i^*}^{s_i^*} \sin \left(\frac{2s^*}{P/d} \right) a^*(s^*) ds^*
 \end{aligned} \tag{3.32}$$

for the streamwise direction, and similarly

$$\begin{aligned}
M_T &= \left[\frac{m}{\rho d^2} + \frac{2 \cos \alpha}{A_0^* U_r l_0^*} \int_{s_i^*}^{s_j^*} \cos \left(\frac{2s^*}{P/d} \right) \int_{s_i^*}^{s_j^*} a^*(\eta) d\eta d\xi ds^* \right] \\
C_T &= i \left[\frac{\delta}{\pi} \frac{m}{\rho d^2} - 4 \cos \alpha \frac{U_r}{A_0^*} \int_{s_i^*}^{s_j^*} \cos \left(\frac{2s^*}{P/d} \right) \int_{s_i^*}^{s_j^*} a^*(\xi) d\xi ds^* \right] \\
K_T &= \frac{m}{\rho d^2} - 2 \cos \alpha \frac{l_0^*}{A_0^*} U_r^3 \int_{s_i^*}^{s_j^*} \cos \left(\frac{2s^*}{P/d} \right) a^*(s^*) ds^*
\end{aligned} \tag{3.33}$$

for the normal direction. The instabilities occur when

- the coefficients C_L and C_T become positive. The system will react to applied pressure so forcefully that it will actually overshoot the stable position on the other side. This kind of behavior is called overstability or dynamic instability. The threshold of dynamic instability occurs when the value of the coefficients are equal to zero.
- the coefficients K_L and K_T become negative. The system becomes statically unstable (divergence) and the threshold occurs when the coefficients are equal to zero.

It is evident that the model enables the solid-liquid system to become unstable in one direction, while retaining stable mode in other direction. In this case lower

threshold will determine the onset of instability. This is a shortcoming, which needs to be addressed in the future.

3.4 RESULTS

A complex arithmetic eigenvalue solver was used to solve the above equations. The nature of the problem is time dependant in contrast to work by others noted before. Our first task was therefore to see what kind of dependency the frequencies ratio has on time and nondimensional velocity. Results are given in figure 7 for the transfer component. The conclusion one draws from this figure is that smaller Re results in a stronger dependency on time. As we approach high Re this dependency vanishes.

The time and velocity are by no means the only parameters. Another parameter is the logarithmic decrement which is a property of the system. Figure 8 examines this aspect. We deduced that the logarithmic decrement played virtually no role in determining the frequency ratio. The logarithmic decrement of property describes the natural logarithm of the ratio of property values in two consecutive cycles.

Accuracy can be assessed quite easily. Since the analytical part of the method doesn't have any errors beside the ones acquired when adopting assumptions the

errors are due to numerical integrations employed. As widely known the error of Simpson's integration is of the order $(n h^5)/90$ where h denotes a step size and n number of points of integration.

Figure 7. Ratio of frequencies as function of velocity and time.

Figure 8. Ratio of frequencies as a function of velocity and log. decrement.

The solutions, presented in figure 9 for divergence and dynamic instabilities, respectively, show trends similar to those predicted by Yetisir and Weaver (1988), although their numerical scheme (employing curvilinear coordinates while calculating solutions to integral equations) differs significantly from this simple algorithm for evaluating eigenvalues in the complex plane. The predictions in this work are closer to experimental measurements than those of Yetisir and Weaver, the reason being the analytical solution employed in this paper. Again, there is a peculiar behavior at the onset of the instability involved, especially with respect to time. Figure 10 depicts the values of the transverse dynamic thresholds calculated at different times and compared to experimental results. A rotated square array with pitch to diameter ratio 1.5 was assumed. It is important to recognize that while flexible cylinder moves (note, the dimensionless time is NOT the Fourier number) the flow conditions change and thus the conditions for the onset of instability are shifted. There are some points where the system is always stable. Nevertheless the results don't change significantly with time and this phenomena can be assessed as quasi-stationary.

Comparison with experiments show excellent agreement in areas with dominant vortex shedding and disagreement at higher reduced velocities which calls for different approach when dealing with fluid elastic instabilities.

Onset of instability Velocity vs. damping

Figure 9. The onset of instability - triangular array.

Figure 10. The onset of instability - rotated square array.

3.5 CONCLUSION

The results presented in figures 7 and 8 show a definite periodic behavior of the cylinder motion. The results could be associated with Strouhal number for any particular flow conditions if a second dimension along z-axis were to be used. This clearly shows the need for a more complex analysis than is presented here. The results also show that at higher values of the nondimensional velocity the temporal character of the frequency ratio vanishes.

The temporal stability analysis shows that there is a 'grey' zone where a steady state analysis would show unstable behavior whereas the 'stripes' of stable lines could bring system back into a stable mode. Therefore the stability phenomena is unsteady and temporal dependance should be considered in this kind of research.

The quasi periodic character of stability boundary at very low reduced velocity and mass damping parameters is attributed to the limitations of the model imposed by assuming one dimensional approximation. If one relaxes this restriction (as in models 2 and 3) the condition cease to exist.

The notion of an unsteady control volume is not often used in a single phase flow. In this particular case it has proven to be very useful. We assumed that the area perturbation behaved as a harmonic function (similarly as vortex shedding), and by using the notion of an unsteady volume, we could easily conserve mass without

affecting the one dimensional character of the model. By solving the equations in a complex plane, we simplified finding the conditions under which the threshold occurs.

4 TWO DIMENSIONAL ANALYSIS INTO NORMAL MODES (SECOND MODEL)

The one dimensional approach seems to be inappropriate for some of the problems encountered. It is adequate for the problem for which it was designed, i.e. for a vibrating cylinder in the field of rigid cylinders. As long as the cylinders don't experience flow conditions leading to fluidelastic instabilities, the model predictions are quite accurate. The model doesn't treat the relationship between adjacent cylinders although it is time dependant (because of the assumptions regarding the area functions).

These problems can be partially resolved through the use of analysis into normal modes as described below. The assembly we will be concerned with in this work is a row consisting of a finite number of cylinders pulsating in the flow. The flow-cylinder configuration is shown in Figure 11. This is the first step in the process of calculating the vibrational characteristics for array of cylinders in fluid flow.

What are the properties that determine the fate of the assembly vibrations? The important parameters are the streaming Reynolds number, the Reynolds number of the incoming flow, the wavenumber and frequency of the oscillations generated by oscillating cylinders and the amplitude of the oscillations. Our task will be to determine what combination of the above mentioned parameters will lead to instability and how they are related to one another. In other words we are going to

carry out a normal modes analysis and extend it with parametric analysis.

Figure 11. The Assembly.

4.1 ASSUMPTIONS

Several assumptions will be employed to simplify the problem. Some of them will be used in the formulation of the problem, some of them later. The assumptions to be used are the following:

- The fluid field is solenoidal.
- There are two coordinate systems used at the same time. A local coordinate system coincides with a streamtube meandering through an assembly. The coordinate system is similar to those in boundary layer problems with x coordinate aligned along the centerline of the streamtube and y coordinate perpendicular to it. The terms in equation of fluid motion arising due to the curvature are neglected as the scaling shows size of second order of perturbation. The global coordinate system, used for evaluation of cylinder motion is a standard Cartesian orthogonal system.
- The cylinder array will be fixed as shown in figure 11. This will effectively keep the first derivative of the mean flow velocity with respect to x near zero. This constraint could be relaxed easily but that would forgo an analytical solution.

- The analysis performed will be a linearized perturbation analysis, and the flow field variables will be decomposed into mean and perturbed flow variables.
- The flow field is two dimensional. This limitation is acceptable since the final results will relate the damping parameter and nondimensional flow velocity (ratio of streaming and ordinary Reynolds numbers). Since the main objective of this work is cross flow induced vibrations as mentioned in first chapter the constraint is justified.
- Volume forces (e.g. gravity) will be neglected.
- The properties of the fluid, such as viscosity or thermal conductivity, will be taken to be constant.
- The mean velocity of the incoming flow will be a constant with regard to space and time. This can be visualized as a streamtube of constant velocity fluid constrained by cylinders. The mean flow is due to an outside applied pressure gradient in the flow field. Therefore the viscosity forces are not controlling the mean flow through the field. Disturbances, however, are solely due to the cylinders motion and the presence of viscosity forces can not be neglected.

- All terms of second order (perturbation squared) will be considered negligible (a standard linear perturbation theory assumption).
- The perturbation will decay within a finite distance of the cylinder array (due to fluid damping).
- The curvature of the streamtube is large compared to its width, allowing a standard Cartesian coordinate system to be used. The x coordinate is aligned along the surface of the cylinder and the y coordinate is perpendicular to the surface.

4.2 GOVERNING EQUATIONS

This approach assumes standard Eulerian notion of the control volume. The control volume is constant and doesn't depend on either time or the local spatial coordinates x and y (as defined in section on assumptions and simplifications).

- Mass Conservation Equation

If the above stated assumptions are utilized, the mass conservation equation reduces to

$$\nabla \cdot \vec{v} = 0 \quad (5.1)$$

which indicates the solenoidal character of the flow field. ∇ represents a gradient operator and velocity vector $\vec{v}$ has two components, $\vec{u}$ and $\vec{v}$ in x and y directions respectively.

- Momentum Conservation Equation

For a Newtonian incompressible fluid the momentum equation becomes

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v} = -\nabla p + \mu \nabla^2 \vec{v} \quad (5.2)$$

where ρ represents the fluid density, p is pressure, t time and μ stands for dynamic viscosity.

The flow field variables are decomposed into mean and disturbance values,

$$\begin{aligned}\bar{u} &= U + u \\ \bar{v} &= V + v\end{aligned}\tag{5.3}$$

where u and v are the perturbed components of the velocity vector and U and V their mean flow counterparts in the x and y direction respectively. The $\bar{u}$ and $\bar{v}$ were defined above. The energy equation is omitted since the momentum equation doesn't depend on temperature and thus the equations are not coupled. By employing the chain rule we obtain the following form of the continuity equation,

$$\frac{\partial U}{\partial x} + \frac{\partial u}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial v}{\partial y} = 0$$

It has been assumed that U remains constant and that V vanishes thus automatically satisfying the continuity equation, u and v satisfy

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\tag{5.5}$$

Note that the pressure was not decomposed into mean and disturbed part. This is caused by assuming $U = \text{const}$ which implies negligible pressure gradient in the mean flow. This is of course not true, however the implication that the vibrations are caused by oscillating pressure in their entirety is not affected by this.

Similarly, the momentum conservation equations takes the form

$$\begin{aligned} \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u \\ \frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla^2 v \end{aligned} \quad (5.6)$$

There are two possibilities for further development of the stability equations. One is to consider the vorticity formulation of the problem (an Orr-Sommerfeld like approach) and a second is to introduce the normal modes of variables into scaled forms of the above equations. The second will be chosen here as it allows a more straightforward extension to two phase flows.

The parameters in the above equations are scaled as follows:

$$\begin{aligned} t - \frac{t^*}{\omega} ; \quad x - x^* d ; \quad y - y^* d \\ u - u^* U ; \quad v - v^* U ; \quad p - p^* \rho U \omega_n \\ F - F^* \rho d^2 U \omega_n \end{aligned} \quad (5.7)$$

with d used for diameter of the tube in array, ω for true and ω_n for natural frequencies of the tube, and F representing the force acting on the cylinder. Asterisks denote dimensionless variables. Note that the units for pressure is Pa/m and for force are N/m due to 2-D model (infinite depth).

The question arises as to why should we scale the time with ω and pressure

with ω_n . The time frame of the event is obviously connected to the time in which one vibration is performed, and this time is inversely proportional to the frequency of vibration. The pressure and forces are governed more by geometry than by oscillations. For instance a small cylinder and a large cylinder vibrating with the same oscillation frequency does not experience the same forces and thus the natural frequency which is dependant on geometry and not on flow properties seems to be more appropriate for scaling of pressure and forces.

For the sake of clarity the superscripts * will be dropped. All the variables appearing in equations for fluid motion from this point on will be in dimensionless form.

- Dimensionless Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5.8)$$

- Dimensionless Momentum Equation

$$\begin{aligned} \left(\frac{\omega}{\omega_n} \right) \frac{\partial u}{\partial t} + \frac{Re_s}{Re} \frac{\partial u}{\partial x} - \frac{\partial p}{\partial x} + \frac{1}{Re} \frac{Re_s}{Re} \nabla^2 u \\ \left(\frac{\omega}{\omega_n} \right) \frac{\partial v}{\partial t} + \frac{Re_s}{Re} \frac{\partial v}{\partial x} - \frac{\partial p}{\partial y} + \frac{1}{Re} \frac{Re_s}{Re} \nabla^2 v \end{aligned} \quad (5.9)$$

where the streaming Reynolds number and Reynolds number are defined as

$$Re = \frac{Ud}{\nu} ; \quad Re_s = \frac{U^2}{\omega_n \nu} \quad (5.10)$$

We have 3 equations and four unknowns, namely velocity components, pressure and frequency ratio. To be able to solve this system we need another set of equations - equations of cylinder motion. These equations are usually found in the form (dimensional)

$$\begin{aligned} m\ddot{X} + c\dot{X} + k_s X - \frac{Re_s}{Re} \Sigma F_x \\ m\ddot{Y} + c\dot{Y} + k_s Y - \frac{Re_s}{Re} \Sigma F_y \end{aligned} \quad (5.11)$$

where m represents a mass of the structure per unit depth, c is its damping parameter and k_s its stiffness. X and Y are cylinder displacements in global x and y directions, $\ddot{}$ and $\dot{}$ are respective second and first time derivatives and Σ represents the sum of all the forces on the cylinder in question.

Using the standard notation (see Lever, Weaver etc.)

$$c = m\delta \frac{\omega_n}{\pi} ; \quad k_s = m\omega_n^2 \quad (5.12)$$

and introducing the scaling given by equation (4.7) yields

$$\begin{aligned}
& \left(\frac{\omega}{\omega_n} \right)^2 \frac{m}{\rho d^2} \ddot{X} + \left(\frac{\omega}{\omega_n} \right) \frac{m \delta}{\rho d^2 \pi} \dot{X} + \frac{m}{\rho d^2} X - \frac{Re}{Re} \Sigma F_x, \\
& \left(\frac{\omega}{\omega_n} \right)^2 \frac{m}{\rho d^2} \ddot{Y} + \left(\frac{\omega}{\omega_n} \right) \frac{m \delta}{\rho d^2 \pi} \dot{Y} + \frac{m}{\rho d^2} Y - \frac{Re}{Re} \Sigma F_y,
\end{aligned} \tag{5.13}$$

The forces on the left hand side of the equations of cylinder motion are obtained by integrating the pressure around the cylinder. The next step is to evaluate the pressure as a function of spatial coordinates and perform the analysis into normal modes.

4.3 ANALYSIS INTO NORMAL MODES

The main direction in which the information travels is assumed to be parallel with the incoming flow direction, in our case that will be x-axis direction. The variables can then be written (noting that ω/ω_n is a complex number) as

$$\begin{aligned}
u &= U(x) e^{iky + \omega t}; \quad v = V(x) e^{iky + \omega t} \\
p &= P(x) e^{iky + \omega t}; \quad X = X(x) e^{iky + \omega t}; \quad Y = Y(x) e^{iky + \omega t}
\end{aligned} \tag{5.14}$$

The symbols U, V, P, X and Y have already been used in the text. Since we don't want to use too many additional symbols, from this point on let those symbols represent some unknown function of x (function yet to be determined). Substituting the normal mode representations into Eqs. (4.12) and (4.13) yields

- Continuity Equation

$$\dot{U} + ikV = 0 \quad (5.15)$$

- x - Momentum Equation

$$\left(\frac{\omega}{\omega_n} \right) U + \frac{Re_s}{Re} \dot{U} - \dot{P} + \frac{Re_s}{Re^2} (\ddot{U} - k^2 U) \quad (5.16)$$

- y - Momentum Equation

$$\left(\frac{\omega}{\omega_n} \right) V + \frac{Re_s}{Re} \dot{V} - ikP + \frac{Re_s}{Re^2} (\ddot{V} - k^2 V) \quad (5.17)$$

- x - Cylinder Motion

$$\left(\frac{\omega}{\omega_n} \right)^2 \frac{m}{\rho d^2} X + \left(\frac{\omega}{\omega_n} \right) \frac{m}{\rho d^2} \frac{\delta}{\pi} \dot{X} + \frac{m}{\rho d^2} X - \frac{Re_s}{Re} \Sigma F_x \quad (5.18)$$

- y - Cylinder Motion

$$\left(\frac{\omega}{\omega_n} \right)^2 \frac{m}{\rho d^2} Y + \left(\frac{\omega}{\omega_n} \right) \frac{m}{\rho d^2} \frac{\delta}{\pi} \dot{Y} + \frac{m}{\rho d^2} Y - \frac{Re_s}{Re} \Sigma F_y, \quad (5.19)$$

As we see above the equations depend also on properties k (dimensionless wavenumber) and ω/ω_n (dimensionless frequency) which determine in detail the actual dependency on other spatial (y-direction) and temporal variable.

