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**Effect of Flux Flow on Current Distribution and Heat Generation
in Composite Superconductors During A Thermal Disturbance***

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Effect of Flux Flow on Current Distribution and Heat Generation in Composite Superconductors During A Thermal Disturbance

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Abstract

An analytical investigation of current distribution and heat generation rate in composite superconductors, incorporating the effects of flux flow during disturbances, is carried out. Equations describing current density in the superconductor and the heat generation rate per unit volume of the composite conductor in the current sharing regime are derived. The results show that when the superconductor is in the flux-flow state, the current density and the heat generation rate depend only on a dimensionless parameter $\phi_f = (\rho_n/\rho_{st})[H/H_{c2}(0)](1-\lambda)/\lambda$. When the thermal disturbance is relatively small and $\phi_f \gg 1$, the current density in the superconductor remains at the critical current density with all the excess transferred to the stabilizer and the heat generation rate is equal to that

usually employed for low temperature superconductors. When the thermal disturbance is large and $\phi_f \gg 1$, the current density in the superconductor can be greater than the critical current density and the heat generation rate equals the critical generation rate, independent of whether the superconductor is in the flux-flow state or the normal state. For moderate and large thermal disturbances and $\phi_f = 1$, which is applicable to high-temperature superconductors because of high $H_{c2}(0)$, the heat generation rate is $q = q_c/2$ if the superconductor is in the flux-flow state and $q = q_c$ if the superconductor is in the normal state. An argument is provided to indicate when and under what circumstances will all the excess current be transferred to the stabilizer while the current in the superconductor remains at the critical current during a thermal disturbance. The differences between high- and low-temperature superconductors and its implication for cryogenic stability are discussed. Some experimental data on critical currents and thermal runaway of sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$ with unoriented grains are presented for temperatures between 64 K and 95 K at zero applied field. Sample calculations on heat generation rate based on the analytical model and the experimental data on critical currents are provided. Limitations on the analytical model are discussed. The need for dissipation data on other types of high-temperature superconductors over a wider temperature range and various applied fields are emphasized.

Keywords: heat generation, transition state, flux flow, current sharing, composite conductors

Introduction

Accurate prediction of heat generation rate in composite conductors is essential in determining the cryogenic stability of superconducting magnets. Simple models usually assume that the superconductor transforms directly from a superconducting state to a normal state during a thermal disturbance, therefore, all the current is transferred to the stabilizer and the total heat generation rate equals that in the stabilizer [1,2]. This assumption is conservative but the result is quite simple. More accurate and sophisticated models assume that there is an intermediate region between zero heat generation rate (superconducting state) and critical generation rate (normal state). The well-known equal-area theorem of Maddock, James, and Norris [3] used a linear relationship between heat generation rate and temperature in this intermediate region. For low-temperature superconductors (LTS), the calculation of the heat generation rate as a function of temperature is based on the following two assumptions, (1) the critical current density decreases linearly with increasing temperature, and (2) in a composite conductor above its critical current and below its critical temperature, the superconductor will continue to carry its critical current, with all the excess transferring to the stabilizer. These two assumptions appeared to work well for LTS and were employed by some researchers [4,5] to calculate the heat generation rate for high-temperature superconductors (HTS). A slightly different approach was reported by Romanovskii [6], who assumes arbitrarily that the effective resistivity of the composite is a linear function of the temperature in the intermediate region. More recently, Abeln, Klemt, and Reiss [7] reported the results of numerical simulations on transient

temperature fields in a composite conductor. Their calculations need the electrical resistivity of the superconductor in all the thermodynamic states in order to determine the heat generation rate. The calculations were performed using a combination of experimentally measured resistivity at 77 K and arbitrary assumptions about the resistivity at other temperatures and current densities.

The previously mentioned intermediate region appears to be a convenient way of calculating the heat generation rate in a composite conductor. However, the results are not general and may not be applicable to HTS. It is well known that, when the current is varied, there is a transition (dissipative) state between the superconducting state and the normal state of a superconductor. For LTS, this transition occurs over a rather narrow current range (as a fraction of the critical current) and flux flow is the dominating mechanism for dissipation. For some of the HTS, data reported by Hascicek and Testardi [8] show that this transition occurs over a relatively wide current range (can be one order of magnitude larger than the critical current). For HTS, the dissipative mechanism in the transition state is more complicated and not clearly understood yet [9].

In this paper, we shall reexamine the second assumption described previously for LTS. We shall provide an explanation why and under what conditions the assumption is valid. We shall calculate the heat generation rate in composite conductors for both the LTS and HTS during thermal disturbances. Implications of the transition state on the cryogenic stability of HTS composites are discussed. We present some preliminary experimental data on critical currents and thermal runaway of a sintered

YBa₂Cu₃O₇ with unoriented grains. Finally, an example on how to calculate the heat generation rate in a composite conductor is provided, based on the present analytical model and the experimental data on critical currents.

