CHARACTERIZATION OF FRACTURE RESERVOIRS USING STATIC AND DYNAMIC DATA: FROM SONIC AND 3D SEISMIC TO PERMEABILITY DISTRIBUTION

Annual Report
March 1998

By
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Christopher L. Hackett
Raymon L. Brown
Hughbert A. Collier
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October 1998

Performed Under Contract Subcontract G4S51731 and Prime Contract No. DE-AC22-94PC91008

Southwest Research Institute
Instrumentation and Space Research Division

National Petroleum Technology Office
U. S. DEPARTMENT OF ENERGY
Tulsa, Oklahoma

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Prepared for
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FOREWORD AND ACKNOWLEDGMENTS

The work reported herein represents the second year of work on the characterization of fracture reservoirs using static and dynamic data from sonic and 3D seismic to permeability distribution. The project coordination and supervision as well as the geophysical studies were performed by Dr. Jorge Parra from the Department of Electronics & Applied Physics, Instrumentation and Space Research Division, Southwest Research Institute. The petrophysical studies were conducted by Dr. Hughbert Collier, consultant to SwRI. The petroleum engineering work was performed by Dr. Akhil Datta-Gupta and Mr. Sriram Peddibhotla under a subcontract agreement between Southwest Research Institute and the University of Texas A&M. Software development, algorithms and computer assistance throughout the project were provided by Dr. Christopher Hackert from Southwest Research Institute. Dr. Raymon Brown from OGS provided assistance on the implementation of the algorithms.

The assistance of Dr. Min Tham, project manager from BDM-Oklahoma, is gratefully acknowledged. We thank Chevron Production USA, in particular, Dr. Mike Morea, for his contribution of Buena Vista Hills field data.

This report contains five sections. Some individual subsections contain their own list of references as well as figures and conclusions when appropriate. The first section includes the introduction and summary of the second-year project efforts. The next section describes the results of the project tasks: (1) Elastic Wave Propagation in Random Geological Media: 1D Solution, (2) Algorithms for Processing Acoustic and Seismic Data, (3) Validation of Processing Acoustic and Seismic Data, (4) Dispersion Analysis of Seismic Waves in Poroelastic Media with Azimuthal Anisotropy, (5) Buena Vista Hills and Lodgepole Data Bases, (6) Petrophysical and Geological Analysis for Reservoir Characterization, and (7) Characterization of Fracture Reservoirs Using Static and Dynamic Data. The last three sections include a summary of accomplishments, technology transfer, and suggestions for follow-on work.
I. INTRODUCTION AND SUMMARY OF PROJECT

A. Background

In low porosity, low permeability zones, natural fractures are the primary source of permeability which affect both production and injection of fluids. The open fractures do not contribute much to porosity, but they provide an increased drainage network to any porosity. They also may connect the borehole to remote zones of better reservoir characteristics. An important approach to characterizing the fracture orientation and fracture permeability of reservoir formations is one based on the effects of such conditions on the propagation of acoustic and seismic waves in the rock.

The objective of this project is to evaluate acoustic logging and surface seismic measurement techniques as well as fluid flow and transport methods for mapping permeability anisotropy and other petrophysical parameters in a fracture reservoir. We will also determine fracture connectivity using multiphase production data.

The work includes theoretical and numerical model studies linked with a balanced petrophysical and engineering program for the development of advanced concepts of borehole and surface seismic to understand the reservoir fracture system and to predict fracture connectivity using multiphase production data.

The project is organized in three phases. Phase I includes the development of theoretical models using deterministic interwell seismic solutions, petrophysical analysis, and rapid simulation of multiphase flow in reservoirs using a semianalytic streamline approach. Phase II includes the development of theoretical models using stochastic solutions, fractured connectivity using inversion of production data, and interwell seismic and sonic logs signal analysis. Phase III includes numerical modeling, analysis, and integration of geophysical and petrophysical data, and application of the inversion method to production data and reconciliation with geophysical data to characterize fracture reservoirs.

In this second year annual report, the topics of research have been slightly modified from those originally proposed. The reasons for the modifications is given below. Thus, the second year of this project involves petrophysical analysis and basic theoretical analysis in geophysics and petroleum engineering. Geophysical techniques include the analysis of plane-harmonic seismic waves in a random geological media, development of scattering correction techniques for extracting the intrinsic properties of the media, and several processing techniques that are required to interpret well logs and interwell seismic data at the Buena Vista Hills field. Petroleum engineering techniques include the development of an inversion method for production data and reconciliation with geophysical data. The petrophysical analysis consists of evaluating the Buena Vista Hills field (partially owned by Chevron) and completing the evaluation of the Twin Creek fractured reservoir (owned by Union Pacific Resources) in the overthrust area of Utah and Wyoming to characterize fractures and rock physical properties for the validation of advanced theoretical concepts developed during this project.
B. Summary of Project Efforts

Our project initially studied Union Pacific Resources’ Lodgepole Field in Summit County, Utah. UPRC was very cooperative, providing a wealth of data on the field. A geological and petrophysical analysis of part of the field was completed for the seismic modeling and in preparation for a crosswell survey. This analysis established the factors controlling fracturing in the reservoir. Integration of the results with a velocity inversion analysis of the 2D surface seismic data established a correlation between a high-velocity anomaly and the main fractured interval in a horizontal well that parallels the seismic line. Initial seismic modeling of the field provided the constraints for a crosswell survey and verified that the technique could be used in the field.

The results of our Lodgepole study were very promising. However, after working on the data for one year, several unforeseen obstacles arose:

1. UPRC decided that the vertical wells that we were initially going to use for a crosswell survey were not going to be available.

2. UPRC lost interest in the play that included Lodgepole Field and significantly reduced their activity. Hydrocarbon production was too low to justify further development of the field.

3. Burke Angstman, the UPRC geophysicist assigned to Lodgepole, left the company and UPRC did not assign another geophysicist to the field.

4. Additional 2D surface seismic lines in the study area were never released to us. The lines were owned by parties other than UPRC and although permission was obtained to use the lines, the owners never followed through with releasing the data.

Thus it was necessary to find another test field. Contacts made at the 1997 International Reservoir Characterization Technical Conference in Houston, Texas led to two possibilities: Marathon’s Yates Field in West Texas and Chevron’s Buena Vista Hills Field in Kern County, California. Yates Field was not used because although it has an extensive database, there is no seismic data.

Our study was switched to Buena Vista Hills Field because a plethora of geological, petrophysical, production, and seismic data is available, including crosswell surveys. Chevron is actively studying the field for their DOE project and they have provided all data that we requested. Data are available for a wide range of scales: from centimeter (FMI and minipermeameter) to hundreds of meters (crosswell surveys). The switch to Buena Vista Hills will allow us to successfully complete our project.

To characterize the Buena Vista Hills field, we have implemented methods of modeling, precessing and interpretation. The modeling methods are based on deterministic and stochastic solutions. Deterministic solutions were developed in Phase I and applied in Phase II to simulate acoustic responses of laminated reservoirs. Specifically, the simulations were aimed at implementing processing techniques to correct P-wave and S-wave velocity logs for scattering effects caused by
thin layering. To validate this method, an example is given of a zone in the Antelope shale at Buena Vista Hills. We are also including a summary of the theory and the processing steps of this new method for predicting intrinsic dispersion and attenuation in Section II. Since the objective for correcting velocity scattering effects is to predict intrinsic dispersion from velocity data, we are presenting an application to illustrate how to relate permeability anisotropy with intrinsic dispersion. Also, the theoretical solution for calculating full waveform dipole sonic that was developed in Phase I was applied to simulate dipole responses at different azimuthal source orientations. The results will be used to interpret the effects of anisotropy associated with the presence of vertical fractures at Buena Vista Hills.

On the other hand, the stochastic solutions were developed to simulate seismic waves propagating in a heterogeneous random medium. Random wavefield simulations are being used to understand dispersion effects associated with heterogeneities at different scales. In addition, new analytical solutions to calculate dispersion and attenuation effects in terms of the standard deviation of rock physical properties are being developed for 1D and 2D random geological media. The final equation for the 1D solution is included in this report, as a summary of the work that remains to be done on the last phase of this project.

The results of the integration of core, well logs, and geology of Buena Vista Hills is also given in Section II. The petrophysics is important for constructing a geological model that is integrated with geophysical data. The results of this integration will be considered as the input model for the inversion technique for processing production data. Section III summarizes accomplishments. In Section IV we present a summary of the technology transfer and promotion efforts associated with this project. Here we include contacts made with industry and work that we are developing with oil companies as well as promotional activity through presentations, papers, and the Internet. In the last section, we address the work to be done in the next six months and future work by applying the processing, modeling and inversion techniques developed in Phases I and II of this project.
II. ACCOMPLISHMENTS

A. Elastic Wave Propagation in Random Geological Media: 1D Solution

The inherently complex nature of geological heterogeneities has produced an increasing interest in using statistical models for describing spatial heterogeneity, particularly in the treatment of subsurface contamination, oil and gas reservoir characterization, and seismic data processing. In the characterization of hydrocarbon reservoirs, seismic methods play an important role. A reservoir formation varies in many different scales: from scales much larger than seismic wavelengths down to much smaller scales. It has been shown that the small-scale variations can have a significant effect on the transmitted wavefield and can give rise to apparent attenuation and dispersion (O’Doherty and Anstey, 1971). In the context of subsurface hydrology and toxic waste contamination, extensive field investigations have shown that the hypothesis of random field heterogeneity is consistent with observed spatial distribution of material properties and with the resulting contamination patterns. A comprehensive review of field measurements, including both saturated and unsaturated as well as continuous and fractured media, can be found in Ababou (1991).

There is a vast amount of literature on wave propagation in heterogeneous media and on statistical approaches to various types of wave propagation phenomena and transport process. Our screening of the literature indicates that heterogeneous wave equations have been treated by a variety of methods (e.g., based on approximate multiple-scale expansions for periodic media (Santosa and Symes, 1991), or on various perturbation methods for statistical continua (Keller, 1994; and Hoffman, 1964). The statistical continuum approach has been used to characterized effective velocity and attenuation (Lerche and Petroy, 1996). Seismic wave propagation in random media applied to dispersion and wave attenuation due to scatterers have been addressed by Korn (1992). There are several authors that have used statistical representation to describe small-scale inhomogeneities in seismological studies (Ikelk et al., 1993; Kneib and Kerner, 1993; Kerner, 1992; and Roth and Korn, 1993). In particular, these authors have used the finite-difference algorithm to simulate seismic waves.

We have developed the fundamental analysis of wave propagation in a randomly heterogeneous geologic medium by treating the most simple stochastic wave propagation problem in a 1D space of plane-harmonic seismic waves propagating in a medium having random material properties distributed in the vertical direction. To demonstrate the applicability of the present solution, numerical models for a medium having random rock physical properties are produced to calculate synthetic seismograms and dispersion of scattering effects associated with such random medium fluctuations.

The topics of this analysis are described in the Second and Third Quarterly Reports of 1997. In addition, we produced a manuscript that was submitted to the Journal of Applied Geophysics. The subject of this manuscript is divided into four parts: (1) formulation of the stochastic wave equation, (2) development of perturbation solutions for the 1D stochastic wave equation, (3) numerical evaluation of the stochastic wavefields, and (4) numerical results. The manuscript also includes two appendices: (A) Fourier space-Green’s function solution using “informal” Fourier representations, and (B) expression of stochastic solution in terms of random Fourier-Stieltjes increments.
1. Formulation of The Stochastic Wave Equation

The constitutive law, mass balance and momentum balance equations were introduced for the development of the stress and displacement solutions for random increments using the Fourier-Stieltjes representation. The random field representation was introduced through the intrinsic rock physical properties of the elastic medium. Each of these intrinsic properties was assumed to have a log-normal probability density function, and the random field representation was expressed in terms of three log-normal probability density functions.

The stochastic wave equation was developed by introducing a perturbation approach based on an infinite series expansion of both the random coefficients and the displacement solution in terms of \( \sigma \)-parameters (standard deviations of random material properties). This perturbation approach was based on work by Ababou and Gehlar (1990). The method yielded the first-order approximation of the particle displacement solution for an incident plane wave in an unbounded heterogeneous medium. The stochastic displacement wavefield was expressed in closed form in terms of the incident deterministic plane wave and in terms of the random material properties of the medium. The complete theoretical development of the solution of the stochastic wave equation was reported in the Second Quarterly Report of 1997. Here we give the final solution and we briefly describe the numerical procedure to evaluate the random wavefield.

2. General Solution of the Stochastic Wave Equation

The general solution of the first order stochastic displacement field in the space-frequency domain is given by

\[
    u'(x, \omega) = u_0(x, \omega) U'(x, \omega)
\]  

where \( u_0(x, \omega) \) is the deterministic right-going plane wave incoming from the far field. The deterministic displacement is

\[
    u_0(x, \omega) = u_0(\omega) e^{-jk_0x},
\]

where \( k_0 = \omega / V_0 \)

\[
    V_0 = \left( \frac{\alpha_G}{\rho_G} \right)^{1/2},
\]

in which

- \( \rho_G \) is the geometric mean density and,
- \( \alpha_G \) is the geometric mean of the 1D stiffness coefficient \( \alpha \), that is, \( \alpha = \lambda + 2\mu \).

