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| A. Study of the Decay of Hot Nuclei Formed in $^{139}$La-Induced Reactions at E/A= 45 MeV by Hybrid Dynamical-Statistical Calculation | |}

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I. Introduction

This report describes the program of research conducted under the direction of Professor Alice Mignerey for the period February 1994 to February 1995. It reflects the efforts of the principal investigator, the postdoctoral research associates Jing Shea and Houria Madani, and graduate students Edmund Garcia-Solis and Daniel Russ. Jing was on part-time maternity leave until May 1994 and Houria remained working with the group on a part-time basis until her departure in October 1994, to join the Nuclear Physics group at the UNAM in Mexico City.

The two main areas of research of intermediate energy heavy-ion reactions and ultrarelativistic heavy-ion reactions are represented in this report. Section II describes the work performed in the intermediate energy regime. Due to a delay in completion of a necessary electronics prototype and the scheduled shutdown of the cyclotron at the Michigan State University National Superconducting Cyclotron Laboratory (MSU-NSCL), our approved experiment at the NSCL has not yet been run and we are scheduled for the first week of April 1995. This experiment will survey the evolution of the reaction mechanisms from $E/A = 30$ to $70$ MeV, using the Xe beam on a Cu target. Section II.A is a report utilizing data obtained from our first preliminary run in spring 1993. Work continues on defining a suitable calibration scheme for the plastic elements of the Maryland Forward Array (MFA) and a summary is presented in Sec. II.C.

The MFA has proven a very successful detector that (with or without the matching annular Si detector) has been used in a number of experiments within our collaboration with Gary Westfall and the $4\pi$ group. In addition, other NSCL groups have suggested collaborative efforts. A very successful example of this is briefly described in Sec. II.B.

One of the biggest challenges in the interpretation of results obtained in intermediate energy heavy-ion reactions is the availability of suitable model calculations to describe reactions in this regime. In his thesis work, Bruce Libby tackled this by applying a Landau-Vlasov model to interpret his data. Appendix A is a preprint of the paper on his work which has been submitted to Phys. Rev. C. My current student Edmund Garcia-Solis has taken this one step further, using the BUU code of Bauer et al., and
developed a cluster recognition model to treat the BUU-generated distributions. His work is presented in Sec. II.D.

Our research in the area of ultrarelativistic reactions has taken a bold new turn. In addition to our collaboration in the RHIC detector project PHOBOS, we are now official members of the current AGS experiment E866. This decision was driven largely by the rejection by the AGS PAC of the PHOBOS at AGS proposal. With the slowdown of the RHIC schedule and the approval of the Conceptual Design Report for PHOBOS, the need and opportunity arose to form more formal ties with a current experimental program at the AGS. Jing has completed her phase of the PHOBOS magnet work and produced a manual to guide future users of the magnetic field code TOSCA. She then spent the summer at MIT assisting in the construction of the New Multiplicity Array (NMA) for the fall running at the AGS. Edmundo joined her at MIT, also working on the LED system described in Sec. III.A. The September-October 1994 run was a success with active participation by Jing, Alice and Edmundo. Jing is now working on the reaction plane definition, using the data taken by the MFA during the E866 run. The hope is to use this to understand the sources of antiproton production in the Au + Au data. A brief report of this work is found in Sec. III.B.

In addition to the experimental endeavors, the principal investigator Alice Mignerey was the organizer of this year's Winter Workshop on Nuclear Dynamics held February 11-18, 1995 in Key West, Florida.
II.A Heavy-Ion Peripheral Collisions at Intermediate Energies

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Studies of heavy-ion-induced reactions at intermediate energies are useful tools to probe the transition from mean-field dominated mechanisms at low bombarding energies to reactions governed by nucleon-nucleon interactions at higher energies. It is generally thought that this transition does not occur at one particular energy, but rather at a continuum of bombarding energies as a function of system size.

The nature and size of the interacting system are important factors in governing the formation of the hot nuclear system produced in the reaction. A number of studies have focused on the role of incomplete fusion in this energy regime. One of the goals of this current study is to explore the role of the deep-inelastic mechanism in the formation of hot systems, with particular focus on the projectile-like products. The first set of systems studied at the Michigan State University National Superconducting Cyclotron Laboratory (MSU-NSCL) by our group was the 50-MeV/u $^{129}$Xe on $^{27}$Al, $^{64}$Cu, $^{139}$La, and $^{165}$Ho reactions. The projectile-like fragments produced in the peripheral collisions of these systems were detected with the Maryland Forward Array (MFA) in coincidence with target-like fragments.
and light charged particles in the MSU-NSCL 4-π array. More details about the experiment, and the detection system, namely the MFA, can be found in reference 4.

The deep-inelastic mechanism, which is prevalent at lower bombarding energies, has been shown to be present in reactions at intermediate energies up to 35 MeV/u\textsuperscript{5,8}. This can be seen in Fig. II.A-1, where the distributions of reconstructed primary projectile-like fragment (PLF) charges, obtained in the 28.2-MeV/u \textsuperscript{129}Xe on \textsuperscript{209}Bi reaction by Baldwin et al.\textsuperscript{6}, are displayed for various bins of neutron multiplicity. The charge distributions, which have nearly Gaussian shapes, become broader with increasing neutron multiplicities. As the increase in neutron multiplicity indicates an increase in energy dissipation, and hence excitation energy, it can be concluded that the features of the charge distributions observed in Fig. II.A-1 are similar to those observed for reactions at lower bombarding energies. To explore to what extent this mechanism persists, the PLF distributions obtained in the 50-MeV/u \textsuperscript{129}Xe on \textsuperscript{27}Al, \textsuperscript{nat}Cu, \textsuperscript{139}La, and \textsuperscript{165}Ho reactions were parameterized in terms of the characteristics of a deep-inelastic process, and some of the results have been previously reported in reference 9.

It is a well known fact that, in a deep-inelastic process, the fragment mass and charge distributions broaden with increasing energy damping. This broadening, which is indicative of nucleon exchanges between the two interacting heavy ions, was not observed for the systems studied here\textsuperscript{9}. In contrast, a broadening of the charge variances with increasing energy dissipation has been reported for the 28.2-MeV \textsuperscript{129}Xe on \textsuperscript{209}Bi system\textsuperscript{6}. It is interesting to compare the 50 MeV/u present data to the results obtained with the lower bombarding energy to explore the changes that occur when the bombarding energy is increased.

A comparison between the charge distribution centroids and variances obtained with the 50-MeV/u \textsuperscript{129}Xe-induced reactions and those obtained with the 28.2-MeV/u \textsuperscript{129}Xe on \textsuperscript{209}Bi reaction, is shown in Fig. II.A-2. The obvious difference between the lower and higher bombarding energy systems is the behavior of the charge variance. For the 28.2-MeV/u \textsuperscript{129}Xe on \textsuperscript{209}Bi, the value of $\sigma^2$ increases with decreasing PLF laboratory energy and reaches a maximum at 1000 MeV. The increase of $\sigma^2$ is achieved with 2 different slopes, the steepest
one being for PLF energies between 1000 and 2500 MeV. The charge centroids show a change of slope at the same PLF laboratory energy. On the other hand, the values of $\sigma_Z^2$ obtained with the 50-MeV/u systems seem to be nearly constant, and the $<Z>$'s decrease nearly linearly with decreasing PLF laboratory energy.

A further exploration of this point is possible by comparing distributions predicted by a nucleon exchange model to the present data and to the data obtained with the 28.2-MeV $^{129}$Xe on $^{209}$Bi. The model of choice in this study was the stochastic exchange model of Tassan-Got$^{10}$, which has shown a good reproduction of low bombarding energy data$^{11}$. Due to the model limitations, centroids and variances of the charge distributions are available only for the lower range of PLF laboratory energy, which approximately corresponds to the range of total kinetic energy loss (TKEL) determined by the entrance channel Coulomb barrier.

The primary distributions (solid lines) predicted by the nucleon exchange model are corrected for light particle evaporation using the decay code GEMINI$^2$, and the results are indicated by the dashed lines. It can be seen that, even with evaporation corrections, the data are not reproduced by the model for the 50-MeV/u $^{129}$Xe-induced reactions, especially in the case of the two heavier targets. However, it is important to point out that preequilibrium emission of light particles, which can be very sizeable at these intermediate energies, has not been taken into account. On the other hand qualitative agreement between the data and the model predictions can be observed for the 28.2-MeV $^{129}$Xe on $^{209}$Bi system. A good quantitative agreement may be possible if preequilibrium emission is taken into account.

Another mechanism that is explored with the present data is incomplete fusion. In the scenario considered here, a piece of the projectile is sheared off and fuses with the target. The remainder proceeds forward with little change in its momentum. It has been previously shown that beam velocity components are present at high, light-charged-particle multiplicities for the present systems$^9$. However, the $V_{PLF}/V_{beam}$ centroids of the velocity distributions obtained for the four systems decrease with decreasing PLF charge, as can be seen in Fig. II.A-3. As no mass parameter was available from the present data, a mass
parameterization was used to determine the experimental PLF velocity. The velocity predicted by the nucleon exchange model for the four systems is also displayed in Fig. II.A-3. It is interesting to note the generally good qualitative agreement between the data and the model calculations for the cases of the two lighter targets (Al, and Cu), although the predictions are consistently lower than the experimental values. In contrast, for the two heavier targets, the behavior of the experimental velocity is reproduced only when the PLF charges are larger than 42. For lower values of \(< Z >\), the dependence of the experimental velocity on \(< Z >\) seems to be steeper than that of the predicted velocity, and the experimental velocities are consistently smaller than those calculated.

The present parameterization of the PLF's obtained in the 50-MeV/u $^{129}$Xe on $^{27}$Al, $^{nat}$Cu,$^{139}$La, and $^{165}$Ho reactions, in terms of a deep-inelastic mechanism, has yielded ambiguous results about the presence of this mechanism in this energy regime. Although the experimental centroids and variances of the charge distributions have not been reproduced, the general behavior of the $V_{PLF}/V_{beam}$ centroids with PLF charge is described for the cases of the Al and Cu targets. The implementation of a preequilibrium emission calculation is underway to better estimate the available excitation energy in these reactions. In addition, calculations based on the Boltzmann-Uehling-Uhlenbeck (BUU) transport equation are being performed (see Section II.D of this report). In these calculations, it is possible to explore the influence of non-binary decay channels. An example is the formation of a projectile-like fragment, a target-like fragment and a smaller fragment from the neck region formed by the overlap of the two nuclei$^{12}$. 

6
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Figure II.A-1. Distributions of reconstructed primary atomic numbers $Z_{prim}$, for fragments detected at $<\theta_{lab}> = 4.3^\circ$, for various bins in neutron multiplicity $m_n$ for the 28.2-MeV $^{129}$Xe on $^{209}$Bi system. Figure taken from reference 6.
Figure II.A-2. Experimental charge centroids and variances as a function of the PLF laboratory energy for the 28.2-MeV $^{129}$Xe on $^{209}$Bi system (top panel), and for the 50-MeV/u $^{129}$Xe-induced reactions (bottom panel). The predictions of the nucleon exchange model before and after evaporation corrections are indicated by the solid and dashed lines, respectively. The data for the Xe on Bi system is courtesy of Baldwin et al.\textsuperscript{13}
Figure II.A-3. Experimental $< V_{PLF}/V_{beam} >$ as a function of PLF secondary charge for the 50-MeV $^{129}$Xe-induced reactions. Tassan-Got's model predictions of $< V_{PLF}/V_{beam} >$ are indicated.
II.B Using the Maryland Forward Array to Measure the Mass and Structure of $^{11}$N

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Exotic proton-rich nuclei offer an excellent opportunity to test nuclear structure calculations and to extend proton-decay data to beyond the proton-drip line. In particular, the structure of $^{11}$N is important to facilitate a comprehensive understanding of the two-proton decay of $^{12}$O. If the $^{11}$N ground-state energy is large, then $^{12}$O probably decays via a one-step, 2-p emission. On the other hand, a low ground state energy may indicate a $^{11}$N intermediate.

The Maryland Forward Array$^1$ (MFA), with its matching annular Si (ASi) detector, was used, in conjunction with a stopping Si telescope, to determine the mass and structure of $^{11}$N. The MFA is an annular arrangement of 16 fast-slow phoswich detectors and can easily identify protons. When used with the ASi, it can achieve a position resolution of better than 2 mm in the radial direction. Our group supported an experiment performed at the Michigan State University National Superconducting Cyclotron (MSU-NSCL). Led by Michael Thoennessen, the experiment used the A1200 and the Reaction Plane Mass Separator (RPMS).

A primary beam of 80-MeV/A $^{16}$O was used to bombard a 1000-mg/cm$^2$ $^{11}$Be target. The reaction products were then focused through the A1200 and the RPMS to select a secondary beam of $^{12}$N. The detector set-up at the end of the RPMS is shown in Figure II.B-1. A 30-mg/cm$^2$ $^{12}$C target was used to produce $^{11}$N in the transfer reaction $^{12}$C($^{12}$N, $^{11}$N)$^{13}$C. The $^{11}$N
emits a proton and decays to $^{10}$C. The MFA, with the ASi, was used to detect the proton in coincidence with a $^{10}$C fragment in the Si stopping telescope, and the mass and half-life of the $^{11}$N could then be calculated by kinematical reconstruction of the event. Daniel Russ and Houria Madani took responsibility for the set-up and operation of the MFA system, both before and during the run.

The data are currently being analyzed at Michigan State as part of the doctoral thesis of Afshin Azhari.