Analysis

Figure 1 shows a typical voltage (V) versus current (I) characteristic for a type 2 superconductor at a constant temperature. Following the usual convention, the electric field (E) and current density (J) are plotted in place of V and I since the former are independent of sample dimensions in the limit that the magnetic field of the current (self field) is ignored. If $J < J_{c1}$ the superconductor is assumed to be a perfect conductor and thus have zero resistance for DC currents. In practice, this condition is rigorously proved only by persistent currents, and so a less strict criterion of a 1 microvolt/cm electric field is usually employed to define J_{c1} . The polycrystalline YBCO sample analyzed in Fig. 1 was at 89 K, and thus exhibited a small J_{c1} value of 18 A/cm². The E-J curve for many superconductors develops a linear regime above J_{c1} , allowing measurement of a differential resistivity, and definition of a different critical current J'_{c1} , from the linear extrapolation to zero voltage. In this case, J'_{c1} has a value of 220 A/cm² in contrast to the much smaller J_{c1} . The pitfalls of these and other schemes are discussed elsewhere [10]. This paper will employ the 1 microvolt/cm definition for J_{c1} , although data for both J_{c1} and J'_{c1} will be shown later for the sample illustrated in Fig. 1. At sufficiently large current density J_{c2} the superconducting state will be destroyed completely, and an ohmic E-J curve will appear as a straight line passing through the

origin. Although one can hypothesize the existence of such a depairing current at a constant temperature below T_c , in fact thermal runaway usually occurs first, pushing the local sample temperature above T_c . An effective J'_{c2} can be defined as the current density which causes the onset of thermal runaway. Between J_{c1} and J_{c2} , the sample is in a transition state which has dissipative and superconducting characteristics.

For LTS, the dominating mechanism for dissipation in this transition state is flux flow and the resistivity ρ_t is equal to the flux-flow resistivity ρ_f . This occurs over a narrow range of current (a fraction of the critical current). For HTS, the transition state occurs over a relatively wide range of current (up to one order of magnitude larger than the critical current). It is not clear what is the mechanism for dissipation in this transition state for HTS. However, a finite resistivity can be measured and it is sometimes referred to as a resistive state [8]. We shall call the resistivity in the resistive state ρ_r . If $J > J_{c2}$, the superconductor is in its normal state and the resistivity is equal to the normal-state resistivity (ρ_n). We shall refer to J_{c1} as the lower critical current density and J_{c2} as the upper critical current density of the superconductor. When the operating field H is greater than the upper critical field H_{c2} , the superconductor is in its normal state, independent of what the current is. Figure 1 describes the characteristic of a type 2 superconductor at a temperature below the critical temperature T_c and an applied field less than H_{c2} . If data (E versus J) were obtained for several temperatures at a given magnetic field, a graph like that shown in Fig. 2 can be constructed. Figure 2 shows the variation of the lower (J_{c1}) and upper (J_{c2}) critical current density with temperature, where the current density is defined as

$$J = I / A \quad (1)$$

and A is the cross-sectional area of the superconductor perpendicular to the direction of the current. Both J_{c1} and J_{c2} are functions of the magnetic field strength H . For LTS, J_{c1} and J_{c2} stay relatively close to each other in Fig. 2 because transition occurs over a narrow range of current. For HTS, J_{c1} and J_{c2} may be quite far apart because transition occurs over a rather wide range of current.

Let point P_0 in Fig. 2 be the operation point of a composite conductor made of a superconductor and a metal stabilizer. The bonding between the superconductor and the stabilizer is assumed to be perfect and the interfacial resistance is negligible. The composite conductor is assumed to be at a uniform temperature T_0 with a current density J_0 . Since J_0 is less than $J_{c1}(T_0)$, the superconductor has zero resistance, therefore all the current passes through the superconductor and there is no heat generation. If a disturbance causes a portion of the composite conductor to move horizontally from P_0 to P_1 or P_2 , then the current density J_0 becomes greater than either $J_{c1}(T_1)$ or $J_{c1}(T_2)$ and the superconductor will no longer be resistanceless. Current is now shared between the superconductor and the stabilizer. An electric field is established in the direction of the current flow. At the same axial location, the electric field is the same in the superconductor and the stabilizer. Heat is generated in both the superconductor and the stabilizer.

In the current sharing regime, the voltage drop is the same in the superconductor (sc) and the stabilizer (st),

$$I_{sc}R_{sc} = I_{st}R_{st} . \quad (2)$$

The sum of the current in the superconductor and the stabilizer is constant and equals I_0 ,

$$I_0 = I_{sc} + I_{st} . \quad (3)$$

The heat generation rate (Q) in the composite is

$$Q = (I_{sc})^2 R_{sc} + (I_{st})^2 R_{st} . \quad (4)$$

Substituting Eqs. 2 and 3 into Eq. 4, we obtain

$$Q = I_0(I_0 - I_{sc})R_{st} . \quad (5)$$

The quantity of interest is the heat generation rate per unit volume (q) of the composite conductor. Dividing Eq. 5 by the unit volume of the composite conductor (sc plus st), we found that

$$q = \rho_{st} J_0^2 \left(\frac{\lambda^2}{1-\lambda} \right) \left(1 - \frac{J_{sc}}{J_0} \right) , \quad (6)$$

where ρ_{st} is the resistivity of stabilizer, λ is the fraction of the superconductor in the composite, and J_{sc} is the current density in the

superconductor. If a critical generation rate per unit volume of the composite conductor is defined as

$$q_c = \rho_{st} J_0^2 \left(\frac{\lambda^2}{1-\lambda} \right), \quad (7)$$

then Eq. 6 can be written as

$$\frac{q}{q_c} = 1 - \frac{J_{sc}}{J_0} = I_{st} / I_0. \quad (8)$$

Equation 8 says that the dimensionless heat generation rate is equal to the fraction of total current in the stabilizer. It should be noted that $J_0(T_0)$ is always larger than J_{sc} in the current sharing regime because some of the current is flowing through the silver substrate. In the literature for LTS [3], it is usually assumed that all the excess is transferred to the stabilizer when the critical current density is exceeded. This is equivalent to assuming that the superconductor will continue to carry its critical current, thus

$$J_{sc} = J_{c1}. \quad (9)$$

We shall see that Eq. 9 is not always valid even though it is a good approximation for many practical applications.