\( U'(x, \omega) \) is the real-value stationary random field related to random material properties described below. In addition, the total stochastic displacement can be determined, to first order, by

\[
    u(x, \omega) = u_0(x, \omega) + u'(x, \omega)
\]

or

\[
    u(x, \omega) = u_0(x, \omega) \left[ 1 + U'(x, \omega) \right].
\]
The function $U'(x, \omega)$ is a frequency-dependent spatial random field that has the following properties: real-valued, zero-mean, and stationary (i.e., with a spectral representation). After we apply the method of Shinozuka (1972) and Shinozuka (1987), the total particle displacement solution is given by,

$$ u(x, \omega) = u_0(x, \omega) + \int e^{-j\omega_0 - kx} u_0(\omega) \sqrt{S(k)} dk \times \left\{ f(k, \omega) e^{j\Phi(k)} + g(k, \omega) e^{j\Psi(k)} \right\} $$

(4) where

$$ f(k, \omega) = \frac{1}{\left( \frac{k}{k_0} - 1 \right)^2 - 1} \left[ \sigma_R + R \sigma_A \left( \frac{k}{k_0} - 1 \right) \right] $$

(5a) and

$$ g(k, \omega) = \frac{1}{\left( \frac{k}{k_0} - 1 \right)^2 - 1} \sqrt{1 - R^2 \sigma_A \left( \frac{k}{k_0} - 1 \right) } $$

(5b)

The functions $\Phi(k)$ and $\Psi(k)$ are the uniformly distributed phase in $[0, 2\pi]$ for any fixed $k \neq 0$. $S(k)$ is the spectral density for the 1D material property coefficients (including the log-density and log-stiffness). Equation (4) is the solution of the 1D stochastic wave equation containing the zero-mean fluctuations of the density and stiffness, and their standard deviations $\sigma_R$ and $\sigma_A$, respectively. In this equation the attenuation was introduced by expressing the wavenumber $k_0$ as a complex variable given by

$$ k_0 = \frac{\omega}{V_0} (1 - j\delta) $$

where $\delta = 1/2 Q$ and $Q$ is the quality factor of the random medium. In Equation (5) the parameter $R$ is the cross-correlation coefficient of the random material properties (i.e., $-1 \leq R \leq 1$).
The particle displacement in the time domain is obtained by applying FFT to Equation (4). To calculate spectral responses and full waveform seismic waves, we use the following density functions:

**Gaussian model:**

\[ S(k) = Dl\ell^2 e^{-\frac{1}{2} (k\ell)^2}, \]

where \( \ell \) is the correlation length. For each \( \gamma \) we have to find the values of D such that \( S(k) \) is normalized by satisfying the relations

\[ \int_0^\infty S(k)dk = \frac{1}{2}. \]

We find that the values of \( \gamma \) and D are given by in the table below:

<table>
<thead>
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<th>( \gamma )</th>
<th>D</th>
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<tr>
<td>0</td>
<td>( \ell/\sqrt{2\pi} )</td>
</tr>
<tr>
<td>1/2</td>
<td>0.485225602( \ell )</td>
</tr>
<tr>
<td>1</td>
<td>( \ell/2 )</td>
</tr>
<tr>
<td>2</td>
<td>( \ell\sqrt{2\pi} )</td>
</tr>
</tbody>
</table>

**Gauss-Markov model:**

\[ S(k) = Dl\ell^2[1 + (k\ell)^2]^{-1}. \]

The following table is obtained for this model,

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( 2\ell/\pi )</td>
</tr>
<tr>
<td>1/2</td>
<td>( \frac{4\ell}{\pi\sqrt{2}} )</td>
</tr>
<tr>
<td>1</td>
<td>( \ell )</td>
</tr>
<tr>
<td>2</td>
<td>( 2\ell/\pi )</td>
</tr>
</tbody>
</table>
3. Numerical Results

To test the random wavefield solution we constructed a model assuming a plane wave propagating in the direction of a vertical borehole. The plane wave propagates with a compressional wave velocity of 2000 m/s in a medium having an average density of 2.7 g/cc and a quality factor \( Q = 50 \). The solution generates a frequency domain transfer function every 5 Hz from 0 to 5115 Hz. There are 11 receiver locations spaced every 1 meter from 1 m to 11 m into the medium. The standard deviation of the fluctuations in the log of the stiffness and density is 0.1 for each property. The length scale \( \ell \) of the medium fluctuations is 0.5 m, corresponding to a characteristic scattering frequency \( \lambda = 4\ell \) of 1000 Hz. Since we used a cross-correlation coefficient of the density and stiffness fluctuations equal to 0.5, each random medium property is generated from the inverse transform of the k-space random field given by

\[
S(k) \left[ e^{i\phi(k)} + e^{i\psi(k)} \right],
\]

where \( \phi(k) \) and \( \psi(k) \) are random phase angles, and \( S(k) = Dk^2 e^{-k^2} \). Different sets of random phase angles were used for each material property. In practice, the symmetric random field is represented by discrete points from \( k\ell = 0.0 \) to 15.0 by 0.001 intervals. This small interval is necessary to resolve a peak in the integrand with a half width on the order of \( \omega \ell/(Q\nu) \). For this reason, the \( Q \) chosen cannot be too large.

Figure 1 shows the density and velocity profiles of a random medium generated using the parameters given above. It is easily observed that there is a roughly 10% variation in the density and velocity, consistent with the 0.1 log variation cited above. In a nonrandom medium, there would be no such variations and both plots would be flat.

Figures 2 and 3 show seismograms for each receiver at frequencies of 400 and 800 Hz, respectively. Note the slight amplitude and pulse width variations, although the \( Q=50 \) forces a general weakening of the pulse with distance at the higher frequencies. The amplitude and pulse width variations away from the main pulse of course would not be seen in a regular nonrandom medium. The appearance of these extra amplitude variations away from the main pulse is seen however in actual field data. (See, for example, Figs. 3, 4 and 7 in the paper by Parra, Zook, and Collier (1996)).

4. Conclusions

The fundamental analysis of wave propagation in a randomly heterogeneous medium was developed by treating the stochastic wave propagation problem in 1D space of plane-harmonic seismic waves traveling in a medium having random material properties distributed in the vertical direction. The 1D random medium solution was developed in terms of standard deviations and random physical properties of velocity and density. A numerical procedure was implemented to calculate synthetic seismograms to assess the dispersion characteristics associated with scattering. The results demonstrate that the solution can be used to simulate scattering attenuation caused by thin layering observed in laminated reservoirs. The solution can be extended to include a plane-wave
incident at arbitrary angles in 2D and 3D random media. The solution has been used to calculate scattering attenuation and velocity dispersion as a function of depth using the amplitude ratio method.

5. References


Figure 1: Density and velocity variations of a random medium as a function of depth. Plots generated using a correlation length $l = 0.5$ m corresponding to a characteristic scattering frequency of 1000 Hz.
Figure 2: Synthetic seismogram of a plane wave propagating in a medium having random rock physical properties given in Figure 1. The source center frequency to produce the seismogram was 400 Hz.
Figure 3: Synthetic seismogram of a plane wave propagating in a medium having random rock physical properties given in Figure 1. The source center frequency to produce the seismogram was 800 Hz.
B. Algorithms for Processing Acoustic and Seismic Data

1. Development of Algorithms For Estimating Components of Velocity Dispersion and Attenuation

Introduction

This section describes the development of algorithms which can be used at available well control within a field to predict the response of the formations at lower frequencies. Specifically, we are using velocity dispersion and attenuation concepts to predict the velocities in reservoir rocks as a function of frequency and angle of transmission. The development of this technology is important because it can potentially be used to: (a) directly map the saturating fluids and permeability within reservoirs, and (b) identify depositional variations within the reservoir.

Background

The velocity dispersion discussed here is defined to be the variation of sound velocity with frequency. Seismologists usually differentiate between waves that travel through the interior or body of a medium ("body waves") and waves that are guided by surfaces ("guided waves"). Guided waves exhibit a type of dispersion ("geometric dispersion") which is the result of constructive interference between the different frequencies involved in the signal. Body waves also exhibit dispersion. It is the nature and potential application of body wave dispersion that is the subject of this research project.

Where body wave dispersion occurs, a related body wave attenuation can be assumed. Knowledge of the body wave attenuation for different frequencies can be used to predict the dispersion (and vice versa) (Futterman, 1962). Our problem is one of understanding and utilizing the frequency dependence of both the attenuation and dispersion of body waves. In particular, we want to identify and isolate those contributions to the velocity dispersion and attenuation that can be related to: (a) the geological environments throughout a reservoir, and (b) the saturating fluids within the reservoir.

Algorithm Development

An implicit assumption in work with dispersion and attenuation is that the dispersion and attenuation vanish at very high frequencies and very low frequencies. We shall follow this line of reasoning and assume that whatever attenuation mechanisms are active from the sonic (kiloHertz) to the seismic (tens of Hertz) frequencies are essentially zero at megaHertz frequencies. Although not stated explicitly, this assumption was used by Sams (1995) to argue that thin layers below the resolution of conventional sonic logs can contribute to the dispersion that takes place between the sonic log measurements and seismic measurements at lower frequencies.

2. Algorithms For Building A High Resolution Velocity Model

The algorithms described in this section were developed to perform various tasks required to build a high resolution model of the formation surrounding the borehole.
Algorithm For The Estimation of High Resolution Logs

Our first step toward understanding velocity dispersion is to build a high resolution model, in this case a thin layer model, of the formations involved based upon available well logs. Since many logs have a resolution of only 2 feet, these logs tend to average over thin layers. We have followed Sams (1995) and used the technique of Nelson and Mitchell (1991) to arrive at a high resolution model of the formation. The idea is to use the high resolution FMI log as a guide to the prediction of the log response of sonic and density logs at a 1 inch (2.54 cm) resolution. The resulting high resolution velocity model can then be used to predict the effects of elastic scattering at lower frequencies.

The idea behind the algorithm for estimating a high resolution velocity model is called resolution matching. Consider for example the case of an FMI log with a resolution of one inch and a sonic log with a resolution of 2 feet. These two logs cannot be compared in this state because their respective resolutions are different. In order to remedy this problem, a filter has been designed which can be used to make the high resolution FMI log appear to have the same type of averaging effect or vertical resolution as the sonic log. After this type of filtering, the FMI log looks a lot smoother. Now that the two logs have the same vertical resolution, we can plot their data points on a graph and fit a curve through the data. This gives us a functional relationship between the filtered FMI data and the sonic data. If we now apply this functional relationship to the unfiltered FMI log data, we will obtain an estimate of what the sonic log would have measured if it had the same resolution as the FMI log. There are some additional changes made to account for different types of problems, e.g., a poor correlation between the sonic and the FMI resistivity. Using this approach, we have been able to take the logs recorded at Buena Vista Hills field and to estimate a high resolution velocity model for the formation surrounding the borehole. The development of this algorithm is the first step toward being able to predict the seismic response at lower frequencies, e.g., sonic frequencies.

Algorithm For Mapping Borehole Position

The injection well at Buena Vista Hills field with all of the modern logs is a deviated hole. In order to handle the problem of deviated holes, an algorithm was written to map the position of the borehole based upon the measurement of measured depth and the tangent angles to the borehole at selected points along the borehole. The resulting algorithm then outputs the x,y,z position of the borehole as a function of measured depth. This algorithm is important in (a) our work to predict a velocity model for the formation surrounding the borehole and (b) for crosswell seismic studies.

Algorithm For Predicting Different Log Responses For Different Borehole Orientations

As frequently occurs, the formation at Buena Vista Hills is dipping in one direction while the borehole is moving in another direction. As a result, the log measured in a deviated borehole through a dipping formation gives a misrepresentation of the layered model that is being constructed for this project. To remedy this problem, we have written an algorithm to correct the logs for both of these effects, i.e., the hole deviation and the dip of the beds. The correction is based upon the assumption that the beds are planar and are all dipping in the same direction. These assumptions allow us to predict the log response for different hole orientations. However, for our study, we have computed the log response that would have been obtained if the borehole was drilled at right angles to the planes of the beds. In this way, our algorithm predicts the log response of the layered medium that we are seeking to model.
3. Algorithms For Predicting Velocity Dispersion/Attenuation Based Upon
Data Measured Within A Well

The algorithms described in this section were developed to estimate the
velocities at different frequencies through the formation surrounding the borehole.

Ray Theoretical Velocity Algorithm

At frequencies where the wavelengths are comparable to or smaller than the one inch beds
determined from the logs, the velocity through a formation is expected to be that expected from
conventional ray theory. A simple ray trace algorithm was written to predict the ray theoretical
velocity. This velocity is the high frequency limit on velocity through a formation. For vertical
transmission through a stack of layers, the velocity is effectively the same as would be obtained by
adding the individual travel times through each layer and dividing the thickness of the package of
layers considered by the sum of the travel times.

Backus Averaging

Backus averaging (1962) is used to obtain the very low frequency character of a layered
formation. An algorithm has been written to compute the effective elastic constants of a layered
transversely isotropic medium. The results are expected to be more accurate as the wavelength gets
much longer than the thickness of the beds involved. This algorithm combined with the ray theory
algorithm can be used to place constraints on the limits of velocity dispersion.

Plane Wave Velocity Algorithm

To compute apparent velocities through layered media at all frequencies, a plane elastic wave
modeling program was written. This algorithm can be used to compute the velocities through a
layered formation at a wide range of frequencies. The end limits are expected to be close to those
predicted by the Ray Theoretical and Backus velocities. However, the actual dispersion between the
end limits is not expected to be a smooth curve except for a specific band of frequencies. The actual
dispersion curve will depend upon the distribution of bed thicknesses within the formation being
studied.

Sonic Attenuation Measurement Algorithm

Next, an algorithm was developed to determine the attenuation over an interval using the
sonic log. The amplitudes of signals measured at two stations were used in a spectral ratio method
of determining the attenuation. Because of the frequencies involved for these attenuation
measurements, the effects of elastic scattering are believed to be negligible (based upon model
studies). As a result, the attenuation measured in this way is believed to represent the intrinsic
attenuation of the formation at this frequency.

4. Independent Measures of Intrinsic Attenuation

Armed with the above algorithms, two methods of estimating the intrinsic
attenuation at Buena Vista Hills have been implemented. The first method is based upon the direct
measurement of attenuation described above for the sonic log. The second method is based upon the
assumption that the velocity dispersion from the high resolution velocity model and the observed
velocities on the sonic logs can be explained in terms of the elastic scattering and the intrinsic attenuation. Since we can compute the effects of elastic scattering, the remaining velocity dispersion is assumed to be due to intrinsic attenuation. We have implemented both of these schemes at Buena Vista Hills and found that the two independent estimates of intrinsic attenuation are in complete agreement. This represents the first stage of predicting the seismic response at lower seismic frequencies.

References


C. Algorithm to Predict A High Resolution Sonic Log  
(method and application)

1. Introduction

The 653 well at Buena Vista Hills has been shown to exhibit very fine layering, with some layers having thicknesses on the order of one inch. This fine scale layering is below the resolution of most logging tools. This is especially true for the sonic velocity logs, which effectively average their measurements over approximately 18” of depth. These small layers can induce additional dispersion at mid-range frequencies, and if not properly accounted for, this will result in errors in the assumed velocity and attenuation profiles. It is important to be able to separate the layering induced attenuation and dispersion from the intrinsic material attenuation and dispersion, since the intrinsic properties are related in some fashion to the rock porosity, saturation, and other properties of interest.

