NEUTRON DETECTOR RESOLUTION FOR SCATTERING

S. A. Kolda, D. Mesh

March 1997

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NEUTRON DETECTOR RESOLUTION FOR SCATTERING

By

Scott A. Kolda

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Approved:

Dr. R. C. Block, Thesis Advisor

Rensselaer Polytechnic Institute

Troy, New York

December, 1996
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A resolution function has been determined for scattered neutron experiments at Rensselaer Polytechnic Institute (RPI). This function accounts for the shifting and broadening of the resonance peak due to the additional path length, traveled by the neutron after scattering and prior to detection, along with the broadening of the resonance peak due to the bounce target. This resolution function has been parameterized both in neutron energy and size of the sample disk.

Monte Carlo Neutron and Photon (MCNP) modeling has been used to determine the shape of the detector resolution function while assuming that the sample nucleus has an infinite mass. The shape of the function for a monoenergetic neutron point source has been compared to the analytical solution. Additionally, the parameterized detector resolution function has been used to broaden the scatter yield calculated from Evaluated Neutron Data File ENDF/B-VI cross section data for $^{238}$U.

The target resolution function has been empirically determined by comparison of the broadened scatter yield and the experimental yield for $^{238}$U.

The combined resolution function can be inserted into the SAMMY code to allow resonance analysis for scattering measurements.
1.1 Objectives

The Rensselaer Polytechnic Institute (RPI) linear accelerator (LINAC) facility uses the neutron time-of-flight method in measuring the transmission, capture and scattering of neutrons. Applying the Oak Ridge National Laboratory neutron data analysis code SAMMY¹, only the resonance parameters from transmission and capture measurements are able to be determined today. The scattered neutrons travel an additional path length after interacting with the nuclei of the material sample which results in a distortion of the resonance shape. This distortion of the resonance shape has been previously undetermined.

The function describing the resonance shape distortion is to be incorporated into the SAMMY code. This function and the scattered neutron detector efficiency are both determined using the standard RPI 2-inch diameter disk sample for a neutron source energy range of 1 to 1000 eV. The resolution function dependence on sample size has been determined for sample sizes ranging from 0.25 to 2 inches in diameter for a neutron source energy of 10 eV since samples may be smaller than the standard size due to reasons of availability or expense.
1.2 Experimental Background

The RPI LINAC produces pulses of high energy electrons. These electrons then strike a tantalum target producing a neutron source. The electrons decelerate in the target through Coulombic interactions resulting in the emission of high energy bremsstrahlung photons. These photons undergo photoneutron interactions with the tantalum, generating free neutrons which are emitted nearly isotropically from the target. A portion of these neutrons are moderated and then pass through the collimated flight tubes to where they interact with the sample material of interest. The transmission, capture and scattering of these neutrons are measured and the resonance parameters are later determined. The configuration for capture and scattering measurements is shown in figure 1.1.

Figure 1.1 - A sketch of the 25 meter path for combined capture and scatter experiments

The detector used at RPI for combined neutron capture and scatter experiments is a sixteen section NaI detector which is shown in figure 1.2.²
Neutrons scattered from the sample are absorbed by boron present in the 99.46% $^{10}$B enriched B$_4$C liner in the following reactions:

\[
^{10}B + n \rightarrow ^7Li^* + \alpha; \quad ^7Li^* \rightarrow ^7Li + \gamma \quad \text{(93%)}
\]

\[
^{10}B + n \rightarrow ^7Li + \alpha \quad \text{(7%)}
\]

The resulting gamma, which is created 93% of the time, has an energy of 477 keV. This gamma can enter the NaI detector crystals, where it is absorbed. If the energy absorbed in the crystal is between the detector discriminator setpoints of 360 and 600 keV, it is then recorded as a scattered neutron. These discriminator setpoints were chosen to limit background in the recorded scattering data.

The Monte Carlo Neutron and Photon (MCNP) code simulation solution is tested through two methods. In the first method, a point source is modeled in MCNP. The time distribution of neutrons on the inner surface of the liner is then compared to the analytical solution. In the second method, the resolution function for a 2-inch diameter isotropic source (infinite mass approximation) is tested for accuracy using experimental scattering data from the $^{238}$U 1 mil (0.001") sample recorded October 11 to 12, 1995.
CHAPTER 2
COMPUTER SIMULATIONS

The primary means of determining the scattered neutron path length distribution has been through the Monte Carlo Neutron and Photon (MCNP) 4A transport code developed at the Los Alamos National Laboratory and distributed through the Radiation and Shielding Information Center (RSIC) at the Oak Ridge National Laboratory. The MCNP code simulates the movement of individual particles and records certain aspects of their behavior. This code is used in determining the time distribution of neutrons on the inner surface of the detector liner, the time distribution of gamma rays exiting the outside radius of the liner, and the scattered neutron detection efficiency.

MCNP allows for both simple and complicated geometries used in detector modeling. The cross sections included in the code allow for both neutron-photon simulations and energy deposition determination. The simple geometry used for the detector liner is shown below in figure 2.1 and the more realistic complicated geometry used for the detector is shown in figure 2.2.

Figure 2.1 - Simple MCNP detector geometry
All dimensions in cm
(not drawn to scale)

Figure 2.2 - Realistic MCNP detector geometry
The detector geometry is an updated version of the model used by Leinweber\(^4\). The liner has been upgraded to a \(^{10}\)B\(_4\)C ceramic that extends beyond the detector by 1.515 cm on each end. Additionally, the NaI detector crystals had been previously modeled as NaSn due to the lack of an I cross section file. The current detector model has \(^{127}\)I included.

2.1 Point Source

The initial simple detector model consists of a monoenergetic 10 eV neutron point source located at the center of the detector and the detector liner. A total of 100,000 case histories have been run with the neutron current entering the inner surface of the liner being scored. The results of this simulation are shown in figure 2.3 with the analytical solution fitted to the MCNP data, where 1 shake is \(10^8\) second. The time distribution of neutrons on the inner surface of the detector liner has an asymptotic nature which is explained by the analytical model.
Figure 2.3 - Point source neutron current incident on the inner liner surface for 10 eV neutrons

2.1.1 Point Source Analytical Solution

Figure 2.4 - Point source solution geometry
The proof follows the method used by Maissel and Glag5 in describing metal deposition on a planar surface after evaporation from a point source. The geometry used for the analytical solution is shown in figure 2.4. The value of \( I_r \) is the value of neutron current intensity across the inner surface of the detector liner, with inner radius \( a \), as a function of path length.

\[
dA_r = \frac{d\Omega \cdot r^2}{\cos \theta}
\]

The distance from the point source to the inner liner surface is \( r \) and the remaining symbols are shown in figure 2.4.

\[
dI_r = \frac{d\Omega}{4\pi} = \frac{dA_r \cdot \cos \theta}{4\pi \cdot r^2}
\]

\[
dA_r = 2\pi a \cdot dx
\]

\[
x = \sqrt{r^2 - a^2}
\]

\[
dx = \frac{r \cdot dr}{\sqrt{r^2 - a^2}}
\]

\[
\cos \theta = \frac{a}{r}
\]

\[
dI_r = \frac{a^2 \cdot dr}{2r^2 \sqrt{r^2 - a^2}}
\]