Strictly speaking, the current in the superconductor is always greater than J_{c1} because otherwise the composite conductor cannot be in the

current sharing regime. If $J_{c2} \geq J_{sc} \geq J_{c1}$, the superconductor is in the transition state, therefore

$$R_f = \rho_f L / A \quad (10a)$$

and

$$R_r = \rho_r L / A \quad (10b)$$

where R_f is the resistance in the flux-flow regime, R_r is the resistance in resistive regime, and L is the axial length in the direction of current flow.

For low-temperature superconductors, flux flow is the dominating mechanism for dissipation, and the Bardeen-Stephen model [11] is expected to apply in the flux-flow regime

$$\rho_f / \rho_n = H / H_{c2}(0) \quad (11)$$

where H is the operating magnetic field, and $H_{c2}(0)$ is the upper critical field of the superconductor at zero temperature. Combining Eqs. 2, 3, 10a, and 11, it can be shown that

$$J_{sc} / J_0 = 1 / (1 + \phi_f) . \quad (12)$$

Substituting Eq. 12 into Eq. 8,

$$q / q_c = 1 / (1 + 1 / \phi_f) , \quad (13)$$

where the dimensionless parameter ϕ_f is

$$\phi_f = (\rho_n / \rho_{st})[H / H_{c2}(0)](1 - \lambda) / \lambda , \quad (14)$$

Similar expressions can be obtained for high-temperature superconductors in the resistive state by using Eq. 10b instead of Eq. 10a,

$$J_{sc} / J_0 = 1 / (1 + \phi_r) , \quad (15)$$

and

$$q / q_c = 1 / (1 + 1 / \phi_r) , \quad (16)$$

where the dimensionless parameter ϕ_r is

$$\phi_r = (\rho_r / \rho_{st})(1 - \lambda) / \lambda . \quad (17)$$

If $J_{sc} > J_{c2}$, the superconductor becomes normal, $\rho_{sc} = \rho_n$, and it can be shown that

$$J_{sc} / J_0 = 1 / (1 + \phi_n) , \quad (18)$$

and

$$q / q_c = 1 / (1 + 1 / \phi_n) , \quad (19)$$

where the dimensionless parameter ϕ_n is

$$\phi_n = (\rho_n / \rho_{st})(1 - \lambda) / \lambda . \quad (20)$$

Discussions

When the superconductor is in the superconducting state, no heat is generated. Thus, if the temperature disturbance is smaller than $T_{g1}-T_0$ in Fig. 2, no heat is generated. When the superconductor is in its normal state ($J_{sc} > J_{c2}$), Eq. 19 shows that heat generation rate in the composite conductor is independent of the magnetic field and the heat generation rate (q/q_c) depends primarily on the resistivity ratio ρ_{st}/ρ_n . If $\rho_{st} \ll \rho_n$, then $q \approx q_c$, heat generation rate becomes equal to the critical generation rate (the generation rate when all the current is in the stabilizer). The superconductor will become normal if the temperature disturbance is very large (point P_3 in Fig. 2).

The more interesting regime is when the superconductor is in the transition state ($J_{c2} \geq J_{sc} \geq J_{c1}$). In this regime, both the current density J_{sc}/J_0 and the heat generation rate per unit volume q/q_c depend only on the dimensionless parameter ϕ_f (Eqs. 12 and 13) if flux flow is the dominating dissipation mechanism, or the dimensionless parameter ϕ_r (Eqs. 15 and 16) for high-temperature superconductors in the resistive state. Figure 3 shows the variations of J_{sc}/J_0 and q/q_c with ϕ_f or ϕ_r . We shall discuss the current sharing regime (transition state) for the LTS and HTS separately.

Low-Temperature Superconductors (LTS)

The dimensionless parameter ϕ_f depends on ρ_n/ρ_{st} , H/H_{c2} , and the conductor fraction λ . Usually ρ_n is several orders of magnitude larger than

ρ_{st} . If the operating (self plus applied) magnetic field is moderate or large, then ϕ_f is very large ($\phi_f \gg 1$). This case is applicable to practical devices operating at a fraction of the upper critical field. On the other hand, if the operating magnetic field H is very small compared to the upper critical field H_{c2} , then ϕ_f is on the order of one ($\phi_f = 1$). One specific application for the case of $\phi \equiv 1$ is high-temperature superconductor current lead operating under the self-field condition. In any case, the dimensionless parameter ϕ_f can range from approximately one to much larger than one. We shall examine the two extreme cases ($\phi_f \gg 1$ and $\phi_f = 1$) and keep in mind that Eqs. 12 and 13 are general and can be applied to calculate the current density J_{sc} and the heat generation rate per unit volume of a composite conductor for any value of ϕ_f as shown in Fig. 3.

