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The Physics of Fast Z Pinches

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The Physics of Fast Z Pinches

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ABSTRACT

The spectacular progress made during the last few years in reaching high energy densities in fast implosions of annular current sheaths (fast Z pinches) opens new possibilities for a broad spectrum of experiments, from x-ray generation to controlled thermonuclear fusion and astrophysics. Presently Z pinches are the most intense laboratory X ray sources (1.8 MJ in 5 ns from a volume 2 mm in diameter and 2 cm tall). Powers in excess of 200 TW have been obtained. This warrants summarizing the present knowledge of physics that governs the behavior of radiating current-carrying plasma in fast Z-pinches. This survey covers essentially all aspects of the physics of fast Z pinches: initiation, instabilities of the early stage, magnetic Rayleigh-Taylor instability in the implosion phase, formation of a transient quasi-equilibrium near the stagnation point, and rebound. Considerable attention is paid to the analysis of hydrodynamic instabilities governing the implosion symmetry. Possible ways of mitigating these instabilities are discussed. Non-magnetohydrodynamic effects (anomalous resistivity, generation of particle beams, etc.) are summarized. Various applications of fast Z pinches are briefly described. Scaling laws governing development of more powerful Z pinches are presented. The survey contains 52 figures and nearly 300 references.

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BASIC NOTATIONS

Throughout this paper we use predominantly the SI system of units. The temperature is measured in energy units (for instance, we write the equation of state for the ideal gas as $p=nT$, with T measured in *Joules*, and not $p=nkT$, with k being the Boltzmann constant and T measured in *Kelvins*). In the "practical" formulas we use mixed system of units (for instance, the temperature will be measured in electron-volts).

B	magnetic field induction
C	radial convergence
E	electric field
I	pinch current
I_w	current in an individual wire
I_Z	ionization energy of an ion in a charge-state Z
L	anode-cathode distance
T	temperature
c	speed of light
g	effective gravity acceleration
h	characteristic thickness of the imploding shell
k	wave vector
m	azimuthal mode number
$\hat{m}$	mass per unit length of the pinch
$\hat{m}_w$	mass per unit length of an individual wire
n	particle number density
p	pressure
r_0	initial pinch radius
r_{min}	pinch radius at a maximum compression
t	time
Γ	growth rate
Λ	the Coulomb logarithm
Π	dimensionless pinch parameter
α	angle between the wave vector and the magnetic field
β	ratio of the plasma pressure and the magnetic field pressure
γ	specific heat ratio
ϵ	electric permittivity of the vacuum
μ	permeability of the vacuum
η	electrical resistivity
λ	wavelength of the perturbation
ρ	mass density
ω	angular frequency

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I. INTRODUCTION

A. A piece of history

Self-constricted plasma configurations are among the most fascinating objects in plasma physics, both because of their natural occurrence in a number of situations, including geophysics (lightning) and astrophysics (current channels at galactic scales), and because of their importance for a variety of applications. The first systematic attempts in the analysis of these configurations began in 1934, with the publication of a paper by J. Bennett (1934) on the equilibrium of streams of charged particles with a finite temperature. L. Tonks (1937) introduced the term "pinch" to describe these configurations.¹ Later, in the 1950s, the prefix "Z" was added to distinguish self-constriction by the axial (z) current from compression of a plasma column by an inductively driven azimuthal (θ) current. Only the former configuration, Z pinch, will be considered in our paper. We note in passing that the other configuration is called a theta (θ) pinch.

A broad attack on the study of Z pinches began in the early 1950s, in conjunction with research on controlled thermonuclear fusion. The idea was to heat a deuterium-tritium mixture by an adiabatic and/or shock compression in a Z pinch and then sustain this system in the equilibrium state until a sufficient amount of fusion energy was released. This early stage of pinch research is covered in a book by Bishop (1959). It was soon discovered, however, that the equilibrium pinch suffers from a large number of magnetohydrodynamic instabilities, including sausage and kink. Current disruptions caused by the development of these instabilities gave rise to voltage surges and the generation of accelerated deuterons that, in turn, produced bursts of neutron radiation. Realization that the neutrons were not of a "noble" thermal origin but were rather a side effect of a disastrous instability, led to widespread pessimism with regard to the chances for Z pinches to produce fusion-relevant plasmas. As a result, Z pinches virtually disappeared from the research programs of large fusion laboratories.

As a legacy of these years, there remained extensive theoretical analyses of the stability of pinch equilibria, summarized in particular in the survey by Kadomtsev (1965), and realization of the role of a so-called "Pease-Braginski current" (Pease, 1957; Braginski, 1958), a current at which radiative losses can be fully compensated by the Ohmic heating (1.4 MA for hydrogen,

¹ Note the title of Sec.V of his more detailed paper (Tonks, 1939): "Constriction of Arc under its Own Magnetic Field - Pinch Effect."

independent of the density and the pinch radius). References to the early studies of Z and θ pinches can be found in Kolb (1960).

B. What are "fast" Z pinches? (What is the scope of this review?)

Interest in Z pinches revived in the mid-1970s and early 1980s, initiated by a rapid development of pulsed-power technology. Various versions of Z pinches were tried, most notably fiber pinches and imploding gas-puffs. For the fiber pinches (see, e.g., Haines, 1982; Hammel, 1983), whose diameter ranged typically from tens of micrometers to a couple of hundred micrometers, the time for establishing the radial equilibrium (a few nanoseconds) was short compared to the duration of the current pulse. In other words, they were evolving along a sequence of the Bennett-type equilibria, where the plasma pressure is approximately balanced against the magnetic forces.

By contrast, the annular gas-puffs (see, e.g., Stallings et al., 1979; Spielman et al., 1985a; Smirnov, 1991) had an initial diameter of a few centimeters, and the driving current pulsewidth was comparable to the implosion time. In this case, a free acceleration of the gaseous shell towards the axis occupies the major part of the total current pulse-width. After having reached a certain minimum radius, the plasma bounces back and ceases to exist; the Bennett-type equilibrium has never been reached. The word "fast" used in the title of this survey refers just to this class of pinch discharges. It specifies the scope of the survey: our prime focus will be discussions of the properties of those pinches where an implosion stage (the on-axis stagnation) is definitive and whose duration is too short to reach the Bennett-type state.

There is a significant difference in the important plasma instabilities for these two systems. Instabilities with an e-folding growth time much longer than the time of the propagation of acoustic signal over the pinch radius are important when considering quasi-equilibrium systems and are, obviously, of much less importance in the behavior of imploding systems. On the other hand, instabilities caused by the presence of large inertial forces (in particular, Rayleigh-Taylor instability, which will be discussed in much detail in this paper) are insignificant for quasi-equilibrium systems and become of a paramount importance for fast Z pinches. A nice discussion of various physics issues related to implosions of thin shells can be found in Turchi and Baker (1973), perhaps, the first paper specifically devoted to fast Z-pinches.

Despite a short life-time (of order ten ns in some cases), the plasma assembled by a fast Z pinch implosion provides unique possibilities for experimentation in a number of areas of physics. The growing interest in this area of research is reflected in particular by the fact that, at a recent conference on high-density Z pinches (Pereira, Davis, and Pulsifer (1997)), more than half of the papers were directly related to fast (in the aforementioned sense) Z pinches.

The quasi-equilibrium self-constricted plasma configurations, of the type of fiber pinches, have their own merits and probably deserve a separate survey. We feel that it would be difficult to cover both subjects in one paper, partly because of the differences in the dominant physical processes, and partly just because of the space limits.

As has already been mentioned, the recent progress in fast Z pinches has been reached, to great extent, because of major breakthroughs in the pulsed-power technology. Pulsed power technology will not be discussed in this survey in any detail. A very brief summary of the pertinent information will be presented in Sec. ID. This review will concentrate on the physics issues of fast Z pinches. We will discuss in some detail simple models of various effects important for pinch performance, so that this survey could be used as a first introduction to the subject. On the other hand, we will consider also more subtle and complicated issues which could be skipped during the first reading of this paper.

The physics of fast Z pinches is an active area of research. Many elements of this complex phenomenon are still not well understood and are the source of scientific disputes. Sometimes, the lack of experimental data and/or of a clear theoretical picture does not allow the discussion to rise above a qualitative, semi-speculative level. Still, even on such occasions, the authors take the risk of presenting their thoughts, with the humble hope that a reader will benefit from comparing her or his viewpoint with authors'.

To give some general impression of the parameter domain of the present-day fast Z pinch experiments, we provide in Table 1 some numbers (a much more detailed discussion will be presented later) that relate not to any specific experiment but rather to some "generic" fast Z pinches. In every particular experiment parameters may vary by a factor of a few.

TABLE 1. Characteristic parameters of a fast Z pinch

Height of the column, L (cm)	1-2
Initial radius of imploding cylinder, r_0 (cm)	2
Convergence, $C \equiv r_0/r_{min}$	10
Mass per unit length, $\hat{m}$ (mg/cm)	1-2
Maximum pinch current, I_{max} (MA)	10
Maximum voltage, V_{max} (MV)	1
Maximum magnetic field on the pinch surface, B_{max} (T)	10^3
Implosion time, t_{imp} (ns)	100
Maximum kinetic energy of the implosion, W_{max} (MJ)	1

By convergence we mean the ratio of the initial pinch radius to the final pinch radius,

$$C = r_0/r_{min}. \quad (1.1)$$

Note that, because of the skin-effect, the voltage is not a well-defined quantity (for instance, inside a highly conducting shell there is no axial electric field at all); in Table 1 we are referring to the integral $E dl$ between the anode and cathode at the distance from the axis equal to the initial pinch radius.

In addition to the most commonly studied parameter domain shown in Table 1, there exists another group of experiments, involving implosions of much heavier liners (with $\hat{m} \sim$ a few gram/cm), with a characteristic implosion time in the range of microseconds (see the end of Sec. I C). These pinches also fall under our definition of "fast" pinches but will be discussed only very briefly in our survey.

The imploding load is often called a "target," similar to the term used in ICF research (e.g., Lindl, 1995). The other traditionally used term is "the liner," which designates a thin imploding annular shell of whatever nature (gas puff, foil, foam, wire-array plasma, etc).

We have tried to limit the references to books and papers in scientific journals that would be easily accessible to the reader. However, in some cases we had to cite conference proceedings.

C. Specific types of fast Z pinches

There exists a variety of initial configurations that are imploded in fast Z pinches. Depending on the application (as will be discussed in more detail in later sections) the initial density profile is chosen to be uniform, annular, or peaked on axis. One initial configuration that we have already mentioned is a supersonic gas jet, with either an annular or uniformly-filled gas density profile, originating from a nozzle situated at one of the electrodes. The gas jet then flows through a fine mesh that serves as the opposite electrode, or there may be simply a hole that receives the jet (Fig. I.1a). More complex multi-shell gaseous jets are also possible.

To create an initial density profile that is more uniform axially between the electrodes, thin annular shells made of metal and plastic foils have also been used for the initial load configuration. Another way of creating the initial configuration is by machining a cylinder from a low-density foam (Fig. I.1b). Development of the aerogel technology allowed experimentalists to produce solid cylinders with average mass density as low as $\sim 1 \text{ mg/cm}^3$ (3 mg/cm^3 have actually been used in experiments) and having very small deviations from cylindrical symmetry (Antolak et al., 1997). Other foams, such as agar have a more coarse structure but have the advantage of being more easily machineable; this allows one to make both uniform and hollow annular cylinders of agar.

For the loads shown in Figs I.1a and I.1b, the substance is initially nonconducting. Before the current will flow through the pinch, the breakdown of the material must occur. Because a breakdown is a statistical process, it may cause considerable initial non-uniformities of the pinch. To try to have a more predictable initiation of the discharge in a foam, one sometimes uses thin conducting coatings on the surface of the foam. In the case of gaseous jets, some method of pre-ionization can be used.

More recently (e.g., Matzen, 1997), considerable progress in the technology of fabricating very fine wire arrays has allowed assembling highly symmetrical cylindrical shells consisting of hundreds of very fine (several micrometers in diameter) metal wires (Fig. I.1c). The initial state of the imploding shell is in this case, obviously, conducting. One may therefore expect a more symmetric initiation of the discharge. For specific applications, a foam cylinder, uniform or annular, or a more complex structure may be inserted into the wire array (Fig. I.1d).

Thus far we have been discussing Z pinches with an implosion time in the range of tens of nanoseconds. There exist devices where the imploding objects are relatively heavy metal shells

and where the implosion time is as long as hundreds of nanoseconds to microseconds. This kind of Z pinch also falls under the aforementioned definition of "fast" Z pinches and will be covered by our survey. As an example, one can mention spherical implosions of metal shells (Baker et al., 1978; Degnan et al., 1995). A schematic of the latter experiment, where quasi-spherical implosions were successfully realized, is shown in Fig. I.1e. Quasispherical targets may be pursued also in lower-mass configurations.

Fig. I.1. Various types of fast z pinches: a) An annular gaseous jet (Stallings et al., 1979); the axis of the diode is horizontal; the nozzle is a cathode, and the mesh is an anode; the plot at the right shows the radial density distribution; b) A cylinder made of agar foam in a Z-pinch diode (Derzon et al, 1997). An anode in this experiment was a transparent wire mesh; the cylinder (1 cm diameter) is surrounded by eight return-current posts; c) a photograph of a 4-cm diameter tungsten wire array used in the PBFA-Z facility (Spielman et al, 1997). The array had 240 wires; the mass per unit length of the array was 2 mg/cm; d) A high-Z liner imploding on a low-density foam (Matzen, 1997). An internal ICF capsule is situated in the center of the foam cylinder; e) A quasi-spherical liner implosion (Degnan et al, 1995). An aluminum liner slides along conical electrodes. The initial radius of the outer surface is 4 cm; the time-sequence is: $t_1=0$; $t_2=12.7 \mu\text{s}$; $t_3=14 \mu\text{s}$. The right column represents the results of 2D MHD computations.

D. Pulsed power

Remarkable progress made during the last few years in fast Z-pinch parameters became possible by progress in pulsed-power technology and in the development of sophisticated diagnostics instrumentation. As has already been emphasized, this review is directed to the discussion of the physics of Z pinches and will not address the equally important issues of pulsed-power technology and pinch diagnostics. However, to give an interested reader some guidance in the pertinent work, we briefly summarize the status of both areas.

The power and current available for Z pinch implosions reached new heights during the last decade: the pulsed-power generator Saturn (Spielman et al., 1989) allowed reaching electrical power of 20 TW and a maximum current of ~ 10 MA. The 50-TW Particle Beam Fusion Accelerator (PBFA) was modified to drive fast Z-pinch implosions at currents of ~ 20 MA. It was renamed 'Z' (and occasionally called PBFA-Z). (Spielman et al., 1996). Both generators are situated at Sandia National Laboratories (Albuquerque, New Mexico, USA). The kinetic energy in imploding liners that were 2-cm long at an initial radius of 2 cm reached ~ 0.35 MJ at Saturn and ~ 1.2 MJ at Z. A schematic of the Z facility is shown in Fig. I. 2. There are other high-power generators used in Z-pinch research. In Russia, the best known is the Angara-5 generator (Al'bikov et al., 1990), with a maximum current of ~ 4 MA and a maximum power to the load of ~ 10 TW. Generators of a similar size include Double Eagle at Physics International, Blackjack 5 at Maxwell Laboratories, and Proto II at Sandia National Laboratories. Worldwide, about 15 generators operating at the current level of 1-3 MA and power to the load of ~ 1 -5 TW are used in Z-pinch research (some of them are described in Camarcat et al., 1985). There are also numerous smaller generators.

Fig. I.2. Schematic of the PBFA-Z facility (Spielman et al, 1996). The diameter of the facility is ~ 30 m. The outermost part is formed by Marx generators. They are connected to 36 transmission lines with water insulation which, in turn, feed magnetically insulated transmission lines converging at the diode. The diode is situated inside the central tank.

Most of larger generators use so-called magnetically insulated transmission lines (MITL) to deliver the power to the axisymmetric diode assembly. These magnetically insulated lines are, in turn, fed by transmission lines insulated with water. The power to the water transmission lines is supplied by high-voltage Marx generators. A wealth of information on these issues can be found in the proceedings of pulsed power conferences. A typical geometry of the diode assembly, where the Z-pinch target is situated, is shown in Fig. I.3.

Fig. I.3. Cross section view of the Z diode with on-axis annular target. There are nine cutaway slots in the current return can for diagnostic access. The power from the magnetically insulating transmission lines flows to the diode through the gap in the lower part of the figure.

To drive the slower and heavier loads of the type shown in Fig. I.1e with the implosion time of the order of $1\ \mu\text{s}$, slower generators are required. Examples of such generators are Shiva Star situated at the Phillips Laboratories, Albuquerque, New Mexico, USA (Degnan et al., 1995), and Pegasus situated at Los Alamos National Laboratory, Los Alamos, USA.

E. Diagnostic instrumentation

Electric parameters of the discharge and the current through the pinch are inferred by measuring the electric and magnetic fields at specific points of the device in the generator with millimeter-size magnetic loops and capacitive probes. To measure the current through the pinch column, it would be necessary to measure the magnetic field inside the return current conductor (see Fig. I.3). This is difficult to do because of very large magnetic fields in a region of strong radiation and heat fluxes. Therefore, the magnetic field (and the current) are usually measured in the MITL, at distances larger than $\sim 3\ \text{cm}$ from the axis of the Z pinch. The voltage is measured

at the insulator stack. A description of the probes and further references can be found in Stygar et al. (1997). A possible way of measuring the magnetic field in the plasma column is the use of a Faraday rotation technique (Branitski et al., 1992 a,b).

