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**EXPERIMENTAL INVESTIGATION OF VIBRATION-INDUCED
BULK SOLIDS TRANSPORT AND SEGREGATION**

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1. ABSTRACT

We report experiments on the rise time T of a single large sphere within a sinusoidally vibrated bed (amplitude a) of uniform particles (diameter d). At fixed acceleration, three distinct behavioral regimes are identified both from visual observations and from the typical increase of T with frequency f . Two convective regimes separated by a critical frequency are found, and for low a and high f , a “non-convective” regime. In the latter, the bed crystallizes and a size dependent rise is evidenced. The relevance of the non-dimensional parameter a/d is shown and a scaling law is deduced which has the form $f \propto d^{-1/2}$.

2. EXECUTIVE SUMMARY

This project is a fundamental study of size segregation in vibrated granular materials with an emphasis on the correlation between observed bulk behavioral regimes and their effects on the motion of a single large particle within a monodisperse medium. The investigation presented in this report is "exploratory" in nature in that it takes a broad look at the phenomenon in the context of recent literature on the subject, which subsequently leads to new and interesting findings.

The experimental system consists of shallow granular bed of monodisperse acrylic spheres of diameter d filling a cylindrical container of the same material, whose floor is a piston mounted onto a vibration exciter system driven by a power amplifier. An important goal of the investigation is to uncover the relevant vibration parameters to properly characterize the segregation process. The time T required for a large acrylic sphere of diameter D to reach the surface of the bed from its initial placement on the center of the piston is studied in detail accounting for variations in the size ratio $\Phi = D/d$, applied vibratory amplitude a , frequency f , velocity $a\omega$, and relative acceleration $\Gamma = a\omega^2/g$. A series of measurements have been carried out by fixing either the amplitude, frequency, velocity or relative acceleration and varying the three remaining parameters, thereby leading to twelve possible combinations of fixed and variable parameters. The most relevant of these cases is the variation of T with f and a at fixed Γ , and with Γ at fixed a or f .

The initial phases of the investigation focused on the effects of room temperature τ on the strength of the convective flow. Results showed a strong correlation of T with τ in that in a span of 65 consecutive trials, fluctuations of T approximately followed fluctuations in τ . Higher temperatures resulted in greater rise times. Additionally, an increase in the room humidity level generally produced longer rise times.

The principal finding of this study is the observation of three behavioral regimes in which the large sphere or *intruder* rises to the surface of the vibrated bed. These regimes are characterized by specific relationships between dimensionless rise time Tf and f . The first regime, where there is bulk convection and surface heaping, occurs for a frequency smaller than $f_{cr} \approx 15$ Hz in which Tf increases exponentially with f when Γ is held constant. In the second regime, typified by bulk convective flow without the formation of a surface heap, Tf again increases with f at fixed Γ , but is now dominated by an exponential of f^2 . Finally the third regime appears at small amplitudes of vibrations $a/d < 0.25$ where convection disappears and the bed is compacted. For $\Gamma > 4$ and $f > 40$ Hz, the spread with Γ is minimal and $Tf \propto e^{\alpha f}$ where α is a constant independent of Γ . Most striking in this regime is a dependence of the rise velocity on the size ratio Φ such that the larger the intruder, the faster is its rise to the surface. To the knowledge of the investigators, this is first experimental evidence of a geometric-based (i.e., in the absence of a bulk convective flow) mechanism of size segregation in three dimensions reported in Monte Carlo simulations of the first author in 1987.

Another significant result concerns the effects of the normalized vibration amplitude (a/d) for which there is a general decay of Tf with a/d , and for $\Gamma > 4$, the data tends to collapse onto a single curve. However, when $\Gamma > 4$ and $a/d < 1$, Tf is seen to increase with Γ at fixed a/d . Furthermore, a/d appears to be a good control parameter characterizing the rapid increase of Tf as a/d is reduced. A relationship between the frequency and bed particle diameter is deduced through comparisons with data appearing in the literature whereby $f \propto 1/\sqrt{d}$.

3. INTRODUCTION

Bulk solids are frequently subjected to vibrations, either intentionally to enhance and promote flows, or as a natural consequence of handling, transport and processing operations [1]. Depending on energetic conditions, a granular bed may display a wide range of behaviors [2-19]. Particulate systems are enormously complex due to the number of factors which affect behavior, such as particle properties and their distributions, constitution of the granular material, flow geometry and the form of the input energy. Despite focused efforts of researchers in this field over approximately the last twenty years, a model capable of predicting flow behavior over the wide range of observed phenomena remains elusive. Consequently, the design of solids handling systems is problematic with regard to scale up from laboratory models and small pilot plants to industrial systems, which generally run at low efficiencies and often fail. This ultimately results in a lack of US competitiveness through reduced productivity in manufacturing processes where particulate transport is involved and losses in energy resources.