One way of including these small length scale effects in the velocity profile is to convolve the relatively coarse velocity log with a very high resolution well log such as the FMI resistivity log. This technique, presented in Nelson and Mitchell (1991), has been shown to be fairly successful in Sams (1995). We have developed and tested a computer program to use this technique in conjunction with the Buena Vista Hills data. The program requires the user to assume a vertical response profile for the velocity and FMI logging tools, and to provide an estimate of the vertical length scale of the layering. It then smooths the high resolution (FMI) data to match the length scale of the velocity log. These two lower resolution curves are correlated over several local windows using linear, power law, and exponential fits. The best fit and window (based on least squares optimization) are kept and used to extrapolate locally higher resolution velocity data. When this is done for every depth within a region of interest, a high resolution velocity log for the entire region may be constructed.

We have obtained FMI logs at one inch sampling intervals, density logs at two inch intervals, and velocity logs at six inch intervals for the 653 well over depths ranging from approximately 3950 to 4850 ft. Information obtained by Dr. Hughbert Collier indicates that the vertical resolution of the FMI tool is less than one inch, the vertical resolution of the density tool is about six inches, and the vertical resolution of the sonic logging tool is about 18 inches. We will use the computer program to produce density and velocity logs extrapolated to the one inch data interval of the FMI log. This detailed information on the material properties will be used in numerical models to predict the effect of the finely layered structure on wave propagation at varying frequencies. Figure 1 demonstrates how this technique has been applied to a section of the Buena Vista Hills.
Figure 1: Low resolution (sonic log measurement) compressional wave velocity and enhanced high resolution compressional wave velocity in the chosen region. The resolution length is reduced from 18 inches to one inch.

2. Use of the Plane-Wave Program to Predict Dispersion using High-Resolution Models (method and application)

Using a plane wave seismic propagation program previously developed for SwRI, we have investigated dispersion and attenuation of vertically propagating waves through a section of the Buena Vista Hills field (4130' to 4160' depth). Taking the enhanced resolution logs described above, we have modeled this 30' vertical section as 360 one inch thick independent layers. This program has the option of accounting for intrinsic attenuation, and we have extended its capabilities to include dispersion due to intrinsic attenuation. The advantage of the plane wave formulation is that it is relatively quick, finishing in about 20 minutes for this particular case. Seismograms are generated in post processing from a frequency domain transfer function, allowing multiple source functions and source frequencies to be modeled from the same run.

We were able to estimate the intrinsic Q values of the formation to use in the model input by examining sonic amplitude logs provided to us by Chevron. Although the eight receivers used in the sonic logging tool are not matched for gain, a reasonable estimate of the local high frequency attenuation was obtained by fitting a line to the average slope of the amplitude fall off with distance for each shot location. The resulting low resolution Q log was then enhanced using the resolution matching technique discussed above.

Once seismograms are obtained for a set of receivers, the effective dispersion and attenuation between receiver pairs is calculated with the spectral ratio method. The resulting velocity and attenuation are recorded as a function of frequency from the phase and amplitude of the Fourier transforms of the time series. By examining different pairs of receivers, we create a log of the dispersive velocity and Q values throughout the section. Because it is inefficient to place receivers too closely together (even for synthetic seismograms), the
resolution of this new log is by necessity somewhat coarse. Receivers could, of course, be placed every six inches or closer, but the accuracy of the spectral ratio method degrades noticeably as the receiver separation is decreased.

Figure 2 shows the resulting velocity curves as a function of frequency as measured across a one meter section of the geology. Even though no intrinsic dispersion is included for one of the curves, the velocity is still changing with frequency as much the other curve. Dispersion models like the one shown here and similar attenuation models are employed in a correction technique that seeks to obtain the properties of the underlying intrinsic medium.

![Figure 2](image_url)

Figure 2: Results of plane wave model showing measured velocity and velocity including scattering dispersion without intrinsic dispersion.

3. **Algorithms and Programs used to Correct for Scattering Effects from Log Data (method and application)**

The log of dispersive velocity and Q values obtained thus far includes unwanted layering effects. Reflections and multiple ray paths obscure the “pure” velocities and intrinsic attenuations that could otherwise be observed. This severely complicates the use of long range seismic (VSP or crosswell) to determine the local medium properties. We are developing a correction technique based on a method found in the open literature that should correct for the scattering effects and yield the true medium dispersion and attenuation.

This method, originally proposed by Ganley and Kanasewich (1980), and later used with real data by Dietrich and Bouchon (1985) and Neep et al. (1996), relies on computational modeling to predict the scattering effects in a medium without intrinsic attenuation. By making the assumption that scattering effects and intrinsic medium effects are independent, the scattering part can be subtracted from the total (measured) values, leaving only
the intrinsic contribution. Expressing this idea mathematically (in a form similar to Brown and Seifert, 1997), we presume that the actual measured velocity at a particular frequency is given by

\[ V_{\text{measured}}(z, \omega) = V_{\text{HF}}(z) + \Delta V_s(z, \omega) + \Delta V_I(z, \omega), \]

where \( z \) denotes depth and \( \omega \) denotes frequency. Here, \( V_{\text{HF}} \) is a baseline velocity (usually that measured by the high frequency sonic log), and the components of the dispersion relative to the baseline are given by \( \Delta V_s \) for the scattering part and \( \Delta V_I \) for the intrinsic dispersion. Similarly, the measured attenuation, \( \alpha \), is given by

\[ \alpha_{\text{measured}}(z, \omega) = \alpha_s(z, \omega) + \alpha_I(z, \omega), \]

where again \( \alpha_s \) denotes changes in amplitude due to scattering and \( \alpha_I \) denotes intrinsic anelastic amplitude losses. It should be noted that as reflection coefficients can change with frequency and be positive or negative, so can the scattering attenuation be positive or negative.

The point of this whole exercise is to confirm that we can extract intrinsic property values from measured values polluted with scattering effects. We have completed a successful test of this concept using synthetic data and hope to be able to apply it to experimental crosswell data shortly. In the synthetic tests, we first ran a test case with a constant \( Q \) value, and were readily able to extract this value from the scattering corrected spectral ratios. Subsequent tests that employed a \( Q \) varying with depth were also successful, but it was found to be more difficult to extract consistent estimates of attenuation and dispersion. The following graph (Figure 3) shows the results of the \( Q \) extraction from synthetic data.

![Figure 3: Intrinsic Q measured over one meter intervals. The one meter average of the Q measurements is plotted as the solid line, results of correction technique plotted as the dashed line.](image-url)
4. Modeling using Random Medium Theory (application)

Understanding how variation and fluctuation in the medium properties affect the wave propagation is quite important to this project. A theory had previously been developed at SwRI to account for how low order variation in the density and material stiffness affect plane wave propagation. A computer code previously written to explore the applications of this theory was rewritten slightly to improve the numerical integration and output format. In addition, the effects of intrinsic dispersion may now also be included in the algorithm. By examining the output of this program, we can confirm the results above that scattering based dispersion and attenuation can be positive or negative in sign depending on the underlying material variations. In fact, the dispersion and attenuation can be expressed in a complex map, varying with both position and frequency.

The program generates a frequency domain transfer function at discrete frequencies for several receiver locations. The test medium is assigned average properties for velocity and density, as well as standard deviations for the fluctuations in the log of the stiffness and density. In addition, the covariance of the density and stiffness fluctuations is set and a constant intrinsic $Q$ factor is assigned. The length scale ($t$) of the medium fluctuations corresponds to a characteristic scattering frequency ($\lambda = 4t$) which separates the low frequency behavior regime from the high frequency behavior regime. The wave field of a plane wave through this random medium is based on an integration of the spectral properties of random medium. This integration must be performed carefully to resolve a peak in the integrand with a half width on the order of $\omega t/(QV)$. For this reason, the $Q$ chosen cannot be too large.

Figure 4 shows a plot of measured dispersion due to random medium effects for a characteristic frequency of 1000 Hz. Intrinsic dispersion due to the assigned $Q$ of 50 is not included in this particular case. The curve at first glance appears to follow a logarithmic pattern, just as the case of constant $Q$ intrinsic dispersion. Closer inspection, however, shows that this curve is more sharply bent than a log profile, with a fairly nondispersive region above 1000 Hz. In fact, at the lower (dispersing) frequencies, the measured velocity varies with spatial location as well as frequency. That is, a different dispersion curve could be calculated between every pair of receivers. It is possible, then, to make a map of effective velocity as $V=V(x, \omega)$ using adjacent receivers to define the $x$ values on the map. This map shows that regions of positive $dV/d\omega$ are interspersed with regions of negative $dV/d\omega$. Thus, strong dispersion in one area is offset by negative dispersion in another. The resulting overall medium is only slightly dispersive, in the limit of this first order theory.
5. Computation of Backus Average Velocity (application)

The high frequency limit of the effective velocity is simply the sum of the material slownesses between the two measuring receivers. In the low frequency (the Backus, or effective medium limit) the determination of effective velocity is more complicated, relying on averages of material properties over distances proportional to the wavelength. Based on information provided to us by Dr. Ray Brown of the Oklahoma Geological Survey, a computer program was constructed to perform this low frequency averaging for plane waves at arbitrary angles of incidence.

6. Method to Predict True Vertical Log Response Using Logs from a Deviated Hole (method and application).

Logs taken in deviated wells or in areas of dipping beds can distort the true geology of the region. If the borehole is not perpendicular to the beds, the well logs will reflect layers that appear to be thicker than they really are. This can have a severe effect on modeling of wave propagation since many of the reflection and scattering characteristics are intimately related to the ratio of bed thickness and wavelength. This in turn can degrade the accuracy of the scattering correction technique described in a previous section. Dr. Ray Brown of the Oklahoma Geological Survey has developed a set of equations based on a second order integral technique that allows the transformation of the well log coordinates from measured depth (arc length along the borehole) to either true vertical depth (actual distance below the surface) or stratigraphic depth (distance perpendicular to a plane bed surface). It is the stratigraphic depth which measures the true thickness of geological features, and that is the coordinate system which should be employed for the modeling of frequency dependent scattering behavior.
7. Time-Frequency Representation of Dipole Sonic Data (application)

In addition, we have also obtained the acoustic waveform data for cross-dipole sonic logs for source depths of about 4340 to 4890 ft. The cross-dipole apparatus consists of two perpendicular dipole sources and eight sets of two perpendicular dipole receivers. We thus have data for four possible source/receiver orientations. The receivers are spaced at half-foot intervals, with the smallest source – receiver separation set at nine feet. Thus, all measured waveforms have traveled through 9 to 12.5 feet of borehole. The asymmetric dipole waveforms will give us information about local anisotropy in the borehole. We will also be able to compare the measured acoustic waveforms against our model, providing experimental validation of this method.

Figures (a) Time-frequency analysis of actual dipole data from Buena Vista Hills (approx. 4370 ft.). (b) Time-frequency analysis of synthetic data (BHTIH) using 1000 Hz source. Note time and frequency axes are different.

Figures: (a) Time-frequency analysis of synthetic data (BHTIH) using 3000 Hz source. The higher source frequency is above the borehole flexural wave cutoff. (b) Same as (a), except rotated 90 degrees in the transversely isotropic earth.
References


D. Dispersion Analysis of Seismic Waves in Poroelastic Media with Azimuthal Anisotropy

A qualitative evaluation of the preferential directions of fluid flow in formations containing hydrocarbons is of great importance in the characterization of fractured reservoirs. Such preferential directions are related to the permeability anisotropy of the reservoir. Reservoirs are considered to be anisotropic when they possess significant variation in physical properties (porosity, permeability, wettability, etc.) in three dimensions. The permeability field at a given point in the rock can be treated as a second-order tensor in directions coincident with the principal permeability planes. In particular, the presence of vertical parallel cracks and fractures in an isotropic rock matrix leads to azimuthal anisotropy, which can be described by a transversely isotropic model with a horizontal axis of symmetry (Crampin, 1985). Azimuthal and incidence variations in P-wave attenuation and phase velocity from reverse VSP, crosswell, and acoustic logging data have the potential to infer parameters associated with the fracture conditions of a reservoir. Recently, analytical studies to relate tensor permeability to attenuation and dispersion of seismic waves has been reported by Parra (1997). The results of this work have led to the development of an analytic solution that estimates the elements of tensor permeability by modifying the constitutive relation of the stress tensor in the pore fluid (Biot, 1955 and Biot, 1962). This constitutive equation was modified to describe the Biot and squirt-flow mechanisms for transversely isotropic poroelastic media. In addition, a field example was presented to test the model and to relate permeability to seismic waves propagating between wells at the Gypsy test site, Oklahoma (Parra et al., 1996). In this field example, horizontal permeability was controlled by cross-bedded and planar-laminated sandstones having an average permeability about three orders of magnitude greater than the average vertical permeability, which was associated with mudstones and siltstone bodies (permeability barriers). In this case the permeability anisotropy was described by a transversely isotropic model with a vertical axis of symmetry.

We have developed the theoretical solution of acoustic wave propagation in poroelastic media (including the Biot and squirt flow mechanisms based on the work of Dvorkin and Nur, 1993) to relate the directional dispersion and attenuation of P- and S-waves with azimuthal permeability anisotropy. In addition, numerical models were produced to evaluate the sensitivity of attenuation and phase velocity to permeability anisotropy for several azimuthal variations and angles of incidence in the frequency range of crosswell seismic measurements and high-resolution reverse VSP (Owen and Parra, 1993). To test the model we used rock physical fluid properties, as well as dispersion data from the Kankakee limestone formation at the Buckhorn test site, Illinois (Parra, 1995).

1. Constitutive Relations

The formulation of the transversely isotropic poroelastic wave equation, including the Biot and squirt flow mechanisms, is based on the constitutive relations (i.e., the total stress tensor of the anisotropic porous medium and the stress tensor in the pore fluid), the momentum balance equation for total stress, and the generalized Darcy’s law, in the framework of Biot’s theory. These equations in the frequency-domain (assuming exp(-jωt) variation) were (Kazi-Azoual et al., 1988; Parra and Xu, 1994; and Parra, 1997) given in the First Annual Report, March 1996.
2. Numerical Modeling

Attenuation and dispersion curves were produced for the model parameters given in Table 1. The curves, as a function of the azimuth angle $\Omega$ (measured from the horizontal x-axis as shown in Figure 1) and the angle of propagation or incident angle (measured from the z-axis), were calculated for the range of frequencies 500-2500 Hz, in steps of 1000 Hz. For angles of propagation $\Phi$ near vertical, as the azimuth angle varies between 0 to 90°, attenuation and phase velocity vary little at each frequency. For example, for a propagation angle of 15°, the phase velocity at 2500 Hz varies from 3117 m/s at $\Omega = 0^\circ$ to 3149 m/s at $\Omega = 90^\circ$, i.e., about one percent. On the other hand, for a propagation angle of 90°, the phase velocity at 2500 Hz varies from 2769 m/s at $\Omega = 0^\circ$ to 3149 m/s at $\Omega = 90^\circ$, i.e., about 14 percent. Attenuation increases more than 50 percent at $\Omega = 90^\circ$, that is, attenuation is the minimum value for propagation angles in the direction parallel to horizontal (minimum) permeability in the frequency range of 500-2500 Hz. At any other propagation angle, attenuation and phase velocity increase as the azimuth angle varies from 0 to 90°.