4.4 SOLUTION TO DYNAMIC INSTABILITY IN STREAMWISE DIRECTION

By using equation (4.15) in equation (4.17) and differentiating equation 4.16 twice, multiplying it by -1 and adding the results we arrive at

$$\left(\frac{\omega}{\omega_n}\right)(\ddot{U}-k^2 U) + \frac{Re_s}{Re}(\ddot{U}-k^2 U) - \frac{Re_s}{Re^2} \left[\left(\frac{d^2}{dx^2} - k^2 \right) (\ddot{U}-k^2 U) \right] \quad (5.20)$$

which has a solution of the form

$$U(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + D_1 \sinh(kx) + D_2 \cosh(kx) \quad (5.21)$$

where

$$\lambda_{1,2} = \frac{1}{2} Re \left\{ 1 \pm \sqrt{1 + 4 \left[\frac{1}{Re_s} \frac{\omega}{\omega_n} + \left(\frac{k}{Re} \right)^2 \right]} \right\} \quad (5.22)$$

The values of the constants C_1 , C_2 , D_1 and D_2 are determined by appropriate boundary conditions. The correct choice is very important and thus we will try to explain and explain the choice in great detail.

Boundary Conditions

Most of the models we found were based on a fixed arrangement of the cylinders. The ideal solution would be modular, it would allow us to build arbitrary arrangements. The first choice for the modular unit would be the cylinder surrounded with imaginative boundaries. This unit is displayed in figure 12.

Figure 12. Example Of Modular Unit.

The first boundary condition is no slip condition on the cylinder's surface. Since the cylinder is moving with some velocity the fluid will match that velocity; the relative velocity of the layer of the fluid close to the surface and surface itself will be zero. This boundary condition is a standard practice in fluid mechanics. Thus, at the cylinder-fluid interface,

$$u - \frac{d\hat{x}}{dt} = U(x) - \frac{\omega}{\omega_n} \frac{Re}{Re_s} X(x) \quad (5.23)$$

where the hat denotes the dimensional displacement in x-direction.

The second boundary condition results from the assumption that the shear generated by the y-component of the oscillation velocity achieves an extreme at the surface. Since the wall shear is the only balancing mechanism it is also safe to assume that the change of shear will be maximum at the interface as well, or

$$\begin{aligned} \frac{\partial^3 v}{\partial x^2 \partial y} - \frac{\partial^3 u}{\partial x^3} &= 0 \rightarrow \\ \frac{d^3 U(x)}{dx^3} &= 0 \end{aligned} \quad (5.24)$$

The third boundary conditions states that the disturbances will remain finite far away from the cylinder. Although it was shown by H. Schlichting (1979) (using C.C.Lin's (1957) analysis) that the disturbances generate substantial flow far away from a still cylinder, it suffices for our purposes that they don't grow. This means that

the constant C_1 must be zero for all positive $\lambda_1 x$.

To generate the last (fourth) boundary condition, let's imagine the boundaries as presented in Figure 12. At the boundary the disturbed velocity is continuous. The velocities generated by cylinders' oscillations must therefore be the same,

$$U_i(x) = U_{i+1}(x) \quad (5.25)$$

where i denotes the cylinder and $i+1$ its neighbor.

Next we must consider that the first and the last cylinder don't have any neighbors. For the first cylinder it's easy to see that the incoming flow will sweep all the disturbances away with the flow. There are two options which might be used for the last cylinder. The first option is to assume that the damping of the fluid is large enough to eliminate any disturbance effects. Despite reasons against such an argument, it could be justified by assuming that the disturbances are much smaller than the mean velocity of the flow going through the bundle. The second option is to assume the velocities don't vanish but remain finite and don't change with distance. First option translates into

$$u(x \rightarrow \pm\infty) = 0 \rightarrow U(x \rightarrow \pm\infty) = 0 \quad (5.26)$$

and second option into

$$\frac{\partial}{\partial x}[u(x \pm \infty)] = 0 \rightarrow \dot{U}(x \pm \infty) = 0 \quad (5.27)$$

The question "What does far away mean?" remains open for the time being. We will denote the distance "far away" simply as a value a_f .

This set of boundary conditions yields the following values for the constants needed to describe the disturbance velocity:

- for the first cylinder - upstream

$$\begin{aligned} D_1 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_a \frac{\left[\cosh(ka_f) - \left(\frac{k}{\lambda_2} \right)^3 e^{\lambda_2(a_f-a)} \sinh(ka) \right]}{H} \\ D_2 &= \frac{\omega}{\omega_n} \frac{Re}{Re_n} X_a \frac{\left[-\sinh(ka_f) + \left(\frac{k}{\lambda_2} \right)^3 e^{\lambda_2(a_f-a)} \cosh(ka) \right]}{H} \\ C_1 &= 0 \\ C_2 &= -e^{-\lambda_2 a} \left(\frac{k}{\lambda_2} \right)^3 [D_1 \cosh(ka) + D_2 \sinh(ka)] \end{aligned} \quad (5.28)$$

where

$$\begin{aligned}
& H \left[\sinh(ka) - \left(\frac{k}{\lambda_2} \right)^3 \cosh(ka) \right] \left[\cosh(kb) - \left(\frac{k}{\lambda_2} \right)^3 e^{\lambda_2(b-a)} \sinh(ka) \right] \\
& - \left[\cosh(ka) - \left(\frac{k}{\lambda_2} \right)^3 \sinh(ka) \right] \left[\sinh(kb) - \left(\frac{k}{\lambda_2} \right)^3 e^{\lambda_2(b-a)} \cosh(ka) \right]
\end{aligned} \quad (5.29)$$

- for any cylinder between first and last

$$\begin{aligned}
D_1 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_a \frac{\left[\cosh(kb) - \left(\frac{k}{\lambda_2} \right)^3 e^{\lambda_2(b-a)} \sinh(ka) \right] - \frac{U_b}{U_a} \left[\cosh(ka) - \left(\frac{k}{\lambda_2} \right)^3 \sinh(ka) \right]}{H} \\
D_2 &= \frac{\omega}{\omega_n} \frac{Re}{Re_n} X_a \frac{\frac{U_b}{U_a} \left[\sinh(ka) - \left(\frac{k}{\lambda_2} \right)^3 \cosh(ka) \right] - \left[\sinh(kb) - \left(\frac{k}{\lambda_2} \right)^3 e^{\lambda_2(b-a)} \cosh(ka) \right]}{H} \quad (5)
\end{aligned}$$

$$C_1 = 0$$

$$C_2 = -e^{-\lambda_2 a} \left(\frac{k}{\lambda_2} \right)^3 [D_1 \cosh(ka) + D_2 \sinh(ka)]$$

where

$$\begin{aligned}
& H \left[\sinh(ka) - \left(\frac{k}{\lambda_2} \right)^3 \cosh(ka) \right] \left[\cosh(kb) - \left(\frac{k}{\lambda_2} \right)^3 e^{\lambda_2(b-a)} \sinh(ka) \right] \\
& - \left[\cosh(ka) - \left(\frac{k}{\lambda_2} \right)^3 \sinh(ka) \right] \left[\sinh(kb) - \left(\frac{k}{\lambda_2} \right)^3 e^{\lambda_2(b-a)} \cosh(ka) \right]
\end{aligned} \quad (5.31)$$

- and for the last cylinder using the notion of small but constant disturbances far away downstream - valid for downstream direction

$$\begin{aligned}
 D_1 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_s \frac{\cosh(ka) - \left(\frac{k}{\lambda_2}\right)^3 \sinh(ka)}{H} \\
 D_2 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_s \frac{e^{\lambda_2(a_f-a)} \left(\frac{k}{\lambda_2}\right)^2 \cosh(ka) - \cosh(ka_f)}{H} \\
 C_1 &= 0 \\
 C_2 &= -e^{-\lambda_2 a_f} \left(\frac{k}{\lambda_2}\right)^3 [D_1 \cosh(ka) + D_2 \sinh(ka)]
 \end{aligned} \tag{5.32}$$

where

$$\begin{aligned}
 H &= \left[\cosh(ka) - \left(\frac{k}{\lambda_2}\right)^3 \sinh(ka) \right] \left[\cosh(ka_f) - e^{\lambda_2(a_f-a)} \left(\frac{k}{\lambda_2}\right)^2 \cosh(ka) \right] \\
 &\quad - \left[\sinh(ka) - \left(\frac{k}{\lambda_2}\right)^3 \cosh(ka) \right] \left[\sinh(ka_f) - e^{\lambda_2(a_f-a)} \left(\frac{k}{\lambda_2}\right)^2 \sinh(ka) \right]
 \end{aligned} \tag{5.33}$$

Pressure

Pressure is computed from, see equation (4.17),

$$k^2 P = \frac{Re_s}{Re^2} \frac{d}{dx} (\tilde{U} - k^2 U) - \frac{Re_s}{Re} \tilde{U} - \frac{\omega}{\omega_n} \dot{U} \quad (5.34)$$

which yields

$$\begin{aligned} \tilde{P} = & \tilde{C}_2 e^{\lambda_2 x} \left(\frac{\lambda_2}{k} \right)^2 \left[\frac{Re_s}{Re^2} \left(\lambda_2 - \frac{k^2}{\lambda_2} \right) - \frac{Re_s}{Re} - \frac{1}{\lambda_2} \frac{\omega}{\omega_n} \right] \\ & - \tilde{D}_1 \left[\frac{Re_s}{Re} \sinh(kx) + \frac{1}{k} \frac{\omega}{\omega_n} \cosh(kx) \right] \\ & - \tilde{D}_2 \left[\frac{Re_s}{Re} \cosh(kx) + \frac{1}{k} \frac{\omega}{\omega_n} \sinh(kx) \right] \end{aligned} \quad (5.35)$$

where the parameters have been made dimensionless by the following scaling:

$$\begin{aligned} C_2 = & \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_a \tilde{C}_2 ; \quad D_1 = \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_a \tilde{D}_1 \\ D_2 = & \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_a \tilde{D}_2 ; \quad P = \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_a \tilde{P} \end{aligned} \quad (5.36)$$

Force in x-direction

The force in x direction is obtained by integrating the pressure along the surface of the cylinder and taking the x component. The force (nondimensional form) is

$$F_x = 2 \int_{a^-}^{a^+} p dx \quad (5.37)$$

Since the cylinder is assumed to be a rigid body, the velocities at a^- and a^+ must be the same. This means

$$\begin{aligned} C_2^+ &= C_2^- e^{\lambda_2(a^+ - a^-)} \\ D_1^+ &= D_1^- \frac{\sinh(ka^-)}{\sinh(ka^+)} \\ D_2^+ &= D_2^- \frac{\cosh(ka^-)}{\cosh(ka^+)} \end{aligned} \quad (5.38)$$

Integrating expression (4.35) produces

$$\begin{aligned} \frac{F_x}{2 \frac{\omega}{\omega_n} \frac{Re}{Re_s} X_a} &= \frac{\tilde{C}_2}{\lambda_2} [1 - e^{\lambda_2(a^+ - a^-)}] \\ &- \frac{\tilde{D}_1}{k} \left\{ \frac{Re}{Re_s} \left[1 - \cosh(ka^-) + \coth(ka^+) \sinh(ka^-) - \frac{\sinh(ka^-)}{\sinh(ka^+)} \right] \right. \\ &\quad \left. + \frac{1}{k} \frac{\omega}{\omega_n} \left[1 - \frac{\sinh(ka^-)}{\sinh(ka^+)} \right] \right\} \\ &- \frac{\tilde{D}_2}{k} \left\{ \frac{Re_s}{Re} \left[1 - \sinh(ka^-) + \tanh(ka^+) \cosh(ka^-) - \frac{\cosh(ka^-)}{\cosh(ka^+)} \right] \right. \\ &\quad \left. + \frac{1}{k} \frac{\omega}{\omega_n} \left[1 - \frac{\cosh(ka^-)}{\cosh(ka^+)} \right] \right\} \end{aligned} \quad (5.39)$$

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The threshold of instability is found by regrouping coefficients in equation (4.18) according to powers of ω/ω_n and that yields the threshold of dynamic instability as a function of mass damping parameter as

$$\begin{aligned}
 0 = & \frac{1}{2} \frac{m}{\rho d^2} \frac{\delta}{\pi} - \frac{\tilde{C}_2}{\lambda_2} [1 - e^{\lambda_2(s^- - a^-)}] \\
 & + \frac{\tilde{D}_1}{k} \left\{ \frac{Re}{Re_s} \left[1 - \cosh(ka^-) + \coth(ka^+) \sinh(ka^-) - \frac{\sinh(ka^-)}{\sinh(ka^+)} \right] \right\} \\
 & + \frac{\tilde{D}_2}{k} \left\{ \frac{Re_s}{Re} \left[1 - \sinh(ka^-) + \tanh(ka^+) \cosh(ka^-) - \frac{\cosh(ka^-)}{\cosh(ka^+)} \right] \right\}
 \end{aligned} \quad (5.40)$$

The effect of neighboring cylinder

Expressions for the constants C_j , D_1 and D_2 contain the velocity U_b , which depends on the neighboring (adjacent) cylinder. If we denote one of the cylinders with i and the adjacent one with $i+1$ then the following results

$$\begin{aligned}
 U_{b,i+1}^- &= U_{b,i-1}^+ \\
 \rightarrow C_{2,i+1}^- e^{\lambda_2 b} &+ D_{1,i+1}^- \sinh(kb) + D_{2,i+1}^- \cosh(kb) - \\
 &C_{2,i}^+ e^{\lambda_2 b} + D_{1,i}^+ \sinh(kb) + D_{2,i}^+ \cosh(kb) \\
 \rightarrow C_{2,i+1}^- - C_{2,i}^+ ; & D_{1,i+1}^- - D_{1,i}^+ ; D_{2,i+1}^- - D_{2,i}^+
 \end{aligned} \quad (5.41)$$

Next we must account for the ratio U_b/U_a present in equation (4.30). First, we assume that the wavenumbers, frequencies and the ratios of adjacent cylinders' differ. Next notice that the mass damping parameter in equation (4.40) depends only on the physical setup. Our cylinder array is comprised of cylinders of the same mass and diameters and is made of the same material. This leads one to conclude that the mass damping parameter at which the instability will occur will be the same for whole array. Having that in mind let us compare the expressions for thresholds of instability for cylinder i and $i+1$. Assume that the tilde values for constants downstream of the i 'th cylinder are known (as is the case with first cylinder). In that case the ratio U_b/U_a can be calculated from relationship described in equation (4.38) as

$$\left(\frac{U_b}{U_a}\right)_i = \frac{\left[\cosh(kb_i^+) - \left(\frac{k}{\lambda_2}\right)^3 e^{\lambda_2(b_i^+ - a_i^+)} \sinh(ka_i^+) \right] - \tilde{D}_{1,i}^+ H_i^+ \frac{\sinh(ka_i^-)}{\sinh(ka_i^+)}}{\cosh(ka_i^+) - \left(\frac{k}{\lambda_2}\right)^3 \sinh(ka_i^+)} \quad (5.42)$$

and using expressions in (4.41)

$$\left(\frac{U_b}{U_a}\right)_{i+1} = \frac{\left[\cosh(kb_{i+1}^+) - \left(\frac{k}{\lambda_2}\right)^3 e^{\lambda_2(b_{i+1}^+ - a_{i+1}^+)} \sinh(ka_{i+1}^+) \right] - \tilde{D}_{1,i+1}^+ H_{i+1}^+ \frac{\omega_i}{\omega_{i+1}} \frac{X_{a_i}^+}{X_{a_{i+1}}^+}}{\cosh(ka_{i+1}^+) - \left(\frac{k}{\lambda_2}\right)^3 \sinh(ka_{i+1}^+)} \quad (5.43)$$

There are two important parameters which determine the above described relationship

$$\frac{\omega_{i+1}}{\omega_i} \frac{X_{a,i+1}}{X_{a,i}} ; k_{i+1} = f(k_i) \quad (5.44)$$

where the second relationship is only implied by assumption of different wavenumbers (and omitted in analysis since it is going to be neglected, see Results of this section).

