HIGH ENERGY COLLIDERS

R.B. Palmer and J.C. Gallardo

Brookhaven National Laboratory
Upton, New York, 11973, U.S.A.

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R. B. Palmer, J. C. Gallardo

Center for Accelerator Physics
Brookhaven National Laboratory
Upton, NY 11973-5000, USA

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Abstract

We consider the high energy physics advantages, disadvantages and luminosity requirements of hadron (pp, pp), lepton (e+e-, μ+μ-) and photon-photon colliders. Technical problems in obtaining increased energy in each type of machine are presented. The machines relative size are also discussed.

1 Introduction

Particle colliders are only the last evolution of a long history of devices used to study the violent collisions of particles on one another. Earlier versions used accelerated beams impinging on fixed targets. Fig. 1 shows the equivalent beam energy of such machines, plotted versus the year of their introduction. The early data given was taken from the original plot by Livingston[1]. For hadron, i.e. proton or proton-antiproton, machines (Fig. 1a), it shows an increase from around 10^5 eV with a rectifier generator in 1930, to 10^{15} eV at the Tevatron (at Fermilab near Chicago) in 1988. This represents an increase of more than a factor of about 33 per decade (the Livingston line, shown as the dash-line) over 6 decades. By 2005 we expect to have the Large Hadron Collider (at CERN, Switzerland) with an equivalent beam energy of 10^{17} eV, which will almost exactly continue this trend. The
SSC, had we built it on schedule, would, by this extrapolation, have been a decade too early!

The rise in energy of electron machines shown (Fig. 1b) is slightly less dramatic; but, as we shall discuss below, the relative effective physics energy of lepton machines is greater than for hadron machines, and thus the effective energy gains for the two types of machine are comparable.

These astounding gains in energy ($\times 10^{12}$) have been partly bought by greater expenditure: increasing from a few thousand dollars for the rectifier, to a few billion dollars for the LHC ($\times 10^6$). The other factor ($\times 10^9$) has come from new ideas. Linear $e^+e^-$, $\gamma - \gamma$, and $\mu^+\mu^-$ colliders are new ideas that we hope will continue this trend, but it will not be easy.

Figure 1: The Livingston Plots: Equivalent beam energy of colliders versus the year of their introduction; (a) for Hadron Machines and (b) for Lepton Machines.
2 Physics Considerations

2.1 General.

Hadron-hadron colliders (\(pp\) or \(\bar{p}p\)) generate interactions between the many constituents of the hadrons (gluons, quarks and antiquarks); the initial states are not defined and most interactions occur at relatively low energy, generating a very large background of uninteresting events. The rate of the highest energy events is a little higher for antiproton-proton machines, because the antiproton contains valence antiquarks that can annihilate on the quarks in the proton. But this is a small effect for colliders above a few TeV, when the interactions are dominated by interactions between quarks and antiquarks in their seas, and between the gluons. In either case the individual parton-parton interaction energies (the energies used for physics) are a relatively small fraction of the total center of mass energy. This is a disadvantage when compared with lepton machines. An advantage, however, is that all final states are accessible. In addition, as we saw in Figs. 1, hadron machines have been available with higher energies than lepton devices, and, as a result, most initial discoveries in Elementary Particle Physics have been made with hadron machines.

In contrast, lepton-antilepton collider generate interactions between the fundamental point-like constituents in their beams, the reactions produced are relatively simple to understand, the full machine energies are available for physics and there is negligible background of low energy events. If the center of mass energy is set equal to the mass of a suitable state of interest, then there can be a large cross section in the s-channel, in which a single state is generated by the interaction. In this case, the mass and quantum numbers of the state are constrained by the initial beams. If the energy spread of the beams is sufficiently narrow, then precision determination of masses and widths are possible.

A gamma-gamma collider, like the lepton-antilepton machines, would have all the machine energy available for physics, and would have well defined initial states, but these states would be different from those with the lepton machines, and thus be complementary to them.

For most purposes (technical considerations aside) \(e^+e^-\) and \(\mu^+\mu^-\) colliders would be equivalent. But in the particular case of s-channel Higgs boson production, the cross section, being proportional to the
mass squared, is more than 40,000 times greater for muons than electrons. When technical considerations are included, the situation is more complicated. Muon beams are harder to polarize and muon colliders will have much higher backgrounds from decay products of the muons. On the other hand muon collider interactions will require less radiative correction and will have less energy spread from beamstrahlung.