When $\phi_f \gg 1$, Eqs. 12 and 13 show that

$$J_{sc} = J_0 / \phi_f \ll J_0 . \quad (21)$$

Equation 21 states that the current density in the superconductor has decreased to only a very small fraction of the original current density before the thermal disturbance. This is because the resistance of the superconductor is much larger than that of the stabilizer when $\phi_f \gg 1$. Recalling that, in the transition state, the superconductor should be able to carry a current density of at least J_{c1} as long as the temperature is below the critical temperature, thus

$$J_{sc} \geq J_{c1} . \quad (22)$$

Which means that the current density in the superconductor cannot be lower than J_{c1} for temperature disturbances greater than $T_{g1}-T_0$. Both Eqs. 21 and 22 must be satisfied in the transition state when $\phi_f \gg 1$. However, Equation 22 with the equal sign has higher priority than Eq. 21 if both of them cannot be satisfied simultaneously. For relatively small thermal disturbances (point P_1 in Fig. 2 with a $\Delta T = T_1 - T_0$), $J_{c1}(T_1)$ may still be a significant fraction of J_0 , and if Eq. 22 is satisfied, then Eq. 21 cannot be satisfied. One can argue that the best solution seems to let $J_{sc} = J_{c1}(T_1)$, which is the minimum requirement dictated by Eq. 22, and then let all the excess current go to the stabilizer. This way Eq. 21 is satisfied as much as possible while Eq. 22 is not violated. Thus, we have provided an explanation why and under what circumstances will all the excess current be transferred to the stabilizer while the current density in the superconductor remains at the critical current density $J_{c1}(T_1)$. This conclusion coincides with the assumption usually made in the literature for low temperature superconductors [3,12]. But this conclusion is not general and is applicable only if the thermal disturbances are relatively small and if $\phi_f \gg 1$. In this case, $J_{sc} = J_{c1}$, and Eq. 8 becomes

$$q / q_c = 1 - J_{c1} / J_0 \quad (23)$$

If one further assumes that J_{c1} decreases linearly with temperature, it can be shown that

$$q / q_c = (T - T_{g1}) / (T_c - T_{g1}) \quad (24)$$

Equation 24 shows that q/q_c varies linearly with T for temperatures between T_{g1} and T_c . This equation is usually employed to calculate the heat generation rate per unit volume for LTS [3,12]. Again it must be emphasized that the derivation of Eq. 24 is based on the assumptions that (1) all the excess current is transferred to the stabilizer which is valid only if the thermal disturbance is relatively small and if $\phi_f \gg 1$, and (2) J_{c1} decreases linearly with temperature which may not be applicable to some HTS.

When the thermal disturbances are relatively large (point P_2 in Fig. 2 with a $\Delta T = T_2 - T_0$), J_{c1} has decreased considerably from the undisturbed value of J_0 to a much lower value of $J_{c1}(T_2)$. Both Eqs. 21 and 22 can be satisfied simultaneously because now $J_{c1}(T_2) \ll J_0$. In this case, The current density J_{sc} is equal to J_0/ϕ_f , which can be greater than $J_{c1}(T_2)$. It should be pointed out that it is possible for point P_2 in Fig. 2 to be still in the flux-flow state, instead of the normal state, provided that $\phi_f \gg 1$. As shown in Eq. 21, $J_{sc} \ll J_0$ if $\phi_f \gg 1$. It is possible that J_{sc} is small enough so that $J_{sc} < J_{c2}(T_2)$, and therefore P_2 is still in the flux-flow state. In this case, the heat generation rate is from Eq. 13 with $\phi_f \gg 1$.

$$q \equiv q_c. \quad (25)$$

Equation 25 is valid if the thermal disturbance is relatively large and if $\phi_f \gg 1$. For large temperature disturbances (point P_2) and $\phi_f \gg 1$, most of the current is in the stabilizer, and consequently most of the heat is generated there and $q \equiv q_c$. This is true whether the superconductor is in the flux-flow state or the normal state.

On the other hand, when $\phi_f = 1$, Eqs. 12 and 13 show that

$$J_{sc} = J_0 / 2, \text{ and} \quad (26)$$

$$q = q_c / 2. \quad (27)$$

The current density in the superconductor has decreased to one-half of that before the thermal disturbance and the heat generation rate is equal to one-half of the critical generation rate. Again Eq. 22 must also be satisfied in the flux-flow regime. Whether Eqs. 22 and 26 can be satisfied simultaneously for point P_1 depends on the magnitude of the temperature disturbance (ΔT) and the shape of J_{c1} in Fig. 2. However, since J_{sc} given by Eq. 26 is much larger than that given by Eq. 21, Eq. 22 can be satisfied for much smaller ΔT . The argument that $J_{sc} = J_{c1}$ and all the excess is transferred to the stabilizer still applies, but it is only needed for much smaller ΔT 's. Thus, for very small ΔT , the superconductor is at the critical current density J_{c1} with all the excess current transferred to the stabilizer, and the heat generation rate is given by Eq. 23 (or Eq. 24 if it is assumed that J_{c1} decreases linearly with temperature). For moderate temperature disturbances (point P_1 in Fig. 2), the superconductor is in the flux-flow state and $q = q_c/2$. For large temperature disturbances (point P_2 in Fig. 2), the superconductor is more likely to be in the normal state because J_{sc} is large and it is likely that $J_{sc} > J_{c2}(T_2)$ and consequently $q \cong q_c$. Thus, the knowledge of the state of the superconductor is important because it determines the heat generation rate of the composite conductor. The state of the superconductor depends on the magnitude of the temperature