With the help of the constants, valid for downstream direction of last cylinder we can calculate one of the remaining parameters. We cannot determine both of them though. Once the parameters are determined the mass damping parameter and non-dimensional flow velocity relationship for instability occurrence can be determined from equation (4.40).

The reason we cannot determine both parameters from the first principles is the lack of a third dimension in the equations for cylinder motion making it homogeneous as far as X_a dependance is concerned. If we add the third dimension there will be one more piece of information (EI product) available.

4.5 SOLUTION IN CROSS STREAM DIRECTION

If we want to address this problem we will have to repeat the steps applied in previous section while simultaneously solving for streamwise direction disturbance velocity components needed to solve for cross stream velocity. In this case, similar manipulation as before produces

$$\left(\frac{\omega}{\omega_n}\right)(\tilde{V}-k^2V)+\frac{Re_s}{Re}\frac{d}{dx}(\tilde{V}-k^2V)-\frac{Re_s}{Re^2}\left[\left(\frac{d^2}{dx^2}-k^2\right)(\tilde{V}-k^2V)\right] \quad (5.45)$$

which has solution of the form

$$V(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} + D_1 \sinh(kx) + D_2 \cosh(kx) \quad (5.46)$$

where

$$\lambda_{1,2} = \frac{1}{2} Re \left\{ 1 \pm \sqrt{1 + 4 \left[\frac{1}{Re_s} \frac{\omega}{\omega_n} + \left(\frac{k}{Re} \right)^2 \right]} \right\} \quad (5.47)$$

Needless to say, the constants in equation (4.46) are not the same as in the equation for $U(x)$ component. The same symbols are retained for simplification. Since this section deals entirely with transversal component of disturbance velocity the constants will be referred to accordingly.

Boundary conditions

Figure 13 represents the boundaries where the boundary conditions are specified.

Figure 13. Modular Unit for Transversal Component.

On the cylinder, the velocity of the fluid matches that of the cylinder (no slip condition).

$$v = \frac{d\tilde{Y}}{dt} \rightarrow V(x) = \frac{\omega}{\omega_s} \frac{Re}{Re_s} Y(x) \quad (5.48)$$

where the tilde indicates the dimensional displacement in y-direction.

The second condition on the interface will be slightly different from above. Since the bulk of the fluid is moving in x-direction we wouldn't expect significant stress to occur between the cylinders, rather we would expect it to occur on the surface of the cylinder. Thus the viscous stress is a maximum on the surface of the cylinder

$$\frac{\partial^2 v}{\partial x^2} = 0 \rightarrow \tilde{V}(x_{surf}) = 0 \quad (5.49)$$

On the boundary between the two cylinders (imaginative divider), the velocity must be continuous,

$$V_{i,i} = V_{i+1,i+1} \quad (5.50)$$

Far away from the cylinder we would expect the velocity to show a non growing characteristic,

$$C_1 = 0 \quad \forall x \geq 0 \quad (5.51)$$

We need to say something about the flow conditions before the first and past the last cylinder in the row. Far upstream, denoted by a_{fu} , we don't expect any disturbances. The information doesn't propagate against the bulk flow of the fluid,

$$v(x-a_{fu})=0 \rightarrow V(x-a_{fu})=0 \quad (5.52)$$

Far downstream (denoted by a_{fd}) we cannot argue the disturbances to die out. We can say, however, that they remain constant,

$$\frac{\partial}{\partial x}[v(x-a_{fd})]=0 \rightarrow \dot{V}(x-a_{fd})=0 \quad (5.53)$$

Constants of Integration

Application of boundary conditions to equation (4.46) produces the following constants

- for the first cylinder in upstream direction

$$\begin{aligned} D_1 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_a \frac{\left[\cosh(ka_p) - \left(\frac{k}{\lambda_2} \right)^2 e^{\lambda_2(a_f-a)} \cosh(ka) \right]}{H} \\ D_2 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_a \frac{\left[-\sinh(ka_p) + \left(\frac{k}{\lambda_2} \right)^2 e^{\lambda_2(a_f-a)} \sinh(ka) \right]}{H} \\ C_1 &= 0 \\ C_2 &= -e^{-\lambda_2 a} \left(\frac{k}{\lambda_2} \right)^2 [D_1 \sinh(ka) + D_2 \cosh(ka)] \end{aligned} \quad (5.54)$$

where

$$\begin{aligned}
 & H \left[\sinh(ka) - \left(\frac{k}{\lambda_2} \right)^2 \sinh(ka) \right] \left[\cosh(ka) - \left(\frac{k}{\lambda_2} \right)^2 e^{\lambda_2(a-r)} \cosh(ka) \right] \\
 & - \left[\cosh(ka) - \left(\frac{k}{\lambda_2} \right)^2 \cosh(ka) \right] \left[\sinh(ka) - \left(\frac{k}{\lambda_2} \right)^2 e^{\lambda_2(a-r)} \sinh(ka) \right]
 \end{aligned} \quad (5.55)$$

- for any cylinder between first and last

$$\begin{aligned}
 D_1 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_a \frac{\left[\cosh(kb) - \left(\frac{k}{\lambda_2} \right)^2 e^{\lambda_2(b-a)} \cosh(ka) \right] - \frac{V_b}{V_a} \left[\cosh(ka) - \left(\frac{k}{\lambda_2} \right)^2 \cosh(ka) \right]}{H} \\
 D_2 &= \frac{\omega}{\omega_n} \frac{Re}{Re_n} Y_a \frac{\frac{V_b}{V_a} \left[\sinh(ka) - \left(\frac{k}{\lambda_2} \right)^2 \sinh(ka) \right] - \left[\sinh(kb) - \left(\frac{k}{\lambda_2} \right)^2 e^{\lambda_2(b-a)} \sinh(ka) \right]}{H} \quad (5.56) \\
 C_1 &= 0 \\
 C_2 &= -\lambda_2^2 \left(\frac{k}{\lambda_2} \right)^2 [D_1 \sinh(ka) + D_2 \cosh(ka)]
 \end{aligned}$$

where

$$\begin{aligned}
 & H \left[\sinh(ka) - \left(\frac{k}{\lambda_2} \right)^2 \sinh(ka) \right] \left[\cosh(kb) - \left(\frac{k}{\lambda_2} \right)^2 e^{\lambda_2(b-a)} \cosh(ka) \right] \\
 & - \left[\cosh(ka) - \left(\frac{k}{\lambda_2} \right)^2 \cosh(ka) \right] \left[\sinh(kb) - \left(\frac{k}{\lambda_2} \right)^2 e^{\lambda_2(b-a)} \sinh(ka) \right]
 \end{aligned} \quad (5.57)$$

- and for the last cylinder using the notion of small but constant disturbances far downstream; valid for downstream direction,

$$\begin{aligned}
 D_1 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_s \frac{\cosh(ka) - \left(\frac{k}{\lambda_2}\right)^2 \cosh(ka)}{H} \\
 D_2 &= \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_s \frac{e^{\lambda_2(a-r)} \left(\frac{k}{\lambda_2}\right) \sinh(ka) - \cosh(ka_p)}{H}
 \end{aligned} \tag{5.58}$$

$$C_1 = 0$$

$$C_2 = -e^{-\lambda_2 a} \left(\frac{k}{\lambda_2}\right)^2 [D_1 \sinh(ka) + D_2 \cosh(ka)]$$

where

$$\begin{aligned}
 H &= \left[\cosh(ka) - \left(\frac{k}{\lambda_2}\right)^2 \cosh(ka) \right] \left[\cosh(ka_p) - e^{\lambda_2(a-r)} \left(\frac{k}{\lambda_2}\right) \sinh(ka) \right] \\
 &\quad - \left[\sinh(ka) - \left(\frac{k}{\lambda_2}\right)^2 \sinh(ka) \right] \left[\sinh(ka_p) - e^{\lambda_2(a-r)} \left(\frac{k}{\lambda_2}\right) \cosh(ka) \right]
 \end{aligned} \tag{5.59}$$

Pressure

Pressure is computed from

$$ikP = \frac{Re_s}{Re^2} (\ddot{V} - k^2 V) - \frac{Re_s}{Re} \dot{V} - \frac{\omega}{\omega_n} V \tag{5.60}$$

which yields

$$\begin{aligned}
 \tilde{P} = & i \left\{ \tilde{C}_2 e^{\lambda_2 x} \left(\frac{\lambda_2}{k} \right) \left[\frac{Re_s}{Re^2} \left(\frac{k^2}{\lambda_2} - \lambda_2 \right) + \frac{Re_s}{Re} + \frac{1}{\lambda_2} \frac{\omega}{\omega_n} \right] \right. \\
 & + \tilde{D}_1 \left[\frac{Re_s}{Re} \sinh(kx) + \frac{1}{k} \frac{\omega}{\omega_n} \cosh(kx) \right] \\
 & \left. + \tilde{D}_2 \left[\frac{Re_s}{Re} \cosh(kx) + \frac{1}{k} \frac{\omega}{\omega_n} \sinh(kx) \right] \right\}
 \end{aligned} \tag{5.61}$$

where the tilde operator operates on constants and pressure as

$$\begin{aligned}
 C_2 = & \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_a \tilde{C}_2 ; \quad D_1 = \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_a \tilde{D}_1 \\
 D_2 = & \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_a \tilde{D}_2 ; \quad P = \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_a \tilde{P}
 \end{aligned} \tag{5.62}$$

Force in the y-direction

The forces are found by integrating the pressure around the cylinder and taking the y component in order to obtain the y-component of the force on the cylinder as (nondimensional form)

$$F_y = \left\{ \int_{\epsilon^-}^{\epsilon^+} p dy \right\}^+ + \left\{ \int_{\epsilon^-}^{\epsilon^+} p dy \right\}^- \tag{5.63}$$

where $\{ \}_+$ and $\{ \}_-$ stand for integration on leeward and flowward surface of the cylinder. Since the cylinder is assumed to be perfect, the connection between both coordinates is expressed as

$$y = y_0 + \sqrt{.25 - (x - x_0)^2} \quad (5.64)$$

This produces

$$\begin{aligned} F_y = \frac{\omega}{\omega_n} \frac{Re}{Re_s} Y_a \int_{a_i^-}^{a_i^+} \sin \left[k \left(y_0 - \sqrt{.25 - (x - x_0)^2} \right) \right] \frac{dx}{\sqrt{[2(x - x_0)]^2 - 1}} \\ \left\{ \tilde{C}_2 e^{\lambda_2 x} \left(\frac{\lambda_2}{k} \right) \left[\frac{Re_s}{Re^2} \left(\frac{k^2}{\lambda_2} - \lambda_2 \right) + \frac{Re_s}{Re} + \frac{1}{\lambda_2} \frac{\omega}{\omega_n} \right] \right. \\ \left. + \tilde{D}_1 \left[\frac{Re_s}{Re} \sinh(kx) + \frac{1}{k} \frac{\omega}{\omega_n} \cosh(kx) \right] \right. \\ \left. + \tilde{D}_2 \left[\frac{Re_s}{Re} \cosh(kx) + \frac{1}{k} \frac{\omega}{\omega_n} \sinh(kx) \right] \right\} \end{aligned} \quad (5.65)$$

where position along the path of integration requires appropriate use of $C_2^\pm$, $D_1^\pm$, $D_2^\pm$ and $C_2^\pm$, $D_1^\pm$, $D_2^\pm$.

The threshold for instability in the cross stream direction is found to be where the constants are the same as in equation (4.65).

$$\begin{aligned}
0 - \frac{m}{\rho d^2} \frac{\delta}{\pi} \int_{a_i^-}^{a_i^+} \sin \left[k \left(y_0 - \sqrt{25 - (x - x_0)^2} \right) \right] \frac{dx}{\sqrt{[2(x - x_0)^2 - 1]}} \\
\left\{ \bar{C}_2 e^{\lambda_2 x} \left(\frac{\lambda_2}{k} \right) \left[\frac{Re_s}{Re^2} \left(\frac{k^2}{\lambda_2} - \lambda_2 \right) + \frac{Re_s}{Re} \right] \right. \\
\left. + \bar{D}_1 \frac{Re_s}{Re} \sinh(kx) + \bar{D}_2 \frac{Re_s}{Re} \cosh(kx) \right\}
\end{aligned} \tag{5.66}$$

The connection between lee and flow-ward sides of the cylinder

The velocity is continuous, which means

$$\begin{aligned}
V_{a_i^+}^* &= V_{a_i^-}^* \\
\rightarrow C_2^* &= C_2^- \\
D_1^* &= D_1^- \\
D_2^* &= D_2^-
\end{aligned} \tag{5.67}$$

resulting in

$$\left(\frac{V_b}{V_a} \right)_i^* = \frac{\left[\cosh(kb_i^*) - \left(\frac{k}{\lambda_2} \right) e^{\lambda_2(a_i^* - a_i^-)} \sinh(ka_i^*) \right] - \bar{D}_1^* H_i^*}{\cosh(ka_i^*) - \left(\frac{k}{\lambda_2} \right)^2 \cosh(ka_i^*)} \tag{5.68}$$

Similary as in section dealing with streamwise threshold the connection between the i 'th and neighboring cylinder manifests itself in expression

$$\left(\frac{V_b}{V_a}\right)_{i+1}^{-} = \frac{\left[\cosh(kb_i^+) - \left(\frac{k}{\lambda_2}\right) e^{\lambda_2(b_i^+ - a_{i+1}^-)} \cosh(ka_{i+1}^-)\right] - \bar{D}_{1,i}^+ H_{i+1}^+ \frac{\omega_i}{\omega_{i+1}} \frac{Y_{a_i}^+}{Y_{a_{i+1}}^-}}{\cosh(ka_{i+1}^-) - \left(\frac{k}{\lambda_2}\right)^2 \cosh(ka_{i+1}^-)} \quad (5.69)$$

4.6 RESULTS

The method described in this chapter cannot describe the entire phenomena as the interaction between tubes cannot be fully modeled. This results in a need for one additional piece of information that must be obtained from elsewhere. That piece of information can either be the ratio of amplitudes of and their corresponding frequencies or the wavenumber of the assembly.

The easiest approach is to choose a wavenumber which corresponds to a known (experimentally determined) relationship between a particular reduced velocity (also known as ratio of streaming and ordinary Reynolds numbers) and mass damping parameter of the solid part of the structure. The next decision which needs to be made is whether there is one dominant wavenumber in the system or is there a single damping parameter equal for each and every cylinder in the assembly. The former yields the slightly different damping parameters for different cylinders

depending on their position and flow conditions, whereas the latter results in a variation of the wavenumbers among the cylinders. Doubtlessly the truth lies somewhere in between. The true flow conditions can be described only for each separate cylinder. Our boundary conditions, however, assume that the damping in the fluid is small enough and thus the fluidelastic mode of instabilities is the dominant one. Having that in mind one can safely deduce that most of the cylinders will be vibrating on the same wavelength and thus the wavenumbers won't differ a great deal. Further the equations for in-flow threshold of instability will yield the ratio of amplitudes. Due to the nature of the boundary condition at the top, the ratio of amplitudes equals 1, which corresponds to perfectly locked-in cylinders. In order to assess this a simple parametric analysis was performed using sin and cos waves as the ratio of amplitudes. The results were very similar to that of the constant amplitude ratio case. Therefore the functional form of amplitude ratio is not of particular importance.

