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Refractive Aiming Corrections for Satellite Observation of Stars

Charles N. Vittitoe, Ron L. Schmidt

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Refractive Aiming Corrections for Satellite Observation of Stars

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Abstract

Standard references describe how apparent zenith angles differ from true zenith angles for observers on the Earth. In fact, correction formulae are available for aiming Earth-based sensors at stars; some corrections give variations as a function of observer altitude. Such corrections have not been available for observers in space. This report develops formulae appropriate for proper aiming from space-based sensors toward the relatively few stars that are near the Earth's limb at any given time. These formulae correct for refractive effects and may be critical for steerable space-borne sensors with fields of view less than one degree, tasked to observe starlight passing near the Earth's surface. Ray tracing in the U. S. Standard Atmosphere, 1976 including H₂O effects, is used to determine relations between the refracted tangent height, the apparent tangent height resulting from observation at the sensor, and the angle through which the detected rays have deviated. Analytic fits of the ray deviation as a function of apparent tangent height allows quick determination of corrections needed for a space-borne sensor. Using those results that apply in the plane of incidence and using the necessary coordinate rotations, alterations in the star's apparent right ascension and declination are evaluated to improve the aim. Examples illustrate that alterations can be larger than one degree, with effects lasting up to a few minutes.

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Introduction

Satellites that observe a bright source on the Earth's surface receive light rays that have been bent by refractive effects in the Earth's atmosphere. Altitude variations in the air pressure, temperature, and constituents (such as changes in humidity) cause variations in the atmospheric index of refraction as well as in the absorption of these rays. The effects are discussed in standard references such as the *Handbook of Geophysics and the Space Environment*, edited by A. S. Jursa of the Air Force Geophysics Laboratory, 1985 (referred to here as HGSE) and *The Infrared & Electro-Optical Systems Handbook*, volume 2, "Atmospheric Propagation of Radiation," edited by F. G. Smith, SPIE Optical Engineering Press, Bellingham, Washington, 1993 (referred to as IR Handbook, V2). The references describe how apparent zenith angles differ from true zenith angles. Corrections are available from these references and from the Astronomical Almanac to help aim correctly for Earth-based observers. The FASCODE propagation code (Air Force Geophysics Laboratory Report AFGL-TR-78-0081, by H. J. P. Smith, D. J. Dube, M. E. Gardner, S. A. Clouth, F. X. Kneizys, and L. S. Rothman, Hanscom AFB, Massachusetts, 16 January 1978) even accounts for refractive elongation of the path when scattering and absorption of the rays are considered in detail. Special atmospheres, with inversions or unusual constituent or temperature gradients, can further complicate refractive effects. (For example, see the report by E. Bauer, *Effects of Atmospheric Refraction on Long-Range, Near-Surface, Electro-Optical Sensing Over Water (Final Report for May 1994 - May 1995)*, Institute for Defense Analysis Report IDA-D-1815, January, 1996.)

Less information is available for refractive effects when viewing a star from space. Views of stars as seen from the ground always include atmospheric effects while views from space only exhibit atmospheric effects from those few stars that are in view at any given time just beyond the limb of the Earth. Because of the few observers in space, less attention for space observation is expected. But the attention is needed for reliable space observation of stars whose rays come near the Earth.

After discussion of problem geometry and coordinate rotations required, a spherically symmetrical atmosphere based on the U. S. Standard Atmosphere, 1976, is used to predict deviations caused by atmospheric refraction. Although the symmetric model is not realistic in a strict sense, it is expected to be as accurate as detailed models when the details are unknown. In most situations, ground-truth atmospheric data for a dynamic atmosphere are not available. The model is expected to improve sensor aiming at stars by predicting the apparent right ascension and declination after refraction occurs. Even though based upon average atmospheric parameters, pointing improvement can be critical for sensors with fields of view less than one degree.

Problem Geometry and Coordinate Rotations

In Figure 1, let $\vec{R}_T = [R_1 \ R_2 \ R_3]^T$ be an ECI* vector to the sensor and α_s, δ_s represent the ECI right ascension and declination to the star. The superscript T on a matrix denotes matrix transpose. A unit vector from the sensor to the star is given by

$$\hat{E} = [E_1 \ E_2 \ E_3]^T = [\cos\delta_s \cos\alpha_s, \cos\delta_s \sin\alpha_s, \sin\delta_s]^T.$$

* ECI (Earth-centered inertial) coordinates are at times called celestial coordinates or geocentric inertial (GCI) coordinates. Because of precession of the equinoxes and rotation of the sun about the center of mass of the Milky Way galaxy, they do not form an inertial system.

*

Star

FIGURE 1. Sensor and event vectors in ECI coordinates.

