SAND 97-0439C

CONF-970726--7

SAND--97-0439C

BRITTLE FAILURE KINETICS MODEL FOR CONCRETE

S. A. Silling

Computational Physics and Mechanics Department

Sandia National Laboratories

P. O. Box 5800, MS-0820

Albuquerque, New Mexico 87185-0820

USA

(505) 844-3973

sasilli@sandia.gov

RECEIVED

MAR 17 1997

OSTI

MASTER

ABSTRACT

A new constitutive model is proposed for the modeling of penetration and large stress waves in concrete. Rate effects are incorporated explicitly into the damage evolution law, hence the term "brittle failure kinetics." The damage variable parameterizes a family of Mohr-Coulomb strength curves. The model, which has been implemented in the CTH code, has been shown to reproduce some distinctive phenomena that occur in penetration of concrete targets. Among these are the sharp spike in deceleration of a rigid penetrator immediately after impact. Another is the size scale effect, which leads to a nonlinear scaling of penetration depth with penetrator size. This paper discusses the theory of the model and some results of an extensive validation effort.

1. INTRODUCTION: NEED FOR A NEW MODEL

Although concrete has been the subject of many constitutive modeling efforts, some unique features of stress waves and penetration in this material have generally been neglected. In particular, in penetration experiments with accelerometers on board the penetrator, a sharp spike in acceleration is found to occur just after the nose of the penetrator breaks the surface of a concrete target. Previous models have apparently failed to predict this. Also, in experiments in which a large spherical blast wave is created from a high explosive charge buried in a concrete slab, the observed stress waves indicate a surprising retention of tensile strength in the material after the initial shock wave has passed. This too has apparently not been reproduced by previous models.

By carrying out computer modeling using a static material model, it is easily demonstrated that these observed effects are inherently dynamic in nature: they cannot be modeled accurately using a constitutive model for concrete that assumes that the dynamic response is identical with the static response. Traditionally, the term "rate dependence" has been used to denote any response which is not identical in static and dynamic experiments. The implication of this phrase is that such differences could be accounted for by modifying a static constitutive

relation for stress by some function that includes strain rate explicitly. This approach has worked well for metals, and some researchers have tried the same approach with concrete. However, the resulting models fail to predict the experimentally observed phenomena discussed in the previous paragraph. This suggests a need for a new approach to modeling the dynamic response of concrete.

It is proposed here that these observed effects, and the reason previous models have not reproduced them adequately, are due to time-dependence in the material response as it undergoes damage. This is different from "rate dependence" because in the present approach, there is no explicit dependence of any quantity on strain rate. Instead, a delay time, called the *time to failure*, is built into the equations that describe evolution of damage. This is the meaning of the term "brittle failure kinetics," or "BFK:" the time dependence of damage accumulation is represented explicitly. The time to failure depends on the local conditions. It also depends on the size and shape of the body. These features are described in more detail below.

In this paper, we first describe the equation of state and strength law in the model. Next, we describe the damage evolution law. Finally, we show some examples and comparison with experimental data.

2. EQUATION OF STATE

Because of limitations on the scope and resources available for developing this model, it was necessary for it to fit easily into the traditional hydrocode framework for material modeling. This means first that the model is isotropic. It also means that the model computes the stress tensor in two distinct parts. The first part is the "equation of state," which finds pressure as a function of mass density, internal energy density, and any internal state variables. The second part is the "strength model," which computes the flow stress as a function of the incremental strain tensor, loading history, and any internal state variables. The flow stress is used in a radial return formulation of J_2 flow theory. Although this formulation suffers from some well-known limitations, it has remained the workhorse in the field of stress wave model-

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, make any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Figure 1. Equation of state reference pressure-volume curves.

ing, and it gives good results in many problems of practical interest.

The equation of state in the present model uses a pressure-volume relation described by a cubic spline representation of the Hugoniot. The cubic spline approach makes it straightforward to represent the complexities in the Hugoniot as well as to modify the model to include new experimental data. The complexities include a linear elastic regime at low stresses, a pore crushing regime at moderate stresses, and a locked (fully crushed) regime at high stresses (Figure 1).