![Diagram](image)

Figure II.B-1: Diagram for the detector set-up

References

II.C Calculation of Light from the Maryland Forward Array Using Specific Luminescence

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Scintillating plastic detectors have recently become very popular in nuclear experiments because they are relatively inexpensive and their timing properties can be tailored to fit specific experimental needs. Calibrations of plastic detectors often use empirical fits to the data to extract particle identification, and energy information. Recently, a secondary electron energy deposition (SEED) model\(^1\) has given a simple expression for the specific luminescence for plastic detectors. In principle, the SEED model calculations can make calibration of plastic detectors simpler and faster than current empirical fitting.

In the SEED model, the light output of a plastic detector is calculated by integrating the specific luminescence over the path of the ion being detected. The first step is to calculate the path of the ion in the detector. The Maryland version of the SEED code uses Zeiglar systematics\(^2\) to calculate the energy loss within a thin piece of plastic. The path is calculated by following the energy loss through the detector. The specific luminescence \(\frac{dL}{dx}\) in the SEED model is given by:

\[
\frac{dL}{dx} = \pi CN \frac{6e^4z^2}{5m_eV^2} \left[ -\frac{5}{6} R^{3/4} + \ln \left( \frac{1+R^{1/4}}{1-R^{1/4}} \right) - 2\tan^{-1}\left( R^{1/4} \right) \right], \tag{1}
\]

where \(C\) is a constant, \(N\) is the number of electrons per unit volume, \(m_e\) is the mass of the electron, \(V\) is the velocity of the ion, \(e\) is the elementary charge on an electron, and \(R\) is the fraction of the energy deposition region that is in the quenching region.

The SEED model has recently been used in an attempt to calibrate the Maryland Forward Array\(^3\) (MFA). A single element of the MFA consists of 1 mm of BC400 fast plastic and 10 cm of BC444 slow plastic sharing a single
photomultiplier tube (PMT). Experimentally, the "fast" light and "slow" light are measured by integrating the signal from the PMT at different times. Overlap between the fast and slow pulses adds light from the fast pulse into the slow and vice versa. The calculation must therefore create a time dependent pulse similar to the signal from the PMT. The pulse shape $P(t)$, used in the Maryland version of the SEED code, is defined as:

$$P(t) = L \left[ \frac{\exp\left(\frac{-t}{t_r}\right) - \exp\left(\frac{-t}{t_f}\right)}{t_r - t_f} \right],$$

where $t_r$ and $t_f$ are the rise and fall times of the plastic, respectively, and $L$ is the light from the detector. Figure II.C-1 shows the calculated pulse shapes for the fast (dashes) and slow (dots) plastics; the solid line in the figure is the sum of the two pulses. The area of the fast and slow pulses are normalized to the SEED light calculation. By integrating the pulse over a time $\Delta t_{\text{fast}}$, for example 20 ns, the SEED calculation includes light from the slow plastic that gets into the fast plastic. Similarly, a slow time gate $\Delta t_{\text{slow}}$ includes some contribution from the fast plastic. The calculated pulse is integrated over the same times as was the signal from the PMT.

![Figure II.C-1: The calculated light pulse for the MFA. The dashed line is the fast pulse. The dotted line is the slow pulse, and the solid line is the sum.](image)

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The effects of the integration are shown in Figure II.A-2. Figure II.A-2(a) is the straight SEED calculation for \( Z = 1 \) to \( Z = 12 \). In this case, the light of the fast plastic is plotted versus the light of the slow plastic. No attempt has been made to include a time dependent pulse. Figure II.A-2(b) is the SEED calculation with a time dependent pulse added. Adding the time dependence helps the calculated plot resemble an actual \( \Delta L \)-L plot obtained on line during the experiment.

![Graphs showing light from fast and slow plastics](image)

Figure II.C-2: Time independent (a) and time dependent (b) light calculations for the MFA fast vs. MFA slow.

There is currently insufficient calibration data to match and test the model. In an experiment scheduled this coming April, more calibration data will be collected, and used to obtain a better description of the pulse shape, the effects of trigger timing on the pulse, and the effective rise time and decay time of the plastic when coupled to the PMT.

References

II.D A Cluster Recognition Model For Intermediate Energy Heavy-Ion Reactions

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There are two main questions concerning the clusterization of the nucleons coming from a BUU-type calculation. One is when to stop the dynamical calculation; the other is how to proceed with actual clusterization given the output variables of the model. To deal with these questions the variables relevant to the computation must be addressed at two different levels.

At one level are the variables that describe the motion of the pseudo nucleons, or test particles, in a set of parallel systems. Provided that the pseudo nucleons obey the Newtonian equations of motion

$$\frac{dp_i^{(n)}}{dt} = -\nabla_r U(r_i^{(n)}; t) \quad \text{and} \quad \frac{dr_i^{(n)}}{dt} = \frac{p_i^{(n)}}{m_i},$$

(1)

where $p_i$ is the particle momentum, $r_i$ is the position, and $m_i$ is the particle mass, the test particles will define a total phase-space distribution. This distribution is in turn a function of the second level variables that govern the evolution of the system as a collective ensemble via the Boltzman-Uehling-Uhlenbeck (BUU) transport equation

$$\left\{ \frac{\partial}{\partial t} + \frac{p}{m} \cdot \nabla_r - \nabla_r U(r; t) \cdot \nabla_p \right\} f(r, p; t) = \overline{I}[f],$$

(2)

where $\overline{I}[f]$ represents the average rate of change of the particle distribution due to two particle collisions, $U(r; t)$ is the density dependent mean field potential, and $f(r, p; t)$ is the phase space distribution.

The collective ensemble described by Eq. (2) will follow a mean field trajectory, from which the average properties of the heavy-ion collision are calculated, and the parallel systems provide the fluctuations about these averages. Recently a consent is emerging that
these fluctuations should play an essential role in the multifragmentation of the system at some point of the dynamical calculation (freeze out time)\textsuperscript{6,12,13}. Therefore, rather than using the mean field characteristics of the calculation, the properties of the parallel systems are compared to the experimental data\textsuperscript{14}.

In order to find the freeze-out or stopping time of the calculation, it is necessary to establish the point when the fluctuations around the mean field trajectory are more pronounced. Fig. 1 shows the evolution of the density function, in the plane of the reaction, as a function of time for the systems \textsuperscript{139}La on \textsuperscript{27}Al and \textsuperscript{nat}Cu at E/A = 45 MeV, as calculated from the BUU using $\mathcal{M} = 300$ pseudo-particles. Distributions are shown for a range of input parameters and calculation times. For the smaller values of the impact parameter, where the projectile and target overlap substantially, a highly compressed system is formed. Then the system will start expanding up to a point when the process is repeated again. This is accompanied by emission of individual pseudo-particles in every cycle. At the end of the first compression, when the mean field applies an inward force toward the source that cancels the outward motion, the system will spend considerable time at a relatively low density and temperature, corresponding to an unstable region in the nuclear phase diagram. Under these conditions, it is expected that large fluctuations in the density will emerge\textsuperscript{17}, giving rise to the condensation of the system into a number of clusters. The description of this process resides outside the extent of the BUU, which deals only with the mean trajectory of the system, and is inadequate for descriptions of unstable evolutions. Therefore, the end of the first expansion indicates an approximate time to stop the dynamical calculation and proceed with the clusterization routine.

Natural variables which can be used to estimate the freeze-out time are the density and the averaged total kinetic energy of the system. The time at which to stop the BUU calculation $t_{sf}$ is then chosen as the time when these variables exhibit their absolute minima. The evolution as a function of time of the averaged total kinetic energy and the highest value of the density distribution computed at each time step for different impact parameters are displayed in Figs. II.D-2 and II.D-3 for the La on Al and Cu systems, respectively. At around 30 fm/c a highly compress system is formed, which subsequently expands, reaching a minimum in density at around 90 fm/c; therefore $t_{sf}$ is set at 90 fm/c for both systems.
The common method to recognize cluster structures is to separate fragments according to the relative position in phase-space of the particles in a system\textsuperscript{15,12,14}. Using this approach, a nucleon belongs to the same cluster if it is sufficiently connected, that is

\[ |r_i - r_i| < D_r \tag{3} \]

or/and

\[ |p_i - p_i| < D_p \tag{4} \]

where \( D_{rp} \) is a parameter which is a function of the local density. Although this method is relatively simple and fast, it is not realistic if the clusters are not sufficiently well separated\textsuperscript{3}. For example, this method is not able to separate two structures that share one surface nucleon. Also strict application of Eqs. (3) and (4) will lead to the exclusion of energetic nucleons in the clusters, therefore miscalculating the amount of internal kinetic energy contained in the cluster.

An alternative approach to finding the clusters without these ambiguities, is by first labeling the interior particles in the parallel systems, then cluster them into fragment "seeds" using Eq. (3). To do this, suppose that at \( t_{st} \) we have a global particle distribution \( \rho(r) \) for the ensemble. If the coordinate space is divided into cubic cells of side \( 2r_b \), every cell in the space can be defined as interior or exterior to a cluster by the following condition

**Condition 1** *The cell \( i \) defined by its center at \( r_i \) will be interior if*

\[ \rho(r_i + r_b x_\mu) < \rho_0 \text{ for every } x_\mu, \mu = 1 \ldots 6 \]

where \( x_\mu \) are six unitary vectors in the direction of the positive and negative axes of a coordinate space, with the positive Z axis directed in parallel to the beam velocity. Taking the position of every nucleon in each parallel system of the ensemble and applying condition 1, it is possible to determine which nucleons are, on the average, entirely surrounded by nuclear matter, tagging them as interior. Then, using the interior nucleons, it is possible to form the cluster "kernel" using a standard diagrammatic approach\textsuperscript{18} based on Eqs. (3) and (4), with \( D_r = r_b = 1.42 \text{ fm, the average nuclear radius.} \)

Note that a nucleon is defined as interior by the surrounding density of its cell, not by the density within the cell. To see how this affects the clustering procedure, take, for
example, the system at the beginning of the dynamical calculation (second square of Fig. II.D-1), if applying a common cluster routine Eq.(3), both nuclei touching each other would be considered a single fragment. On the other, hand if the interior nucleons are separated first, we would have, after clustering, two separated fragments, plus a number of exterior nucleons that can be treated differently.

Once the configuration of the cluster seeds for every parallel system is established, the corresponding surrounding nucleons tagged as exterior are tested by the condition

**Condition 2** The exterior nucleon $i$ belongs to the cluster $j$ if

$$s_i \leq R_{cm}^j + r_b$$

and

$$p(s_i)_{frm}^2 + 2m_i[BE + E_{co}^i] \leq P_{cm}^j,$$

where $R_{cm}^j$ and $P_{cm}^j$ are the positions and momenta of the center of mass for the $j$ cluster, respectively, and $p(s_i)_{frm}$, $m_i$ and $s_i$ are the Fermi momenta, mass, and relative distance to the cluster center of the nucleon $i$, respectively. The nucleon binding energy is given by an average value of $BE = 8.0$ MeV and the neutron-incremented-energy $E_{co}^i = 5$ MeV for $i$ neutrons and 0 MeV for protons. For nucleons that, by Condition 2, belong to two clusters, a random decision is taken. After the first pass using Condition 2, the procedure is repeated for the remaining exterior nucleons, computing the new values of the center-of-mass position and momentum at every iteration, until convergence to a constant mass of the clusters is achieved. The nucleons that, at this point, do not belong to any cluster are tagged as free. Finally, for those remaining free nucleons a coalescence check is done by using Eqs. (2) and (3).

When the configuration of the cluster is established, the collective properties for every cluster, such as translational kinetic energy, angular momentum, excitation energy, etc., are computed by using standard semi-classical formulas. Due to the instability of the cluster formed, is not possible to know the exact zero point of the potential energy. Thus, a parameter had to be introduced to calculate the excitation energy

$$E^* = E_{kin}^* - \chi E_{fermi}^*$$

(5)

where $E_{kin}^*$ is the excitation energy due to the internal kinetic energy of the test particles,
$E_{\text{fermi}}$ is the average fermi energy of the nuclei and $\chi$ is a parameter. This parameter is chosen in such way that the ground state $E^* = 0$ for the nuclei before the interaction.

As an application of this clustering model, the reactions $^{139}\text{La}$ on $^{27}\text{Al}$ and $^{64}\text{Cu}$ at $E/A = 45$ MeV are simulated. The results are then compared to experimental data\textsuperscript{19} after running through the evaporation code GEMINI\textsuperscript{16}. The model cross sections and angular distributions, as a function of the detected charge, were calculated directly from the output of the evaporation code. On the other hand, the velocity distribution, and sum charge yield for different multiplicity gates were filtered according to the correspondent experimental set-up.

The results of the model calculation, together with the experimental data, are shown in Figs. II.D-4 to II.D-7. Figs. II.D-4 and II.D-5 show the angular distributions and integrated charge distributions for the $\text{Al}$ and $\text{Cu}$ target, respectively. For the $\text{Al}$ target, the angular distributions (Fig. II.D-4) for the heavier fragments $Z = 42$ down to $Z = 34$ are well reproduced by the calculations. On the other hand, for the range $Z = 34$ to $Z = 22$, the model overpredicts the yield of forward angles. After $\theta_{\text{cm}} = 50^\circ$ the calculations again correspond to the experimental data. For $Z$ values smaller than 20, the contrary trend is found; the model replicates well the data up to $\theta_{\text{cm}} = 50^\circ$ and after this it underpredicts the yield.