Optical measurements are useful for the characterization of the early stage of the Z-pinch implosion, when the X-ray radiation doesn't yet overwhelm the optical detection system. Optical interferometry and holography allow one to detect low-density blow-off plasma at an early stage, as well as to observe instabilities of individual wires in the wire arrays. These measurements have a spatial resolution of a few tens of micrometers to millimeters and temporal resolution ~ 1 ns. Further details and references can be found, e.g., in Haines et al. (1997); Muron, Hurst, and Derzon (1997); and Deeney, McGurn et al. (1997a). Emission tomography is described by Veretennikov et al. (1992).

For later stages of the implosion, the x radiation becomes significant and is successfully used for characterization of the pinch. Total radiated energy (less than ~ 5 keV) is typically measured with bolometers. Calorimeters can also be used to measure the total radiation energy. X-ray diodes (XRDs), with ~ 1 ns resolution, and photoconducting detectors (PCDs) are used to make broadband time-resolved measurements of x-ray spectra (Spielman et al., 1997). Filtered scintillation detectors have also been developed (Averkiev et al., 1992). X-ray spectroscopy is used to characterize the radiation in x-ray lines for obtaining plasma temperatures and ionization states (Leeper et al., 1997; Pikuz et al., 1997). X-ray framing cameras provide a spatial resolution as small as 100 micrometers, with exposure time duration as small as 100 picoseconds.

The high-energy electron beams sometimes generated in Z pinches can be detected by the high-energy photon radiation from electrodes and the pinch, and high-energy ion production can be observed through gamma spectroscopy of activated materials. The presence of fast deuterons is inferred from neutron radiation.

A complete description of the status of diagnostic instrumentation would require the inclusion of many tens if not hundreds of additional references and would lead us well beyond the intended scope of this paper. We just point out that many diagnostics papers can be found in the January (1997) issue of the *Review of Scientific Instruments* and in the proceedings of the 1997 Conference on Dense Z pinches (Pereira, Davis, and Pulsifer (1997)).

F. Applications

Although this paper contains a special section on pinch applications (Sec. X), we believe that a brief summary would be appropriate here.

Fast Z pinches are traditionally a point of interest in research on controlled thermonuclear fusion. One approach consists of imploding a DT gas-puff (e.g., Nedoseev, 1990, and references therein), possibly on a DT cryogenic fiber. A more sophisticated version of this approach is based on the so-called staged Z pinch (Rahman et al., 1995), where a conducting shell is imploded on the current-carrying DT fiber, causing fast compression of the azimuthal magnetic field at the later stage of the implosion. Another approach employs the capability of the fast Z pinch to generate powerful thermal radiation. If a cylinder is nested within the imploding liner, then the intense radiation is generated at the instant of the strike between the two (Smirnov, 1991). This radiation fills the interior of the inner cylinder. If a capsule filled with DT is situated inside the inner cylinder, then the radiation flux causes ablation and implosion of the capsule, leading to a fast compression of the DT fuel. This is the concept of the so called "dynamic hohlraum" (Matzen, 1997; see Fig. I.1d).

The other "traditional" application of fast Z pinches is that of a radiation source (results in this area obtained before 1988 have been covered in a survey by Pereira and Davis, 1988). Depending on the parameters of the system (and, in particular, on the elemental composition of the pinch), the radiation spectrum can be controlled to a considerable degree. The pinch can serve as a source of photons in the range from tens of electron volts to ~ 10 kilo-electron volts. As a rule, the targets made of higher atomic number material result in pinches that produce softer radiation. Conversely, targets of lower atomic number materials produce lower density pinches where higher plasma temperatures may be achieved. After stagnation at the cylindrical axis of symmetry, a considerable fraction of both kinetic and magnetic energy gets converted into x-ray radiation. In the case of PBFA-Z, approximately 2 MJ of total radiation output has been produced in pulses shorter than 10 ns (Spielman, 1998). This certainly opens new horizons for a number of applications.

Fast Z pinches provide an opportunity to study properties of matter in extreme conditions. In particular, by using intense radiation to cause ablation from one side of a sample under study, one can excite a strong shock wave in the sample and obtain information about the equation of state (Branitski et al., 1996; Olson et al., 1997). Extreme states of matter achievable in fast Z pinches can be used to obtain information about opacities for astrophysics (Springer et al., 1997);

they are also of interest for pulsed VUV or x-ray lasers (Spielman et al., 1985a; Dangor, 1986; Porter et al., 1992a). One more application is based on the compression of the initial bias magnetic field by a conducting shell. In this way one can obtain very high magnetic fields, at the level of 20 MG. The corresponding information can be found in the proceedings of megagauss conferences.

II. GENERAL DESCRIPTION OF THE PHENOMENON AND IDENTIFICATION OF THE KEY PHYSICS ISSUES

In this section we summarize – sometimes just in a verbal form – the most important phenomena that affect pinch performance. Later in the paper we give a more detailed description, but here we intend to provide the reader with general guidance in how a particular phenomenon fits into the overall picture. We will generally follow a natural time-sequence of a pinch event: from the initialization of the breakdown, through the run-in phase, to the on-axis stagnation and the following rebound.

In the major part of this section we assume that collision rates in the plasma are high enough to allow an adequate description of the plasma in terms of macroscopic quantities, like density, temperature, pressure, etc. However, in the implosion of gas puffs and in a lower-density plasma that may be formed inside the wire array, the collisionality may become small, and considerable deviations from the Maxwellian distribution may develop, requiring more sophisticated tools for the plasma description. These issues are discussed outside the “chronological” sequence in the last subsection (II G) and, partly, in subsection II A.

A. Breakdown (for non-conducting loads)

If one deals with the loads that are initially non-conducting, then ionization of the load should occur to allow a current to flow. Gas puffs are a common example of this type of load. For noble gases with a density in the range of 10^{18} cm^{-3} , typical for gas-puff implosions (Fig. I.1a) and the pinch height $L \sim 1\text{-}2 \text{ cm}$, the breakdown voltage, according to the Paschen law (e.g., Raizer, 1991), lies in the range of 300-500 V (Fig. II.1). Certainly, so low a voltage arrives at the gap early in the pulse (note that, prior to the onset of a considerable axial current, the voltage is a well-defined quantity; see the end of the Section I C) and the breakdown condition is easily met. However, one should remember that the Paschen law corresponds to a quasi-steady-state version of the breakdown where the electrons and ions cross the gap in a very short time compared to the time during which the voltage is sustained. As we show in Sec. V, the situation for the aforementioned set of parameters is just the opposite, and at the voltage of a few hundred volts, development of the discharge would take time well in excess of 10^{-7} s (i.e., the total duration of the current pulse). In reality, the voltage reaches the values much greater than a few hundred volts within a short time, approximately a few nanoseconds. Therefore, the breakdown and ionization will occur in a non-steady-state fashion, at voltages considerably exceeding the Paschen's value, when the ionization time becomes comparable to the characteristic time of the voltage growth,

$V/\dot{V}$. We conclude that the gap will remain relatively weakly ionized and essentially non-conducting until the voltage reaches a value considerably higher than the Paschen's breakdown voltage, and then the ionization occurs very rapidly.

Fig. II.1. Paschen Law for some gases.

A phenomenon that interferes with the ionization of the gas-puff is electron magnetization. Indeed, for the gas-puff radius of 2 cm, at an optimum (in the Paschen's sense) density of $\sim 10^{17} \text{ cm}^{-3}$, the electron gyro-radius for 10–20 eV electrons becomes less than λ_{ea} (the mean free path with respect to the electron-atom collisions) at a very weak magnetic field $\sim 0.1 \text{ T}$ (this corresponds to the current through $r=2 \text{ cm}$ gas-puff of the order of 10 kA, orders of magnitude less than the “characteristic” pinch currents, which reach mega-amperes). As soon as electrons get magnetized, further avalanching and current growth get inhibited and, with the voltage continuing to grow, the discharge shifts to the higher-density regions. One can conceive this process as a consecutive formation of current-carrying shells, starting from the zone near the optimum density, and propagating deeper into the gas-puff with the voltage growing. Eventually, a highly conducting shell (sometimes called sheath) with the thickness comparable to the skin-layer will be formed that will “screen” the penetration of the electric field into the region inside it. The ionization process with account for the skin-effect was considered by Vikhrev and Braginski (1987).

Because the initial gas-puff is never perfectly axisymmetric (the gas flow is turbulent) and because the breakdown itself, even in a perfectly axisymmetric gas column, is a statistical process, it may happen that, inside the sheath, there will remain current channels produced by the early breakdowns. These channels may seed instabilities during the further implosion, whence the importance of the initialization phase in the whole picture of the shot. Preionization

of the gas puff can probably make the discharge more uniform (Baksht, Russkikh and Fedyunin, 1995; Baksht, Russkikh and Chagin, 1997).

B. Instabilities of the cold initial plasma

The onset of magnetization may cause development of fast instabilities related to the Hall effect (Gordeev, Kingsep, Rudakov, 1994). These instabilities give rise to the formation of a complex 3-D structure of the current flowing in the ionized part of the column. The other group of instabilities of the early stage of the discharge is a group of overheating and ionization instabilities (Branitski et al., 1991; Afonin, 1995). Their origin can be understood as follows. Assume that the axial electric field is maintained constant, and that a conductivity of some thin channel of the discharge plasma is slightly increased. Then, the current and the Ohmic dissipation in this channel also increase, causing the temperature to rise. If the conductivity grows with a growing temperature, this process will self-accelerate. This is a possible origin of a filamentation instability that causes the current to concentrate in a number of thin filaments.

There is a preferential azimuthal mode number m for this instability determined by the ratio of the skin-depth, δ_{skin} , to the plasma radius:

$$m \sim r / \delta_{skin} . \quad (2.1)$$

Indeed, at a smaller azimuthal mode number, the current is forced to remain almost azimuthally uniform by inductive effects (see Sec. V for more detail); at higher mode numbers, the stabilizing role of the thermal conduction increases. Note that, as will be shown in Sec. VI, the high- m perturbations are relatively well-behaved in terms of the Rayleigh-Taylor instability. The filaments therefore persist until the late phase of the implosion, without causing disintegration of the pinch.

In some cases, axisymmetric ($m=0$) structures (called also "striations") have also been detected at the early phase of the discharge of the gas-puff (Branitski et al., 1991). The simplest mechanism hypothesized for their appearance is also the temperature variation of the resistivity. However, striations, being axisymmetric structures, are connected "in series" (not "in parallel" as in the case of the filaments); therefore, the joule heating increases when the resistivity *increases*. In other words, this instability is possible when the resistivity grows with temperature. Such a situation may be met in a weakly ionized plasma (i.e., at the early stage of the discharge preceding the appearance of filamentation), where an increase of temperature causes an increase

of the ionization degree and, accordingly, an increase of the contribution of the Coulomb collisions to plasma resistivity. Because the cross section of the Coulomb collisions at low temperatures is much higher than the cross section for electron-atom collisions [at $T \sim 1$ eV the electron-ion cross section is $3 \cdot 10^{-13} \text{ cm}^3$] (see Sec. IV), whereas typical electron-atom cross-section is $\sim 10^{-15} \text{ cm}^3$, the resistivity (in the domain of temperatures $\sim 1-3$ eV) may have a positive temperature derivative, making the striation instability possible.

To conclude this section: a relatively cold and weakly ionized plasma present during the early phase of the discharge may be susceptible to specific instabilities caused by a strong temperature dependence of the resistivity, ionization state, and radiation power. These instabilities, generally speaking, do not require any mass motions and may, therefore, develop much faster than hydrodynamic instabilities dominant at the later stage of the discharge. They can produce filamentary current distribution that will seed $m \neq 0$ instabilities and produce axial striations that will seed the axisymmetric $m=0$ mode.

C. Breakdown of a foam load

Now we briefly discuss the breakdown of a foam load. At the density 10 mg/cm^3 , agar foam (approximate stoichiometric formula CHO) and the SiO_2 aerogel correspond to approximately $1 \cdot 10^{20} \text{ atoms/cm}^3$ (averaged over the volume). The foam has a strongly nonuniform structure, with the voids occupying most of the volume. Still, the aforementioned average density is within an order of magnitude of the gas density at atmospheric pressure and one can expect a similar mechanism of the streamer development. (For the theory of gas breakdown at high pressures, see Raizer, 1991; Lozanski and Firsov, 1975.) The breakdown voltage of gases at such a density in gaps 1-2 cm tall is quite high, in the range of 100 kV.

If the foam cylinder is too tall (and the breakdown voltage is too high), the breakdown may occur in the gap of the magnetically insulated transmission line (MITL), and not across the foam. The paper by Volkov et al. (1993), which describes, among other things, implosions of NaSiO_3 "porous solids," states that when the target height was larger than the gap of the MITL, the discharge across the foam could not be initiated at all.

Returning to situations where breakdown does occur, and using the aforementioned analogy with breakdowns in dense gases, one can predict that breakdown should develop in the form of a streamer that creates a conducting channel in the foam. A localized channel may be formed also

by a surface breakdown that sometimes occurs at a lower voltage (very difficult to predict quantitatively). Therefore, one can expect that the initial distribution of the current in the foam will be nonuniform, and this will have a negative effect on the symmetry of the implosion. A conducting coating on the surface of the foam appears to provide better conditions for the initiation of the discharge. More detail on these issues will be presented in Sec. V.

D. Early phase of the discharge in a wire array

A typical wire array used in fast Z-pinch experiments has initial radius $r_0 \sim 1-2$ cm, height $L \sim 1-2$ cm, the number of wires 100-300, with the wire radius $r_w \sim 3-10$ μm . The most common materials of the wires are tungsten, aluminum, nickel, and some other metals. With a wire radius of a few microns, the current very quickly, within a couple of nanoseconds, penetrates into the wires. In other words, the skin-effect does not play any role at the early stage of the discharge. The wires get heated and evaporated within the first few nanoseconds of the current pulse. Note that axisymmetric "striation" instabilities can in principle develop in separate wires, but they will be uncorrelated between the neighboring wires and will not produce a coherent axisymmetric structure of the type mentioned in Sec. II B. During the transition from liquid metal to metal vapor, the resistivity should increase considerably and the heating process should accelerate.

Within a few nanoseconds (we are referring to experimental parameters similar to the ones listed in Table 1), the temperature of the wire reaches 2-3 eV and the wires become plasma. Individual plasma channels become susceptible to the familiar sausage and kink magnetohydrodynamic instabilities (e.g., Kadomtsev, 1965). The fastest growing modes correspond to the axial wave number $k_z \sim 1/r_w$ and have a growth rate

$$\Gamma \sim \sqrt{\frac{\pi}{\mu} \frac{B_w}{\sqrt{\hat{m}_w}}}, \quad (2.2)$$

where μ is the magnetic permittivity of the vacuum, B_w is the magnetic field produced by the current through the wire at the wire surface, and $\hat{m}_w$ is the wire mass per unit length. The growing perturbations randomly distributed over the wire length cause a gradual broadening of the wire by a kind of a random motion. The expansion velocity can be evaluated from energy considerations as $v \sim \Gamma r$. As we show in Sec. V, this random motion of the wires eventually causes their merging; if the number of the wires is large (more than 200), the merging occurs relatively early in the pulse. For small number of wires (less than 20-30) the merging occurs at a late stage of the

implosion, when the wire array radius has already considerably decreased compared to the initial radius (Sanford et al., 1996).

If the MHD mechanism of the wire merging is indeed dominant, one can make some predictions with regard to the thus-formed quasi-continuous shell: it will be grossly non-uniform, with the spatial scale of the non-uniformities of the order of a half of the inter-wire distance. These non-uniformities will be both axial and azimuthal and will serve as seeds for the Rayleigh-Taylor instability. Experiments with individual wires indeed show development of the sausage and kink perturbations and do not contradict the aforementioned picture of the wire merging (see Beg et al., 1997; Bartnik et al., 1990). Fig. II.2 shows a shadowgraph of a single wire explosion at the Magpie generator (Beg et al., 1997).

Fig. II.2. A set of single-shot schlieren photographs recorded for $7\ \mu\text{m}$ carbon fibres (Beg et al., 1997). The maximum current through the fibre was $\sim 100\ \text{kA}$. The exposure time was $7\ \text{ns}$.

In single wire explosions it is possible that almost all the current be carried by a low-density plasma corona. This phenomenon can be particularly pronounced if wires contain absorbed gases that are released early in the pulse. The resistivity of the corona becomes small because of the high temperature; coronal plasma cannot freely expand because of the presence of the confining magnetic field. The core of the wire may remain cold and dense until very late stages of the discharge (Sasorov, 1991; Haines et al., 1997). There exists, however, a significant difference between the discharge in a single wire and a discharge in a wire array, even if the current per wire is the same as in the case of a single wire. The difference stems from the fact that, in the case of a

wire array, there exists a common azimuthal magnetic field enclosing the whole wire array and exerting a force directed to the axis of the array (not to the axis of a individual wire!). The halo plasma (which is very light) would be blown off past the cores of the wires, and the current would be forced by inductive effects to switch to the wire cores. Therefore, the role of the halo plasma in case of fine enough wire arrays should be relatively insignificant: the current is forced to flow through the wire core. In the wire arrays consisting of a small number of wires, however, a plasma blown into the interior of the wire array can intercept a considerable amount of current (see, e.g., Aivazov et al., 1988, where such an effect was clearly visible in the implosion of an array of 6 wires).