It is well-known that vibratory disturbances of a homogeneous particulate mixture can result in separation of the constituents, the most ubiquitous phenomenon of which is termed "size segregation" [20-39]. Often this de-mixing is undesirable especially when a final end product requires a precise blend of various constituents to achieve a specific effect, such as in the pharmaceutical industry. Conversely, vibrations can also be used to assist in mixing through induced convection. Relationships between mechanisms which cause segregation and those that assist in mixing are not well-understood.

This initial objective of this project was to explore the effects of granular flows induced through vibrations on size segregation over a range of parameters such as wall conditions, particle properties, geometry and vibratory parameters. During the course of the research, observations and measurements permitted us to identify specific critical parameters thereby allowing a reduction in the scope of the experiments. Consequently, we report on our results where detailed data has been obtained to study the behavior of a single large sphere in the bed over a broad range of vibration parameters as well as size ratios. In so doing, we have been able to identify several regimes in the vibrated bed in which different mechanisms are responsible for the motion of the large sphere. Most striking is our finding that the mode or type of applied excitation has a pronounced effect on behavior, an observation which previously has not been reported to the knowledge of the investigators.

The starting point of this study is to describe the evolution of the rise time* T of the large sphere with the vibration parameters and the possible connections to other commonly observed phenomenon in vibrated beds, such as heaping, surface waves, subharmonic instabilities, convection and compaction. In what follows, the objective of the experiment is given, together with a description of the apparatus, experimental procedures, and our analysis and interpretation of the rise time data. The report is concluded with a summary of the main results.

* The *rise time* is the time required for the large sphere to reach the surface of the bed.

4. DISCUSSION OF RESULTS

4.1 Objective of the Experiment

The experiment is designed for the observation of a large sphere rising in a vibrated granular medium. The approach is to measure the time T required for it to reach the surface from its initial position. In addition, observations of the global bed response to the input vibration are made. When a wide range of excitation parameters is involved, it is possible to observe most of the common features displayed by a vibrated granular bed, such as heaping, arching, surface waves, convection, fluidization or compaction. The apparent bed behavior is observed visually sometimes with the help of a stroboscope and sometimes by following the motion of dyed tracer particles.

An important goal of the investigation is to find the relevant vibration parameters able to properly describe the process whereby the large sphere rises in the bed. The obvious parameters are the amplitude and frequency; but the vibration velocity or the relative acceleration are two others which we have found to be meaningful. Consequently, a series of measurements have been made by fixing either the amplitude, the frequency, the velocity or the relative acceleration and varying the three remaining parameters. Hence there are twelve possible combinations of fixed and variable parameters. It appeared that the most interesting cases are the variation of rise time with frequency and amplitude at fixed relative acceleration, and the variation of rise time with relative acceleration at fixed amplitude or fixed frequency.

4.2 Description of the Experimental Setup and Parameters

The granular bed is composed of monodisperse acrylic spheres filling a cylindrical container of the same materials whose floor is a piston. The piston is mounted onto a vibration exciter driven by a power amplifier. A Bruel & Kjaer accelerometer is fixed on the piston base and connected to the exciter controller, which is a device containing a digitally controlled generator. This provides an input signal to the power amplifier, (a vibration meter enabling accurate measurement and control of any vibration parameters), and a compressor for regulation of the vibration excitation. During an experiment, the vibration exciter control permits an easy variation of the vibration frequency at fixed amplitude, velocity or acceleration.

Since a granular bed is such a complex material, its properties are highly dependent on the environment and are not easily predictable. The temperature and humidity level in the closed room in which the experiments were carried out had significant effects on the bed response. Consequently, in order to maintain some control of these parameters, a fan was used to reduce the temperature increase caused by heating of the vibration exciter, while an air conditioned was used to reduce the humidity. Digital measurements of the temperature and humidity were recorded. Another undesirable factor is the build-up of static charge on the surface of the particles as a result of numerous collisions during shaking. When particles are charged, they repel each and usually adhere to the walls of the cylindrical container whose charge is opposite. Therefore, particle-particle and particle-wall interactions are not simple since they include long distance electrostatic interactions which cannot be easily quantified or controlled. To avoid the build-up of static electricity, particles are treated with a common household anti-static sheet which was effective over the period of several months.

The cylinder has a diameter $D_{\text{cyl}} = 4.5''$ while the floor piston diameter was slightly smaller so that it could move freely without rubbing against the walls. Acrylic bed sphere of $d = 1/8''$ were used filling the cylinder to an initial depth of H such that $H/D_{\text{cyl}} = 0.44$. Since a shallow bed was chosen for these studies, it is likely that there will be strong interactions between the behavior of the bulk and what is observed on the surface. The ratio of the diameter of the large sphere to the bed particles, denoted as $\Phi = D/d$, varies from unity to twelve. The coefficient of friction of the particles was approximated to be $\mu = 0.20$ by measuring the angle in which three cemented spheres would begin to slide down an acrylic plate. Normal restitution

coefficient was determined to be greater than 0.9 by simply dropped spheres onto an acrylic plate and measuring their rebound height.