Points $\hat{n}$ at a distance l from the sensor in direction $\hat{E}$ are given by

$$\hat{n} = \vec{R}_T + l [E_1 \ E_2 \ E_3]^T.$$

The length of the projection of $\vec{R}_T$ onto the line is

$$l_{proj R_T} = R_1 E_1 + R_2 E_2 + R_3 E_3.$$

The distance from the origin to the line is

$$d_c = (R_1^2 + R_2^2 + R_3^2 - l_{proj R_T}^2)^{1/2}.$$

When $d_c > R_{eq}$, where R_{eq} is 6378.137 km, the equatorial radius of the Earth, the ray cannot intersect the Earth. With or without refraction, rays having $d_c > R_{eq}$ will reach the sensor. With refraction, some smaller values of d_c will also reach the sensor. Refraction occurs in a plane defined by the Earth's center, the sensor, and the star. The plane is called the refraction plane or the plane of incidence.

Rotation by angle $\psi = \text{atan}(-R_1/R_3)$ about $\hat{y}$ places the new vector R_T'' in the $y'' - z''$ plane. ($R_1 = R_3 = 0$ cases are excluded and R_3'' is forced to positive values.)

$$R_T'' = \begin{bmatrix} \cos \psi & 0 & \sin \psi \\ 0 & 1 & 0 \\ -\sin \psi & 0 & \cos \psi \end{bmatrix} \cdot \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} 0 \\ R_2 \\ \sqrt{R_1^2 + R_3^2} \end{bmatrix}$$

$$\hat{E}'' = \begin{bmatrix} \cos \psi & 0 & \sin \psi \\ 0 & 1 & 0 \\ -\sin \psi & 0 & \cos \psi \end{bmatrix} \cdot \begin{bmatrix} \cos \delta_s \cdot \cos \alpha_s \\ \cos \delta_s \cdot \sin \alpha_s \\ \sin \delta_s \end{bmatrix} = \begin{bmatrix} \frac{R_3 \cos \delta_s \cdot \cos \alpha_s - R_1 \sin \delta_s}{\sqrt{R_1^2 + R_3^2}} \\ \cos \delta_s \cdot \sin \alpha_s \\ \frac{R_1 \cos \delta_s \cdot \cos \alpha_s + R_3 \sin \delta_s}{\sqrt{R_1^2 + R_3^2}} \end{bmatrix} = \begin{bmatrix} E_x'' \\ E_y'' \\ E_z'' \end{bmatrix}.$$

To complete the rotations to the refraction plane, rotate by angle η (to be determined) about R_T'' (thus keeping R_T'' in the $y''-z''$ plane) to place the new $\hat{E}'' = \hat{E}'$ vector in the $y'-z'$ plane. The direction cosines of $\hat{R}_T''$ are $c_1 = 0$, $c_2 = R_2/|\hat{R}_T|$, and

$$c_3 = (\sqrt{(R_1^2 + R_3^2)})/|\hat{R}_T|.$$

The transformation matrix described by rotation through angle η about an axis described by c_1, c_2, c_3 is given by G. A. Korn and T. M. Korn, *Mathematical Handbook for Scientists and Engineers*, McGraw-Hill, 1968, p. 475:

$$A = \cos\eta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + (1 - \cos\eta) \begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_2c_1 & c_2^2 & c_2c_3 \\ c_3c_1 & c_3c_2 & c_3^2 \end{bmatrix} + \sin\eta \begin{bmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix}.$$

The x' component of the transformed unit vector is

$$E_1' = \cos\eta E_1'' + c_1^2 E_1'' - \cos\eta c_1^2 E_1'' + c_1c_2 E_2'' - \cos\eta c_1c_2 E_2'' \\ - \sin\eta c_3 E_2'' + c_1c_3 E_3'' - \cos\eta c_1c_3 E_3'' + \sin\eta c_2 E_3''.$$

Because $c_1 = 0$, when this component is set to zero (to place the primed axes so that $\hat{E}'$ is in the $y'-z'$ plane) the equation for η simplifies to

$$\tan\eta = \frac{E_1''}{c_3 E_2'' - c_2 E_3''}.$$

Two values of η satisfy the goal of $E_1' = 0$; the values differ by π . To make the solution unique, when $\eta < 0$, π is added to its value. After η is evaluated, the rotation about R_T'' can be performed. The new unit vector $\hat{E}'$ in the $y'-z'$ plane is

$$\hat{E}' = \begin{bmatrix} 0 \\ E_2' \\ E_3' \end{bmatrix} = \begin{bmatrix} 0 \\ \cos\delta' \\ \sin\delta' \end{bmatrix},$$

where the latter representation is in terms of the latitude, δ' , in the primed coordinate system.

$$E_2' = \cos\delta' = c_3 E_1'' \sin\eta + E_2'' [\cos\eta + (1 - \cos\eta)c_2^2] + c_2c_3 E_3'' (1 - \cos\eta)$$

$$E_3' = \sin\delta' = -c_2 E_1'' \sin\eta + c_2c_3 E_2'' (1 - \cos\eta) + E_3'' [\cos\eta + (1 - \cos\eta)c_3^2].$$

In this primed coordinate system, refraction is easily accounted for by replacing δ' by $\delta' + \Delta\delta$ where $\Delta\delta$ is the deviation caused by refraction. Only the latitude is altered by refraction.

$$\delta'_{Aim} = \delta'_{Star} + \Delta\delta$$

Because at optical wavelengths the atmospheric index of refraction is >1 and decreases with altitude, $\Delta\delta$ has a sign that increases the aim point distance above the limb of the Earth. The sign is discussed in the following section.

