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A Stability Tool for Use Within TELEGRIP

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A Report on ,

A Stability Tool for Use Within TELEGRIP

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Abstract

This paper describes a graphical tool referred to as *Stab Tool*, developed within a commercial software package, TELEGRIP. The basis of *Stab Tool* is that it allows functions that can determine the various types of stabilities of an arbitrarily shaped object held by a fixture, robotic gripper, or of a set of objects piled randomly together. Several modes of operation are possible: (1) stability tests - *given the locations and surface normal directions at the contacts and the set of external*

force that could act on the object, determine if the object is stable, (2) fixture/grasp design - given the geometry of the object and the set of external forces that could act on the object, determine an optimal set of locations of contacts with fixture elements/fingers that will stabilize the object in the face of any force from the given set. Illustrative examples for using these functionalities are included by using graphic models.

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1. INTRODUCTION

During the assembly of a product, it is vital that the partially-completed assembly be stable. To guarantee this, we must ensure that contacts among the parts and the fixtures are sufficient to stabilize the assembly. Thus, it would be desirable to have an efficient method for testing an assembly stability, and, if this is not possible, generating a set of additional fixture contact points, known as "fixels," that will stabilize it.

We can apply this method to the situation of safe handling of special nuclear material (SNM). To have these functionalities would help improve the safety and enhance the performance of special nuclear material (SNM) handling and storage operations, since some methods are needed for gripping objects in a stable manner. Also, we may need a way to find a pit-holding fixture inserted into containers.

In this paper, we present a stability tool, which we call *Stab Tool*, which was developed to test the stability of objects grasped by robotic hands, objects placed in fixtures, or sets of objects piled randomly on top of one another. *Stab Tool* runs on top of a commercial software package, TELEGRIP, which is used for geometry modeling and motion creation.

The successful development of the stability depends strongly on TELEGRIP's ability to compute the distances between pairs of three-dimensional bodies in the simulated environment. While TELEGRIP provides a yes/no collision test, it does not prove the distance between bodies prior to collision, nor the penetration distance after collision. In addition, this tool requires the location of contact points and contact normals. Again, while this information is embedded in TELEGRIP's data structures, it is not easily available through TELEGRIP's collision detection algorithm.

Our interbody distance computation tool takes advantage of the polyhedral representations of bodies used by TELEGRIP and of linear programming techniques to yield an efficient algorithm. It works as follows: a pair of bodies is identified for distance computation; then a linear program is formulated whose constraints are the inequality constraints defining the interiors of the bodies.

To these constraints, we add a slack variable that represents the distance that a (variable) point in space is away from each inequality constraint surface. The objective then is to find the point in space that minimizes this distance. The optimal point (found by solving a linear program) is an approximation of the actual separation (if positive) or penetration (if negative) distance of the two bodies.

The stability tests implemented in *Stab Tool* are actually a straight-forward application of techniques previously developed for grasping and dexterous manipulation applications (Trinkle and Zheng, 1995; Trinkle, 1992) and has been formulated, but not implemented, by Palmer (1987). The test requires only the solution of a set of linear equations and inequalities and can be done rather quickly using a linear program solution algorithm, e.g., the simplex algorithm. Between the velocity-domain formulation and the dual force-domain formulation, the latter has been used for the implementation.

1.1 Overview

Within *Stab Tool*, several different stability tests have been implemented: a *static test without friction*, a *dynamic test without friction*, and a *static test with friction*. The static frictionless test requires the knowledge of the points of contact, the contact normals among all the bodies, and the external forces acting on the bodies (e.g., gravitational forces) to formulate a linear program. The solution of the linear program (obtained by a routine available in the IMSL library) indicates whether or not the system of bodies is first-order stable. First-order stability can be likened to a marble in a bowl shaped like an inverted pyramid. No infinitesimal perturbing force can cause the marble to move. Thus, first-order stability is stronger or more conservative than the standard stability definition. The frictionless assumption was thought to be useful for a similar reason: if a set of bodies in contact are stable without friction, then it is surely stable with contact friction.