In all experiments, the initial placement of the large sphere is at the center of the piston surface and in general, it remains centered during its rise to the bed surface. The temperature and humidity of the room are noted before starting to shake the piston. A sinusoidal vibration $a\sin(\omega t)$ is applied to the piston, where $\omega = 2\pi f$ and f is varied between 5 and 75 Hz. The largest amplitude the vibration exciter can provide is $a = 0.25'' = 2d$. As a control parameter, the relative acceleration Γ is defined as the ratio of the maximum vibration acceleration to gravity, i.e., $\Gamma = a\omega^2/g$ which varied from 0 to 13.

We point out that the sinusoidal signal used is a *constant current drive*. A constant voltage drive is usually used because the mechanical resonance is damped and that improves the sound quality (as in a loudspeaker, for instance). The vibration of a granular bed is another problem. First, there is no need to ensure a flat response in frequency since the acceleration is directly measured and the power supply adjusted to obtain the desired level of vibration. Secondly, when the bed collides with the piston, the sinusoidal motion is perturbed regardless of the type of signal used. In fact, we observed that the response of a granular bed changes when a sinusoidal vibration of given acceleration and frequency is applied by either using a constant voltage or constant current drive. The manner in which the piston reacts to the collision is at the origin of this difference. In the case of a constant voltage drive, the system tends to resist the collision by increasing the current (and thus the applied force), which is controlled mainly by electrical and magnetic parameters such as inductance, resistance, magnetic field and coil length. In the case of constant current drive, a constant sinusoidal force is applied to the piston and the collision can be described by a purely mechanical equation which is unaffected by a change in voltage during the collision (i.e., no electrical-mechanical coupling [40]). Hence, the choice of excitation is important. The constant current drive used in these experiments has the advantage of providing a better defined system in the sense that the applied force is always known, and it appears that this gives the closest agreement with previous studies [20, 29], where the type of excitation was not specified.

4.3 Rise Time Studies

The rise time T is defined as the time required for the large sphere to move from its initial position to the bed surface after the piston vibration begins. This is measured using a simple stopwatch. The instant when the vibration begins is well-defined, but the time when the ball reaches the surface depends on the criterion used. Three different times were considered: when the top of the sphere just reaches the level of the bed surface; when the ball is half embedded and half outside of the bed; and when it attains its highest altitude above the piston floor. The latter criteria was used as the rise time for the majority of the experiments discussed in this section.

The results reported here are limited by the fact that measurements were very sensitive to experimental conditions (i.e., temperature and humidity conditions in the room, changes in the particle and wall friction properties, etc.) and as such, they contribute to fluctuations in values. During the initial stages of the investigation, repeatability was a serious problem until it was realized that changes in temperature and humidity strongly affected the bed response to the piston vibration. As an example, it was surprising that at the end of a full day of experiments, convection strength was much lower than in the morning for the same experiment, and that at the beginning of the following day, convection recovered its original strength. We noted that after several hours, heat released by the vibration system could increase the room temperature 5°C.

Fig. 1: Rise time T versus experiment number.

Figure 2: Rise time T (left axis) and room temperature (right axis) versus experiment number.

This effect is demonstrated in Fig. 1 which shows the measured rise time T when $f = 20$ Hz and $\Gamma = 4$ over 56 runs. After the first consecutive 27 trials, the system is allowed to cool down for a period of 2 hours and then another 29 consecutive experiments are performed. Despite the large fluctuations in the plot, there is a clear tendency of T to increase with the number of trials. After the system cools down, the values of T are much lower than before, but again, there is a marked increase with the experiment number. Several repeated measurements of T with the corresponding room temperature were made in order to clarify the role of temperature on T . The bed was vibrated at $f = 12$ Hz and $\Gamma = 4$. In Fig. 2, room temperature and T are plotted on the same graph as a function of the experiment number. Although the temperature variation is only approximately 2°C , T varies between 10 to 50 seconds. Moreover, fluctuations in the rise time approximately follow fluctuations of the temperature excluding the first 10 runs. We therefore conclude that temperature had a significant effect on the results, although it is not clear why. In addition, it was seen that an increase of humidity promoted longer rise times.

As reported in the literature [3, 20], the friction properties of the container walls influences macroscopic behavior. An important consideration is to understand how wall friction changes when subjected to continuous collisions with the particles over an extended period of time. In order to approach this problem, it is necessary to control the wall properties by, for example, cementing a layer of particles to the cylinder walls so as to maintain its rugosity over a period of time. Unfortunately, it was not possible to easily apply this technique without major redesign of the apparatus due to the necessity of having the piston slide along the inside of the cylinder walls with a close tolerance.

4.3A Observed Rise Regimes

If the rise time T (or equivalently the nondimensional value Tf^2 [Ⓢ]) is plotted as a function of frequency for several values of fixed relative acceleration, it is possible to distinguish three main domains, each of which corresponds to a different macroscopic behavior of the bed: convection with heaping, convection with no heaping and crystallization. In these studies, the diameter ratio $\Phi = D/d = 8$.