Permanent crushing and partial recovery of the pore space following compression are treated by having separate unloading curves from the point of maximum stress on the Hugoniot. The point at which the unloading curve, or "release" curve, intersects the horizontal axis determines how much of the pore space is permanently crushed out. As material is expanded along the release curve, it eventually fractures, as shown in the figure. The fracture stress depends linearly on the scalar damage variable, whose evolution is discussed in Section 4 below. The Hugoniot, together with the unloading curve, is called the *reference curve*. It may be represented in the form $p_{\text{ref}}(v, \bar{v})$, where v is the specific volume and $\bar{v}$ is the minimum specific volume that has been attained during the loading history at a given material particle.

Dependence of the pressure on internal energy density is modeled using the standard Gruneisen term, which takes the thermodynamic state off of the reference curve according to the internal energy density. Thus the full equation of state for pressure is of the following form:

$$p(v, \bar{v}, e) = p_{\text{ref}}(v, \bar{v}) + \frac{\Gamma_0}{v_0} [e - e_{\text{ref}}(v, \bar{v})] \quad (1)$$

where Γ_0 is a constant called the *Gruneisen coefficient*, v_0 is the specific volume of unstressed virgin material, and $e_{\text{ref}}(v, \bar{v})$ is the internal energy density associated with the reference curves. This function is found from the energy jump condition along the Hugoniot together with the First Law expression for isentropic processes along the unloading curve:

Figure 2. Strength curves.

$$e_{\text{ref}}(v, \bar{v}) - e_0 = \frac{1}{2}(p_{\text{ref}}(\bar{v}, \bar{v}) + p_0)(v_0 - \bar{v}) - \int_{\bar{v}}^v p_{\text{ref}}(v', \bar{v}) dv', \quad v \geq \bar{v} \quad (2)$$

where p_0 and e_0 are the pressure and internal energy density of unstressed virgin material. In this equation, note that $p_{\text{ref}}(\bar{v}, \bar{v})$ is the pressure on the Hugoniot at the point of maximum compression, from which unloading is assumed to occur.

At high pressures, the Hugoniot, along with the release curves, is measured using plate impact experiments designed to shock load a sample and then allow it to release. At lower pressures (up to about 500 MPa), the hydrostatic response may be measured quasi-statically in a triaxial test apparatus.

3. STRENGTH MODEL

As discussed in the previous section, the purpose of the strength model is to produce a value of flow stress based on local conditions and loading history. This is accomplished using a family of Mohr-Coulomb surfaces, called the *strength curves*, parameterized by the scalar damage parameter, ϕ , whose evolution is discussed in the next section. As shown in Figure 2, the strength curves form a family of straight lines on the T_1 - Y plane, where Y is the flow stress and T_1 is the maximum (most tensile) principal stress.

The traditional approach to modeling the strength of geological materials is to use mean stress, rather than maximum principal stress, on the horizontal axis of a graph such as the one shown in Figure 2. However, for concrete, the present approach is used because a large compressive load in one direction cannot compensate, in its effect on strength, for tensile loads applied in a perpendicular direction. Therefore the most tensile load, i.e., the maximum principal stress, is a better index to the overall strength of the material than mean stress.

The strength curves, regardless of the damage of parameter, reach a maximum value Y_{max} in the limit of large confining pressure, because it is unreasonable physically to assume that an increase in

Figure 3. Triaxial failure envelope (solid line) and its relation to the strength curve for virgin ($\phi=0$) material.

strength can continue indefinitely in this limit. Due to experimental limitations, it is currently impossible to conduct triaxial tests at sufficiently high confining pressure to confirm or refute this view, but it appears to be a reasonable way to construct a model of this type.

As discussed in the next section, not all portions of the strength curves shown in Figure 2 are accessible in practice, because of the accumulation of damage. The strength curves shown should at present be regarded as hypothetical in this sense. They show the strength that the material would exhibit if it were possible to freeze the damage in time and prevent further accumulation.