The frictionless dynamic test amounts to computing the acceleration of the bodies when there is no contact friction. This test cannot be formulated as a linear program. Rather, the best we can do is formulate it as a positive

semi-definite quadratic program. The quadratic program gives no additional information other than the linear program when the system is first-order stable, but provides us with accurate motion information otherwise. This information is useful when a user is designing a fixture or stable robotic grasp, interactively.

The third test is a generalization of the static frictionless test. For this test, we formulate a linear program that checks whether or not equilibrium is possible when Coulomb friction is active at the contacts. The Coulomb friction cone at each contact point is approximated by an inscribed polyhedron, thus making all relevant relationships linear. If the linear program is feasible, then it is possible to balance all externally applied forces (e.g., gravitational forces) without sliding.

There is another useful concept we can consider on top of those stability tests. When assembling a product, it is necessary to ensure that all the parts already inserted remain in place. To guarantee this we must provide a fixture that will stabilize the assembly. The problem of developing automated methods for designing fixtures for parts assemblies is, therefore, of great practical interest. An efficient method (Wolter and Trinkle, 1994) is

available for generating a set of additional fixture contact points that will stabilize it. This method works by parameterizing possible fixel locations over all reachable faces of the parts and solving a linear program over the unknown contact forces and fixel locations that will be solvable if the assembly can be stabilized by any combination of the added fixels. A simplex algorithm is applied to minimize the sum of the magnitudes of the contact forces acting throughout the added fixels. All fixels with zero contact forces are discarded, leaving a small set of fixels that stabilizes the assembly with small contact forces. Within *Stab Tool*, we have taken a simplified approach by trying not to find an optimized set of fixtures. Instead, it allows the user to go through an iterative process so that the user can find a better set as the iteration is increased upon user selection of fixtures at each run.

A tool to determine the stability of virtual models like *Stab Tool* is cheaper and more efficient than manufacturing prototypes. Specifically, we can apply these stability tests to the case of gripping an object in a stable manner in order to improve safety and the performance of special nuclear material (SNM) handling and storage operations.

2. THE SYSTEM MODEL

2.1 Contacts

The first stage in analyzing the stability of an assembly is to discover the locations and orientations of the contacts among the parts. This is fundamentally problematic, because a geometric model of an assembly can never be a precise representation of the real assembly. It is quite possible for small inaccuracies to significantly affect the stability of the assembly. For the purposes of our analysis, however, we will assume that the locations and normals of the contacts can be derived from the geometric model with sufficient accuracy.

Parts will be labeled by indices $1 \leq k \leq n_c$, where n_{cont} is the number of contacts among all the parts and fixtures. We define $P(k)$ to be the set of movable parts involved in the k^{th} contact. This will always be a set of size one (if the contact is between a movable part and an unmovable part) or two (if the contact is between two movable parts).

Let k be a contact and $i \in P(k)$ be a movable part involved in that contact. That contact will be characterized by its location $r_{k,i}$ and its normal $\hat{n}_{k,i}$ of unit length and directed inward with respect to part i . Thus, if $P(k) = \{i, j\}$, then $(r_{k,i}, \hat{n}_{k,i})$ and $(r_{k,j}, \hat{n}_{k,j})$ describe the same contact with oppositely directed normals represented in possibly different frames. Place right-handed contact frame $C_{k,i}$ with axes $\hat{n}_{k,i}$, $\hat{t}_{k,i}$, and $\hat{o}_{k,i}$ such that its origin coincides with contact point k and its normal axis, $\hat{n}_{k,i}$, points inward with respect to body i . Here, $\hat{n}_{k,i}$, $\hat{t}_{k,i}$, and $\hat{o}_{k,i}$ are mutually orthogonal 3-vectors of unit length with $\hat{n}_{k,i} \times \hat{t}_{k,i} = \hat{o}_{k,i}$. Place left-handed contact frame $C_{k,j}$ with axes $\hat{n}_{k,j}$, $\hat{t}_{k,j}$, and $\hat{o}_{k,j}$ such that its origin coincides with contact point k , but direct its axes oppositely (i.e., $\hat{n}_{k,j} = -\hat{n}_{k,i}$, $\hat{t}_{k,j} = -\hat{t}_{k,i}$, and $\hat{o}_{k,j} = -\hat{o}_{k,i}$).

