GYPSY FIELD PROJECT
IN RESERVOIR CHARACTERIZATION

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Objectives

The overall objective of this project is to use the extensive Gypsy Field laboratory and data set as a focus for developing and testing reservoir characterization methods that are targeted at improved recovery of conventional oil.

The Gypsy Field laboratory, as described by Doyle, O’Meara, and Witterholt (1992), consists of coupled outcrop and subsurface sites which have been characterized to a degree of detail not possible in a production operation. Data from these sites entail geological descriptions, core measurements, well logs, vertical seismic surveys, a 3D seismic survey, crosswell seismic surveys, and pressure transient well tests.

The overall project consists of four interdisciplinary sub–projects which are closely interlinked:

1. Modeling depositional environments.
2. Upscaling.
4. Tracer testing.

The first of these aims at improving our ability to model complex depositional environments which trap movable oil. The second entails testing the usefulness of current methods for upscaling from complex geological models to models which are more tractable for standard reservoir simulators. The third investigates the usefulness of numerical techniques for identifying unswept oil through rapid calculation of sweep efficiency in large reservoir models. The fourth explores what can be learned from tracer tests in complex depositional environments, particularly those which are fluvial dominated.

Summary of Technical Progress

During this quarter, the main activities involved the “Modeling depositional environments” sub–project, for which the progress is reported below.
IMPLEMENTATION OF TENSOR FORM OF PERMEABILITY DISTRIBUTION IN MULTIGRID SIMULATOR

During the last quarter of the year the research has been focused on:

- modifying the developed multigrid simulator for pressure equation in reservoir simulation to be able to consider permeability as a tensor function of coordinates rather than scalar function as it has been implemented in the current version.

Multigrid methods have proved to be very efficient at solving the non-separable diffusion equation defined in up to three dimensions (Scott 1985, Behie, and Forsyth 1983). This type of equation plays a central role in the theory of flow through porous media and provides a basis for setting up the governing partial differential equations of oil reservoir simulation. Allowing for discretisation in time, the diffusion equation at each time step may be written compactly as a self-adjoint boundary value problem,

\[-\nabla K \nabla u + \sigma u = f \tag{1.1}\]

where \(K\) is proportional to the permeability tensor with strong spatial variation, \(u\) is the dependent variable, in this case pressure, \(\sigma\) is a constant and \(f\) is a functional term evaluated at the previous time step. The problem is to solve for \(u\) implicitly throughout the reservoir domain subject to prescribed boundary conditions (usually no-flow Neumann conditions) on the boundary.

We are considering only the situation when permeability tensor function \(K_{ijk}\) has diagonal form:

\[
K_{ijk}(x,y,z) = \begin{bmatrix}
K_x(x,y,z) & 0 & 0 \\
0 & K_y(x,y,z) & 0 \\
0 & 0 & K_z(x,y,z)
\end{bmatrix} \tag{1.2}
\]

For the three-dimensional case the problem may be stated as follows. On the box \(R:=\{(x,y,z): 0 \leq x \leq h_x, 0 \leq y \leq h_y, 0 \leq z \leq h_z\}\) find the function, \(u(x,y,z)\), such that

\[
(L_x + L_y + L_z)u(x,y,z) + \sigma u(x,y,z) = f(x,y,z) \tag{1.3}
\]

where
\[ L_x = -\frac{\partial}{\partial x} K_x(x,y,z) \frac{\partial}{\partial x} \]
\[ L_y = -\frac{\partial}{\partial y} K_y(x,y,z) \frac{\partial}{\partial y} \]
\[ L_z = -\frac{\partial}{\partial z} K_z(x,y,z) \frac{\partial}{\partial z} \]  \hspace{1cm} (1.4)

with Neumann boundary conditions: \( \frac{\partial \mu}{\partial n} = 0 \)

The approximate solution to (1.3) is the Rayleigh-Ritz-Galerkin projection of the true solution onto \( S_{m,n}^k(R) \). This projection is defined by requiring the residual in the differential equation to be orthogonal to the space of splines \( S_{m,n}^k(R) \).

We approximate \( u(x,y,z) \) by

\[ u(x,y,z) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} u_{ijk} B_i(x)B_j(y)B_k(z) \]  \hspace{1cm} (1.5)

The approximate solution must satisfy

\[ \iiint \{(L_x + L_y + L_z + \sigma)u(x,y,z) - f(x,y,z)\}B_\lambda(x)B_\mu(y)B_\nu(z)dv = 0 \]  \hspace{1cm} (1.6)

for \( 1 \leq \lambda \leq m, 1 \leq \mu \leq n, 1 \leq \nu \leq p \)