Note that for $\phi=1$, i.e., fully damaged material, no tension can be sustained. This is consistent with the definition of damage for purposes of the model: current tensile strength divided by the tensile strength of virgin material (fixing $\phi=0$).

4. DAMAGE EVOLUTION

In Figure 3, we show the strength curve for virgin material together with the failure envelope that would be measured in a triaxial test. To the left of the line labeled "brittle-ductile" transition, the two curves coincide. To the right of this line, brittle behavior predominates, and the two curves diverge because the failure envelope includes damaged states, while the strength curve shown is for damage fixed at zero.

The current state of stress, as shown in the figure, may lie to the right of the triaxial failure envelope for some interval of time in a dynamic problem. This is possible because damage accumulates only at a finite rate. To help specify this rate, we define a variable Ω , called the *overload*. As shown in Figure 3, the overload is the horizontal distance from the failure envelope to the current state of stress in the plane shown.

The evolution law for damage is given by

$$\dot{\phi} = \dot{\phi}_b + \dot{\phi}_d + \dot{\phi}_c \quad (3)$$

where $\dot{\phi}_b$, $\dot{\phi}_d$, and $\dot{\phi}_c$ are the brittle, ductile, and compressive damage rates respectively. The brittle damage rate is given by

Figure 4. Dependence of time to failure on local stress and body size.

$$\dot{\phi}_b = \begin{cases} \frac{1}{\tau(T_1, \mathcal{B})}, & \Omega \geq 0 \\ 0, & \Omega < 0 \end{cases} \quad (4)$$

where τ is the *time to failure*, a function of the maximum principal stress and the geometry of the body $\mathcal{B}$. The dependence of τ is illustrated in Figure 4. Note that for a given body, as the maximum principal stress becomes increasingly tensile, the time to failure decreases. This decrease continues until T_1 reaches a constant called the spall stress, T_{spall} ; beyond which the time to failure is negligible, i.e., failure is immediate. The spall stress can be measured by plate impact experiments. Note that the spall stress differs from the fracture stress p_* in that the latter is a function of the damage variable, while the former is a material property.

The dependence of τ on body size may be investigated by picturing a long rod of length L and density ρ subjected to a given tensile stress σ_0 which exceeds the quasi-static tensile strength. Suppose the rod breaks up symmetrically into fragments of length $L/2$. How much time, τ , does it take for the breakup to occur? To investigate this, we construct a lumped mass model for the fragments (Figure 5). Each fragment has a displacement U measured away from the cross-sectional plane in which a crack forms. The interaction between the fragments is assumed to be described through the following relation between the traction across the crack plane, σ_{gap} , and U :

$$\sigma_{\text{gap}} = \sigma_0 \left(1 - \frac{2U}{\delta} \right), \quad 0 \leq U \leq \delta/2 \quad (5)$$

where δ is the gap distance at which no traction is present, which we define as completion of the breakup process. Combining this with Newton's Second Law applied to each fragment, and assuming an initial defect (gap width e) at time zero, we arrive at the following initial value problem for fragment displacement:

$$\ddot{U} = \frac{4\sigma_0 U}{L\rho\delta}, \quad U(0) = e/2, \quad \dot{U}(0) = 0. \quad (6)$$

Solving this initial value problem, we find

Figure 5. Lumped mass model for the breakup of a rod under tension.

$$U(t) = \frac{e}{2} \cosh\left(\sqrt{\frac{4\sigma_0}{L\rho\delta}} t\right). \quad (7)$$

Let us define the *fundamental time to failure*, τ_0 , by $U(\tau_0) = \delta/2$. Then

$$\tau_0 = \sqrt{\frac{L\rho\delta}{4\sigma_0}} \cosh^{-1}\left(\frac{\delta}{e}\right). \quad (8)$$

The quantities δ and e are not measurable; nevertheless this analysis suggests that

$$\tau_0 = C\sqrt{L}. \quad (9)$$

where C is a constant. Similar analysis may be carried out for radial cracks in an axisymmetric geometry, which is the predominant failure mode in penetration problems. The constant of proportionality C in Equation (9) may be evaluated through computer simulations of split Hopkinson bar experiments together with the experimental data. The best value of C results in the best agreement on a graph of peak stress as a function of strain rate.