As we have mentioned in the Introduction, the power supply system for fast Z pinches is a complex multi-stage device, with the gradual power amplification (and pulse shortening) from stage to stage. As the stages are coupled with each other, a relatively long and low voltage pulse may arrive at the electrodes of the pinch chamber before the main high-power pulse arrives. This early, low-voltage slow pulse is commonly called a "pre-pulse." Despite a very small energy delivered to the load in the pre-pulse stage, the presence of pre-pulse can have a considerable influence on the further course of events. For example, it can cause an early evaporation of the wires and thereby modify initial conditions at the time when the main pulse arrives. It can also affect breakdown of the gas-puffs and foam loads discussed in Sections II A-II C.

As a summary to this section, one can state that the wires of the array experience hydrodynamic instabilities causing their average broadening and eventual merging. These instabilities produce a non-uniform shell, with the scale of non-uniformities approaching the inter-wire distance, thereby setting initial conditions for the Rayleigh-Taylor instability during the run-in phase. Before the wires merge, some small amount of the low-density halo plasma may be "blown" through the inter-wire gaps into the inner part of the wire array.

E. Run-in phase: implosion of a thin shell vs. implosion of gas puffs; shocks; radiation losses and snow-plow model; hydrodynamic stability

As soon as the current-carrying sheath (consisting, possibly, of numerous separate channels as in the case of wire arrays) is formed, it begins to accelerate to the axis. To provide a good efficiency of converting the energy stored in the pulsed power generator into kinetic energy of the imploding pinch, one should choose an optimum mass of the pinch material. This mass should be such that the implosion time be approximately equal to the time within which the current reaches its maximum value: if the mass is too large, the current pulse ends before the pinching

occurs; if the mass is too small, the pinching occurs before the current reaches its maximum, also meaning a poor efficiency.

In the simplest case of a thin annular shell (like the one formed by the wire array), the equation of the radial motion can be written as:

$$\frac{\hat{m}}{2\pi r} \ddot{r} = -\frac{B^2}{2\mu} = -\frac{\mu I^2}{8\pi^2 r^2}, \quad (2.3)$$

where $B=B(t)$ is the magnetic field at surface of the pinch, and $I=I(t)$ is the pinch current. Let us measure the current in the units of the maximum current, $I_{\max}$, the time in the units of the time τ within which the current reaches its maximum, and the radius in the units of the initial radius r_0 . In other words, we introduce dimensionless variables

$$\tilde{r} = r/r_0, \quad \tilde{t} = t/\tau, \quad \tilde{I} = I/I_{\max}. \quad (2.4)$$

Rewritten in these variables, Eq. (2.3) converts to:

$$\tilde{r} \ddot{\tilde{r}} = -\Pi \tilde{I}^2, \quad (2.5)$$

where

$$\Pi = \frac{\mu I_{\max}^2 \tau^2}{4\pi \hat{m} r_0^2} \quad (2.6)$$

is a dimensionless scaling parameter of the problem (see also an instructive analysis by Katzenstein, 1981, where the effects of the external circuit are directly taken into account). The two implosions with the same functional dependence of the current vs. time (i.e., with the same dependence $\tilde{I} = \tilde{I}(\tilde{t})$) occur in a similar fashion if the parameter Π for them is the same. In particular, the time instant when the pinch collapses on the axis, measured in the units of τ , is the same for both implosions. We discuss the consequences of the aforementioned similarity in more detail in Secs. III and XI.

The minimum height of the pinch is determined by the inductance of the external circuit: there is no sense in reducing the height considerably below the value where the inductance of the diode becomes less than the inductance of the external circuit.

In case of uniform (not annular) cylinders, like uniform gas-puffs or foam cylinders, another important element of the picture emerges: the shock wave propagates ahead the current-carrying sheath and implodes on axis considerably earlier than the sheath. We imply that the skin-depth is small compared to the radius of the imploding column, as is the case in real situations.

The converging cylindrical shock, if it possesses a good symmetry, can by itself produce a strong increase of density and temperature on axis, formally singular (Guderley, 1942; see also a nice presentation of Guderley's results in Whitham, 1974). After the shock is reflected from the axis and reaches again the surface of the cylinder, a kind of an adiabatic compression begins, where the plasma pressure is approximately equal to the magnetic pressure, and the sound speed in the plasma is comparable to the Alfvén velocity. This means that a kind of a transient Bennett-type equilibrium is formed for a few acoustic transit times.

In this discussion of the implosion of a uniform density (not annular) column, we ignored the role of radiation. This can be the case with imploding DT gas puffs. If, however, one deals with implosions of heavier elements (for details see Sec. IV), then the radiation of the plasma behind the shock can become important. In the extreme case of very strong radiation losses, the plasma behind the shock radiates its energy much faster than the time within which the shock could propagate across the radius of the pinch. In this extreme case the shock will not be formed at all, and all the material impacted by the magnetic piston will just stick to the piston. It corresponds to a so-called snow-plow model studied in much detail in the early years of pinch research (1950's). We will use the term "snow-plow" just in this sense, to designate a simple model where all the material swept by the magnetic piston merely sticks to it. In more recent years, this model was improved by Potter (1978) and Miyamoto (1984) to include the possible presence of the shock-heated layer in front of the piston.

Equations of motion of the magnetic piston sweeping the plasma as a snow-plow reads as:

$$\frac{\dot{m}^*}{2\pi r} \ddot{r} - \rho(r) \dot{r}^2 = -\frac{\mu I^2}{8\pi^2 r^2} \quad (2.7)$$

$$\dot{m}^* = -2\pi r \dot{r} \rho(r) \quad (2.8)$$

where $\rho(r)$ is the initial density distribution and $\hat{m}^* = \dot{m}^*(t)$ is the instantaneous mass accreted at the piston. The second term in the l.h.s. of Eq. (2.7) describes the momentum imparted to the piston by the accreting material. We assume that the density at a certain radius remains intact until the very moment of the magnetic piston arrival. For two implosions with the initial density

distributions having the same functional dependence on r/r_0 , there exists the same similarity law as for the equation (2.3), i.e., the two implosions are similar if the parameter Π for them is the same.

Consider in some more detail the initial stage of the implosion of a thin shell (Eq. 2.3). When the pinch radius has not yet decreased considerably with respect to its initial value r_0 , one can present r as $r=r_0-\Delta r$, with Δr small. In all our semi-quantitative estimates we will use a simple model of the current through the pinch,

$$I = I_{\max} \sin^2\left(\frac{\pi t}{2\tau}\right). \quad (2.9)$$

At an early stage of the implosion, the pinch current can be with a reasonable accuracy approximated by a parabolic dependence vs. time,

$$I = I_{\max} \left(\frac{\pi t}{2\tau}\right)^2, \quad (2.10)$$

For the dependence (2.9) one easily obtains for Δr :

$$\frac{\Delta r}{r} = \frac{\pi^4 \Pi}{480} \left(\frac{t}{\tau}\right)^6. \quad (2.11)$$

Note that the pinch radius departs from its initial value very slowly, $\sim t^6$. For a load with a mass corresponding to the collapse at $t=\tau$, even at $t=(2/3)\tau$ the pinch radius is decreased by a mere 10% of its initial value. This discussion shows that a pinch has a long "latent" phase followed by a very fast collapse that occurs within a small fraction ($\sim 0.1-0.2$) of the total implosion time τ . Calculated time-histories of the wire array radius and the pinch current in one of the shots at the Z device are shown in Fig. II.3.

Fig. II.3. Measured current (dots), and wire radius calculated with Screamer code (dashes), and Kimfol-filtered (200-280 eV) x-ray diode signal (solid line), for Z shot 104. Courtesy of K. Struve, SNL. A long “latent period” during which the shell radius decreases very slowly is clearly visible.

Consider now an early stage of the implosion of a uniform cylinder in the framework of the snow-plow model. In this case, at the same mass per unit length as in a thin shell, the current sheath moves towards the axis faster than in the case of a thin shell. The reason is merely the smaller mass involved in the implosion at an early stage. To illustrate this point more quantitatively, consider an analog of Eq. (2.11) assuming that the initial density distribution is uniform, i.e., $\rho = \hat{m}/\pi r_0^2$. One now has:

$$\frac{\Delta r}{r_0} = \frac{\pi^2}{4} \sqrt{\frac{\Pi}{30}} \left(\frac{t}{\tau} \right)^3 \quad (2.12)$$

This solution is valid until Δr is less than, roughly speaking, $r_0/3$. We see that implosion begins faster than for a thin shell. The collapse on axis also occurs earlier than for the thin shell of the same mass per unit length. Still, the latent period is present in this case, too.

We have discussed three limiting cases of the implosion: that of a thin shell, that of non-radiating uniform (not annular) column and that of a strongly radiating uniform column (snow-plow model). Of course, a whole range of intermediate cases is also possible.

So far, we were considering implosions that are stable and maintain a perfect cylindrical symmetry. However, in reality, the imploding shell suffers from hydrodynamic instabilities. The most important class of instabilities is related to the presence of a fast acceleration directed

inward during most of the run-in. This acceleration is driven by the magnetic field. In the frame co-moving toward the center together with some element of the shell, one finds a situation of a fluid supported from below by a magnetic field against the effective gravity force. There exists a free gravitational energy in the system and instability sets in that tries to cause the fluid and the magnetic field to change places. For this reason, this instability is often called “interchange instability”. A more common name used in the area of fast Z pinches, is the “Rayleigh-Taylor instability”, because of its close similarity to the instability of a heavy fluid supported from below by a light fluid, described by Rayleigh (1900) and Taylor (1950). A detailed analysis of this instability in incompressible fluids can be found in a celebrated monograph by Chandrasekhar (1961). Modern surveys of the theoretical issues have been published by Sharp (1984) and Kull (1993). Experimental aspects of the instability in the ICF environment have been discussed in the survey paper by Kilkenny et al. (1995).

The Rayleigh-Taylor instability causes development of considerable perturbations of the shape of the liner, breaking cylindrical symmetry of the implosion. The most dangerous modes are axisymmetric modes: axisymmetric perturbations of the surface, of the type shown in Fig. II.4a causes increase of the magnetic field at the “tip” of the perturbation, while development of the azimuthal perturbation of the type shown in Fig. II.4b causes decrease of the magnetic pressure at the tip of the perturbation.

Fig. II.4. Axisymmetric mode (a) and the z -independent azimuthal mode (b). Dashed lines depict magnetic field lines.

Considerable distortions caused by these perturbations lead to a non-simultaneous arrival on axis of different parts of the pinch — a detrimental effect for a number of applications (for example, it leads to a broadening of the radiation pulse). It is, therefore, desirable to eliminate or mitigate the Rayleigh-Taylor instability (Sec. VIII).

A summary of this section: the run-in phase consists of a long "latent" period (70-90% of the current pulse-width), during which the current shell experiences only a small displacement to the axis, followed by a very rapid implosion on axis (10-30% of the current pulse-width). In the implosions of uniform targets, depending on the role of radiation, one can meet the situation with a clearly discernible shock-wave propagating in front of the magnetic piston, and the situation where all the material essentially "sticks" to the piston (a snow-plow model). The imploding shell may suffer from the Rayleigh-Taylor instability that breaks the cylindrical symmetry and causes an effective broadening of the shell.

F. Stagnation and rebound: hydrodynamic stability; 3-D effects; possible generation of intense particle beams

When a thin plasma shell stagnates on axis, the kinetic energy gets transformed into a thermal energy, producing a high-temperature, high-density plasma. For example, in the implosions of wire arrays, temperatures in the range of hundreds electron-volts and electron densities in the range of 10^{22} cm^{-3} are typical (Sanford et al., 1996; Maxon et al., 1996; Matzen, 1997; Deeney, Nash et al., 1997). In a high-Z plasma, the thermal energy may be quickly radiated from the plasma column: this is a principle of the black-body (or nearly black-body) radiation sources. In the implosions of lighter materials, the energy can be converted into the line-radiation of hydrogen-like ions. In such a situation, electron temperature can exceed 1 keV.

In the implosion of a high-quality cylindrical shell the plasma pressure at the instant of stagnation is universally higher than the instantaneous magnetic pressure at the surface of the pinch (see Sec. III A). Therefore, even putting aside effects of hydrodynamic instability, one concludes that the plasma has to expand after collapse on axis. Only very strong radiation can cool the plasma down rapidly enough to prevent it from expanding. In the course of expansion and radiation, the plasma pressure decreases, and the magnetic pressure begins to compress the plasma again. However, this second implosion, if it occurs, is of less interest because the pinch becomes very diffuse.

Let us discuss in some more detail the effect of plasma stability on the quality of the implosion, taking as an example an implosion of a thin shell. For the situation shown in Fig. II.5a, which depicts the possible effect of a strong sausage instability, one can easily conceive a formation of a narrow neck at some instant of time (Fig. II.5b). Formation of the neck will be accompanied by ejecting part of the material from the neck region along the axis, as shown in Fig.

II.5b. A virtual break-up of the pinch at the neck point is possible, with only a low-density plasma remaining in the “bridge” that connects the two sides of the gap (Fig. II.5c). At a low plasma density, development of various plasma instabilities leading to the “turning-on” of the anomalous resistance is possible (see Sections II G and IX A). This causes a rapid decrease of the current through the pinch and formation of a very high inductive voltage across the gap. This, in turn, may lead to the generation of high-energy particle beams, with the energies exceeding the applied voltage by the order of magnitude or more. Specifics of this complex phenomenon is discussed in the survey papers by Trubnikov (1986) and Vikhrev (1986).

Fig. II.5. Formation of the neck (description in the text).

If the instabilities at the run-in phase are not too detrimental, the plasma radius in the imploded state is much smaller than the pinch length. One might expect that this thin column might be subject to MHD instabilities typical for the Bennett pinch, in particular, to the kink instability (which can hardly be of importance at the earlier stages, where pinch radius is comparable with or greater than the pinch height). However, as has been already mentioned, the magnetic pressure is small compared to the pressure of the imploded plasma, and these instabilities just do not have enough time to develop: the plasma rebounds back before anything happens. This comment once again underlines a fundamental difference of the transient, “fast” pinches that are discussed in this survey, and quasi-equilibrium pinches.

On the other hand, purely hydrodynamic instabilities of the Rayleigh-Taylor type are still possible even at this point, depending on the density distribution in the plasma column; if there is a region where the density is growing in the outward radial direction, the instability is very

probable. This may happen, for instance, if a wire array of dense material implodes onto a light foam cylinder. The source of the instability is a deceleration of the dense material by the pressure force of the compressed foam cylinder.

Above, we were discussing an implosion of a thin shell. In the other extreme case where the initial state is that of a uniform density and weak radiation losses, the implosion occurs in a different way (see previous section): near the maximum compression point it becomes close to quasistatic, and the plasma pressure becomes of the order of the magnetic pressure. In such a situation, development of sausage and kink instabilities of the quasi-equilibrium pinch becomes more probable. These instabilities are different from the Rayleigh-Taylor instability in that they are not particularly sensitive to the radial mass distribution. Their growth rate is of the order of the inverse acoustic time. Therefore, in order these instabilities to show themselves, the quasi-steady phase should have a duration at least of several acoustic times.

In the case of a uniform and strongly radiating load that can be described by a snow-plow model, all the energy delivered to the pinch material (minus the radiated energy) is stored in the kinetic energy of the imploding shell. If the stability is good enough, the collapse on axis occurs more or less in the same way as in the implosion of a thin shell.

To summarize: the shape and the size of the plasma column near the stagnation point is determined by the growth of instabilities during the run-in phase. Disruption of the plasma column by the $m=0$ instability may cause formation of particle beams. In the implosions of weakly-radiating uniform loads, formation of quasi-equilibrium state is possible, with a life-time of a few of acoustic times. This quasi-equilibrium state may suffer from sausage and kink instabilities.

G. Z pinches with lower density and higher temperature. The loss of local thermodynamic equilibrium. Microinstabilities. Self-generated magnetic fields.

This sub-section breaks the previously followed chronological order and addresses a group of non-MHD phenomena that may become important on any stage of the discharge. In those zones of the Z pinch where the plasma density is low and the plasma temperature is high, the rate of establishing a thermodynamic equilibrium may become too low compared to the implosion time. Such situations can be easily met in the implosion of low density gas puffs, in the corona area formed around separate wires in the wire arrays, and in the blow-off plasma that

can penetrate inside the wire array before the collapse of the array itself (this list is far from exhaustive).