Each type of collider has its own advantages and disadvantages for High Energy Physics: they would be complementary.

2.2 Required Luminosity for Lepton Colliders.

In lepton machines the full center of mass of the leptons is available for the final state of interest and a “physics energy” $E_{\text{phy}}$ can be defined that is equal to the total center of mass energy.

$$E_{\text{phy}} = E_{\text{c of } m} \quad (1)$$

Since fundamental cross sections fall as the square of the center of mass energies involved, so, for a given rate of events, the luminosity of a collider must rise as the square of its energy. A reasonable target luminosity is one that would give 10,000 events per unit of $R$ per year (the cross section for lepton pair production is one $R$, the total cross section is about 20 $R$, and somewhat energy dependent as new channels open up):

$$\mathcal{L}_{\text{req.}} \approx 10^{34} \text{ (cm}^{-2}\text{s}^{-1}) \left( \frac{E_{\text{phy}}}{1 \text{ (TeV)}} \right)^2 \quad (2)$$

2.3 The Effective Physics Energies of Hadron Colliders.

Hadrons, being composite, have their energy divided between their various constituents. A typical collision of constituents will thus have significantly less energy than that of the initial hadrons. Studies done in Snowmass 82 and 96 suggest that, for a range of studies, and given the required luminosity (as defined in Eq. 2), then the hadron machine’s effective “physics” energy is between about 1/3 and 1/10 of its total. We will take a value of 1/7:

$$E_{\text{phy}}(\mathcal{L} = \mathcal{L}_{\text{req.}}) \approx \frac{E_{\text{c of } m}}{7}$$
The same studies have also concluded that a factor of 10 in luminosity is worth about a factor of 2 in effective physics energy, this being approximately equivalent to:

$$E_{\text{phy}}(\mathcal{L}) = E_{\text{phy}}(\mathcal{L} = \mathcal{L}_{\text{req.}}) \left( \frac{\mathcal{L}}{\mathcal{L}_{\text{req.}}} \right)^{0.3}$$

From which, with Eq. 2, one obtains:

$$E_{\text{phy}} \approx \left( \frac{E_{\text{c of m}}}{7(\text{TeV})} \right)^{0.6} \left( \frac{\mathcal{L}}{10^{34}(\text{cm}^{-2}\text{s}^{-1})} \right)^{0.2} (\text{TeV})$$

(3)

Table 1 gives some examples of this approximate "physics" energy.

<table>
<thead>
<tr>
<th>Machine</th>
<th>C of M Energy</th>
<th>Luminosity</th>
<th>Physics Energy</th>
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<tr>
<td>ISR</td>
<td>0.056</td>
<td>$10^{32}$</td>
<td>0.02</td>
</tr>
<tr>
<td>Tevatron</td>
<td>1.8</td>
<td>$7 \times 10^{31}$</td>
<td>0.16</td>
</tr>
<tr>
<td>LHC</td>
<td>14</td>
<td>$10^{34}$</td>
<td>1.5</td>
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<tr>
<td>VLHC</td>
<td>60</td>
<td>$10^{34}$</td>
<td>3.6</td>
</tr>
</tbody>
</table>

It must be emphasized that this effective physics energy is not a well defined quantity. It should depend on the physics being studied. The initial discovery of a new quark, like the top, can be made with a significantly lower "physics" energy than that given here. And the capabilities of different types of machines have intrinsic differences. The above analysis is useful only in making very broad comparisons between machine types.

3 Hadron-Hadron Machines

3.1 Luminosity.

An antiproton-proton collider requires only one ring, compared with the two needed for a proton-proton machine (the antiproton has the opposite charge to the proton and can thus rotate in the same magnet ring in the opposite direction - protons going in opposite directions require two rings with bending fields of the opposite sign), but the luminosity of an antiproton-proton collider is limited by the constraints.
in antiproton production. A luminosity of at least $10^{32} \text{ cm}^{-2}\text{s}^{-1}$ is expected at the antiproton-proton Tevatron; and a luminosity of $10^{33} \text{ cm}^{-2}\text{s}^{-1}$ may be achievable, but the LHC, a proton-proton machine, is planned to have a luminosity of $10^{34} \text{ cm}^{-2}\text{s}^{-1}$: an order of magnitude higher. Since the required luminosity rises with energy, proton-proton machines seem to be favored for future hadron colliders.