The ductile and compressive components of rate of damage, $\dot{\phi}_d$ and $\dot{\phi}_c$, model the destruction of cohesive bonds between grains at the microstructural level due to large shear deformation or crushing of the void space. This effect is represented by

$$\dot{\phi}_d = \frac{\dot{\epsilon}}{\epsilon_{d1} - \epsilon_{d2}}, \quad \epsilon_{d1} \leq \epsilon \quad (10)$$

where ϵ is equivalent plastic strain and ϵ_{d1} and ϵ_{d2} are constants. Similarly,

$$\dot{\phi}_c = \frac{\dot{p}}{p_{d1} - p_{d2}}, \quad p_{d1} \leq p \quad (11)$$

where p_{d1} and p_{d2} are constants. These constants may be found from Hugoniot data, which show fairly clearly the pressure at the onset of pore crushing and the pressure at which pore crushing is complete. Moreover, the volumetric strains at these pressures provide estimates of

Figure 6. Experimental and computed penetration depth for long steel rods into concrete.

ϵ_{d1} and ϵ_{d2} , under the assumption that the damage mechanisms are similar between the cases of shear deformation and compression.

5. EXAMPLE PROBLEMS

We now present results of some computer simulations making use of the BFK model described above. The code used was CTH (McGlaun, Thompson, and Elrick, 1990), an Eulerian finite difference code that is used in some parts of the penetration mechanics community.

5.1. Deep penetration of steel rods into concrete

Forrestal, Frew, Hanchak, and Brar (1996) conducted experiments in which steel, ogive-nosed rods with $L/D=10$ were shot into deep concrete targets over a wide range of impact velocities. The target material was "SAC-5" concrete, which has been the subject of extensive material testing, and whose parameters are the default for the BFK concrete model. The nominal unconfined compressive strength of this material is 41.4 MPa (6000 psi). A comparison of the experimentally observed depth of penetration and the computed values is shown in Figure 6. The experimental data points at the highest two velocities are thought to be affected by an abrasion mechanism that was not modeled in the code. Not shown is an experimental data point with impact velocity 1224 m/s, in which the projectile broke up during penetration.

5.2. Stress waves from a buried high explosive charge

In this experiment (Keller and Lesko, 1992), an 0.473 kg (1.0 lbm) spherical high explosive charge was buried at the center of a SAC-5 concrete cylinder of height 1.83 m (6.0 ft) and diameter 2.44 m (8.0 ft). The explosive was Pentolite 50/50. Radial velocity gauges were buried along the midplane of the cylinder at distances of 0.457 m (18 in), 0.914 m (36 in), and 1.067 m (42 in) from the center. The test was car-

Figure 7. Comparison of calculated and experimental gauge velocity data in the buried explosive charge experiment.

ried out four times, showing a high degree of reproducibility of the velocity data between the tests. Figure 7 shows a comparison of the experimental data from a typical test with the CTH simulation using the BFK model.

Part of the significance of this result for validation of the model lies in the trough that is seen in the velocity history at about 325 μ s. This feature is due to tensile hoop stresses in the concrete after the shock wave has passed through it. These stresses are on the order of 35 MPa, which far exceeds the quasi-static tensile strength of the material, which is about 3 MPa. The model correctly reproduces this trough because of the delay in the introduction of damage, as described above. Alternative models that have been tested against this problem do not exhibit this feature of the experimental data.

5.3. Perforation of a concrete slab

Cargile, Giltrud, and Luk (1993) carried out experiments in which ogive-nosed steel projectiles with mass 2.34 kg (5.15 lbm), diameter 0.0508 m (2.0 in), and $L/D=7$ were shot into thin SAC-5 concrete slabs. The experiment included on-board accelerometers. For one of the shots, for which the impact velocity was 309 m/s (1014 ft/s) and the target thickness was 0.254 m (10 in), filtered (4000 Hz) accelerometer data was made available to the author by private communication. A comparison of the experimental and computed accelerometer data is shown in Figure 8.