Among three-dimensional polyhedra, contacts can be classified into four types: vertex-face, face-face, edge-edge, and vertex-edge cases. We find the contacts between two polyhedral parts i and j by searching for intersecting pairs of surface elements (i.e., faces, edges, or vertices). In most cases it is simple to model the kinematic constraints arising from such a pair by a set of one or

more point contacts. Some examples are shown in Figure 1 to illustrate different types of contacts that can occur during assembly situation.

Figure 1: Types of Contacts in 3D

2.2 Kinematics, Dynamics, and Coulomb Friction Constraints

The existence of a contact means that there can be no relative motion between the contacting parts that causes interpenetration, i.e., the relative linear velocity of the contact point on part i with respect to the contact point on body j may not have component in the direction $\hat{n}_{k,i}$ (the contact normal directed away from i and j).

At contact k , sliding and separation are allowed, but penetration is not. Let $\dot{q}_i$ and $\ddot{q}_i$ be the 6-vectors representing the generalized velocity and acceleration of the center of mass of part i with respect to the initial frame U . Let c_k denote the force applied through contact k either by part j on part i expressed with respect to frame $C_{k,i}$, or by part i on part j expressed with respect to frame $C_{k,j}$. Then the 6-vector, $(w_n)_{k,i}$, is the unit wrench due to the normal vectors, $\hat{n}_{k,i}$, associated with contact point k on body i expressed with respect to frame B_i whose origin coincides with the center of gravity of part i and whose axes are parallel to those of U :

$$(w_n)_{k,i} = \begin{bmatrix} \hat{n}_{k,i} \\ r_{k,i} \times \hat{n}_{k,i} \end{bmatrix}$$

Having expressed the contact accelerations in the contact frames, the kinematically admissible accelerations of the assembly are those that obey the following simple inequality, the non-penetration constraints, at

all the contacts, and this can be formulated as follows (Wolter and Trinkle, 1994):

$$\mathbf{W}_n^T \ddot{\mathbf{q}} \geq 0 \quad (1)$$

where the $6n_{part}$ -vector, $\ddot{\mathbf{q}}$, is the assembly acceleration vector, formed by the vertical concatenation of the individual acceleration vectors of the parts and the $(n_{cont} \times 6n_{part})$ matrix $\mathbf{W}_n^T$ can be constructed from the individual unit wrenches, $(\mathbf{w}_n)_{k,i}$. Specifically, row k of the $\mathbf{W}_n^T$ is formed by the horizontal concatenation of n_{part} row vectors of length 6; the i^{th} and j^{th} vectors are $(\mathbf{w}_n)_{k,i}^T$ and $(\mathbf{w}_n)_{k,i}^T$, respectively. All other vectors in row k are zero vectors. The other components of the contact accelerations are not constrained, that is, sliding or rolling is allowed at each contact.

Also, let $\mathbf{g}_i$ be the external wrench acting on part i expressed with respect to $\mathbf{B}_i$, and $\mathbf{M}_i$ be the block diagonal with the upper left (3×3) block equal to the mass, $\mathbf{m}$, times the identity matrix and the lower right (3×3) block equal to the inertia matrix of part i . Let $\mathbf{c}_k$ denote the force applied through contact k either by part j on part i expressed with respect to frame $\mathbf{C}_{k,i}$, or by part i on part j expressed with respect to frame $\mathbf{C}_{k,j}$. Then, the acceleration of the parts must be consistent with the contact and external wrenches acting on them. Combining the Newton-Euler

equations for all the parts and casting them in matrix form yields the following compact representation:

$$\mathbf{W}_n \mathbf{c}_n - \mathbf{M} \ddot{\mathbf{q}} = \mathbf{g}_{ext}$$

where the $6n_{part}$ -vector, $\mathbf{g}_{ext}$, is the vertical concatenation of the individual external wrenches, $\mathbf{g}_i$, the $(6n_{part} \times 6n_{part})$ matrix, $\mathbf{M}$, is the block diagonal assembly inertia matrix defined by $\mathbf{M} = \text{blockdiag.} \{ \mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_{n_{part}} \}$, and n_{cont} -vectors, $\mathbf{c}_n$, is the vertical concatenation of the contact force components, $(\mathbf{C}_n)_k$.