In the bottom panel of Fig. II.D-4 the experimental $Z$ distribution in cross section is compared to the model predictions. The dashed lines in this figure represent the cross sections integrated for different intervals of impact parameter. In general, the cross sections agree reasonably well with the experimental data. However, for $Z = 7$ to $Z = 10$ the model underpredicts the yield, and it is possible to observe a bump around $Z = 30$. This bump grows proportionally to the interval of integration and may indicate that the fitting of the experimental angular distribution used to calculate the cross section was biased to exclude the fragments from more peripheral reactions, which are very forward peaked.

The angular distributions for the Cu target show the same general behavior as for the Al target, except for the overprediction of the calculation for fragments with $Z \geq 32$ at the backward angles. For the smaller fragments, the calculation better fits the data for $\theta_{\text{cm}} \geq 100^\circ$. The bump in the integrated cross sections around $Z = 32$ remains almost constant with
increasing integration interval, and the cross section increases for larger integration intervals for fragments with $Z \geq 36$. This component of the spectra may again be interpreted as evaporated residues of the projectile-like fragments, coming from more peripheral reactions.

The sum detected charge and the source velocity distributions, for all coincidence events with multiplicity $n = 2$ and $n = 3$, are shown in Fig. II.D-6 and II.D-7. The dashed lines represent the calculated distributions and the solid lines the data. In general, for the $n = 2$ events, the peak in the $Z_{\text{tot}}$ distribution is well reproduced by the calculation. However, the tail of the distribution is overpredicted for the $Al$ target, and the central part of the distribution is underpredicted for the $Cu$ target. For $n=3$ events, the model is able to predict the peak of the distribution; however, it underestimates the width and the tale of the distribution. On the other hand, the model is able to reproduce the peaks of the $V_S$ distributions within a few percent, also reproducing the width and the tail for $n = 2$ events for the $Cu$ target. However, the width is overpredicted for the $Al$ target. For $n = 3$ events, the velocity distribution widths are well reproduced, but the peak is shifted to higher velocities for both targets.

The results obtained confirm that it is possible to describe the fragment distributions produced in intermediate energy heavy-ion reactions considering statistical and dynamical features. In this work, a dynamic description of the nucleon collisions was coupled with a subsequent statistical decay of the primary source through a clustering subroutine. The specific clustering criterion is a reasonable approach, not only for dealing with dynamically separated clusters, but also for generally dense stages of the reaction. This feature makes a significant improvement to the predictions when compared with the results from other clustering models$^{2,12,14}$. Finally, it should be pointed out that, in spite of the reasonably good predictions that are obtained, this clustering routine is not optimal because it sharply stops the dynamical calculation. Further work in this area should focus on obtaining the clusterization directly from the dynamical evolution of the density distribution of the system. To this end, the formation of clusters from the ensemble systems is being investigated by increasing the fluctuations in the dynamical evolution, and performing a renormalization of the density distribution function by artificially inserting cells of empty space within the density volume.
REFERENCES


Figure II.D-1. Evolution predicted by the BUU simulation of the density in the reaction plane, as a function of the time and impact parameter, for the systems $^{139}$La on $^{27}$Al and $^{64}$Cu at E/A = 45 MeV.
Figure II.D-2. Predicted average total kinetic energy (top) and maximum value of the density function (bottom), as a function of time, for the system $^{139}La$ on $^{27}Al$ at $E/A = 45$ MeV. Each line represents the computation for different impact parameters (b).
Figure II.D-3. Predicted average total kinetic energy (top) and maximum value of the density function (bottom), as a function of time, for the system $^{130}\text{La}$ on $^{nat}\text{Cu}$ at $E/A = 45$ MeV. Each line represents the computation for different impact parameters ($b$).
Figure II.D-4. Angular distribution for selected Z values (top) and integrated (bottom) charge distributions for the reaction La + Al. For the angular distributions, the experimental (solid line) and calculated (dash lines) values are defined in the center of mass. For the total cross section, the experimental data are represented by diamonds, and the calculated distributions are represented by solid, dashed and dotted lines, for integration over impact parameters from 1 to 5 and 7 and 9 fm, respectively.
Figure II.D-5. Angular distribution for selected Z values (top) and integrated (bottom) charge distributions for the reaction La + Cu. For the angular distributions, the experimental (solid line) and calculated (dash lines) values are defined in the center of mass. For the total cross section, the experimental data are represented by diamonds, and the calculated distributions are represented by solid, dashed and dotted lines, for integration over impact parameters from 1 to 5 and 7 and 9 fm, respectively.
Figure II.D-6. Experimental (solid lines) and calculated (dashed lines) sum of detected charge and source velocity distributions; expressed in velocity relative to the beam velocity $v_s$; for $n=2$ and 3 events for the reaction La + Al.
45 MeV/u La + Cu

Figure II.D-7. Experimental (solid line) and calculated (dashed line) sum of detected charge and source velocity distributions; expressed in velocity relative to the beam velocity $v_\text{b}$; for $n=2$ and 3 events for the reaction La + Cu.
III.A The LED System for the NMA in E866 at the BNL AGS

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The New Multiplicity Array (NMA) system was designed to measure the high multiplicity of fast particles produced in the collision of Au on Au at 10.6 A GeV/c utilizing the improved E802 spectrometer\(^1\) at the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL). The NMA consist of 400 threshold Cerenkov modules, each made of UV-transparent lucite. A light Emitting diode (LED) system was built for the NMA for monitoring. Each of the NMA modules has an LED embedded at the back face of the lucite. The LED signal is read by the same photomultiplier tube as the real Cerenkov light produced by the incoming particles. Each LED is pulsed by a positive 2-V pulse of 150-ns duration. The resultant light output from the LED is a negative 40-mV pulse of 1.5-\(\mu\)s duration when the photomultiplier tube is biased -800 V.

Figure III.A-1 is a schematic drawing of the LED/NMA system for one module. The LED's are driven by a common LED driver. The pulse height and the width of the LED output can be adjusted by varying the amplitude and the width of the input pulse. In this application, a 4 V x 600 ns square pulse from the pulsar gives the proper light output from the LED's. The LED system was installed and used for the E866 experiment at BNL in fall 1994.

The LED used for the NMA is the low current LED (HLMP1790) commercially purchased from the Hewlett Packard (HP) company. The peak wavelength of the emission spectrum is 560 nm (green). The Cerenkov light emitted from the lucite is about 420 nm (blue). At first, the blue LED (HLMP-
Figure III.A-1. One NMA module with an LED embedded inside the lucite.

DB15) was tested but was found to have a longer rise time (about 1 μs) and less light output as compared to the green LED (HLMP1790). With the same input to the LED's, the ratio of the output pulse height for the green LED to the blue LED was 1.46. Therefore, even though the blue LED better matches the photomultiplier tube's response, the green LED was chosen for the application. A typical light output from the green LED (HLMP1790) and the pulse input are shown in Fig. III.A-2.
The LED driver is a modified copy of the voltage follower used for the tracking chambers in E802. The circuit layout is shown in Fig. III.A-3 for each follower. There are a total of 12 voltage followers in parallel, which share a common pulser input. Each voltage follower drives 40 LED's in parallel. Fig. III.A-4 shows a schematic flow chart for the LED system. The 40 Molex connectors are connected to one of 12 followers, and each Molex connector has two wires going to the LED leads at the back face of the NMA module. The power supply for the LED driver is a 3-A, dual-output, ±15-V, linear power supply. The current limit was estimated as follows: If the LED has a threshold voltage of 1.8 V, and normally 2 V are used for pulsing the LED, then the current going through each LED is about 4 mA. Therefore, each follower should supply 160 mA of current to the load of 40 LEDs, and about 2 A for the entire LED system.
Figure IV.A-3. Circuit diagram of the voltage follower.
Figure V.A-4. A schematic flow chart for the LED system.

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III.B The E866 Calibration: Antiprotons and the NMA

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1. Proposed Antiproton Measurements

Many studies in relativistic heavy-ion collisions have attempted to look for specific signatures for quark matter formation \(^1\). Amongst the signatures proposed for the detection of a quark-gluon plasma is the enhanced production of antibaryons \(^2\), \(^3\), \(^4\). Since it annihilates with large probability in a baryon rich environment \(^5\), the antiproton has been suggested as a sensitive probe of hot and dense hadronic matter being created in heavy-ion collisions. Several experiments at the Brookhaven AGS have measured the spectra of antiprotons with O and Si beams at AGS energies \(^6\)-\(^12\). In the E859 experiment, the antiproton yields, the transverse momenta \(M_t\) spectra, and the rapidity density distributions were all measured with improved statistics as compared with the E802 data. Furthermore, the first antilambda yields were measured, and it was found that a large portion (~63\%) of antiprotons come from antilambda decay. However, a clear picture of how antiprotons are produced or annihilated has not yet emerged \(^7\). Theoretical studies of antiproton production at AGS energies using relativistic quantum molecular dynamics (RQMD) \(^13\) and a relativistic cascade code ARC \(^14\) have been used as aids to describe available data. These models attempt to follow the space-time development of the nucleus-nucleus collision, and the production and annihilation of antiprotons. The RQMD is able to describe available data by relying on the enhanced production of antiprotons, followed by the annihilation of a large fraction of the produced antiprotons, Conversely ARC describes the data by producing less antiprotons initially, but the annihilation of antiprotons is "screened" in the high density environment of the collision on account of collisions with mesons. It is then of great interest to study the antiproton yields from the E866 experiment with the Au + Au collisions. Particularly,
detection of any asymmetry of the final antiproton yields relative to the collision reaction plane could be useful in determining how much antiproton absorption is occurring in the spectator material. The proposed study for the E866 experiment is to utilize the Henry Higgins (HH) spectrometer and the Forward Spectrometers (FS) to measure the antiproton yields, $M_t$ spectra, and the rapidity density distributions was done in E859. The important addition in the current E866 experiment is using the NMA to define the reaction plane to search for any asymmetry in the antiproton yields.

Table 1 shows the calculated ARC results for the antiproton yields at different HH angle settings with a minimum bias and central trigger.

<table>
<thead>
<tr>
<th>HH angle setting</th>
<th>pbars/hour (minimum bias)</th>
<th>pbars/hour (central)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$14^\circ$</td>
<td>17</td>
<td>52</td>
</tr>
<tr>
<td>$24^\circ$</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>$34^\circ$</td>
<td>1</td>
<td>3.7</td>
</tr>
<tr>
<td>$44^\circ$</td>
<td>0.6</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table III.B-1. Predicted number of antiprotons per hour as detected by HH at different angle settings with the minimum bias and central trigger, calculated from ARC.

2. Preliminary NMA Data

The NMA data to be shown here were taken in fall 1994 at the Brookhaven AGS using the Au beam at 10.6 A GeV/c on a stationary Au target. Fig. III.B-1 shows the schematic projective view of the NMA modules on the vertical plane at the target position. The beam direction in the drawing is going out of plane. The NMA modules are grouped as rings with a sector for every module; the module number is chosen to have the ring number followed by the sector number. The ring number is related to the $\theta$ angle and the sector
number is related to \( \phi \) angle. For example, module number 901 means the module is located in ring number 9 and sector number 1. The unmarked module positions are empty. The HH is located on the \( \phi = 0^\circ \) side and the FS is on the \( \phi = 180^\circ \) side. The \( \theta \) angle is defined from \( 0^\circ \) (upstream) to \( 180^\circ \) (downstream) with increasing ring numbers, and the \( \phi \) angle is defined from \( 0^\circ \) (HH side) counterclockwise to \( 180^\circ \) (FS side) from top then to \( 360^\circ \) (HH side) from the bottom. The gaps between neighboring modules in \( \theta \) and \( \phi \) are subtracted and hits are randomized within each module. For simplicity, the data shown here are counting from ring number four and above. Figs. III.B-2 and III.B-3 show histograms of the \( \theta \) and \( \phi \) angular distribution, respectively, with a 10 degree bin size, and for a total of 300 events.

![Diagram of NMA modules](image)

Figure III.B-1. A projective view of the NMA modules on the vertical plane at the target position. A module which is marked with an "x" is
active, otherwise the position is empty. The beam direction is going out of the paper.

Figure III.B-2. Histogram of the $\theta$ angular distribution for 300 events.
The algorithm for finding the reaction plane is still under development. As the first attempt, a cosine function with three parameters ($P_1 \cos(\phi) - P_2 + P_3$) is being used to fit the $\phi$ histogram event by event. The selected ranges for fitting are $35^\circ < \phi < 145^\circ$ and $215^\circ < \phi < 325^\circ$. The parameters $P_1$ and $P_3$ are allowed to vary from 0 to 100, and $P_2$ is allowed to vary from $0^\circ$ to $360^\circ$. Figure III.B-4 displays the single event $\phi$ histograms for different multiplicities. After fitting a total of 300 individual events, the three fitting parameters $P_1$, $P_2$ and $P_3$ are show in Fig. III.B-5. The top panel is $P_1$, the middle panel is $P_2$, and the
bottom panel is $P_3$. The angle parameter $P_2$ distribution over the 300 events has a minimum at around channel 250. Figure III.B-6 is a 2-dimensional plot of $P_1$ vs. $P_2$. Again, the density of points is lower at $P_2 \sim 250$. The algorithm looks hopeful, but many problems need to be solved and this line of work is still underway.

Figure III.B-4. Single event $\phi$ histograms with different multiplicities ($m$).
Figure III.B-5. The histograms of the first fitting parameter $P_1$ (top panel), the second fitting parameter $P_2$ (middle panel), and the third fitting parameter $P_3$ (bottom panel), for a total of 300 individual events.
Figure III.B-6. The two dimensional histogram of the fitting parameters $P_1$ vs. $P_2$, for a total of 300 events.