The proper declination and right ascension for aiming is retrieved by rotating about $\hat{R}_T'$ by angle $-\eta$ back to the double-primed coordinate system and a further rotation by angle $-\psi$ about the y'' axis to get back to ECI coordinates. The resulting $\hat{E}$ determines the declination and right ascension for aiming.

$$\delta_{Aim} = \arcsin E_3$$

$$\alpha_{Aim} = \arctan \frac{E_2}{E_1}.$$

In summary, the method has been outlined to find the declination and right ascension to aim toward in order to see the star when refraction effects occur. The required input is the sensor position and δ_{star} , α_{star} . Remaining tasks discussed in the following sections include:

- Determine $\Delta\delta$ for a given $\hat{R}_T'$ and $\hat{E}'$
- Develop Earth intersection tests for an ellipsoidal Earth shape
- Make estimates of the size of refraction effects for a sample star

Deviation Determination by $\hat{R}_T'$ and $\hat{E}'$

Within a given atmosphere, deviations caused by atmospheric refraction are completely determined by sensor position and the direction to the star. The changes in direction occur in a plane defined by $\hat{R}_T'$ and the direction to the star, $\hat{E}'$. Because this section describes phenomena that are always in the primed coordinate system, the primes are omitted from all vectors and from declinations and right ascensions within this section to simplify notation. The advantage of simpler notation is expected to outweigh the disadvantage of possible confusion with ECI coordinates (that are also unprimed).

The refraction plane is illustrated in Figure 2. If H_i' , the apparent height of the refracted ray, is greater than 65 km, the deviation is less than 0.002 degree (35 μ radian, 7.2 second), and no corrections are likely to be required. If $H_i' > 31$ km, deviations are less than 0.02 degree (0.35 mradian). The largest corrections occur for $H_i' < 1$ km where deviations can reach 1.1 degree (19.2 mradian). Depending on the application, some of these deviations require correction. With refraction, with the U. S. Standard Atmosphere, 1976, with a spherical Earth of radius $R_e = 6371$ km, and with $R_T = 25942$ km, when $d_c > R_e - 558.5$ km, the apparent height (H_i') of the refracted ray is greater than 0.4 km, and the ray reaches the sensor. With the same atmosphere and Earth and $R_T = 25990$ km, when $d_c < R_e - 604.8$ km, $H_i' < -0.29$ km and the ray is intercepted by the Earth and not received by the sensor. At a fixed $R_T = R_e + 200$ km, where $75.82^\circ < \theta_{inc} < 83.60^\circ$, the apparent heights vary from $H_i' = 0$ km to $H_i' = 160$ km. Such details vary with R_T and with assumptions about the atmospheric model and about the shape of the Earth; there is even a slight wavelength dependence. At larger values of R_T than the case illustrated in Figure 2, the ϕ_T , θ_{inc} , l , and d_c values are all altered. For $R_T > R_e + 200$ km with the illustrated H_i' ; however, the deviations are unchanged. Let us return to the geometry to continue the description of how such calculations are done.

A vector from the origin to the point on the unrefracted ray (directed toward the sensor) that is closest to the Earth's center is

$$\vec{d}_c = \vec{R}_T + l_{projR_T}(\hat{x}\cos\alpha_x + \hat{y}\cos\alpha_y + \hat{z}\cos\alpha_z).$$

The $\vec{d}_c$ does not depend on the distance to the source at $\vec{E}$. That source distance enters only as the direction cosines are formed. When a star is viewed, these direction cosines can be evaluated from the star's right ascension α_s and declination δ_s . With E_i for $i = x, y, z$ representing the components of the unit vector along $\vec{E}$, *

$$\cos\alpha_x = \frac{\hat{x} \cdot \vec{E}}{|\vec{E}|} = \cos\delta_s \cdot \cos\alpha_s = E_x, \quad \cos\alpha_y = \frac{\hat{y} \cdot \vec{E}}{|\vec{E}|} = \cos\delta_s \cdot \sin\alpha_s = E_y,$$

$$\cos\alpha_z = \frac{\hat{z} \cdot \vec{E}}{|\vec{E}|} = \sin\delta_s = E_z.$$

* Recall that primes have been omitted.

FIGURE 2. Illustration of the plane of refraction, $y' - z'$ plane.

The star is so far away that its direction cosines measured from the Earth's center are the same as those measured from the Earth-orbiting satellite. In Figure 2, θ_{inc} is found from

$$\theta_{inc} = \arccos(-\vec{R}_T \cdot \hat{E}/R_T) \text{ or } \theta_{inc} = \arcsin(d_c/R_T),$$

where $0 < \theta_{inc} < \pi$. Values of $\theta_{inc} > \pi/2$ occur when the sensor is on the same side of the Earth as the star and refraction effects do not occur. Angles where refraction is of concern are limited to $0 < \theta_{inc} < \pi/2$.