The LHC and other future proton-proton machines might\cite{2} be upgradable to $10^{35} \text{ cm}^{-2}\text{s}^{-1}$, but radiation damage to a detector would then be a severe problem. The 60 TeV Really Large Hadron Colliders (RLHC: high and low field versions) discussed at Snowmass are being designed as proton-proton machines with luminosities of $10^{34} \text{ cm}^{-2}\text{s}^{-1}$ and it seems reasonable to assume that this is the highest practical value.

### 3.2 Size and Cost.

The size of hadron-hadron machines is limited by the field of the magnets used in their arcs. A cost minimum is obtained when a balance is achieved between costs that are linear in length, and those that rise with magnetic field. The optimum field will depend on the technologies used both for the the linear components (tunnel, access, distribution, survey, position monitors, mountings, magnet ends, etc) and those of the magnets themselves, including the type of superconductor used.

The first hadron collider, the 60 GeV ISR at CERN, used conventional iron pole magnets at a field less than 2 T. The only current hadron collider, the 2 TeV Tevatron, at FNAL, uses NbTi superconducting magnets at approximately $4^\circ K$ giving a bending field of about 4.5 T. The 14 TeV Large Hadron Collider (LHC), under construction at CERN, plans to use the same material at $1.8^\circ K$ yielding bending fields of about 8.5 T.

Future colliders may use new materials allowing even higher magnetic fields. Model magnets have been made with Nb$_3$Sn, and studies are underway on the use of high $T_c$ superconductor. Bi$_2$Sr$_2$Ca$_1$Cu$_2$O$_8$ (BSCCO) material is currently available in useful lengths as powder-in-Ag tube processed tape. It has a higher critical temperature and field than conventional superconductors, but, even at $4^\circ K$, its current density is less than Nb$_3$Sn at all fields below 15 T. It is thus unsuitable for most accelerator magnets. In contrast YBa$_2$Cu$_3$O$_7$ (YBCO) ma-
terial has a current density above that for Nb$_3$Sn (4°K), at all fields and temperatures below 20 °K. But this material must be deposited on specially treated metallic substrates and is not yet available in lengths greater than 1 m. It is reasonable to assume, however, that it will be available in useful lengths in the not too distant future. it mean for hadron colliders.

![Graph](image-url)

Figure 2: Relative costs of a collider as a function of its bending magnetic field, for different superconductors and operating temperatures.

A parametric study was undertaken to learn what the use of such materials might do for the cost of colliders. 2-in-1 cosine theta superconducting magnet cross sections (in which the two magnet coils are circular in cross section, have a cosine theta current distributions and are both enclosed in a single iron yoke) were calculated using fixed criteria for margin, packing fraction, quench protection, support and field return. Material costs were taken to be linear in the weights of superconductor, copper stabilizer, aluminum collars, iron
yoke and stainless steel support tube. The cryogenic costs were taken to be inversely proportional to the operating temperature, and linear in the outer surface area of the cold mass. Tunnel, access, vacuum, alignment, focusing, and diagnostic costs were taken to be linear with tunnel length. The relative values of the cost dependencies were scaled from LHC estimates.

Results are shown in Fig. 2. Costs were calculated assuming NbTi at (a) 4 °K, and (b) 1.8 °K, Nb3 Sn at (c) 4 °K and YBCO High Tc at 20 °K (d) and (e). NbTi and Nb3 Sn costs per unit weight were taken to be the same; YBCO was taken to be either equal to NbTi (in (d)), or 4 times NbTi (in (e)). It is seen that the optimum field moves from about 6 T for NbTi at 4 °K to about 12 T for YBCO at 20 °K; while the total cost falls by almost a factor of 2.

One may note that the optimized cost per unit length remains approximately constant. This might have been expected: at the cost minimum, the cost of linear and field dependent terms are matched, and the total remains about twice that of the linear terms.

The above study assumes this particular type of magnet and may not be indicative of the optimization for radically different designs. A group at FNAL[3] is considering an iron dominated, alternating gradient, continuous, single turn collider magnet design (Low field RLHC). Its field would be only 2 T and circumference very large (350 km for 60 TeV), but with its simplicity and with tunneling innovations, it is hoped to make its cost lower than the smaller high field designs. There are however greater problems in achieving high luminosity with such a machine than with the higher field designs.