The conversion of equation (2.7) from path length space to time space is fairly simple since the neutrons are monoenergetic. By substituting \( r = vt \) and \( a = vt_0 \) into equation (2.7), where \( v \) is the velocity, the time distribution of intensity becomes:

\[
dI_t = \frac{t_0^2 dt}{2r^2 \sqrt{l^2 - t_0^2}}
\]
The curve in figure 2.3 is the fit to the MCNP data using equation (2.8)

2.2 Disk Source Energy Simulations

The next series of simple geometric simulations were carried out for a 2 inch diameter disk sample as shown in figure 2.2 and with a parallel beam of incoming neutrons from the left. These neutrons scatter isotropically from the sample nuclei which are assumed to have infinite mass. Monoenergetic beam neutrons of 1, 10, 100, and 1000 eV were used in four separate simulations. For these simulations, 1 million neutron case histories were followed into the B$_4$C liner and the photon current exiting the outer liner radius was scored. The sample input for the 10 eV case is presented in Appendix A.1. In figures 2.5 and 2.6, the time distribution of the photon current is shown for normalized 1 and 1000 eV source neutrons; these are displayed as a function of the product of time and square root of energy. There is an apparent shift toward longer path length for the higher energy neutrons. This will be shown to be a minor effect in Chapter 3.

Figure 2.5 - Neutron current distribution in path length space for 1 eV neutrons emitted from a 2-inch disk source.
2.3 Disk Source Size Simulations

The following series of simulations concern the relationship between scattered neutron path length and sample size. Simulations were carried out for monoenergetic 10 eV neutron isotropic sources of 0.25, 0.5, 1, 1.5 and 2 inches in diameter. The point source is excluded due to the singularity in intensity. For these simulations, 1 million neutron case histories were followed to the inner surface of the liner, where the neutron current was scored. Figure 2.7 demonstrates the change in neutron current intensity over time as a function of disk diameter. The significant effect of disk size on the distribution of path lengths is apparent. Disk sizes less than 1 inch in diameter show a relatively narrow peak when compared to the standard 2-inch disk size. This makes it difficult to find a universal function to describe these curves. That process will be covered in Chapter 3.
2.4 Energy Deposition Simulations

Not all photons exiting the outer B$_4$C liner surface are going to be detected. First of all, the liner extends beyond the detector crystals. The number of neutrons captured in this region is small. Second, the photons could travel through the crystal without interaction. This is unlikely for the NaI crystal. Third, not all energy depositions in the crystal are recorded as scattering events. Discriminator setpoints of 360 keV and 600 keV have been established to prevent corruption of the scattering signal. The higher limit denotes the cutoff between the low energy scattering signal photons, which have a birth
energy of 477 keV, and the high energy capture signal photons, which are born at energies greater than 1 MeV. The lower setpoint was established to minimize the effects of system background. Approximately 28% of the scatter signal photons are Compton scattered in the liner to energies less than 360 keV.

The energy deposition simulations were carried out for monoenergetic scattered neutron energies of 1, 10, 100 and 1000 eV neutrons. For each simulation, the scattered neutron was followed into the liner, where the radiative absorption took place. The photons were then transported into the crystals, where the energy deposition was scored. One million neutron case histories were run at each of the neutron energies. The simulation input for the 10 eV neutron source is included in Appendix A.2. The simulation results are shown in Table 2.1.

<table>
<thead>
<tr>
<th>Neutron source energy (eV)</th>
<th>Photon energy deposition range (keV)</th>
<th>Fraction of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt; 360</td>
<td>0.229 ± 0.0015</td>
</tr>
<tr>
<td></td>
<td>360 - 680</td>
<td>0.587 ± 0.0008</td>
</tr>
<tr>
<td>10</td>
<td>&lt; 360</td>
<td>0.295 ± 0.0015</td>
</tr>
<tr>
<td></td>
<td>360 - 680</td>
<td>0.575 ± 0.0009</td>
</tr>
<tr>
<td>100</td>
<td>&lt; 360</td>
<td>0.288 ± 0.0016</td>
</tr>
<tr>
<td></td>
<td>360 - 680</td>
<td>0.550 ± 0.0009</td>
</tr>
<tr>
<td>1000</td>
<td>&lt; 360</td>
<td>0.260 ± 0.0017</td>
</tr>
<tr>
<td></td>
<td>360 - 680</td>
<td>0.489 ± 0.0010</td>
</tr>
</tbody>
</table>

Table 2.1 - MCNP determined photon energy deposition efficiency
As can be seen in Table 2.1, approximately 16% of the neutrons below 100 eV emitted from the source do not deposit any energy into the detector crystals. MCNP simulations measuring different tallies have been performed to determine the nature of the losses. Since the detector liner is 36.05 cm long, MCNP simulations reveal that 1.8% of the neutrons escape out the ends of the detector liner without being captured. As mentioned earlier, 7% of the neutron capture reactions with $^{10}$B do not produce photons and will not be counted. Additionally, 1 - 2% of the photons that have been produced by the $^{10}$B captures in the liner escape out the ends of the detector liner. The remaining signal loss of approximately 4% is internal to the detector.

The additional signal loss at higher neutron energies is due to neutron leakage through the detector liner. This neutron leakage has been determined for the 2-inch disk source by using the simple model. The neutron current exiting the outer liner surface has been scored for one million case histories. The detector liner captures a very high percentage of the incident neutrons over the energy range of interest, 1 to 200 eV. This can be seen in Table 2.2.

<table>
<thead>
<tr>
<th>Neutron source energy (eV)</th>
<th>Fraction of neutrons exiting liner</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.000327 ± 0.0542</td>
</tr>
<tr>
<td>1000</td>
<td>0.0555 ± 0.004</td>
</tr>
</tbody>
</table>

Table 2.2 - Neutron leakage through the B$_4$C liner
CHAPTER 3
RESOLUTION FUNCTION PARAMETERIZATION

Successful incorporation of a scattering resolution function into the SAMMY code for the standard RPI sample size of a 2-inch diameter disk requires energy-dependent parameterized curves. To allow for different size samples to be used, size-parameterized curves for the scattering resolution function will also be determined.