Once θ_{inc} is determined, Snell's Law provides a means to determine the deviation. The Law says that the product of the index of refraction, the distance from the origin, and the sine of the angle between $\hat{r}$ and the element of path length is a constant for the path taken by a ray in a spherically symmetric medium. When applied to the point at the sensor: $\hat{r} = \vec{R}_T$, the angle of interest is ϕ_T , the index of refraction is assumed to be unity. Sensor altitudes must be sufficient for this assumption to be valid. Here sensor altitudes are taken as $\geq 200\text{km}$; all refractive effects occur at lower altitudes. The constant in Snell's Law becomes $p = R_T \sin \phi_T$. From Figure 2, with the apparent altitude of the received ray, H'_t ,

$$p = R_T \sin \phi_T = R_e + H'_t.$$

Both the ϕ_T and the H_t' have a fixed relationship with the deviation, $\Delta\delta$.
 $\phi_T = \theta_{inc} + \Delta\delta$. The relationship with apparent height, $H_t'(\Delta\delta)$, is developed later.

$$R_T \sin(\Delta\delta + \theta_{inc}) = R_e + H_t'(\Delta\delta).$$

Because deviations are small, the equation simplifies to

$$\Delta\delta \cos \theta_{inc} + \sin \theta_{inc} (1 - \Delta\delta^2/2) = (R_e + H_t'(\Delta\delta))/R_T.$$

A newton's method may be used to solve for the deviation. Confusion is possible concerning the proper radius of the Earth to use. The development using Snell's Law assumes a spherically symmetric Earth and atmosphere. The U. S. Standard Atmosphere, 1976, that is later used for ray tracing to develop the $\Delta\delta$ dependence of H_t' is a spherically symmetric representation of the atmosphere. The radius chosen here is the average radius, 6371 km. Once $\Delta\delta$ is identified, results from previous sections outline rotations back to ECI coordinates and evaluation of the aiming parameters δ_{Aim} and α_{Aim} .

Sign determination for $\delta'_{Aim} = \delta'_{Star} \pm \Delta\delta$

As illustrated in Figure 2, refraction causes the aim point to be raised further above the limb of the Earth. Recall that in primed coordinates, the z' components of $\vec{R}_T$ are always ≥ 0 . δ'_{Star} is negative for any starlight experiencing refraction effects. When the z' component of $\vec{d}_c$ is positive, refraction raises the aim, and the positive sign is required. When the z' component of $\vec{d}_c$ is negative, the star is in a direction where $\vec{E}'$ and $\vec{d}_c$ have opposing y' components. Raising the aim requires the negative sign. *In summary, the positive sign is appropriate when $R_3' + |l_{projR_T}|E_3' \geq 0$; otherwise, the negative sign is used.*

There are subtle cases that give complications. A negative sign, for example, might make the star's declination less than $-\pi/2$. In such a situation, after the subtraction, the star's declination is replaced by $\delta'_{Star} + 2|\delta'_{Star} + \pi/2|$, and its right ascension is altered by the replacements $E_1' \rightarrow -E_1'$ and $E_2' \rightarrow -E_2'$. This is the exception to the earlier statement that in the primed coordinate system, only starlight declinations are altered by refraction.

Ray tracing to determine apparent height, $H_t'(\Delta\delta)$

The ray path received at the sensor is illustrated in Figure 2. The distance of closest approach to the Earth is indicated as H_t . HGSE refers to H_t as the refracted tangent height. As a convenience, measure positions on the path by the distance from the origin, r , and the angle ζ between $\vec{r}$ and a vector to the point of closest approach. ϕ remains the angle between $\vec{r}$ and an element of path length. $n(r)$ represents the r variation of the index of refraction. With this notation, Snell's Law becomes

$$n(r)r \sin \phi = p,$$

where p is a constant. At the position $\vec{r} = \vec{r}_0$ on the path where ζ is zero, ϕ is $\pi/2$ and $p = n(r_0)(R_e + H_t)$. H_t is the Earth-tangent height; it is the minimum altitude of the ray that reaches the sensor. This minimum-altitude position is a convenient initial condition for ray tracing to determine the path and the apparent height. **By choosing a series of H_t values, ray tracing determines the apparent height and deviation for each corresponding Earth-tangent height.**

The element of path length is

$$ds = \frac{dr}{\cos \phi} = \sqrt{r^2 d\zeta^2 + dr^2}.$$

Because of Snell's Law, $\cos \phi = \sqrt{1 - \frac{p^2}{n^2 r^2}}$ and because the initial value of ζ is zero,

$$\zeta = \int_{r_0}^{r_0+1\text{km}} \frac{pdr}{r\sqrt{n^2 r^2 - p^2}} + \int_{r_0+1\text{km}}^r \frac{pdr}{r\sqrt{n^2 r^2 - p^2}}.$$

The integral has been broken into two components, ζ_1 and ζ_2 , corresponding to portions below and above 1-km away from r_0 . This separation is convenient because the integrand is singular at the lower limit of integration; integration difficulty is isolated in the first integral.