Usually, temperature equilibrium between the ions and electrons is violated most easily, because the energy equilibration time between these two species contains a large parameter m_i/m_e , where m_i and m_e are the ion and electron masses, respectively. This equilibration time can be conveniently represented by the following expression derived from Braginski's (1965) paper (for the value of the Coulomb logarithm $\Lambda=10$):

$$\tau_{ei}^{(E)}(s) \approx 0.53 \cdot 10^{-13} \frac{[T(eV)]^{3/2} A^2}{\rho(mg/cm^3) Z_{eff}^2}, \quad (2.13)$$

where A is the atomic weight of the plasma ions, and Z_{eff} is effective charge of the plasma ions (in case of a fully stripped ions it is equal to just the atomic number Z). Taking, as an example, the density of 0.1 mg/cm^3 , the temperature $T=1 \text{ keV}$, and $A=12$, $Z_{eff}=6$ (fully ionized carbon plasma), one finds that $\tau_{ei}^{(E)}$ is $\sim 70 \text{ ns}$. As the kinetic energy of the imploding plasma shell is carried predominantly by the ions, decoupling of the electron and ion temperatures in the stagnation phase leads to a reduction of the electron temperature. This, in turn, affects the rate of atomic collisions, changes plasma resistivity and other transport coefficients, etc. As in a plasma with non-fully stripped ions the tail of the electron distribution function experiences losses caused by excitation and ionization events, the tail may get depleted, affecting the rate of the excitation and the radiation intensity (DeGroot, 1997). An example of the analysis of the experimental data on x-ray spectra from implosions of the foam loads on the Saturn device, with non-equilibrium effects included, can be found in MacFarlane et al. (1997).

At small densities, it may happen that electron collision rate may become less than electron cyclotron frequency ω_{ce} . This occurs if

$$B(MG) > 10^3 \frac{Z_{eff}^2 \rho(mg/cm^3)}{A[T(eV)]^{3/2}}. \quad (2.14)$$

We assume that $\Lambda=10$. Under this condition, the Hall terms in the Ohm's law become dominant. When this happens, plasma instabilities caused by the Hall effect may turn on (Gordeev, Rudakov, Kingsep, 1994), leading to current filamentation.

At lower densities, the relative velocity of electrons and ions (which is proportional to the current density divided by the plasma density) may become higher than the threshold of one or another current-driven instability. Among these instabilities, the lowest threshold has a so called

"lower hybrid instability" driven by the current perpendicular to the magnetic field (Krall and Liewer, 1971). It increases the plasma resistivity above the classical level. An even stronger increase can be caused by the ion acoustic instability which develops when the relative electron-ion velocity exceeds the ion thermal velocity (see, e.g., a survey by Vedenov and Ryutov, 1975). This instability can particularly easily be excited in a heavy ion plasma, because of a small thermal velocity of the ions. An obvious effect of these instabilities is a broadening of the skin layer.

In a low-density blow-off plasma that will fill the dynamic hohlraum (Sec. I E), the presence of large electron heat fluxes (and corresponding strong deviation of the distribution function from the Maxwellian) may give rise to a host of electromagnetic instabilities, leading to the generation of magnetic fields, which, in turn affect the heat transport. All these phenomena, which may play an important role in the pinch performance, will be discussed in Sec. IX.

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III. IMPLOSION IN THE IDEAL CASE OF THE ABSENCE OF INSTABILITIES

A. Implosion of a thin shell

In this section we discuss the case of an "ideal" implosion that might occur in the absence of instabilities. This will give us a kind of a reference point that would allow us to more clearly see effects of possible complications caused by instabilities. Stable purely cylindrical implosion can be numerically simulated with a great deal of detail and a considerable amount of information obtained in such simulations is available in published literature (e.g., Hammer et al., 1996). However, we prefer to concentrate on simple analytical models that allow the reader to more easily follow the chain of causes and effects. Working in this spirit, we start from the analysis of the simplest possible system, the structureless perfectly conducting thin shell. After that, we gradually add complicating factors, like the finite conductivity, possible rotation, etc.

The implosion of a thin shell is described by Eq. (2.3). We find the kinetic energy W_{kin} of the shell at the point where it reaches some desired minimum radius r_{min} . This can be a radius determined by a finite thickness h of the shell (i.e., $r_{min} \sim h$). To find W_{kin} , one can use the energy relationship that can be obtained by multiplying Eq. (2.3) by $\dot{r}$ and integrating by parts, from $t=0$ to $t=t^*$, where t^* corresponds to the point where $r=r_{min}$. One finds:

$$W_{kin} = \frac{\mu}{4\pi} \int_0^{t^*} \frac{dI^2}{dt} \ln\left(\frac{r}{r_{min}}\right) dt; \quad (3.1)$$

W_{kin} is a kinetic energy per unit length of the pinch at $t=t^*$. Let us discuss what mass of the liner would correspond to a maximum of W_{kin} for a given current waveform $I=I(t)$ and a given convergence (Eq. 1.1). The mass of the liner enters Eq. (3.1) only implicitly through the implosion time $t^*=t^*(\hat{m})$, and through the dependence of the radius vs. time, $r=r(t, \hat{m})$

At large convergences, one can obtain a simple approximate expression for W_{kin} . To do that, one should note that the logarithm in the integrand of Eq. (3.1) is almost constant and equal to $\ln C$ for the most part of the implosion. The contribution of the part where the logarithm begins to change (roughly speaking, after the pinch radius reaches $r_0/2$), is small because the time within which pinch implodes from $r_0/2$ to r_{min} is very short compared to the total current pulse-width (see Fig. II.3 and comments after Eq. (2.11)). So, replacing the logarithm in the integrand of (3.1) by a constant value $\ln C$ (corresponding to the final convergence $C=r_0/r_{min}$, i.e., typically, to $C \sim 10-20$), one finds:

$$W_{kin} \approx \frac{\mu I^2(t^*) \ln C}{4\pi} . \quad (3.2)$$

One sees that the maximum of the transferred energy corresponds to such a mass that the time t^* roughly corresponds to the maximum of the current, in agreement with the conclusion drawn in Sec. II E. This statement has a so called "logarithmic accuracy," i.e., it is valid up to the terms of order of $1/\ln C$. To make a more accurate estimate of the optimum implosion time, one should take a derivative of W_{kin} over $\hat{m}$. From Eq. (3.1) one finds:

$$\frac{\partial W_{kin}}{\partial \hat{m}} = \frac{\mu}{4\pi} \int_0^{t^*} \frac{dI^2}{dt} \frac{1}{r} \frac{\partial r}{\partial \hat{m}} dt . \quad (3.3)$$

Obviously, the derivative $\partial r / \partial \hat{m}$ is positive: a heavier liner implodes slower, and at a given time, has a larger radius. Therefore, if t^* corresponds to the current maximum, the derivative $\partial W_{kin} / \partial \hat{m}$ is positive. It reaches zero (i.e., the kinetic energy reaches a maximum) at some point beyond the maximum of the current. This point usually corresponds to the current that is 20-30% less than the current maximum. For the current waveform as in Eq. (2.9), the optimum value of the parameter Π is equal to approximately 4.

In a real life, for a given pulse-power generator, the current waveform cannot be considered as independent of the parameters of the load (because of the contribution of the load impedance to the overall impedance of the circuit). This circumstance can be taken into account by solving equations for the pinch together with the circuit equations (see, e.g., Katzenstein, 1981; Struve et al., 1997). One should also emphasize that the kinetic energy is not necessarily an appropriate figure of merit. For example, in the experiments on generation of radiation the figure of merit could be a radiated energy. The contribution to the radiated energy comes not only from the kinetic energy of the pinch but also from Joule heating (see Sec. X A). Still, the kinetic energy is an important and easily tractable characteristic of the implosion and we will concentrate our discussion on it.

Using Eq. (3.2), one can derive an expression for the volume density of the kinetic energy w_{kin} at the stagnation point in the case of an empty liner:

$$w_{kin} = \frac{W_{kin}}{\pi r_{min}^2} \approx 2 p_m^* \ln C , \quad (3.4)$$

where p_m^* is the magnetic pressure at the surface of the pinch at $r=r_{min}$. If this energy is converted to the thermal energy of the monatomic ideal gas, then the gas pressure will be $p=(2/3)w_{kin}$, or

$$p = \frac{4}{3} p_m^* \ln C \quad (3.5)$$

One sees that, with such a scenario, the gas pressure is indeed considerably higher than the magnetic pressure, and the pinch will rapidly expand after stagnation. The pinch rebound was detected in the experiments with aluminum wire arrays on the Saturn facility (Sanford et al., 1997).

If one deals with a liner made of heavy elements, then, in fact, a considerable amount of energy will be spent on the ionization, reducing p compared to the estimate (3.5). The radiative losses act in the same direction. If the radiation loss time is considerably shorter than the acoustic time r_{min}/c_s (where c_s is the sound speed), then, in the absence of instabilities, the pinch might experience a collapse to ever smaller radii (a radiative collapse, see, e.g., Meierovich, 1986). The Joule heating, especially with the anomalous resistance included, works in the opposite direction (Robson, 1991), as well as the radiation imprisonment and effects of electron degeneracy (Haines, 1989; Chittenden, Haines, 1990). The radiative collapse in a real situation may be prevented from happening also by the constraints imposed by circuit equations (a fast increase of the pinch inductance may decrease the current, Haines, 1989; Choi, 1997).

B. A snow-plow model (strongly radiating column)

In this section we switch to the discussion of implosions of the targets with initially uniform density distribution, like foam cylinders, or smooth radial density distributions, like, e.g., gas puffs. As was pointed out in Sec. II E, there exist two very different domains of parameters, depending on the role of radiation. If the radiation is strong, then a very thin shell of the material swept by the piston is formed. This case is similar to the one discussed in the previous section and we will start from it, leaving discussion of the second possibility (weak radiation) until the next section.

For the snow-plow model described by Eq. (2.7), the analog of Eq. (3.1) reads as:

$$W_{kin} = \frac{\mu}{4\pi\hat{m}} \int_0^{t^*} \frac{d(\hat{m}^* I^2)}{dt} \ln \frac{r}{r_{min}} dt, \quad (3.6)$$

where $\hat{m}$ is the total mass per unit length, and $\hat{m}^*$ is the mass swept by the current sheath by the time t ,

$$\hat{m}^* = 2\pi \int_r^{r_0} r' \rho(r') dr'. \quad (3.7)$$

We assume here that one deals with the implosion of a simple gas-puff or a foam cylinder, without an external shell (otherwise, one would have to add the mass of this shell both to $\hat{m}$ and $\hat{m}^*$ in Eq. (2.7)). Using the same arguments as in the case of a thin shell, one can show, that, at high convergences and smooth density distributions, W_{kin} is still determined by Eq. (3.2). From elementary mechanical considerations one can find that the power released in the inelastic interaction of the initially resting substance with a moving piston is (per unit length) $\hat{m}^* \dot{r}^2 / 2$. Integrating this expression over the time, one finds that, at $C \gg 1$, the part of the energy that was radiated from the accreted mass, is, by the order of magnitude, $1/\ln C$ of the final kinetic energy (i.e., relatively small). This is so because most of the mass is accreted before the liner reaches the radius, say, $r_0/3$, when the liner velocity is still small compared to its final velocity at $r=r_{min}$.

An interesting feature of Eq. (2.7) consists in that, at the properly chosen radial density distribution, one can provide conditions where the surface of the pinch does not experience radial acceleration. Such a regime may be desirable for the reason that it may be stable with respect to the Rayleigh-Taylor instability. This idea has been explored by Hammer et al. (1996). To approach the state of a constant velocity, the outermost part of the pinch should experience a sudden kick that would impart to it the desired velocity v . As soon as this state has been reached, the further evolution of the system is described by Eq. (2.7) with the first term in the left-hand-side of this equation omitted. One obtains the following equation for the desired density profile (Hammer et al., 1996):

$$\rho(r) = \frac{\mu I^2 \left(\frac{r_0 - r}{v} \right)}{16\pi^2 v^2 r^2}. \quad (3.8)$$

Note that the density should rapidly increase near the axis ($\propto 1/r^2$), and essentially all the mass should be concentrated within the radius $r \sim 2r_{min}$. Fig. III.1 depicts the required density distributions for two implosion velocities.

Fig. III.1. Density distributions for constant velocity implosions. Shown are the profiles with velocities $3 \cdot 10^7$ cm/s and $6 \cdot 10^7$ cm/s. The current waveform was determined self-consistently for the Saturn circuit equations.

For so sharply varying density distributions one cannot use Eq. (3.2) to estimate the kinetic energy. The final kinetic energy in this case is equal to:

$$W_{kin} = \frac{\hat{m}v^2}{2}, \quad (3.9)$$

with the mass $\hat{m}$ related to the implosion velocity by Eqs. (3.7) and (3.8), with $r=r_{min}$ in the former equation. The optimum velocity (at which W_{kin} is maximum) is the velocity at which $r=r_{min}$ is reached at the current maximum. Also important is to note that the radiative losses W_{rad} in this case are considerable. Using the same arguments as the ones that followed Eq. (3.6) one can show that the total radiated energy is $W_{rad}=W_{kin}$ (Cf. Hammer et al., 1996).

C. Weakly radiating plasma

In this case, the shock wave splits from the piston and propagates in front of it, heating and compressing the plasma. As the initial temperature of the pre-shocked plasma is small, the shock has a very large Mach number and can be considered as a strong shock (Landau and Lifshitz, 1987). For the gas with adiabatic index γ , the plasma density (ρ_I) and plasma pressure (p_I) behind the shock are:

$$\rho_1 = \frac{\gamma+1}{\gamma-1} \rho, \quad p_1 = \frac{2}{\gamma+1} \rho u^2, \quad (3.10)$$

where u is the shock velocity. If the magnetic piston is moving with some constant velocity v , there is a simple relationship between u and v , a direct consequence of the mass conservation equation, $\rho u = \rho_1(u-v)$:

$$u = \frac{\gamma+1}{2} v. \quad (3.11)$$

For the ideal monatomic gas ($\gamma=5/3$), $u=(4/3)v$, i.e., the shock velocity is approximately 30% higher than the velocity of the piston. This means that the shock converges on axis when the pinch radius is equal, roughly speaking, to $r_0/3$. This is a crude estimate, as it doesn't take into account variation of v and effects of the cylindrical geometry. Still, it doesn't differ strongly from more elaborate analyses, in particular from the "slug" model by Potter (1978).

Potter's model assumes that the plasma behind the shock is uniform, with parameters related to the parameters in front of the shock by Eqs. (3.10). The pressure p_1 is, on the other hand, equal to the magnetic pressure. This allows one to close the set of equations and to find the position of the shock and the piston as a function of time. The prediction is that the shock reaches the axis at $r \approx 0.31r_0$. After the shock is reflected from the axis and reaches the piston, a quasi-equilibrium state is formed in which the plasma pressure is approximately equal to the magnetic pressure. One sees that this is quite a different situation from implosions of thin shells where the particle pressure in the final state is much higher than the magnetic pressure.

Such solutions are of interest in the implosions of low- Z targets, in particular, DT gas-puffs and deuterated carbon foams. If one deals with the targets of heavier materials, they may remain not fully ionized behind the shock (see Sec. IV). In this case the use of the power law adiabats may break down. As a considerable amount of energy is spent on the ionization, the temperature behind the shock is lower than in the fully ionized case, and the density higher (Zeldovich, Raizer, 1967). In such a situation, the separation between the piston and the shock front reduces compared to what has been discussed above, and the snow-plow model of Sec. III B becomes relevant.

D. Effect of rotation

The idea of introducing initial rotation in the imploding liner goes back to the early 1970s when it was proposed as a method to stabilize imploding liners of liquid lithium in the concept of the fusion reactor LINUS (Book and Winsor, 1974; Barcion, Book and Cooper, 1974). The idea is that the centrifugal acceleration has an opposite direction to the radial acceleration (directed inward) and may have a stabilizing effect on the Rayleigh-Taylor instability. We will return to the stabilization effects later, in Sec. VIII, while here we will briefly describe the influence of rotation on the dynamics of a thin liner, assuming that any deviations from the cylindrical symmetry are absent. The problem of how one can introduce rotation will be discussed in Sec. VIII.

We will neglect the loss of the angular momentum in the course of implosion and will characterize the rotation velocity by an initial rotation energy $W_r^{(0)}$. Because of the momentum conservation, the rotation energy scales as C^2 ; this means that, at $C \sim 10$, the rotation energy will become an important player in the dynamics of the liner even if $W_r^{(0)}$ was quite small, $\sim 1\%$ of the final kinetic energy.

Eq. (2.3) should be modified by adding a centrifugal force to the r.h.s.; namely, the following equation should now be used:

$$\hat{m}\ddot{r} = -\frac{\mu I^2}{4\pi r} + 2W_r^{(0)} \frac{r_0^2}{r^3}. \quad (3.12)$$

The centrifugal term (the last term in r.h.s.) grows rapidly as the liner implodes to small radii. The reversal of the liner acceleration occurs at $r=r_r$ where

$$r_r^2 = r_0^2 \frac{8\pi W_r^{(0)}}{\mu I^2}. \quad (3.13)$$

Note that the liner continues to implode at this point. The rotation energy at $r=r_r$ is

$$W_r(r=r_r) = \frac{\mu I^2}{8\pi} \quad (3.14)$$

On the approach to the point $r=r_r$, the growth rate of instability decreases.

Assume that the goal of a particular experiment is to reach a certain value of the convergence C corresponding to some r_{min} . If one chooses $r_r = r_{min}$, one sees that the energy penalty is very modest: the rotation energy (3.14) is factor of $2\ln C$ less than the kinetic energy (3.2) of the non-rotating liner. If one wants the acceleration to change sign earlier, and thereby produce even stronger stabilizing effect, the energy penalty grows. For example, if one chooses $r_r = 2^{1/2} r_{min}$, the rotation energy at $r = r_{min}$ is only $\ln C$ less than the energy of non-rotating liner. At even faster rotation, the dynamics of implosion changes in such a way that the liner does not even reach the desired point $r = r_{min}$. We conclude that spinning the liner up provides an opportunity for stabilizing the Rayleigh-Taylor instability at the last stretch of the run-in, and the energy penalty (for large convergences, $C \sim 10$) is moderate, ~ 20 -40%.