Regime 1

In the first regime, surface heaping and convection are observed for frequencies $f < 15 \pm 1$ Hz. Fig. 3 shows Tf plotted on a logarithmic scale as a function of frequency where each curve corresponds to a fixed relative acceleration Γ ranging from 1.5 to 9.0. For each value of Γ , $\ln(Tf)$ increases linearly with f while the curves obtained when Γ is varied essentially remain parallel to each other. The data can be fit with a linear law of the form,

$$\ln(Tf) = \alpha(\Gamma)f + \beta(\Gamma) \quad (1)$$

where the coefficient $\alpha(\Gamma)$ is a constant almost equal to unity for $1.5 < \Gamma < 3.5$ while $\beta(\Gamma)$ is very small and describes the variation of Tf with Γ . When f is fixed, Tf decreases with Γ .

The upward motion of the large sphere is attributed to a strong internal convective flow and the increase of Tf with f is associated with a reduction in convection speed. When the frequency is close to 15 Hz, the flow is diminished, and a heap, initiated near the walls, expands toward the center of the cylinder in a similar way observed in the two-dimensional experiments of Clement et al. [3]. Also, heaping was seen in our experiments for relative acceleration values $\Gamma < 1$ and this at first appeared to be at odds with the heaping threshold value reported in the literature, i.e., $\Gamma_{crit} \cong 1.2$. However, this value applied to an experiment where the entire cylindrical cell was vibrated in contrast to our apparatus where only the piston is oscillated. Consequently, if there is sufficient side wall friction, the imposed relative motion between

[Ⓢ] We stress here that plotting Tf rather than T versus f does not change the qualitative features observed.

the bed and the fixed walls creates a shearing forces promoting heaping even for Γ values less than unity. In the case when the entire cell is vibrated, relative motion between the bed and walls occurs only when the bed is in flight which occurs when $\Gamma > 1$.

Fig. 3: Dimensionless rise time Tf as a function of frequency f , for a range of relative acceleration Γ : ($\diamond$) = 1.5, ($\blacksquare$) = 2, ($\bullet$) = 2.5, ($\blacktriangle$) = 3, ($\times$) = 3.5, ($\circ$) = 4.5, ($\diamond$) = 5, (—) = 6, ($\square$) = 7, ($+$) = 8, ($\triangle$) = 9. The vertical line delimits the first and second regime. Curves connect data of same Γ .

Regime 2

The second regime is observed for $f > 15$ Hz where there is no more heaping. The top surface appears to be flat; however, observation of the bulk motion using a black tracer particle show that convection still exists. The tracer, which is initially placed at the center of container, will rise and move towards the cylinder wall where it is carried down along the wall to an undetermined depth. Sometime later it reappears on the top surface. Because the width of the downward flow field near the walls is approximately 2 ~ 3 bed particle diameters, the size large sphere ($\Phi = 8$) is such as to prohibit it from re-entering the flow near the wall. This geometric effect, reminiscent of several early studies of Rosato et al. [33-35], is a segregation mechanism which is active at the bed surface.

Fig. 4. Dimensionless rise time Tf as a function of dimensionless amplitude a/d , for a range of relative acceleration Γ : ($\diamond$) = 0.6, ($\blacksquare$) = 0.8, ($\blacktriangle$) = 1, ($\boxtimes$) = 1.25, ($*$) = 1.5, ($\bullet$) = 2, ($\otimes$) = 3, ($\blacksquare$) = 4, ($\blacksquare$) = 5, ($\diamond$) = 6, ($\square$) = 7, (Δ) = 8, ($\times$) = 9, (+) = 10, ($\circ$) = 11. For clarity, symbols are not connected in the third regime; only a single line shows the linear trend.

The variation of Tf in this regime is different from what was observed in the first regime. Fig. 4 shows that the Tf curves at fixed relative acceleration Γ varies slowly near $f = 15$ Hz, but grows more rapidly as f is increases. For $\Gamma = 2.5, 3.0$ and 3.5 , the break between the first and second regime is clearly shown by the change in shape of the Tf versus f curves before and after the transition. At fixed frequency, there is general decrease of Tf when Γ is reduced. For $\Gamma \geq 3.5$, the curves all have essentially the same shape. The following equations have been used to fit the curves in this regime:

$$Tf = \frac{\alpha}{[\Gamma - \Gamma_c]^\beta} \left[\exp\left(\frac{f}{f_c}\right)^2 - 1 \right] \quad (2)$$

$$Tf = \frac{\alpha}{[\Gamma - \Gamma_c]^\beta} \exp\left(\frac{f}{f_c} - 1\right)^2 \quad (3)$$

The form of equation (1) is a simplified version of Knight et al.'s [19] phenomenological law while equation (2) has been introduced to account for the fact that, close to the transition, the rise time varies slowly. Indeed, equation (2) expresses the additional condition that $\frac{d(Tf)}{df} = 0$ at $f = f_c$, where f_c is expected to be close to 15 Hz. Values of the constants in the above equations which lead to a good fit are given by:

Parameter	Γ_c	f_c (Hz)	β	α
Equation 1	~ 1.0	15.7	1.39	300
Equation 2	~ 1.0	11.7	1.21	570

We note that the transition frequency f_c is underestimated in the second equation. Knight et al.'s [19] original phenomenological law is given by,

$$Tf = \tau \left[\exp\left(\frac{H}{\xi}\right) - 1 \right] \quad (4a)$$

where

$$\tau = \frac{\alpha}{[\Gamma - \Gamma_c]^\beta} \exp(c_1 f + c_2) \quad (4b)$$

$$\xi = \frac{1}{c_3 f^2 + c_4} \quad (4c)$$

In attempting to fit our experimental data to the above law, we found that the parameters c_1 , c_2 and c_4 were negligible while c_3 , which was taken to be a function of Γ in [19], was actually independent of Γ . The reason for the difference in scaling may be a consequence of the differences between experimental procedures as highlighted in the chart below.