disturbance and on the shape of the curve J_{c2} in Fig. 2. This situation is quite different from the case described previously for $\phi_f \gg 1$. It should be noted that the case of $\phi_f = 1$ is not likely to be encountered in practical devices using LTS, because the ratio $H/H_{c2}(0)$ is usually not very small for low-temperature superconductors. As we shall see later, the case of $\phi_f = 1$ is more appropriate for HTS because $H_{c2}(0)$ is usually very large for most high-temperature superconductors.

High-Temperature Superconductors (HTS)

There are several major differences between HTS and LTS. First, the dissipation mechanism in the transition state may be different. Second, the transition state (space between curves J_{c1} and J_{c2} in Fig. 2) of HTS occurs over a much wider range of current than that of LTS. Finally, the upper critical field $H_{c2}(0)$ of HTS is usually much higher than that of the LTS. Following is a discussion of the implications of these differences.

Recent experimental results on $\text{YBa}_2\text{Cu}_3\text{O}_7$ wires reported by Askew, et al. [13] show that the resistivity in the transition state depends on the microstructure of the samples. The resistivity of unoriented (sintered) ceramic samples showed no dependence on magnetic field between 0.05 T and 0.5 T and a slight dependence on temperature. On the contrary, the resistivity of grain-aligned (directionally solidified) samples shows almost no dependence on temperature and a strong dependence on magnetic field. The resistivity of the grain-aligned sample is much smaller (two orders of magnitude) than that of the unoriented sample. Although it is not clear

whether the dissipation mechanism in the transition state for the grain-aligned samples is directly related to flux flow (Lorentz-force dependent), the experimental data seems to agree in general with the Bardeen-Stephen model for LTS. Therefore, the results described previously for LTS can be applied to HTS with grain-aligned samples (or more specifically, to HTS with resistivity in the transition state that follows the Bardeen-Stephen model). In this case, Eqs. 12 to 14 are expected to apply to HTS with grain-aligned specimens. The only difference is that $H_{c2}(0)$ is quite large (typically 500 T) for HTS. This would make the dimensionless parameter ϕ_f in Eqs. 12 and 13 much smaller in practical devices and the situation $\phi_f = 1$ becomes more likely for HTS. For HTS which does not follow the Bardeen-Stephen relation described by Eq. 11, one can still use the measured resistivity ρ_r in the transition state and Eqs. 15 to 17 to calculate the current distribution and heat generation rate in the composite conductor. In this case, the current distribution and the heat generation rate are independent of the magnetic field.

The combination of $\phi_f \cong 1$ and a larger $\Delta J = J_{c2} - J_{c1}$ for HTS offers some advantage over the LTS as far as cryogenic stability is concerned in addition to a wider operating temperature range and other advantages [2] for HTS. For a given thermal disturbance, the HTS may still be in the transition state while the LTS may be in the normal state. If the dissipation mechanism is flux flow in both the LTS and HTS (such as the grain-aligned HTS that follows the Bardeen-Stephen model), the heat generation rate is $q_c/2$ for HTS and q_c for LTS. In fact, $\phi = 1$ means that the resistance is the same in the superconductor and the stabilizer. This is the case for the grain-aligned HTS where the resistivity of the transition

state is about the same as that of the stabilizer. The heat generated in the HTS will be half of that in the LTS. This is because current is shared equally (if $\lambda = 0.5$) in the HTS and the stabilizer compared to all the current is in the stabilizer for LTS. The latter generates more (twice as much) heat than the former and therefore, is less stable cryogenically. On the other hand, if the resistivity of the transition state of the HTS is much larger than that of the stabilizer (such as the unoriented, sintered samples), most current will be in the stabilizer. Then the heat generated in HTS and LTS is not much different. In practical devices, it is most likely that grain-aligned HTS will be used because it also has much higher critical current density. To actually evaluate the heat generated in HTS composites during a thermal disturbance, one needs to know the resistivity of the transition state (ρ_f or ρ_r), the curves J_{c1} and J_{c2} in Fig. 2 for the operating magnetic field, and the upper critical field $H_{c2}(0)$ and the normal state resistivity ρ_n of the HTS if the dissipation follows the Bardeen-Stephen model.

Experimental data on J_{c1} and J_{c2} for a Sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$

The most commonly measured critical current density is J_{c1} which is usually determined by the criterion of $1 \mu\text{V}/\text{cm}$ for the electric field strength. However, there is little information on the current density J_{c2} (the current density at which the superconductor becomes normal independent of what the temperature is) and the data on the variation of J_{c2} with temperature like that shown in Fig. 2 simply does not exist. The primary reasons for the lack of data on J_{c2} for HTS are (1) that most researchers are focusing on raising J_{c1} , and (2) that there is some

experimental difficulties in measuring J_{c2} (particularly for bulk samples) because the samples may heat up and consequently internal temperature gradient may develop at these high current densities [13]. In the following paragraphs, we describe the attempt to measure J_{c2} of a sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$.