E. 3D implosions

As has been already mentioned in Sec. I C, one may deliberately implode shells with a geometry different from cylindrical. Fig. I.1e depicts an implosion of an approximately spherical shell whose polar areas are sliding along the conical electrodes. This scheme was successfully realized in experiments by Degnan et al. (1995). The magnetic pressure at the surface of the shell is larger in the polar areas (because of the smaller distance from the axis). To compensate this effect (that would lead to deviations from the spherical implosion) the shell thickness was made larger in the polar parts. To avoid jetting at the point of the sliding contacts of the shell with the electrodes, the angle at their apex was made greater than 45° .

The other technique for producing quasi-spherical implosions is a proper tailoring of the thickness of the initially cylindrical liner, with the thickness decreasing from the equatorial plane to the ends (Fig. III.2). The liner implosion then occurs as shown in Figs. III.2 a-d, with the volume inside the liner experiencing (at the stages following Fig. III.2 c) a 3-D compression. Such scheme has been successfully tested in experiments with relatively massive aluminum liners by Alikhanov et al. (1977). 3D implosions are of interest in conjunction with some fusion schemes, which we will briefly discuss in Sec. X.

Fig. III.2. Producing a quasi-spherical implosion with an initially cylindrical liner with varying thickness. The liner is thicker near the equatorial plane.

F. Electrode phenomena

In Secs. III A-D we have been considering problems with a pure cylindrical symmetry, i.e., problems where all parameters depend only on r . A perfect cylindrical symmetry cannot be reproduced in a real life, even if the system is MHD-stable. Indeed, the pinch always has a finite length, and there is a contact between the pinch plasma and the electrodes. This obviously creates some axial non-uniformity and thereby violates pure cylindrical symmetry. The presence of the electrodes may affect the pinch performance in a number of ways.

First, there is some friction between the liner and the surface of the electrode. We will see that this effect may be significant for high temperature and low density pinch plasmas. The presence of the tangential shear flow in the transition region between the electrode and the liner may excite the Kelvin-Helmholtz instability (see Chandrasekhar, 1961) and shear-flow turbulence, which can enhance the liner-electrode friction. We will discuss these issues in more detail in Sec. IV.

Second, there is a heat flux to cold massive electrodes. Its significance is again determined by the density and temperature of the liner. This heat flux causes an axial variation of the plasma

temperature near the electrodes, thereby violating the cylindrical symmetry of the implosion. Under certain circumstances this variation may be considerable (see Sec. IV).

Third, there may occur some mass influx from the surface of the electrode that makes the end part of the pinch heavier and causes its lagging with respect to the equatorial part of the pinch.

Fourth, conducting electrodes impose a "frozen-in" condition on the normal component of the magnetic field. To illustrate the possible role of this effect, consider a liner implosion in the presence of a weak initial axial magnetic field B_{z0} (such geometries are supposed to be used in experiments on the generation of strong magnetic fields; see, e.g., Alikhanov et al., 1967). If (as it usually is) the bias magnetic field is small compared to the azimuthal magnetic field of the pinch, it does not considerably affect the pinch dynamics during the run-in phase. Consider, however, what happens to the magnetic field itself. The axial magnetic flux through every element of the electrode and liner is conserved (because of their high conductivity). This creates a situation shown in Fig. III.3: a thin near-electrode layer appears where the embedded magnetic field becomes almost radial in direction; the thickness of this layer is of the order of the skin-depth δ , which is very small compared to the pinch radius. From the conservation of the magnetic flux, the following estimate of the radial magnetic field inside the skin-layer follows:

$$B_r \sim B_{z0} \frac{r}{\delta} \quad (3.15)$$

Fig. III.3. Distortion of the axial magnetic field in the course of a liner implosion (δ is the skin-depth). Strong enhancement of the initial magnetic field occurs within a skin layer; as the perfectly conducting liner moves towards the axis, the magnetic flux initially enclosed by the liner has to be transferred through a thin skin layer. Thick lines depict a cylindrical liner, thin lines with arrows are magnetic field lines.

This estimate corresponds to the intermediate stage of the run-in, where the pinch radius is equal to, say, a half of the initial radius. For a typical implosion time ~ 30 ns, the skin-depth in the electrode material is of the order of 10^{-3} cm and, for $r \sim 1$ cm, one obtains $B_r \sim 10^3 B_{z0}$. Even if the bias magnetic field is small, say, 10^4 G, the magnetic field in the skin is very large and may become comparable with the self magnetic field of the pinch. This, in turn, will cause a thermal explosion of the electrode skin-layer (because of very high current density), and also cause a change in the dynamics of the liner implosion near the surface of the electrodes.

In some experimental settings, in order to provide a better diagnostic access, one of the electrodes is made with a hollow central part (Fig. III.4). This may lead to a different phenomenon, similar in some sense to the plasma focus effect (for the description of the latter see, e.g., Sec. 4 in Vikhrev and Braginski, 1987). When, in the course of the implosion, the liner slides past the edge of the hole, some current-carrying "bridge" should be formed from the materials of both the liner and the electrode (panel 2). The density of this bridge is presumably less than the density of the liner itself. Anomalous resistivity may turn on, causing considerable heating and radiation from this region. The bridge experiences a magnetic pressure directed downward and begins to evolve as shown in panels 3-4. The further evolution of the bridge should lead to its early self-implosion in some axial point, formation of the "neck" (panel 5) and, possibly, the break-up of the current channel, with generation of high-energy particle beams. The collapse should be accompanied by injection of the material in both directions from the collapse point. Bright features appearing near the anode hole relatively early in the pulse were observed by Derzon et al. (1997) and they persisted till late time (Fig. III.5). These bright features, i.e. an emissive jet out the anode, are believed to be due to the scenario depicted in Fig. III.4.

Fig. III.4. Possible scenario of the implosion of a wire array in the case of a hollow anode. The picture is made deliberately asymmetric to emphasize statistical character of the bridge formation.

Fig III.5. Scale size rendition of target compared to time-integrated and gated x-ray images of the foam target. Plasma jetting along the axis outside of the pinch and near single mode structure between the electrodes is observed. The diameter of the anode ring is 1 cm.

G. Structure of the imploding shell

The knowledge of the structure of an imploding shell is required for the stability analysis that will be made in the further sections of this report. Consider the implosion of a thin shell whose

thickness h is such that the time of propagation of the compression wave through the distance h is small compared to the characteristic time of the implosion process. This assumption is certainly valid for thin enough shells. Then the shell can be considered as being in a quasi-steady-state mechanical equilibrium, which is governed by the following equation:

$$\rho g = -\frac{\partial}{\partial x} \left(p + \frac{B^2}{2\mu} \right), \quad (3.16)$$

where g is the effective gravity acceleration (in the co-moving frame) and x is a coordinate directed towards the axis. We are using a planar model because of a small thickness of the shell. The geometry of the problem is illustrated by Fig. III. 6.

Fig. III.6. The slab geometry used in the stability analysis; $\mathbf{g} \equiv -\mathbf{e}_z g$ ($g > 0$).

There is no magnetic field at the inner surface of the shell (no current inside the shell) and, therefore, the acceleration is related to the magnetic field B_0 at the outer surface of the shell via equation:

$$\rho g h \sim \frac{B_0^2}{2\mu}, \quad (3.17)$$

where h is the shell thickness.

First assume that the temperature of the shell is determined by the Ohmic heating. A rough estimate of the thermal energy delivered to the unit area of the shell can be made by multiplying the Poynting vector by the characteristic time τ of the implosion process:

$$\frac{E_0 B_0 \tau}{\mu}. \quad (3.18)$$

The electric field on the surface of the shell depends on the relationship between the skin-depth h_{skin} and the total thickness h of the shell:

$$E_0 \sim \frac{B_0 h_{skin}}{\tau} \max\left(1, \frac{h_{skin}}{h}\right). \quad (3.19)$$

Assume first that heat losses via radiation are negligible. Then, the thermal energy per unit area of the shell is

$$\frac{B_0^2}{2\mu} h_{skin} \max\left(1, \frac{h_{skin}}{h}\right). \quad (3.20)$$

This thermal energy determines the plasma pressure inside the shell. As one can see from Eqs. (3.17) and (3.20), in this quasi-equilibrium state, the shell thickness is necessarily of the order of the skin-depth h_{skin} (Fig. III.7 a).

Consider now the other possibilities. If the shell experiences some turbulent motions produced by hydrodynamic instabilities, then a new source of heating becomes available and may deliver much more thermal energy to the shell than follows from Eq. (3.20). One more situation where thermal energy may be large is an implosion of sufficiently thick gas-puff or foam annulus, where the shock wave propagates before the piston and heats the plasma. When the thermal energy of the shell is considerably greater than the energy delivered by the Ohmic heating, an equilibrium with the shell thickness much greater than the skin-depth h_{skin} becomes possible (Fig. III.7 b). In the region beyond the skin-layer, the magnetic pressure in Eq. (3.16) is negligible. At a uniform temperature and a uniform plasma composition, one obtains then a familiar exponential density distribution,

$$\rho \propto \exp(-x/h), \quad (3.21)$$

with the scale-length h defined as

$$h = \frac{T}{\bar{m}g}, \quad (3.22)$$

and $\bar{m}$ being an average atomic weight (half a proton mass for the hydrogen plasma).

Fig. III.7. Density distribution in the shell: a) the case where the density distribution can be characterized by a single length-scale h ; b) thin skin-layer, $h_{skin} \ll h$; c) skin-layer much thicker than the shell thickness.

The opposite limiting case is that of very fast radiative losses, where the plasma thermal energy is much less than it would be according to Eq. (3.20). This case is of a particular interest for the wire arrays and we discuss it in some detail, following general ideas of the papers by Hussey, Roderick and Kloc (1980), and Hussey, Roderick (1981), Grigor'ev and Zakharov (1987), Chukbar (1993a), and Hammer et al. (1996).

The plasma thermal energy in this case becomes much smaller than (3.20), leading to a corresponding decrease of the plasma temperature. At low temperatures, the skin-depth becomes greater than the shell thickness, $h_{skin} > h$. The axial electric field is then uniform over the shell thickness. We assume that the temperature is also uniform, providing a uniform

conductivity and a uniform axial current. The latter, in turn, means that the magnetic field varies linearly over x . As it must vanish at the inner side of the shell (we assume that there is no axial current inside the imploding shell), we find that

$$B = B_0 \left(1 - \frac{x}{h}\right) \quad (3.23)$$

where h is the shell thickness. Substituting this expression into the equilibrium equation (3.16) and using relationship $p = s_{iso}^2 \rho$, with $s_{iso} = \sqrt{T/\bar{m}}$ being a speed of the isothermal sound waves, we find the density distribution in the form:

$$\rho = \rho_0 \left[\left(1 - \exp\left(-\frac{gx}{s_{iso}^2}\right)\right) - \frac{x}{h} \left(1 - \exp\left(-\frac{gh}{s_{iso}^2}\right)\right) \right] \quad (3.24)$$

We have taken into account that the density should become zero at $x=0$ and $x=h$. Note that h now is *not* determined by Eq. (3.22) because the magnetic force is now dominant in the pressure balance. The constant ρ_0 is to be determined from the known mass per unit area of the shell, $\Sigma = \hat{m}/2\pi r$, which, in turn, is related to the driving magnetic field via relationship

$$\Sigma \equiv \int_0^h \rho dx = \frac{B_0^2}{2\mu g} \quad (3.25)$$

As follows from Eq. (3.20), the assumption of the skin-depth being greater than the shell thickness implies that almost all the energy delivered to the shell by Ohmic heating is radiated to make plasma pressure less than the magnetic pressure, $\rho s_{iso}^2 \ll B_0^2/2\mu$. In this case, as follows from Eqs. (3.24) and (3.25), one automatically has $gh \gg s_{iso}^2$. This allows one to obtain the following simple expression for ρ_0 :

$$\rho_0 = 2\Sigma/h \quad (3.26)$$

The density distribution (3.24) is shown in Fig. III.7 c. The scale-length h , in turn, is set by the thermal equilibrium condition, Joule heating = radiative losses.

IV. BASIC PLASMA PROCESSES

A. Ionization

At typical electron densities $\sim 10^{20}$ - 10^{21} particles/cm³ and temperatures exceeding several electron-volts, the plasma can be considered as an ideal gas. The equation of state is determined by the average charge state Z_{eff} of the ions:

$$p = (1 + Z_{eff})n_i T, \quad (4.1)$$

where n_i is a number density of the ions. At densities of interest, deviations of the electron distribution function from the Maxwellian are negligibly small, even in the domain of electron energies $\sim (4-5)T$. In other words, electron collision frequencies for the "tail" electrons are still high enough to maintain a Maxwellian distribution, despite the energy losses. At $n_e \sim 10^{20}$ - 10^{21} particle/cm³, the phase volume available for the free electrons is much greater than that for the bound electrons. Because of this, the average charge of the ions will correspond to the ionization state where the ionization potential is of order of several electron temperatures,

$$I_{Z_{eff}} = KT, \quad (4.2)$$

where $I_{Z_{eff}}$ is the ionization potential of the ion in the charge state Z_{eff} and K is a numerical coefficient that very weakly (logarithmically) depends on the parameters of the system (see, e.g., Landau and Lifshitz, 1980). At electron densities $\sim 10^{20}$ cm⁻³ it is ~ 5 . For the materials common in the Z-pinch studies (C, Al, Ar, Ni, W) the first six ionization potentials (in eV) are (see Kikoin, 1976): 11.3, 24.4, 47.9, 64.5, 392, and 489.8 (for C); 6, 18.8, 28.4, 120, 153.8, and 190.4 (for Al); 15.8, 27.6, 40.9, 59.8, 75, and 91.3 (for Ar); 7.6, 18.1, 36.2, 56, 78, and 110 (for Ni); 8, 17.7, 24.1, 35.4, 48, and 61 (for W). At small Z_{eff} , ionization potentials can be roughly approximated as a linear function of Z_{eff} , $I_{Z_{eff}} \approx \text{const} \cdot I_1 Z_{eff}$, where $I_1 = 13.6$ eV. At higher temperature, when the ions are approaching a fully stripped state, the model with $I_{Z_{eff}} \approx I_1 Z_{eff}^2$ is more accurate. In the intermediate range of the charge states, one can use the Thomas-Fermi model (Landau and Lifshitz, 1977).

By summing up the ionization energies on the way to the final state Z_{eff} , one finds the total ionization energy I per ion. Within any power law approximation for the ionization potential, $I_{Z_{eff}} \approx \text{const} Z_{eff}^\alpha$, this total energy is roughly equal to $K Z_{eff} T / (1 + \alpha)$. Here as well as above, we imply that Z_{eff} is less than the charge Z of a fully stripped ion.

The energy required to bring the gas from a non-ionized cold state to the state where it has a temperature T and the effective charge state Z_{eff} , is

$$w_i = \frac{3}{2} n_i (1 + Z_{eff}) T + \frac{K Z_{eff} n_i T}{1 + \alpha} \quad (4.3)$$

This expression formally looks as though additional degrees of freedom were added to the electrons ($2K/3(1+\alpha)$ per electron). This, in turn, means that the compression rate in a strong shock will be higher than a factor of 4 (as it was for an ideal monatomic gas, see, Eq. (3.10)).

The ionization rate can be evaluated on the basis of the Thomson formula for the ionization cross-section (see, e.g., Smirnov, 1975; Raizer, 1991) which provides a reasonable accuracy. At the energy of the impinging electron not far above the ionization potential, one has roughly

$$\sigma_i \approx \frac{e^4 N_{out}}{16\pi\epsilon^2 I_{Zeff}^2} \frac{(mv^2/2) - I_{Zeff}}{I_{Zeff}}, \quad (4.4)$$

where ϵ is an electric permittivity of the vacuum, N_{out} is the number of electrons in the outer shell, and v is velocity of the impinging electron. The ionization reactivity $\langle \sigma_i v \rangle$ averaged over the Maxwellian distribution is

$$\langle \sigma_i v \rangle \approx \frac{\sqrt{2\pi} e^4 N_{out}}{16\pi\epsilon^2 I_{Zeff}^2} \sqrt{\frac{2T}{m}} \exp\left(-\frac{I_{Zeff}}{T}\right). \quad (4.5)$$

Again, this estimate may give an error ~ 2 in every particular case but it is useful in quick scoping studies.

In implosions of tungsten wire arrays with a mass $\mu \sim 1$ mg/cm at the Z facility (Spielman et al., 1997), the final radius r_{min} of the pinch was ~ 1 mm, and the electron temperature was ~ 200 eV. In this case, one can expect $Z_{eff} \sim 15$, and electron density $n_e \sim 5 \cdot 10^{20} \text{ cm}^{-3}$. Then, according to (4.2), and (4.5), the ionization time $1/n_e \langle \sigma_i v \rangle$ would be ~ 5 ns, in other words, somewhat longer than the duration of a strongly compressed state (which is ~ 3 -4 ns). The plasma in the final state may not reach the ionization equilibrium (see Fig. IV.1). This is even more true for the implosion of gas-puffs where typical temperatures are higher and the densities lower. The issues related to the absence of the local thermodynamic equilibrium have been discussed in DeGroot et al., 1997. The slowness of the ionization process makes the plasma equation of state "stiffer" and allows reaching higher final temperatures compared to the situation where the ionization equilibrium is reached.