Figure 14 presents the results for rotated square array with pitch to diameter ratio of 1.5. They are used for comparison between one dimensional analysis performed by Marn and Catton (1990), experimental results obtained by Weaver and Fitzpatrick (1988), Paidoussis et al. (1988) and results presented on previous pages. The results are very good for the ranges of damping parameters and reduced velocities the models were designed for. The one dimensional model deals with a single flexible cylinder in array of rigid cylinders. This approach cannot model fluid elastic instabilities but is very good for approximating the damping controlled modes

of instabilities. The border between both modes is at a value of 10 for the reduced velocity. The two dimensional approach, with the boundary conditions described in this work is on the other hand best suited for modelling instabilities controlled by fluidelastic forces.

The fluidelastic forces are more evident when observing in-flow direction than cross-flow direction, which is apparently false result, due to the assumption of periodicity in direction perpendicular to streamtube centerline. This is shown in figure 15. There is a distinct peak when the reduced velocity reaches value of approximately 200. Above and below this value the cross-flow mode is much more stable than in-flow mode.

Reduced Velocity vs. Damping, $P/d=1.5$

Figure 14. Comparison between 1-D and 2-D analysis.

Reduced Velocity vs. Damping, $P/d=1.5$

Figure 15. In-flow and cross-flow instabilities.

4.7 CONCLUSION

Single phase flow induced vibrations were described in the preceding contribution. For reduced flow velocities below 10, a one-dimensional unsteady integral approach, which simulates damping controlled instabilities, should be used. For velocities over 10, a two dimensional approach simulating instabilities controlled by fluidelastic forces would be the best tool to use. The strength of described model is in its simplicity and versatility. Since the results are given analytically it can be used for quick best-estimate analyses for various pitch to diameter ratios. The weakness of the model is that results in cross stream direction cannot be fully trusted, the results in streamwise direction, however, show good agreement with experimental results.

5 FLOW INDUCED VIBRATION STABILITY ANALYSIS USING VORTICITY TRANSPORT EQUATION (THIRD MODEL)

The equations used in this section of the dissertation are based on vorticity formulation of the momentum equation and coupled with equations of motion for cylinders. The governing equations for fluid motion using linearized perturbation theory and coupled with equations of motions for cylinders are displayed to yield the threshold of dynamic instability for the entire array. Simple parametric analysis is performed to optimize the instability threshold for a rotated square array with a pitch to diameter ratio of 1.5. The results are compared with theoretical and experimental results for similar arrays. The chapter is concluded with the discussion of discrepancies.

The main drawback of the models described before is that it seems to be of little help in dealing with the real geometries found in steam generators, the main application for which this research is being performed. This chapter concentrates on semi infinite and finite arrays of staggered cylinders. Calculations are performed on semi-infinite (finite depth, infinite width) and finite (44 cylinders) arrays.

5.1 GEOMETRICAL DESCRIPTION AND GENERAL ASSUMPTIONS

Figure 16 depicts the geometrical array we were concerned with. All of our semi-infinite array calculations were performed for an array of 11 cylinders deep and infinitely wide. There is no particular reason why number 11 was chosen for depth of the field. Figure 16 presents cylinders in rotated square array. Note that by varying the angle between two adjacent cylinders one can describe many other popular geometries (e.g. triangular). The flow is incoming from the left and exiting at the right with constant velocity. Inlet constant velocity seems to be consistent with the real flow conditions since the pressure fluctuations tend to be equalized in the steam generator downcomer.

Figure 16. The Geometry.

The problem described is a complex one and as such requires several assumptions. To evaluate the instability phenomena one has to establish the proper criteria to detect such an occurrence. Instability of the above structure manifests itself by rapidly growing amplitudes. More about this will be found in the section describing the governing equations.

The first assumption we used was that the phenomena of importance can be treated as quasi stationary. The validity of this assumption was shown in a previous chapters and is de facto used by most of the experimentalists while determining the onset of instability. If one evaluates the data reported, one discovers that the threshold of instability is reported as a function of reduced velocity and not as a function of apparent position of the tube at the time of instability occurrence. The latter would probably be impossible to achieve as the researcher needs to detect the instability by measuring the acceleration or actual displacement of the tube.

Next, the flowfield was assumed incompressible, which is a reasonable assumption as the fluid entering the lower part of steam generator is in form of liquid water. Probably the most controversial assumption is the assumption that if the tubes were fixed the flow through the field will be constant everywhere. Experimental work by Simonin and Barcouda (1986) nicely justifies this assumption. Similar work was performed at UCLA for square array by Leicher, Marn and Catton. The results for semi-infinite array suggest that an investigation be performed for finite array as they seem to be too conservative. Figure 17 shows the part of the streamtube used

in our calculations. As can be seen, the streamtube attaches to the tube and detaches at some angle of attachment. By using this assumption one effectively takes into account the wake (and vortices) between the cylinders and treats them as a rigid body with a slip surface.

Figure 17. The Streamtube and Cylinder.

The next question one must answer before attempting to solve the problem is what causes the fluctuations in the first place and what kind of behavior (stable or unstable) is the fluid-solid structure experiencing.

To answer this question let us perform a simple mind experiment. Let us picture the flow through the array of cylinders without imperfections. What happens if we increase the Reynolds number of incoming flow? The cylinders yield to increased body form drag but there is no net lift experienced, regardless of the Re number (as long as the cylinder does not experience vortex shedding). This proves that in absence of fluctuations simple increasing of Re number of flow does not warrant the onset of instability. Thus all of the disturbances are due exclusively to the disturbance part of velocity field. By solving the disturbed velocity field (superimposed on mean velocity field) one obtains the necessary information to predict the occurrence of instabilities. The motion of the tubes is used for vorticity generation and this produces the change in pressure along the perturbed cylinder surface, which in turn causes the net force acting on the cylinder and subsequently an instability occurrence when the instability criteria becomes fulfilled.

Neither fluid nor solid need to be in an unstable mode for instabilities to occur. In general there are four possibilities; 1) the fluid is stable and the structure unstable, 2) the solid is stable and the fluid unstable, 3) both are unstable or 4) some combination in which both participants are in a stable mode yet the whole structure is in an unstable mode.

Let us first examine the fluid instability. The single phase flow field can be assumed to be incompressible as the pressure variations are too small to cause significant density variations. Unstable fluid would therefore vibrate with uniform frequency and impose that frequency on otherwise stable solid parts of the structure. Chen (1987) reviews the result stating that the frequency spectrum of the solid vibrations vary considerably thus rejecting the possibility of an unstable fluid mode.

Next, consider unstable solid and stable fluid. The unstable solid part of the structure would most likely vibrate with one of the modes of its natural frequency while becoming unstable. Above (or below) such frequency the tubes would be stable. This would preclude the occurrence of a threshold of instability associated with any particular nondimensional velocity. This kind of behavior (occurring and vanishing unstable modes associated with a particular nondimensional velocity) is true for vortex shedding induced vibrations, but cannot be extended to fluidelastic instability due to the irregular frequency spectrum measured by various researchers and observed threshold of nondimensional velocity above which all velocities render the structure unstable.

The only remaining possibility is the combination of otherwise stable modes of both fluid and solid parts of the unstable structure. How is that possible? One can imagine the tubes in the fluid as connected to the walls by springs and dampers. In addition to structural damping there is also a strong damping caused by the surrounding fluid. These two effects compete with each other. Moreover, the fluid

damping depends on the ratio of streaming and ordinary Reynolds number. If the flow conditions result in equal but opposite damping of both fluid and solid parts of the structure or overdamping of the structure by the fluid part then:

- a growing solution to the equations of structural motion is possible as there is no net damping effect,
- such conditions can be associated with a threshold of nondimensional velocity above which all the velocities result in unstable motion of the structure (instability and overstability of the structure).

The problem statement could be formulated as follows: given the ratio of ordinary and streaming Reynolds numbers and the geometry of the structure, find such combination of frequencies and amplitudes of the solid parts of the structure that the damping parameter of any given tube equals the calculated (velocity flow field dependent) fluid damping parameter.

5.2 NUMERICAL CONSIDERATIONS

Figure 17 shows the basic cell on which calculations are performed. The geometry used will be a cylindrical geometry. By coupling a sufficient number of cells together, one can describe the whole streamtube meandering between the cylinders. This geometry is also nearly orthogonal (in case of vibrations) which helps with simplification of the governing equations. One can assign properties to each cell separately thus achieving a finite array or use two rows and mirror them over x axis - this would accomplish semi-infinite array.

The ideal algorithm for this task would be composed of the following steps:

- Assume a cylinder perturbation in x,y direction.
- Calculate the vorticity generation caused by such a perturbation invoking appropriate assumptions.
- Calculate components of the disturbed velocity field using the vorticity definition and the mass conservation equation.
- Calculate the pressure distribution on the cylinder surface using the momentum equation.

- Integrate the pressure along the cylinder surface to calculate the components of force acting on the cylinder.
- Perform an optimization procedure to determine the distribution of cylinders perturbations and frequency ratios that yield the lowest possible threshold of fluidelastic instability.

5.3 MODEL EQUATIONS

Fluid part

In accordance with the assumptions noted above the velocity field is divided into a mean and perturbed part. The same is done for the vorticity field. Since the mean flow through the streamtubes is a constant, the only vorticity generated will be that of perturbed velocity components,

$$\begin{aligned} \nabla \times \vec{v} &= \nabla \times (\vec{V} + \vec{u}) \\ &= \nabla \times \vec{V} + \nabla \times \vec{u} = \nabla \times \vec{u} = \eta \end{aligned} \quad (5.1)$$

where $\vec{V}$ stands for mean velocity and $\vec{u}$ for perturbed velocity vectors.

The vorticity transport equation for incompressible fluid can be found in literature (Panton, 1984) and is transformed using equation (5.1) to

$$\frac{\partial \eta}{\partial t} + \frac{V_\theta}{r} \frac{\partial \eta}{\partial \theta} - \nu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \eta}{\partial r} \right) + \nu \frac{1}{r^2} \frac{\partial^2 \eta}{\partial \theta^2} \quad (5.2)$$

where η stands for vorticity, r and θ are the coordinates and V_θ is the angular component of the mean velocity.

The following scaling will be used throughout this chapter:

$$\begin{aligned} t &= \frac{t^*}{\omega} ; \quad r = r^* d ; \quad x = x^* d ; \quad y = y^* d ; \quad \vec{u} = \vec{u}^* V_\theta \\ p &= p^* \rho U \omega_n ; \quad F = F^* \rho U d^2 \omega_n ; \quad \eta = \eta^* \frac{V_\theta}{d} \\ X &= X^* d ; \quad Y = Y^* d \end{aligned} \quad (5.3)$$

where x and y represent Cartesian coordinates used in evaluation of cylinder movements and X and Y are the amplitudes in the respective directions. Note that due to 2-D approximation units of pressure become Pa/m and force is expressed in N/m.

If one recalls the stationary character of the problem, one may use the following ansatz for vorticity, velocity and cylinder motion amplitudes,

$$\begin{aligned} X &= X(x,y) e^{i\omega t} ; \quad Y = Y(x,y) e^{i\omega t} \\ \eta &= \eta(r,\theta) e^{i\omega t} ; \quad v_\theta = v_\theta(r,\theta) e^{i\omega t} ; \quad v_r = v_r(r,\theta) e^{i\omega t} \end{aligned} \quad (5.4)$$

Scaling the governing equations and dropping asterisks for convenience one arrives at following dimensionless equation for disturbed vorticity

$$\frac{\omega}{\omega_n} i\eta + \frac{Re_s}{rRe} \frac{\partial \eta}{\partial \theta} - \frac{Re_s}{Re^2} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \eta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \eta}{\partial \theta^2} \right] \quad (5.5)$$

The following dimensionless parameters appearing in the equations to be solved,

$$Re = \frac{V_0 d}{\nu} ; \quad Re_s = \frac{V_0^2}{\omega_n d} \quad (5.6)$$

appear as the ratio of above parameters, namely the ratio of streaming Reynolds to ordinary Reynolds number. The ratio yields the so called reduced velocity, used as one of primary parameters for determining the threshold of instability.

The next step is to evaluate the boundary conditions. On the cylinder surface the velocity of the fluid matches that of the cylinder (no slip condition). The motion itself generates vorticity. At the outer boundary of the fluid cell a wake exists. As mentioned before, it will be treated as a rigid body with slip, which implies the normal derivatives to be 0. Entrance and exit of the fluid cell will be matched with previous and next fluid cells, thus implying the angular derivatives to be equal 0. Or in algebraic terms,

$$\begin{aligned}
@ r=r_{inn} &\rightarrow \eta - i \frac{\omega}{\omega_n} \frac{Re}{Re_s} \left\{ \frac{\partial}{\partial y} [X(r, \theta)] - \frac{\partial}{\partial x} [Y(r, \theta)] \right\} \\
@ r=r_{out} &\rightarrow \frac{\partial \eta}{\partial r} = 0 \\
@ \theta = \pm \theta_0 &\rightarrow \frac{\partial \eta}{\partial \theta} = 0
\end{aligned} \tag{5.7}$$

This problem is complete and can be solved numerically.

Next step is to evaluate the angular component of the velocity, using the mass conservation equation in cylindrical coordinates,

$$\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_\theta}{\partial \theta} = 0 \tag{5.8}$$

and the definition of vorticity,

$$(\nabla \times \vec{u})_z = \frac{1}{r} \left[\frac{\partial}{\partial r}(rv_\theta) - \frac{\partial v_r}{\partial \theta} \right] \tag{5.9}$$

Combining eqs. (5.8) and (5.9) and some algebraic manipulation produces the following equation for evaluation of angular velocity component,

$$2\eta + r \frac{\partial \eta}{\partial r} - 3 \frac{\partial v_\theta}{\partial r} + r \frac{\partial^2 v_\theta}{\partial r^2} + \frac{v_\theta}{r} + \frac{1}{r} \frac{\partial^2 v_\theta}{\partial \theta^2} \tag{5.10}$$

with boundary conditions

$$\begin{aligned}
 @ r=r_{in} & \rightarrow v_{\theta} - i \frac{\omega}{\omega_n} \frac{Re}{Re_s} [-Y \sin \theta + X \cos \theta] \\
 @ r=r_{out} & \rightarrow \frac{\partial v_{\theta}}{\partial r} = 0 \\
 @ \theta = \pm \theta_0 & \rightarrow \frac{\partial v_{\theta}}{\partial \theta} = 0
 \end{aligned} \tag{5.11}$$

The pressure is evaluated using the momentum equation without terms associated with ratio of true and natural frequencies (as suggested by development of stability criterion below),

$$\begin{aligned}
 \frac{1}{r} \frac{\partial p}{\partial \theta} - \frac{Re_s}{Re^2} \left[\frac{\partial^2 v_{\theta}}{\partial r^2} + \frac{1}{r} \frac{\partial v_{\theta}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_{\theta}}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right] \\
 - \frac{Re_s}{Re} \left[\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta} + \frac{v_r}{r} \right]
 \end{aligned} \tag{5.12}$$

Use the conservation of mass to derive the integral equation for evaluation of pressure on the cylinder's surface as

$$p = \int_{-\theta_0}^{\theta_0} \left[\frac{Re_s}{Re^2} \left(r_0 \frac{\partial^2 v_{\theta}}{\partial r^2} + \frac{\partial v_{\theta}}{\partial r} \right) \right] d\theta \tag{5.13}$$

When solving for the pressure distribution around a cylinder in an array of cylinders, one needs to consider the differences in the flow relative to that of a single cylinder. Irrotational flow over a single cylinder produces a pressure distribution

resembling part of a cosine curve. The reason is evident if one thinks about the pattern of streamlines surrounding a cylinder. Far away from the cylinder, the streamlines are parallel to the centerline of the cylinder. On the cylinder surface the streamlines depict the shape of the cylinder. Simple Bernoulli-type of analysis for streamtubes would require that the pressure (neglecting viscous losses which are, according to D'Alembert paradox, zero) increase with the square of the velocity. Velocity increases linearly with decreasing height of the each particular streamtube. On the top of cylinder the density of streamlines is the largest thus resulting in lowest pressure. The opposite is true for both the forward and aft stagnation points. The variation in between will be proportional to the square root of the distance from the center of the cylinder multiplied by the sine of the running angle. Such a function indeed resembles the cosine curve (Figure 18).