$$\mathbf{M} = \text{blockdiag.} [\mathbf{M}_1 \ \mathbf{M}_2 \ \dots, \ \mathbf{M}_{n_{parts}}] \quad (2)$$

$$\ddot{\mathbf{q}} = [\ddot{\mathbf{q}}_1^T \ \ddot{\mathbf{q}}_2^T \ \dots, \ \ddot{\mathbf{q}}_{n_{parts}}^T]^T \quad (3)$$

$$\mathbf{g}_{ext} = [\mathbf{g}_{ext_1}^T \ \mathbf{g}_{ext_2}^T \ \dots \ \mathbf{g}_{ext_{n_{parts}}}^T]^T \quad (4)$$

The Coulomb's model of friction requires that k^{th} contact force stay within or on a friction cone which can be written as follows:

$$\mathbf{c}_k^T \mathbf{D}_k \mathbf{c}_k \geq 0 \quad (5)$$

$$(\mathbf{c}_n)_k \geq 0 \quad (6)$$

where $\mathbf{D}_k = \text{diag}\{\mu_k^2, -1, -1\}$ and μ_k is the coefficient of friction at the k^{th} contact point.

3. THE STABILITY TEST

3.1 Assembly Stability

Consider a system in a configuration achieving static equilibrium. Such a system would be considered to be dynamically stable if, when perturbed, it returned to its initial configuration. This common definition of stability is best suited to systems without dry friction or Coulomb, because dry friction usually prevents a system from returning to the original equilibrium configuration. For rigid body systems with significant dry friction, stability is often better characterized by the system's inability to spontaneously accelerate away from its current configuration. This type of stability is referred to as static stability. To clarify the distinction between dynamic and static stability, compare the following two scenarios: a marble sitting in the bottom of a hemisphere bowl is dynamically stable. If disturbed, it will eventually return to its starting position. By contrast, a book placed on a table is statically stable. If untouched, the book will remain at rest. However, perturbing the book by pushing it leaves it in a new location from which it will not independently return to its initial position. (In this report, we view assembly stability from the static point of view as did Palmer (1987).) In other words, an assembly is unstable under a given set of external forces and moments (such as gravity or part insertion forces) if those cause any of its parts to move. This occurs if there exists an infinitesimal motion of the parts and a set of contact forces between the parts such that: (1) the motion does not result in any part penetrating another; (2) the instantaneous direction of motion of each part is consistent with the forces and moments on that part; and, (3) the energy input by the external forces and moments exceeds the energy dissipated by sliding friction. If no such fall-down motion exists, then we will consider the assembly stable. (Note that this definition does not imply that a perturbed stable system eventually returns to the original equilibrium configuration – only static stability is required here.)

To describe the stability test methods we have used to implement the *Stab Tool*, we assume that we are given a complete list of the contacts between a set of parts and a support structure, as well as the masses and centroids

of the parts. As has been stated previously, the *Stab Tool* has capabilities of performing several different stability tests as follows.

3.2 The Static Frictionless Stability Test

For the static frictionless stability test, we will determine if the parts will accelerate if they begin at rest and there is no friction. There are two alternative formulations of the test. One is “velocity formulation” and the other is “force formulation” (Wolter and Trinkle, 1994). Their physical interpretations are first, the equations of equilibrium must be satisfied with compressive contact forces; and the second, if there exists a kinematically admissible velocity for which the virtual work is positive, then the assembly is unstable. These two LPs are dual, and the existence of a motion with positive virtual work is synonymous with the infeasibility of the equilibrium equation under the non-negativity constraint on the wrench intensity vector, c_n . Here, we use the “force formulation” for the static frictionless stability testing, and this is given as follows:

$$\text{Minimize: } 0 \quad (7)$$

$$\text{Subject to: } W_n = -g_{\text{ext}} \quad (8)$$

$$c_n \geq 0 \quad (9)$$

where the unknown, c , is the vector of force magnitudes. Specifically, the k^{th} element of c is the magnitude of the force at the contact point k . Notice that the objective function of the force formulation is independent of the contact force magnitudes. This implies that the requirement for stability is that a c exist which satisfies the constraints of Equations (9) and (10). The former constraint is the set of equilibrium equations and the latter requires that the contacts be compressive force. Thus, a frictionless assembly is stable only if equilibrium can be satisfied with compressive contact forces. If at least $3n_p$ elements of c_n are positive, then the assembly is stable. In other words, when the assembly is unstable, the force formulation will be infeasible.