Fig. IV.1. General characterization of various kinetic processes. The solid lines correspond to the ionization time equal to the number shown by the curve. Below these curves, ionization equilibrium cannot be reached. Dashed lines correspond to the solutions of Eqs. (4.8) and (4.10). The arrows indicate domain where the radiation time is shorter than the numbers by the curve. The dotted line corresponds to the electron-ion energy exchange time equal to 50 ns. Above the curve, the actual energy exchange time is shorter. All the curves correspond to tungsten, with the assumption that the plasma thickness is $h=1$ mm.

In the run-in phase of the wire array implosions, where the temperature is lower and the time scale is longer, the ionization is close to the equilibrium. In the gas-puff either of the situations is possible, depending on the specific conditions.

B. Radiative losses

In a partially ionized plasma, where the temperature is related to the ionization potential via Eq. (4.2), the plasma radiation is determined predominantly by free-bound transitions and by the line radiation. The free-free radiation becomes dominant at higher temperatures, where the plasma is (almost) fully ionized, $Z_{eff} \sim Z$. The evaluation of the line radiation would require detailed calculations of atomic processes and radiation transport, a problem that goes well beyond the scope of this survey.

We present here estimates for the free-bound radiation. For the optically thin plasma, the power radiated via free-bound transitions per unit volume can be evaluated as (e.g., Huba, 1994):

$$P_{rad}(W/cm^3) \approx 1.7 \cdot 10^{-32} Z_{eff} [n_e(cm^{-3})]^2 \sqrt{T(eV)} \left(\frac{I_{Z_{eff}-1}}{T} \right), \quad (4.6)$$

where $I_{Z_{eff}}$ is an ionization potential from a charge state $Z_{eff}-1$ to a charge state Z_{eff} . The free-bound radiation is dominant over the free-free radiation if the plasma temperature is lower than $I_{Z_{eff}}$. As the temperature is related to $I_{Z_{eff}}$ via Eq. (4.2) with $K \sim 5$, this is automatically true in our case. Using (4.2), one can rewrite (4.6) as:

$$P_{rad}(W/cm^3) \approx 1.7 \cdot 10^{-32} Z_{eff} K [n_e(cm^{-3})]^2 \sqrt{T(eV)}. \quad (4.7)$$

A convenient parameter characterizing the radiation losses is the radiation loss time τ_{rad} defined as the ratio of the plasma thermal energy, $(3/2)n_e T$, to radiation power:

$$\tau_{rad}(ns) \approx \frac{1.5 \cdot 10^{22} \sqrt{T(eV)}}{n_e(cm^{-3}) Z_{eff} K}. \quad (4.8)$$

At the temperature $T \sim 25$ eV and the density $n_e = 10^{20} cm^{-3}$ typical for implosions of metal wire arrays, this estimate yields a radiation time ~ 30 ns ($Z_{eff} \sim 5$).

The plasma remains optically thin until the line-integrated radiation power, hP_{rad} , remains less than the black-body radiation power through two plasma surfaces, $2\sigma T^4$, with σ being the Stefan-Boltzmann constant, $\sigma \approx 10^5 W/cm^2 eV^4$ (Zel'dovich and Raizer, 1967). The transition from optically thin to optically thick plasma can be described by interpolation equations presented in Mosher and Colombant, 1992.

The condition when the system becomes optically thick with respect to the free-bound radiation is (Huba, 1994):

$$h(cm) > 1.2 \cdot 10^{37} \frac{[T(eV)]^{7/2}}{Z_{eff} [n_e(cm^{-3})]^2}. \quad (4.9)$$

Inequality (4.9) can be relatively easily satisfied early in the run-in phase of wire array implosions, at the temperatures below ~ 5 eV. At higher temperatures, the plasma becomes transparent to free-bound radiation. It is also transparent during most of a gas-puff implosion.

If the plasma is optically thick, the heat to the radiating surface is delivered from the bulk of the plasma by radiative transport. In a typical Z-pinch setting, the optical depth remains moderate. This point is exemplified, in particular, by relationship (4.9): although the optical

depth may become greater than 1, it does not reach values of many thousands, at which the photon diffusion time exceeds the other characteristic times of the process. Accordingly, the time of the radiative transport over the plasma is short compared to the other processes; this allows one to use a bulk plasma temperature as the temperature of a radiating surface. We introduce the radiation cooling time with respect to the black-body losses in the same way as we have introduced (4.8), i.e., by dividing the line density of plasma energy, $(3/2)n_eTh$, by $2\sigma T^4$ (the factor "2" takes into account radiation from the inner and the outer sides of the imploding shell; for the typical height-to-radius ratio of the imploding shell, the radiation from the inner side gets absorbed predominantly by the electrodes, not by the shell). This yields:

$$\tau_{rad}(s) = 1.2 \cdot 10^{-24} \frac{n_e(cm^{-3})h(cm)}{[T(eV)]^3} . \quad (4.10)$$

At $h=0.1$ cm and $n_e=10^{20}$ cm⁻³ the thus defined radiation time is less than a typical duration of the run-in phase (~ 50 ns) even at a modest temperature of 6-7 eV (Fig. IV.1). Estimates (4.8) and (4.10) cover the extreme ends of possible radiative loss scenarios and both show that the radiative cooling is a significant factor. Thermal equilibrium of the imploding shell is provided by balancing the radiative losses against some heating mechanism, for instance, the Ohmic heating (in the case of wire array implosions) or shock heating (in the case of gas puffs and dense foam loads). In the latter case, the presence of fast radiative losses from shocked material leads to its considerable additional compression; the shocked material virtually "sticks" to the piston. This may serve as another justification for using a snow-plow model.

In case of the wire arrays, the typical temperatures of the sheath plasma in the middle of the run-in phase, determined from the balance between the Ohmic heating and radiative losses is 25-30 eV (Peterson et al., 1997).

One should emphasize that, although radiation losses are important in establishing the thermal balance of the imploding shell during the run-in phase, they are insignificant in the overall energy balance of the system: the Ohmic losses in the shell are small compared to the kinetic energy imparted to it. Conversely, in the case of uniform loads (like uniform gas-puffs), where the material gets heated by the shock, radiative losses are significant even in the overall energy balance (see Secs. II E, III A, and III B).

C. Electron-ion temperature equilibration

Finiteness of the time for establishing electron-ion equilibrium may affect the processes in the situations where the density is low or the temperature is high, or both. The energy transferred to the electrons from the ions, per electron per unit time, is (Braginski, 1965):

$$\frac{3}{2} \frac{T_i - T_e}{\tau_{ei}^{(E)}} \quad (4.11)$$

where $\tau_{ei}^{(E)}$ is given by Eq. (2.13). Shown in the dash-dotted line in Fig. IV.1 is the solution of the equation $\tau_{ei}^{(E)} = 50$ ns (a typical duration of the run-in phase on Saturn facility). Above this line electron and ion temperatures are equal. One sees that, typically, during the run-in phase, this condition holds.

Let us consider in more detail the energy equilibration near the stagnation point of the wire-array implosion. Initially, essentially all the energy is carried by the mechanical motion of the ions. As soon as the implosion occurs, the ions become Maxwellian essentially instantaneously, because of a very large Coulomb cross-section (which scales as Z_{eff}^4). The energy exchange then causes the transfer of the ion energy to the electrons which, in turn, exchange it on the ionization and radiation.

If the radiation power is high, it keeps electron temperature at a relatively low level and increases the rate at which the energy is transferred from the ions to electrons. In this case, energy stored in the electrons at any particular moment is small, and one can neglect T_e in the expression (4.11). The duration of the high-density state is of the order of the h/u , where u is the implosion velocity related to the implosion energy per ion (W) by $m_i u^2/2 = W$. The radiated power (per unit surface area), in the case of a strong ion-electron coupling is, accordingly, $n_i W u$. With black-body radiation, one can write the following equation that determines the electron temperature:

$$2\sigma T_e^4 \sim n_i u W. \quad (4.12)$$

The coupling between ions and electrons is strong if the power flux from the electrons to ions, which, according to Eq. (4.11), is $\sim n_e W / \tau_{ei}^{(E)}$ per unit volume exceeds the ion kinetic energy released per unit volume per unit time, $n_i u W / h$ or, in other words, if the inequality

$$\tau_{ei}^{(E)} < Z_{eff} h / u \quad (4.13)$$

holds.

Solutions of these equations are presented in Fig. IV.2. In the case shown (tungsten, implosion energy $W=400$ keV, mass per unit length $\hat{m}=1.5$ mg/cm), these conditions are compatible with each other. Had the inequality (4.13) been violated, the electron temperature would remain at the level below the one determined from Eq. (4.12), and efficiency of conversion of the liner kinetic energy into radiation would decrease. What we presented here is a rough qualitative characterization of possible chain of causes and effects in the problem of the radiation efficiency. Detailed quantitative numerical analyses are available (see, e.g., Peterson et al., 1997).

D. Transport coefficients

Of importance for the problem under consideration are three of the transport coefficients: electrical resistivity, plasma viscosity, and electron thermal conductivity. For a plasma made of a mixture of several ion species, their mutual diffusion coefficients may also be needed.

1. Electrical resistivity

The electrical resistivity can be expressed as (see Braginski, 1965):

$$\eta(\text{Ohm} \cdot \text{m}) = \frac{5 \cdot 10^{-4} Z_{\text{eff}}}{[T(\text{eV})]^{3/2}} \quad (4.14)$$

Fig. IV.2. Upper curve: electron temperature as determined by the balance of the black-body radiation losses and energy exchange term (Eq. (4.13)). Lower curve: electron temperature determined by the balance of a black-body radiation and influx of a mechanical energy (Eq. (4.12)). Assumptions: tungsten, $W=400$ keV, $\hat{m}=1.5$ mg/cm.

Related to η is magnetic diffusivity $D_m = \eta / \mu$, which determines the current penetration into the plasma (see Sec. III G):

$$D_M(\text{cm}^2/\text{s}) = \frac{3.9 \cdot 10^6 Z_{\text{eff}}}{[T(\text{eV})]^{3/2}}. \quad (4.15)$$

Taking as an example $T=25$ eV, $Z_{\text{eff}}=5$, one finds that $D_M \sim 10^5$ cm²/s. For the duration of the run-in phase ~ 50 ns, the skin-depth $h_{\text{skin}} = (2\tau D_M)^{1/2}$ is ~ 1 mm.

Gordeev (1987) contends that in a low-density gas-puff implosion of a multi-species plasma, the mutual friction between the ion components may cause an enhanced penetration of the magnetic field.

2. Plasma viscosity

Plasma kinematic viscosity is (Braginski, 1965):

$$\nu(cm^2/s) = \frac{3.4 \cdot 10^{-3} [T(eV)]^{5/2} A^{1/2}}{Z_{eff}^4 \rho(mg/cm^3)}. \quad (4.16)$$

Consider possible role of a viscous friction of the imploding shell against the resting electrodes. The thickness of the near-wall layer affected by viscous drag is (Landau and Lifshitz, 1987) $\sim(\tau\nu)^{1/2}$, where τ is the characteristic time of the process. Taking $\tau \sim 50$ ns, $T \sim 25$ eV, $A=180$, $Z_{eff}=5$, $\rho \sim 1$ mg/cm³, one finds this thickness to be very small, $\sim 10^{-4}$ cm. In other words, the viscous effects can hardly play any role during the run-in phase of the wire array implosions. They may become more important near the stagnation point, where the plasma temperature is higher, or in the implosions of lower density gas puffs.

For the substances containing a considerable fraction of hydrogen, e.g., the agar, kinematic viscosity may be larger than (4.16), because the hydrogen free-path length is Z_{eff}^2 times greater than that of the high- Z ions (Ryutov, 1996). To take into account this effect, one should add to (4.16) the "hydrogen" term,

$$\nu_H(cm^2/s) = \alpha_H \frac{3.4 \cdot 10^{-3} [T(eV)]^{5/2}}{Z_{eff}^2 \rho(mg/cm^3)}, \quad (4.17)$$

where α_H is the number of hydrogen atoms per a heavier atom (e.g., in CHO, $\alpha_H=1/2$); when evaluating the contribution (4.16) of heavier atoms to the viscosity of the hydrogen-containing substance, one should identify A with the average atomic weight of heavy atoms only (e.g., in case of CHO, $A=(12+16)/2=14$). Strictly speaking, expression (4.17) should contain a numerical coefficient of order of 1 that depends on Z_{eff} . But for the first general survey, Eq. (4.17) provides a sufficient accuracy.

In a pure heavy-ion plasma with large enough Z_{eff} , it may happen that the main contribution to viscosity will come from electrons (Haines, private communication, 1997). The reason for this is the same as in the case of the hydrogen contribution: the free path length of electrons is Z_{eff}^2 times larger than that of the heavy ions; in addition, electrons are Z_{eff} times more numerous. Of course, electron contribution gets strongly suppressed if electrons are magnetized (see Eq. (2.14)).

Generally, as we have already mentioned in conjunction with Eq. (4.16), the laminar friction between the plasma and resting electrodes is small. In the transition layer a shear-flow turbulence may develop and increase the momentum transfer. This will be a turbulence of supersonic flow (the plasma velocity far from the wall is much higher than the sound speed in the plasma shell), with strong radiative losses. Very little is known about the momentum transfer under such circumstances. First observation of the Kelvin-Helmholtz instability in the ICF-relevant environment has been reported by Hammel et al. (1994).

3. Mutual diffusion of different ion species

In situations where the plasma contains the ions of more than one species, for example, a plasma of CH and CH₂ plastics, or a plasma of agar foam, one has to consider the process of the mutual diffusion of the ion components. To be specific, we present estimates for the diffusion of hydrogen (or its isotopes) in a plasma of a heavy element. One has:

$$D_H(\text{cm}^2/\text{s}) \approx 5 \cdot 10^{18} \frac{[T(\text{eV})]^{5/2}}{Z_{\text{eff}}^2 n_Z(\text{cm}^{-3})} \quad (4.18)$$

Because of the diffusion of the hydrogen component, its relative concentration in a particular point may change with time, thereby affecting plasma properties (e.g., plasma viscosity). The presence of the high effective gravity force may cause a gravitational separation of the elements on the diffusion time scale. As a numerical example, consider the following set of parameters (representative for the wire array implosions): $T=25$ eV, $n_Z=2 \cdot 10^{18} \text{ cm}^{-3}$, $Z_{\text{eff}}=5$. One has then $D_H=360 \text{ cm}^2/\text{s}$. For a typical duration of the run-in phase, $\tau \sim 50$ ns, this corresponds to the diffusion length $h_{\text{diff}} = (2D_H\tau)^{1/2} \sim 0.06$ mm, which is too small to cause any significant redistribution of hydrogen. On the other hand, in the lower density, higher temperature gas puff implosions, redistribution of the light ions may become important. Bailey et al. (1982) observed separation of deuterium and argon.

4. Electron thermal conductivity

Electron thermal diffusivity, according to Braginski, 1965, can be evaluated as:

$$\chi_e(\text{cm}^2/\text{s}) = 0.27 \frac{[T_e(\text{eV})]^{5/2} A}{Z_{\text{eff}}^2 \rho(\text{mg}/\text{cm}^3)} \quad (4.19)$$

The numerical coefficient corresponds roughly to the range of Z_{eff} from 3 to 6. The characteristic time for electron heat transport over some distance x is x^2/χ_e . Taking $x=1mm$, $T=25\text{ eV}$, $\rho=1\text{ mg/cm}^3$, $A=180$, one finds that this time is $\sim 10^{-5}\text{ s}$, much longer than the pulse-width. Electron heat conduction may become even less significant if electrons get magnetized.

If one deals with a hotter and lower density plasma typical for the gas puffs, one may enter the domain where electron thermal losses to the end walls (electrodes) become important. However, in a lower-density and hotter plasma, electrons are magnetized much earlier (see (2.14)) and this channel of heat losses from the current-carrying sheath becomes again unimportant. On the other hand, the inner part of the gas-puff that is heated by the shock wave, and where the magnetic field is absent, may experience fast heat losses to the ends, especially in the case of lighter gases. Eq. (4.19) provides the reader with the tool for estimating this loss channel. In quasistatic pinches, an important channel of the heat loss is an enthalpy flow to the electrodes related to the pinch current (Haines, 1960). In fast pinches this channel is usually sub-dominant.

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V. AN EARLY STAGE OF THE DISCHARGE

A. Breakdown of gas-puffs

Although at present the fast Z-pinch research is concentrated on the wire array implosions, the other fast Z pinches, in particular, gas-puffs (see, e.g., Stallings et al., 1979; Branitski et al., 1991, 1992 a,b; Deeney et al., 1993; Baksht et al., 1995) and pinches with foam targets (Derzon et al., 1997) are also of a considerable interest. Therefore, we start this section with the issues of initializing these types of pinches. What we present here is not a quantitative theory: it is rather a broad qualitative discussion aimed at the identifying critical physics issues.