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III. C PHOBOS Magnet Work

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The magnet work done for PHOBOS was described extensively in last year's Progress Report\(^1\). The magnet design was changed and the new configuration was entered into the program TOSCA for the field calculations. These were included in the PHOBOS Conceptual Design Report\(^2\) (CDR). The magnetic field calculations for the current CDR approved by the RHIC committee are the same as shown these in last year's Progress Report. An introduction manual was written to document how the TOSCA calculation was done for the PHOBOS magnet design work. It is a tutorial manual to assist other members in the collaboration that may need to use the TOSCA program for further PHOBOS magnet work.

References


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V. Publications 1993-1995

A. Articles in Refereed Journals


4. Mass and Charge Distributions in Fe-Induced Reactions, H. Madani, A.C. Mignerey, A.A. Marchetti, A.P. Weston-Dawkes, W.L. Kehoe, F. Obenshain, Physical Review C (accepted for publication)


B. Conference Proceedings


C. Published Abstracts


VI. Acknowledgments

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The experimental programs described in this report are integral parts of several collaborations. At MSU we are working with the $4\pi$ group of Gary Westfall, and have assisted in an experiment with Michael Thoennessen. At the AGS and MIT we are a part of the E866 collaboration and the PHOBOS detector collaboration. The support of our colleagues within these collaborations is greatly appreciated.
APPENDIX A

Study of the Decay of Hot Nuclei Formed in $^{139}\text{La}$-Induced Reactions at $E/A = 45\text{ MeV}$ by a Hybrid Dynamical-Statistical Calculation

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ABSTRACT

The reactions $^{139}\text{La} + ^{27}\text{Al}$ and $^{139}\text{La} + ^{nat}\text{Cu}$ at $E/A = 45\text{ MeV}$ have been modelled by combining a Landau-Vlasov dynamical model calculation with a statistical sequential binary decay model code. For the reaction $^{139}\text{La} + ^{27}\text{Al}$, the major features of the experimental data are adequately described by the model calculations. These features include the inclusive fragment cross sections and the total charge and source velocity distributions of multiple fragment events. Other finer features, such as charge-Dalitz plots and the branching ratios between events of different multiplicity, are not reproduced by the calculation. The failure of the calculations is even greater for the reaction $^{139}\text{La} + ^{nat}\text{Cu}$, in which only the inclusive source velocity distributions are reproduced. Because source velocity can be a measure of how much of the target is incorporated into the projectile in inverse kinematics reactions, this indicates that the earliest stages of the reaction, covered by the dynamical calculation, are adequately characterized by the model. It is the latter stages of the reaction, when fragments are emitted, where the model calculation particularly fails. However, there are some indications that statistical decay of the source has occurred, whether by sequential binary decays or some type of prompt multifragmentation.

I. INTRODUCTION

In recent years, the emission of complex fragments ($Z > 2$), also known as intermediate mass fragments (IMF's), in intermediate energy heavy-ion reactions has been extensively studied, both experimentally$^{(1-16)}$ and theoretically$^{(17-24)}$. The emission of several (more than two) complex fragments has been loosely dubbed
"multifragmentation". However, many fragment final states can arise both by sequential emission\(^{(1-4)}\) and by simultaneous breakup of a highly excited nucleus\(^{(18)}\). Differentiating between these competing mechanisms experimentally can be difficult, which leads to the use of decay simulations\(^{(3)}\) and reaction model calculations\(^{(25-28)}\) to help interpret the experimental data. Additionally, the same data can be interpreted by calculations from competing reaction models\(^{(1,5,20)}\), further muddying the picture. Aside from differentiating between sequential and simultaneous decay mechanisms, research has concentrated on the roles of dynamics and statistics in the mechanism of complex fragment emission, and any possible interplay between the two\(^{(2,12,25-28)}\).

In order to explore the competition between sequential binary decay and "true" prompt multifragmentation, and the interplay between statistics and dynamics, it is necessary to model reactions in a way that both dynamical effects and statistical decay are treated. Unfortunately, dynamical models cannot account for statistical emission of fragments\(^{(29)}\), and statistical decay models cannot include the dynamical effects that can lead to complex fragment emission before equilibrium is reached. One way around this problem is to try to include fluctuations in the dynamics\(^{(24,30,31)}\). Another method is to use a dynamical model to follow the early stages of the reaction, until equilibrium is reached. A statistical decay model can then be applied to follow the decay of any hot primary fragment or fragments present. In this way, a better understanding of the different processes that can lead to complex fragment emission in heavy-ion collisions at intermediate energies can be attained. These "hybrid" models have gained popularity in helping to interpret experimental data\(^{(32)}\).

One of the more prevalent methods of determining the dynamical evolution of the reacting system is to solve the Landau-Vlasov (LV) (also known as the BNV, BUU or VUU) equation\(^{(33-37)}\). These models are all essentially semi-classical versions of time-dependent Hartree-Fock calculations, and have the form:

\[
D_t f = \frac{df}{dt} + [f, H] = I_{\text{coll}}. \tag{1}
\]

This equation includes a term for the time evolution of the mean fields of the colliding nuclei, along with the collision integral \(I_{\text{coll}}\), which treats individual nucleon-nucleon collisions. The solution of the LV equation can be done in a full ensemble method\(^{(38)}\), in which a substantial number of test particles per nucleon is used to map the phase space occupation. A sufficient number of particles needs to be used to reduce the effects that are due to numerical fluctuations caused by Monte-Carlo sampling of the phase space. Since most results are quantities averaged over the entire phase space, numerical fluctuations should not cause large problems in the interpretation of the calculations\(^{(38)}\).

In the following work, the LV calculations are performed over a range of impact parameters, and the results are then used to parameterize inputs for the GEMINI
statistical decay code\(^{39}\). In GEMINI, all possible binary decays are included, and decay chains are followed until the final nuclei no longer emit particles. The results of the GEMINI calculation are then filtered through the geometry and velocity acceptances of the detector system\(^{40,41}\) to allow for comparison between the calculated and the experimental results.

II. PERFORMANCE OF THE MODEL CALCULATIONS

Performance of the dynamical model calculations requires the determination of several parameters in the model- the number of test particles per nucleon, their widths in position and momentum spaces, the compressibility of the nucleus, and the nucleon-nucleon cross section. A Skyrme interaction that yields a compressibility constant of 200 MeV was used. Widths in position and momentum spaces were chosen to be 1.444 fm and 0.346 MeV/c, respectively, which reproduce the binding energies and radii of the target and projectile within 20 percent. Stability of the systems at non-reacting (very large) impact parameters was achieved by using 40 test particles per nucleon, and the free nucleon-nucleon cross section with its energy and angular dependence was used.

The calculations were performed for the systems \(^{139}\)La + \(^{27}\)Al and \(^{nat}\)Cu at E/A = 45 MeV over a range of impact parameters and time steps of 10 fm/c, up to a time of 210 fm/c. In some cases, the dynamics were followed to longer times to attempt to verify certain features of the calculations. Once the dynamical calculations had been performed, a clusterization routine\(^{42}\) was used to determine the properties of any fragment(s) present in the calculated results. These properties include the charge, mass, excitation energy, angular momentum, emission angle, and source velocity of the fragment(s). It should be noted that these calculations can account for fast-fission, deep-inelastic reactions, participant-spectator-like reactions, and (possibly) multifragmentation, so there may be several fragments present at some, though not all, impact parameters. The clusterization routine also determines the energies and angles of particles (protons and neutrons) not included in any cluster.

In order to combine the dynamical calculations with the statistical decay model GEMINI, it is necessary to determine the time at which to end the dynamics and switch on statistical decay. In other words, at what time does the reacting system reach equilibrium? Because the dynamical model includes light particle emission at all stages of the reaction, the determination of this time is very important. If it is too early in the reaction, then equilibrium has not been reached and GEMINI is not applicable. If it is too late, then the "pre-equilibrium" stage will also include some emission of light particles after equilibrium, and the properties of the fragment(s) will not be correctly determined for application to GEMINI.
In order to determine the time at which to "freeze-out" the properties of the clusters and start the GEMINI calculations, the calculated mean energy of the light particles emitted was determined as a function of time. Figure 1 shows this quantity for the reactions $^{139}\text{La} + ^{27}\text{Al}$ and $^{nat}\text{Cu}$ at E/A = 45 MeV and b = 1 fm. The lines are to guide the eye. For the reaction $^{139}\text{La} + ^{27}\text{Al}$, the mean energy of the light particles decreases until about 90 fm/c, after which the energy is essentially constant. This change in the mean energy indicates the time at which equilibrium emission of light particles starts, and is consistent with previously published results of the same system at a higher energy\cite{25}. For the reaction $^{139}\text{La} + ^{nat}\text{Cu}$, the freeze-out time is longer, approximately 100 to 110 fm/c. This longer time may be due to the increase in the available energy for the reaction on the copper target. For larger energies, it should take more nucleon-nucleon collisions to thermalize the energy, thus more time. The fluctuations in the mean energy of the light particles at longer times in the reaction $^{139}\text{La} + ^{nat}\text{Cu}$ are due to oscillations in the density of the fusion residue, shown as a function of time in Figure 2. The composite system undergoes a compression-expansion stage, with density fluctuations that eventually damp out.

III. RESULTS AND COMPARISON TO EXPERIMENTAL DATA

A. The Reaction $^{139}\text{La} + ^{27}\text{Al}$ at E/A = 45 MeV

Contour plots of the time evolution of the density distribution of nucleons in space for the reaction $^{139}\text{La} + ^{27}\text{Al}$ at E/A = 45 MeV for a range of impact parameters are shown in Figure 3. In this figure, which is shown in the center-of-mass system, the projectile is at the left and the target is at the right at t=0. For the most central collisions, a hot fused system is formed. This system is highly deformed because the target and nucleus compress as they react and fuse. This distorted nuclear system will eventually reach thermal and shape equilibria and then decay statistically. At large impact parameters, the reaction is more reminiscent of deep-inelastic collisions, in which a short-lived, rotating dinuclear system forms and then the projectile- and target-like fragments reseparate. There can be considerable exchange of nucleons between the target and projectile, along with excitation of the fragments, which subsequently decay by the emission of light particles. The picture is less clear for the intermediate (b = 3 and 4 fm) impact parameters.

While at first glance at Figure 3 the reaction at b = 3 fm seems to be similar to the more central collisions, this may not be the case. The possibility of a fast-fission reaction mechanism\cite{43} at these impact parameters warrants a closer look at the density distribution of nucleons in space for these reactions. An extra-push model calculation\cite{44} indicates that for this reaction fast-fission should occur at $\ell$-waves corresponding to an impact parameter between 2 and 3 fm. To further study
the possibility of a fast-fission-like component in the dynamical calculations in this reaction, contour plots of the density of nucleons in space for various times at \( b = 3 \) fm are shown in Figure 4. It is clear that there are two regions of high density of nucleons for the reaction at this impact parameter at a time scale that is consistent with asymmetric fast-fission for systems at similar masses and energies\(^{43}\). The two regions of high density are still present at times of up to 300 fm/c\(^{46}\), but are washed out by evaporative effects at larger times.

In order to determine whether a fast-fission type of reaction is reflected in the experimental data, the properties (charge, mass, angular momentum, and excitation energy) of the system were determined at \( b = 3 \) and 4 fm by considering the hot source produced in the calculation to be a single hot nucleus and also by dividing in space between the two centers of density to form two hot fragments. The properties of the fragment(s) were then parameterized, including the information from the other impact parameters, and GEMINI calculations were performed. The experimental fragment cross sections \( \sigma(Z) \), and the cross sections calculated both with and without the fast-fission scenario are shown in Figure 5. It is clear that including fast-fission better reproduces the cross sections for fragments with \( Z \leq 20 \). The absolute magnitude of the distribution is reproduced by the calculations for a substantial fraction of the emitted fragments, and the general shape of the distribution is reproduced over the entire range of fragments studied. For the rest of the results concerning this reaction, the fast fission scenario will be used.

Figure 6 shows the total charge and source velocity distributions for the experimental data (solid line) and the model calculations (dashed line) for events with a complex fragment multiplicity \( n \) equal to 2 and 3. The calculated distributions were filtered through the detector angular and velocity acceptances to compare to the experimental data; the arrow is at the source velocity for complete fusion. For the \( n = 2 \) events, the peak in the total detected charge distributions \( Z_{\text{tot}} \) is well reproduced by the calculation, but the tail at low \( Z_{\text{tot}} \) is vastly underpredicted. This effect is most likely due to underestimating the number of \( n=3 \) events, because the low \( Z_{\text{tot}} \) tail is interpreted as due to \( n=3 \) events for which only two fragments were detected\(^{47,45}\). For \( n = 3 \) events, the model overpredicts the charge at the peak of the distribution and underestimates both the width and the tail. Virtually no \( n = 4 \) events were produced by the calculation. The model reproduces the peaks in the \( V_S \) distributions for both the \( n = 2 \) and 3 events, but underestimates the tail of the distribution for the \( n=3 \) events. The double peak in the \( n = 2 V_S \) distribution is due to the abrupt change in the reaction mechanism between \( b = 2 \) and 3 fm from fusion (or incomplete fusion) to fast-fission. This leads to a discontinuity in the parameterizations of the properties of the fragment(s) at the \( \ell \)-wave of the transition.