The curve fitting program of TableCurve $2D^6$ was used to assist in this task. This program uses automated statistical methods to determine the best curve fit from a library of 3456 linear and nonlinear equations. The equation which showed the most promise for fitting the MCNP resultant time distribution of neutrons or photons over all disk sizes and energies is the asymmetric double cumulative (ADC) equation. The curve, the product of two error functions, has the equation and shape shown below, where $a$, $b$, $c$, $d$, $e$ and $f$ are the constants determined during fitting.

\[ y = a + \frac{b}{4} \left[ 1 + \text{erf} \left( \frac{x - c + \frac{d}{2}}{e\sqrt{2}} \right) \right] \left[ 1 - \text{erf} \left( \frac{x - c - \frac{d}{2}}{f\sqrt{2}} \right) \right] \]  

(3.1)
The error function, erf(x), is described by equations 3.2 and 3.3 below. Figures 3.2 and 3.3 demonstrate the behavior of the error function for two different ranges of the independent parameter. As can be seen below, the value of erf(x) approaches ±1 as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt$$  \hspace{1cm} (3.2)

$$erf(-x) = -erf(x)$$  \hspace{1cm} (3.3)
the value of $x$ approaches infinity in either direction. Therefore, the value of the ADC curve approaches the value of ‘a’ in this situation. The ADC function has been modified in the following manner to use as a resolution function. The value of ‘a’ has been set to zero to have the curve cover a finite area as $x$ approaches infinity. The resulting function has been termed the “bulge” equation and has the form of equation (3.4) below.

\[
y = b \left[ 1 + \text{erf} \left( \frac{x - c + d/2}{e \sqrt{2}} \right) \right] \left[ 1 - \text{erf} \left( \frac{x - c - d/2}{f \sqrt{2}} \right) \right] \tag{3.4}
\]

The bulge equation has been fitted to a selected portion of the MCNP curves by using the Origin\textsuperscript{8} curve fitting program. This method allows the bulge curve to reproduce as much of the MCNP curve as possible. The comparison of the two curves is shown in figure 3.4.
As can be seen above, the bulge curve drops off faster than the MCNP simulation resultant shape. Figure 3.5 below shows the shape of the difference between the two curves for the greater time region. This shape has been fitted in Origin using a sum of two exponentials in the form of equation (3.5). This equation, previously used by Moretti\(^9\) to modify the neutron target resolution equation in SAMMY, has been termed the “tail” curve and is shown in equation (3.5) below, where \(\alpha, \beta, \gamma, \delta, \varepsilon\) and \(\tau\) are the constants determined during fitting.

\[
tail(t) = \alpha\left\{ \beta \exp[\gamma(t + \tau)] + \delta \cdot \exp[\varepsilon(t + \tau)] \right\}
\]  
(3.5)
Figure 3.5 - Best fit of sum of exponentials curve for the longer time tail region for a 2-inch diameter disk 10 eV neutron source

Following the method of Moretti, the two equations are summed together starting at the point where the value of the tail equation is zero. A plot of the MCNP and combined tail and bulge curves is shown in figure 3.6. As can be seen, the MCNP curve has been well represented as a sum of the two equations.

Figure 3.6 - Combined curve fit of MCNP curve for neutron current incident upon inner detector liner surface for a 2-inch diameter disk 10 eV neutron source
3.1 Energy Dependence

The constants in both the bulge and tail curves were parameterized as a function of energy over an energy range of 1 to 1000 eV for a 2-inch neutron disk source for photon current exiting the outer liner surface. The points used in the parameterization were the 1, 10, 100 and 1000 eV values for the parameters in equations (3.4) and (3.5). The result is shown in table 3.1. The parameterized curves were determined by Origin for the simpler curves and TableCurve 2D for the more sensitive parameters.

<table>
<thead>
<tr>
<th>BULGE CURVE</th>
</tr>
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<tr>
<td>( b = 7.932 \times 10^{-4} \cdot E^{-0.4563} - 1.843 \times 10^{-4} )</td>
</tr>
<tr>
<td>( c = 1.877 \cdot E^{-0.5} + 0.0054 )</td>
</tr>
<tr>
<td>( d = 3.468 \cdot E^{-0.5} + 0.00432 )</td>
</tr>
<tr>
<td>( e = 0.216 \cdot E^{-0.5} + 0.00563 )</td>
</tr>
<tr>
<td>( f = 0.854 \cdot E^{-0.5} + 4.827 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TAIL CURVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = -0.2742 \cdot E^{-0.00486} + 0.2751 )</td>
</tr>
<tr>
<td>( \beta = 1.6044 \times 10^{-4} - 0.5876 )</td>
</tr>
<tr>
<td>( \gamma = -1.221 \cdot E^{0.5} - 0.1788 )</td>
</tr>
<tr>
<td>( \delta = \frac{1}{1.9612 - 0.1557 \cdot \ln(E) - 0.08541 \cdot E^{-2}} )</td>
</tr>
<tr>
<td>( \varepsilon = -0.5266 \cdot E^{0.5} + 0.02379 )</td>
</tr>
<tr>
<td>( \tau = -4.057 \cdot E^{-0.5} - 0.05542 )</td>
</tr>
</tbody>
</table>

Note: \( E \) is energy in eV

Table 3.1 - Energy parameterized values

Parameter sensitivity was determined by visually comparing the MCNP fitted curves with the parameterized value curves in Mathcad. The dependence of the values would be expected to be a function of energy in the manner of \( E^{0.5} \) or \( E^{-0.5} \) since the time required
for the scattered neutrons to reach the inner surface of the liner is proportional to $E^{-0.5}$. This is the case for most of the parameters. As a note, the areas of the curves have not been normalized to a value of 1 and need to be normalized prior to application. The applicability of these curves will be explored in chapter 4 where an interpolated curve will be tested against the $^{238}$U 36 eV, 66 eV, 102 eV and 189 eV resonance scattering data.

3.2 Size Dependence

The constants in both the bulge and tail curves were parameterized over a diameter range from 0.25 to 2 inches for a 10 eV neutron disk source for neutron current entering the inner liner surface. The fitted curve is shown in Figure 3.7 for the 1-inch disk.

![Figure 3.7](image_url)

Figure 3.7 - Combined curve fit of MCNP curve for neutron current incident upon the inner detector liner surface for a 1-inch diameter disk 10 eV neutron source
The points used in the parameterization were the 0.25, 0.5, 1.0, 1.5 and 2.0-inch disk values for the parameters in equations (3.4) and (3.5). The result is shown in table 3.2. The parameterized curves were determined by Origin for the simpler curves and TableCurve 2D for the more sensitive parameters. Parameter sensitivity was determined by visually comparing the MCNP fitted curves with the parameterized value curves in Mathcad.

### BULGE CURVE

\[
b = \frac{1}{475.81 \cdot D^{0.5} - 197.55}
\]

\[
c = \frac{1}{0.4425 + \frac{2.4911}{D}}
\]

\[
d = \sqrt[3]{0.3097 \cdot D^2 - 0.01796}
\]

\[
e = 0.1131 + 0.05961 \cdot D \ln D - 0.02113 \cdot D^{2.5} + 0.001941 \cdot \frac{\ln D}{D^2}
\]

\[
f = 0.03049 \cdot D^{2.634} + 0.07878
\]

### TAIL CURVE

\[
\alpha = \sqrt[3]{1.0101(10^{-5}) - 2.1283(10^{-5}) \cdot D + 1.3606(10^{-5}) \cdot D^2 - 9.6377(10^{-7}) \cdot D^3}
\]

\[
\beta = \frac{1}{0.1661 - 13.070 \cdot \exp(-D)}
\]

\[
\gamma = -8.2784 + 5.900 \cdot \ln D
\]

\[
\delta = -1.1780 \cdot D + 2.9307
\]

\[
\epsilon = 2.3820 \cdot D^{0.5} - 5.0290
\]

\[
\tau = -1.5676 + 1.4699 \cdot \exp(-D)
\]

Note: D is disk diameter in inches

Table 3.2 - Size parameterized values
CHAPTER 4
EXPERIMENTAL ANALYSIS

The resolution function was tested by comparing scattered neutron data measured for $^{238}\text{U}$ on October 11 - 12, 1995 with ENDF/B-VI (Evaluated Neutron Data File B, version VI) cross sections. Uranium was chosen since its resonance parameters are known to a fairly high level of confidence. Also, the heavy nucleus can be approximated as having an infinite mass. The 1 mil thick sample data was chosen for analysis to minimize scattered neutron signal loss due to multiple scattering, where 1 mil is 0.001 inches. The strong scattering resonances at 36 eV, 66 eV, 102 eV and 189 eV were analyzed.