Knight et al. [19]	Our Experiments
Entire container subjected to shaking.	Only piston is vibrated and walls are fixed.
Container tapped followed a relaxation period	Continuous sinusoidal vibration applied.
$H/D_{cyl} = 1, 2, 3$	$H/D_{cyl} = 0.44$
Report from MRI experiments indicate that the top 10-15 layers of the bed do not follow equation (3) very well.	$H/d = 12$

Despite these dissimilarities, equation (1) and (3) contain a contribution growing as $\exp(f^2)$, which is the essential term describing the quadratic shape of the curves appearing in Fig. 4 for $f > 15$ Hz. It must be pointed out here that the use of a constant current drive results in a piston vibration which is far from sinusoidal at low frequencies. Consequently, the reported transition detected between the first and second regimes may be promoted by the dependence of the vibration shape on frequency. Further investigations are required to understand this effect.

Regime 3

The third regime is observed at small amplitudes of vibrations, where convection disappears. The first effect of the small shaking amplitude is to compact or densify the bed which, because it is composed of monodisperse spheres, tends to produce a very ordered structure appearing to be hexagonal in nature. The effect of this crystallization-like phenomenon can be seen in Fig. 5 which shows a view of the bed surface. Although most of the particles in the bed are "frozen" in position, they still have significant rotational motion. In addition, there appears to be convection rolls at the walls where spheres are either "absorbed" or "ejected". Some free particles move randomly on the crystallized surface seeking a potential well - for example, a hole in the structure or location very close to the walls where the rolls prevent the formation of a crystal.

Fig. 5: View of the surface of the vibrated bed shows a crystallized structure. The large sphere, dyed in black, has reached the surface here.

The bed crystallizes when $a/d < 0.25$, that is, when the piston amplitude is much smaller than the diameter of the bed particles. This is interesting since this observation supports the results of our recent simulations [18] which clearly showed little or no convective flow when $a/d < 0.25$. The rise time corresponding to this regime appears in the high frequency domain of Fig. 4, which also roughly shows the first and second regimes for comparison. For $\Gamma > 4$ and $f > 40$ Hz, the spread with Γ is minimal and the

points almost lie on a straight line. Hence (like the first regime) the rise time variation with frequency is exponential and can be fit with a simple linear relation of the form:

$$\ln(Tf) = \alpha f + \beta \quad (5)$$

where α and β are independent of Γ . The best fit is obtained when $\alpha = 0.107$ and $\beta = 4.02$.

Despite the existence of very small convection rolls, our experimental evidence strongly indicates that the mechanism whereby the large sphere rises is non-convective. A clear distinction from the previously discussed convective regimes was seen in the experiments upon observing the motion of the bed particles surrounding the intruder near the surface. In addition, as shown in Fig. 6 for the case where $f = 50$ Hz and $a/d = 0.1$, the mean velocity of the large sphere depends on the size ratio $\Phi = D/d$. This means that the larger the intruder, the faster it rises to the surface, a fact which is in contradiction with the occurrence of a convective flow which drags particles in the center of the bed upwards at nearly the same velocity regardless of size, density or shape. Previously, effects of the size ratio had been seen experimentally in a two-dimensional bed [30], but the extension of this result to three dimensions (3D) was until now not straightforward. In a 2D system, disks tend to naturally pack into a triangular mesh unlike their counterpart in 3D. Consequently, in 2D, two rising behaviors have been observed, one purely convective and the other dependent on size ratio [30, 31], while in 3D only convection has been experimentally reported as the mechanism causing a large sphere to rise [19, 20]. Since our experiments demonstrate that it is possible to obtain a strongly ordered structure by shaking a very small amplitude and high frequencies in 3D, we were able to see this size ratio dependence and apparently non-convective regime.

Fig. 6: Average Rise velocity ($\mu\text{m/sec}$) versus size ratio Φ for the case where $f = 50$ Hz and $a/d = 0.1$

4.3B Scaling Properties of the Vibration Amplitude

In order to examine the effect of vibration amplitude, the same data as in Fig. 3 is shown in Fig. 7 replotted against normalized amplitude a/d for fixed value of Γ . There is a general decay of Tf with a/d and for Γ greater than approximately 4, the data tends to collapse onto a single curve, while for $\Gamma < 4$, there is more scatter representing the first regime of Fig. 3. The curve exhibits the important feature that Tf increases quickly as a/d is reduced.