3.3 The Dynamic Frictionless Stability Test

According to our definition of assembly stability, if there exists contact forces and

virtual displacements consistent with the Newton-Euler equations of motion and the contact model is such that the virtual work is positive, then the assembly is unstable. Otherwise, the assembly is stable. This means that an assembly is unstable if and only if there exists a nonzero assembly acceleration, $\ddot{\mathbf{q}}$, satisfying the equations:

$$\mathbf{W}_n \mathbf{c}_n - \mathbf{M} \ddot{\mathbf{q}} = -\mathbf{g} \quad (10)$$

$$\mathbf{W}_n^T \ddot{\mathbf{q}} \geq 0 \quad (11)$$

$$\mathbf{c}_n \geq 0 \quad (12)$$

$$\mathbf{c}_n^T \mathbf{W}_n^T \ddot{\mathbf{q}} = 0 \quad (13)$$

The Equations (11), (12), (13), and (14) are kinematic and dynamic constraints acting on the assembly, respectively. These constraints constitute mixed linear complementary problem (MLCP) (Cottle, Pang, and Stone, 1992) in $\mathbf{c}_n$ and $\ddot{\mathbf{q}}$. However, since $\mathbf{M}$ is positive definite and symmetric, it is invertible. Thus $\ddot{\mathbf{q}}$ can be eliminated to yield an equivalent linear complementary problem (LCP) in $\mathbf{c}_n$ as follows:

$$\mathbf{W}_n^T \mathbf{M}^{-1} \mathbf{g} + \mathbf{W}_n^T \mathbf{M}^{-1} \mathbf{W}_{ng} \mathbf{c}_n \geq 0 \quad (14)$$

$$\mathbf{c}_n \geq 0 \quad (15)$$

$$\mathbf{c}_n^T (\mathbf{W}_n^T \mathbf{M}^{-1} \mathbf{g} + \mathbf{W}_n^T \mathbf{M}^{-1} \mathbf{W}_{ng} \mathbf{c}_n) = 0 \quad (16)$$

This in turn can be recast as a quadratic program (QP) as follows:

$$\text{Minimize: } \mathbf{g}^T \mathbf{M}^{-1} \mathbf{W}_n^T \mathbf{c}_n + \frac{1}{2} \mathbf{c}_n^T \mathbf{W}_n^T \mathbf{M}^{-1} \mathbf{W}_{ng} \mathbf{c}_n$$

$$\text{Subject to: } \mathbf{c}_n \geq 0$$

Since the product $\mathbf{W}_n^T \mathbf{M}^{-1} \mathbf{W}_{ng}$ is only positive semi-definite, the QP may have a non-unique solution (Wolter and Trinkle, 1982). However, it was shown by Lotstedt (1982),

that even though the vector $\mathbf{c}_n$ which minimizes the QP is non-unique, the corresponding assembly acceleration $\ddot{\mathbf{q}}$ is unique. This fact allows one to compute the assembly acceleration as follows:

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1} (\mathbf{W}_n^* \mathbf{c}_n + \mathbf{g})$$

where $\mathbf{c}_n^*$ is any value of $\mathbf{c}_n$ minimizing the quadratic program (19) and (20). Thus, if and only if there exists $\ddot{\mathbf{q}} \neq \mathbf{0}$ and satisfying above constraints, then the assembly is unstable. Otherwise, the assembly is stable. The physical implications of the above observations are that for frictionless assemblies, the solution of the quadratic program determines whether or not the assembly is stable and if it is not, it determines the exact fall-down mode.