Data are taken from the U. S. Standard Atmosphere, 1976, to give atmospheric pressure, temperature, and water vapor content, in 1-km increments up to 85 km and in larger increments at higher altitudes. These data are needed to model the index of refraction and are illustrated in Appendix A.

At each r_0 (corresponding to each choice in the series of H_t values), the index of refraction may be specified in terms of the refractivity, N .

$$n(r_0) = 1 + N(r_0) \times 10^{-6}.$$

Over any small interval from r_0 to $r_0 + \Delta h$, the refractivity is approximated by

$$N(r) = N(r_0) \exp\left[\frac{-(r-r_0)}{k}\right].$$

Within the interval Δh , the scale height, k , is approximated by

$$k = \Delta h \ln\left(\frac{N(r_0)}{N(r_0 + \Delta h)}\right).$$

Because the scale heights and Δh are small compared to r_0 and because $n(r_0)$ is near 1, ζ_1 can be evaluated analytically:

$$\zeta_1 = \frac{\sqrt{2p\Delta h}}{r_0 \sqrt{1 - r_0 N(r_0) \times 10^{-6} / k}} \left(1 - \frac{\Delta h}{3r_0}\right).$$

Larger values of ζ are obtained by straightforward numerical integration of ζ_2 . The integrals are carried to the minimum allowed sensor altitude, $h = 200$ km, where $R_T = 200$ km and where $\phi = \phi_T$ and $\zeta = \zeta_T$. At this altitude, the index of refraction exceeds unity only by about 10^{-13} .

Initial values for each integration fix the initial values $\zeta = 0$, $r = R_e + H_t$, $\phi = \pi/2$, and $p = rn(r)$. Final values of $\phi = \phi_T$ and $r = R_e + 200\text{km}$ are initially known from $p = (R_e + 200\text{km}) \sin \phi_T$ and from the "final" altitude of 200 km. The apparent height is also known (with the initial conditions) because $R_T \sin \phi_T = R_e + H_t'$. The resulting $\Delta\delta$ is determined by the integration over $d\zeta$.

Results for a series of H_t values are seen in Figure 3 where the error in apparent height ($H_t' - H_t$) of the Earth-tangent ray is given as a function of the ray's minimum height, H_t . A similar result has been published in HGSE, pages 18 - 67. There is a discontinuity in slope that occurs near $H_T = 11$ km. More obvious discontinuities occur in some other parameters (for example, see Figure 4a) as the stable tropopause altitude is passed. Because $p = n(H_t) \cdot (R_e + H_t) = R_e + H_t'$, the equation for the curve in Figure 3 is

$$H_t' - H_t = [n(H_t) - 1] \cdot (H_t + R_e).$$

At $H_t = 0$, the ray intercepts the spherical Earth and no refracted ray is detected. Appendix B describes an alternative intercept test that accounts for some features of the shape of the Earth.

FIGURE 3. Error in Earth-tangent height vs refracted tangent height predicted for the U. S. Standard Atmosphere, 1976.

For present purposes, the deviation, $\Delta\delta = \varphi_T - \theta_{inc}$, in Figure 2 is the required parameter. θ_{inc} values are available from $\pi/2 - \theta_{inc} = \zeta_T + \beta$ and $2\beta - \varphi_T = -\theta_{inc}$. Solving for $\Delta\delta$ gives

$$\Delta\delta = 2\varphi_T + 2\zeta_T - \pi.$$

The $\Delta\delta$ is needed as a function of the apparent height, H'_T . Results are illustrated in Figures 4a, 4b, and 4c.

FIGURE 4a. Deviation caused by atmospheric refraction as a function of apparent tangent height, H'_T , predicted for the U. S. Standard Atmosphere, 1976.

FIGURE 4b. Results for the interval $30 \leq H_t' \leq 80$ km.

FIGURE 4c. Results for the interval $60 \leq H_t' \leq 190$ km.

The deviations in Figure 4 are also illustrated in Figure 5 along with the analytic representation of the data. With deviations in degrees and apparent altitudes in km, the analytic fits (+ signs) illustrated are

$$\Delta\delta = 1.0864 \exp\left[\frac{-(H_t' - 1.8403)}{8.9}\right] \text{ for } H_t' < 10.59 \text{ km}$$

$$\Delta\delta = 0.3261 \exp\left[\frac{-(H_t' - 12.4455)}{6.1}\right] \text{ for } H_t' > 12.45 \text{ km}$$

$$\Delta\delta = 0.4067 \exp\left[\frac{-(H_t' - 10.5905)}{14.0}\right] \text{ for } 10.59 \leq H_t' \leq 12.45 \text{ km}.$$

Note the scale heights. Near the tropopause, the scale height is largest at 14 km. At lower altitudes, the scale height is 8.9 km, while at higher altitudes the scale height is 6.1 km.

FIGURE 5. Comparison of analytic fit to $\Delta\delta$ data as a function of H_t' .