Another way of characterizing reactions in which three complex fragments are detected is to construct a Dalitz plot of the atomic numbers of the three fragments. A schematic of a charge-Dalitz plot is shown in Figure 7. The scales in the figure
run from the edges of the triangle to the opposite vertex and have a value of $Z_i/Z_{\text{tot}}$, in which $Z_i$ is the charge of the $i$th fragment ($i = 1, 2, \text{ or } 3$) and $Z_{\text{tot}}$ is the total detected charge of the event. The fragments are randomized as to which is 1, 2, or 3. In addition, a minimum total detected charge of thirty is required to ensure good kinematic characterization of the event. In a charge-Dalitz plot, if the yield is concentrated at the vertices, then the event has one large and two small fragments. An event with three nearly equal sized fragments would show up in the center of the plot. Experimental and calculated charge-Dalitz plots are shown in Figure 8. The general trend of the experimental data, in which the charge-Dalitz plot is populated predominantly at the vertices, corresponding to asymmetric decays, is reproduced by the calculation. However, for the experimental data, the central region, corresponding to symmetric decays, shows some yield. On the other hand, the central region of the charge-Dalitz space for the model calculations shows a hole- or a lack of symmetric decays produced by the modelling of this reaction.

The agreement or disagreement between the experimental and calculated results can also be examined by determining the branching ratios of the multiple fragment events, shown in Table 1. It is apparent that the model vastly underpredicts the amount of $n = 3$ and 4 events. By comparing the results of Figures 5 and 6 with those presented in Figure 8 and Table 1, it is apparent the agreement between the experimental and calculated results depends on which observables are examined. While global features, such as the inclusive fragment cross sections and total charge and source velocity distributions of multiple fragment events have been adequately described by the calculations, finer features, such as the event branching ratios, have not.

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Table 1: Experimental and calculated proportions of multiple fragment events for the reaction $^{139}$La + $^{27}$Al at $E/A = 45$ MeV.

B. The Reaction $^{139}$La + $^{\text{nat}}$Cu at $E/A = 45$ MeV

Contour plots of the distribution of nucleons in space for the reaction $^{139}$La + $^{\text{nat}}$Cu for a series of impact parameters are shown in Figure 9. The evolution of the
mechanism with increasing impact parameter is similar to that for the reaction $^{139}$La + $^{27}$Al. However, for the most central collisions ($b = 1$ fm), a new mechanism may be occurring that is not seen for the more asymmetric system $^{139}$La + $^{27}$Al.

The density distributions of nucleons in space for this reaction at $b = 1$ fm are shown in Figure 10. At $t = 60$ fm/c, the system is hot and very compressed. As the system expands ($t = 100$ fm/c) fluctuations in the density distribution start to form. The fluctuations seem to produce clusters of nucleons in space ($t = 140$ fm/c). This might be the onset of some type of multifragmentation. However, these clusters do not separate but condense back into a highly distorted system by $t = 300$ fm/c.

Figure 9 also shows that for the central collisions the highly compressed system is approaching a disk-like shape. The formation of a disk of nucleons and the subsequent multifragmentation may be due to Rayleigh-Taylor-like surface instabilities\cite{46}. In these instabilities, multifragmentation occurs because of the interactions between the two surfaces of the disk. The disk breaks into spherical fragments which have a lower total surface energy than the disk. Other dynamical model calculations for reactions of similar systems and energies show the formation of bubbles, rings, or even donuts of nucleons in space\cite{46-49}. The formation of fragments in this reaction could also be related to spinodal decomposition of the system\cite{50}. This means that the initial compression-expansion process leads to a region of negative compressibility, in which the system is then unstable.

It is very difficult to determine the properties of the several fragments ($Z$, $A$, velocity, and angle) produced in the multifragmentation scenario for central collisions. Therefore, the central collisions were considered to proceed as in the reaction $^{139}$La + $^{27}$Al, in which a single hot source was formed, and the GEMINI inputs were then parameterized. The fragment cross sections were then determined and compared to the actual experimental cross sections, shown in Figure 11. By not requiring the source in central collisions to undergo fragmentation, the yield of the heavier fragments calculated by the model is higher than the experimental yield, while that of the lightest fragments is lower. A multifragmentation scenario in the model would cause a decrease in the cross section of the heaviest fragments and an increase in the cross section of the lighter products. The peak in the calculated cross section distribution can be attributed to the fission of the hot nuclear system that was formed in the central collision. It is clear that assuming that this source does not fragment before the application of the statistical decay model may not be valid. However, it is not possible to adequately treat the multifragmentation scenario by this model.

The disagreement between the calculated and experimental fragment cross sections in the reaction $^{139}$La + $^{nat}$Cu is in contrast to the agreement observed for the reaction $^{139}$La + $^{27}$Al, in which both the magnitude and the shape of the fragment cross section distribution were adequately described. This change in the results shows the evolution of possible mechanisms for the emission of complex fragments as the reacting system becomes more symmetric and heavier, and as the available energy increases.
To obtain a better understanding of where the calculations may be failing for the heavier target, the total detected charge and source velocity distributions of multiple fragment events were determined, and then compared to the experimental data after filtering through the detector acceptance, as shown in Figure 12. Surprisingly, the source velocity distributions for the n = 2 and 3 events are fairly well reproduced by the calculation. This is very similar to the results of the calculation for the reaction $^{139}$La + $^{27}$Al. In an incomplete fusion model of inverse kinematics reactions, source velocity is a function of the degree of fusion between the target and projectile. The agreement between the experimental and calculated source velocity distributions shows that the earliest stages of the reaction are effectively treated by the dynamical calculation. On the other hand, neither the peaks nor the tails of the $Z_{\text{tot}}$ distribution are reproduced by the calculation. For the n = 2 events, the experimental distribution is peaked at a much lower value than the calculation. Recalling that the modeling of this reaction did not use a possible multifragmentation scenario that may be occurring for central collisions, the lack of agreement between the calculation and the experiment could be interpreted as evidence that some type of multifragmentation is occurring for this reaction. The mechanism of this multifragmentation process is an important question that has not been answered by these calculations.

IV. DISCUSSION

The experimental results shown in this paper have been previously presented in the context of the systematics of $^{139}$La-induced reactions at intermediate energies\(^{(7,45)}\). In these reactions, there is an indication of statistical decay of highly excited nuclei, shown by the dependence of the multiple fragment event branching ratios only on the inferred excitation energy per nucleon of the decaying system, not on the projectile-target combination or the bombarding energy\(^{(7,46,51)}\). Chemical reactions often follow an Arrhenius rate law, in which the rate is proportional to $\exp(1/T)$, where $T$ is the temperature. In nuclear matter, $T$ is proportional to $E^{1/2}$. Thus, the statistical nature of the emission of many fragments can also be demonstrated if a plot of the natural logarithm of the ratio $P(n)/P(2)$ as a function of $1/\sqrt{E}$ is linear\(^{(52,53)}\), with $P(n)$ the probability of decay into $n$ fragments and $P(2)$ the probability of decay into two fragments. Such a plot is shown in Figure 13 for the reactions $^{139}$La + $^{27}$Al, $^{51}$V, $^{nat}$Cu, and $^{139}$La at $E/A = 45$ MeV\(^{(7,45)}\). These yields have not been corrected for efficiency as was done in reference 40. However, the main result of the efficiency correction was to change the magnitudes of the ratios, but not the general trend of the data\(^{(52,53)}\).

The data in Figure 13 are quite striking. Except for the reaction $^{139}$La + $^{27}$Al, the data for the different targets are both linear at the different multiplicities and overlap, indicating the possible role of statistical decay in the production of complex
fragments in these reactions. The high excitation energy bins for the $^{27}$Al target correspond to the tail of the source velocity distribution$^{(7,45)}$, and may not be well determined. The slope, related to the "potential energy" of the n-body system at freeze-out$^{(52,53)}$, is steeper for the $n = 4$ events than the $n = 3$ events, showing that the "barrier" for the production of four complex fragments is greater than that for the production of three.

The use of a hybrid dynamical-statistical calculation to help interpret experimental data has been used to study other systems with varying degrees of success. This approach seems to work best in studying the reactions of very asymmetric systems, such as the reaction $^{139}$La + $^{27}$Al at E/A = 45 MeV in this paper and at E/A = 55 MeV$^{(25,26)}$. These reactions have been shown to proceed by an incomplete fusion mechanism$^{(7,45,51)}$, with a source velocity between that of the projectile and the complete fusion product. For these reactions, the available energy in the center of mass is still fairly low, on the order of 3 - 5 MeV/A (depending on the degree of fusion) and the emitting source can be well characterized in terms of source velocity and total charge.$^{(7,45)}$ The study of the reaction $^{139}$La + $^{27}$Al at E/A = 55 MeV$^{(25,26)}$ reached similar results to those presented in this paper. The hybrid calculation reproduces experimental features such as the inclusive fragment cross sections and the total charge and source velocity distributions of multiple fragment events.

As the reacting system becomes more symmetric, the available energy increases, and the hybrid approach of dynamics plus sequential binary decay may no longer be applicable. Results similar to those presented in Figure 10 were also exhibited in the study of the reactions $^{139}$La + $^{63}$Cu at E/A = 55 MeV$^{(26)}$ and $^{131}$Xe + natCu at E/A = 45 MeV$^{(27,28)}$.

There is some agreement between the calculated results of the reaction $^{139}$La + natCu at E/A = 45 MeV presented in this paper and the reaction $^{131}$Xe + natCu at E/A = 45 MeV$^{(27,28)}$. Differences in the experimental results arise from differences in the detector system thresholds and angular coverages for the two studies. In the Lanthanum-induced reaction, the detector system covered angles close to the grazing angle (1.7°), so the heaviest fragments were easily detected$^{45}$. Thus, the predominant decay channel detected for the n=3 events was to one heavy and two light fragments$^{(45)}$. On the other hand, the study of the reaction $^{131}$Xe + natCu showed symmetric fragments in the n=3 events$^{(27,28)}$. However, the study of the Xe-induced reaction did not include fragments emitted at laboratory angles less than 5.5°. In reverse kinematics reactions, the heavy fragments are emitted at small laboratory angles. By not including fragments emitted at small laboratory angles, the asymmetric decay channels have not been properly accounted for, leaving only the more symmetric decays.

In the comparison between experimental and calculated results in the Xe-induced reaction, similar results to those presented in this paper were shown. These results included a failure to reproduce the magnitudes of the inclusive fragment cross sections
and the total charge distributions of the $n = 3$ events. However, the general shape of the fragment cross sections was reproduced. By using a statistical multifragmentation calculation linked to the dynamical one, better agreement between the experimental and calculated results was achieved.²⁸ By showing that the source velocity distribution for the $n=3$ events in the Lanthanum-induced reaction is adequately reproduced, this paper has explicitly shown that the early stages of the reaction are well characterized by the dynamical calculation. It is only at the fragment emission stage that this calculation used fails to reproduce the results sensitive to the mechanism of fragment emission.

V. SUMMARY

The present study shows the importance of determining the source velocity distribution of the multiple fragment events. For the reaction $^{139}$La + $^{nat}$Cu at $E/A = 45$ MeV, for which there is not a match between fragment cross sections or total charge distributions determined experimentally and by the model, there is good agreement in the source velocity distributions. In the geometric incomplete fusion model, the source velocity is a measure of the degree of fusion between the target and projectile. In other words, the source velocity is a variable sensitive to the reaction dynamics. The agreement between the experimental data and the calculated data in the determination of the source velocity for this reaction shows that the failure in the reproduction of the other experimental observables, such as the fragment cross sections, is due to a poor description of the fragment emission stage, whether due to a dynamical disassembly of a highly excited source or to limitations of the sequential binary decay calculation.

The results of Figure 13 and Reference 51 suggest that complex fragments are emitted in a statistical way, but figures 11 and 12 show the limitations in GEMINI in handling reactions of heavier systems (higher available energy) after a dynamical calculation prepares the decaying system. However, for reactions involving the more symmetric system, the model is able to reproduce the source velocity distribution, which is the experimental observable that depends on the dynamics of the reaction. The observables that depend on the break-up phase were only reproduced for the more asymmetric system studied. Regardless, this paper has shown the strength of the dynamical portion of the hybrid model used and the weakness of the statistical portion in attempting to explain complex fragment emission in reactions of other than highly asymmetric systems.

ACKNOWLEDGMENTS

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(d) Present Address: GANIL BP 5027 F14021 CAEN, France.

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La+X $E/A=45$ MeV
mean energy of light particles
$b=1$ fm

![Graph showing energy of light particles as a function of time.]