The above result regarding the behavior of Tf with a/d suggests that perhaps it may be used as a control parameter. We first remark that experiments of Ahmad et al. [29] also implied that Tf decreased with

vibration amplitude; however the scaling role of the dimensionless amplitude a/d was not noticed. They placed a large sphere in a vibrated bed of sand (mean particle diameter $d_A = 0.5$ mm) contained within a cylinder of 20.32 cm in diameter. Their bed aspect ratio $H/D_{cyl} = 0.625$ was close to that used in our experiments, i.e., $H/D_{cyl} = 0.44$. From their data, it is possible to compute the values of a/d_A at which there is an increase of about an order of magnitude in T , from the location where T varies slowly with f (see Fig. 4 in Ref. [29]). This provides a measure of the change in the large sphere's rise behavior due to the decrease of vibration amplitude. For a range of Γ values between 4 and 10, the computed values of a/d_A are quite consistent with a mean $a/d_A \equiv 0.37 \pm 0.08$. It is striking that while there is more than a factor of two between the smallest and largest values of Γ , the distribution of a/d_A values is only 20% around the mean. By following the same procedure with our data shown in Fig. 7, it was found that $a/d \equiv 0.4 \pm 0.1$ which is in reasonably good agreement with the mean value extracted from Ahmad et al.'s data. Consequently, it appears that the ratio a/d may be able to correctly characterize the divergence of T for small vibration amplitudes. Another striking observation is that the frequencies corresponding to the computed a/d values are completely different in the two experiments. In fact, it can be shown that they depend on the diameter of the bed particles. In order to see this, if we stipulate that the vibration state is well defined by the values of a/d and Γ , then the experiments of Ahmad et al. are comparable and even equivalent to ours if these parameters are the same for both systems. Consequently, there is a relationship between the frequencies and the ratio of bed particle diameters given by,

$$\frac{f}{f_A} = \sqrt{\frac{d}{d_A}} \quad (5)$$

Upon substitution of the numerical values of the diameters, we obtain that $f_A \equiv 2.5f$, which agrees reasonably well with the actual scaling between the two systems. This last result enforces the idea that the amplitude must be scaled by the diameter of the bed particles. The validity of the previous analysis relies on the hypothesis that disparities in particles properties or in geometry can be neglected as well as the additional effects of shearing at the walls caused by the motion of the piston.

Fig. 7: Dimensionless rise time Tf as a function of dimensionless amplitude a/d , for a range of relative acceleration Γ : ($\diamond$) = 1.5, ($\blacksquare$) = 2, ($\bullet$) = 2.5, ($\blacktriangle$) = 3, ($\times$) = 3.5, ($\circ$) = 4.5, ($\diamond$) = 5, ($\blacksquare$) = 6, ($\square$) = 7, (+) = 8, (Δ) = 9.

4.3C Further Observations on the Role Played by the Relative Acceleration

The rise time curves at fixed relative acceleration Γ have particularly interesting properties. On these curves, there is a monotonic variation of Tf with frequency, and when Γ is varied, they are regularly deformed in a continuous fashion. The fact that this behavior was not seen when curves of fixed amplitude or velocity were plotted indicates the importance of Γ^* . However, the situation is complex since rise time can be quite different for a fixed Γ with different values of a/d and frequency, the latter playing the role as a transition parameter. In what follows, we describe the effect of Γ when either frequency or amplitude are fixed.

When the frequency is fixed, rise time T decreases with an increase of relative acceleration as can be deduced from Fig. 3 or 5. This is equivalent to stating that T is reduced with an increase of amplitude. However, the latter statement is only true when frequency f or relative acceleration Γ is fixed, but not when the velocity $a\omega$ is held constant. (see Fig. 8). Alternatively, the rise time will always decrease with an increase in Γ at fixed f , but not at fixed amplitude as explained below.

Fig. 8: Dimensionless rise time Tf as a function of dimensionless vibration amplitude a/d for three different vibration velocities $a\omega$. (Δ) = 0.27 m/sec, ($\square$) = 0.3 m/sec, ($\diamond$) = 0.5 m/sec. Notice the irregularity of the constant velocity curves.

* This is not surprising since other work reported in the literature also describe Γ as a control parameter.

When the amplitude is constant, the rise time variation with Γ appears as in Fig. 9. For $a/d \geq 1$, Tf decreases with Γ ; but when $a/d < 1$ and $\Gamma > 4$, Tf increases with Γ . No clear explanation has been found to account for this strange behavior. However, it appears that this irregularity is a consequence of the heaping/non-heaping transition. Indeed, the conditions at which the irregularity occurs ($a/d < 1$ and $\Gamma > 4$) yields that $f > \frac{1}{\pi} \sqrt{g/d}$ which implies that $f > f_{\min}$ where $f_{\min} = \frac{1}{\pi} \sqrt{g/d} = 17.7$ Hz. Interestingly, the irregular behavior appears only when there is no heaping. In Fig. 8, the jump observed in the curves of fixed velocity amplitude has the same origin. When $a\omega = 0.3$ m/sec, the irregularity occurs for $a/d \approx 0.7$ which corresponds to a value of the relative acceleration $\Gamma \approx 4.1$.