The total resolution function is due to several factors, including the neutron burst width, the channel width, the target contribution, and the detector contribution. Each component in the resolution function varies in importance, with the primary contribution coming from the additional path length to the detector for scattered neutrons at lower energies. At higher energies, the target contribution also becomes significant.

Accurate determination of the resolution shape for the target and detector are critical for the determination of the resonance parameters since all resolution effects broaden the experimental data and alter the resonance shape. Currently the SAMMY inputs for channel width and burst width are valid, while the target moderator and detector shapes are not included. For scattered neutron analysis in SAMMY, these effects need to be included. MCNP modeling has determined the detector resolution shape due to the additional scattered neutron path lengths. These path lengths were parameterized to
allow for use over the MCNP analyzed energy range. The scattering target resolution function was determined by comparison with experimental data using the method of Moretti\textsuperscript{9}.

For the testing of the resolution shape, resonance data that is well documented and known with a high degree of confidence was chosen. This is the reason for using \( ^{238}\text{U} \) data. The chosen data is \( ^{238}\text{U} \) data from ENDF/B-VI Doppler broadened to 300K. The data are Doppler broadened to room temperature for reasons outlined by Moretti\textsuperscript{9}.

4.1 Broadening Process

Both the total cross section and capture cross section data were downloaded from the Brookhaven National Laboratory to calculate scattering yield. These data were processed using a version of the capture analysis Mathcad programs, developed by Moretti\textsuperscript{9}, which were altered for scattering analysis. The first alteration is the calculation of single-interaction scattering yield from ENDF data, which is shown in equation (4.2), where \( E \) is the energy, \( n \) is the sample number density of \( 1.3854 \times 10^4 \) atom/barn, \( \sigma_{\text{tot}} \) is the total cross section, \( \sigma_{\text{cap}} \) is the capture cross section, \( Y_{\text{the}} \) is the Doppler-broadened ENDF determined single-interaction scattering yield and \( T \) is the calculated transmission.

\[
T(E) = \exp(-n \cdot \sigma_{\text{tot}}(E)) \quad (4.1)
\]

\[
Y_{\text{the}}(E) = (1 - T(E)) \cdot \frac{\sigma_{\text{tot}}(E) - \sigma_{\text{cap}}(E)}{\sigma_{\text{tot}}(E)} \quad (4.2)
\]
The Doppler-broadened ENDF calculated yield is shown in figure 4.1 for energies of 30 to 115 eV.

![Graph showing scattering yield vs energy](image)

**Figure 4.1** - Calculated yield from Doppler-broadened ENDF cross sections for the 1 mil $^{238}$U sample

The Doppler-broadened ENDF calculated yield were additionally broadened at all resonances with the channel width square function and the burst width gaussian function, which are the forms used by Moretti$^9$. The resultant function was broadened by the calculated detector function and multiplied by an efficiency value that creates the best fit to the experimental data by having the lowest $\chi^2$. The starting point for the efficiency determination was a value in the range of the MCNP efficiency values listed in Table 2.1. Further broadening was performed to determine the target resolution function for neutron scattering through comparison of the broadened ENDF scattering yield and the experimental results.
4.2 Experimental Setup

The data used for resolution function analysis comes from $^{238}$U capture/scattering experiments on October 11 - 12, 1995. The samples and number of triggers for each sample are listed in table 4.1.

<table>
<thead>
<tr>
<th>Sample type and thickness</th>
<th>Triggers (x100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{238}$U 1-mil</td>
<td>2400</td>
</tr>
<tr>
<td>Empty can</td>
<td>500</td>
</tr>
<tr>
<td>$^{238}$U 5-mil</td>
<td>1000</td>
</tr>
<tr>
<td>Empty can</td>
<td>100</td>
</tr>
<tr>
<td>$^{238}$U 10-mil</td>
<td>800</td>
</tr>
<tr>
<td>Empty can</td>
<td>200</td>
</tr>
<tr>
<td>$^{238}$U 20-mil</td>
<td>800</td>
</tr>
</tbody>
</table>

Table 4.1 - Sample and number of triggers in capture/scatter experiment

For these experiments, the base channel width was set to 0.03125 μsec and the LINAC burst width was 0.070 μsec. The data from the experiments were checked for statistical fluctuations, deadtime corrected, normalized and summed into a single file for each sample using the data processing programs on the HP-1000 computer for capture data.

4.3 Experimental Data Processing

Due to the radioactivity of the $^{238}$U sample, background determination is not simple. The background was subtracted following the method of Moretti$^9$. The empty can background was first subtracted on a channel-by-channel basis. The sample contribution to the background was then determined at each resonance through linear interpolation of the mean signal far out on the resonance wings. Errors were propagated throughout the
process. The flux normalization factor was determined to be $13.60 \pm 0.16$ by following the method of Moretti$^9$.

4.4 Experimental Results

The resolution function fits were determined for four resonances of the 1 mil $^{238}$U sample to test the resolution function. Figure 4.2 below shows the result for the 102 eV resonance of the 1 mil $^{238}$U sample. The results for all of the resonances are shown in Appendix B. The ENDF calculated yield was burst width, channel width, and path length resolution broadened. For the 102 eV resonance, the scattered neutron detection efficiency value of 0.562 was determined to have the minimum $\chi^2$ value of 5.93. The main portion of the curve is well represented by the resolution function. The

![Graph showing scattering yield vs energy](image)

Figure 4.2 - Comparison of the resolution broadened ENDF calculated scattering yield and experimental scattering yield for the 102 eV resonance for the 1 mil $^{238}$U sample.
peak and low energy tail regions are not as well represented. These two regions will be covered in detail in the next section. Table 4.2 below is a compilation of the scattered neutron efficiencies for the minimum $\chi^2$ along with the $\chi^2$ values for the compared resonances. Figure 4.3 shows the sensitivity of $\chi^2$ for the 102 eV resonance.

<table>
<thead>
<tr>
<th>Resonance Energy (eV)</th>
<th>Detector Efficiency</th>
<th>Minimum $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.68</td>
<td>0.552</td>
<td>2.99</td>
</tr>
<tr>
<td>66.03</td>
<td>0.580</td>
<td>2.34</td>
</tr>
<tr>
<td>102.54</td>
<td>0.562</td>
<td>6.36</td>
</tr>
<tr>
<td>189.7</td>
<td>0.560</td>
<td>8.24</td>
</tr>
</tbody>
</table>

Table 4.2 - Resonance energy detector efficiency and fit values

Figure 4.3 - Scatter detection efficiency sensitivity for the 102 eV resonance for the 1 mil $^{238}\text{U}$ sample
From MCNP modeling, the expected detector efficiency is in the range of 0.55 to 0.575 between 10 and 100 eV. Also, as expected, the trend is decreasing efficiency as energy increases. The values in Table 4.2 for detector efficiency are between these two values, except for the 66 eV resonance, which is approximately 1% higher than the maximum value. Overall, the detector efficiency remains a relatively constant value of 0.56. The lower value determined at 36 eV could be due to capture of scattered neutrons by the strong capture resonance. This will be covered further in Section 4.5.1.