Application of newton's method to solve

$$R_T \sin(\Delta\delta + \theta_{inc}) = R_e + H_t'$$

for $\Delta\delta$ is more convenient using the apparent height as a function of deviation rather than the deviation as a function of apparent height described above. Solving for the apparent height gives

$$H_t' = 1.8403 + 8.9 \ln \frac{1.0864}{\Delta\delta} \text{ for small } H_t' \text{ and } \Delta\delta > 0.4067 \text{ degree.}$$

$$H_t' = 10.5905 + 14.0 \ln \frac{0.4067}{\Delta\delta} \text{ for intermediate } H_t' \text{ and } 0.3261 < \Delta\delta < 0.4067 \text{ degree.}$$

$$H_t' = 12.4455 + 6.1 \ln \frac{0.3261}{\Delta\delta} \text{ for large } H_t' \text{ and } 0.0030 < \delta < 0.3261 \text{ degree.}$$

Other values of deviation are assigned an artificial value of $H_t' = 200 \text{ km}$ that corresponds to $\Delta\delta = 0$.

Differences between the analytic fit and the ray-trace data in Figure 5 are less than 0.003 degree (52 μ radian, 10.8"). This error can be reduced by using a more involved fitting function. Because of the dynamic nature of the atmosphere, a detailed fit of deviation as a function of the apparent tangent height with errors less than 0.003 degree is not justified. If standard deviations in atmospheric parameters were available as a function of altitude, Monte Carlo methods could be used to find expected variabilities in $H_t'(\Delta\delta)$ and $\Delta\delta(H_t')$. An even larger difference between the analytic fit and ray-trace data might not be a cause for alarm.

Aiming correction review with given $\vec{R}_T$, $\hat{E}$

Let us outline the aiming correction procedure that applies when true right ascension and declination are known. Coordinate rotations transform the $\vec{R}_T$ and $\hat{E}$ into the incident plane (primed coordinates). θ_{inc} is evaluated from $\text{acos}(-\vec{R}_T \cdot \hat{E}/R_T)$. Deviation is found by applying Newton's method to solve $R_T \sin(\Delta\delta + \theta_{inc}) = R_e + H_t'(\Delta\delta)$ for $\Delta\delta$. The deviation is added to the latitude in the primed coordinate system, with care in the choice of sign. Coordinate transformations back to ECI coordinates give the apparent right ascension and declination desired for aiming the sensor.

Target-position correction with given $\vec{R}_T$, incident plane, ϕ_T

There may be times when the true right ascension and declination are unknown and refraction corrections are used to improve estimates of a source location. In such situations, the sensor position and aim direction are known. The image location relative to the optical center of the image plane determines the incident plane and ϕ_T . The apparent tangent height is found from $H_t' = R_T \sin \phi_T - R_e$. The deviation as a function of the apparent tangent height, $\Delta\delta(H_t')$, specifies the deviation. This type of correction is useful for a system used to find new stars and specify their location. Such a correction is not considered here.

Aiming-Correction Examples for Viewing Alpha Centauri from Space

To illustrate refractive effects, let us examine the aiming corrections required to view $\alpha^1 Cen$ from a sensor in space. The examples chosen position the sensor in a Keplerian orbit with the following initial conditions:

Right ascension of ascending node: $\Omega = 40^\circ$

True anomaly: $v = 0^\circ$

Inclination angle: $i = 90^\circ$

Argument of perigee: $\omega = -80^\circ$

Altitude = 200 km.

Orbit eccentricities are taken as $\epsilon = 0, 0.50$, and 0.75 for three examples. Each orbit starts at perigee in the southern hemisphere. The orbit's Ω coupled with the inclination places the orbit nearly in a plane containing $\alpha^1 Cen$. Thus, any alterations in apparent right ascension are small. Figure 6 displays a portion of the three orbits: a dashed line for $\epsilon = 0$, a dash-dot line for $\epsilon = 0.5$, and a solid line for $\epsilon = 0.75$. Distances are illustrated in km. In Figure 6, $\alpha^1 Cen$ is nearly in the same plane with its declination near -60° . The $\epsilon = 0.5$ and 0.75 cases

are calculated to the largest values illustrated in Figure 6; the $\epsilon = 0$ orbit is calculated just beyond a full revolution.

FIGURE 6. Sample orbits for refraction tests.

As the sensor traverses its orbit, alterations occur in the apparent declination angle. Figures 7, 8, and 9 show the effect. Examination of the $\epsilon = 0$ case in Figure 7 shows that refractive effects disturb a small fraction of the orbit. When the effects occur, they last for about 60 seconds and can cause deviations slightly larger than 1° . With other orbit choices, the changes in right ascension also can be up to about 1° and last a few times as long.