Fig. 9: Dimensionless rise time Tf as a function of relative acceleration Γ for a range of fixed dimensionless vibration amplitudes a/d : ($\circ$) = 0.1, ($\square$) = 0.25, ($\triangle$) = 0.5, ($\blacktriangle$) = 1, ($\blacksquare$) = 1.5, ($\bullet$) = 2.

5. CONCLUSIONS

The main finding in this investigation is the existence of three regimes in which a single large sphere can rise to the surface of a vibrated bed of uniform particles. The first regime corresponds to the formation of a heap, which coexists with convection, and the rise time is seen to increase exponentially with frequency when relative acceleration is kept fixed. This first regime occurs for a frequency smaller than a critical frequency. In the second regime, the rise time is also increasing with frequency at fixed relative acceleration, but the variation is now dominated by an exponential of the square of the frequency. Also, the large sphere is carried upwards by a convective flow where heaping no longer occurs. Thus, a distinction must be made between a convective regime where heaping is observed, and a convective regime without heaping. In the third regime,

the bed crystallizes and the rise time varies again exponentially with frequency. Here, the rise velocity depends on the size ratio and because particles are confined to a very stable crystal-like structure, it is likely that the rise phenomenon is controlled by structural defects promoted by the presence of the large sphere (or intruder). Then, the more perturbed the structure, i.e., the larger the intruder, the faster is its rise.

The rise of a large sphere in a vibrated bed appears to be strongly dependent on the amplitude of vibration. For fixed relative accelerations, the smaller the amplitude, the longer the rise time. More important is the fact that a decrease in amplitude leads to the same relative increase of rise time whatever is the value of relative acceleration. In fact, comparisons with Ahmad et al.'s experiments strongly suggests that a/d , the amplitude nondimensionalized by the diameter of the bed spheres, is the appropriate control parameter. Then, the rapid increase of rise time when the amplitude decreases can be understood as a finite size effect due to the discrete nature of the bed.

6. REFERENCES

1. B. Ennis, J. Green and R. Davies, "Particle Technology: A Legacy of Neglect," *Chem. Eng. Progress* **90**, 32 (1994).
2. P. Evesque and J. Rajchenbach, "Instability in a Sand Heap," *Phys. Rev. Lett.* **62**, 44 (1989).
3. E. Clement, J. Duran and J. Rajchenbach, "Experimental Study of Heaping in a Two-Dimensional Sandpile," *Phys. Rev. Lett.* **69**, 1189 (1992).
4. T. G. Owe Berg, R. L. McDonald and R. J. Trainor, Jr., "The Packing of Spheres," *Powder Technology* **3**, 183 (1969); J. B. Knight, C. G. Fandrich, C. N. Lau, H. M. Jaeger and S. R. Nagel, "Density Relaxation in a Vibrated Granular Material," *Phys. Rev. E* **51**, 3957 (1995).
5. S. B. Savage, "Streaming Motions in a Bed of Vibrationally Fluidized Dry Granular Material," *J. Fluid Mech.* **194**, 457 (1988).
6. C. Laroche, S. Douady and S. Fauve, "Convective Flow of Granular Masses Under Vertical Vibrations," *J. Phys. France* **50**, 699 (1989).
7. E. Clement and J. Rajchenbach, "Fluidization of a Bidimensional Powder," *Europhys. Lett.* **16**(2), 133 (1991).
8. P. Evesque, E. Szmatala and J. P. Denis, "Surface Fluidization of a Sand Pile," *Europhys. Lett.* **12**(7), 623 (1990).
9. S. Warr, J. M. Huntley and G. T. H. Jacques, "Fluidization of a Two-Dimensional Granular System: Experimental Study and Scaling Behavior," *Phys. Rev. E* **52**, 5583 (1995).
10. H. K. Pak and R. P. Behringer, "Surface Waves in Vertically Vibrated Granular Materials," *Phys. Rev. Lett.* **71**, 1832 (1993).
11. F. Melo, P. B. Umbanhowar and H. L. Swinney, "Hexagons, Kinks and Disorder in Oscillated Granular Layers," *Phys. Rev. Lett.* **75**, 3838 (1995).
12. E. Clement, L. Vanel, J. Rajchenbach and J. Duran, "Pattern Formation in a Vibrated Granular Layer," *Phys. Rev. E* **53**, 2972 (1996).
13. S. Douady, S. Fauve and C. Laroche, "Subharmonic Instabilities and Defects in a Granular Layer Under Vertical Vibrations," *Europhys. Lett.* **8** (7), 621 (1989).
14. C. R. Wassgren, C.E. Brennen, M.L. Hunt "A Discrete Element Simulation of a Deep Bed of Granular Material Subjected to Vertical, Sinusoidal Vibration," preprint (1996).
15. C. R. Wassgren, M. L. Hunt and C. E. Brennen, "Vertical Vibration of a Deep Bed of Granular Materials in a Container," *J. Appl. Mech.* **63**(3), 712-719 (1996).
16. Y-h. Taguchi, "New Origin of a Convective Motion: Elastically Induced Convection in Granular Materials," *Phys. Rev. Lett.* **69**, 1367 (1992).