The phase velocity curves follow the same pattern as those of the attenuation. That is, as the azimuthal axis becomes parallel to the direction of maximum permeability, the phase velocity approaches the propagation velocity parallel to the fracture plane, which corresponds to the velocity associated with the parameter $c_{11}$. When the azimuthal axis is in a direction parallel to horizontal permeability, the phase velocity approaches the velocity associated with the stiffness constant $c_{33}$.

In the next example we analyze the effects of propagation angle and azimuth on attenuation and phase velocity for the selected frequency of 1500 Hz, when horizontal permeability is varied. The figures show that, as the incident angle $\Phi$ becomes perpendicular to maximum permeability (90°), attenuation is the minimum value. In this case, the motion is controlled by the squirt flow element, $s_3$, associated with vertical permeability, $= 1000$ md. For a vertical permeability of 1000 md, the attenuation peak corresponds to a frequency of about 100 kHz. For a horizontal permeability of $= 2.5$ md, the attenuation peak is at a frequency of about 1 kHz. Since the models that we are analyzing are in the frequency range of 500-2500 Hz, the attenuation associated with vertical permeability of 1000 md is negligible. This analysis suggests that high-frequency information will be required to measure attenuation that is sensitive to vertical permeability at the angle of propagation $\Phi = 90^\circ$.

When the vertical permeability value cannot be evaluated directly from attenuation, either in the frequency range of crosswell seismic measurements or acoustic logging, we can infer preferential fluid flow directions by analyzing dispersion and attenuation curves for different azimuth and incidence angles. For example, curves produced in the frequency range of 50 to 10,000 Hz for the azimuth angles 0, 30, 60, and 90°, and for incident angles of 45 and 90°, show that as the angle of incidence approaches 90°, there is a decrease in attenuation for azimuth angles less than 90°. In particular, for $\Phi = 90^\circ$ and $\Omega = 0^\circ$, attenuation is practically zero.

To predict horizontal permeability, $k_x$, one must understand the propagation characteristics at the angle of incidence, $\Phi = 90^\circ$ (when the azimuthal axis is in the plane parallel to maximum permeability or perpendicular to the axis of symmetry). This condition corresponds to maximum attenuation in which attenuation is controlled by the horizontal squirt flow element, $s_3$. 

27
associated with horizontal permeability, $k_x$. This suggests that processing techniques may be developed to extract horizontal permeability from attenuation measurements at an azimuth of 90° measured from the horizontal axis of symmetry.

3. Dispersion of Seismic Waves in the Kankakee Limestone Formation

The applicability of the dispersion and attenuation solution was demonstrated using interwell seismic data recorded in a hydrocarbon bearing formation at the Buckhorn test site, Illinois. Travel-time tomography imaging of a porous zone (low-velocity heterogeneity) within the Kankakee has been reported by Saito (1991), and an integration of well logs, reverse VSP, and interwell seismic data is given by Parra (1995). According to the exploration and production histories of the field, the oil reserve is the porous zone of the Silurian Kankakee limestone formation, which is horizontally distributed at a depth of 200 m and is less than 8 m thick.

To correlate permeability to seismic waves, two seismic traces were selected. The first trace was recorded in receiver well D, 20 m from source well A, and the second trace was recorded in receiver well B, 46 m from source well A. The source and the two receiver wells were located in the same vertical plane. Both receivers and the source were placed at a depth of 196 m. The spectra of the seismic signatures are given in Figure 2. The phase velocity dispersion curves obtained using the spectra ratio method are plotted, together with theoretical phase velocity curves, in Figure 3. Theoretical curves were produced for azimuthal angles of 0, 30, 40, and 90° based on the model parameters of an oil saturated formation given in Table 2. The dispersion curve calculated at an azimuthal angle of 90° fit the observed curve best at frequencies greater than 2000 Hz. This suggests that the waves travel in the direction of maximum permeability, parallel to the fractured Kankakee formation, which surrounds the low-velocity heterogeneity.

The modeling demonstrates that flow is controlled by vertical fractures in the host rock matrix rather than by porosity. On the other hand, the observed dispersion curve (for frequencies less than 2000 Hz) shows that flow is controlled by porosity in the low-velocity heterogeneity within the Kankakee. Here the phase velocity varies from about 3500 m/s to 4500 m/s, a range consistent with the low-velocity anomaly delineated with travel-time tomography and P-wave velocity logs (Saito, 1991).

4. Conclusions

The numerical results show that in the frequency range of 500-2500 Hz, the attenuation and phase velocity of the P-wave are sensitive to the azimuth and propagation angles. For angles of propagation in the direction of maximum permeability, the attenuation is maximum and the motion is controlled by the squirt flow element associated with minimum permeability (which is in the direction of the horizontal axis of symmetry). In this case, the effect of the azimuth angle on attenuation is negligible. This suggests that single borehole acoustic measurements such as long-space logging may be appropriate to predict the maximum vertical fluid flow in a reservoir by analyzing attenuation and dispersion of acoustic signatures. Furthermore, to predict other preferential directions of fluid flow in a reservoir, the changes in attenuation and dispersion of acoustic waves may be useful if measurements are conducted at different azimuth and incident angles. For instance, as the angle of incidence changes from 0 to 90°, there is a decrease in attenuation, and as the azimuth angle goes to
zero (which is in the direction of the horizontal axis of symmetry), the attenuation is very small. This suggests that the motion is in the direction of minimum permeability (in this case the motion is controlled by the squirt flow element associated with the maximum permeability), which controls the fluid-flow distribution in the horizontal direction. As a result, high-resolution reverse VSP measurements conducted at different azimuth and incident angles have the potential to provide the data necessary to calculate the directional attenuation and dispersion to predict the horizontal directions of the fluid in a reservoir.

Table I. Transversely Isotropic Poroelastic Formation Parameters

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Table II. Formation and Saturant Parameters, Kankakee Model

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<td>Darcy</td>
<td>2</td>
</tr>
<tr>
<td>$Sq_h$</td>
<td>(mm)</td>
<td>4</td>
</tr>
<tr>
<td>$Sq_v$</td>
<td>(mm)</td>
<td>6</td>
</tr>
</tbody>
</table>

REFERENCES


Figure 1: 3-Dimensional View of An Anisotropic Medium With Axis of Symmetry Perpendicular To The Vertical Z-Axis, Showing Orientations of Azimuthal And Propagation Angles
Figure 2: Spectra of Common Source Waveforms Recorded at 20 m and 46 m From Source Well, Fractured Kankakee Reservoir, Illinois
Figure 3: Comparison Between Experimental And Theoretical Phase Velocity Curves Produced At Azimuthal Angles of 20°, 30°, 60°, And 90°, Fractured Kankakee Reservoir, Illinois
E. Buena Vista Hills and Lodgepole Data Bases

The following accomplishments were achieved on compiling and analyzing the data from the Buena Vista and Lodgepole reservoir. The objective was to obtain the required data from Chevron and Union Pacific Resources oil companies to construct geological models for the integration of geophysical and production data as well as to validate the different technologies that were developed in Phase I and Phase II.

1. Buena Vista Hills Database

Chevron provided a plethora of geological and petrophysical data (hard copy and digital) for their pilot CO₂ injector project in the Buena Vista Hills field, Kern County, California. This database is being used as our field site. Data and/or services have been donated by Chevron, Schlumberger, and Accutech.

a. An extensive petrophysical database was amassed for the new well (653Z), which was drilled to be the injector well. The producing interval (952.8 feet) was cored and 99.5% of the core was recovered. A plethora of tests have been run on the core and a comprehensive suite of state-of-the-art wireline logs was run in the well. Crosswell seismic surveys were made of the pilot study wells. The following data have been provided:

(1) Core analyses: lithologic description, fracture analysis, white light and UV light photographs, porosity, permeability, grain density, fluid saturation, minipermeameter, wettability, mineralogy, mercury porosimetry, core plug P and S wave measurements, and hydraulic flow units.

(2) Logging suite: array induction, dipole sonic, formation microimager, magnetic resonance, density, neutron, carbon/oxygen, deviation survey, attenuation, and borehole imaging analysis of the FMI.

(3) Seismic data: shear-wave birefringence VSP and cross-well seismic acquisition and processing.

b. A petrophysical database consisting of well logs, deviation surveys, production data, and formation tops has been assembled for the four other wells in the pilot study area.

c. A petrophysical database consisting of porosity and permeability values calculated from the SP curve for fifty-six wells was provided.

d. A digital database of the pilot study area and fifty-two other wells in the area has been constructed. This task included quality controlling the digitized well logs and standardizing the curve names.

e. Two nonfractured intervals were analyzed to provide the petrophysical parameters for the seismic modeling.
f. The FMI resistivity was processed to provide a calibrated FMI resistivity curve. This processing is seldom done, but we are using it to calibrate to the minipermeameter and acoustic data. FMI resistivity has much better thin bed resolution capabilities than the other logging tools.

g. A two-dimensional forward and inverse modeling analysis of four old electric logs was completed and the results plotted. This is a technique that can be applied to logs that predate the 1950's.

Most of the wells in the Buena Vista Hills field, including four of the five wells in the pilot study area, were drilled in the 1950's and only have old electric logs. On these logs it is difficult to resolve the thin (1 mm to 25 cm) turbidite beds and the thin carbonate rich beds scattered throughout the Antelope Shale. Some modern logs, such as the array induction, do a much better job of resolving thin beds. There is one modern logging suite (well 653Z-26B) in the pilot study area.

To better resolve the thin beds and better correlate the old electric logs with the 653Z's array induction, an investigation was made into the possibility of reprocessing the old logs with AccuTech’s two-dimensional forward and inverse modeling analysis software. The process involves an iterative forward modeling of tool response and accommodates non-linear tool response functions. The program corrects for borehole, invaded zone, and shoulder-bed effects, resulting in bed boundaries with a resolution of less than one foot. The program also calculates an equivalent 6FF40 induction response, which can be used to correlate to induction logs in the 653Z and 563 wells.

The four old electric logs in the pilot study area were reprocessed from the top of the Miocene Chert (approximately 3,900 feet) to TD, and a 6FF40 induction curve was also generated. Reprocessing the curves enhanced the definition of the thin beds. The laminar nature of the formation is now more evident, especially on the reprocessed short normal curve. The result is a better match to the thin bed resolution of the array induction and the thin beds seen in the core photos. Also, the apparent resistivity values of beds of all thicknesses have been refined significantly to more truly match flushed zone (Rxo) and uninvaded zone (Rt).

A log display for well 553 is included as an example of the processing. The log is annotated with Chevron's marker beds. The log plot includes the original and the reprocessed curves. Tracks 1-3 is a standard presentation of the original logs: spontaneous potential (SP), 16 inch normal (RES16Nor), 10 foot lateral (RES10Lat), and 19 foot lateral (RES19Lat). Track 4 has the original (RES16Nor) and the reprocessed short normal (RXO-A) curves. Track 5 has the original (RES19Lat) and the reprocessed (RT-A) 19 foot laterals. Track 6 has the RXO-A curve.

h. Accutech is applying its proprietary processing to the raw array induction data in an attempt to improve thin bed resolution. The results will be compared with Schlumberger’s processing. Accutech is doing the work for free.
2. Lodgepole Field Database

a. Union Pacific Resources provided hardcopy and digital data for Lodgepole Field in Summit County, Utah. UPRC was very cooperative, providing a wealth of data on the field:

(1) Six horizontal and twelve vertical boreholes have been drilled in Lodgepole field.

(2) 2D seismic lines both parallel and perpendicular to the fracture orientation.

(3) UPRC conducted an outcrop study of the fracture orientation in the Twin Creek formation.

(4) Resistivity, density, neutron, sonic, caliper, and gamma ray logs for almost all the vertical wells. MWD gamma ray logs for every horizontal well and formation microscanner (FMS) logs for three of them (35-2H, 4-1H and 34-1H).

(5) A velocity survey and synthetic seismogram are available for the 35-1.

(6) Cuttings for most wells.

b. A geological and petrophysical analysis of part of the field was completed for the seismic modeling and in preparation for a crosswell survey. This analysis established that fracturing in the Twin Creek Formation is selectively controlled by lithology and is concentrated in dolomitized intervals. Thin section analysis shows these intervals to be dolomitized mudstones.

c. Integration of the results with a velocity inversion analysis of the 2D surface seismic data established a correlation between a high-velocity anomaly and the main fractured interval in a horizontal well that parallels the seismic line.

d. Initial seismic modeling of the field provided the constraints for a crosswell survey and verified that the technique could be used in the field.
F. Petrophysical and Geological Analysis for Reservoir Characterization

An important part of our work is to be able to create a database including core, well logs, acoustic and seismic data to produce geological cross sections. The objective is to construct an initial reservoir model for the integration of geophysical and production data to predict permeability distribution. Below we present an example of integration between cores and well logs.

Field History

The Buena Vista Hills field is 25 miles southwest of Bakersfield in Kern County, California (Figure 1). DOE and Chevron are currently funding an advanced reservoir characterization study of the field to evaluate the feasibility of initiating CO₂ enhanced oil recovery. Their study focuses on a new infill well (653Z-26B) and the four 1950’s-vintage wells that surround it (Figure 1). The five wells will be used for a CO₂ pilot flood, with the new well serving as the injector.

An extensive petrophysical database is being amassed for the 653Z well. The entire producing interval (952.8 feet) was cored and 99.5% of the core was recovered. Numerous tests have been run on the core and a comprehensive suite of state-of-the-art wireline logs were run in the well. Crosswell seismic surveys were made of the pilot study wells.