Figure 7 shows that at about 1000 s from perigee, the line of sight to $\alpha^1 Cen$ is beginning to skim the atmosphere. As viewed from the sensor, the star's declination appears more negative. As the line of sight comes closer to the Earth, the apparent declination continues to decrease until it reaches a minimum. The following abrupt increase (back to a change of zero) represents the line of sight being cut off by the Earth's surface. Later in the orbit, the line of sight becomes nearly tangent to the Earth's surface in the northern hemisphere and the apparent declination jumps to its maximum value and subsequently decreases at a slower rate as seen near 3150 s from perigee in Figure 7.

Results for the $\epsilon = 0.5$ are illustrated in Figure 8. More time is required to reach sensor positions where refraction effects occur, but effects are similar to $\epsilon = 0$ results in polarity, magnitude, and duration. The maximum apparent change near 0.6° at about 5400 s from perigee results from orbit positions being calculated at 1-s intervals. Finer time resolution would include a ray that just skims the Earth's surface, and deviations would reach about 1.3° as seen in Figure 7.

FIGURE 7. Variation in declination angle with time past perigee for the $\epsilon = 0$ orbit.

FIGURE 8. Variation in declination angle with time past perigee for the $\epsilon = 0.5$ orbit.

Data for the more eccentric orbit, $\epsilon = 0.75$ in Figure 9, illustrates refractive effects lasting about 170 s. Even longer times can be encountered with special selection of orbit parameters. For the $\epsilon = 0.75$ case illustrated in Figure 6, refraction effects start when the sensor nears $\sqrt{x^2 + y^2} = 14000\text{km}$, and $z = 13000\text{km}$, and begin again when $\sqrt{x^2 + y^2} = 10700\text{km}$ and $z = 32000\text{km}$.

FIGURE 9. Variation in declination angle with time past perigee for the $\varepsilon = 0.75$ orbit.

Discussion of Results

Refractive aiming corrections have been developed to improve satellite observation of stars for geometries where the line of sight is nearly tangent to the Earth's surface. Required inputs are satellite position and direction to the target star. Predictions are based on ray tracing in the U. S. Standard Atmosphere, 1976, including H_2O effects. Necessary coordinate rotations and atmospheric parameters are described. Line of sight cutoff by the Earth is determined from the WGS 84 Earth-ellipsoid model.

Predictions of expected refractive effects were presented for several satellite orbits. Refraction can alter stellar right ascensions and declinations by more than 1° . The orbits considered give refractive-effect durations typically from ~1 minute to ~3 minutes. These time intervals suggest that the chosen satellites have lines of sight to the target star that effectively pass through the ~35 km height of the atmosphere in a few minutes. Results have a negligible wavelength dependence for micron-wide bands in the range $1\text{--}10\mu\text{m}$. Wavelength-specific treatments would improve results at shorter wavelengths or at narrow bands near $2.6738\mu\text{m}$ or $6.2696\mu\text{m}$ where the refractivity becomes large.

Appendix A: Models for Atmospheric Parameters

A series of models is illustrated for atmospheric parameters required to find the refractive aiming corrections. The first model described is that of the atmospheric refractivity, $N(h)$, as a function of altitude, h . Using the IR Handbook, V2 model and R. J. Hill and R. S. Lawrence, *Refractive Index of Water Vapor in Infrared Window*, Infrared Physics 26, pp. 371-376, 1986, the refractivity is

$$N = \left[237.2 + \frac{526.3\nu_1^2}{\nu_1^2 - \nu^2} + \frac{11.69\nu_2^2}{\nu_2^2 - \nu^2} \right] \cdot \frac{P_{dry}}{T(K)} + \left[\frac{651.5\nu_3^2}{\nu_3^2 - \nu^2} + \frac{7.4\nu_4^2}{\nu_4^2 - \nu^2} + \frac{57.9\nu_5^2}{\nu_5^2 - \nu^2} \right] \cdot \frac{P_{H_2O}}{T(K)} +$$

$$\left[\frac{3.744 \times 10^6 \nu_6^2}{\nu_6^2 - \nu^2} \right] \cdot \frac{P_{H_2O}}{T(K)^2}.$$

Pressures are in kPascals, temperatures in K, wave numbers, $\nu = 1/(\text{wavelength})$, are in cm^{-1} . Mode wave numbers are listed in Table 1.

TABLE 1. Constant parameters in refractivity model

Index i	Mode Numbers ν_i, cm^{-1}
1	114000
2	62400
3	111575
4	3740
5	1595
6	125

At wave number $\nu = 1 \mu\text{m}^{-1}$, N is illustrated in Figure 10 based on pressure and temperature profiles in the U. S. Standard Atmosphere, 1976. The U. S. Standard Atmosphere profiles are illustrated in Figures 11 through 13.

FIGURE 10. Predicted atmospheric refractivity based on the U. S. Standard Atmosphere, 1976, the IR Handbook, V2, and data from R. J. Hill and R. S. Lawrence.

FIGURE 11. U. S. Standard Atmosphere, 1976, estimate of dry atmospheric pressure as a function of altitude.

FIGURE 12. U. S. Standard Atmosphere, 1976, estimate of atmospheric temperature as a function of altitude.

FIGURE 13. U. S. Standard Atmosphere, 1976, estimate of atmospheric water-vapor pressure as a function of altitude.