17. J. A. C. Gallas, H. J. Herrmann and S. Sokolowski, "Convection Cells in Vibrating Granular Media," *Phys. Rev. Lett.* **69**, 1371 (1992).
18. Y. Lan and A. D. Rosato, "Convection Related Phenomena in Vibrated Beds," under review *Phys. Fluids* (1997).
19. J. B. Knight, E. E. Ehrichs, V. Y. Kuperman, J. K. Flint, H. M. Jaeger and S. R. Nagel, "An Experimental Study of Granular Convection," *Phys. Rev. E* **54**, 5726 (1996).
20. J. B. Knight, H. M. Jaeger and S. R. Nagel, "Vibration-Induced Size Separation in Granular Media: The Convection Connection," *Phys. Rev. Lett.* **70**, 3728 (1993).
21. T. Poschel and H. J. Herrmann, "Size Segregation and Convection," *Europhys. Lett.* **29**(2), 123 (1995).
22. J. A. C. Gallas, H. J. Herrmann, T. Poschel and S. Sokolowski, "Molecular Dynamics of Size Segregation in Three Dimensions," *J. Stat. Phys.* **82**, 443 (1996).
23. G. C. Barker and A. Mehta, "Size Segregation Mechanisms," *Nature* **364** (1993).
24. R. L. Brown, "The Fundamental Principles of Segregation," *J. Inst. Fuel* **13**, 15 (1939).
25. J. L. Olsen and E. G. Rippie, "Segregation Kinetics of Particulate Solids Systems: I, Influence of Particle Size and Particle Shape Distribution," *J. Pharm. Sci.* **53**, 147 (1964).
26. E. G. Rippie, J. L. Olsen, M. D. Faiman, "Segregation Kinetics of Particulate Solids Systems: II, Particle Density-size Interactions and Wall Effects," *J. Pharm. Sci.* **53**, 1360 (1964).
27. M. D. Faiman and E. G. Rippie, "Segregation Kinetics of Particulate Solids Systems: III, Dependence on Agitation Intensity," *J. Pharm. Sci.* **54**, 719 (1965).
28. J. C. Williams, "Segregation of Powders and Granular Materials," *Fuel Soc. J. (Univ. of Sheffield)* **14**, 29 (1963).
29. K. Ahmad and I. J. Smalley, "Observation of Particle Segregation in Vibrated Granular Systems," *Powder Technology* **8**, 69 (1973).
30. J. Duran, T. Maozi, E. Clement and J. Rajchenbach, "Size Segregation in a Two-Dimensional Sandpile: Convection and Arching Effects," *Phys. Rev. E* **50**, 5138 (1994).
31. W. Cooke, S. Warr, J. M. Huntley and R. C. Ball, "Particle Size Segregation in a Two-Dimensional Bed Undergoing Vertical Vibration," *Phys. Rev. E* **53**, 2812 (1996).
32. P. K. Haff and B. T. Werner, "Computer Simulation of the Mechanical Sorting of Grains," *Powder Technology* **48**, 239 (1986).
33. A. Rosato, F. Prinz, K. J. Strandburg and R. H. Swendsen, "Monte Carlo Simulation of Particulate Matter Segregation," *Powder Technology* **49**, 59 (1986).
34. A. Rosato, K. J. Strandburg, F. Prinz and R. H. Swendsen, "Why the Brazil Nuts are on Top: Segregation of Particulate Matter by Shaking," *Phys. Rev. Lett.* **58**, 1038 (1987).
35. A. Rosato, Y. Lan and D. Wang, "Sorting in Multi-Component Systems," *Powder Technology* **66**, 149 (1991).
36. R. Jullien, P. Meakin and A. Pavlovitch, "Three-dimensional Model for Particle Size-Segregation by Shaking," *Phys. Rev. Lett.* **69**, 640 (1992).
37. R. Jullien and P. Meakin, "A Mechanism for Particle Size-Segregation in Three Dimensions," *Nature* **334**, 425 (1990).
38. R. Jullien and P. Meakin, "Particle Size-Segregation by Shaking in Two-Dimensional Disk Packings," *Europhys. Lett.* **22**(7), 523 (1993).
39. S. Dippel and S. Luding, "Simulations on Size Segregation: Geometrical Effects in the Absence of Convection," *J. Phys. I France* **5**, 1527 (1995).
40. J. Meisel, *Principles of Electromechanical-Energy Conversion*, R. E. Krieger (Malabar, FL, 1984).

7. APPENDIX: PUBLICATIONS RESULTING FROM RESEARCH

Loic Vanel, Anthony D. Rosato, and Rajesh N. Dave, "Rise-Time Regimes of a Large Sphere in Vibrated Bulk Solids," *Physical Review Letters* preprint (1997).

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