If the flow velocity is increased, the creeping flow assumption is no longer valid. A standing pair of Föepl vortices would yield an effectively elongated body across which the pressure needs to be measured. The values for the stagnation and minimum pressure won't change. The position of the minimum pressure will shift slightly to the rear portion of the cylinder. The pressure distribution on the rear of the cylinder is stretched over the effective body changing the pressure on the cylinder itself, see Figure 19.

Figure 18. Creeping Flow Around Cylinder.

Figure 19. Flow Around Cylinder with Foepl Vortex Pair.

A cylinder in an array will experience yet different pressure distribution, especially if displaced from its neutral position. If one assumes all four neighbors of a flexible cylinder to remain rigid, then the thickest density of streamlines occurs on the side of maximum cylinder displacement and the lowest on the opposite side. Since the streamlines of the wakesheet still resemble the annulus, the anticipated pressure distribution will not be the same as that valid for flow over single cylinder. The streamlines sketched in Figure 20 would lead one to expect an almost linear variation around the cylinder. This is for the cylinder positioned deep in the bundle, at least three rows behind the first cylinder. As opposed to the flow over single cylinder, the maximum velocity (yielding minimum pressure) does not occur at the top of cylinder but at the narrowest passage between the cylinders (as indicated in Figure 20 by V_{max}). Unfortunately the experiments measuring the oscillating pressure on the time scale of single oscillation are at the present moment impossible to perform, therefore one needs to verify the assumption of streamline form using velocity measurements. intuition was correct, experiments were carried out.

The experiment consisted of measuring the mean velocity in cylinder array using Laser Doppler Anemometry. The setup and theoretical assumptions are described in Leicher et al. (1991). The main conclusions were that streamlines resemble the figure 20 not only in rotated square but also in square arrays (phenomena known as jet switch depicting the streamlines bending rather than their expansion after the pair of cylinders). The results for rotated square array were confirmed also by others (Simonin and Barcoudi, 1986, Halim and Turner, 1986).

Figure 20. Flow Around Cylinder in Array.

In order to simplify the calculations an additional assumption was used. We assumed that there is no radial pressure change on the line connecting the centers of two adjacent cylinders. Since the pitch to diameter ratios are rather small and the working fluid is incompressible this shouldn't be too restrictive assumption.

Lastly we evaluated the forces in x and y direction as

$$F_x = \int_{-\theta_0}^{\theta_0} p \sin \theta \, d\theta ; \quad F_y = \int_{-\theta_0}^{\theta_0} p \cos \theta \, d\theta \quad (5.14)$$

With this step the solution for fluid part of the governing equation is completed. Now the solid part needs to be matched in order to obtain the threshold of instability.

Solid part

To describe the solid motion we need a different set of equations - equations of cylinder motion, which can usually be found in the following form

$$\begin{aligned} m\ddot{X} + c\dot{X} + kX - \Sigma F_x(x, y, t) \\ m\ddot{Y} + c\dot{Y} + kY - \Sigma F_y(x, y, t) \end{aligned} \quad (5.15)$$

where c denotes damping parameter and k denotes structure stiffness. They can also be described as

$$c = m \delta \frac{\omega_n}{\pi}; \quad k = m \omega_n^2 \quad (5.16)$$

Using the notation, scaling and assumptions introduced in equations (5.3) and (5.5), one arrives at (stars are dropped for convenience)

The value for amplitude has meaning only if it is real. It can be shown by analysis of a simpler problem (chapter 3) that equation (5.17) is homogeneous and

$$\begin{aligned} & \left(\frac{\omega}{\omega_n} \right)^2 \frac{m}{\rho d^2} X + i \frac{\omega}{\omega_n} \frac{m \delta}{\rho d^2 \pi} X + \frac{m}{\rho d^2} X - \frac{Re_s}{Re} \Sigma F_x(x,y) \\ & \left(\frac{\omega}{\omega_n} \right)^2 \frac{m}{\rho d^2} Y + i \frac{\omega}{\omega_n} \frac{m \delta}{\rho d^2 \pi} Y + \frac{m}{\rho d^2} Y - \frac{Re_s}{Re} \Sigma F_y(x,y) \end{aligned} \quad (5.17)$$

can be rearranged in two possible ways to show the two possible modes of instability,

$$\begin{aligned} & X - \frac{0}{\frac{\Im \left(\frac{Re_s}{Re} \Sigma F_x \right)}{X} - i \frac{\omega}{\omega_n} \frac{m \delta}{\pi \rho d^2}} \\ & X - \frac{0}{\frac{\Re \left(\frac{Re_s}{Re} \Sigma F_x \right)}{X} - \frac{m}{\rho d^2} \left[1 - \left(\frac{\omega}{\omega_n} \right)^2 \right]} \end{aligned} \quad (5.18)$$

where $\Re$ denotes real part of the force and $\Im$ an imaginary part.

Only the streamwise direction amplitudes are shown in equations (5.18) and (5.19) for the sake of clarity. Considering equation (5.18) there are only two solutions possible: either $X=0$ for any nonzero denominators or infinite amplitude for zero denominators (i.e. threshold of instability). The equation is homogeneous only for a two dimensional approximation. For three dimensions, the conditions for the threshold of instability become much more evident. The upper portion of eq. (5.18) describes the dynamic instability whereas the lower portion deals with static instability. In the limiting case of undisturbed flow field (fluid force on cylinder is

negligible) the static instability condition transforms into condition for Strouhal type of instability.

Equation (5.18) can thus be rewritten to yield the explicit formulation of threshold of dynamic and static instabilities as

$$\frac{\delta}{\pi} \frac{m}{\rho d^2} - \frac{\Im\left(\frac{Re_s}{Re} \Sigma F_x\right)}{Xi \frac{\omega}{\omega_n}} ; \frac{m}{\rho d^2} - \frac{\Re\left(\frac{Re_s}{Re} \Sigma F_x\right)}{X \left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]} \quad (5.19)$$

We are concerned with dynamic instability, i.e. one associated with the first power of ratios of frequencies, and this is a reason why we kept only the first powers of ω/ω_n throughout the pressure integration (the ω/ω_n dependency stems from the applications of the boundary conditions).

5.4 RESULTS FOR SEMI-INFINITE ARRAY

Unfortunately the equations presented cannot be solved analytically, since the geometrical domain changes with moving cylinders and due to the complexity of the boundary conditions. We have employed the easiest scheme to approach these kind of problems with high enough accuracy as to warrant trustworthy results, i.e. finite difference discretization with alternate direction implicit (ADI) split scheme, which preserves second order accuracy if used with central differencing. The normal derivatives at the boundaries were evaluated using first order accurate discretization, although second order upwind discretization was utilized without much difference in results.

Results are presented in the form of graphs depicting the onset of instability for various sets of parameters. The basic case we were investigating was a rotated square array with pitch to diameter ratio of 1.5 and Reynolds number of 10^4 . We took a closer look at a differences arising from using different disturbance patterns, changing Reynolds numbers, varying pitch to diameter ratio, changing the setup patterns and comparing crosstream and streamwise instabilities.

Instability as a function of disturbance patterns

Figure 21 shows a theoretical prediction for different disturbance patterns. The cylinders in the array under investigation can position themselves in many

different patterns. This figure presents the cases with cylinders arranged in 1) sine pattern in x and cosine in y direction, 2) sine pattern in both directions and 3) uniformly displaced in x and moving toward each other in y direction. This was an attempt to manually optimize the pattern to obtain the most dangerous setup which causes the dynamic instabilities, but more intensive investigation algorithms for numerical optimization is pending. All of the instabilities investigated were in cross stream direction as the results were consistently showing lower (and thus more dangerous) threshold of instability in cross stream than in streamwise direction.

The figure proves that the case of tubes oscillating against each other is not a probable one, which can be expected. For the rest of the patterns the results don't differ very much. Sine-cosine pattern was used as a basic case (the most dangerous) for other parametric analyses.

Influence of different Reynolds numbers

Figure 22 shows an effect of incoming flow Reynolds number as a parameter to determine onset of instability. The calculations were made using $Re = 10^3$, 10^4 and 10^5 . The results show that if smaller diameter tubes are chosen (lower Re number), stiffness (and subsequently ω_n) of the tubes needs to be substantially increased in order to preserve the same critical mass damping parameter. This is (from heat transfer point of view) not a desirably property of the system. The relationship between increased velocity (and subsequently corresponding mass flux) and diameter

is linear, i.e. if one increases velocity for, say, 50%, one has to increase the diameter of the tubes for the same amount to keep the same reduced velocity and thus a position of the Figure 22.

Influence of geometrical patterns

Figure 23 presents instability thresholds for different geometrical arrangements. Angle of 30° means equilateral triangular array, angle 45° stands for rotated square array and angle 15° means triangular array where three adjacent cylinders form a triangle with one 30° and two 75° angles.

As anticipated the rotated square array was more stable than triangular array. The most stable array was the 15° one. The reason seems to be the effect the cylinder displacement has on the streamtube - 30° attachment angle seems to be the most volatile one and should be avoided in heat exchanger design.

Instability as a Function of Different Disturbance Patterns

Figure 21. Instability as a Function of Different Disturbance Patterns.

Reduced Velocity vs. Reynolds and Streaming Reynolds Numbers

Mass Damping Parameter = 100, 300

Reynolds Str. Reynolds
MDP=300 MDP=300
——— - - - -

Reynolds Str. Reynolds
MDP=100 MDP=100
..... - . - .

Figure 22. Instability as a Function of Different Reynolds Numbers.

Figure 23. Instability as a Function of Streamline Attachment Angle.

Effect of pitch to diameter ratio

Figure 24 presents thresholds of instability for different pitch to diameter ratio. With decreasing P/d ratio causes the structure to become less and less stable. In addition to reported experimental results (Blevins, 1990) which predict the same behavior, this is also intuitively correct: decreased tube spacing leads to more pronounced effects between adjacent tubes.

For pitch to diameter of 2 we were not able to predict any instabilities (i.e. dynamic instabilities) at all. Price et al., 1987, report an absence of fluidelastic instabilities for pitch to diameter ratios above 2.12, so our conclusion seems to be a viable one.

Comparison of various methods

Figure 25 shows comparison of theoretical and experimental predictions. All data were taken for rotated square array with pitch to diameter ratio of 1.5, these data are utilized in experimental (Weaver and Fitzpatrick, 1988, Paidoussis et al, 1988) and theoretical predictions as well. The theoretical models we are comparing our vorticity formulated model with are 1-D model reported by Marn and Catton, 1990 and based on Yetisir and Weaver work, 1984 and 2-D analysis into normal modes, reported by Marn and Catton, 1991.

Figure presents four predictions by current model: the onset of first unstable cylinder, onset of last unstable cylinder and some averaged value for cross stream and streamwise direction. We think that the averaged results are the ones which have to be taken into consideration for designing heat exchangers. The sole tube vibrating in the otherwise unstable array will be quickly damped. The last onset, on the other hand, means that all of the tubes are vibrating, which has to be avoided at all cost. The average result is valid for vibration of around 30% of the tubes and this is a potentially uncontrollable situation. Note that 30% figure stems from the arithmetic averaging of results, whereas the phenomena itself is clearly geometrical.

Taking the average result as a valid one yields a conservative prediction of the onset of instability. This is attributable to two factors: 1) it has been shown that 2-D predictions for instability threshold produce more conservative results than 3-D models (Squire's Theorem) and 2) the results in lower reduced velocity range are not due only to multiple tube dynamic instability (where tubes actively influence each other), but also due to divergence and single mode fluidelastic instability (high damped fluids) as reported earlier (Marn and Catton, 1991).

Reduced Velocity vs. Pitch to Diameter Ratio

Mass Damping Parameter = 100, 300

Figure 24. Instability as a Function of Pitch to Diameter Ratio.

Figure 25. Comparison of Threshold of Instability Predictions.

5.5 RESULTS FOR FINITE ARRAY OF CYLINDERS

The designer of the heat exchanger can vary only a limited set of parameters important for the design. These are:

- type of setup (angle of attachment)
- pitch to diameter ratio
- Reynolds and streaming Reynolds numbers
- mass damping parameter

The results presented in this section were calculated with the designer's needs in mind. The graphs presented will try to be of as much service to the person actually utilizing the model to predict the onset of instability as possible. But before the results are explained let us once more again underline the basic presumption used for presentation: the array will be deemed unstable when approximately 50% of the tubes in most dangerous amplitude ratio positions become unstable.

Figure 26 represents the comparison between some experimental results (Weaver and Fitzpatrick, 1988 and Paidoussis et al, 1988) and corresponding theoretical predictions for 1-D model (Marn and Catton, 1990), 2-D analysis into normal modes (Marn and Catton, 1991) and vorticity formulation presented in this paper. It is worth noting that both, in stream and cross stream predictions almost coincide and are roughly proportional to the mass damping parameter to the power

0.5. The results presented assumed a constant streaming Reynolds number of 400,000. This number was obtained from an experimentally determined point, but could be determined from the actual design by utilizing the design criteria. The figure includes the lower threshold of instability (labeled minimum), i.e. when the first tube becomes unstable, and the upper threshold (denoted maximum), where all the tubes will be in instable mode for a given mass damping parameter.

**Reduced Velocity vs. Mass Damping Parameter
Finite Array, Str. Re.=400,000**

Figure 26. The Comparison Between Theoretical Predictions and Experiments.

Figure 27 shows the dependence of reduced velocity on streaming Reynolds number using two values for mass damping parameter, 300 (roughly air), 100 and 1 (roughly water). The paragraph at the end of this section will give appropriate dependency on both, Reynolds and streaming Reynolds numbers, although the experimental data fittings suggest that use of latter seems to be more appropriate.

Figure 28 compares the results, obtained by keeping constant Reynolds number with those obtained by keeping the streaming Reynolds constant. The results are somewhat puzzling to the authors. It seems as the experimentalists deliberately (or inherently) kept streaming Reynolds constant while increasing the reduced velocity. Blevins (1990) noted in his book that none of the current models were capable of predicting the instability of an untested heat exchanger tube array than the band of data presented in Mulcahy et al. (1986). We found that our model follows the data fitting in the above reference almost exactly.

The question about cluster of data in lower left corner of figures 26 and 28 remains unanswered by this model. The reason is different vibration mechanism, namely for the mass damping parameter below 0.7 vortex shedding prevail over fluid elastic instability. The former mechanism was addressed in chapter 4, combined models would be therefore the best solution to overall problem.

Reduced Velocity vs. Streaming Reynolds Number

Figure 27. The Influence of Streaming Reynolds Number.

Reduced Velocity vs. Mass Damping Parameter

Finite Array

$Re=10,000$ and $Str.Re.=400,000$

Figure 28. Comparison Between Experimental Data and Theoretical Predictions

Obtained by Keeping Reynolds and Streaming Reynolds Constant.

Figure 29 shows the weak importance of pitch to diameter ratio with regard to threshold of fluid elastic instability. This suggests lowering the same as greater density of tubes doubtlessly enhances the heat transfer and consequently increases the overall efficiency of the exchanger.

Figure 29. The Influence of Pitch to Diameter Ratio.

Finally, the results presented suggest the following formula to be applied as a threshold for dynamic instability of an arbitrary staggered array, first as a function of streaming Reynolds number:

$$\frac{U}{\omega_n d} - 0.0174 \left[\frac{m\delta}{\rho d^2 \pi} \right]^{\frac{1}{2}} (Re_s)^{\frac{2}{3}} \ln \left(70 \frac{P}{d} \right) \quad (5.20)$$

and as function of Reynolds number for $500 < Re < 40,000$

$$\frac{U}{\omega_n d} - 0.06 \left[\frac{m\delta}{\rho d^2 \pi} \right]^{1.01} e^{\left(1.05 \frac{P}{d} - 0.01 \pi + 1.2 \times 10^{-4} Re \right)} \quad (5.21)$$

5.6 CONCLUSIONS

A vorticity formulation of the instability problem was used in this chapter. The perturbation equations were developed and solved numerically. The fluid forces were obtained by integrating the pressure along the surface of the cylinders and then incorporated as RHS of the equations of cylinder motion to predict a threshold of dynamic instability. Criteria for instability occurrence were shortly explained and a simple optimization and parametric study was implemented varying disturbance patterns, Reynolds numbers, geometrical setups and pitch to diameter ratios.