4 Circular e+e−Machines

4.1 Luminosity.

The luminosities of most circular electron-positron colliders have been between $10^{31}$ and $10^{32}$ cm$^{-2}$s$^{-1}$, CESR is fast approaching $10^{33}$ cm$^{-2}$s$^{-1}$ and machines are now being constructed with even higher values. Thus, at least in principle, luminosity does not seem to be a limitation (although it may be noted that the 0.2 TeV electron-positron collider LEP has a luminosity below the requirement of Eq.2).
4.2 Size and Cost.

At energies below 100 MeV, using a reasonable bending field, the size and cost of a circular electron machine is approximately proportional to its energy. But at higher energies, if the bending field $B$ is maintained, the energy lost $\Delta V_{\text{turn}}$ to synchrotron radiation rises rapidly

$$\Delta V_{\text{turn}} \propto \frac{E^4}{R \cdot m^4} \propto \frac{E^3 B}{m^4}$$

and soon becomes excessive ($R$ is the radius of the ring). A cost minimum is then obtained when the cost of the ring is balanced by the cost of the rf needed to replace the synchrotron energy loss. If the radius of the ring is proportional to its circumference, and the rf is proportional to its voltage then the size and cost of an optimized machine rises as the square of its energy.

The highest energy circular $e^+e^-$ collider is the LEP at CERN which has a circumference of 27 km, and will achieve a maximum center of mass energy of about 0.2 TeV. Using the predicted scaling, a 0.5 TeV circular collider would have to have a 170 km circumference, and would be very expensive.

5 $e^+e^-$ Linear Colliders

For energies much above that of LEP (0.2 TeV) it is probably impractical to build a circular electron collider. The only possibility then is to build two electron linacs facing one another. Interactions occur at the center, and the electrons, after they have interacted, must be discarded. The size of such colliders is now dominated by the length of the two linacs and is inversely proportional to the average accelerating gradient in those structures. In current proposals[4] using conventional rf, these lengths are far greater than the circumspheres of hadron machines of the same beam energy, but as noted in section 2.3, the effective physics energy of a lepton machine is higher than that of a hadron machine with the same beam energy, thus offsetting some of this disadvantage.

5.1 Luminosity.

The luminosity $\mathcal{L}$ of a linear collider can be written:

$$\mathcal{L} = \frac{1}{4\pi E} \frac{N}{\sigma_x} \frac{P_{\text{beam}}}{\sigma_y} n_{\text{collisions}}$$

(5)
where $\sigma_x$ and $\sigma_y$ are average beam spot sizes including any pinch effects, and we take $\sigma_x$ to be much greater than $\sigma_y$. $E$ is the beam energy, $P_{\text{beam}}$ is the total beam power, and, in this case, $n_{\text{collisions}} = 1$. This can be expressed \cite{8} as,

$$L \approx \frac{1}{4\pi E} \frac{n_\gamma}{2r_o\alpha} \frac{P_{\text{beam}}}{\sigma_y} \frac{U(T)}{\sigma_x}$$  \hspace{1cm} (6)$$

where the quantum correction $U(T)$ is given by

$$U(T) \approx \sqrt{\frac{1}{1 + T^{2/3}}}$$  \hspace{1cm} (7)$$

with

$$T \approx \frac{2F_2r_o^2}{\alpha} \frac{N\gamma}{\sigma_x \sigma_y}$$  \hspace{1cm} (8)$$

$F_2 \approx 0.43$, $r_o$ is the classical electromagnetic radius, $\alpha$ is the fine-structure constant, and $\sigma_z$ is the $\text{rms}$ bunch length. The quantum correction $T$ is close to unity for all proposed machines with energy less than 2 TeV, and this term is often omitted \cite{5}. Even in a 5 TeV design \cite{6}, an $T$ of 21 gives a suppression factor of only 3. $n_{\gamma}$ is the number of photons emitted by one electron as it passes through the other bunch. If $n_{\gamma}$ is significantly greater than one, then problems are encountered with backgrounds of electron pairs and mini-jets, or with unacceptable beamstrahlung energy loss. Thus $n_{\gamma}$ can be taken as a rough criterion of these effects and constrained to a fixed value. We then find:

$$L \propto \frac{1}{E} \frac{P_{\text{beam}}}{\sigma_y} \frac{U(T)}{U(T)}$$

which may be compared to the required luminosity that increases as the square of energy, giving the requirement:

$$\frac{P_{\text{beam}}}{\sigma_y U(T)} \propto E^3.$$  \hspace{1cm} (9)$$

It is this requirement that makes it hard to design very high energy linear colliders. High beam power demands high efficiencies and heavy wall power consumption. A small $\sigma_y$ requires tight tolerances, low beam emittances and strong final focus. And a small value of $U(T)$ is hard to obtain because of its weak dependence on $T$ ($\propto T^{-1/3}$).
5.2 Conventional RF.