Primary production in the field dates back to 1952. Initial oil production in the pilot study area ranged from 50 to 220 BOPD. Current oil production in this part of the field averages 20 BOEPD. Oil gravity averages approximately 30E API in the pilot area (Toronyi, 1997). The reservoir is predominately water-wet (Morea, 1997b).

Geology

Buena Vista Hills is a northwest-southeast trending, elongated, doubly-plunging anticline with two structural highs on the crest (Toronyi, 1997). The CO₂ pilot area is located on the west high, which is called West Dome.

The producing interval is a 1,300 thick section of the Upper Miocene Monterey Formation. Locally, the interval is referred to as the Antelope Shale and is subdivided into three units: the Brown Shale (upper quarter), the Upper Antelope Shale (middle half), and the Lower Antelope Shale (lower quarter). Only the Brown and Upper Antelope Shales were cored in the 653Z well. Geochemical analysis of the oil suggests that most of the present production comes from the Brown and Upper Antelope Shales (Morea, 1998).

The Antelope Shale is an unusual and unlikely reservoir rock (Morea, 1997a, 1997b; Toronyi, 1997):

1. Approximately 95% of the rock is thin (1 to 5 cm), graded, clay-free, siliceous shale beds.
2. Very thin (1 mm to 25 cm) clayey sand laminae are intercalated with the siliceous shale. The 653Z core contains 748 sand laminae; all but one are in the Upper Antelope Shale.

3. Porosity averages 28%, while permeability averages only 0.07 md. Permeability of the siliceous shale is in the microdarcy range.

4. The rock was deposited in a restricted basin as distal turbidite and hemipelagic siliceous beds. Diagenetically altered diatom tests were the source of the silica.

5. There is extensive lateral stratigraphic continuity across the field, except where sands are present.

Petrographically, the siliceous shale is silty, dolomitic, opaline mudstone. Very finely crystalline dolomite is dispersed in a matrix of isotropic opal-ct. Detrital grains are feldspar, clay, quartz, pyrite, and dolomite (Morea, 1997b).

Thin section analysis revealed that most of the porosity is matrix microporosity. Moldic and fracture pore types are also present. Interconnectedness of all pore types is poor. Pore throat radii of the siliceous shale are less than 0.5 microns and up to 2.5 - 10 microns for the sands (Morea, 1997b).

Mercury porosimetry tests indicate that at reservoir conditions only the sands are capable of producing oil (Morea, 1997b). The sands, however, constitute only 5% of the rock, so it appears that fractures may be contributing significantly to hydrocarbon production.

The extent to which the Antelope Shale is fractured and the significance of those fractures for fluid migration is open to debate. Pressure build-up analyses suggest that the fracture system is not pervasive (Toronyi, 1997). However, cores, crosswell seismic images, and satellite data all point to significant fracturing in the Buena Vista Hills field. Fracturing occurs on a variety of scales in the core and in the FMI images. Regardless of their relationship to fluid production, fractures are prevalent enough to affect borehole and interwell seismic measurements in most of the reservoir.

The Study Interval

The petrophysical parameters of the interval from 4,130 to 4,160 feet were used for the seismic modeling. This interval was chosen because it is homogeneous and has few fractures (see the FMI image in Figure 2). In fact, it is one of the least fractured intervals in the core. Core recovery was 100%. The interval is within the lower portion of the Brown Shale. Stratigraphic marker NA occurs at 4,147 feet.

The entire interval is siliceous shale, with no sand or carbonate beds. Laminations are distinct from 4,133 to 4,144 feet. They are mm’s to cm’s thick and most are parallel. From 4,144 to 4,160 feet the rock has a slightly mottled appearance in UV light. In plain light it looks very homogeneous, with only a few laminations. Most of the petrophysical properties are very consistent in the homogeneous zone, while the laminated interval has more variation (Figure 2).
The laminated portion has some fractures. Most of them are parallel and vertical to near vertical. The fractures are only a few inches long and terminate at bed boundaries. A fine grained, clay-rich material fills most of the fractures (Morea, 1997a).

Table 1 summarizes the petrophysical properties plotted in Figure 2 and used for the seismic modeling. The abbreviation for each property is the one used in Figure 2. In this figure, the second group of waves are resistivity logs. These logs were determined from array induction measurements, and they have depths of investigations of 90, 20, and 10 inches.

Table 1. Petrophysical Properties of the Study Interval (4,130 - 4,160 feet)

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Mean</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain Density*</td>
<td>g/cc</td>
<td>2.3</td>
<td>2.18 - 2.42</td>
</tr>
<tr>
<td>Permeability (Perm)</td>
<td>md</td>
<td>0.042</td>
<td>0.01 - 0.13</td>
</tr>
<tr>
<td>Porosity (POR)</td>
<td>%</td>
<td>0.26</td>
<td>0.19 - 0.33</td>
</tr>
<tr>
<td>Bulk Density (RHOB)</td>
<td>g/cc</td>
<td>1.94</td>
<td>1.82 - 2.06</td>
</tr>
<tr>
<td>Compressional Velocity (VEL COMP)</td>
<td>ft/sec</td>
<td>8,313</td>
<td>7,930 - 8,770</td>
</tr>
<tr>
<td>Shear Velocity (VEL SH)</td>
<td>ft/sec</td>
<td>4,638</td>
<td>4,360 - 5,040</td>
</tr>
</tbody>
</table>

*A grain density curve was not included in Figure 2.

In summary, the study interval is low density (2.3 g/cc), high porosity (26%), very low permeability (0.04 md), slightly fractured, laminated, siliceous shale. The mean compressional velocity is approximately 8,300 ft/sec and the shear velocity is 4,600 ft/sec.

Bibliography


Morea, M.F., 1997a, Advanced reservoir characterization in the Antelope Shale to establish the viability of CO2 enhanced oil recovery in California’s Monterey Formation siliceous shales, 1997 Second Quarter Report to DOE.

Morea, M.F., 1997b, Advanced reservoir characterization in the Antelope Shale to establish the viability of CO2 enhanced oil recovery in California’s Monterey Formation siliceous shales, 1997 Third Quarter Report to DOE.

Morea, M.F., 1998, Advanced reservoir characterization in the Antelope Shale to establish the viability of CO2 enhanced oil recovery in California’s Monterey Formation siliceous shales, 1997 Fourth Quarter Report to DOE.

Figure 1. a.) Map of Buena Vista Hills Field, b.) Map of the CO₂ Pilot Flood Area.

Figure 2. Logs and FMI image of the study interval in the 653-26B well. Porosity (POR) and Permeability (PERM) are core values; all others are log curves. All curves have been depth corrected to the AO10 array induction curve.
Characterization of Fracture Reservoirs Using Static and Dynamic Data

1. Introduction

The integration of dynamic data into petroleum reservoir characterization has been an area of active research in recent years. Previous work includes incorporation of pressure transient tests\textsuperscript{1-4}, multiwell pressure interference tests\textsuperscript{5}, and tracer tests\textsuperscript{6}. However, much of the work has been limited to single-phase flow. Recently, Vasco et al.\textsuperscript{7} integrated multiphase water-cut history into stochastic reservoir characterization. Integration of multiphase production history is particularly important since it is the most widely prevalent dynamic data.

We have investigated various inversion techniques with the objective of integrating multiphase production history into reservoir models. Our approach combines elements of geostatistics within the framework of inverse modeling. The methodology for solving the inverse problem has two principal components: a 3-D, multiphase streamline simulator as a forward model and simulated annealing/conjugate gradient methods as the minimization scheme. Some of the techniques that we have looked at are as follows.

a. Traditional and Very Fast Simulated Annealing

This approach involves randomly perturbing permeabilities at various spatial locations until a misfit criteria is satisfied. The misfit criteria can be, for example, matching the observed and predicted production data within a certain tolerance. Simulated annealing is used as the minimization procedure. Briefly, in such an approach, the permeability is first altered at a randomly selected location and the change in misfit is computed. If the misfit is reduced, then the perturbation is accepted. If the misfit increases, then the perturbation is accepted with a probability given by the Gibb's distribution. By allowing for the selection of unfavorable moves, such an approach provides a mechanism for reaching the global minimum.

b. Structure Preserving Inversion Schemes

Because of the very large computational times associated with simulated annealing, we have also explored alternative gradient-based approaches. This approach involves a gradient-based iterative minimization procedure such as the conjugate gradient method that perturbs reservoir properties (for example, permeability) only at selected pilot locations to match the production history. The resulting changes in properties at the pilot locations are then transferred to other locations by kriging that preserves the initial structure such as the permeability covariance matrix. Typically only about 10-15\% of the grid points are used as pilot locations resulting in orders of magnitude savings in computation time compared to simulated annealing. We have developed a two-stage approach to data integration. First, we generate conditional simulations of permeability fields based on static data such cores, logs and seismic response. Next, we calibrate the resulting models to dynamic data using a novel approach called structure-preserving inversion. The principal advantages of our approach are: (1) it preserves the underlying structure derived based on the static data, (2) it dramatically reduces the computation time compared to full perturbation inversion by significantly reducing the parameter space, and (3) it offers flexibility in incorporating subjective information based on prior knowledge of the reservoir.
It is well known that results obtained through inverse modeling are non-unique in general. To address the non-uniqueness, an ensemble analysis is carried out. In this way, the most likely model and the associated uncertainty can be determined.

2. Theoretical Background

In this section the theoretical basis for the Simulated Annealing and Structure Preserving Inversion Schemes is discussed in some detail.

a. Minimization Scheme: Simulated Annealing

The objective of the inverse problem in reservoir characterization is to minimize the error function, \( E(m) \), where \( m \) is a model vector. It can be porosity, permeability or other static data of reservoir. The problem with the solution of linear/linearized methods is that these methods will always find the minimum closest to the starting model since the updating model is only accepted if the current error is less than the error computed for the previous model. It is called a greedy algorithm in which the model may be trapped in the local minimum.

This problem is then solved by using the minimization scheme that can find the global minimization of a function \( E(m) \). One of the methods is Simulated Annealing (SA). This method has been used in variety of optimization problems that involve finding optimum (minimum or maximum) values of a function of a very large number of independent variables. The simulation is adopted from the annealing process in which a solid in a “heat bath” is warmed by increasing the temperature, followed by slow cooling (annealing) until it forms a crystal. The formation of a crystal is related to reaching the global minimum of the error energy in an optimization problem. If the cooling is rapid (quenching), a meta stable glass can be formed, which is parallel to getting trapped in a local minimum.

In simulated annealing, at each temperature, the solid is allowed to reach thermal equilibrium where the probability of being in a state \( i \) with energy \( E_i \) can be represented by the Gibbs or Boltzmann probability density function:

\[
p_{E_i} = \frac{\exp \left[ -\frac{E_i}{KT} \right]}{\sum_{j \in S} \exp \left[ -\frac{E_j}{KT} \right]} = \frac{1}{Z(T)} \exp \left[ -\frac{E_i}{KT} \right],
\]

(1)

where the set \( S \) consists of all possible configurations, \( K \) is Boltzmann’s constant, \( T \) is temperature, and \( Z(T) \) is the partition function that is defined as:
The key of this process is the requirement of equilibrium. As mentioned in the previous paragraph, the process will be annealing or quenching depending on the cooling schedule. Many proposed algorithms have been reviewed in many references. The popular SA algorithm in solving variety of problems is Metropolis Algorithm.

b. Metropolis Algorithm

The Metropolis algorithm has been introduced by Metropolis\textsuperscript{8} to simulate the evolution of a solid in heat bath to thermal equilibrium. Then Kirkpatrick et al\textsuperscript{9} formulated this algorithm and used in wide variety of applications. The algorithm starts with initial model \( m_i \) with energy \( E(m_i) \), then a perturbation to \( m_i \) is made by replacing one cell of model \( m_i \) with random variables. The random variable is taken from normal (Gaussian) distribution, which has known mean and standard deviation. The new model, let say \( m_j \), is then compared with the previous model, \( m_i \), and evaluated the error \( \Delta E_{ij} \) as follows:

\[
\Delta E_{ij} = E(m_j) - E(m_i),
\]

Then the new model is accepted or rejected depended on the value of the error \( \Delta E_{ij} \). If \( \Delta E_{ij} = 0 \) then the new model is always accepted. However, if \( \Delta E_{ij} > 0 \) then the new model is accepted with Gibb’s probability:

\[
p = \exp \left[ - \frac{\Delta E_{ij}}{T} \right],
\]

where \( T \) is the “temperature” or we call control parameter which is initially high and reduced slowly according to specified a cooling schedule. Geman and Geman\textsuperscript{10} showed that a necessary and sufficient condition for convergence to the global minimum of SA is given by the following cooling schedule:

\[
T(k) = \frac{T_0}{\ln k},
\]

where \( T_0 \) is initial temperature and \( k \) is number of iteration.
c. Fast Simulated Annealing (FSA) Algorithm

Fast Simulated Annealing was introduced to modify Metropolis Algorithm. This new algorithm is very similar to the Metropolis SA except that it uses a Cauchy-like distribution to generate the models for testing in the Metropolis criterion\textsuperscript{11}. The Cauchy-like distribution is defined as follows:

\[ f(\Delta m) \propto \frac{T}{(\Delta m_i^2 + T^2)} \tag{6} \]

where \( T \) is analogous to temperature and it is lowered to specific cooling schedule and \( \Delta m_i \) is the perturbation in the model parameter. This Cauchy-like distribution allows for selection of models far from the current position because of its long tail, while at low temperature it prefers models in the close neighborhood of the current location. The distribution has a flatter tail than a normal (Gaussian) distribution. Therefore, it has a better chance of getting out of local minima\textsuperscript{11,12}.

Because of the choice of generation function given by Eq. 6, the cooling schedule required for convergence is no longer logarithmic but it is now inversely proportional to iteration number\textsuperscript{11}:

\[ T(k) = \frac{T_0}{k} \tag{7} \]

d. Structure Preserving Inversion

Characterizing heterogeneous permeable media using dynamic production data generally involves the solution of an inverse problem. It is well known that such inverse problems are ill-posed in general and require ‘regularization’ which restricts the high frequency content in the degrees of freedom of the admissible solutions. Two approaches to such regularization are (1) incorporation of a prior permeability covariance\textsuperscript{13} when the statistics of the permeability field can be estimated from the field data or (2) imposing smoothness conditions specified by the discrete model Laplacian, a finite difference approximation to the second spatial derivative of the permeability field.\textsuperscript{7,14} The equivalence between the two approaches for an exponentially decaying covariance function has been demonstrated by Tarantola.\textsuperscript{14}

The structure preserving inversion is similar in principle to the pilot point method originally proposed by de Marsily et al.\textsuperscript{15} and incorporates prior information in a different way in the sense that the covariance matrix is embedded in the parameterization of the permeability field itself.\textsuperscript{16} The approach incorporates two distinct types of point measurements (1) actual permeability measurements obtained or inferred based on field data and (2) estimated permeability at carefully selected locations known as pilot points. The pilot point values will be
obtained by matching the dynamic data in a least squares sense as discussed below. The final permeability field is derived from these point measurements through the use of the kriging algorithm.