The $\text{YBa}_2\text{Cu}_3\text{O}_7$ sample is 25 mm long and 0.87 mm in diameter. The method used to measure J_{c1} and J_{c2} is the standard four-probe technique with a current pulsing system [13]. The current pulsing system is capable of delivering 25 A down to 50 μs durations and is particularly suitable for the present measurement because of the heat up problem mentioned previously. Tests are conducted by forcing a feedback-controlled current through the sample which ramps up and then holds at a fixed value, as shown in Fig. 4. The voltage across the sample is amplified and digitized simultaneously with the current ramp, producing the raw data file shown in Fig. 4. Division of the voltage curve by the current curve produced the result shown in Fig. 1. If the current is increased to sufficiently large values, thermal runaway is observed, even on a submillisecond time scale. This is observed as an upward voltage spike, produced because the differential resistivity of the sample increases with temperature in this region [13]. This controlled thermal runaway was observed in the 15-25 amp region, depending on sample temperature. Some method was needed to reproducibly define the onset current I'_{c2} for this process. This was done by varying the level at which the current was held after the ramp (shown as I'_{c2} in Fig. 4) so that the corresponding voltage curve (which would be flat if the sample were at a constant temperature) shows an increase of 2 mV over 0.2 msec. This was accomplished by watching the

individual bits flip on the digitizer, with remaining unfiltered noise supplying an appropriate "dither" voltage. An exaggerated angle is shown in Fig. 4 below the relevant spot in the voltage curve. It should be noted that, unlike J_{c2} , J'_{c2} is not an intrinsic property of the superconductor. J'_{c2} depends on the size, the geometry, and the method of cooling in addition to temperature and applied field.

The results of measured J_{c1} , J'_{c1} , and J'_{c2} at zero applied field over the temperature range from 64 K to 95 K are shown in Fig. 5. J_{c1} is a significant fraction of J'_{c1} except when J_{c1} becomes quite small at higher temperatures (>80 K), then J'_{c1} can be two or three times as large as J_{c1} . The relation between J_{c1} and temperature appears to be fairly linear over the temperature range shown in Fig. 5. J_{c1} approaches zero at 89 K and J'_{c1} approaches zero at approximately 92 K. The current density J'_{c2} decreases slowly with temperature until approximately 85 K, it then begins to drop sharply with temperature. Below 95 K two distinct plateaus appear in the J'_{c2} curve. The first starts at 89 K, which is the temperature where J_{c1} drops below 100 mA (see Fig. 1) and continues until 90.6 K where the first non-zero resistivity is detected with a 100 microamp probe current. In this temperature range, the junctions between the individual grains of superconductor have effectively gone normal while the grains are still superconducting. Another plateau occurs above 92 K, where the grains themselves are starting to lose their superconducting properties. This last plateau ends at 93.2 K where the effect shown in Fig. 4 ends abruptly, just as the resistivity curve begins its final bend over to the normal state properties.

To see how to evaluate the heat generation rate in a composite conductor subjected to a thermal disturbance described previously, we shall use the data for J_{c1} and J'_{c2} in Fig. 5. Using J'_{c2} to evaluate the heat generation rate is conservative because J_{c2} is always larger than J'_{c2} . To help explain the procedure, we replotted only J_{c1} and J'_{c2} as functions of temperature in Fig. 6. Since the data is for sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$, the dissipation in the transition regime is probably not due to flux flow and we cannot use Eqs. 11 to 14. In this case, we should use Eqs. 15 to 17 to calculate the current density and the heat generation rate in the composite conductor. Assuming that the composite conductor is operating at 65 K (T_0) with a current density $J_0(T_0)$ of 1,400 A/cm² (point P_0 in Fig. 6). We want to find out what are the current density in the superconductor and the heat generation rate per unit volume of the composite if a thermal disturbance causes the temperature of the superconductor to increase to 75 K (point P_1), 85 K (point P_2), and 95 K (point P_3). To do so, we need to know the value of ϕ_r given by Eq. 17. We shall calculate two cases by using two different values of ϕ_r .

For the first case, we assume that the resistivity of the superconductor in the transition state ρ_r is large compared to that of the stabilizer ρ_{st} and $\phi_r = 99$. If a thermal disturbance causes the temperature of the superconductor to increase by 10 K, the superconductor (and the stabilizer) is now at 75 K and is represented by point P_1 . From Eq. 15, we have

$$J_{sc} / J_0 = 0.01 . \quad (28)$$

However, at $T = 75$ K, $J_{c1} \approx 1,000$ A/cm², and from Eq. 22

$$J_{sc} / J_0 \geq 0.71 . \quad (29)$$

Equations 28 and 29 cannot be satisfied simultaneously. In this situation, the equal sign in Eq. 29 has higher priority and should be used. To calculate the heat generation rate, we must use Eq. 8 instead of Eq. 16, and the result is

$$q / q_c = 1 - 0.71 = 0.29 . \quad (30)$$

If the thermal disturbance causes the temperature of the conductor to increase by 20 K (point P₂ in Fig. 6), Eq. 28 remains the same ($\phi_r = 99$). But from Fig. 6 at T = 85 K, $J_{c1} = 232 \text{ A/cm}^2$ and Eq. 22 gives

$$J_{sc} / J_0 \geq 0.16 . \quad (31)$$

Again, Eqs. 28 and 31 cannot be satisfied simultaneously and the equal sign in Eq. 31 has higher priority. The heat generation rate is

$$q / q_c = 1 - 0.16 = 0.84 . \quad (32)$$

If the thermal disturbance causes the temperature of the superconductor to increase by 30 K (point P₃ in Fig. 6), the superconductor becomes normal and if the normal state resistivity of the superconductor is much larger than that of the stabilizer, then from Eq. 20

$$q / q_c \equiv 1$$

These results with $\phi_r \gg 1$ is familiar to LTS researchers because it is identical to the results obtained by simply assuming that the superconductor is carrying its critical current with all the excess transferred to the stabilizer.