4.5 Discussion of Results

4.5.1 Peak Reduction

The experimental resonance peak shown in figure 4.2 is distorted in shape. Similar effects were seen for all of the resonances, with the 36 eV resonance having a flat top. This distortion could be due to capture of some of the scattered neutrons which would create a reduction of the scattering signal primarily at the peak.

Equation (4.1) was used to determine the fraction of the neutron beam which was transmitted through the sample. This value subtracted from 1 is the fraction of the beam interacting with the sample and is shown in Table 4.3 for the 1 mil $^{238}$U sample for the Doppler broadened ENDF resonance peak total cross section values and an atomic density value of $1.385 \times 10^{-4}$ atoms per barn. Multiple interactions are of concern when
<table>
<thead>
<tr>
<th>Resonance energy (eV)</th>
<th>ENDF $\sigma_{\text{tot}}$ (barns)</th>
<th>Fraction of beam interacting with sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.68</td>
<td>13,491.1</td>
<td>0.846</td>
</tr>
<tr>
<td>66.03</td>
<td>4387.4</td>
<td>0.445</td>
</tr>
<tr>
<td>102.54</td>
<td>6068.0</td>
<td>0.568</td>
</tr>
<tr>
<td>189.7</td>
<td>5224.7</td>
<td>0.515</td>
</tr>
</tbody>
</table>

Table 4.3 - Fraction of beam interacting at $^{238}$U resonances for 1 mil sample

more than approximately 10% of the beam interacts with the sample. This is certainly the case for the $^{238}$U resonances of interest and should show up as a reduction of the scattering signal, primarily at the peak. This effect will have to be included for a complete scattering resolution function for thick samples.

4.5.2 Lower Energy Wing Enhancement

For each of the scattering resonances, the lower energy wing has a larger signal than that of the higher energy wing, while the Doppler-broadened ENDF curve has a lower energy wing signal smaller than that of the higher energy wing, shown in figure 4.1.

Figure 4.4 - Collision in the center-of-mass system where M1 is the incident particle
The energy transferred to the $^{238}\text{U}$ results in a lowering of the energy of the scattered neutron, which would result in an increase of the signal at the lower energy wing of the resonance at the expense of the neutrons with energies on the high energy side of the resonance. Figure 4.4 shows a collision in the center-of-mass (CM) system between an incoming mass $M_1$ and a target mass $M_2$ as well as the scattering angle $\theta$ in the CM system. The energy transferred from $M_1$ to $M_2$ is given by equation 4.3 where $\Lambda$ is defined in equation 4.4\textsuperscript{12}. Equation (4.4) below shows the amount of energy

$$E_{\text{transfer}}(\theta) = \frac{1}{2} \Lambda E(1 - \cos(\theta))$$

$$\Lambda = \frac{4M_1 M_2}{(M_1 + M_2)^2} \tag{4.4}$$

transferred through elastic collisions\textsuperscript{12}. The value of $E_{\text{transfer}}(\theta)$ is the function of energy transferred to $M_2$ as a function of the scattering angle in the center-of-mass system. This has been determined by solving the equations of conservation of momentum and energy for the elastic collision between particles $M_1$ and $M_2$. The minimum value of $\theta = 0^\circ$ represents forward scattering of $M_1$, while the maximum value of $\theta = 180^\circ$ represents backward scattering of $M_1$.

Calculations have been performed for collisions between a neutron and $^{238}\text{U}$ nucleus for neutrons scattering at angles of $0^\circ$ and $180^\circ$ in the center-of-mass system to determine the maximum effect on the measured energy. Equations (4.3), (4.5) and (4.6) were used for the calculation of expected measured neutron energy. The value used for

$$E_{\text{scatter}}(\theta) = E - E_{\text{transfer}}(\theta) \tag{4.5}$$
\[ E = \left( \frac{72.2997 \cdot 25.5784}{t - t_0} \right)^2 \]  

Equation (4.6)

t_0, 3.78562 \mu s, was determined from a gaussian fit to the gamma flash curve. The path length of the neutron down the flight tube is 25.5784 m. The path length for the scattered neutrons is 16.814 cm, which is the distance to the B_4C liner inner surface directly below the ends of the detector and approximately the maximum path length traveled by a detected neutron. Equation (4.6) is the equation used to determine the experimentally measured energy of the neutron.

The results of the calculations are shown in Table 4.4. The size of the time bins for these experiments at this energy range is 0.0625 \mu s.

<table>
<thead>
<tr>
<th>Incident neutron energy (eV)</th>
<th>Scattering angle, ( \theta ) (degrees)</th>
<th>( E_{\text{scatter}}(\theta) )</th>
<th>Time shift (( \mu s ))</th>
<th>Experimental determined energy (eV)</th>
<th>Maximum energy shift (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.68</td>
<td>0</td>
<td>36.68</td>
<td>2.007</td>
<td>36.203</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>36.06</td>
<td>2.024</td>
<td>36.199</td>
<td></td>
</tr>
<tr>
<td>66.03</td>
<td>0</td>
<td>66.03</td>
<td>1.496</td>
<td>65.171</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>64.91</td>
<td>1.509</td>
<td>65.163</td>
<td></td>
</tr>
<tr>
<td>102.2</td>
<td>0</td>
<td>102.2</td>
<td>1.202</td>
<td>100.870</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>100.5</td>
<td>1.213</td>
<td>100.858</td>
<td></td>
</tr>
<tr>
<td>189.7</td>
<td>0</td>
<td>189.7</td>
<td>0.883</td>
<td>187.23</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>186.5</td>
<td>0.890</td>
<td>187.21</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4 - Calculated measured energy shift due to nuclear recoil for \(^{238}\text{U}\)
Recoil calculations were performed for the lighter metals of elemental Molybdenum ($M = 95.94$ AMU) and Iron ($M = 55.847$ AMU). Tables 4.5 and 4.6 display the results. In all cases, the experimental energy shift due to recoil of the target nucleus is negligible.