In our discussion of gas-puffs, we will present most of the numerical estimates for the density range $3 \cdot 10^{17}$ - $3 \cdot 10^{18}$ cm⁻³. We mean here peak densities, in the middle of the gas stream. Taking as a representative value for the cross-section of electron scattering on atoms

$$\sigma_a \sim 10^{-15} \text{ cm}^2, \quad (5.1)$$

one finds that the mean-free path for electron scattering,

$$\lambda_{ea} = 1/n_a \sigma_a, \quad (5.2)$$

where n_a is a neutral atom density, is much shorter than the typical height of the pinch. If the applied voltage is such that the energy acquired by the electron between two collisions is small compared to the ionization potential, the electron avalanches would develop quite slowly. The seed electron would experience a random walk with a superimposed average drift in the direction of the anode; its energy would gradually increase and reach the excitation threshold; at this point, with a high probability, it would lose energy through excitation and only with a small probability would reach the ionization threshold I_{ion} (this, incidentally, is a standard picture of the gas breakdown at densities above the Paschen pressure minimum, see Meek and Craggs, 1978). In the situation of fast Z pinches, where the voltage is rapidly growing, before the aforementioned process produces sufficient electron multiplication, the voltage reaches the level where electrons acquire the ionization energy already between two successive collisions:

$$eE\lambda_{ea} > I_{ion}, \quad (5.3)$$

where E is the electric field strength. Then, a typical e-folding time for avalanching will be only $1/v_e n_a \sigma_i$, where σ_i is the ionization cross-section, see Sec. IV A.

Before going further, we note that in gas-puff experiments the radial density distribution is relatively smooth, with a gradual transition from the nominal density inside the jet to much lower density at the jet periphery (Fig. I.1a). Therefore, with voltage growing, the condition (5.3) will be first satisfied at low densities. But the density cannot be too low: in order to produce a considerable charge multiplication on its way to the anode, the electron would have to experience at least several ionizing collisions, i.e., the product $Ln_a\sigma_i$ (where L is an anode-cathode distance) should be greater than, say, 10:

$$Ln_a\sigma_i > 10. \quad (5.4)$$

Eq. (5.4) imposes a lower limit on the density. At $\sigma_i \sim 3 \cdot 10^{-16} \text{ cm}^2$ and $L \sim 1.5 \text{ cm}$ the required densities are $\sim 2 \cdot 10^{16} \text{ cm}^{-3}$, and this is where breakdown will occur first. The time for developing a considerable ionization is $\sim 10/v_e n_a \sigma_i$ ($\sim 5 \text{ ns}$ at $n_a = 2 \cdot 10^{16} \text{ cm}^{-3}$).

With applied voltage rapidly growing, the inequality in Eq. (5.3) will be met in the deeper layers of the jet and the ionization front will move towards the higher densities. Eventually, if the conductivity of the outer current-carrying shell becomes so high that the resistive component of the voltage ceases to grow, the further propagation of the breakdown front towards the higher densities stops. In other words, the skin-effect becomes important and, since this time, the further increase of the current occurs in the outer layers of the gas-puff (this skin-dominated stage of ionization has been analyzed by Vikhrev and Braginski, 1987). After that, the further ionization of the inner layers occurs under the action of radiation from the current-carrying shell and, at the later stages of the implosion, by the shock heating.

In gas-puffs, the isodensity surfaces are usually not cylindrical but rather conical, because of the divergence of the jet (see Hussey, Matzen and Roderick, 1986). Deeney et al. (1994) used special nozzles producing almost cylindrical jets. Superimposed on the regular flow, smaller-scale density fluctuations produced by the hydrodynamic turbulence may be present. This brings additional complications to the picture of the breakdown. The aforementioned arguments are helpful in the analysis of this situation, too. We will leave, however, this part of the problem to the interested reader.

In very low density pinches, where even the maximum density of the jet is less than roughly 10^{16} cm^{-3} , the electron multiplication factor becomes insufficient (see Eq. (5.4)) and a different mechanism of breakdown should come into action. It should strongly depend on the generation of electrons at the cathode (Baksht, Russkikh, Chagin, 1997), via, probably, photoemission.

Note that, at low gas densities, even a very weak current may cause the magnetization of electrons, thereby affecting the avalanching process. At the density 10^{16} cm^{-3} the electron-neutral elastic collision frequency of, say, 30-eV electrons is $3 \cdot 10^9 \text{ s}^{-1}$, and becomes lower than electron gyrofrequency at the magnetic field of only 0.015 T. In a 4-cm diameter column such a magnetic field would be created by the current of only $\sim 1.5 \text{ kA}$ (!). Therefore, even a relatively weak axial magnetic field (weak compared to the pinch azimuthal field at the maximum current) may affect the breakdown process and, thereby, the overall pinch performance. The favorable effect of an axial magnetic field $\sim 0.3 \text{ T}$ has been recorded in experiments by Baksht, Russkikh, and Chagin (1997), and Gasque et al. (1996). Of course, we do not claim that the bias magnetic field does not have other effects on the pinch physics (in particular, on the pinch stability at the later stages of the implosion, where it becomes considerably higher because of the radial compression). We just emphasize that even a very weak field can influence breakdown of the gas-puffs and make it more "regular".

The gas breakdown itself is a statistical process and may lead to a formation of azimuthally-asymmetric current-carrying channels, especially at lower densities where electrode effects become important (and bring about a new source of non-uniformities). Therefore, the pre-ionization of the gas by some external source might be beneficial. This is shown in the papers by Stallings et al. (1979), Baksht, Russkikh and Fedyunin (1995), and Baksht, Russkikh, Chagin (1997).

The presence of a long enough pre-pulse may also be beneficial for creating a uniformly ionized column. Effect of the pre-pulse is determined by its time duration and its voltage. In particular, in Baksht, Russkikh and Fedyunin (1995) the prepulse (1.5 μs , 1 kV) did not cause the breakdown because the axial line density $n_0 L < 3.6 \cdot 10^{15} \text{ cm}^{-2}$ was well below the Paschen optimum for Ar. Such a prepulse would have caused a breakdown of Ar with a density an order of magnitude higher. Whether this would be beneficial for the further fast implosion is not quite clear, because during the long pre-pulse numerous ionization-radiation instabilities (see Sec. V C) could develop and lead to strong perturbation of the initial state. The pre-pulse breakdown was reported by Smith and Dogget (1985) who also studied the current distribution of argon gas-puffs with a density below 10^{16} cm^{-3} during the first 20 ns of the discharge. They correctly identified an important role of the early electron magnetization, although their use of the magnetized resistivity to explain a large skin depth does not look right.

B. Breakdown of the foam

As the commonly used foams of CH_x , SiO_2 , and agar ($\approx \text{CH}_2\text{O}$) are insulators, the question of the time and quality of the breakdown exists for these loads, too. In particular, it would be important to know whether breakdown occurs at the outer surface, or some discharge channels are formed in the bulk of the foam. Very little is known at the moment on these issues and we have to limit ourselves to merely a verbal discussion.

To be more specific, we discuss breakdown of SiO_2 foam. Breakdown voltage for the SiO_2 foam could be quite large. This can be understood from the following qualitative considerations. If we substitute the foam with a gas with the same average particle density, i.e., with $6 \cdot 10^{20} \rho_0$ (mg/cm^3)/ A particles per cm^3 , the gas density would be quite high. For example, for $\rho_0 = 5 \text{ mg}/\text{cm}^3$ and $A = 20$ the particle density would be $1.5 \cdot 10^{20} \text{ cm}^{-3}$. At room temperature, this density would correspond to a pressure of approximately 5.5 atm; the Paschen product (pressure times length) will be then, roughly speaking, 1000 times higher than its optimum value for the majority of gases. This would correspond to breakdown voltages in the range of a hundred kilovolts. High voltages needed for initiation of discharge in the foam loads and the corresponding delay for the onset of the current flow may cause strong leaks and even a closure of the gap in the magnetically insulating transmission line (see Fig. I.3).

As soon as the voltage reaches $\sim 100 \text{ kV}$, breakdown occurs. At high densities, it has a tendency to develop in the form of a narrow channel which, generally speaking, is not straight (Lozanski, Firsov, 1975). The energy required to ionize a breakdown channel to a singly-charged state is very small. An estimate from below for this quantity can be presented as:

$$W_{ion} > \pi L a_0^2 I_{ion} \rho / A m_p, \quad (5.5)$$

where L is the column length, a_0 is its radius, I_{ion} is the ionization energy, ρ_0 is the foam density, m_p is proton mass, and A is an average atomic weight. In "practical" units,

$$W_{ion}(J) \approx 3 \cdot 10^3 L(\text{cm}) [r_0(\text{cm})]^2 \rho_0(\text{mg}/\text{cm}^3) / A. \quad (5.6)$$

Taking $L = 1 \text{ cm}$, $a_0 = 0.05 \text{ cm}$, $\rho_0 = 5 \text{ mg}/\text{cm}^3$ and $A = 20$, one obtains $W_{ion} \approx 0.2 \text{ J}$. After the first breakdown channel is formed, new breakdowns may still occur, because the inductive voltage induced in the bulk of the dielectric can be quite high until 3-5 channels are formed. So, one can expect that, after this first phase of the shot, the column will carry several discharge channels. Though the energy released in the breakdown is very small compared to the total energy delivered to the pinch during the whole implosion process, the consequences of the formation of non-

axisymmetric breakdown channels can be quite severe because the channel will have a density different from that of external medium and will serve as a strong perturbation during the MHD phase of the implosion. The role of appearance of thin breakdown channels in the performance of fiber pinches, with the dielectric (frozen deuterium) fibers was discussed by Meierovich and Sukhorukov (1992).

In the state of a single ionization, the plasma should have temperature $\sim 2-3$ eV. This would correspond to relatively low magnetic diffusivity (see Eq.(4.15)), $D_M \sim 5 \cdot 10^5$ cm²/s. With this diffusivity, the resistive broadening of the current channel should be slow (for $t=10$ ns the broadening would be ~ 1 mm). In other words, as soon as several highly conducting channels are formed, the initial current will be trapped in them. With the continuing ionization of the column, the further current buildup will occur in thin skin-layer at the surface of the column. But, as we have already emphasized, the initial current will remain trapped within several narrow channels inside the column. The presence of this current will cause some distortions of the equilibrium; the trapped magnetic field will grow proportionally to the convergence. Formation of several channels was observed in the experiments with gaseous liners on ANGARA-5-1 facility (Volkov et al., 1993). The reason for their formation and persistence during the whole implosion event could be just the one suggested above.

An ideal situation would be, of course, the one where breakdown is produced uniformly over the surface of the cylinder and intercepts so much current that the voltage drop inside the cylinder becomes insufficient to produce any internal breakdowns. In this sense, an interesting option is the use of a thin conducting coating. The coating should not necessarily be thicker than the skin-depth. What is sufficient (and relatively easily achievable even for the metal coatings with a thickness of order a fraction of a micron), is that the L/R time of the circuit be considerably greater than 10-20 ns. However, experimentally, such coatings may become oxidized and lose their conductivity (G. Allhouse, 1996, private communication). Still, there is experimental evidence that conductive coatings and prepulse had a favorable effect on the quality of the discharge (Nash et al., 1997a and Derzon et al., 1997)

One more solution could be the use of the discharge configuration shown in Fig. V.1 where the inner part of the liner assembly would be protected from the early stray discharges by relatively thick insulating disks. At the later stages of the discharge, when the pinching begins, the surface of the dielectric would become conducting and serve as an electrode. The discharge over the outer surface could be facilitated by using a circular "rib" on the electrode (Fig. V.1) that

would cause field enhancement near the edge of this "rib". Of some concern could then be possible effect of the rib on the implosion velocity near the electrodes.

Fig. V.1. Protection of the inner part of the foam cylinder from stray discharges.

The presence of a pre-pulse can have a considerable effect on the breakdown of the coated foam. Even if the voltage during the prepulse is in the range of only a few kilovolts, it is sufficient, in the time-frame $\sim 1 \mu\text{s}$, to fully evaporate conducting coating. Depending on the voltage and the pre-pulse length, the evaporated material can expand by a few millimeters. The conductivity of this relatively cold vapor will be low and, probably, insufficient to inductively shield the liner. On the other hand, the presence of a gaseous corona around the foam load may turn on the same breakdown mechanism as in the case of gas-puffs (Sec. V.A). A small axial magnetic field ($\sim 0.3 \text{ T}$) may be beneficial in producing a more symmetric current-carrying shell.

The aforementioned scenario of evaporation of the coating without its ionization corresponds just to one possible shape of the prepulse, with a long "pedestal" of a low voltage. If the prepulse is shorter and with a higher voltage, then a fast transition from vapor to a highly ionized plasma may occur.

C. Thermal instabilities; filamentation and striations

It has been known since the mid-1980s that, early in the pulse, surface layers of the gas-puff pinch experience fast instabilities that cause formation of bright stripes perpendicular to the axis and (usually later) parallel to the axis. We call the first of them “striations” and the second of them “filaments”. Such patterns, for instance, have been clearly observed in the study by Branitski et al. (1991) at the Angara-5-1 facility (Fig. V.2). In this particular experiment the maximum current was 3 MA, voltage 0.4-0.6 MV, and current rise-time ~ 100 ns. The loads were usually Xe gas puffs with the height 1 cm and initial diameter 3 cm and mass per unit length ~ 0.1 mg/cm. Ripples with the wavelength ~ 1 mm were formed immediately after arrival of the current pulse and were gradually replaced by filamentary structure with the same wave length; these filaments persisted till the middle of the pulse.

Fig. V.2. Laser shadowgraphs of the Xe liner implosion at the Angara-5 facility (Branitski et al., 1991). The frames are separated by 30 ns; the current was 1.6 MA. The axis of the discharge is vertical. The cathode is at the bottom. The anode (a thick dark strip in the middle of the figure) was made of a mesh, so that the plasma penetrates beyond the anode and produces some perturbations there.

These modes develop very rapidly compared to the Rayleigh-Taylor instability and should have a different nature. They are usually identified with the thermal instabilities where, once the temperature in some fluid element increases (decreases), it continues to increase (decrease).

Possible causes of such behavior are an increase of radiation losses at decreasing temperature, and positive feedback in the Joule heating. In its "pure" form this instability does not require mass redistribution and may occur at the times short compared to the acoustic time for the spatial scale of the instability. In this formulation, the thermal instability was considered by Afonin (1995).

If parameters of the system are such that the growth rate of the thermal instability becomes comparable with the growth rate of Rayleigh-Taylor instability, they get coupled. Such a situation was, in particular, discussed by Imshennik and Neudachin (1987, 1988). In the absence of the gravity force (and, whence, the R-T instability) the slow thermal instability gets coupled with acoustic motions. This instability can be called "radiative-condensation instability" because of the formation of clumps of colder matter at its nonlinear stage (Aranson et al., 1993).

We discuss here these instabilities for the case where the thickness of the cylindrical conducting shell is much less than the skin-depth, and where the mechanical motions of the shell can be neglected ("fast" thermal instability). We start from the electrodynamical part of the problem by solving which one can relate the current perturbation to the perturbation of the resistivity. For a thin shell, its properties can be characterized by the "surface conductivity" σ_s , which is the product of the shell thickness and the conductivity proper, or by a "surface resistivity" $\eta_s \equiv 1/\sigma_s$. The surface current can be presented as

$$J_t = E_t / \eta_s, \quad (5.7)$$

where E_t is the tangential component of the electric field. For perturbations, one has:

$$\delta J_t = \delta E_t / \eta_s - (\delta \eta_s / \eta_s^2) E_p, \quad (5.8)$$

where the sign " δ " designates the perturbations, and the unperturbed quantities do not bear this sign. As we are going to consider perturbations with the scale length shorter than the shell radius, we replace the cylindrical geometry by the planar one, with the axis x corresponding to the radial coordinate, and the axis y corresponding to the azimuthal coordinate. In the unperturbed state, the current and the electric field have only the z components, while the magnetic field has only the y component. The unperturbed magnetic field is zero inside the shell (in our geometry, at $x < 0$).

As there are no currents outside the shell, the magnetic field there is curl-free and can be presented as a gradient of some scalar function ψ , $\delta \mathbf{B} = -\nabla \psi$; this function satisfies the Laplace equation, $\nabla^2 \psi = 0$. We consider the perturbations in the form $\exp(\Gamma t + ik_y y + ik_z z)$, where $\text{Re} \Gamma$ is a

growth rate. In addition to Laplace equation for ψ , we will need the x component of the Faraday's law,

$$ik_y \delta E_z - ik_z \delta E_y = -\Gamma \delta B_x. \quad (5.9)$$

The solution of the Laplace equation for the $x>0$ ($x<0$) half-space reads as:

$$\psi_{\pm} = A_{\pm} \exp(\mp kx), \quad (5.10)$$

where $k = \sqrt{k_y^2 + k_z^2}$. We need to supplement these equations with the boundary conditions at the $x=0$ surface: the continuity of the normal component of the magnetic field (this yields $A_+ = -A_-$ in (5.10)), and the jump condition for the tangential components of the magnetic field in terms of the surface current (5.8). Using these conditions, after some elementary algebra one finds the following expression for the perturbation of the current:

$$\delta J_z = -J_z \frac{k_y^2}{k^2} \frac{1}{1 + (\Gamma/\Gamma_0)} \frac{\delta \eta_s}{\eta_s}, \quad (5.11)$$

where $1/\Gamma_0$ is a characteristic decay time for the current perturbations,

$$\Gamma_0 = \frac{2k\eta_s}{\mu} \equiv \frac{2k\eta}{\mu h}. \quad (5.12)$$

The y component of the current perturbation is: $\delta J_y = -(k_z/k_y) \delta J_z$. We remind that k_y is related to the azimuthal mode number m : $k_y = m/r$. It is also convenient to introduce the angle α between the unperturbed magnetic field and the wave vector: $\sin \alpha = k_z/k$.