Next, we consider the case where the resistivity of the superconductor in the transition state is only several times larger than that of the stabilizer and assume that $\phi_r = 3$. In this case, we have (Eq. 15)

$$J_{sc} / J_0 = 0.25 . \quad (33)$$

At point P_1 ($T = 75$ K), Eq. 29 must be satisfied. Again, Eqs. 29 and 33 cannot be satisfied simultaneously and we choose the equal sign in Eq. 29. The heat generation rate is then given by Eq. 30. Thus, at point P_1 , the heat generation rate is independent of the value of ϕ_r . If the temperature disturbance is increased to 20 K (point P_2), then $J_{c1} = 232$ A/cm² and from 22 (or from Eq. 31)

$$J_{sc} / J_0 \geq 0.16 . \quad (34)$$

It is evident that if the current in the superconductor is given by Eq. 33, Eq. 34 is automatically satisfied. In this situation (point P_2), the current density in the superconductor is greater than the critical current density corresponding to the temperature at P_2 . The heat generation rate is

$$q / q_c = 1 - 0.25 = 0.75 . \quad (35)$$

Comparing Eq. 35 ($\phi_r = 3$) to Eq. 32 ($\phi_r = 99$) indicates that heat generation rate does depend on ϕ_r for a temperature disturbance of 20 K. This is different from the result obtained for a temperature disturbance of 10 K.

It should be noted that the data shown in Fig. 5 is for temperatures above 64 K at zero applied field. There are definite needs for more dissipation data at lower temperatures and various applied field. Furthermore, the sample calculations provided here is based on data for sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$ (with unoriented grains and relatively low critical current density). The more interesting and practical superconductors are the grain-aligned $\text{YBa}_2\text{Cu}_3\text{O}_7$ (which follows the Bardeen-Stephen model for dissipation) and other types of high-temperature superconductors (such as the bismuth based ceramic superconductor made from the powder-in-tube processing). Unfortunately, there is no data (like those shown in Fig. 5) available at this time for these superconductors. These high-temperature superconductors usually have higher critical current densities and are more likely to be used in practical devices. Their dissipation characteristics and consequently the heat generation rate may be quite different from that of the sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$.

Summary

Equations describing the steady-state heat generation rates in the current sharing regimes for composite conductors are derived based on the assumption that the conductor is at a uniform temperature. When the superconductor is in the normal state, the heat generation rate depends

primarily on the resistivity ratio ρ_{st}/ρ_n , and is independent of the magnetic field (Eq. 19). When the superconductor is in the transition state, the current density in the superconductor and the heat generation rate per unit volume of the composite conductor depend on the dimensionless parameter ϕ_f (Eqs. 12 and 13) if flux flow is the dominating dissipation mechanism. This case is applicable to LTS and to HTS with grain-aligned structure that follows the Bardeen-Stephen model for dissipation. The parameter ϕ_f depends on the resistivity ratio ρ_n/ρ_{st} , the magnetic field ratio $H/H_{c2}(0)$, and the conductor fraction λ (Eq. 14). When the thermal disturbance is relatively small and $\phi_f \gg 1$, the superconductor is maintained at the critical current density J_{c1} and the heat generation rate is given by Eq. 23. This situation is more likely to exist in most LTS devices and in some HTS devices operating at very high magnetic fields. If it is further assumed that J_{c1} decreases linearly with temperature, then the heat generation rate is given by Eq. 24, which is identical to the equation usually employed to calculate the heat generation rate for LTS. When the thermal disturbance is relatively large and $\phi_f \gg 1$, most of the current is in the stabilizer and $q = q_c$, independent of whether the superconductor is in the flux-flow state or the normal state. On the other hand, when $\phi_f = 1$, both the current density in the superconductor and the heat generation rate per unit volume of the composite conductor depend on the magnitude of the temperature disturbance and the state of the superconductor. When the superconductor is in the flux-flow state, $q = q_c/2$. When the superconductor is in the normal state, $q \equiv q_c$. This situation is more likely for HTS devices operating at low or moderate magnetic fields because $H_{c2}(0)$ is quite large for most HTS. For HTS with unoriented, sintered samples, the dissipation mechanism is not from flux

flow, however, the current distribution and the heat generation rate can be calculated by using Eqs. 15 and 16 if the resistivity of the transition state is known. The variations of the current density (J_{sc}/J_0) and the heat generation rate (q/q_c) over a wide range of ϕ_f or ϕ_r is shown in Fig. 3.