FIG. 1. Calculated mean energy of light particles emitted in the reactions $^{135}\text{La}$ + $^{27}\text{Al}$ (diamonds) and $^{nat}\text{Cu}$ (triangles) at an impact parameter of $b = 1$ fm as a function of time. The lines are to guide the eye.
FIG. 2. Density of the composite system (in fm$^{-3}$) as a function of time for the reaction $^{139}$La + $^{nat}$Cu at $b = 1$ fm. The line is to guide the eye.
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FIG. 3. Contour plots of the distribution of nucleons in space as a function of time for the reaction $^{139}$La + $^{27}$Al for several impact parameters. The time steps are in units of fm/c, and the impact parameters (b) are in units of fermis.
FIG. 4. Contour plots of the distribution of nucleons in space for the reaction $^{139}\text{La} + ^{27}\text{Al}$ at $b = 3$ fm and times of 120, 140, 160, and 180 fm/c. $Z$ is the beam direction; $Y$ is the out-of-plane axis.
FIG. 5. Experimental (diamonds) and calculated (circles and squares) fragment cross sections for the reaction $^{139}\text{La} + ^{27}\text{Al}$ at $E/A = 45$ MeV. The circles are for the scenario not including fast-fission; the squares include fast-fission. The error bars indicate the uncertainty associated with the fitting procedure of the inclusive angular distributions.
FIG. 6. Experimental (solid line) and calculated (dashed line) total detected charge and source velocity distributions for $n = 2$ and 3 events for the reaction $^{139}\text{La} + ^{27}\text{Al}$ at $E/A = 45$ MeV. The arrow is at the source velocity for complete fusion between the target and projectile.
FIG. 7. Schematic diagram of a charge-Dalitz plot.
FIG. 8. Experimental (top) and calculated (bottom) charge-Dalitz plots for the reaction $^{139}$La + $^{27}$Al at E/A = 45 MeV.
$45 \text{ MeV/u La} + \text{Cu}$

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FIG. 9. Same as Fig. 3 for the reaction $^{139}\text{La} + \text{nat Cu}$. 
FIG. 10. Distribution of nucleons in space for the reaction $^{139}$La + $^{nat}$Cu at $b$ = 1 fm and times of 60, 100, 140, and 300 fm/c. $X$ is the in-plane axis; $Y$ is the out-of-plane axis.
FIG. 11. Experimental (diamonds) and calculated (squares and stars) fragment cross sections for the reaction $^{139}\text{La} + ^{\text{nat}}\text{Cu}$ at $E/A = 45$ MeV. The calculated cross sections are for two different values of the freeze-out time. The error bars are the same as in Fig. 5.
FIG. 12. Same as Fig. 6 for the reaction $^{139}$La + $^{nat}$Cu at E/A = 45 MeV.
FIG. 13. Natural logarithm of the ratio $P(n)/P(2)$ as a function of $1/E^{*1/2}$ for the reactions $^{139}\text{La} + ^{27}\text{Al}$, $^{51}\text{V}$, $^{nat}\text{Cu}$, and $^{139}\text{La}$ at $E/A = 45$ MeV.
APPENDIX B

Mass and Charge Distributions for the Reaction

\[ {}^{40}\text{Ca} + {}^{209}\text{Bi} \text{ at } 600 \text{ MeV} \]

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Abstract

The charge and mass of the projectile-like fragments produced in the 15-MeV per nucleon \(^{40}\text{Ca} + ^{209}\text{Bi} \) reaction were determined for products detected near the grazing angle. Neutron number-charge \((N - Z)\) distributions were generated as a function of the total kinetic energy loss and parameterized by their centroids, variances and correlation coefficients. Though the initial system is very asymmetric, after the interaction, a drift of the charge and mass centroids towards further asymmetry is observed. The production of projectile-like fragments is consistent with a tendency of the projectile-like fragments to retain the projectile neutron-to-proton ratio \(< N > / < Z > \approx 1\). The correlation coefficient remains well below 1.0 for the entire range of total kinetic energy lost. Predictions of two nucleon exchange models, Randrup’s and Tassan-Got’s, are compared to the experimental results. The models are not able to reproduce the evolution of the experimental distributions, especially the fact that the variances reach a maximum and then decrease as function of the energy loss. This behavior supports the hypothesis that some form of projectile-like fragmentation or cluster emission is perturbing the product distribution from that expected from a

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damped mechanism.

1. Introduction

A significant boundary in the description of heavy ion-collisions is the Coulomb and centrifugal barriers. When two nuclei approach each other, they must overcome these barriers to make contact. At convergent energies slightly higher than the Coulomb and centrifugal barriers, the product nuclei, in spite of the large energy conversion from relative kinetic to excitation energy, will preserve the binary character of the initial system. These kinds of reactions are termed damped or deep-inelastic. Even though there have been extensive studies of these reactions, several aspects are still not completely understood. Among the open questions are two that are crucial for the understanding of the reaction mechanisms: how the excitation energy is divided between projectile- and target-like fragments and the direction of the drifts of the product distribution in the mass-charge plane.

It is commonly accepted that the transformation of kinetic energy and angular momentum into excitation energy and intrinsic spin of the products is caused by nucleon exchange between the two reaction partners\textsuperscript{1,2}; however, recent experimental evidence suggests that nucleon transfer alone may not account for all of the excitation energy produced in these types of reactions\textsuperscript{4–6}. Energy dissipation may be due to the excitation of collective surface and giant resonance modes\textsuperscript{3}. For the case of the drifts of proton and neutron number, it has been suggested that they are driven by the static potential created by the binuclear potential\textsuperscript{11}, which would result in transfer of nucleons to produce more symmetric systems. Nevertheless, ambiguous tendencies have been observed. The study of the reaction $^{58}Ni$ on $^{197}Au$ at 887 MeV\textsuperscript{10} resulted in agreement between the evolution of the drift and the predictions of the potential energy surface (PES) gradient only when equal excitation energy division between the reaction components was assumed. On the other hand, in the experiments $^{64}Ni$ on $^{238}U$ at E/A = 8.3 MeV\textsuperscript{6}, $^{56}Fe$ on $^{165}Ho$ at 672 MeV\textsuperscript{31}, and $^{35}Cl$ on $^{209}Bi$ at 528 MeV\textsuperscript{30}, the drift of the average proton and neutron numbers($<Z>$ and$<N>$) does not follow the direction that minimizes the potential of the composite system.

In order to study this behavior, a quantitative analysis of the N-Z plane must be made, and characteristic parameters have to be deduced. The resemblance of the spectra to three
dimensional binomial distributions permits description in terms of the centroids (approximated by the averages $< N >$ and $< Z >$), the variances $\sigma_N^2$ and $\sigma_Z^2$, and the correlation coefficients $\rho_{NZ}$ of the distributions\textsuperscript{4}. The drifts in $< Z >$ and $< N >$ are expected to be dominated by the gradient of the binuclear potential created during the interaction time; whereas the values of the distribution variances are influenced by the multiplicity of the stochastic interchange of nucleons. On the other hand, the correlation coefficient indicates the degree of correlation in proton and neutron number for the nucleons exchanged to produce the detected projectile-like fragments. Generally, $\rho_{NZ}$ tends to exhibit a smooth transition from uncorrelated neutron-proton exchange at values of energy loss close to zero, to correlated exchange for larger values of energy loss\textsuperscript{6}. Finally, the $N/Z$-equilibration is another feature of damped reactions that is usually studied. It is generally found that the $< N > / < Z >$ values for the projectile-like fragments, as a function of the energy loss, tend to the $N/Z$ of the composite system\textsuperscript{7,8}.

To test the validity of the physical ideas used to describe the reaction mechanisms of the deep-inelastic collisions, several models have been developed. Some of these models are based on the concept of energy dissipation by collective nuclear modes\textsuperscript{12–15}, and others are supported by the hypothesis of stochastic exchange of nucleons between the colliding ions\textsuperscript{16–20}. Given that the distributions predicted by the models often describe the emitted fragments in their exited state after the interaction, and that the experimental data are from secondary fragments, evaporation corrections usually have to be taken into account to make comparisons between experimental data and model predictions.

Generally, the collective models lack consistency and completeness. Because of their restriction to particular aspects of the reaction mechanisms and their dependence on relevant features by auxiliary theories, it is difficult to judge the precision of the basic concepts that these models propose\textsuperscript{21}. Over all, the nucleon-exchange models are better able to reproduce the characteristics of the damped reactions. Based on stochastic transport theories, most of these models predict the evolution of the N-Z centroid distributions, as well as their variances, in a mode suitable for comparison with experiments.

In this work, the secondary mass and charge distributions of the projectile-like fragments of the system $^{40}$Ca on $^{209}$Bi at $E/A = 15$ MeV were measured. The proximity of the de-
tection array to the grazing angle and the wide range of energy loss produced, permits the study of the aspects of interest for characterization of the deep-inelastic reaction component. The experimental set-up and the details of the data acquisition and reduction are given in Sec. 2, which is followed by a presentation of the experimental results and the evaporation corrections in Sec. 3. The comparison to model predictions and other systems is made in Sec. 4. Finally a summary and the final conclusions are given in Sec. 5.

2. The Experiment and Data Reduction

The experiment was performed at the Holifield Ion Research Facility (HHIRF) of Oak Ridge National Laboratory. A 598-MeV $^{40}$Ca beam was used to produce the reaction on a self-supporting $^{209}$Bi (1 mg/cm$^2$) target. The beam charge current was kept between 10 and 15 nA during the experiment. The projectile-like fragments were detected at 9.35° using time of flight (TOF) to identify their mass number and a $\Delta E - E$ system to deduce their atomic number and kinetic energy.

The experiment was performed in a 30.5-cm diameter scattering chamber connected to a TOF arm. The start and stop detectors were transmission-type parallel plate avalanche counters (PPAC). In addition to timing information, the stop PPAC was used to provide position in the detection plane. The $\Delta E - E$ energies of the projectile-like fragments were measured using a gas ionization chamber, which was segmented by four nodes. The data were collected using the CAMAC data acquisition system$^{22}$, and the off-line data analysis was performed using the program LISA$^{23}$. This study was part of a series of similar experiments; a detailed description of the set-up may be found in the references 30 and 31.

The kinetic energy calibration of the projectile-like fragments was performed using elastic events. The gains of the first, second and fourth elements of the ionization chamber $\Delta E_i$ $i=1,2,4$ were normalized to that of the third element $\Delta E_3$. The absolute calibration was fixed by equating the sum of the relative energies from the four $\Delta E$ elements to the energy computed with STOPX$^{23}$, a kinematics program. It was found that the measured $\Delta E$ energies had a dependence on the position where the projectile-like fragments (PLFs) hit the detector surface ($X - Y$) perpendicular to the beam direction ($Z$). Due to this dependence, corrections to the $\Delta E$ signals, and thus to the total energy ($E_{tot}$), were made via a matrix
correction.

The values of the TOF also had an $X - Y$ dependence, so a matrix correction method was again employed. The resolution for the total energy and time of flight after corrections was calculated by the full width at half maximum (FWHM) for the elastic peak, giving $\delta E_{\text{tot}} \simeq 12.6$ MeV and $\delta TOF \simeq 849$ ps.

The atomic number identification was done following the $\Delta E - E$ technique. The series of curves ($Z$-curves) from the $\Delta E$ vs. $E$ plane were mapped until the charge values were independent of the fragment energy. The second element of the ionization chamber was used as the $\Delta E$, so events that stopped in the first or second element of the ionization chamber were not considered for the data analysis, setting a cut off in the energy of 10 MeV for Ca products. The calibration was set by a polynomial fit based on the $Z$ of Ca. The resolution achieved for the charge using the FWHM of the elastic peak was $\delta Z \simeq 0.36$ charge units.

Since the mass of a projectile-like fragment is proportional to its kinetic energy times the square of the time of flight, the determination of the mass is, in principle, a simple task. However, due to the resolution obtained for the time of flight, the elastic mass resolution was about 1.3 mass units. In consequence, the centroid positions of the mass spectrum could not be established with certainty. To solve this problem, a deconvolution method was used to intensify and identify the mass peaks$^{24}$. By setting a window for each $Z$, projections on the mass spectra were made for energy intervals of about 15 MeV. The position of the mass peaks was then established by deconvolution of the spectra, and the mass number was calibrated by a quadratic function using the elastic peak as absolute reference.

Once the mass number, the charge, and the kinetic energy of the secondary projectile-like fragments were established, the computation of the total kinetic energy lost (TKEL) in the reaction was performed$^{30,31}$. Due to the particle evaporation of the excited primary products, the measured mass differed from the primary mass and it was necessary to introduce a correction to the mass of the measured PLF’s. Two possibilities were considered for the excitation energy parameter when running the evaporation corrections. In the first case, it was supposed that there was an equal energy division of the total available excitation energy $E^{*}_{TOT}$ between the PLF and the TLF ($E^{*}_{PLF} = E^{*}_{TLF}$). In the second case, the
two fragments were assumed to reach statistical equilibrium; therefore $E^{*}_{TOT}$ was divided between the reaction fragments in proportion to their mass ratios, as given by the equation

$$E^{*}_{PLF} = E^{*}_{TLF} \frac{M_{PLF}}{M_{TLF}}.$$  \hspace{1cm} (1)

For each event, the procedure to correct the TKEL was done using the iteration method described extensively in other studies\textsuperscript{30,31}, with the difference that the evaporation correction was performed, not only by incrementing the mass of the secondary PLF ($\delta M$), but also by increasing its charge value ($\delta Z$).

The evaporation functions $\delta M = \delta M(Z, M, E^{*}_{PLF})$ and $\delta Z = \delta Z(Z, M, E^{*}_{PLF})$ were calculated using the Projection Angular-Momentum Coupled Evaporation Code PACE II\textsuperscript{25}. Except for the spin and the angular momentum, the default values of the program were used. Calculations have shown that the amounts of charge and mass evaporated are not strongly dependent of the spin values\textsuperscript{26}. Therefore, only an estimate of the initial spin of the primary PLF was used. On the other hand, the angular momentum of the PLF was computed assuming the classical sticking model, using a linear interpolation between its higher (grazing) and lower (fusion) limits\textsuperscript{26,28}.

After the TKEL corrections were performed, the N-Z distributions were generated by plotting the relative yield of projectile-like fragments as a function of their atomic and neutron numbers. Examples of the distributions for different cuts of total kinetic energy loss are displayed in Fig. 1. The N-Z plane is binned in cells of 0.20 X 0.18 neutron-charge units. The resemblance of the spectra to two-dimensional binomial distributions allows the parameterization of the N-Z plane in terms of centroids, variances and correlation coefficients\textsuperscript{6}. Employing consecutive 10- to 30-MeV bins of TKEL, moment analysis was used to calculate the parameters:

$$< Z > = \frac{1}{n} \sum_{i=1}^{n} Z_i,$$ \hspace{1cm} (2)

and

$$\sigma_Z^2 = \frac{1}{n-1} \sum_{i=1}^{n} (Z_i - < Z >)^2.$$ \hspace{1cm} (3)

Analogous equations were used for the neutron number. The correlation factor is given by
\[ \rho_{NZ} = \frac{\sigma_{NZ}}{\sigma_N \sigma_Z}, \]  

(4)

with

\[ \sigma_{NZ} = \frac{1}{n-1} \sum_{i=1}^{n} (Z_i - <Z>)(N_i - <N>). \]  

(5)

The resulting parameters are summarized in Tables I and II. Corrections for finite resolution derived from the experimental distributions were made. The uncertainties were calculated by the statistical error propagation method.