3.4 The Static Stability Test with Friction

This test checks whether or not an equilibrium is possible for an assembly when Coulomb friction is active at the contacts by determining if it is possible to balance all externally applied forces (e.g., gravitational forces) without sliding. Assuming point contact with Coulomb friction, $\mathbf{c}_k$ may have three nonzero components, $(\mathbf{c}_n)_k, (\mathbf{c}_t)_k, (\mathbf{c}_o)_k$, corresponding to the directions, $\hat{n}_k, \hat{t}_k, \hat{o}_k$. The equilibrium equations that need to be satisfied can be represented as follows:

$$\mathbf{W}_n \mathbf{c}_n + \mathbf{W}_t \mathbf{c}_t + \mathbf{W}_o \mathbf{c}_o = -\mathbf{g}$$

where the n_{part} -vectors, $\mathbf{c}_n, \mathbf{c}_t, \mathbf{c}_o$, are the vertical concatenations of the contact force components, $(\mathbf{c}_n)_k, (\mathbf{c}_t)_k, (\mathbf{c}_o)_k$, respectively. Since the Coulomb friction cone at each contact point is approximated by an inscribed polyhedron, the equation is a linear program. Then, we will say that the object is stable if there exists contact forces $\mathbf{c}$ which satisfies the following equations:

$$\sum \mathbf{e}_{ij} \mathbf{c}_{ij} = -\mathbf{g}_{obj} \quad (17)$$

$$\mathbf{c}_{ij} \geq 0 \quad (18)$$

The above equation means that the contact force at the contact point must lie within an imaginary friction cone that has its origin at the

contact point, height equal to contact normal force, and base radius equal to frictional force magnitude. For example, we can approximate the friction cone by a four-sided pyramid. Then we have:

$$\mathbf{e}_{ij} = \{ \hat{n}_i + \mu_i \hat{t}_i, \hat{n}_i + \mu_i \hat{\theta}_i, \hat{n}_i - \mu_i \hat{t}_i, \hat{n}_i - \mu_i \hat{\theta}_i \} \quad (19)$$

where the four-sided friction cone at each contact can be described in Figure 2.

Figure 2: Friction Cone Approximation by Four-Sided Polyhedron

In other words, if this linear program is feasible, then it is possible to balance all external applied forces (e.g., gravitational forces) without sliding.

4. ROBUST FIXTURE AND GRASP DESIGN

Given an unstable assembly, we wish to find a small set of fixels that can be placed on the open faces of the parts to stabilize the assembly. An open face is any portion of the surface of any part that is not already in contact with another part. One sure method of fixturing an assembly would be to cover every open face with contacts. This would be the equivalent of embedding the assembly in a block of concrete. This strategy would obviously stabilize any assembly, but we would prefer to find a smaller set of contacts although they may not be an optimal one. To find this set, we take a four stage approach:

Step 1: Add a finite number of fixels that can be moved anywhere on the open faces of the assembly.

Step 2: Construct a linear program using the force-formulation that incorporated these fixels as contact points.

Step 3: Solve the linear program (perform a stability test).

Step 4: If the solution is feasible, then the system is stable meaning that the chosen fixels constitute fixture design.

The implementation of this approach uses a modification of the force formulation presented in the previous section. Repeating the above-mentioned process many times, we would be able to get some small set of contacts that will stabilize the assembly, thus, finding a robust fixturing is an iterative design process in the

assembly situation. Finding a set of fixture points could be applied to synthesizing a stable grasp of an object by a robotic hand. Once the set of fixtures is determined using the above-mentioned process, then we can consider each fixture point as the position where a robot finger tip should be placed for grasping.

4.1 Human-Aided Design

As explained in the previous section, the robust fixture/grasp design is enabled by applying the stability testing algorithm repeatedly to the chosen set of fixels on the open faces of the parts in the assembly until is found a stabilized assembly. From the viewpoint of users, this requires one to have the capability to move fixels and finger contact points on the open faces and re-evaluate stability. In *Stab Tool*, this is achieved through mouse-clicking. Once a user-selected, multi-rigid body system model has been placed on the screen, the user can either perform tests for the three types of stabilities previously described or enter into the human-aided fixture design phase. Fixture design can be considered as an iterative design process with human input. By mouse-clicking, the user can select arbitrary fixture-points on any face of the body to be tested. (The user can rotate the workpart(s) to be fixtured, so he/she can put fixels on faces not visible in the first view.) Then, the user can invoke the stability test on the screen to determine whether the user-selected fixture-points can stabilize the geometric model selected. This process can be applied repeatedly until the user can find a good small set of the fixels which stabilize the system. We call this process of determining robust fixture/grasp design process as that of a *human-aided iterative process*.