Let \( y_n = (y_m, y_p) \) denote the extended vector containing both measurements and computed pilot point values. Similarly, let \( u_n = (u_m, u_p) \) contain values at the corresponding location in the initial permeability field, \( Y_m \). Typically, \( Y_m \) will be obtained using stochastic simulation techniques such as the sequential Gaussian method and will incorporate the static data. If \( G_n \) is the matrix containing the kriging weights, then integration of dynamic data can be expressed as a conditional simulation as follows

\[
Y_{cc} = Y_n + (Y_{us} - U_n)
\]
\[
= Y_{us} + G_n (y_n - u_n)
\]
\[
= Y_{us} + G_n \Delta y_p
\]

where \( Y_{cc} \) is the permeability field after calibration using the dynamic data and

\[
Y_n = G_n y_n
\]
\[
U_n = G_n u_n
\]

The elements of the matrix \( G_n \) can be obtained by solving a set of kriging equations. Let \( \lambda^T \) denote a row of matrix \( G_n \). Then we have

\[
\lambda = \begin{bmatrix} C_{y_m y_m} & C_{y_m y_p} \\ C_{y_p y_m} & C_{y_p y_p} + \mu I \end{bmatrix}
\]

where, \( C_{y_m y_m} \) contains the covariance between the elements of \( y_n \) while \( C_{y_p y_m} \) contains the covariance between \( y_m \) (log permeability at the location to be estimated) and elements of \( y_n \), and \( \mu^T \) is the Lagrange parameter. Eq. 3, represents the kriging estimator without and with non-bias conditions.

All that is left at this point is to estimate the values at the pilot point locations. This is accomplished by minimizing the performance index

\[
J(y_p) = (d - g(y_p))^T C_D^{-1} (d - g(y_p))
\]

where \( y_p \) is the vector of parameter values at the pilot locations, \( d \) is the observed data vector, \( g(y_p) \) is the model prediction based on the forward model, \( g \) and \( C_D \) is the data weighting matrix containing the measurement errors.
The primary advantage of the structure preserving approach is that it can generate physically realistic permeability fields that are conditioned to both static and dynamic measurements. It also provides significant computational advantage over full perturbation methods\textsuperscript{1,7,13} since typically only 10-15\% of the grid locations are used as pilot points, thus significantly reducing the parameter space. By carefully selecting the location of the pilot points, the calibrated permeability field can be made to closely replicate the covariance structure of the permeability field derived based on the static data. Finally, the approach offers a flexible way to incorporate qualitative geological information based on the prior knowledge of the reservoir. For example, placement of pilot points along a known fault can convey information regarding the ‘sealing’ or ‘non-sealing’ nature of the fault. In what follows, we discuss application of the structure preserving approach to integration of dynamic production data during reservoir characterization. Some variations of the pilot point approach have been applied for integrating transient pressure data by other authors.\textsuperscript{17,18} However, to our knowledge, this is the first time such an approach has been used to condition stochastic reservoir models to multiphase production history.

3. Application

a. Results of a Synthetic Model by Using Simulated Annealing

In this case our synthetic model is a permeability field of 21x21x1 generated by using the sequential Gaussian method, specified by a given mean, standard deviation, and variogram. See Fig. 1. Fig. 2 shows the water-cut history of production wells from a nine-spot pattern (8 production wells and one injection well) placed on the synthetic permeability field. These results generated by using the 3-D multiphase streamline simulator were discussed in our earlier reports. The coordinates of the wells are as follows:

<table>
<thead>
<tr>
<th>Wells No.</th>
<th>x</th>
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</tr>
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<tr>
<td>1</td>
<td>1</td>
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</tr>
<tr>
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<td>1</td>
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</tr>
<tr>
<td>9</td>
<td>11</td>
<td>11</td>
<td>1</td>
</tr>
</tbody>
</table>

Running inverse modeling by honoring the data of the 9 wells resulted in a permeability realization as shown in Fig. 3. The matches to the water-cut history are shown in Fig. 4. This inverse modeling uses the Simulated Annealing (SA) Metropolis algorithm as the minimization scheme. Fig. 5 shows three different realizations of the permeability field using the same of water-cut history. As it is observed, the individual models generated by using the production data do not reproduce the reference model at all.
The following step is to carry out a series of inversions (up to 50 inversions) and use the ensemble analysis to examine underlying features shared by all the models. Fig. 6 shows the ensemble median model corresponding to different number of realizations. In this stage, the median model reproduces the features of the reference permeability field quite well. Fig. 7 shows the isotropic semivariogram of median model compared to the reference permeability field.

b. **Results of a Synthetic Model by Using Structure-Preserving Inversion**

In this case our synthetic model is a single-layer heterogeneous reservoir model with 25x25 grid cells as shown in Fig. 8 and Fig. 9. A waterflood is simulated using a 9-spot pattern with one injection well in the center and eight producing wells. The principal feature of the reference permeability field (Fig. 9) is the low permeability region in the southwest corner and our objective is to reproduce this low permeability region based on the waterflood response. Our approach to dynamic data integration assumes that we already have a reasonable starting model based on the static data. Thus, we start with an initial permeability field that contains most of the features of the reference model except for the low permeability region (Fig. 10).

Up to 1200 days of water-cut data were generated for the eight producing wells from the reference model using the streamline simulator as shown in Fig. 11. Also shown in the plot are the water-cut predictions from the initial input model. Due to the missing low permeability region in the initial input model, the water-cut response exhibits significantly earlier water breakthrough in producing well 1, whereas breakthrough is delayed in wells 2, 4, 5, 6, 7, and 8 compared to that from the reference model. Water did not breakthrough in well 3 for up to 1200 days for both reference and input models.

We will examine three scenarios which correspond to three sets of pilot points used for permeability perturbation: Case 1, uses all the cells except for those which are at or in the close vicinity of well locations, and is thus similar to conventional full perturbation; Case 2, uses 15 percent of the cells as pilot points (94 locations) and, Case 3 uses 7.5 percent of the cells (46 locations).

c. **Case 1: Full Perturbation Inversion**

In this case all the cells are selected as pilot point locations except for those which are close to the injectors and producers. The reason for not selecting these cells is that the objective function is generally much more sensitive to changes in permeability values at these locations compared to other locations. Hence, the gradient-based inversion methods tend to have a “selective preference” in perturbing permeability at these locations to match the observed production data. Another reason is that the permeability values at or in the close vicinity of the wells are generally known or can be reasonably estimated. An integral part of any gradient-based inversion is the parameter sensitivity coefficients. For structure-preserving inversion in particular, such sensitivities can convey very useful information regarding the location of the pilot points. Fig. 12 displays sensitivity coefficients that indicate change in the objective function with respect to changes in natural log permeability at each cell location. A positive value of the sensitivity coefficient indicates that a decrease in permeability in that cell location will result in a reduction of the objective function and
Thus, to match the observed water-cut history, permeability values in cell locations with positive sensitivity coefficients should be reduced, while in cells with negative sensitivity coefficients need to be increased. From Fig. 12, it is clear that the permeability values in the initial input model should be reduced in the southwest corner to match the water-cut data. Such an indication makes sense because the initial permeability field (Fig. 10) does not have the low permeability region compared to the reference model (Fig. 9). The magnitude and distribution of sensitivity coefficients also suggest that more pilot point locations should be positioned in the south side of the model compared to the north side. This strategy will be adopted in Case 2.

Fig. 13 shows the water-cut match after 10 iterations and Fig. 14 shows the inverted permeability field after the match. A visual comparison with the initial permeability field indicates that the main structure of the two permeability fields has remained much the same. However, the inverted permeability field does exhibit a reduced permeability region in the southwest corner as present in the reference model.

d. Case 2: Pilot Point Inversion (15% of Cells)

Here we chose 15% of the total cells as pilot points (94 pilot locations). Fig. 15 shows the distribution of pilot locations and well positions. The pilots are distributed in a regular fashion with more in the south side as suggested by the sensitivity analysis in Case 1.

Following the proposed structure preserving inversion procedure, we iteratively perturb the permeability at the selected pilot point locations and then transfer the change to other locations using the kriging algorithm. The variogram used in kriging is derived from the initial model with a correlation length of 9 cell units. Fig. 16 shows the inverted permeability filed after matching the water-cut history. Notice that the low permeability distribution in the reference model is resolved in a smoothed fashion in the southwest side of the inverted model similar to Case 1 (Fig. 14).

Variograms of the initial, reference, and inverted permeability fields are shown in Fig. 17 to examine the effect of pilot point inversion on the underlying structure of the permeability field. The variogram of the inverted model is between those of the initial model and the reference model. The inverted model captures the main structure of the initial model while at the same time reflects the structure of the reference model as a result of integrating water-cut response from the reference model.

e. Case 3: Pilot Point Inversion (7.5% of Cells)

In order to examine how further reduction in pilot points will affect the inversion results, 46 pilot points, about 7.5% of total cells, are used in Case 3 as shown in Fig. 18. The inverted permeability field after matching water-cut data is shown in Fig. 19. Similar to the previous cases, the low permeability distribution in the reference model is resolved as a smoothed low permeability region in the southwest side of the inverted permeability field.
f. Comparison of Computational Efficiency

One important criteria for evaluating inversion techniques is computational efficiency. Table 1 shows a comparison of computer time used in the three cases for the same convergence criteria. The overall performance of Case 2 is about 4.5 time faster compared to the full perturbation Case 1. It is expected that for large-scale models savings in computer time by using the pilot point inversion would be much more significant. Table 1 also indicates that using less pilot points does not necessarily guarantee better computational efficiency since it requires additional number of iterations to reach the acceptable misfit.

4. Field Examples

We are currently examining the field data from two fractured reservoirs for possible application of the structure preserving inversion scheme. The relevant information from these fields are summarized below.

a. Yates Field

The Yates Field is located in Pecos and Crocket Counties of West Texas, approximately 80 miles south of Midland. It was discovered in 1926 and had produced nearly 1.3 billion barrels of oil until the beginning of 1993. The recovery at that time was approximately 30% of the estimated OOIP. At the end of 1992 a total of 1100 production and 57 injection wells were active. See Fig. 20.

The Yates Field produces from four formations\(^{19}\) (Seven Rivers, Queen, Grayburg and San Andres). The Seven Rivers interval is composed of interbedded sandstone, siltstone and some dolomite. The Queen consists of siltstones and fine sandstones interbedded with silty dolomites. The Grayburg is composed of interbedded dolomite and siltstone. Unlike the overlying Seven Rivers and Queen, the Grayburg produces from fractured dolomite.

The San Andres is the thickest and most prolific formation within the Yates Field. The unique reservoir quality of the San Andres is a function of depositional carbonate shoal facies, extensive natural fracturing, karst, dolomitization, and precipitation of secondary calcite cement

b. Main Events in Yates Field

Before 1976, the Yates Field was operated under depletion, producing mainly from the fractured Grayburg and San Andres formations. As pressure depleted free gas evolved and migrated rapidly through the natural fracture system to the structural top to form a slow expanding gas cap\(^{19}\).

Following the Yates Field unitization in 1976, a gas injection pressure maintenance program was instituted to retard the invasion of water into the oil producing portion of the reservoir and to conserve energy. This allowed greater utilization of the efficient gas cap gravity drainage mechanism.
Additional pressure maintenance and infill development well programs were implemented during the 1980's to improve recovery. Starting with a pilot waterflood on the western flanks of the field in 1979, a pattern polymer flood was expanded into additional portions of the westside oil column from 1983 to 1986. The polymer flood operations were suspended on schedule in 1989 while pattern waterflooding continued. During late 1985, carbon dioxide injection commenced in the northern, eastern and crestal areas of the field. Carbon dioxide injection was abandoned in 1991.

A Co-Production Project was initiated in late 1992. The main goal of this project was to de-water reservoir areas containing oil bypassed by water encroachment by lowering the gas-oil and water-oil contacts using high-volume water withdrawals and gas cap inflation (increase reservoir pressure using methane and nitrogen injection). A second effect is the initiation of gas-oil gravity drainage within the expanded gas cap.

The water invasion occurred in the field's fracture network bypassing the oil located in the highly oil saturated matrix. The oil can only be recovered if the water is removed from the fracture network allowing the oil to flow from the matrix into the fracture system. During the Co-Production Project the aquifer encroachment was reversed and a significant amount of oil was recovered that could not have been produced by conventional primary or secondary production methods.

In addition to the Co-Production Project, since 1992 the natural fracture network within the Yates Field has been studied in detail. It was shown that all the wells in the field receive matrix fluids from conduits in the extensive natural fracture network. By focusing on the areas where the fracture network is most developed, it was possible to take advantage of the natural drainage system by maximizing withdrawals from high rate, high efficiency wells in these areas. Between 1992 and 1994 almost 400 wells where shut in while keeping a stable total daily oil production rate.

For the existing wells it was possible to identify those located in highly fractured areas of the reservoir based on their production history. In 1993-1994 more than 30 new short-radius horizontal wells were drilled. A discrete fracture network modeling approach: *FracMan* was used to study the spatial distribution of fractures in the field and to optimize location and orientation of the new horizontal wells.

c.  **Difficulties Encountered**

Because San Andres formation is the target, efforts to gather information related to the field have been performed. It has been found that most wells in the Yates field are drilled only to relatively shallow depths and do not penetrate the entire San Andres. Therefore, properties of the reservoir, extrapolated between wells in the geologic model, are more reliable just for the upper San Andres. The same limitation was found in the cored wells and in the well completion information.
d. Data Bases

With the information downloaded, a set of data bases have been organized for Queen, Graybour and San Andres formations, in Tract 49. The data bases elaborated include: well test completion data, petrophysical analysis of cored wells, well logs, structural cross-sections (NW-SE & NE-SW), and total production of the field.

e. Additional Information Required

Some additional information that could be beneficial is listed below. We are attempting to get this information from the operator.