An argument is provided to explain why and under what circumstances will all the excess current be transferred to the stabilizer while the current density in the superconductor remains at the critical current density J_{c1} during a thermal disturbance. It is demonstrated that all the excess current will be transferred to the stabilizer only if the temperature disturbance is relatively small and $\phi_f \gg 1$ or if the temperature disturbance is even smaller and $\phi_f = 1$. Otherwise, the current density in the superconductor will be greater than the critical current density J_{c1} . It is pointed out that the combination of $\phi_f \equiv 1$ and larger $\Delta J = J_{c2} - J_{c1}$ for HTS is advantageous from the point of view of cryogenic stability because the superconductor is more likely to be in the transition state which generates less heat than the normal state for a given thermal disturbance provided that the dissipation in the superconductor follows the Bardeen-Stephen model. One specific example of $\phi_f \equiv 1$ is the application of high-temperature superconductor current lead operating under self-field condition.

Some preliminary data on dissipation for a sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$ (with unoriented grains) are presented (Fig. 5). Attempts were made to measure the upper critical density J_{c2} (the current density at which the superconductor becomes normal at constant temperature). However, thermal runaway as a result of internal heating occurred before J_{c2} . The

current density at thermal runaway is referred to as J'_{c2} which is always less than J_{c2} . J'_{c2} is not an intrinsic property of the superconductor because it depends on the size, geometry, and method of cooling in addition to temperature and applied field. Sample calculations on heat generation rates are presented based on the data for sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$. It is pointed out that the dissipation data reported here for sintered is from 64 K to 95 K at zero applied field. There is a need to obtain data at lower temperatures and various applied magnetic fields. Furthermore, there is a lack of data on the more interesting and practical superconductors such as the grain-aligned $\text{YBa}_2\text{Cu}_3\text{O}_7$ (which follows the Bardeen-Stephen model for dissipation) and the bismuth based ceramic superconductor made from the powder-in-tube processing. These high-temperature superconductors usually have higher critical current densities and are more likely to be used in practical devices. Their dissipation characteristics and consequently the heat generation rate may be quite different from that of the sintered $\text{YBa}_2\text{Cu}_3\text{O}_7$.

Finally, it should be emphasized that the present analysis is based on the assumption that the temperature of the superconductor is uniform. As the thickness and dissipation (internal heat generation) of the superconductor increases, this assumption may no longer be valid because a transverse temperature gradient may develop. A model including transverse heat conduction in the superconductor will be more appropriate and will also help in understanding the phenomenon of thermal runaway described previously.

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Nomenclature

A	cross-sectional area of superconductor, m^2
H	magnetic field strength, A/m
H_{c2}	upper critical field, A/m
I	current, A
J	current density, A/m^2
J_{c1}	lower critical current density defined by the $1 \mu\text{V}/\text{cm}$ criterion, A/m^2
J'_{c1}	lower critical current density given defined in Fig. 1, A/m^2
J_{c2}	upper critical current density defined as the current density at which the superconductor becomes normal at constant temperature, A/m^2
J'_{c2}	critical current density at thermal runaway condition (Fig. 4), A/m^2
L	axial length in the direction of current, m
Q	heat generation rate, W
q	heat generation rate per unit volume of the composite conductor, W/m^3

q_c	critical heat generation rate per unit volume of the composite conductor (Eq. 7), W/m^3
R	electrical resistance, Ω
T_{g1}	temperature at the intersection of J_0 and J_{c1} (Fig. 2), K
T_{g2}	temperature at the intersection of J_0 and J_{c2} (Fig. 2), K
V	voltage, V
ΔT	magnitude of temperature disturbance, K
ρ	electrical resistivity, $\Omega\cdot m$
λ	fraction of superconductor in the composite, dimensionless
ϕ_f	a dimensionless parameter defined in Eq. 14.
ϕ_r	a dimensionless parameter defined in Eq. 17.
ϕ_n	a dimensionless parameter defined in Eq. 20.

Subscripts

f	flux-flow state
n	normal state
r	resistive state
sc	superconductor
st	stabilizer
t	transition state
0	operating point P_0 in Figs. 2 and 6
1	point P_1 in Figs. 2 and 6
2	point P_2 in Figs. 2 and 6
3	point P_3 in Figs. 2 and 6

Figure Captions

Fig. 1 E versus J characteristic of a type 2 superconductor at a constant temperature.

Fig. 2 Variations of the lower (J_{c1}) and upper (J_{c2}) critical current densities with temperature at constant magnetic field. Point P_0 is the operating point, points P_1 , P_2 , and P_3 are the states of the composite conductor after thermal disturbances.

Fig. 3 Variations of the current density ratio J_{sc}/J_0 and the heating rate ratio q/q_c with the dimensionless parameter ϕ_f or ϕ_r .

Fig. 4 Experimental data on the variations of the voltage and current with time at 89 K and zero applied field for a sintered $YBa_2Cu_3O_7$.

Fig. 5 Experimental data on the variations of current densities J_{c1} , J'_{c1} , J'_{c2} , and resistance R with temperature at zero applied field for a sintered $YBa_2Cu_3O_7$.

Fig. 6 Experimental data on the variations of current densities J_{c1} and J'_{c2} with temperature at zero applied field for a sintered $YBa_2Cu_3O_7$. These data are used in the calculations of the heat generation rate in a composite conductor (point P_0 is the operating point, points P_1 , P_2 , and P_3 are the states of the composite conductor after thermal disturbances).

Current Density

Temperature

Fig. 2

Fig. 5