3. The Experimental Results

Graphical representation of the experimental parameters given in Tables I and II are shown in Figs. 2 and 3. The squares represent the correction of the TKEL assuming thermal equilibrium of the primary fragments and the circles represent the correction assuming equal energy division. If the error bars in the figures were smaller than the plot symbols they were not included. As can be seen in Fig. 2, the \(<Z>\) and \(<N>\) decrease with increasing energy loss for both cases of TKEL correction. The \(<N> / <Z>\) ratio remains around 0.9 for all the TKEL values, well below the ratio of the composite system (1.4). Around 170 MeV of TKEL, \(<Z>\) and \(<N>\) have lost about 5 Z and 7 N units, respectively. At low energy loss, the differences between the \(<Z>\) and \(<N>\) corresponding to the two types of TKEL correction are not significant; however, around 80 MeV of TKEL these differences increase slightly. The thermal equilibrium correction yields average values of charge and neutron number that are larger than those obtained by the equal energy division correction. This result is expected since the equal division of excitation energy will deposit more excitation in the lighter PLF for the same TKEL than will the thermal division, causing the increased effect of evaporation of the primary products.

The values of \(\sigma_{Z}^2\), \(\sigma_{N}^2\), and \(\rho_{NZ}\) are displayed in Fig. 3. The general development of these parameters as a function of the TKEL is similar for the equal excitation energy division and the thermal equilibrium corrections. The \(\sigma_{Z}^2\) increases to around 1.8 at TKEL of 200 MeV, then starts decreasing until about 320 MeV of energy loss. After this point the trend of the data is not clear. The maximum TKEL imposed by the spherical entrance channel Coulomb
barrier is 324 MeV. While products of different charge and deformation will have different
barriers, this value serves as a guide for the expected limit for the deep-inelastic products.
In general $\sigma^2_N$ follows the same pattern as $\sigma^2_Z$; however, its overall value and its rate of
increase are larger. Starting at around 2 for 20 MeV of TKEL, $\sigma^2_N$ reaches a maximum at
around 300 MeV of TKEL. Some kind of discontinuity is present around 50 MeV of TKEL,
especially for values of $\sigma^2_N$. It is interesting to note that this energy is in the same region
where a change in the slope of $<Z>$ and $<N>$ is observed.

The correlation coefficient $\rho_{NZ}$ is about 0.3 at 20 MeV of TKEL. It then increases to
its maximum of 0.6 at 180 MeV of TKEL, after which it decreases slowly. Although the
evaporation process disturbs the significance of the $\rho_{NZ}$, a small correlation between the
proton and neutron exchange may be inferred from the observed trend.

4. Model Calculations and Comparison to Other Systems

Two nucleon exchange models have been used to interpret the experimental results: Randrup’s model\textsuperscript{18}, and the Tassan-Got model\textsuperscript{19}. In these calculations, the physical picture of
the reaction is generally the same. The projectile and target approach each other along
Coulomb trajectories until they are within the interaction radius. At this point the system is represented as two Fermi-Dirac gases which exchange nucleons, energy and angular
momentum. After the interaction, the primary PLF and TLF follow separate Coulombian
trajectories and decay by evaporation into secondary residues.

The models utilize different approaches to determine the nucleon transference. For the
Tassan-Got model, when the participant nuclei are within range of their potential, a window
opens and stochastic transfers may occur. The transfer probability is calculated via a phase-
space integral, which, accounting for the Pauli blocking, incorporates the phase-space flux
term, the barrier penetrability and the occupation probabilities. This transfer produces the
variation of mass, charge, excitation energy and spin. On the other hand, in Randrup’s
model, the dynamical variables are determined by treating the interacting nuclei as two
spheres interconnected by a small cylindrical neck. The mean trajectory of these variables is
derived from the Lagrange-Rayleigh equations of motion and the fluctuations are found using
a Fokker-Planck type transport equation. Nucleon-nucleon collisions are mostly prohibited
by the Pauli principle, so the relative angular momenta are generated via the interaction of the system with the mean field.

These models were implemented in computer codes and, after appropriate treatment of their output\textsuperscript{31,30}, primary distributions were generated. The evaporation of the primary fragments was performed using the code PACE\textsuperscript{25} to obtain the secondary PLF's. The averages and variances were then calculated by moment analysis, as was done for the experimental data. The results are shown in Figs. 2 and 3, where the dashed lines and solid lines represent the model predictions using Tassan-Got's and Randrup's models, respectively.

A. Drift and Centroid Distributions

Once the primary nucleon averages are defined, the secondary distributions are a function of the excitation energy distributions predicted by each model. In Fig. 4 the average excitation energy deposited in the projectile-like fragments and its proportion to the total excitation energy, as a function of TKEL, are shown. The average excitation energy deposited in the primary PLF shows the same trend for both models, increasing steadily up to about 180 MeV, then decreasing from that value. From the values of the excitation energy ratio, it can be seen that the total excitation energy of the system, as generated by the models, is approximately equally shared between the two fragments during the first 180 MeV of TKEL. After that the system tends towards a thermalized state with increasing energy loss.

Given the average excitation energy predicted by Randrup's and Tassan's models, together with the measured secondary distributions, it is possible to use the iterative procedure described in Sec. 3 to reconstruct, from the measured experimental data, a reasonable approximation of the average properties of the primary distributions of the projectile-like fragments. The results of this computation are shown in Fig. 5, where nucleon drifts (average measured atomic and neutron number minus the projectile atomic and neutron number) and the $<N>/<Z>$ ratio of the primary fragments are plotted as function of the TKEL. The diamonds represent the reconstructed primary distributions using the excitation energy function supplied by Randrup's model. The corresponding distributions for Tassan-Got's model were not significantly different, so they are not shown in the this figure.
The drift values of the primary fragments shown in Fig. 5 indicate a net transfer of protons and neutrons from the projectile to the target. Up to 200 MeV of TKEL the proton transfer is small, with $< Z > - Z_p \simeq 2$. After this point the drift rapidly increases, with up to 8 protons transferred at 360 MeV of TKEL. Contrary to this evolution, the $< N > - N_p$ values decrease until up to 200 MeV of TKEL, where $< N > - N_p \simeq 4$, and then remain constant at around -6 neutrons. There is a corresponding change of sign of the slope of the variances (Fig. 2) also around 200 MeV. The bottom panel of Fig. 5 shows the evolution of the $N/Z$ ratio for the reconstructed primary distributions; as for the ratio of the secondary distributions, there is a tendency to maintain the $N/Z = 1$ of the projectile.

In Fig. 5 the model predictions corresponding to the nucleon drift values for the primary distributions are also displayed. In both Randrup’s and Tassan-Got’s calculations, the $< N > / < Z >$ evolution of the primary distributions exhibit a tendency of the system toward the neutron-to-proton ratio of the composite system. However, for each model this is achieved in different ways. In Randrup’s model a net neutron transfer to the PLF increases steadily with TKEL, until it reaches its maximum at around 300 MeV, while the charge flow between the PLF and the TLF is almost nil. On the other hand, Tassan-Got’s model produces a substantially smaller transfer of neutrons from the TLF to the PLF, compensated by a strong charge flow in the opposite direction. The evolution of the proton drift of the reconstructed primary distributions is closer to the prediction by Tassan-Got’s model; however, the proton drift is overpredicted by this model. In the case of Randrup’s prediction, both the neutron and proton drifts are overpredicted by the model.

The evolution in the N-Z plane of the projectile-like fragment centroids, is displayed in Fig. 6 for increasing values of TKEL. The centroids of the experimental distributions are represented by the circles for secondary and diamonds for the primary fragments. Starting at the injection point of the initial system, the lines show the predictions of Tassan-Got’s and Randrup’s models. The composite system $N/Z$, the $N = Z$ and the $\beta$ stability lines are also shown. The primary distributions predicted by both models evolve towards the composite system line. However, the path followed by Randrup’s prediction is driven by the mean field potential of the system and drifts toward symmetry. For Tassan-Got’s model, the path is the result of a localized interaction\textsuperscript{31}. It is clear that, for both models, evaporation
leads the distributions towards the valley of $\beta$ stability, while the data only approach the $N = Z$ line.

**B. Distribution Variances**

The model calculations for the secondary values of $\sigma_2^2$, $\sigma_N^2$, and $\rho_{NZ}$ are displayed in Fig. 3. Both models predict an increase of the variances with energy loss. For $\sigma_2^2$, Randrup's calculation diverges abruptly from the experimental results at the first few MeV of TKEL, increasing steadily up to 20 at around 320 MeV (off the scale of the figure). Tassan-Got's predictions also overestimate the experimental values, although not by much. The values for $\sigma_N^2$ better resemble the experimental distribution, however, both models underpredict the experimental results. It is interesting to note that both the $\sigma_2^2$ and $\sigma_N^2$ values obtained from Tassan-Got's model appear to saturate at energy losses corresponding to the maxima of the excitation energy deposited in the primary PLF (Fig. 4). Finally, the $\rho_{NZ}$ computed by Randrup's calculation increases steadily with TKEL. The evolution does not reflect at all the trend found for the data, underpredicting the experimental values up to 200 MeV of TKEL, after which it greatly overpredicts the same. On the other hand, Tassan-Got's prediction for $\rho_{NZ}$ is consistently larger than that calculated from the data, for all but the lowest values of TKEL. Although the calculation does level off, it does so at a much larger value of $\rho_{NZ}$ than the data shows.

It is instructive to compare the evolution of the variances in terms of the data from studies at similar energies. The following experiments were used: $^{37}$Cl + $^{209}$Bi at 15 MeV/n$^{30}$, $^{58}$Ni + $^{165}$Ho at 16 MeV/n$^{29}$, and $^{56}$Fe + $^{165}$Ho at 12 MeV/n$^{31}$. Since the deep-inelastic mechanism evolves within the range of the total available kinetic energy, the TKEL scale between different systems is not equivalent. To perform a direct comparison between the systems, a rescaling of the TKEL was performed.

According to the nucleon exchange model$^{21}$, the energy loss per nucleon exchanged is proportional to the square root of the relative kinetic energy ($T^{1/2}$), defined by

$$T^{1/2} = (E_{c.m} - TKEL - V_c)^{1/2},$$

(6)

where $E_{c.m}$ is the center-of-mass kinetic energy and $V_c$ is the Coulomb barrier. Therefore,
the initial relative kinetic energy of the system $T_0^{1/2}$ is

$$T_0^{1/2} = (E_{c.m} - V_c)^{1/2}. \quad (7)$$

It is possible to rescale the TKEL by introducing the variable $\tau$

$$\tau = 1 - (T^{1/2}/T_0^{1/2}), \quad (8)$$

defined in the interval [0,1] from zero total kinetic energy loss to the limit of the kinetic energy available imposed by the exit channel Coulomb barrier, in this work approximated by that for the spherical entrance channel.

Figures 7 and 8 display the values of $\sigma_N^2$ and $\sigma_Z^2$, respectively, as a function of $\tau$ for different systems. Also included in the figure is the $N/Z$ value for each composite system and, between parenthesis, the $N/Z$ of the correspondent PLF. For the systems $Cl + Bi$ and $Ni + Ho$, the entire possible range of available energy was not reported, probably due to a lack of statistics for higher values of $\tau$ (TKEL). The changes in magnitude of the variances for the different systems are remarkable, especially for the $\sigma_N^2$ values, which have maximum values ranging from 10 at $\tau$ around 0.6 units for the system $Ca + Bi$ to about 35 units for the $Ni + Ho$. The different ranges of the variances between systems as a function of $N/Z$ have also been studied by Souza et al.\textsuperscript{16} without any conclusive results.

The general trend of the $\sigma_N^2$ values (Fig. 7) is to increase as a function of $\tau$. However, the slope of this change shows a discontinuity, leading to a maximum for the $Cl + Bi$ system and to an inflection point for the other systems at around 0.3 units of $\tau$. The $\sigma_Z^2$ values (Fig. 8) seem to follow the same evolution, with an abrupt change in slope leading to maxima for the $Cl + Bi$ and $Ca + Bi$ systems, and to a clear inflection point for the other systems.

It is hard to give an interpretation of the evolution of the variances in the systems described, especially for the values where the neutron and charge variance begin decreasing. In the models discussed in this paper, an increase in the variances is correlated with an increase in the number of nucleons exchanged by the participant nuclei. The variances are expected to increase with increasing $\tau$ or TKEL (proportional to the interaction time). Following this picture, a decrease in the variance, correlated with a change of slope of the drift, may be interpreted as a reflection of a sudden decrease of nucleon flow. Furthermore, at around 0.4 $\tau$ units for $Ca + Bi$ and 0.18 units for $Cl + Bi$\textsuperscript{30}, the models indicate that
a transition from equal energy division to thermalization is also taking place, indication of some kind of saturation process for the deep-inelastic interaction. These observations could point to a different kind of interaction mechanism. A break up of the PLF is one possible explanation for these findings\textsuperscript{27}. In particular, the break-up via $\alpha$ particles would be consistent with the tendency of the system towards the $N = Z$ line.