The gradients for structures have limits that are frequency dependent, but the real limit on accelerating gradients in these designs come from a trade off between the cost of rf power against the cost of length. The use of high frequencies reduces the stored energy in the cavities, reducing the rf costs and allowing higher accelerating gradients: the optimized gradients being roughly proportional to the frequency up to a limit of approximately 250 MeV/m at a frequency of the order of 100 GHz. One might thus conclude then that higher frequencies should always be preferred. There are however counterbalancing considerations from the requirements of luminosity.

![Graphs showing dependencies of various parameters on rf frequency](image)

Figure 3: Dependence of some sensitive parameters of 0.5 TeV proposed linear colliders as a function of their rf frequencies.
Fig. 3, using parameters from current 0.5 TeV linear collider proposals [4], plots some relevant parameters against the rf frequency. One sees that as the frequencies rise,

- the required alignment tolerances get tighter;
- the resolution of beam position monitors must also be better; and
- despite these better alignments, the calculated emittance growth during acceleration is worse; and
- the wall-power to beam-power efficiencies are also less.

Thus while length and cost considerations may favor high frequencies, yet luminosity considerations would prefer lower frequencies.

5.3 Superconducting RF.

If, however, the rf costs can be reduced, for instance when superconducting cavities are used, then there will be no trade off between rf power cost and length and higher gradients would lower the length and thus the cost. Unfortunately the gradients achievable in currently operating niobium superconducting cavities is lower than that planned in the higher frequency conventional rf colliders. Theoretically the limit is about 40 MV/m, but practically 25 MV/m is as high as seems possible. Nb$_3$Sn and high Tc materials may allow higher field gradients in the future.

The removal of the requirements for very high peak rf power allows the choice of longer wavelengths (the TESLA collaboration is proposing 23 cm at 1.3 GHz) thus greatly relieving the emittance requirements and tolerances, for a given luminosity.

At the current 25 MeV per meter gradients, the length and cost of a superconducting machine is probably higher than for the conventional rf designs. With greater luminosity more certain, its proponents can argue that it is worth the greater price. If, using new superconductors, higher gradients become possible, thus reducing lengths and costs, then the advantages of a superconducting solution might become overwhelming.

5.4 At Higher Energies.

At higher energies (as expected from Eq. 9), obtaining the required luminosity gets harder. Fig.4 shows the dependency of some example
machine parameters with energy. SLC is taken as the example at 0.1 TeV, NLC parameters at 0.5 and 1 TeV, and 5 and 10 TeV examples are taken from a review paper by one of the authors[6]. One sees that:

- the assumed beam power rises approximately as $E$;
- the vertical spot sizes fall approximately as $E^{-2}$;
- the vertical normalized emittances fall even faster: $E^{-2.5}$; and
- the momentum spread due to beamstrahlung has been allowed to rise.

These trends are independent of the acceleration method, frequency, etc, and indicate that as the energy and required luminosity rise, so the required beam powers, efficiencies, emittances and tolerances will all get harder to achieve. The use of higher frequencies or exotic technologies that would allow the gradient to rise, will, in general, make the achievement of the required luminosity even more difficult. It may well prove impractical to construct linear electron-positron colliders, with adequate luminosity, at energies above a few TeV.

6 $\gamma - \gamma$ Colliders

A gamma-gamma collider[9] would use opposing electron linacs, as in a linear electron collider, but just prior to the collision point, laser beams would be Compton backscattered off the electrons to generate photon beams that would collide at the IP instead of the electrons. If suitable geometries are used, the mean photon-photon energy could be 80% or more of that of the electrons, with a luminosity about 1/10th.

If the electron beams, after they have backscattered the photons, are deflected, then backgrounds from beamstrahlung can be eliminated. The constraint on $N/\sigma_x$ in Eq.5 is thus removed and one might hope that higher luminosities would now be possible by raising $N$ and lowering $\sigma_x$. Unfortunately, to do this, one needs sources of bunches with larger numbers of electrons and smaller emittances, and one must find ways to accelerate and focus such beams without excessive emittance growth. Conventional damping rings will have difficulty doing this[10]. Exotic electron sources would be needed, and methods using lasers to generate[11] or cool[12] the electrons and positrons are under consideration.
Figure 4: Dependence of some sensitive parameters on linear collider energy, with comparison of same parameters for $\mu^+\mu^-$ colliders.