The peak in acceleration occurs when the nose of the projectile is only about one third of the way through the slab. After that, the damage accumulation reduces the strength of the target so that acceleration is reduced. The BFK model reproduces the timing and height of this peak because of the explicit dependence of damage on time. Alternative models that have been tested fail to reproduce these features of the experimental result.

The second peak in the experimental data shown in Figure 8 (at about 1.2 ms may be due to interaction of the penetrator with a large

Figure 8. Calculated and experimental acceleration data for perforation of a concrete slab.

fragment of target material that has been released from the back surface. A continuum model such as the one discussed here cannot easily reproduce such an interaction because it involves the random and discrete nature of fragments.

6. DISCUSSION

The sharp peak in acceleration of a projectile just after it breaks the surface of a concrete target, which has been seen experimentally as discussed in the preceding paragraph, is an important feature of penetration into concrete, because this peak determines the maximum load on a penetrator. Therefore, a computational model that correctly predicts this peak would be of value to designers of penetrating munitions. Efforts are currently underway at Sandia National Laboratories to provide further experimental measurement of this peak in acceleration in larger scale projectiles than the one described in the preceding section.

The BFK model has been combined with a cratering and fragmentation model. This attempts to predict the gross features of damaged concrete targets. Recall that the overload measures how far out of the failure envelope a state of stress is. We assume that the greater this parameter is, the more gross damage and fragmentation the material is subjected to. A new variable, called the *brittle fracture impulse*, is defined at each fully damaged material particle by:

$$\bar{\Omega} = \theta(s) \int_0^1 \Omega d\phi \quad (12)$$

where s is the distance to the nearest free surface, and θ is a function that accounts for the greater mobility of cracks and fragments near a free surface. The brittle fracture impulse is used as a measure of the gross damage and fragmentation that a body undergoes as it fails.

An interesting feature of concrete penetration that the BFK model predicts is the "size scale effect," which causes depth of penetration to scale nonlinearly with penetrator size if the impact velocity is held fixed. Specific experimental data regarding this effect were apparently

Figure 9. Size scale effect for deep penetration into concrete.

collected during the 1940's but then lost. However, an Army Technical Manual (1965) presents an empirical correlation to these data. The BFK model provides qualitative agreement with this correlation. Figure 9 shows the predicted effect for deep penetration of $L/D=4$ projectiles striking thick concrete targets at 732 m/s (2400 ft/s).

An unusual feature of the BFK model is that, as discussed previously, the parameters depend on the size and shape of the body. This is a reflection of the nature of brittle failure: the body fails as whole,

rather than locally. In this sense the model is nonlocal.

ACKNOWLEDGMENTS

This work was supported by the United States Department of Energy under Contract DE-AC04-94AL85000. Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy. Support from the Joint DoD/DOE Munitions Technology Development Program, the Defense Special Weapons Agency, and the Naval Surface Warfare Center, Dahlgren Division, is gratefully acknowledged.

REFERENCES

- J. D. Cargile, M. E. Giltrud, and V. K. Luk, 1993, "Perforation of thin unreinforced concrete slabs," Sixth International Symposium on Response of Structures to Non-nuclear Munitions, Panama City, FL
- M. J. Forrestal, D. J. Frew, S. J. Hanchak, and N. S. Brar, 1996, "Penetration of grout and concrete targets with ogive-nose steel projectiles," *Int. J. Impact Engng.*, Vol. 18, pp. 465-476
- "Fundamentals of protective design (non-nuclear)," 1965, Department of the Army Technical Manual, TM5-855-1, Washington, D.C., p. 3-14
- J. A. Keller and J. Lesko, 1992, "Experimental results for material modeling and breaching dynamics of thick concrete slabs," FR 85801, Denver Research Institute, Denver, CO
- J. M. McGlaun, S. L. Thompson, and M. G. Elrick, 1990, "CTH: a three-dimensional shock wave physics code," *Int. J. Impact Engng.*, Vol. 10, pp. 351-360