<table>
<thead>
<tr>
<th>Incident neutron energy (eV)</th>
<th>Scattering angle, $\theta$ (degrees)</th>
<th>$E_{\text{scatter}}(\theta)$</th>
<th>Time shift ($\mu$s)</th>
<th>Experimental determined energy (eV)</th>
<th>Maximum energy shift (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0</td>
<td>20.00</td>
<td>2.718</td>
<td>20.478</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>19.18</td>
<td>2.775</td>
<td>20.472</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>200.00</td>
<td>0.860</td>
<td>197.394</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>191.83</td>
<td>0.878</td>
<td>197.340</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5 - Calculated measured energy shift due to nuclear recoil for Mo

<table>
<thead>
<tr>
<th>Incident neutron energy (eV)</th>
<th>Scattering angle, $\theta$ (degrees)</th>
<th>$E_{\text{scatter}}(\theta)$</th>
<th>Time shift ($\mu$s)</th>
<th>Experimental determined energy (eV)</th>
<th>Maximum energy shift (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0</td>
<td>20.00</td>
<td>2.718</td>
<td>20.478</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>18.62</td>
<td>2.817</td>
<td>20.468</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>200.00</td>
<td>0.860</td>
<td>197.394</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>186.18</td>
<td>0.891</td>
<td>197.301</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6 - Calculated measured energy shift due to nuclear recoil for Fe

Table 4.4 demonstrates that the approximation of the $^{238}$U nucleus having an infinite mass is acceptable, since the maximum time shift due to recoil energy loss is much less than the size of the time bins used in the experiments. The recoil calculations
performed for Molybdenum and Iron show that the maximum recoil losses for the lower energy neutrons can result in a time shift equal to or greater than one channel width. This can distort the experimental data. Nuclear recoil effects will have to be taken into account for scattering measurements of samples with lighter nuclei.

The previous calculations have not taken into account the delay in detection of scattered neutrons resulting from the path length traveled by the photons created in the detector liner prior to detection. Using dimensions from the detector model of figure 2.2, the maximum path length traveled by a photon prior to detection is determined to be 40.28 cm. Since the photon travels at the speed of light, the resultant maximum time shift is $1.34 \times 10^{-9}$ seconds. This shift is too small to be detected, which justifies the neglecting of the detection path length of the photons created in the liner.

4.5.3 Scattering Target Resolution Function

Only the burst width, channel width and detector resolution functions were considered. The fits for the lower energy scattering resonances are reasonably good, while the fits degrade at resonance energy values greater than 100 eV. Since nuclear recoil energy and photon path length effects are minimal, this must be due primarily to the target resolution.

To determine the target resolution, the effects of the other three factors have to be known to a high degree of confidence. Both channel width and burst width effects are previously known. The scattering detector resolution function determined earlier is reasonably well-known for the following reasons. First, the time distribution of neutron
intensity on the inner liner surface resulting from a monoenergetic neutron point source matches the shape of the analytical solution. Second, the MCNP generated curves for the time distribution of photon intensity for the monoenergetic 2-inch disk source are similar in path length space and show the additional path length traveled into the liner by the higher energy neutrons. Third, the shifting of the resonance peaks is accurately determined by the MCNP model, as shown in the curves in Appendix B. Fourth, the detector efficiency determined from MCNP energy deposition calculations is in good agreement with the values that result in the best correlation between the broadened ENDF calculated scattering yield and the experimental yield. Fifth, the fits for the lower energy resonances are already reasonably close.

The starting point for the scattering target resolution function is the capture resolution function determined by Moretti for the bounce target. The following chi-square plus sum of exponentials function from Moretti was used to generate the target resolution function, \( R(t) \), as a function of time:

\[
R(t) = \frac{(t + \text{shift})^2}{2 \cdot \Lambda^2} \exp \left( \frac{-(t + \text{shift})}{\Lambda} \right) + A_1 (A_2 \exp(A_3(t + 0.94) + A_4 \exp(A_5(t + 0.94)))
\]

The value for \( \Lambda \) as a function of energy was determined from the following equation:

\[
\Lambda(E) = 0.3017 - 0.08739 \cdot \ln(E) + 0.00706 \cdot (\ln(E))^2
\]

The value for ‘shift’ as a function of energy was determined from the following equation:

\[
\text{shift}(E) = 0.381 \cdot \exp(-0.019 \cdot E) + 0.133 \cdot \exp(-0.094 \cdot E) + 0.105
\]

The value for \( A_1 \) as a function of energy was determined from the following equation:

\[
A_1(E) = -1.106 \cdot \exp(-0.0058 \cdot E) + 47.52 \cdot \exp(-65.083 \cdot E) + 1.264
\]
The values used for the constants ‘A₂’ through ‘A₅’ are listed in Table 4.7.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A₂</td>
<td>A₃</td>
<td>A₄</td>
<td>A₅</td>
</tr>
<tr>
<td>-65.64</td>
<td>-5</td>
<td>0.3898</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

Table 4.7 - Constants used in capture and scattering target resolution functions

The resultant resolution function is shown below in Figure 4.5 for a neutron energy of 189.7 eV.

![Figure 4.5 - Capture target resolution function at 189.7 eV](image)

The application of this resolution function to the previously broadened calculated ENDF scattering yield results in a resonance curve which is both exceedingly broadened and shifted toward higher energy. The reason for the shift is yet to be determined. The excessive broadening of the calculated resonance curve is the result of the sum of exponentials tail being too large in magnitude. Also, the curve has a secondary peak due to the sum of exponentials tail. The tail has been shifted to make the resolution function smooth. Both the shift of the chi-square function and the magnitude of the tail have also
been adjusted to create good agreement between the experimental data and the broadened
ENDF parameters.

The resulting fit for the 189.7 eV resonance using the modified scattering target
resolution function is shown in Figure 4.6. The fits for all of the resonances are
displayed in Appendix C. Table 4.8 is a listing of the adjusted parameters and the
goodness-of-fit values for all of the resonances.

<table>
<thead>
<tr>
<th>Energy (eV)</th>
<th>shift</th>
<th>$A_1$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.68</td>
<td>0.223</td>
<td>0.064</td>
<td>1.796</td>
</tr>
<tr>
<td>66.03</td>
<td>0.185</td>
<td>0.116</td>
<td>1.384</td>
</tr>
<tr>
<td>102.2</td>
<td>0.155</td>
<td>0.180</td>
<td>2.617</td>
</tr>
<tr>
<td>189.7</td>
<td>0.120</td>
<td>0.333</td>
<td>2.653</td>
</tr>
</tbody>
</table>

Table 4.8 - Modified target results
The modified target resolution function for the 189.7 eV resonance is shown in Figure 4.7.

![Graph showing the modified target resolution function for 189.7 eV.](image)

Figure 4.7 - Scattering target resolution function for 189.7 eV

The change to the resolution equation of (4.7) is the constant that shifts the peak of the exponentials. Equation (4.11) with the additional shift replaces equation (4.7).

\[ R(t) = \frac{(t+\text{shift})^2}{2\cdot \Lambda^3} \exp \left( \frac{-(t+\text{shift})}{\Lambda} \right) + A_1 (A_2 \exp(A_3(t+1.207) + A_4 \exp(A_5(t+1.207))) \]  

(4.11)

The values for 'shift' have been fitted by equation (4.12), which replaces equation (4.9).

\[ \text{shift}(E) = 0.52419 - 0.0979 \cdot \ln(E) + 0.00394 \cdot (\ln(E))^2 \]  

(4.12)

Since the magnitude of the tail is small, equation (4.13) has been used to replace equation (4.10). Further refinement of the relationship is possible, although this simple relationship provides good results for energies up to 190 eV.

\[ A_1(E) = \frac{0.3333 \cdot E}{189.7} \]  

(4.13)

These modifications to the previously determined capture resolution functions provide excellent matches to the $^{238}$U experimental scattering data over the analyzed
energy range. Currently, there is no explanation for the difficulties experienced with the previously determined resolution functions, although the modifications do work. The neutron tail is smaller than the previously determined tail. Possibly this is due to the decoupling of the target and detector resolution effects.
CONCLUSIONS

This thesis presents a resolution function applicable to low energy neutron scattering measurements performed at RPI. This resolution function has been parameterized both in neutron energy and sample diameter. Scattered neutron detection efficiency has also been determined.