Geological interpretation: base map, correlation of zones of interest, structure map, structure cross sections.

Stratigraphic cross sections.

Petrophysical evaluation: core-log adjustment, $\phi$-$k$ relationships, electrical properties, rock compressibility, capillary pressure curves, relative permeability curves, wettability, and interfacial tension.

Original fluid levels.

Net productive thickness: $\phi$, k, Sw cutoffs used, net thickness isopach maps.

Porosity and permeability variations, isopermeability and isoporosity maps.

Well test data initial productivity, pressure and level tests, buildup and drawdown tests, productivity index measurements, production tests, injectivity tests.

Water-oil and gas-oil contact movement history.

Fracture network mapping.

f. Future Plan

Build a geostatistical model for the reservoir description
Perform flow simulation using the streamline model
Incorporate production data using inverse modeling.

g. Buena Vista Field

We are also looking at the possibility of applying structure preserving inversion to the Buena Vista field. The primary objective here is to conduct advanced reservoir characterization and modeling studies in the Antelope Shale reservoir. Characterization studies will then be used to determine the technical feasibility of implementing a CO$_2$ enhanced oil recovery project in the Antelope Shale in Buena Vista Hills Field.
h. Available Information

The study is going to be focused in the pilot area shown in Fig. 21. For this pilot the available information are as follows:

- LAS files for wells: 553, 554, 563, 564, and 653Z.
- Corrected LAS files for wells: 553, 554, 563, and 564.
- Deviation information for well 653Z.
- Production information (Excel) for wells: 553, 554, 563, and 564.
- Core information of well 653Z (Excel).

i. Future Plan

The available information has already been loaded in $RC^2$, a 3-D geostatistical reservoir characterization software. The first step is to relate permeability to resistivity measurements and spontaneous potential (sp) using core data and well logs from the injector well. After having a correlation for permeability at each well, this information can be used as an initial model to invert production data. The next step is to determine the relationship between the wave velocities and the permeability at the borehole scale by using wave velocities corrected for scattering effects. This could be used to invert the production data for new values of permeability at the interwell scale.

5. References


Oliver, D. S.: “Incorporation of Transient Pressure Data into Reservoir Characterization,” In Situ (1994), Vol. 18, 243-75.


Web Page of Fractured Reservoir Discrete Feature Network Technologies, General Information.

Software developed by Golder Associates Inc., FracMan Development Team.
Figure 1  Reference permeability field of synthetic data

Figure 2  Water-cut history of reference model
Figure 3  Permeability field from a single realization

Figure 4  Water-cut history matching from a single realization
Figure 5  Single Model using SA without smoothing from different realization
Figure 6  Median Model using SA without smoothing from ensemble models
Figure 7: The Isotropic Semivariogram of medium models from SA Metropolis
Fig. 9 Permeability field used as the reference (true) model.

Fig. 10 Permeability field used as the initial input for inversion.
Fig. 11  Water-cut response from the reference model and the input model for a nine-spot.

Fig. 12  Sensitivity coefficients of the objective function (gradient) for full perturbation, Case 1.
Fig. 13  Water-cut match after 10 iterations using full perturbation inversion, Case 1.

Fig. 14  Permeability field derived from inversion of water-cut history using full perturbation inversion, Case 1.
Fig. 15  Pilot point locations for Case 2.

Fig. 16  Permeability field derived from inversion of water-cut history using pilot point inversion, Case 2. Notice the low permeability region in southwest side.
Plot point locations for Case 3.

Comparison of variograms for the reference, initial, and inverted logs.
Fig. 19  Permeability field derived from inversion of water-cut history using pilot point inversion, Case 3. Notice the low permeability region in the southwest side.

Figure 20  Location of Yates Field.
Figure 21  Structure Contour in Point “P”, Buenavista Field

TABLE 1 - COMPARISON OF COMPUTATIONAL EFFICIENCY FOR SYNTHETIC CASE STUDY*

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h,m:s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>15 %</td>
<td>7.5 %</td>
<td></td>
</tr>
<tr>
<td>of Cells</td>
<td>(5 iter.)</td>
<td>(6 iter.)</td>
<td>(13 iter.)</td>
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<tr>
<td>cells</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(5 iter.)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Real  1.44:10  16:59  38:35
User   44:36   10:23  20:40

*conducted at SGI machine
III. SUMMARY OF ACCOMPLISHMENTS

A. Elastic Wave Propagation in Random Geological Media

We have investigated the problem of elastic wave propagation in random media. This is important to understand because almost all geologic media is spatially heterogeneous by nature, consisting of a random agglomerate of many-sized rocks, soil and strata. In this formulation, a plane-harmonic seismic wave propagates in a medium having random material properties in the vertical direction. The random field representation was introduced through the intrinsic rock physical properties of the elastic medium. Each of these intrinsic properties was assumed to have a log-normal probability density function, and the random field representation is expressed in terms of these log-normal probability density functions. The constitutive law, the mass balance and the moment balance equations were written in the Fourier-Stieltjes representation using random Lamé coefficients and random mass density. The stochastic wave equation was developed by introducing a perturbation approach based on an infinite series expansion of both random coefficients and the displacement solution in terms of σ-parameters (standard deviations of the random material properties). The method had yield an integral representation of the displacement wavefield based on the Green’s function. This representation was expressed in terms of the random rock physical properties of the medium. To test the displacement wave solution, synthetic seismograms and dispersion due to scattering effects were calculated for stiffness and density fluctuations of the random medium.

B. Application of Algorithms for Processing Well Log Data

Attenuation of seismic energy and velocity anisotropy consists of an intrinsic component and apparent component caused by scattering from velocity and density heterogeneity. The intrinsic component of attenuation is very important in determining fluid properties through a reservoir. However, it may be difficult to distinguish between the apparent (layer-induced) effects caused by elastic scattering and intrinsic components when thin layers are present, unless detailed information is available on the velocity structure. We have implemented a scattering correction technique to predict the intrinsic dispersion and intrinsic attenuation from sonic velocity data. We constructed a high-resolution velocity model based on FMI logs and quality factors derived from sonic log data recorded at Buena Vista Hills field. The wave response for intervals at different frequencies were obtained using a viscoelastic plane-wave modeling code to model a finely layered region. Intrinsic attenuation is determined from synthetic seismograms using the two station spectral ratio method, and a scattering correction techniques based on the contribution of the reflection coefficients that is caused by thin layers. This scattering correction technique is also applied to the total velocity dispersion to predict intrinsic dispersion. The applicability of the method is demonstrated by comparing the predict intrinsic attenuation results based on the layered model with the intrinsic attenuation derived from sonic logs.

In summary, a methodology has been developed for building a high resolution velocity model within available well control. The high resolution velocity model is used to predict the component of dispersion that is due to elastic scattering. The rest of the difference between the observed sonic velocity and the ray theoretical velocity through the formation is assumed to be due to intrinsic attenuation. Since there are some intrinsic mechanisms which are believed to be active at sonic frequencies, e.g., the Biot mechanism, our high frequency model of the formation is assumed.
to still have some intrinsic attenuation at the frequencies characteristic of the ray theoretical velocities. Thus, when the wavelengths become comparable to the thickness of the beds involved, we are assuming that some amount of intrinsic attenuation still exists. It is this intrinsic attenuation that we measured with our sonic attenuation measurements. The velocity differences between the ray theoretical velocity and the observed velocity are attributed to elastic scattering and intrinsic attenuation. We computed the elastic scattering component by using our high resolution model of the formation and assumed the rest of the velocity dispersion was caused by intrinsic attenuation. The attenuation associated with the velocity dispersion obtained in this manner was in excellent agreement with the intrinsic attenuation measured directly from the full wave sonic. Our work has therefore led to a self-consistent velocity/attenuation model which can be used to predict the velocities for seismic experiments related to the well control. This will allow us to better predict permeability distribution from crosswell and/or surface seismic data. Such permeability distribution can be integrated with production data to obtain the optimum characterization of a reservoir.

C. Dispersion Analysis of Seismic Waves in Poroelastic Media with Azimuthal Anisotropy

A transversely isotropic model with a horizontal axis of symmetry, based on the Biot and squirt flow mechanisms, predicts acoustic waves in poroelastic media. The model estimates dispersion and attenuation of waves propagating in the frequency range of crosswell and high-resolution reverse VSP for vertical permeability values much greater than horizontal permeability parameters. The model simulations demonstrate that the preferential direction of fluid flow in a reservoir can be determined by analyzing the phase velocity and attenuation of acoustic waves propagating at different azimuth and incident angles. As a result the compressional wave can be related to permeability anisotropy to predict the preferential directions of flow in a reservoir. Theoretical predictions and dispersion of interwell seismic waves in the fractured Kankakee formation at the Buckhorn test site demonstrate that fluid flow in the rock matrix surrounding a low-velocity heterogeneity is controlled by vertical fractures. On the other hand, flow in the low-velocity heterogeneity is controlled by porosity.

D. Petrophysical and Geological Analysis for Reservoir Characterization

We cataloged and analyzed petrophysical, geological and seismic data from two reservoirs, Buena Vista Hills and Lodgepole Databases. These data have been used to validate geophysical and petroleum engineering concepts for reservoir characterization. Also the data have been used to demonstrate applications by means of poster and paper presentation and publications. The data are being interpreted to produce geological and geophysical models to be integrated with production data to map permeability distributions between wells.

An application using Lodgepole data. Surface seismic interpretation with well logs at the Lodgepole field delineated the major geological boundary members of the Twin Creek reservoir. The petrophysical analysis provided the rock physical properties and thickness of the Leeds Creek, Watton Canyon, Boundary Ridge, and Rich Members of the Twin Creek Formation. The surface seismic, and the horizontal well log information provided the extension of the fracture zone in the Watton Canyon Member. The petrophysical analysis including thin section cuttings samples,
and FMS images demonstrated that the major fractures occur in the dolomitic mudstone rocks. Since
the Watton Canyon Member is predominantly dolomitic mudstone, it is the member of major probability
of having fractures. In addition, the velocity inversion results show that a high-velocity anomaly in
the image correlated with a fracture zone intercepted by the horizontal well. This suggested that
velocity image can be used to map fracture zones in the Watton Canyon at the Lodgepole.

An application using Buena Vista Hills Data. Attenuation of seismic energy and
velocity anisotropy consists of an intrinsic component and apparent component caused by scattering
from velocity and density heterogeneity. The intrinsic component of attenuation is very important in
determining fluid properties through a reservoir. However, it may be difficult to distinguish between
the apparent (layer-induced) effects caused by elastic scattering and intrinsic components when thin
layers are present, unless detailed information is available on the velocity structure. We have
implemented a scattering correction technique to predict the intrinsic dispersion and intrinsic
attenuation from sonic velocity data. We constructed a high-resolution velocity model based on FMI
logs and quality factors derived from sonic log data recorded at Buena Vista Hills field. 360-inch
layers using a viscoelastic plane-wave modeling code modeled a finely layered region. Intrinsic
attenuation is determined from synthetic seismograms using the two station spectral ratio method,
and a scattering correction techniques based on the contribution of the reflection coefficients that is
caused by thin layers. This scattering correction technique is also applied to the total velocity
dispersion to predict intrinsic dispersion. The applicability of the method is demonstrated by
comparing the predict intrinsic attenuation results based on the layered model with the intrinsic
attenuation derived from sonic logs.

E. Characterization of Hydrocarbon Reservoirs Using Static and Dynamic Data

A reservoir model derived from the static data such as geologic, well, and seismic
data, will result in fluid flow predictions that do not necessarily match the observed field production
history. Since our ultimate objective is to build a reservoir model for future performance predictions,
it is imperative that such models adequately reproduce the past performance history. Thus, the model
needs to be further improved by integrating dynamic data such as transient pressure and tracer
response and multiphase production history. This is particularly critical for fractured reservoirs since
the fluid flow may be governed by a few dominant fractures.

Conditioning reservoir models to dynamic data such as transient pressure, tracer, and
multiphase production history typically requires the solution of an inverse problem. Such inverse
problems are generally computationally intensive. More importantly, while satisfying dynamic data,
inverse methods often result in reservoir models that do not conform to the spatial patterns derived
from the static data. The key objective for dynamic data integration is to construct stochastic
reservoir models that are consistent with the static data while reproducing available dynamic data.

The focus of our efforts during the second year has been devoted to investigate
various inversion techniques with the objective of integrating multiphase production history into
reservoir models. Our approach combines elements of geostatistics within the framework of inverse
modeling. The methodology for solving the inverse problem has two principal components: a 3-D,
multiphase streamline simulator as a forward model and simulated annealing/ conjugate gradient
methods as the minimization scheme. The major accomplishments during this period can be summarized as follows:

Developed a new procedure called ‘structure preserving inversion’ to efficiently integrate dynamic data such as transient pressure, tracer and multiphase production history into stochastic reservoir models. A computer program has been developed to apply the structure preserving inversion technique. The program utilizes a previously developed 3-D multiphase streamline simulator as a forward model and a conjugate gradient based minimization scheme for inversion of dynamic data. The computer program has been tested on synthetic data and is currently being applied to field data. Two technical papers have been written outlining the forward model and the inversion procedure. These are as follows:


IV. TECHNOLOGY TRANSFER

A. Marathon

A meeting was scheduled with Marathon’s Denver office regarding problems they were having with the reservoir at Yates field in West Texas. Jorge Parra, Southwest Research Institute (SwRI) and Ray Brown, Oklahoma Geological Survey (OGS), flew to Littleton, Colorado in order to discuss the potential application of the technologies being developed during this DOE project and to review the types of exploitation problems Marathon is facing in fractured reservoirs. At the same time an examination was made of the data available for characterizing the fracture systems at Marathon’s Yeats field in west Texas.

Yeats field is producing from the San Andres formation, a fractured dolomite. Although the matrix porosity is an important element, the production is dominated by large fracture zones and caverns within the reservoir. The reservoir consists of a gas cap over an oil leg resting on top of water. Marathon intends to exploit the reservoir by injecting steam into the gas cap in order to accelerate the production of the reservoir. The problem occurs in steam flooding when nearby fractures direct the steam along unknown paths throughout the reservoir. Marathon wants to map: (a) the movement of steam throughout the reservoir; and (b) the fracture systems responsible for redirecting the steam.