The results suggest that the rotated square array should be used in heat exchangers as it is shown to be more stable than triangular array. The structure is shown to require higher mass damping parameter for the same reduced velocity when decreasing pitch to diameter ratio. The Reynolds number's variations influence the results in predictable manner, assuming the flow regime remains stable. The comparison of results with those predicted earlier and with experimental results shows a conservative prediction of instability occurrence for semi-infinite and very good for finite array. There is no "data fitting" implemented in the model, the only parameters imported in the code are pitch to diameter ratio, angle of attachment, Reynolds and streaming Reynolds numbers, reduced velocity and diameter of the tubes in the array.

If Reynolds number is kept constant while varying the reduced velocity the latter is found linearly dependent on mass damping parameter (which is in disagreement with published experimental results where the importance of Reynolds number has been neglected), but the constant streaming Reynolds number yield excellent agreement with available experimental data.

This chapter presents an overview of coupled equation of fluid and solid motion coupled at the interface and applied toward prediction of threshold of instability. The main conclusion is summarized in formulas linking reduced velocity to mass damping parameter, Reynolds or streaming Reynolds number, angle of attachment as an indication of a geometrical array used and pitch to diameter ratio.

6 INSTABILITY OF FINITE CYLINDER ARRAY SUBJECT TO AIR-WATER MIXTURE CROSS FLOW (FOURTH MODEL)

In U shape heat exchanger design there are two characteristic places where cross flow induced vibrations become important factor for its design: 1) the bottom part of the exchanger where the flow consists of mainly single phase and 2) at the top part of steam exchanger with two phase flow cross flow. There has been a few papers published in domain of theoretical modelling of single phase cross flow but almost none in two phase cross flow. This contribution is addressing the air-water cross flow over array of cylinders in rotated triangular array.

Why did we choose air-water rather than steam-water mixture as a subject of our investigation? First, the steam-water mixture warrants coupling of energy and momentum equations. Two phase flow by itself is very complicated phenomena and is not fully understood. Going step by step helps to pinpoint the importance of different mechanisms as to direct future experimental and theoretical research. Secondly, there were not enough experimental data available to verify the theoretical results for steam-water mixtures.

This work also reflect the authors conviction that complicated phenomena cannot be addressed by using very complicated methods, they should be addressed as simply as possible, gradually working the way out to more sophisticated tools, but doing that with extreme caution not to oversimplify the problem.

6.1 ASSUMPTIONS AND SIMPLIFICATIONS

As mentioned above it would be unrealistic to expect solving of this complicated problem without a number of assumptions. Most of the assumptions are derived from either common knowledge about the fluid flows or by recent experimental observations.

The flow is assumed to be horizontal to exclude gravity effects. The gravity would affect the slip between the liquid and gaseous phases and as such possibly change the assumptions used in the development of the model. The improved model should contain this provision in light of the fact that U portion of the steam generator faces upward and is thus a subject to buoyancy effects.

Since the model at hand uses TRAC equations as a starting point it is useful to point out some of the assumptions used in its development. In particular TRAC equations and correlations are not capable of predicting the void fraction distribution. This seems to be the only free parameter and thus suitable for tuning of our model according to available experimental data.

One of the assumptions used throughout the development of these models is that of mean velocity. Similarly as before (Yetisir and Weaver, 1988, previous chapters) the flow field is assumed to be decomposed into mean and perturbed flow. Constant velocity was used as a mean angular component and mean radial

component was assumed to be zero. There are a number of experimental observations available to support this assumption in single phase flow, but the experimental data for actual velocity profile in two phase flow is not readily accessible.

The fluid elastic instability for single phase flow has been shown to be quasi stationary phenomena. Although the phase change affects the drag and lift as well as interfacial shear there is no evidence that it affects the temporal character of perturbation motion. This hints the use of $e^{i\omega t}$ form as before. The underlying hypothesis about perturbation velocity being cause of fluid elastic instability is preserved.

The model assumes that the mean flow satisfies all governing equations.

6.2 GOVERNING EQUATIONS

The governing equations are taken from TRAC manual (1979) as

- liquid mass conservation equation

$$\frac{\partial(1-\alpha)\rho_f}{\partial t} + \nabla \cdot [(1-\alpha)\rho_f \vec{v}_f] = -\Gamma \quad (6.1)$$

- gaseous mass conservation equation

$$\frac{\partial \alpha \rho_g}{\partial t} + \nabla \cdot (\alpha \rho_g \vec{v}_g) = \Gamma \quad (6.2)$$

- conservation of liquid phase momentum

$$\begin{aligned} \frac{\partial \vec{v}_f}{\partial t} + \vec{v}_f \cdot \nabla \vec{v}_f = & -\frac{1}{\rho_f} \nabla p + \frac{c_l}{(1-\alpha)\rho_f} (\vec{v}_g - \vec{v}_f) |\vec{v}_g - \vec{v}_f| \\ & - \frac{\Gamma}{(1-\alpha)\rho_f} (\vec{v}_g - \vec{v}_f) - \frac{c_{wf}}{(1-\alpha)\rho_f} \vec{v}_f |\vec{v}_f| + \vec{g} \end{aligned} \quad (6.3)$$

- conservation of vapor phase momentum

$$\begin{aligned} \frac{\partial \vec{v}_g}{\partial t} + \vec{v}_g \cdot \nabla \vec{v}_g = & -\frac{1}{\rho_g} \nabla p + \frac{c_l}{\alpha \rho_g} (\vec{v}_g - \vec{v}_f) |\vec{v}_g - \vec{v}_f| \\ & - \frac{\Gamma}{\alpha \rho_g} (\vec{v}_g - \vec{v}_f) - \frac{c_{wg}}{\alpha \rho_g} \vec{v}_g |\vec{v}_g| + \vec{g} \end{aligned} \quad (6.4)$$

- conservation of energy equation

$$\begin{aligned} \frac{\partial}{\partial t}[(1-\alpha)\rho_f e_f + \alpha\rho_s e_s] + \nabla \cdot [(1-\alpha)\rho_f e_f \vec{v}_f + \alpha\rho_s e_s \vec{v}_s] \\ - -p\nabla \cdot [(1-\alpha)\vec{v}_f + \alpha\vec{v}_s] + q_{wf} + q_{ws} \end{aligned} \quad (6.5)$$

The air-water mixture is used thus avoiding the energy equation. It is mentioned here for completeness. Also, the previous models have shown the character of the solution and unimportance of imaginary component in transformed governing equation and suggesting to omit the unsteady term in development of the model. Note

$$\tilde{v}_\theta = V_\theta + v_\theta ; \quad \tilde{v}_r = V_r + v_r - v_r \quad (6.6)$$

Introducing the decomposition in mass conservation equation and subtracting the mean flow variables and ignoring the time dependant term results in

$$\frac{\partial v_\theta}{\partial \theta} - -\frac{1}{\alpha} \frac{\partial}{\partial r} (\alpha r v_r) - -r \frac{\partial v_r}{\partial r} - v_r - r v_r \frac{1}{\alpha} \frac{\partial \alpha}{\partial r} \quad (6.7)$$

for gaseous phase.

The flow has been assumed horizontal. After carefully examining available correlations for slip (drift flux) one has to conclude that for bubbly and slug flow gravity plays deciding role. According to Rohatghi et al.(1982, eqs 5.1 and 5.2) there

is no relative velocity between gaseous and liquid phases for flows with absence of gravity. This, assumption about negligible wall shear, the fact that there is no phase change in the flowfield and the fashion in which void fraction distribution in the flow field discussed below was chosen result in following form of gaseous momentum equation for radial and angular perturbation velocities,

$$\begin{aligned} \frac{V_0}{r} \frac{\partial v_r}{\partial \theta} - 2 \frac{V_0}{r} v_\theta - \frac{1}{\rho} \frac{\partial p}{\partial r} \\ \frac{V_0}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{V_0}{r} v_r - \frac{1}{r\rho} \frac{\partial p}{\partial \theta} \end{aligned} \quad (6.8)$$

Combining both equations results in

$$\frac{1}{r} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{\partial^2 v_\theta}{\partial \theta \partial r} - \frac{2}{r} \frac{\partial v_\theta}{\partial \theta} - \frac{\partial v_r}{\partial r} = 0 \quad (6.9)$$

As a first approximation the void fraction dependence along the streamtube is assumed to be negligible. Dhir (1991) reports the change of void distribution within the first few rows of the bundle. Clearly there is a room for improvement as far as this assumption is concerned, but that would have to be addressed in the future.

Using this assumption and substituting terms in Eq. (6.9) utilizing mass conservation equation yields

$$\begin{aligned}
& v_r \left[\frac{2}{r^2} + \frac{3}{r\alpha} \frac{\partial \alpha}{\partial r} + \frac{1}{\alpha} \frac{\partial^2 \alpha}{\partial r^2} - \left(\frac{1}{\alpha} \frac{\partial \alpha}{\partial r} \right)^2 \right] \\
& + \frac{\partial v_r}{\partial r} \left(\frac{3}{r} + \frac{1}{\alpha} \frac{\partial \alpha}{\partial r} \right) + \frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} = 0
\end{aligned} \tag{6.10}$$

This equation has the same form in both dimensionless and dimensional configurations. Since the dimensionless cast is more appropriate for numerical solutions the former will be used.

The scaling parameters used throughout the text are

$$\begin{aligned}
t &= \frac{t^*}{\omega} ; \quad r = r^* d ; \quad x = x^* d ; \quad y = y^* d ; \quad \bar{u} = \bar{u}^* V_0 \\
p &= p^* \rho U d \omega_n ; \quad F = F^* \rho U d^2 \omega_n \\
X &= X^* d ; \quad Y = Y^* d
\end{aligned}$$

where * denotes the dimensionless numbers. For the sake of clarity the stars in this text will be omitted.

Quasi stationary phenomena ansatz is used to accommodate the time dependency:

$$\begin{aligned}
X &= X(x,y) e^{i\omega t} ; \quad Y = Y(x,y) e^{i\omega t} \\
v_\theta &= v_\theta(r,\theta) e^{i\omega t} ; \quad v_r = v_r(r,\theta) e^{i\omega t}
\end{aligned} \tag{6.12}$$

Following dimensionless groups are found to be important

$$Re = \frac{V_0 d}{\nu} ; \quad Re_s = \frac{V_0^2}{\omega_n d} \quad (6.13)$$

The boundary conditions for the radial perturbation velocity require that the velocity of the fluid on the cylinder surface matches that of the adjacent fluid,

$$\begin{aligned} @r=r_{inn} &\rightarrow v_r = i \frac{\omega}{\omega_n} \frac{Re}{Re_s} [X \sin \theta + Y \cos \theta] \\ @r=r_{out} &\rightarrow \frac{\partial v_r}{\partial \theta} = 0 \\ @\theta = \pm \theta_0 &\rightarrow \frac{\partial v_r}{\partial \theta} = 0 \end{aligned} \quad (6.14)$$

Finally, the model must include the void fraction distribution. This is one of the TRAC inherent disadvantages. As mentioned before one expects the void fraction to attain minimum in the middle of the streamtube and to be more or less symmetric. In addition, the fine tuning of the model to experimental results can be achieved by changing the void fraction distribution. The best results (and at the same time satisfying the conditions for void fraction mentioned above) were found by assuming

$$\frac{1}{\alpha} \frac{\partial^2 \alpha}{\partial r^2} = \text{const.} - \left(\frac{V_0}{\omega_n d} \right)^s \rightarrow$$

$$\alpha = \bar{\alpha} \cosh \left[\left(\frac{V_0}{\omega_n d} \right)^{\frac{s}{2}} \left| r - \frac{r_{\text{inn}} + r_{\text{out}}}{2} \right| \right]; \quad (6.15)$$

$$0.0 \leq \alpha \leq 1.0$$

where exponent s was found to be 1.5 and bar corresponds to centerline (i.e. minimum) void fraction. This distribution represents anecdotal evidence of researchers who reported increase in void fraction toward the outer diameters of the tubes.

In order to calculate the forces acting on the cylinders the pressure needs to be computed from the angular momentum equation. Using all the above mentioned assumptions and utilization of scaling produces

$$p = \frac{Re_s}{Re} \int_0^{\theta_0} r_0 \left(\frac{\partial v_r}{\partial r} + \frac{v_r}{\alpha} \frac{\partial \alpha}{\partial r} \right) d\theta \quad (6.16)$$

for pressure calculations. The forces are obtained by integrating the pressure along the cylinders surface.

6.3 THRESHOLD OF INSTABILITY

The conditions for instability do not have a direct bearing on the type of fluid flow, i.e. the difference between single and multiphase flow. There is an influence on natural frequency and mass ratio of the system, granted but the form of equations don't change. For that reason the equations from previous chapters adopted.

The governing dimensionless equations for cylinder motion are

$$\begin{aligned} \left(\frac{\omega}{\omega_n}\right)^2 \frac{m}{\rho d^2} X + i \frac{\omega}{\omega_n} \frac{m \delta}{\rho d^2 \pi} X + \frac{m}{\rho d^2} X - \frac{Re_s}{Re} \Sigma F_x(x,y) \\ \left(\frac{\omega}{\omega_n}\right)^2 \frac{m}{\rho d^2} Y + i \frac{\omega}{\omega_n} \frac{m \delta}{\rho d^2 \pi} Y + \frac{m}{\rho d^2} Y - \frac{Re_s}{Re} \Sigma F_y(x,y) \end{aligned} \quad (6.17)$$

with following definitions and assumptions, such as mass damping parameter

$$c = \frac{\delta}{\pi} \frac{m}{\rho d^2} \quad (6.18)$$

and the conditions for instability is

$$\frac{m}{\rho d^2} \frac{\delta}{\pi} - \frac{\omega_n}{\omega} \geq \frac{\left(\frac{Re_s}{Re} \Sigma F_x\right)}{X} \quad (6.19)$$

in x (global coordinates) direction. Similar expression can be obtained for y direction as well.

The static instability (divergence) is not of interest as it is not believed to produce any significant impact on structural instability. Fluid elastic (dynamic) instability, associated with first power of ω/ω_n ratio is of major concern and is the final result of this investigation.

6.4 GEOMETRY AND NUMERICAL DOMAIN

The geometry the method discussed above will be applied to is a finite array of circular cylinders presented in Figure 30. The array consists of 44 cylinders which can be arranged in number of different setups. Of 44 cylinders the outer 12 are assumed rigid the rest being flexible. The two combinations for relative positions of the cylinders used in this report are normal triangular and parallel triangular arrays with pitch to diameter of 1.32 and 1.47. The reason for these particular values is compatibility with available experimental data.

The computational domain consists of a cylinder surrounded by an arc of fluid flow (denoted as a block). Number of block are fitted together to describe the array. The compatibility conditions between blocks are continuity of velocity and pressure.

Figure 30. The Flowfield.

6.5 RESULTS

The solutions to the governing equations were obtained by using Alternate Direction Implicit finite difference scheme and applying above described temporal ansatz. Figures 31 and 32 show the critical reduced velocity dependence on mass damping parameter. There seems to be the incompatibility between Pettigrew et al. (1988) and this definition of mass damping parameter. In fact, the inconsistency arises through the scaling of equations for cylinder motion. This model cannot treat the logarithmic decrement separately as it lacks the solid dynamic background to do it. The mass damping parameter is treated as a single parameter governing the ω/ω_n to the first power term on the LHS of equations of the cylinder motion. Figures present the theoretical predictions for normal and parallel triangular arrays and the experimental results obtained by Pettigrew et al. (1988). While at the higher reduced velocity region data seem to agree nicely with predictions there is a sizeable discrepancy at the lower reduced velocities. There are several possible explanations:

- the model presented does not account for viscous forces which may be important for low reduced velocity flows. The best remedy would be to repeat the analysis using true two fluid models and rigorous relations between phases and wall. Since that is hardly possible due to extreme complexity of the problem one could attempt to solve the problem by using appropriate available wall shear and interfacial correlations.

- The high reduced velocity flows coincided with intermittent (annular) flows where the profiles probably coincided with the cosh profile used in this model. At the lower reduced velocity region the flow regime was denoted as bubbly and the void distribution may not conform to the profile assumed.
- Experiments dismissed the early detection of fluid elastic instability as noise and recorded its occurrence at higher reduced velocity flows.