Some features of the wavelength dependence are illustrated in Figure 14. Wavelength differences are small in the wavelength interval from 1 to 10 microns; differences become significant in the interval 0.1 to 1 micron. At 0 km in Figure 10, the 1-micron refractivity is just above 200, while at 10 microns in Figure 14 the refractivity is about 1% smaller. If the detected wavelength bands are small, such as for hyperspectral sensing, wavelength can make considerable difference in the 1 to 10 micron region. The ν_4 in Table 1 causes a significant change in refractivity at $\lambda = 2.6738 \pm 0.0038 \mu m$ (where N becomes quite large). The parameter ν_5 gives changes near $\lambda = 6.2696 \mu m$.

FIGURE 14. Predicted differences in atmospheric refractivity based on the U. S. Standard Atmosphere, 1976, the IR Handbook, V2, and data from R. J. Hill and R. S. Lawrence. The dashed line represents $N(\lambda = 0.1 \mu m) - N(\lambda = 1 \mu m)$; the solid line illustrates $N(\lambda = 1 \mu m) - N(\lambda = 10 \mu m)$ data.

Appendix B: Earth-Intersection Test

The solution of $R_T \sin(\Delta\delta + \theta_{inc}) = R_e + H'_i$ to find the proper $\Delta\delta$ for a particular sensor position is based on the spherically symmetric U. S. Standard Atmosphere, 1976. Thus, it is logical to use a spherical Earth to check for starlight blocking. The solution could be extended to include an ellipsoidal model of the atmosphere coupled with the WGS 84 Earth-shape ellipsoid for blocking checks and for the several km changes in R_e over the Earth's surface. Including such details is not presently justified. However, times at which stars pass behind the Earth can be modeled more accurately by accounting for the ellipsoidal shape.

Points on the WGS 84 Earth-shape ellipsoid in the x-z (ECI coordinate) plane has coordinates related by

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1,$$

where the semimajor axis a is 6378.137 km, the semiminor axis b is 6356.752 km, the eccentricity is 0.08181919. In terms of the latitude δ of points on the ellipsoid surface,

$$x = \left[\frac{1}{a^2} + \frac{\tan^2 \delta}{b^2} \right]^{-1/2}$$

$$z = x \tan \delta.$$

The distance from the origin to the ellipsoid at given δ is

$$d_{\text{ellipsoid}} = (x^2 + x^2 \tan^2 \delta)^{1/2} = \sqrt{1 + \tan^2 \delta} \cdot \left[\frac{1}{a^2} + \frac{\tan^2 \delta}{b^2} \right]^{-1/2}.$$

In primed coordinates (in the plane of refraction),

$$\vec{R}_T' = \vec{R}_T'' = \begin{bmatrix} 0 \\ R_2 \\ \sqrt{R_1^2 + R_3^2} \end{bmatrix},$$

where R_1, R_2, R_3 are the original ECI coordinates of the sensor. A coordinate rotation about $\hat{x}'$ through angle $-\zeta_T$ * and division by $|\vec{R}_T| = R_T$ gives a unit vector in the direction of the refracted Earth-tangent point.

$$\hat{R}_{H_i}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \zeta_T & \sin \zeta_T \\ 0 & -\sin \zeta_T & \cos \zeta_T \end{bmatrix} \cdot \begin{bmatrix} 0 \\ R_2 / \sqrt{R_1^2 + R_2^2 + R_3^2} \\ (\sqrt{R_1^2 + R_3^2}) / \sqrt{R_1^2 + R_2^2 + R_3^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{R_2}{|\vec{R}_T|} \cos \zeta_T + \frac{\sqrt{R_1^2 + R_3^2}}{|\vec{R}_T|} \sin \zeta_T \\ \frac{-R_2}{|\vec{R}_T|} \sin \zeta_T + \frac{\sqrt{R_1^2 + R_3^2}}{|\vec{R}_T|} \cos \zeta_T \end{bmatrix}.$$

The same coordinate transformations (rotation about $\hat{R}_T'$ by angle $-\eta$ and about the y'' axis through angle $-\psi$) that transformed $\hat{E}'$ back to $\hat{E}$ (in ECI coordinates) are applied to $\hat{R}_{H_i}'$, generating the ECI components of $\hat{R}_{H_i}$. The latitude is found from these new coordinates:

$$\delta = \text{atan} \left(\frac{R_{H_i,3}}{\sqrt{R_{H_i,1}^2 + R_{H_i,2}^2}} \right).$$

To decide if the ray path intercepts the Earth, this latitude of the Earth-tangent point is calculated and the resulting $d_{\text{ellipsoid}}$ compared with $H_i + R_e$. Those paths with $H_i + R_e < d_{\text{ellipsoid}}$ are blocked by the Earth.

* ζ_T that were evaluated during the ray trace are dependent on the chosen "final" altitude of 200 km. At any position on the ray path above 200 km the sum $\varphi_T + \zeta_T$ is constant at $(\pi + \Delta\delta)/2$.

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