The experimentally determined scattering resonance has a peak which is lower in energy and broader than the corresponding capture resonance peak. This is primarily due to the additional and varied distance traveled by the scattered neutrons prior to detection. MCNP models were used to determine the distribution of the additional path lengths traveled by the scattered neutrons prior to detection. These models used isotropic sources, which simulate s-wave neutron scattering with nuclear recoil neglected. The time distribution of neutron current entering the inner surface of the B₄C liner and photon current exiting the outer surface of the B₄C liner were determined in this manner.

The simplest model of the monoenergetic neutron source is the isotropic point source. The MCNP results are in good agreement with the analytical solution.

The next series of MCNP models consist of monoenergetic isotropic disk sources of varying size and energy. For the varying energy, the size was fixed at the standard RPI sample diameter of 2 inches. For the varying size, the energy was fixed at 10 eV. The resulting curves were successfully reproduced and parameterized using an asymmetric double cumulative curve combined with a sum of exponentials tail.
The energy parameterized curves were tested against experimental uranium scattering measurements. The lower energy curves were well reproduced. However, the fit degraded at higher energies due to the scattering target resolution.

The scattering target resolution function was determined for the $^{238}$U resonances up to 187.9 eV. This function is a modified form of the capture target resolution function for the bounce target. The modifications produced an excellent fit to the experimental data.

There are additional potential improvements to the scattering resolution function which were not addressed in this thesis. For example, nuclear recoil effects could be taken into account to allow for determination of scattering resonance parameters for lighter nuclei samples, the scattering target resolution could be improved, and the scattering resolution function may also be tested against different materials to ensure further applicability.
REFERENCES


APPENDIX A

SAMPLE MCNP MODELING INPUT FILES

A.1 - Sample MCNP input file for disk source neutron current on inner liner surface

2.0 inch diameter 10 eV neutron photon problem

cells:

c
1 1 -2.31 3 -4 1 -2 $ b4c liner
2 0 -1 3 -4 $ inner void
3 0 2 -3 4 $ outside world

c surfaces:

c
1 cz 3.18
2 cz 4.18
3 pz -1.515
4 pz 34.535

mode np
sdef cel=2 erg=1.0e-5 pos=0 0 16.4973 rad=d1 axs=0 0 1 ext=0
sil 0 2.54127
imp:n 1 1 0
imp:p 1 1 0
ml 5010 0.79568 5011 0.00443 6012 .1978 6013 .0022 $ b4c
fl:p 2
el .36 .68 100
tl 18 20 21.4 22.8 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41
42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64
65 66 67 68 69 70 71 72 73 74 75 76 77 78 79
80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98
99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114
115 116 117 118 119 120 121 122 123 124 125 126 128 130 132 134
136 138 140 142 144 146 148 150 154 158 162 166 170 174
178 186 195 207 213 227 241 260 290 385 1E33
nps 1000000
print
A.2 - Sample MCNP input file for energy deposition determination

sodium iodide detector w/b4c liner efficiency

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40 5 -2.50 33 -14 -5 4
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1 cz 3.18
2 cz 4.18
3 cz 0.0
4 cz 4.2672
5 cz 4.445
6 cz 15.24
7 cz 16.0528
8 pz 0.0
9 pz 0.9525
10 pz 16.1925
11 pz 16.379825
12 pz 16.640175
13 pz 16.8275
14 pz 32.0675
15 pz 33.02
16 p 0.3827 -0.9239 0 0
17 p 0.9239 -0.3827 0 0
18 p 0.9239 0.3827 0 0
19 p 0.3827 0.9239 0 0
20 cz 2.54127
21 cz 2.5908
22 cz 2.4892
23 cz 2.8385
24 cz 2.8893
25 pz 16.329025
26 pz 16.690975
27 pz 18.227675
28 pz 18.545175
29 cz 2.2225
30 cz 15.4178
31 pz 0.635
32 pz 16.3576
33 pz 16.6624
34 pz 32.385
35 pz -1.515
36 pz 34.535

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sdef cel 21 pos 0 0 16.4973 axs 0 0 1 rad d1 ext 0 erg 1.e-5
s11 0 2.54127
imp:n 1 39r 0
imp:p 1 39r 0
vol 1280 15r
m1 13027 1
m2 5010 0.19568 5011 0.00432 6012 .1978 6013 .0022
m3 11023 0.5 53127 0.5
m4 5010 0.1592 5011 0.6408 6012 .1978 6013 .0022
m5 6012 .989 6013 .011
f18:p (1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16)
e18 0 .36 .68 100
nps 1000000
print

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c
c
c
c
sodium iodide detector with b4c liner
c sample: b4c; thickness = .1025 inch
c
c
mcnp cell definitions

c
c
+/
right of or outside

/-/
left of or inside

/#!
omitted regions

c
c
no. material description

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c 2 nai detector pie segment #2 17 -16 9 -10 5 -6

c 3 nai detector pie segment #3 16 19 9 -10 5 -6

c 4 nai detector pie segment #4 18 -19 9 -10 5 -6

c 5 nai detector pie segment #5 17 -18 9 -10 5 -6

c 6 nai detector pie segment #6 16 -17 9 -10 5 -6

c 7 nai detector pie segment #7 -16 -19 9 -10 5 -6

c 8 nai detector pie segment #8 19 -18 9 -10 5 -6

c 9 nai detector pie segment #9 18 -17 13 -14 5 -6

c 10 nai detector pie segment #10 17 -16 13 -14 5 -6

c 11 nai detector pie segment #11 16 19 13 -14 5 -6

c 12 nai detector pie segment #12 18 -19 13 -14 5 -6

c 13 nai detector pie segment #13 17 -18 13 -14 5 -6

c 14 nai detector pie segment #14 16 -17 13 -14 5 -6

c 15 nai detector pie segment #15 -16 -19 13 -14 5 -6

c 16 nai detector pie segment #16 19 -18 13 -14 5 -6

c 17 aluminum front face 8 -31 4 -30
sodium detector w/b4c liner
sample: b4c; thickness = .1025 inch

mcnp surface definitions

c surf no. description type distance from centerline or radius from front plane of detector, cm.
c 1 b4c liner inside radius cylinder 3.18
c 2 b4c liner outside radius cylinder 4.18
c 3 not used cylinder 0.0
c 4 detector inner can outside radius cylinder 4.2672
c 5 nai crystal inside radius cylinder 4.445

c 6 nai crystal outside radius cylinder 15.24
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APPENDIX B $^{238}$U RESONANCE FITS FOR DETECTOR RESOLUTION

Bounce target data: Resolution fit to 1 mil Uranium sample at 36 eV Resonance

SCATTER DATA

Yield vs Energy

TB$_{total}$: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width, burst width and detector functions. Detector efficiency used is 0.552.