5. EXAMPLES

5.1 Stability Testing

Once a geometric model of some grasping or sub-assembly has been created and placed, we can perform different kinds of stability testing as described in the previous section. An example of the running of the stability formulations is shown in Figure 3 for an artificially-constructed subassembly situation. Gravity is straight down the page, and the bottom-most part is assumed to be unmovable. This subassembly has the contacts found among the parts in their current positions. The force formulation determines whether the subassembly is statically stable or not. For this specific configuration, if the solution is unbounded, the subassembly is stable in the sense that the cap is not moving in the configuration while the tetrahedra is fixed. If we have used velocity formulation, we can determine the stability by computing $\dot{q}$ and determine if it is zero. In this way, the velocity formulation can also be used to find a feasible falldown motion for an assembly.

This specific example was determined to be statically stable, since the solution is feasible and the meaning contact force is compressive, whereas it is not stable by the dynamic frictionless test. This is because the values of $\dot{q}$ are nonzero and the cap portion is rising slightly while the tetrahedra are descending. As a matter of fact, the stability result is very sensitive to the geometry and not always easily assessed by a human.

Figure 3: Example of StabTool Operation: Determining the Stability of Staked Objects

5.2 Fixture Design

A simple structure has been created as shown in Figure 4 to illustrate the process of determining a set of fixture points for an object. We can consider this geometric model as a simplified pit-in-a-container structure, where a pit is held within a container and its stability should be maintained by a set of fixture points which can be considered as supporting fixtures made between the surfaces of the pit and inner facets of the container. We are assuming that the enclosing container is an unmoveable wall with respect to the pit to be fixtured. Within *Stab Tool*, we can try to select a set of fixture points (which supposedly stabilizes the pit) by mouse-clicking on the surface of the pit and performing a stability test. We can repeat this process by reselecting the set of fixture elements until we find a small set of fixture contacts which stabilizes the pit. This process is, as previously defined, a human-aided iterative fixture design process.

Figure 4: Example of StabTool Operation: Determining Stabilizing Fixturing

5.3 *Grasp Design*

Finding a set of fixture points could be applied to synthesizing a stable grasp of an object by a robotic hand. Once the set of fixtures are determined using the above-mentioned process, we can consider each fixture point as the position where a robot finger tip should be placed for grasping. A grasp design for a cylindrical object by a 4-finger gripper is illustrated in Figure 5.

Figure 5: Example of StabTool Operation: Determining the Stability of Object-Grasping

6. CONCLUSION

We have developed a prototype tool for testing the stability of an arbitrarily shaped object held by a fixture, robotic gripper, or a set of objects piled randomly together. This tool, called *Stab Tool*, is embedded into the commercial software package TELEGRIP, and uses various built-in features that it provides. Several different modes of operation are possible: stability tests and fixture/grasp design. The fixture/grasp design is achieved through an iterative human-aided process by utilizing the CAD features built within TELEGRIP. The reason for the frictionless assumption in *Stab Tool* is its conservativeness: if the system is stable without friction, then it must be with friction, since friction can only dissipate as fast as it

enters a system. Also, robust fixture/grasp design capability facilitates an interactive design process which determines stable system configurations by having the ability to find fixel positions with the help of human input.

Stability tests for virtual models using a tool like *Stab Tool*, are cheaper and more efficient than manufacturing prototypes. Some methods are needed for gripping objects in a stable manner and we may need a way to find a pit-holding fixture inserted into containers. Thus, we can apply this method to the situation of safe handling of special nuclear material (SNM). To have these functionalities would help improve the safety and enhance the performance of handling and storage operations of special nuclear material (SNM).

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