Clearly, although gamma-gamma collisions can and should be made available at any future electron-positron linear collider, to add physics capability, whether they can give higher luminosity for a given beam power is less clear.

7 $\mu^+\mu^-$ Colliders

7.1 Advantages and Disadvantages
The possibility of muon colliders was introduced by Skrinsky et al.[13] and Neuffer[14] and has been aggressively developed over the past two
years in a series of meetings and workshops[15, 16, 17, 18, 19].

The main advantages of muons, as opposed to electrons, for a lepton collider are:

- The synchrotron radiation, that forces high energy electron colliders to be linear, is (see Eq. 4) inversely proportional to the fourth power of mass: It is negligible in muon colliders. Thus a muon collider can be circular. In practice this means in can be smaller. The linacs for the SLAC proposal for a 0.5 TeV Next Linear Collider would be 20 km long. The ring for a muon collider of the same energy would be only about 1.3 km circumference.

- The luminosity of a muon collider is given by the same formula (Eq. 5) as given above for an electron positron collider, but there are two significant changes: 1) The classical radius $r_\circ$ is now that for the muon and is 200 times smaller; and 2) the number of collisions a bunch can make $n_{\text{collisions}}$ is no longer 1, but is now limited only by the muon lifetime and becomes related to the average bending field in the muon collider ring, with

$$n_{\text{collisions}} \approx 150 B_{\text{ave}}$$

With an average field of 6 Tesla, $n_{\text{collisions}} \approx 900$. These two effects give muons an in principle luminosity advantage of more than $10^6$. As a result, for the same luminosity, the required beam power, spot sizes, emittances and energy spread are far less in $\mu^+\mu^-$ colliders than in $e^+e^-$ machines of the same energy. The comparison is made in Fig. 4 above.

- The suppression of synchrotron radiation induced by the opposite bunch (beamstrahlung) allows the use of beams with lower momentum spread, and QED radiation is reduced.

- $s$-channel Higgs production is enhanced by a factor of $(m_\mu/m_e)^2 \approx 40,000$. This combined with the lower momentum spreads would allow more precise determination of Higgs masses, widths and branching ratios.

But there are problems with the use of muons:

- Muons can be obtained from the decay of pions, made by higher energy protons impinging on a target. But in order to obtain enough muons, a high intensity proton source is required with very efficient capture of the pions, and muons from their decay.
The selection of fully polarized muons is inconsistent with the requirements for efficient collection. Polarizations only up to 50% are practical, and some loss of luminosity is inevitable (e^+e^- machines can polarize the e^-s up to \( \approx 85\% \)).

Because the muons are made with very large emittance, they must be cooled, and this must be done very rapidly because of their short lifetime. Conventional synchrotron, electron, or stochastic cooling is too slow. Ionization cooling\(^{20}\) is the only clear possibility, but does not cool to very low emittances.

Because of their short lifetime, conventional synchrotron acceleration would be too slow. Recirculating accelerators or pulsed synchrotrons must be used.

Because they decay while stored in the collider, muons radiate the ring and detector with decay electrons. Shielding is essential and backgrounds will be high.

### 7.2 Design Studies

A collaboration, lead by BNL, FNAL and LBNL, with contributions from 18 institutions has been studying a 4 TeV, high luminosity scenario and presented a Feasibility Study\(^{19}\) to the 1996 Snowmass Workshop. The basic parameters of this machine are shown schematically in Fig. 5 and given in Table 2. Fig. 6 shows a possible layout of such a machine.

Table 2 also gives the parameters of a 0.5 TeV demonstration machine based on the AGS as an injector. It is assumed that a demonstration version based on upgrades of the FERMILAB, or CERN machines would also be possible.

The main components of the 4 TeV collider would be:

- A proton source with KAON\(^{21}\) like parameters (30 GeV, \(10^{14}\) protons per pulse, at 15 Hz).

- A liquid metal target surrounded by a 20 T hybrid solenoid to make and capture pions.