The first reason stated seems to make the best sense as far as physics of the system is concerned. It should therefore serve as a guidance for future endeavors.

Figure 33 shows the dependence of critical reduced velocity on angle of attachment. The angle of attachment is used to describe several different setups. 30° is in literature denoted as parallel triangular array, 45° as rotated square and 60° as normal triangular array. Similarly as in single phase case the stability tends to increase with falling angle of attachment but with almost insignificant effect. This result is in disagreement with data of Pettigrew et al. (1988) whose results favor the normal triangular array as the most stable of all. The possible explanation for that is in the next chapter.

Figure 34 depicts the relationship between critical reduced velocity and pitch to diameter ratio. Similarly as for angle of attachment this parameter was found to be rather insignificant for higher reduced flow velocities which is confirmed by

experimental data. Both, data and predictions, show the difference at lower reduced velocity.

Figure 31. Threshold of Instability for Normal Triangular Array.

Figure 32. Threshold of Instability for Parallel Triangular Array.

Figure 33. The Reduced Velocity as a Function of Angle of Attachment.

Figure 34. The Reduced Velocity as a Function of Pitch to Diameter Ratio.

For higher critical mass damping parameter (7.00) the following correlations to link the effects of angle of attachment and pitch to diameter velocity was found to hold

$$\frac{V_{0.2}}{\omega_n d} = \frac{C}{\frac{P}{d} \alpha^{0.3}} \quad (6.20)$$

where C depends on mass damping parameter and is usually expressed in terms of constant K and exponent n .

This chapter does not offer power regression for reduced velocity as a function of mass damping parameter as the border between the regions of validity of certain correlation are not clear. The reader is encouraged to use the model in its entirety.

6.6 CONCLUSION

The two phase cross flow instability of finite cylinder array was presented in this chapter. The model is philosophically similar to the ones developed for single phase flow. The governing equations are the ones used for TRAC computer code and a number of simplifying assumptions has been implemented.

The model is used to calculate the threshold of fluid elastic instability for a finite array consisting of 44 flexible cylinders. The results are compared to experimental data of Pettigrew et al. (1988). The results and data show good agreement for higher critical reduced velocities/mass damping parameters but offer different solution for lower values of the parameters. The possible causes for the discrepancy are identified and some general guidance to overcome them is offered.

The upper stability boundary for two phase flow does not depend on either skin friction or interfacial drag. It is a momentum (i.e. inertia) controlled mechanism. This can be concluded if one compares results in figures 32 and 38. The results shown in figure 32 neglect the skin friction while those in figure 38 include skin friction by using the friction factor recommended in TRAC review by Rohatghi et al. (1982). The upper stability boundary doesn't change while friction seems to enhance the stability at lower values of the mass damping parameter, although not dramatically. There are two important differences between figures 32 and 38: 1) different (slightly) form of void fraction distribution is assumed (that can be ruled out

as a contributor to lower stability boundary change if one compares figures 36 and 34) and 2) absence of friction in figure 32. The most probable answer to this comparison is that the skin friction stabilizes the flow thus rendering the solid-fluid system more stable. This shifts the lower stability boundary to the left, closer to experimental results.

In both cases (figure 32 and 38) several assumptions were made with regard to void fraction distribution, and this was found to be the decisive factor for the lower stability boundary. It seems that two different stability mechanisms are in place when two phase flow is considered: first is inertia controlled mechanism (similar as single phase flow), and prevails at higher reduced velocity while second governs the lower reduced velocity region and is governed by skin friction and void fraction distribution. The multiple stability boundaries are in my opinion due to coincidence of wave number with void fraction distribution (which was assumed). This shows a need for more research concentrating on friction factor and local void fraction distribution in arrays of cylinders.

7 VOID FRACTION DISTRIBUTION AND WALL SHEAR EFFECTS ON INSTABILITY OF WATER-AIR CROSS FLOW MIXTURE

The model based on TRAC governing equations (described in the previous chapter) which assumed no effect of the wall has been found to predict the fluidelastic instability occurrence for lower mass damping parameter region with some degree of inaccuracy. This was attributed to lack of wall shear influence in predominantly bubbly flow regime and to possibly erroneous assumptions for void fraction distribution in this regime. The third assumption which could influence the solution is the assumption of the constant mean angular velocity.

This chapter will try to address all three questions by 1) rederivation of governing equations and 2) performing some kind of parametric analysis applying different void fraction distributions and their influence on the solutions of governing equations.

7.1 ASSUMPTIONS AND SIMPLIFICATIONS

Similarly as in earlier chapters the linearized perturbation analysis will be performed. The flowfield will be decomposed in the mean and perturbed flowfield with underlying assumption that both field satisfy the governing equations independently of each other. In fact, only the mean flow is truly independent as the

mean velocity plays important role in development of perturbed momentum equation.

The mean flow is assumed to have a constant angular component and radial component equal to zero. This does not imply a potential flow as the flow does not satisfy the conditions for irrotational flow. This is an assumption based on observations and originated in work of Lever and Weaver (1984), and was later confirmed by several Laser Doppler Anemometry measurements. Unfortunately this kind of behavior has been observed in the case of single phase flow only, there is still not enough data available in the case of two phase flow to come to similar conclusions on the basis of data alone. The implications of non constant angular component of the velocity will be addressed later.

The void fraction distribution is assumed constant in angular direction. Unfortunately this assumption cannot be relaxed at this time as not sufficient evidence or knowledge about downstream behavior of two phase flow through the bundle is available.

The liquid phase is assumed incompressible and the flow is horizontal to exclude effects of gravity. TRAC correlations suggest no slip between the phases for slug and bubbly flow regimes in absence of gravity thus neglecting the interfacial drag.

All derivation will be performed for liquid phase. Since there is no phase transition, the energy equation is decoupled from the momentum and there is no correspondence between the liquid and gaseous components of the flow other than through the void fraction.

7.2 GOVERNING EQUATIONS

The governing equations are taken from TRAC manual (1979). They were already presented in the previous chapter but are repeated here for completeness (for liquid phase only) as

- liquid mass conservation equation

$$\frac{\partial(1-\alpha)\rho_f}{\partial t} + \nabla \cdot [(1-\alpha)\rho_f \vec{v}_f] = -\Gamma \quad (7.1)$$

- conservation of liquid phase momentum

$$\begin{aligned} \frac{\partial \vec{v}_f}{\partial t} + \vec{v}_f \cdot \nabla \vec{v}_f = & -\frac{1}{\rho_f} \nabla p + \frac{c_i}{(1-\alpha)\rho_f} (\vec{v}_s - \vec{v}_f) |\vec{v}_s - \vec{v}_f| \\ & - \frac{\Gamma}{(1-\alpha)\rho_f} (\vec{v}_s - \vec{v}_f) - \frac{c_w}{(1-\alpha)\rho_f} \vec{v}_f |\vec{v}_f| + \vec{g} \end{aligned} \quad (7.2)$$

Decomposing the flow field

$$\tilde{v}_\theta = V_\theta + v_\theta ; \tilde{v}_r = V_r + v_r - v_r \quad (7.3)$$

and using simplifying assumptions

$$\frac{\partial \alpha}{\partial \theta} = 0 ; \frac{\partial V_\theta}{\partial \theta} = 0 ; V_r = 0 \quad (7.4)$$

one arrives at the following form of liquid phase momentum equation (note that subscripts f have been dropped for convenience)

$$\begin{aligned} & \frac{\partial v_r}{\partial t} + \frac{V_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{1}{\rho} \frac{\partial p}{\partial r} \\ & \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial v_\theta}{\partial \theta} - \frac{1}{r\rho} + \frac{2c_{wf}}{(1-\alpha)\rho} v_\theta V_\theta \end{aligned} \quad (7.5)$$

In the above equation the standard linearized theory assumption about neglecting the components of order perturbation squared has been utilized.

The liquid conservation of mass results in the following

$$v_\theta = \int_{\theta_0}^{\theta} -\frac{1}{(1-\alpha)} \frac{\partial}{\partial r} [(1-\alpha)v_r] d\theta \quad (7.6)$$

These equations are combined in a single differential-integral equation for radial component of perturbed velocity. This equation is solved in the computational domain presented in Figure 35. 77 of these basic units were considered for each calculation (representing 44 cylinders in the finite flowfield).

Figure 35. Computational Domain.

The equation (with omitting interfacial drag for horizontal flow) reads as

$$\begin{aligned}
 & V_0 \left\{ v_r \left[\frac{1}{(1-\alpha)^2} \left(\frac{\partial(1-\alpha)}{\partial r} \right)^2 - \frac{3}{r(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} - \frac{1}{(1-\alpha)} \frac{\partial^2(1-\alpha)}{\partial r^2} - \frac{2}{r^2} \right] \right. \\
 & \left. - \frac{\partial v_r}{\partial r} \left[\frac{3}{r} + \frac{1}{(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} \right] - \frac{\partial^2 v_r}{\partial r^2} - \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} \right\} + v_r \frac{\partial V_0}{\partial r} \left[\frac{1}{r} - \frac{1}{(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} \right] \\
 & + v_r \frac{\partial^2 V_0}{\partial r^2} = \\
 & \frac{2c_w}{\rho(1-\alpha)} \left\{ V_0 \left[\left(\frac{1}{r} - \frac{1}{(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} \right) \int_{\theta_0}^{\theta} \left(v_r + \frac{rv_r}{(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} + r \frac{\partial v_r}{\partial r} \right) d\theta \right. \right. \\
 & \left. \left. + \int_{\theta_0}^{\theta} v_r \left(\frac{1}{(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} + \frac{r}{(1-\alpha)} \frac{\partial^2(1-\alpha)}{\partial r^2} - \frac{r}{(1-\alpha)^2} \left(\frac{\partial(1-\alpha)}{\partial r} \right)^2 \right) \right. \right. \\
 & \left. \left. + \frac{\partial v_r}{\partial r} \left(2 + \frac{r}{(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} \right) + r \frac{\partial^2 v_r}{\partial r^2} \right] d\theta + \frac{\partial V_0}{\partial r} \int_{\theta_0}^{\theta} \left(v_r + \frac{rv_r}{(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} + r \frac{\partial v_r}{\partial r} \right) d\theta \right\}
 \end{aligned} \tag{7.7}$$

Note that equation simplifies considerably by using assumptions in chapter 6.

Boundary Conditions

The boundary conditions used are similar as before. On the cylinder, the perturbation velocity matches that of the cylinder. On the left and right borders the continuity of the velocity field was assumed, on the top of the computational unit the perturbation was assumed to be damped out. The last boundary condition is rather speculative and might be the reason for the peculiar behavior of the solution for low reduced velocities. Similar behavior was reported by Lever and Weaver (1984) in the case of one dimensional analytical approach and was later attributed to linearized

perturbation analysis.

How do these boundary conditions pertain to the reality? According to Dhir (1991) there is a layer of almost pure liquid on the cylinder surrounded by high void fraction fluid. The void fraction then drops towards the midchannel where it achieves the minimum. The liquid on the interface is incompressible but this is not necessarily so for the high void fraction fluid adjacent to the liquid layer. The width of the liquid layer is assumed negligible in this analysis. This suggests that the radial perturbation velocity on the interface may not match with the cylinder velocity as the compressibility effects will become apparent. The true velocity at the interface will thus be less than the velocity of the cylinder. This assumption was used in one of the cases by halving the velocity of the fluid on the interface and the results are compared in the result section.

Second questionable boundary condition is the shape of the flowfield, in particular the detachment and attachment points. After the results have been compared to the experimental ones it became clear that the assumption of the flowfield in the shape of an annulus holds in the case of rotated (also known as parallel) triangular array but is less valid in case of normal triangular array. There is not enough data taken in the case of rotated square array to warrant comparison.

Calculation of pressure, fluid forces and effects of wall shear

The wall shear used in TRAC should be implemented with caution when solving Eq. (7.7). It is very unclear what will be the effect of wall shear multiplier as defined in TRAC on the flowfield between the tubes. Moreover, the question whether the same factor should be used for the calculation of mean and perturbed velocity is not addressed. Decomposition of the flowfield into mean and perturbed flowfield supports this assertion, nevertheless the validity of it (as always with semi empirical based derivations) must be questioned in the future).

The pressure is computed using the angular form of momentum equation as

$$p = \frac{Re_s}{Re} \int_0^\theta \left\{ V_\theta \left[\frac{rv_r}{(1-\alpha)} \frac{\partial(1-\alpha)}{\partial r} + r \frac{\partial v_r}{\partial r} - \frac{2rc_{wf}}{(1-\alpha)\rho_f} v_\theta \right] - rv_r \frac{\partial V_\theta}{\partial r} \right\} d\theta \quad (7.8)$$

The fluid forces are calculated by integrating the pressure around the cylinder. Instability condition is applied (similarly as in the chapter 6) and threshold of dynamic instability is computed.

Void fraction distribution

As mentioned earlier the TRAC governing equations require the user to supply own void fraction distribution. The local void fraction has not yet been measured although Leicher (1991) reports that the measurements are in progress. In

particular, the question whether the void fraction distribution is affected by perturbed velocities remain unclear. The researchers reported higher concentration of bubbles close to the solid-liquid interface and lower toward the center. The assumption used in this part is

$$\frac{1}{\alpha} \frac{\partial^2 \alpha}{\partial r^2} - C = 1 \quad (7.9)$$

The boundary conditions are:

- global minimum is attained at the centerline
- the value of the void fraction at any particular point cannot exceed 1.0 and the average void fraction equals the bulk void fraction
- the local maximum is achieved at the solid liquid interface and at the vortex sheet position

The equation suggests the solution of the form

$$\alpha = C_1 \sinh(r) + C_2 \cosh(r) \quad (7.10)$$

and the boundary condition result in final form used in the calculations (different from the form used in the chapter 6)

$$\alpha = \frac{\bar{\alpha} (r_2 - r_1) \left[\cosh(r) - \sinh(r) \tanh\left(\frac{r_{inn} + r_{out}}{2}\right) \right]}{[\cosh(r_{inn}) - \cosh(r_{out})] \tanh\left(\frac{r_{inn} + r_{out}}{2}\right) + \sinh(r_{out}) - \sinh(r_{inn})} \quad (7.11)$$

Fine tuning of the model later revealed that theoretical predictions conform better to experimental results if the constant C relates to the bulk fluid velocity as

$$C = C_0 U_0^s \quad (7.12)$$

and the exponent $s = 0.10$.

Since the void fraction distribution was found to play a major role in determining the character of the solution the reliability of the model will remain questionable as long as the effects of vibrations on void fraction distribution and the local void fraction distribution are not explored in greater detail.

Effect of constant mean angular velocity

The constant mean angular velocity assumption was used throughout this analysis. Equation (7) enables one to take into consideration the change of mean angular velocity in the radial direction. The assumption of well developed flow is still preserved (the angular partial derivative of mean velocity equals zero). The mean velocity profile in the streamtube was taken to have the following properties,

$$\frac{1}{V_0} \frac{\partial^2 V_0}{\partial r^2} = C - 1 ; \quad @ r = \frac{r_{in} + r_{out}}{2} \rightarrow \frac{\partial V_0}{\partial r} = 0 \quad (7.13)$$

and the integral of the velocity over the streamtube should equal the critical interstitial bulk velocity used for determining the Reynolds and streaming Reynolds numbers.

The velocity profile found to satisfy the equation is

$$V_{\theta} = C_1 \sin(r) + C_2 \cos(r) \quad (7.14)$$

and the boundary conditions determine the final form of the expression as

$$V_{\theta} = \frac{U_{cr} (r_{out} - r_{inn}) \left[\sin(r) \tan\left(\frac{r_{inn} + r_{out}}{2}\right) + \cos(r) \right]}{[\cos(r_{inn}) - \cos(r_{out})] \tan\left(\frac{r_{inn} + r_{out}}{2}\right) + \sin(r_{out}) - \sin(r_{inn})} \quad (7.15)$$

where the U_{cr} stands for critical interstitial velocity.

Alternatively the parabolic velocity profile could be employed. Since the true mean flow profile was found to be almost constant the *sin* seems to be more appropriate function than a parabola.

The effects of this assumption were explored in the parametric study and results showed small difference when compared to the constant profile.