With regard to the mapping of the steam movement throughout the reservoir, Marathon representatives listed the following possible causes of velocity variations within the reservoir associated with the presence of the steam front:

- changes in reservoir temperatures from 80 °F to 470 °F can cause a decrease in porosity;
- closure of small aperture fractures;
- 400+°F causes a change in bulk modulus and shear modulus;
- “steam hits”- water in formation flashes (goes to zero, i.e., evaporates);
- water condenses away from steam front;
- distillation of oil (fractional vaporization);
- oil draining;
- water draining;
- carbonate dissolution;
- changes in background stress change due to changes in pressure within the reservoir.

(Note: Due to pressure differences within the reservoir between two wells 1100 feet apart, Marathon detected a 10 degree rotation of the maximum principal stress direction. The ambient pressure of the reservoir is 450 PSI while the applied pressure near the steam front is 500 PSI. The implication here is that the steam front introduces a rotation in the stress axes and possibly affects the anisotropy of the reservoir).

The fracture systems seem to consist of antithetic fractures and caverns associated with the fractures rather than a single uni-directional fracture pattern. In addition, the fractures range in aperture from the mm range up to 20 cm or more. Although the smaller fractures are of interest,
the “high capacity channels” and associated caverns appear to be the major concern in steam flooding for this field.

The initial study of this field could be used to: (a) address issues of which types of waves can best be used to probe the reservoir; (b) evaluate simple models with fractures; and (c) quantify and qualify the differences between “total porosity velocity” determined by a suite of well logs and “the primary porosity velocity” determined by porosity.

“Total porosity” is used above to describe the combined porosity due to primary and secondary processes. Marathon, has developed a method of computing the velocity every six inches within the hole that would be measured (at lower frequencies than those used by the sonic log) if a low frequency sonic were available. One interpretation of the difference between the high frequency measurement (the sonic) and the low frequency measurement is the effect of elastic scattering and attenuation. If so, then the problem remains of estimating the frequency differences between the “total porosity velocity” and the “sonic velocities”. If the velocity drift (dispersion) could be unraveled, it might be the key to developing seismic methods which could be used to map fracture porosity throughout the reservoir.

B. Conoco

A meeting and exchange of presentations was made with representatives of Conoco at the Southwest Research Institute. Members of the research team representing the Southwest Research Institute gave presentations on different aspects of the research being funded by the DOE. Then a general discussion of problems being faced by Conoco were discussed. Conoco representatives gave the following list of research topics that they want investigated:

- depth imaging
- processing-application (anisotropy and attenuation)
- AVO inversion
- geostatistics
- geopressure prediction
- geophysical petrophysical integration
- 3D footprints/3D geostatistics
- time-lapse seismic

Conoco representatives indicated an interest in this project for two reasons: (1) the prediction of the frequency-dependence of AVO effects; and (2) the dispersion of borehole seismic measurements

C. Texaco/Halliburton/LandMark

One of the members of the research team, R. Brown, made a presentation on velocity dispersion to the New Orleans chapter of the Society of Exploration Geophysics. After the presentation, a meeting was set up with Texaco and a representative of LandMark regarding some of their problems in the state waters/offshore Gulf of Mexico blocks. The engineer involved feels that Texaco is leaving a large percentage of the petroleum reserve in the ground because of problems in
mapping and understanding the geometry of the reservoirs. Texaco indicated a real interest in this research and were willing to share data with the project.

Next, R. Brown had a meeting with Halliburton and a representative of LandMark there in New Orleans. Halliburton is very interested in this project because it could mean the creation of a great deal of business with Texaco. LandMark is interested because they want to be on the leading edge in the development of integrating well control with 3D seismic grids.

In summary, the technology transfer for this project has been very successful. Marathon, Conoco, Texaco, Halliburton and LandMark all see potential application of ideas developed during this study.

D. List of Papers and Presentation


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Collier, H. A., 1997, "Steps for Creating a Digital Well Log Database" was presented at the AAPG Southwest Section Meeting in San Angelo, TX (June 5-6). The presentation included data from Lodgepole Field.

Collier, H. A, 1998, "Creating a Digital Groundwater Database" was presented at the Texas Ground Water Association Convention in Galveston, TX (January 28-29). Some of the techniques used in our study of Lodgepole Field were presented in the poster session as examples of techniques that can be applied to groundwater studies.

Owen, T.E., 1997, Characterization of fracture reservoirs using static and dynamic data: from sonic and 3D-seismic to permeability distribution: Project Progress Newsletter No. 1, Southwest Research Institute, San Antonio, Texas.


E. Internet postings on project or software to download.

Software to calculate multiphase streamlines in 3D (forward modeling) is available from the anonymous ftp site at drillbit.tamu.edu. The forward modeling software is being used by BDM-Oklahoma, U. of Tulsa, Western Atlas, RC2, Chevron, University of Texas.

The project objective, newsletters, and capabilities available for reservoir characterization are being posted in the Internet at http://www.space.swri.edu/geophysics. A second newsletter describing accomplishments and future project directions will be issued in April 1998.

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V. SUGGESTIONS FOR FOLLOW-ON WORK

In this project the taxpayers have received the development of new techniques for exploring and developing oil and gas fields. The ultimate payoff will be in the form of increased revenues in the form of taxes paid by the petroleum industry. This work should be continued because it represents the small fragment of research left to support the oil and gas industry. In addition, this work represents the cutting edge of new research for actually determining where the oil and gas are located (rather than just finding a possible trap). Our work is critical to the detection and monitoring of fluid movement within the subsurface. As a result, our studies will help the DOE achieve its mission of increasing domestic production and delaying abandonment in an environmentally sound manner. In particular, The application of velocity dispersion is the technology being developed as a result of this project. Velocity dispersion is only beginning to be utilized by the industry and there is a big learning curve in order to transfer the technology to the industry. However, the technology will impact those companies with large holdings which have not been able to accurately detect and monitor the reservoir fluids. Velocity dispersion can potentially be used to map the fluid distribution within reservoirs. We have learned from this study how to estimate the components of dispersion at the well control. This is the first step in utilizing well control in order to predict the effects at lower frequencies. This will allow us to better predict permeability distribution from crosswell and/or surface seismic data. Such permeability distribution can be integrated with production data to obtain the optimum characterization of a reservoir.

Scaling Methodology Using Single Borehole and Broadband Interwell Data

At this point we have developed a number of critical algorithms and methods which can be used to predict the components of dispersion or attenuation as a function of measured depth along the borehole. This is a first step to integrating seismic measurements made at different frequencies. Now we have to look at expanding these concepts and our local borehole model to crosswell and surface seismic measurements. The difficulty here will be to explain the anisotropic effects of both the dispersion and the attenuation. In addition to the difficulty of anisotropy, we hope to be able to study the nature of intrinsic attenuation at seismic frequencies. This is not something which has been accomplished in a laboratory environment because of the length scales believed to be associated with some popular models for attenuation at these frequencies. Our goal then is to use our current analysis techniques to identify the components of dispersion at the well control and then to extend this representation of the subsurface in terms of attenuation components between the wells using interwell and surface seismic data. Once this has been accomplished, the relationship of these data with critical reservoir parameters will be established.

Petrophysical and Geological Analysis for Reservoir Characterization

Chevron has now provided all the analyses that they have performed to date in the Buena Vista Hills Field. This database will be used for a detailed petrophysical/geological analysis of selected intervals in the reservoir for seismic modeling and to investigate several petrophysical techniques. The following task will be performed during the final year of this project:
• Construct a petrophysical/geological model of selected fractured intervals in the fractured, producing portion of the reservoir in the 653-26B well. The analysis will incorporate the core, log, and seismic data. Petrophysical parameters will be derived for the seismic modeling.

• Permeability variations in the reservoir will be scaled to the resolution of the FMI resistivity data by examining the minipermeameter data and the permeability calculated from the Rxo curve. These data will be scaled up to the resolution of the sonic logs.

• Permeability and porosity variations across the pilot study area will be correlated and mapped. These analyses will be used to interpret the crosswell seismic profiles. This will be accomplished by using the porosity and permeability curves that Chevron calculated from the SP curves.

• Two techniques for two-dimensional forward and inverse modeling analysis of old electric logs will be evaluated. These techniques provide better resolution of thin beds and better resistivity values, both of which will improve mapping of units for the crosswell seismic interpretation.

• Investigate the accuracy of the SP-derived porosity and permeability transforms.

• Investigate two techniques (Accutech’s and Schlumberger’s) for processing array resistivity data for thin bed resolution.

• Additional data will be acquired from Chevron as it becomes available.

Long Range. The techniques used in this study will be applied to the reservoir characterization of an old carbonate field in the Texas Panhandle. The operator, Springfield Exploration, is cooperating with us in a study of the field to site additional infield wells.

Integration of geophysical and engineering data from Buena Vista Hills Reservoir

The optimum goal of this project is to develop an approximate 3D geometry of the reservoir using geological cross sections, well logs, and the results of the modeling and seismic processing. The next step is to integrate this 3D geometry and the petrophysics with dynamic data (e.g., oil production data, water cuts at individual wells, etc.) To produce an initial model. The following step is to utilize the 3D streamline simulator (developed in Phase I) as a forward model and a conjugate gradient based minimization scheme for inversion of dynamic data that was developed in Phase II. Finally, at the end of the inversion we will have a three dimensional description of the reservoir (e.g., the permeability distribution through the reservoir).
Stochastic Modeling of 2D Seismic Data

In this period, work began on a second-order 2D solution for propagation of elastic waves in random media. The work basically followed the approach used for the 1D solution. The 2-D "Hooke's Law" constitutive relation was expressed in terms of stresses and strains. On the right hand side, strains and material properties were expressed as series in $\sigma_A$ and $\sigma_B$, where the $\sigma_A$ and $\sigma_B$ are the standard deviations for random variation in elastic constants and $\mu(x)$, respectively. On the left hand side, the stress is represented also as a series in $\sigma_A$ and $\sigma_B$.

Because the equation is for two dimensions, a matrix equation actually applies, of the form

$$S = \begin{pmatrix} S_{xx} & S_{xz} \\ S_{xz} & S_{zz} \end{pmatrix} = \begin{pmatrix} \mathbf{a}^T \mathbf{u} & \mathbf{c}^T \mathbf{u} \end{pmatrix}.$$  \hspace{1cm} (1)

where the vectors $\mathbf{u}$, $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are defined as

$$\mathbf{u}^T = \{u^{(x)}, u^{(z)}\},$$

$$\mathbf{a}^T = \left\{\alpha \frac{\partial}{\partial x}, \lambda \frac{\partial}{\partial z}\right\},$$

$$\mathbf{b}^T = \left\{\lambda \frac{\partial}{\partial x}, \alpha \frac{\partial}{\partial z}\right\},$$

$$\mathbf{c}^T = \left\{\mu \frac{\partial}{\partial z}, \mu \frac{\partial}{\partial x}\right\}.$$  \hspace{1cm} (2a-d)

Thus, for example,

$$S_{xx} = \alpha \frac{\partial u^{(x)}}{\partial x} + \lambda \frac{\partial u^{(z)}}{\partial z} = a e_{xx} + (\alpha - 2\mu) e_{zz}. \hspace{1cm} (3a)$$

This kind of expression we then expand as

$$\sigma_{xx} = S_{xx}^{(0)} + S_{xx}^{(A)} \sigma_A + S_{xx}^{(B)} \sigma_B + S_{xx}^{(AA)} \sigma_A^2 + S_{xx}^{(AB)} \sigma_A \sigma_B + S_{xx}^{(BB)} \sigma_B^2. \hspace{1cm} (4a)$$

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and, on the right hand, expand as

\[ \alpha = \alpha_0(1 + \sigma_A^2 A_\omega + \frac{1}{2} \sigma_A^4 (A_\omega)^2), \]  
\[ \mu = u_0(1 + \sigma_B^2 B_\Omega + \frac{1}{2} \sigma_B^4 (B_\Omega)^2), \]  
\[ u^{(6)} = u_0^{(6)} + U_A^{(6)} \sigma_A + U_B^{(6)} \sigma_B + U_{AA}^{(6)} \sigma_A \sigma_B + U_{BB}^{(6)} \sigma_B^2. \]  

(5a)  
(5b)  
(5c)

We do this for all the Sij.

We then take a statistical average for which the first order terms average to zero, yielding

\[ \langle S_{ij} \rangle = S_{ij}^{(0)} + \langle S_{ij}^{(AA)} \rangle \sigma_A^2 + \langle S_{ij}^{(AB)} \rangle \sigma_B + \langle S_{ij}^{(BB)} \rangle \sigma_B^2. \]  

(6)

Using eq. (1), the evaluation of second order terms \( \langle S_{ij}^{(AA)} \rangle, \langle S_{ij}^{(AB)} \rangle, \langle S_{ij}^{(BB)} \rangle \) involves evaluation of terms like

\[ \langle A^{(1)} e_{xx}^{(A)} \rangle, \langle A^{(1)} e_{xx}^{(A)} \rangle, \langle A^{(1)} e_{xx}^{(B)} \rangle, \langle A^{(1)} e_{xx}^{(B)} \rangle, \]
\[ \langle B^{(1)} e_{xx}^{(A)} \rangle, \langle B^{(1)} e_{xx}^{(A)} \rangle, \langle B^{(1)} e_{xx}^{(B)} \rangle, \langle B^{(1)} e_{xx}^{(B)} \rangle, \]
\[ \langle B^{(1)} e_{xx}^{(A)} \rangle \]

and

\[ \langle e_{xx}^{(AA)} \rangle, \langle e_{xx}^{(AA)} \rangle, \langle e_{xx}^{(AA)} \rangle, \langle e_{xx}^{(BB)} \rangle, \langle e_{xx}^{(BB)} \rangle, \langle e_{xx}^{(BB)} \rangle, \]
\[ \langle e_{xx}^{(BB)} \rangle, \langle e_{xx}^{(BB)} \rangle, \langle e_{xx}^{(BB)} \rangle. \]  

(7a)  
(7b)

The set of averages given by the set (7a) are integrals and have been evaluated using first order two-dimensional solutions for the wave functions obtained previously. The second set of averages given by set (7b) must be evaluated using second order two-dimensional solutions for the wave functions. These second order two-dimensional solutions are being currently obtained.

This model solution will be used to simulate seismic waves in 2D random media. The 2D random media will include 2D heterogeneities based on sonic data from Buena Vista Hills field. We will produce synthetic seismograms to analyze travel-time fluctuation, and amplitude fluctuation. Also, attention and dispersion effects associated with 2D scattering will be used to interpret interwell seismic data recorded between wells at Buena Vista Hills.