5. Summary and Conclusions

The mass and charge distributions for the secondary PLF’s from the reaction $^{40}$Ca + $^{209}$Bi at $E/A=15$ MeV have been measured. The distributions have been parameterized in terms of the first and second moments as a function of the TKEL. Presuming evaporation mechanisms for correction of the TKEL, the primary distributions for the PLF have been reconstructed, using the division of the excitation energy between the participant nuclei as predicted by Randrup’s model. The evolution of the centroids, variances and correlation coefficients with the TKEL has been compared to the predictions from Tassan-Got’s and Randrup’s models. The trends of the charge and neutron variances have been compared to the variances of the distributions found in other experiments.

From the $<N>/<Z>$ experimental values, it is clear that there is a strong tendency of the PLF to maintain the neutron-to-proton ratio of the original projectile up to very high values of TKEL. As a result, an intense flow of protons can be observed from the PLF to the TLF, together with a small net transfer of neutrons in the same direction.

For the second moments, a small correlation between the charge and the neutron number is observed, in agreement with the Tassan-Got model. On the other hand, the variances show a peculiar tendency to decrease after reaching a maximum. The same behavior is found when comparing this evolution with the trends of the variances for other systems. This, together with an evolution of the centroids and drifts towards the $N = Z$ may be a sign that the system is driven not only by a deep-inelastic-like reaction mechanism, but may be also influenced by processes such projectile fragmentation or break up.

The recent surge in experimental techniques using $4\pi$ and forward angle detection systems seems ideal to confirm these findings and to study alternative mechanisms for damped reactions. The forward detectors would be able to characterize projectile-like break-up frag-
ments, and the detection of the target-like fragments using a $4\pi$ array would permit the reconstruction of the primary fragments.

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Fig. 1. Representative contour plots of the measured atomic number (Z) versus the measured neutron number (N) for four different bins of TKEL: 20-50 MeV, 100-130 MeV, 160-190 MeV and 300-320 MeV.
Fig. 2. Experimental and theoretical values of $<Z>$, $<N>$ and $<N>/<Z>$ corresponding to the secondary distributions of the projectile-like fragments. The circles and squares represent the experimental results after correcting the energy-loss scale, assuming equal and thermal division of the excitation energy, respectively. The theoretical results predicted by Randrup’s and Tassan-Got’s models are represented by lines. The dotted lines represent the theoretical calculations from Tassan-Got’s model and the solid lines Randrup’s predictions. The dashed horizontal lines represent the value of the $N/Z$ for the composite system.
Fig. 3. Experimental and theoretical values for the variances $\sigma_Z^2$ and $\sigma_N^2$ and the correlation factor $\rho_{NZ}$ corresponding to the secondary distributions of the projectile-like fragments. The circles and squares represent the experimental results after correcting the energy-loss scale, assuming equal and thermal division of the excitation energy, respectively. The theoretical results predicted by Randrup's and Tassan-Got's models are represented by lines. The dotted lines represent the theoretical calculations of Tassan-Got's model and the solid lines Randrup's predictions.
Fig. 4. Model calculations for the average excitation energy stored in the primary PLF (top) and percent ratio of the average excitation energy in the PLF to the average total excitation energy of the compound system (bottom) as a function of the TKEL. The dotted line refers to the Tassan-Got calculations, the solid lines represent Randrup’s model prediction, and the dashed horizontal lines represent limits of equipartition of excitation energy and thermal equilibrium between the two reaction partners.
Fig. 5. Reconstructed and theoretical nucleon drift (average atomic and neutron number minus the projectile atomic and neutron number) and $\langle N \rangle / \langle Z \rangle$ ratio for primary fragments, as function of the energy loss. The dashed and the solid lines represent primary distributions calculated with Tassan-Got's and Randrup's models, respectively. The dashed horizontal lines represent the value of the $N/Z$ for the composite system.
Fig. 6. Evolution of the nuclide distribution in the N-Z plane of the projectile like fragments as a function of TKEL. Experimental secondary distributions are represented by diamonds, reconstructed primary distributions are represented by circles. Primary and secondary distributions predicted by Randrup’s model are indicated by solid and dashed lines, respectively. The calculated primary and secondary distributions for Tassan-Got’s model are represented by dot-dashed and dotted lines, respectively.
Fig. 7. Experimental values for the variances $\sigma^2_N$ as a function of the parameter $\tau$, corresponding to the secondary distributions of the projectile-like fragments for the systems (from the top to the bottom) $^{37}$Cl + $^{209}$Bi at 15 MeV/n$^{30}$, $^{40}$Ca + $^{209}$Bi at 15 MeV/n, $^{56}$Fe + $^{165}$Ho at 12 MeV/n$^{31}$ and $^{58}$Ni + $^{165}$Ho at 16 MeV/n$^{29}$. Included in the figure is the N/Z value for each composite system and, between parenthesis, the N/Z of the correspondent PLF.
Fig. 8. Experimental values for the variances $\sigma_z^2$ as a function of the parameter $\tau$, corresponding to the secondary distributions of the projectile-like fragments for the systems (from the top to the bottom) $^{37}$Cl + $^{209}$Bi at 15 MeV/n$^3$, $^{40}$Ca + $^{209}$Bi at 15 MeV/n, $^{56}$Fe + $^{165}$Ho at 12 MeV/n$^3$ and $^{58}$Ni + $^{165}$Ho at 16 MeV/n$^3$. Included in the figure is the N/Z value for each composite system and, between parenthesis, the N/Z of the correspondent PLF.
TABLES

Table 1. The secondary centroids \(< Z >\) and \(< N >\), the variances \(\sigma_Z^2\) and \(\sigma_N^2\), and correlation factor \(\rho_{NZ}\), for the secondary distributions obtained from the reaction \(^{40}\text{Ca} + ^{209}\text{Bi}\) at 600 MeV. The energy loss scale TKEL has been corrected for evaporation assuming equal excitation energy division between the reaction fragments. The TLEL values of the center of the bin are quoted and reflect consecutive bin intervals.

<table>
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<th>TKEL</th>
<th>(&lt; Z &gt;)</th>
<th>(&lt; N &gt;)</th>
<th>(&lt; N &gt; / \langle Z \rangle)</th>
<th>(\sigma_Z^2)</th>
<th>(\sigma_N^2)</th>
<th>(\rho_{NZ})</th>
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<td>1.066±0.002</td>
<td>0.152±0.023</td>
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<td>0.259±0.035</td>
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<td>2.919±0.429</td>
<td>0.428±0.018</td>
</tr>
<tr>
<td>95</td>
<td>18.87±0.01</td>
<td>17.26±0.01</td>
<td>0.966±0.001</td>
<td>0.394±0.021</td>
<td>2.084±0.435</td>
<td>0.435±0.019</td>
</tr>
<tr>
<td>105</td>
<td>17.56±0.01</td>
<td>16.87±0.01</td>
<td>0.960±0.001</td>
<td>0.737±0.028</td>
<td>2.494±0.543</td>
<td>0.543±0.018</td>
</tr>
<tr>
<td>115</td>
<td>17.21±0.01</td>
<td>16.37±0.01</td>
<td>0.951±0.002</td>
<td>0.757±0.030</td>
<td>2.934±0.508</td>
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<td>15.26±0.01</td>
<td>0.931±0.001</td>
<td>1.135±0.025</td>
<td>4.118±0.525</td>
<td>0.525±0.013</td>
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<tr>
<td>165</td>
<td>15.38±0.01</td>
<td>14.21±0.01</td>
<td>0.924±0.001</td>
<td>1.479±0.036</td>
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<tr>
<td>200</td>
<td>14.36±0.01</td>
<td>13.08±0.01</td>
<td>0.911±0.002</td>
<td>1.260±0.043</td>
<td>6.109±0.454</td>
<td>0.454±0.017</td>
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<tr>
<td>240</td>
<td>13.06±0.02</td>
<td>12.00±0.02</td>
<td>0.919±0.003</td>
<td>0.965±0.059</td>
<td>6.513±0.385</td>
<td>0.385±0.023</td>
</tr>
<tr>
<td>280</td>
<td>11.69±0.04</td>
<td>11.49±0.05</td>
<td>0.983±0.008</td>
<td>0.585±0.091</td>
<td>7.077±0.288</td>
<td>0.288±0.037</td>
</tr>
<tr>
<td>320</td>
<td>10.20±0.10</td>
<td>9.37±0.11</td>
<td>0.919±0.021</td>
<td>0.706±0.185</td>
<td>4.289±0.406</td>
<td>0.406±0.110</td>
</tr>
<tr>
<td>360</td>
<td>9.55±0.38</td>
<td>8.40±0.56</td>
<td>0.879±0.094</td>
<td>0.259±0.471</td>
<td>4.184±0.249</td>
<td>0.249±0.372</td>
</tr>
<tr>
<td>400</td>
<td>8.72±0.78</td>
<td>7.69±1.03</td>
<td>0.881±0.262</td>
<td>0.026±0.726</td>
<td>0.751±0.639</td>
<td>0.639±0.472</td>
</tr>
</tbody>
</table>
Table 2. The secondary centroids $< Z >$ and $< N >$, the variances $\sigma_Z^2$ and $\sigma_N^2$, and correlation factor $\rho_{NZ}$, for the secondary distributions obtained with the reaction $^{40}\text{Ca} + ^{209}\text{Bi}$ at 600 MeV. The energy loss scale TKEL has been corrected for evaporation assuming thermal division of excitation energy between the reaction fragments. The values of the center of the bin are quoted and reflect consecutive bin intervals.

<table>
<thead>
<tr>
<th>TKEL</th>
<th>$&lt; Z &gt;$</th>
<th>$&lt; N &gt;$</th>
<th>$&lt; N &gt; / &lt; Z &gt;$</th>
<th>$\sigma_Z^2$</th>
<th>$\sigma_N^2$</th>
<th>$\rho_{NZ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>19.90±0.03</td>
<td>21.12±0.03</td>
<td>1.062±0.003</td>
<td>0.216±0.050</td>
<td>1.705±0.118</td>
<td>0.327±0.055</td>
</tr>
<tr>
<td>45</td>
<td>19.45±0.01</td>
<td>20.19±0.01</td>
<td>1.038±0.001</td>
<td>0.377±0.022</td>
<td>2.073±0.053</td>
<td>0.426±0.018</td>
</tr>
<tr>
<td>65</td>
<td>19.20±0.02</td>
<td>19.67±0.02</td>
<td>1.025±0.002</td>
<td>0.481±0.028</td>
<td>2.311±0.074</td>
<td>0.456±0.020</td>
</tr>
<tr>
<td>75</td>
<td>18.74±0.01</td>
<td>19.05±0.01</td>
<td>1.015±0.002</td>
<td>0.590±0.029</td>
<td>3.019±0.091</td>
<td>0.442±0.017</td>
</tr>
<tr>
<td>85</td>
<td>18.47±0.01</td>
<td>18.52±0.01</td>
<td>1.004±0.001</td>
<td>0.668±0.030</td>
<td>3.687±0.099</td>
<td>0.426±0.016</td>
</tr>
<tr>
<td>95</td>
<td>18.17±0.01</td>
<td>17.81±0.01</td>
<td>0.980±0.002</td>
<td>0.664±0.033</td>
<td>2.996±0.082</td>
<td>0.471±0.017</td>
</tr>
<tr>
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<td>17.94±0.01</td>
<td>17.41±0.01</td>
<td>0.970±0.002</td>
<td>0.803±0.038</td>
<td>2.887±0.080</td>
<td>0.527±0.018</td>
</tr>
<tr>
<td>115</td>
<td>17.74±0.01</td>
<td>17.12±0.01</td>
<td>0.965±0.002</td>
<td>0.985±0.024</td>
<td>3.082±0.086</td>
<td>0.565±0.019</td>
</tr>
<tr>
<td>135</td>
<td>17.04±0.01</td>
<td>16.10±0.01</td>
<td>0.945±0.001</td>
<td>1.061±0.034</td>
<td>3.692±0.066</td>
<td>0.536±0.012</td>
</tr>
<tr>
<td>165</td>
<td>16.38±0.01</td>
<td>15.10±0.01</td>
<td>0.922±0.001</td>
<td>1.396±0.040</td>
<td>4.692±0.093</td>
<td>0.545±0.015</td>
</tr>
<tr>
<td>200</td>
<td>15.28±0.01</td>
<td>14.12±0.01</td>
<td>0.924±0.001</td>
<td>1.669±0.042</td>
<td>5.454±0.103</td>
<td>0.553±0.015</td>
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<tr>
<td>240</td>
<td>14.18±0.01</td>
<td>12.61±0.01</td>
<td>0.889±0.002</td>
<td>1.115±0.046</td>
<td>6.316±0.163</td>
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<tr>
<td>280</td>
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<td>12.04±0.02</td>
<td>0.919±0.003</td>
<td>0.720±0.073</td>
<td>7.012±0.265</td>
<td>0.320±0.020</td>
</tr>
<tr>
<td>320</td>
<td>11.97±0.04</td>
<td>11.78±0.04</td>
<td>0.984±0.007</td>
<td>0.533±0.288</td>
<td>6.998±0.403</td>
<td>0.276±0.032</td>
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<tr>
<td>360</td>
<td>10.53±0.08</td>
<td>9.83±0.09</td>
<td>0.933±0.016</td>
<td>0.703±0.160</td>
<td>5.098±0.761</td>
<td>0.371±0.083</td>
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<tr>
<td>400</td>
<td>9.52±0.20</td>
<td>8.38±0.25</td>
<td>0.881±0.045</td>
<td>0.388±0.288</td>
<td>4.798±1.455</td>
<td>0.285±0.177</td>
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