- A 5 T solenoidal channel to allow the pions to decay into muons, with rf cavities to, at the same time, decelerate the fast ones that come first, while accelerating the slower ones that come later. Muons from pions in the 100-500 MeV range emerge in a 6 m long bunch at 150 ± 30 MeV.
Figure 5: Overview of a 4 TeV Muon Collider
0.25 - 1.3 TeV & 1.3 - 2 TeV Accelerators

25-250 GeV Accelerator

2.5-25 GeV

30 GeV
p Driver

cooling

1 km

Figure 6: Layout of the collider and accelerator rings.

- A solenoidal snake and collimator to select the momentum, and thus the polarization, of the muons.
- A sequence of 20 ionization cooling stages, each consisting of: a) energy loss material in a strong focusing environment for transverse cooling; b) linac reacceleration and c) lithium wedges in dispersive environments for cooling in momentum space.
- A linac and/or recirculating linac pre-accelerator, followed by a sequence of pulsed field synchrotron accelerators using superconducting linacs for rf.
- An isochronous collider ring with locally corrected low beta ($\beta=3$ mm) insertion.

7.3 Status and Required R and D

Muon Colliders are promising, but they are in a far less developed state than hadron or $e^+e^-$ machines. No muon collider has ever been built. Much theoretical and experimental work will be needed be-
Table 2: Parameters of Collider Rings

<table>
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<tr>
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<th>Value 2</th>
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<td>rms I.P. beam size</td>
<td>$\mu$m</td>
<td>2.8</td>
</tr>
<tr>
<td>Chromaticity</td>
<td></td>
<td>2000-4000</td>
</tr>
<tr>
<td>$\beta_{\text{max}}$</td>
<td>km</td>
<td>200-400</td>
</tr>
<tr>
<td>Luminosity</td>
<td>cm^{-2}s^{-1}</td>
<td>10^{35}</td>
</tr>
</tbody>
</table>

Therefore one will even know if they are possible. In particular, theoretical work is needed on the cooling sequence, on the collider ring, and on estimations of background in the detectors. The highest priority experimental work is:

- Demonstration of ionization cooling;
- Demonstration of liquid targets, solenoid pion capture and the use of rf near such a source;
- Construction of model pulsed magnets for the accelerator and large aperture superconducting quadrupoles for the intersection region of the collider.
8 Comparison of Machines

In Fig. 7, the effective physics energies (as defined by Eq. 3) of representative machines are plotted against their total tunnel lengths. We note:

- Hadrons Colliders: It is seen that the energies of machines rise with their size, and that this rise is faster than linear ($E_{\text{eff}} \propto L^{1.3}$). This extra rise is a reflection of the increases in bending magnetic field used, as new technologies and materials have become available.

![Figure 7: Effective physics energies of colliders as a function of their total length.](image)
- Circular Electron-Positron Colliders: The energies of these machines rise approximately as the square root of their size, as expected from the cost optimization discussed in section 4 above.

- Linear Electron-Positron Colliders: The SLAC Linear Collider is the only existing machine of this type. One example of a proposed machine (the NLC) is plotted. The line drawn has the same slope as for the hadron machines and implies a similar rise in accelerating gradient, as technologies advance.

- Muon-Muon Colliders: Only the 4 TeV collider, discussed above, and the 0.5 TeV demonstration machine have been plotted. The line drawn has the same slope as for the hadron machines.

It is noted that the muon collider offers the greatest energy per unit length. This is also apparent in Fig. 8, in which the footprints of a number of proposed machines are given on the same scale.

Figure 8: Approximate sizes of some possible future colliders.
9 Conclusions

Our conclusions, with the caveat that they are indeed only our opinions, are:

- The LHC is a well optimized and appropriate next step towards high effective physics energy.

- A Very Large Hadron Collider with energy greater than the SSC (e.g. 60 TeV c-of-m) and cost somewhat less than the SSC, may well be possible with the use of high Tc superconductors that may become available.

- A "Next Linear Collider" is the only clean way to complement the LHC with a lepton machine, and the only way to do so soon. But it appears that even a 0.5 TeV collider may be more expensive than the LHC, has significantly less effective physics energy, and will be technically challenging. Obtaining the design luminosity may not be easy.

- Extrapolating conventional rf $e^+e^-$linear colliders to energies above 1 or 2 TeV will be very difficult. Raising the rf frequency can reduce length and probably cost for a given energy, but obtaining luminosity increasing as the square of energy, as required, may not be feasible.