The effects of the angle of attachment

The model consistently overpredicts the threshold of instability for large angles of attachment. This means that the vortex sheet separation point varies with void fraction. While the results for single phase flow agree with experimental for a large scale of various angles of attachment the results for two phase air water mixture agree very well only within limited angle of attachment region (30° to 45° - parallel /rotated/ triangular to rotated square array). Since the convective heat transfer is taking place within the region between the attachment and detachment points it seems that 45° (rotated square) array is the most desirable from stability and heat transfer performance point of views.

7.3 METHOD OF SOLUTION

Similarly as before the Alternate Direction Implicit (ADI) method is used for solution of the resulting equations. There is an increased number of iterations due to the integral part of the equation but this does not decrease the overall stability of the method. The solution to the perturbed radial component is then used in angular momentum equation to calculate the pressure (perturbed) distribution on the cylinder and the resulting forces. The forces are used in the equation for cylinder motion and the instability criteria is applied.

The model is capable of analyzing virtually any size of domain and is limited with computer memory.

7.4 RESULTS

Results are in the form of graphs comparing the critical reduced (gap) velocity with mass damping parameter while changing different parameters. The theoretical predictions are compared with experimental results of Pettigrew (1988) for parallel triangular array (30° angle of attachment). The angle of attachment effect has already been discussed above.

Void fraction distribution

The proposed void fraction distribution is compared to the experimental results in Figure 36. It is evident that the results show better agreement with the data as the result presented in the preceding section for mass damping parameter higher than 5.0. Both, one in previous section as well as current one, assumptions will have to be tested in greater detail as discussed above.

Boundary condition effects

The effects of compressible layer of fluid close to the interface (higher local

void fraction) can be approximated by reducing the perturbed velocity on the cylinder interface when compared to the velocity of the moving cylinder. Several proportionality factors were assumed in the Figure 37. The effect of this were found to be important in the lower reduced velocity region while at the high reduced velocity region the effects of void fraction distribution remained the most decisive factor.

Figure 36. The Effect of Void Fraction Distribution.

Figure 37. Boundary Condition Effects.

Wall shear effects

As proposed the c_{w} coefficient for wall shear was used in solving Equation (7). The values for the coefficient were taken out of the TRAC manual. The results were not much different as if the coefficient was neglected. Increasing the wall shear coefficient to the value much higher than suggested by the authors of TRAC resulted in extremely good agreement with experiment. This is shown in Figure 38. This leads to the conclusion that the interfacial drag which was not accounted for but is an order of magnitude or more higher and has very similar character as the wall shear could be the linear mechanism for suppressing the instability at lower mass damping parameters.

The regions of interchanging stable and unstable areas observed in this and previous sections could be explained with use of linearized perturbation theory. Although the theory predicts instability in reality the instability is not observed. The reason are nonlinear effects which suppress vibrations. The predictions of the onset of instability are therefore conservative.

Figure 38. Effects of Wall Shear.

7.5 CONCLUSION

Two phase flow over a finite cylinder array with pitch to diameter ratio of 1.47 and angle of attachment of 30° was a subject under consideration. The flow was a mixture of air and water.

Model equations were transformed into single partial differential equations solved numerically using Alternate Direction Implicit method (ADI). The equations themselves allowed for non constant mean angular velocity and for arbitrary void fraction distribution. Void fraction distribution which accounts for the minimum in the midchannel was proposed and mean angular velocity distribution (with respect to the radial coordinate component) with a maximum in the midchannel was utilized. The effect of compressibility close to the interface was simulated by arbitrarily reducing the velocity at the interface for some factor when compared to the velocity of the cylinder.

The contribution of mean angular velocity distribution was found to be of minor importance, similarly the effects of wall shear as long one uses the values proposed by authors of TRAC. If the wall shear coefficient was increased for an order of magnitude the effects of increased damping were apparent and results coincided with an experimental observations.

8 CONCLUSION AND FUTURE WORK

Previous chapters showed four distinct models to deal with the same phenomena: flow induced vibrations in staggered cylindrical structures, three subject to single phase and one subject to two (air-water) cross flow.

The first model showed good agreement with experimental data for low reduced velocity range. The vibrations are assumed to be caused by periodic excitation (e.g. vortex shedding). The model assumes harmonic oscillation of the flexible streamtube meandering between the cylinders. The structure responds to such source of excitation with vibrations and when the conditions are fulfilled the instability occurs. Blevins (1990) showed in his book an example of decaying amplitudes caused by vortex shedding with increased mass damping parameter decay. This means that the vortex shedding does not play important role for "real life" applications.

The second model is an analytical model trying to assess fluid elastic instability of cylindrical structure. The motion of neighboring cylinders is taken into consideration through application of boundary conditions. The results give reasonable predictions valid for ranges of reduced velocity between 10 and 100. More importantly, the model shows that the equations are homogeneous with respect to amplitude motion and underscores the importance of relative amplitude motions of adjacent cylinders.

The third model uses vorticity formulation of transport equation and solves the problem numerically, limiting important parameters to physical properties only (no adjustments are made to accommodate experimental data). The results are in excellent agreement with available experimental data for values of reduced velocity above 2.5 and mass damping parameters above 1.0, which covers most of the everyday occurring flows (typical value for water flow being 4.0, and 300 for air flow). The model is summarized in formula linking critical reduced velocity, streaming Reynolds number, mass damping parameter and pitch to diameter ratio.

The fourth model attempts to analyze the water-air flow over the finite cylinder domain. The results are very good for the reduced velocity over 7.0 and show right trend below that value. The reason for the lower discrepancy is attributed to the linearized perturbation analysis. In the real experimental setup the nonlinear effects tend to damp the vibrations even the linear stability analysis predicts the instability. The comparison with available experimental data show that the detachment point moves toward the attachment point for cylinder arrays with angle of attachment greater than 45° . This suggests that the use of normal triangular arrays should be discouraged as the mean flow drag increases without the benefit of increased heat transfer.

All of the models assume constant mean flow velocity (with respect to local coordinate system) throughout the flowfield while allowing the perturbed velocity to account for the instabilities. The fourth model shows that the variation of the mean

angular velocity with respect to radial coordinate does not change the results significantly. The assumption of constant mean flow velocity was shown to be reasonable one after second row of tubes.

What should be done in the future?

In the case of single phase flow I will reformulate the question: "What would I do differently if I were to start from the scratch?"

I would use Boundary Element method rather than Finite Differences. The algorithm would address the problem in three steps. First, the nonlinear mean flow problem would be solved including the wake region behind the cylinders. Second, the BEM would be applied to solve for linearized perturbation equations. Lastly, the results would be optimized in global coordinate space to get the most dangerous (and thus conservative) relative movements of the cylinders.

As far as physics of the single phase flow is concerned the experiments performed should confirm (or deny) the importance of streaming Reynolds number as reported in this work (rather than looking into possible importance of Reynolds number).

There remains much more to be achieved in the field of two phase flows. I would consider an influence of local void fraction on reduced velocity (and streaming

Reynolds number) just outside the boundary layer, say in first third of the gap. By increasing velocity the phases would tend to separate to form three distinct jets (excluding the boundary layer) and the flow streaming Reynolds number should take into consideration increased drag between the phases. This streaming Reynolds number (or its equivalent which will depend on artificial viscosity, void fraction and velocity of the mean flow) could then be used in the above models to predict the onset of instability of cylindrical structures subject to cross flow.

This will not be possible unless the behavior of local void fraction is better understood. There are experiments underway measuring the local void fraction using the optical sonds. Also the thesis of quasistationary character of void fraction distribution while accounting for its influence on perturbed flowfield should be either confirmed or denied.

Lastly, the energy equation should be incorporated in the analysis. The experiments showed that the boiling/condensation does not affect the onset of instability but the phase change does.

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APPENDIX A

DERIVATION OF GOVERNING EQUATIONS FOR NON-STEADY CONTROL VOLUME

Usually the control volume is fixed in the space in time. However, if we expect the unit volume to change its shape periodically it may be beneficially to develop the governing equations based on that kind of property. For example, if we have a pulsating tube, we can reduce the dimensions of the system from 2 to 1 by assuming the tube motion, and defining the borders of control volume as borders of the system.

Continuity

Denote the rate at which the mass is being accumulated in the control volume by RA . In that case,

$$RA = \lim_{\Delta t \rightarrow 0} \frac{-m|_t + m|_{t+\Delta T}}{\Delta t} \quad (A.1)$$

If mass of the control volume can be assumed as continuous function in time (which is certainly true for our purposes), it can be approximated with Taylor series as

$$m|_{t+\Delta t} = m|_t + \frac{\partial m}{\partial t} \Delta t + \frac{1}{2} \frac{\partial^2 m}{\partial t^2} (\Delta t)^2 + \dots \quad (\text{A.2})$$

Now, assume the fluid to be incompressible, to get the rate of mass accumulation to be

$$RA = \lim_{\Delta t \rightarrow 0} \left(\rho \frac{\partial V}{\partial t} \right) = \rho \frac{\partial V}{\partial t} = \rho \Delta x \frac{\partial A}{\partial t} \quad (\text{A.3})$$

The conservation of mass requires

$$\begin{array}{ccccc} \text{Rate} & & & \text{Rate} & \text{Rate} \\ \text{of Mass} & = & & \text{of} & \text{of} \\ \text{Accumulation} & & & \text{Inflow} & \text{Outflow} \end{array}$$

Denote rate of inflow with RI, rate of outflow with RO, to get

$$\begin{aligned} RI &= \rho u A|_x \\ RO &= \rho u A|_{x+\Delta x} = \rho u A|_x + \frac{\partial}{\partial x} (\rho u A|_x) \Delta x + \dots \end{aligned} \quad (\text{A.4})$$

Combining equations (A.4) and (A.5) one gets the non-steady form of continuity

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x} (\rho u A) = 0 \quad (\text{A.5})$$

Momentum

Use the same rationale as when developing continuity, denote rate of change of momentum as RM, inflow of momentum as IM, outflow of momentum as OM, and forces, acting on the fluid as FF. The momentum conservation equation can be expressed as

$$\begin{array}{ccccccc} \text{Rate} & & \text{Inflow} & & \text{Outflow} & & \text{Forces} \\ \text{of Change} & = & \text{of} & + & \text{of} & + & \text{Acting} \\ \text{of Momentum} & & \text{Momentum} & & \text{Momentum} & & \text{on Fluid} \end{array}$$

The specific terms are, recalling the notion of one-dimensionality and incompressibility, as well as inviscid fluid,

$$\begin{aligned} RM &= \frac{\partial}{\partial t}(um) = \rho \frac{\partial}{\partial t}(uV) = \rho \Delta x \frac{\partial}{\partial t}(uA) \\ IM &= \rho u u A|_x \\ OM &= \rho u u A|_{x+\Delta x} - \rho u u A|_x + \frac{\partial}{\partial x}(\rho u u A|_x) \Delta x + \dots \\ FF &= -pA|_x + pA|_{x+\Delta x} = -pA|_x + pA|_x + \frac{\partial}{\partial x}(pA|_x) \Delta x + \dots \end{aligned} \tag{A.6}$$

if the volume is continuous function in space and time, and the final form of momentum

$$\rho \frac{\partial}{\partial t}(uA) + \frac{\partial}{\partial x}(\rho u u A) = -\frac{\partial}{\partial x}(pA) \tag{A.7}$$

or in vector form

Continuity

$$\frac{\partial V}{\partial t} + \nabla \cdot (\bar{u} V) = 0 \quad (\text{A.8})$$

Momentum

$$\frac{\partial}{\partial t}(\bar{u} V) + \nabla (\rho \bar{u} \bar{u} V) = -\frac{1}{\rho} \nabla p V \quad (\text{A.9})$$

APPENDIX B

EXAMPLE

The usefulness of the third model (see chapter 5) is demonstrated by the following example:

Problem:

An engineer is to design a cross flow heat exchanger. The transport media on both, primary and secondary, sides is liquid water. The average temperature on the primary side is 90°C and the pressure is 1.5 bar whereas the average temperature on the secondary side is 40°C and the pressure is 1.0 bar. Thermal analysis shows that the necessary mass flow on each side of the exchanger is 13 kg/s. The exchanger consists of 81 tubes which are to be placed in a bundle with a cross section not larger than a square with a base of 0.125 m ($L_0 \times w$). The secondary flow is into the exchanger through a duct with the same width as the exchanger and a height of 0.15 m (h). Is this exchanger operating in a regime where 50% or more of the tubes will likely experience fluid elastic instabilities? How does one design or redesign the exchanger to operate at the threshold of instability?

Answer:

a) physical properties:

$$T=90^{\circ}\text{C}, p=1.5 \text{ bar} \quad \rho=965 \text{ kg/m}^3 \quad \nu=3.171 \times 10^{-7} \text{ m}^2/\text{s}$$

$$T=40^{\circ}\text{C}, p=1.0 \text{ bar} \quad \rho=992 \text{ kg/m}^3 \quad \nu=6.539 \times 10^{-7} \text{ m}^2/\text{s}$$

use carbon steel tubes, $\rho=7854 \text{ kg/m}^3$

b) kinematic conditions:

outer diameter (calculated from simple geometric formula):

$$d_{out} = \frac{L_{out}}{1 + \frac{P}{d} \sqrt{2} (N-1)} = \frac{125}{1 + 1.5 \cdot \sqrt{2} \cdot 4} = 0.013 m$$

3/8 inch tubes with $d_{out} = 0.013 m$ are chosen. The natural frequency of the tube filled with water needs to be measured but is for this problem estimated to be 50 Hz.

primary flow velocity:

$$V_{in} = \frac{4\dot{m}}{\rho_{in} \pi d_{in}^2 n_{tubes}} = \frac{4 \cdot 13}{965 \cdot \pi \cdot 0.008^2 \cdot 81} = 3.30 \frac{m}{s}$$

secondary flow velocity (between the tubes):

$$V_{out} = \frac{\dot{m}}{\rho_{out} h w} \frac{1}{1 - \frac{d}{P}} = \frac{13}{992 \cdot 0.15 \cdot 0.125} \frac{1}{1 - \frac{1}{1.5}} = 2.1 \frac{m}{s}$$

outer Reynolds and streaming Reynolds numbers:

$$Re = \frac{Vd}{\nu} = \frac{2.1 \cdot 0.013}{6.54 \times 10^{-7}} = 4.2 \times 10^4$$

$$Re_s = \frac{V^2}{\omega_s \nu} = \frac{2.1^2}{50 \cdot 6.54 \times 10^{-7}} = 1.35 \times 10^5$$

mass damping parameter

$$c = \frac{d_{in}^2 \rho_{in} + (d_{out}^2 - d_{in}^2) \rho_{Fe}}{d_{out}^2 \rho_{out}} = \frac{0.01^2 \cdot 965 + (0.013^2 - 0.01^2) \cdot 7854}{0.013^2 \cdot 992} = 3.81$$

where δ is assumed to be 1.0 for the purpose of this problem. In reality δ needs to be measured.

outer reduced velocity

$$\frac{V_{out}}{\omega_n d} = \frac{2.1}{50 \cdot 0.013} = 3.23$$

c) results

the model predicts the following critical values (i.e. with given mass damping parameter and geometrical setup the critical values can be determined):

$$\begin{aligned} \frac{V}{\omega_n d} &= 6.9196 \\ Re &= 8.94 \times 10^4 \\ Re_c &= 6.186 \times 10^5 \end{aligned}$$

d) optimization

The critical values are higher than the proposed parameters thus the design can be approved as far as flow induced vibrations are concerned. The most efficient

way to narrow the gap between critical and proposed velocity is to decrease P/d parameter to, say, 1.2 (this would result in decrease of heat exchanger size). This yields reduced velocity of 4.2. But change in P/d ratio influences the calculation of critical values and one has to repeat the step c):

the following critical values are predicted for mass damping parameter of 3.81 and pitch to diameter ratio of 1.20

$$\begin{aligned}\frac{V}{\omega_n d} &= 5.20 \\ Re &= 6.722 \times 10^4 \\ Re_c &= 3.49 \times 10^5\end{aligned}$$

and the suggested P/d diameter is thus justified. One would have to repeat the calculation of other pertinent heat exchanger data but this is beyond the scope of this particular exercise.