Y$_{exp}$: Experimental scattering yield from LINAC run on 11-12 Oct 95 with the 1 mil Uranium sample.

Y$_{thc}$: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width and burst width.

Determine the chi-square error:

$$\chi^2 = \frac{TB_{total} - Y_{exp}(E_1)}{Y_{thc}(E_1)}$$

$$\sigma Y_{ld}$$

$$\chi^2_{total} = \frac{\sum (TB_{total} - Y_{exp}(E_1))^2}{\sigma Y_{ld}}$$

$E_1$: Energy for the calculation

$\sigma Y_{ld}$: Error in the experimental yield

$$\chi^2_{total} = 2.98618337$$
Bounce target data: Resolution fit to 1 mil Uranium sample at 66 eV Resonance

SCATTER DATA

TB_{total_h}
Y_{exp(E_{th})}
Y_{h_{i=0}}

TB_{total}: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width, burst width and detector functions. Detector efficiency used is 0.589.

Y_{exp}: Experimental scattering yield from LINAC run on 11-12 Oct 95 with the 1 mil Uranium sample.

Y_{thc}: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width and burst width.

Determine the chi-square error:

E_{1_{3336}} = 64.98989497
E_{1_{3386}} = 66.79649977

\[ \sum_{h = 3336}^{3386} \left( \frac{TB_{total_h} - Y_{exp(E_{th})}}{\sigma Y_{ld_h}} \right)^2 \]

\[ \text{Chi}^2_{\text{total}} = 2.33736117 \]
Bounce target data: Resolution fit to 1 mil Uranium sample at 102 eV Resonance

SCATTER DATA

Yield vs Energy

TB_{total}: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width, burst width and detector functions. Detector efficiency used is 0.564.

Y_{exp}: Experimental scattering yield from LINAC run on 11-12 Oct 95 with the 1 mil Uranium sample.

Y_{thc}: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width and burst width.

Determine the chi-square error:

\[ E_{1_{4060}} = 100.82906834 \]
\[ E_{1_{4110}} = 104.3381339 \]
\[ \chi^2_{total} = \frac{\sum_{h=4060}^{4110} \left[ \frac{TB_{total(h)} - Y_{exp}\left(E_{1_{h}}\right)}{\sigma Y_{ld(h)}} \right]^2}{48} \]
\[ \chi^2_{total} = 6.36303562 \]
Bounce target data: Resolution fit to 1 mil Uranium sample at 187 eV Resonance

SCATTER DATA

Yield vs Energy

TB_{total}^{h_k}: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width, burst width and detector functions. Detector efficiency used is 0.56.

Y_{exp}(E_{l_h}^{h}): Experimental scattering yield from LINAC run on 11-12 Oct 95 with the 1 mil Uranium sample.

Y_{thc}(E_{l_h}^{h}): Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width and burst width.

Determine the chi-square error:

\[ E_{14830} = 184.71672219 \]
\[ E_{14880} = 193.49824494 \]

\[ \chi^2_{\text{total}} = \frac{\sum_{h=4830}^{4880} \left[ \frac{TB_{total}^{h} - Y_{exp}(E_{l_h}^{h})}{\sigma Y_{ld}^{h}} \right]^2}{48} \]

\[ \chi^2_{\text{total}} = 8.23984141 \]
Bounce target data: Resolution fit to 1 mil Uranium sample at 36 eV Resonance

**SCATTER DATA**

![Graph showing yield vs energy](image_url)

**TBtotal:** Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of all four factors. Detector efficiency used is 0.58.

**Yexp:** Experimental scattering yield from LINAC run on 11-12 Oct 95 with the 1 mil Uranium sample.

**Yth:** Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width, burst width and target functions.

Determine the chi-square error:

$$E_1^{2165} = 37.362$$

$$E_1^{2075} = 36.011$$

$$\sigma_{Yld} = \text{Error in the experimental yield}$$

$$E_1 : \text{Energy for the calculation}$$

$$\sum_{h=2075}^{2165} \frac{\left( TB_{total,h} - Y_{exp}(E_{1,h}) \right)^2}{\sigma_{Yld,h}}$$

$$\text{Chi}^2_{total} = \frac{1.796}{88}$$

$$\text{Chi}^2_{total} = 1.796$$
Bounce target data: Resolution fit to 1 mil Uranium sample at 66 eV Resonance

SCATTER DATA

TBtotal: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of all four factors. Detector efficiency used is 0.58.

Yexp: Experimental scattering yield from LINAC run on 11-12 Oct 95 with the 1 mil Uranium sample.

Yth: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width, burst width and target functions.

Determine the chi-square error:

\[ E_{1336} = 64.99 \]
\[ E_{1386} = 66.796 \]

\[ E_{1} = \text{Energy for the calculation} \]

\[ \sigma_{Yld} = \text{Error in the experimental yield} \]

\[ \sum_{h=3336}^{3386} \left( \frac{[TB_{\text{total}}_h - Yexp(E_{1h})]^2}{\sigma_{Yld}_h} \right) \]

Chi\textsubscript{total} = 1.384
Bounce target data: Resolution fit to 1 mil Uranium sample at 102 eV Resonance

SCATTER DATA

Yield vs Energy

TB_{total,h}

Y_{exp}(E_{1,h})

Y_{th}(E_{1,h})

TB_{total}: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of all four factors. Detector efficiency used is 0.562.

Y_{exp}: Experimental scattering yield from LINAC run on 11-12 Oct 95 with the 1 mil Uranium sample.

Y_{th}: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width, burst width and target functions.

Determine the chi-square error:

\begin{align*}
E_{1,4060} &= 100.829 \\
E_{1,4110} &= 104.338 \\
\sum_{h=4060}^{4110} \left[ \frac{TB_{total,h} - Y_{exp}(E_{1,h})}{\sigma Y_{ld,h}} \right]^2 &= \frac{\text{Chi}_2^2_{total}}{48} \\
\text{Chi}_2^2_{total} &= 2.617
\end{align*}

E_1: Energy for the calculation

\sigma Y_{ld}: Error in the experimental yield
Bounce target data: Resolution fit to 1 mil Uranium sample at 189 eV Resonance

SCATTER DATA

Yield vs Energy

TB$_{\text{total}}$: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of all four factors. Detector efficiency used is 0.56.

Y$_{\text{exp}}$: Experimental scattering yield from LINAC run on 11-12 Oct 95 with the 1 mil Uranium sample.

Y$_{\text{th}}$: Scattering yield calculated from Doppler broadened ENDF cross sections using a combined resolution function consisting of channel width, burst width and target functions.

Determine the chi-square error:

E$_{4835}$ = 185.739

E$_{4872}$ = 192.232

\[
\sum_{h=4835}^{4872} \frac{\left[ TB_{\text{total}}(E_h) - Y_{\text{exp}}(E_h) \right]^2}{\sigma Y_{\text{ld}}(E_h)}
\]

Chi$^2_{\text{total}} := \frac{\sum_{h=4835}^{4872} \left[ TB_{\text{total}}(E_h) - Y_{\text{exp}}(E_h) \right]^2}{\sigma Y_{\text{ld}}(E_h) \sum_{h=4835}^{4872}}

\]

Chi$^2_{\text{total}} = 2.653$