- Laser driven accelerators are becoming more realistic and can be expected to have a significantly lower cost per TeV. But the ratio of luminosity to wall power is likely to be significantly worse than for conventional rf driven machines. Colliders using such technologies are thus unlikely to achieve the very high luminosities needed for physics research at higher energies.

- A higher gradient superconducting Linac collider using Nb$_3$Sn or high Tc materials, if it became technically possible, could be the most economical way to attain the required luminosities in a higher energy $e^+e^-$collider.

- Gamma-gamma collisions can and should be obtained at any future electron-positron linear collider. They would add physics capability to such a machine, but, despite their freedom from the beamstrahlung constraint, may not achieve higher luminosity.

- A Muon Collider, being circular, could be far smaller than a conventional rf $e^+e^-$linear collider of the same energy. Very preliminary estimates suggest that it would also be significantly
less expensive. The ratio of luminosity to wall power for such machines, above 2 TeV, may be better than that for electron positron machines, and extrapolation to a center of mass energy of 4 TeV does not seem unreasonable. If research and development can show that it is practical, then a 0.5 TeV muon collider could be a useful complement to $e^+e^-$-colliders, and, at higher energies (e.g. 4 TeV), could be a viable alternative.

10 Acknowledgments

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11 Discussion

R.Taylor: I was afraid if it was going to run over but it worked well because we spent hardly any time on what is wrong with muon colliders compared to the length of time we spent on what was wrong with linear colliders. (laugh)

K.Henry: So what’s wrong with the muon colliders? (laugh)

Palmer: I can tell you which parts of the muon collider keep me awake at night. That changes, of course, from week to week. Enormous progress has been made even in last few months. The collider lattice had been a problem, but doesn’t worry me any more. We also had a serious difficulty in the transverse cooling lattices. When we first tried tracking particles through, some muons never came out. They were hitting resonances. Now we understand that problem and have tracked through transverse cooling sections that work.

But we have not done energy cooling yet. We know theoretically how to do it, but we haven’t got a realistic lattice and tracked muons.
through it. Having been burnt once, I will have sleepless nights until we get past that hurdle.

The collider ring may have instability problems that are not fully understood. We think that it will have to have BNS damping applied by using RF quadrupoles, but we haven’t worked that out.

We haven’t done many things that need to be done, but I do not yet see any insuperable problems. I do not sleep that badly. (laugh)

Erich Vogt: Have you considered using surface muons which have been considered at KEK as an alternative muon source?

Palmer: Yes, but we need bunches with very large numbers of muons in order to get luminosity. It seems to be difficult to get them from surface muons. And there is a more basic problem, we need both charges, I do not think this is possible with surface muons.

Edward Witten: What fraction of muons decay before entering the ring?

Palmer: With the parameters we’ve considered, about three-fourths are lost. Half decay during the cooling sequence, and half of the remainder decay during acceleration.

Alfred Mann: It would be interesting to hear about the shielding problems that arise in the muon-muon collider.

Palmer: I think I know what you’re trying to get at. (laugh) The radiation from decay electrons in the ring itself can be shielded relatively easily. Dumping 2 Tev muons is more difficult because it takes 2 km of concrete to stop them, but that too is ok. The problem you may be hinting at, which I didn’t mention because we are not yet sure about the calculation, is radiation from neutrinos. Muons decaying in a straight section of the collider ring produce a neutrino beam with opening angle of $1/\gamma$ that, for a 4 TeV collider, is only a meter or two wide 35 km away. The neutrino cross section is small, but rising linearly with energy, and there are 20 mega-watts of power in that beam. The resulting radiation level appears to be close to the legal limit. You can’t shield it and it always breaks ground somewhere because unfortunately the earth is round. (laugh) It rises as the fourth power of the energy and is only inverse with the machine depth. Thus, even if a 4 TeV $\mu^+\mu^-$ collider is just ok, a 10 Tev collider is probably impractical.

Leon Lederman: Going back to the beginning of your talk on the Livingston Plot, you said that there is a $10^6$ rise of cost rise for a $10^{12}$ rise in energy so we are $10^6$ cleverer. (laugh) Did you include inflation in those numbers?
Palmer: Yes, I did, but I may not have done it right. Down the bottom I had 100 Kev and I said to myself what I could buy that now for a few thousand dollars. This is not fair because in 1930 you could not buy one and it must have taken quite a bit of labor to build one. I did not try and estimate that cost.

References


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