Substituting (1.5) into (1.6) and using (1.4) with Neumann boundary conditions we get a linear system of equations

\[ (H + Q)v = f \]  \hspace{1cm} (1.7)

where

\[ H_{ijk,v} = \iiint (K_x(x,y,z)B'_i(x)B'_j(x)B'_k(y)B_\mu(y)B_\nu(z)B_\lambda(z)) + K_x(x,y,z)B'_i(x)B'_j(y)B_\mu(x)B_\nu(z)B_\lambda(z) + K_x(x,y,z)B'_i(x)B'_j(z)B_\mu(x)B_\nu(y)B_\lambda(z))dxdydz \]  \hspace{1cm} (1.8)

\[ Q_{ijk,v} = \sigma \iiint B_\lambda(x)B_i(x)B_\mu(y)B_j(y)B_\nu(z)B_\lambda(z)dxdydz \]  \hspace{1cm} (1.9)

and

\[ f_{ijk} = \iiint f(x,y,z)B_i(x)B_j(y)B_k(z)dxdydz \]  \hspace{1cm} (1.10)
The main difference between considering permeability as a scalar function of coordinates or as a tensor function is in the form of matrix $H$. In scalar case, when $K_z = K_y = K_x$, variations in permeability function give input to the different entries of stiffness matrix $H$. That means that change in permeability magnitude in the neighboring elements results in change of the absolute value of the corresponding entries of the stiffness matrix. In case of tensor form of the permeability change in different components of the tensor in the neighboring elements may not result in significant change of stiffness matrix entries. This property of the system (1.7) is especially important in considering practically important situation when geological structures form some kind of intersecting channels or layers and, thus, giving rise to described above distribution in permeability tensor.

In the computer simulator being developed which was described in the previous report the permeability tensor for the finest grid of the region is generated by defining three independent functions $K_{x_{i,j,k}} = K_x(x_i,y_j,z_k)$, $K_{y_{i,j,k}} = K_y(x_i,y_j,z_k)$, and $K_{z_{i,j,k}} = K_z(x_i,y_j,z_k)$ of the cells centers coordinates. The stiffness matrix $L^M$ and right-hand side vector $f^M$ are formed and only non-zero elements are stored in the memory.

For the smoothing and final iteration procedure Gauss-Seidel algorithm is used (Varga, 1962). The exact form of the Gauss-Seidel method depends on the ordering chosen for the mesh points. For the pressure distribution equation typical in reservoir simulations, red-black ordering is used, making one pass through the mesh updating the "even" points and another pass updating the "odd" points. When quantities in permeability tensor are more strongly coupled along one dimension than another, the user can change algorithm and relax a whole line along that dimension simultaneously. The simulator uses "zebra" relaxation procedure in this case, relaxing odd and even lines on successive passes.

The key issue in implementing successful multigrid procedure to the reservoir simulation is the choice of interpolation operators, $I_{M-1}^M$, residual transfer operator, $I_{M}^{M-1}$, and the approximation of the stiffness matrix on coarse grids $L^{M-1}$. The residual transfer operator is defined so that each of the 27 neighboring fine grid node values are weighted and their averaged values are prescribed to the corresponding coarse grid nodes. The particular choice for 27 weight coefficients implemented in the simulator was given in previous report. Only these coefficients are stored in the memory and are used to perform restriction from fine grid to coarse one.

It is known from the literature (Nicolaides, 1977; Alcouffe et al., 1981) that if permeability matrix may have large jump discontinuities interpolation operator $I_{M-1}^M$ it is save (in sense of convergence) to choose interpolation operator to be adjoint to the residual transfer one $I_{M}^{M-1}$, thus, $I_{M-1}^M = (I_{M}^{M-1})^*$. There is no need to store matrix $I_{M-1}^M$ of the interpolation operator, the result of the application of this operator was found analytically by "Mathematica" software (Wolfram, 1991) and then saved as a FORTRAN subroutine.

Given the fine-grid stiffness operator $L^M$, the residual operator $I_{M}^{M-1}$ and hence the interpolation operator $I_{M-1}^M$, following Nicolaides (1977), an automatic way to construct the coarse-grid stiffness is by the following:
\[ L^{M-1} = I_{M}^{M-1} L^{M} (I_{M}^{M-1})^* \]

The action of the product of operators on the right hand side can be calculated by considering its effect on some coarse-grid function \( u_{IK} \). The derivation of the coarse-grid difference operator coefficients is a tedious task and involves lengthy algebraic formalism. This was done again with the use of "Mathematica" software (Wolfram, 1991) and the results were transformed into FORTRAN subroutine. In the simulator non-zero elements of the operators \( L_{M-k}^{M} \) (k means grid level) are formed and stored in the memory while performing multigrid downward steps in V-cycle and are used again in the upward steps.

It has been shown in the previous report that for the case of scalar permeability the approach (1.11) has resulted in excellent convergence of the numerical algorithm for all practically important (but limited by the assumption of scalar form of permeability function) reservoir situations. Unfortunately, this is not the case for the tensor form of permeability function. The reason is in the form of projecting operation (1.11) of the stiffness matrix from fine grid to coarse. Application of the projection operation (1.11) results in loosing information about tensor properties of permeability matrix on coarse grids. In other words on coarse grid for the each cell the components of the permeability tensor are being averaged automatically giving rise to scalar form of permeability on subsequent grids. The consequence is loose of convergence for certain geological structures described above or unrealistic change in pressure field.

Two ways of overcoming this problem can be suggested. First approach is to choose another form of interpolation \( I_{M-1}^{M} \), and residual transfer \( I_{M}^{M-1} \) operators in order to maintain not the continuity of the pressure distribution between neighboring cells (as it is implemented in current version of the code) but rather to maintain continuity of fluxes and, thus, taking into account characteristic features permeability tensor on coarse grids. Second way is to form stiffness matrix for each grid independently using renormalization procedure for permeability tensor. The disadvantage of this approach is that it very resources consuming procedure.

References