Note that, according to (5.11), for highly conductive shells, where the resistive decay time is very long compared to the time of the process, the relative current perturbation, $\delta J_z / J_z$, can be small even for considerable perturbation of the resistivity, $(\delta \eta_s / \eta_s) \sim 1$.

Now we switch to the equation of the thermal balance. It can be presented in the form:

$$\dot{Q}(T) = \eta_s(T) J^2 - q(T), \quad (5.13)$$

where Q is a plasma energy content per unit area of the shell, and q is the power loss (radiation) per unit area. The equilibrium state corresponds to the balance of the two terms in the r.h.s.:

$$\eta_s(T_0) J^2 - q(T_0) = 0. \quad (5.14)$$

Equation for the temperature perturbation, written with the account for the equilibrium condition (5.14), is:

$$C_v \frac{\partial \delta T}{\partial t} = 2q \frac{\delta J_z}{J_z} + q \frac{\eta'_s}{\eta_s} \delta T - q' \delta T, \quad (5.15)$$

where the prime designates the derivative of the corresponding quantity over the temperature, and C_v is the heat capacity per the unit area of the shell. Using Eq. (5.11), one then obtains the following dispersion relation:

$$\Gamma = \frac{q \eta'_s}{C_v \eta_s} \left[1 - \frac{2 \cos^2 \alpha}{1 + (\Gamma / \Gamma_0)} \right] - \frac{q'}{q C_v}, \quad (5.16)$$

with $\sin \alpha \equiv k_z / k$.

Instead of exactly solving this (quadratic) equation, we present a qualitative discussion of possible instabilities described by it in some limiting cases. One extreme case is that of a strong temperature dependence of the heat losses and a weak temperature dependence of the resistivity. In this case, one can neglect the first term in the r.h.s. of Eq. (5.16). The remaining term predicts the instability if $q' < 0$, or in other words, if the radiative losses decrease with the temperature. This may happen in the optically thin plasma dominated by free-bound radiation or by line radiation, where some strong transitions disappear with the growing temperature because of the change of the ionization state.

In the opposite limiting case where the temperature dependence of the resistivity is dominant, one can neglect the last term in the r.h.s. Instabilities present in this case are driven by the temperature dependence of the resistivity. Somewhat paradoxically, these instabilities are present at either sign of η'_s ; what depends on the sign is their spatial structure. At $\eta'_s > 0$, to make the r.h.s. as large as possible (at Γ positive), one has to choose $\alpha = \pi/2$ ($m=0$). In other words, the fastest growing modes at a positive temperature dependence of the resistivity are axisymmetric modes ("striations"). At $\eta'_s < 0$, the most unstable modes correspond to $\alpha=0$, in other words, to the purely azimuthal perturbations. For them one has (with $q' = 0$):

$$\Gamma = \frac{q |\eta'_s| |1 - (\Gamma / \Gamma_0)|}{C_v \eta_s 1 + (\Gamma / \Gamma_0)}. \quad (5.17)$$

The largest growth rate corresponds to $\Gamma_0 \rightarrow \infty$, in other words, according to (5.12), to large m number (thin filaments stretched along the axis). Our simple model does not include the thermal

conductivity along the surface of a layer (see Sec. IV). If included, it will limit from below the size of both striations and filaments.

The positive dependence of resistivity vs. temperature is typical for low temperatures, where the ionization degree grows and electron-neutral collisions get replaced by the Coulomb collisions with much higher cross-section (Sec. II B). Accordingly, striations should form predominantly at the early stage of the pulse. This instability, as well as the other thermal instabilities, can reach a strongly nonlinear stage. They will eventually cause redistribution of matter (the process that we have not included into the analysis presented above). Such nonlinear structures may exist much longer than η'_s is positive and seed the Rayleigh-Taylor instability.

The negative dependence of resistivity vs. temperature takes over later, when plasma gets singly ionized. Therefore, filamentation should develop later than striations - in agreement with experimental data of Branitski et al. (1991), who specifically studied an early stage of gas-puff implosion on Angara-5-1 facility (3 MA, 0.4-0.6 MV, linear mass ~ 0.1 mg/cm, Xe). The ripple propagating from cathode to anode later in the pulse was replaced by filaments that persisted till the middle of the pulse (Fig.V.2). Azimuthal instabilities were not affected by replacement of Xe by Ne. The presence of azimuthally asymmetric structures in the foam loads was recorded on the Saturn device (Lazier et al., 1997).

D. Early stage of the wire array discharge; merging of the wires.

Excitation of pulsed currents in metal wires and dielectric fibers was subject to numerous experimental studies: Aranchuk et al. (1986); Bartnik et al. (1990), Beg et al. (1997), Mosher and Colombant (1992), Sarkisov and Etlicher (1995); Sarkisov et al. (1995 a,b), Sethian et al. (1987), Skowronek and Romeas (1985). Theoretical analyses were published by Coppins et al. (1988), Rosenau et al. (1988), Bud'ko et al. (1990), Neudachin and Sasorov (1991), Sasorov (1991), and Bobrova et al. (1992).

An important feature of the discharge in a single wire is that the wire core, at least for thick wires, may remain cold and expand very slowly. The core is surrounded by a plasma "corona" that contains only a small fraction of mass but carries almost all the current. This conclusion was made in the paper by Aranchuk et al. (1986) that was specifically devoted to experimental studies of single wire explosions (see also an earlier paper by Aranchuk, Bogolyubskii and Tel'kovskaya, 1985). They have found that, in the explosions of 20- μm diameter copper wires, only 2-7% of the total mass were carrying the current and radiating. The rest of the mass remained cold. This corona was subject to violent unstable motions, while the core remained more or less cylindrical. The maximum current through the wire was 0.5 MA and the current rise time was approximately 100 ns. The halo plasma can be formed because of desorption during a prepulse (a point made in Bartnik et al., 1990) or just because of the evaporation of the whole wire. This relatively low-density halo provides better conditions for the breakdown (Cf. Sec. V.A) if the prepulse is long enough. A strong effect of the wire cleanliness on formation of the corona was reported by Bartnik et al. (1994). A long-lasting core of the exploded wire was observed by Kalantar and Hammer (1993). Sarkisov et al. (1995 a,b) detected a thick core in explosions of 20- μm copper wires; they used absorption of 532-nm laser light to detect the evolution of the exploding wire. Beg et al. (1997) performed a very detailed study of explosions of carbon (7- and 33- μm diameter) and aluminum (25- μm diameter) wires at the maximum current ~ 100 kA and current rise time 55 ns. For the thicker wires, the core existed at least until the current maximum, whereas for the 7- μm carbon wire it disappeared within ~ 10 ns. We note in passing that the paper by Beg et al. (1997) contains a wealth of information on the wire pinches, including detailed characterization of the instabilities in coronal plasma, and detection of electron beams. Detailed numerical simulations of development of the $m=0$ instability have been recently published by Chittenden et al. (1997). The theory attributes formation of the corona to the Ohmic heating of the low-density plasma to the anomalous resistivity (Sasorov, 1991; Haines, 1996). We will return to this issue in Sec. IX.A.

As has already been mentioned in Sec. II B, behavior of the plasma corona of the wires assembled in the cylindrical array is very different from that of separate wires. The reason is the presence of a strong common magnetic field. As a curiosity, one can mention the following: the common magnetic field of the wire array near its surface is $\mu I/2\pi r$, where r is the array's radius. The magnetic field produced by a certain wire in the location of its closest neighbor (i.e., at the distance $2\pi r/N$, where N is the number of wires in the array), is $\mu I/(2\pi)^2 r$, i.e., universally smaller by a factor of 2π than the common field.

This common field accelerates the light coronal plasma past the wire cores towards the center of the array and forces the current to switch to the cores. Therefore, there is a reason to believe that the current will be forced to flow in the wire cores. This observation may serve as a justification of the following model of the wire merging which implies that essentially all the mass of the wire is involved into hydrodynamic motions.

As is well known (see Kadomtsev, 1965; Bateman, 1978), a wire is unstable with respect to the MHD sausage and kink modes. For perturbations with wavelengths exceeding the wire radius, $k_z < r_w^{-1}$, growth-rates are:

$$\Gamma \sim \sqrt{\frac{B_w^2 k_z^2}{\mu(\hat{m}/N\pi r_w^2)}}, \quad (5.18)$$

where B_w is the magnetic field intensity at the surface of the wire (we ignore the factor 2-3 difference between the growth rates of the kink and sausage modes). Numerical results pertaining to specific radial profiles of the current and the density can be found, e.g., in Felber (1982) and Pereira, Rostoker, and Pearlman (1984).

According to what has been said above, we assume that all the wire mass is involved in the motion. The modes with $k_z \sim 1/r_w$, where r_w is the instantaneous radius of an individual wire, create perturbations randomly distributed over the wire length and cause a gradual broadening of the wire (the increase of the effective r_w). The broadening velocity can be evaluated (in particular from the dimensional considerations) as $v \sim \Gamma r_w$. Using expression (5.18) and noting that $B_w = \mu I/2\pi N r_w$, one finds:

$$v \sim I \sqrt{\frac{\mu}{4\pi \hat{m} N}}. \quad (5.19)$$

In “practical” units,

$$v(cm/s) \sim \frac{3 \cdot 10^6 I(MA)}{\sqrt{N \cdot \hat{m}(mg/cm)}}. \quad (5.20)$$

Solving equation $\dot{r} = v$ with I as in (2.9), one finds that r_w reaches a half of the inter-wire gap, $\pi r/N$, at

$$\frac{t}{\tau} \sim 15 \left[\frac{\hat{m}(mg/cm)}{N} \right]^{1/6} \left[\frac{r(cm)}{\tau(ns)I_0(MA)} \right]^{1/3}. \quad (5.21)$$

For the wire array with the “standard” Z parameters, t/τ is ~ 0.5 . Therefore, development of the MHD instabilities of the wires can, in principle, cause an early merging of the wires. Note that at $t/\tau \sim 0.5$ the current in the wires reaches only a quarter of its maximum value, and the array’s diameter experiences only a very small change. This is why we neglected change of r in the preceding discussion. In a more sophisticated version of this analysis, one should take convergence into account.

If the MHD mechanism of the wire merging is indeed the dominant one, one can make some predictions with regard to the initial state of the thus formed liner: it will be grossly non-uniform, with the spatial scale of the non-uniformities of the order of a half of the inter-wire distance. These non-uniformities will be both axial and azimuthal. This observation may be of some value for the numerical simulations of the R-T instability of the type carried out by Peterson. Experimentally, an increase of the number of wires had a favorable effect on the implosion symmetry (Sanford et al., 1996). The authors of this paper relate improved performance to an early formation of a continuous shell in the case of a large number of the wires.

To conclude this section: Most probably, the merging of the wires occurs in a turbulent fashion, with development of the perturbations with the scale of the order of the instantaneous wire radius. When merging occurs, the thus formed shell has a thickness of order of the interwire distance and non-uniformities of the same scale. The amplitude of non-uniformities is of order of 1. This sets the stage for the further evolution of the liner, where two competing processes occur: smoothing out the inhomogeneities by virtue of hydrodynamic motions and thermal conductivity, and enhancement of those modes that are Rayleigh-Taylor unstable.

VI. HYDRODYNAMIC STABILITY OF AN IMPLoding LINER

The Rayleigh-Taylor (R-T) instability plays an important role in essentially all high-energy-density experiments, including experiments with fast Z pinches and ICF capsules. This instability is very universal and very difficult to stabilize. It is a key factor among those setting the limit for the performance of fast Z pinches and other pulsed power devices. No surprise, there exist hundreds of publications devoted to the studies of this instability in general and in the pulsed plasma systems in particular. We will certainly not be able to cover all the relevant results in this relatively compact paper. The interested reader can find further references in the surveys by Sharp (1984), Kull (1993), and Lindl (1995), the latter survey considering specifically the physics of ICF capsules. A summary of experimental results for ICF capsules was given by Kilkenny (1995). As a good general introduction, one could recommend Chandrasekhar's book (1961) which, however, deals only with incompressible systems.

In this section we discuss the instability of an ideal fluid, without accounting for dissipative processes like viscosity, thermal conductivity, and electrical resistivity in the body of the fluid (although we allow for the presence of shock waves, which are, of course, dissipative structures). Dissipative effects are discussed in Sec. VII.

Generally, theoretical analysis of the magnetic Rayleigh-Taylor instability involves very lengthy calculations that do not match the format of this survey. Still, to help the reader more closely follow some important arguments, we present a complete derivation of the growth rates for one, relatively simple, system: a slab of a uniform incompressible perfectly conducting fluid supported from below by a horizontal magnetic field (Harris, 1962). After that, mostly on the qualitative level, we will add new elements to the picture of stability of a thin shell (Sections VI.B and C). In Sections VI.D and E, we discuss effects of accretion and the shock wave in implosions of uniform loads. In Section VI.F effects of a cylindrical convergence are analyzed. Section VI.G deals with nonlinear effects and a turbulent sheath broadening. In Sec. VI H we revisit an issue of a dominant role of axisymmetric perturbations.

One should remember that, in the implosions of thick metal shells of the type used by Degnan et al. (1995), the structural strength of the material can have a considerable stabilizing effect at the early stage of the implosion process. These effects have not yet been studied in great detail and we will not discuss them below. Some further information and pertinent references can be found in Atchinson et al. (1997).

A. Stability of a slab of an incompressible fluid

The geometry of the problem is illustrated by Fig. III.6: the slab thickness is h , the gravity force is directed downward in the x direction, with $\mathbf{g}_x = -\mathbf{e}_x g$, and $g > 0$; the unperturbed magnetic field B occupies the lower half-space, $x < 0$, and is parallel to the axis y . In the geometry of a cylindrical implosion of a thin shell, x would correspond to the radial coordinate (directed in this case to the axis), y would correspond to the azimuthal (θ) coordinate, and z to the axial coordinate. The unperturbed magnetic field that supports the slab is related to the gravity force and the fluid density by the obvious relationship:

$$\rho g h = \frac{B^2}{2\mu} \equiv p_m, \quad (6.1)$$

where we use a notation p_m to designate the magnetic pressure.

As the unperturbed state does not depend on time and on the coordinates y and z , one can seek the solution of the problem in the form of the harmonic perturbations in these variables, i.e., in the form

$$f(x) \exp(-i\omega t + ik_y y + ik_z z). \quad (6.2)$$

The instability corresponds to $\text{Im}\omega > 0$. Sometimes, instead of ω , we use the growth rate,

$$\Gamma = -i\omega \quad (6.3)$$

The linearized hydrodynamics equations in the slab are:

$$-\omega^2 \rho \xi = -\nabla \delta p \quad (6.4)$$

$$\nabla \cdot \xi = 0 \quad (6.5)$$

where ξ is the displacement of the fluid element with respect to its unperturbed position, and δp is the pressure perturbation. These equations yield $\nabla^2 \delta p = 0$, with the solution

$$\delta p = A \exp(kx) + B \exp(-kx) \quad (6.6)$$

where

$$k = \sqrt{k_x^2 + k_y^2}, \quad (6.7)$$

and A and B are arbitrary constants. One finds then from (6.4) that

$$\xi_x = \frac{k}{\rho\omega^2} [A \exp(kx) - B \exp(-kx)]. \quad (6.8)$$

At the upper and lower boundaries of the slab, one should impose boundary conditions of the pressure balance at the perturbed boundary. These conditions are:

$$\delta p - \rho g \xi_x = 0, \text{ the upper boundary,} \quad (6.9)$$

$$\delta p - \rho g \xi_x = \delta p_m, \text{ the lower boundary,} \quad (6.10)$$

with $\delta p_m = B \delta B_y / \mu$. To find the magnetic field perturbation at the perfectly conducting surface, one should use the condition that the magnetic field has a zero normal component at the surface, or, in other words, that

$$\mathbf{n} \cdot \delta \mathbf{B} + \mathbf{B} \cdot \delta \mathbf{n} \equiv -\delta B_x + B \delta n_y = 0 \quad (6.11)$$

where n is the unperturbed outer normal to the lower surface, $n = (-1, 0, 0)$, and

$$\delta n_y = \frac{\partial \xi_x}{\partial y} = i k_y \xi_x. \quad (6.12)$$

Perturbation of the vacuum magnetic field is curl-free, whence

$$\delta \mathbf{B} = -\nabla \psi \quad (6.13)$$

The scalar potential satisfies the Laplace equation,

$$\nabla^2 \psi \equiv \frac{\partial^2 \psi}{\partial x^2} - k^2 \psi = 0. \quad (6.14)$$

Its solution evanescent at $x \rightarrow -\infty$ is

$$\psi = C \exp(kx) \quad (6.15)$$

where C is another arbitrary constant. Substituting this solution into (6.13) to find δB_x , substituting the resulting expression for δB_x into (6.11) to express C in terms of the value of ξ_x at the lower boundary, and returning to Eq. (6.13) to express δB_y in terms of ξ_x , one finds that the magnetic pressure perturbation at the lower boundary is:

$$\delta p_m = -2 \frac{k_y^2}{k} p_m \xi_x. \quad (6.16)$$

This equation shows that magnetic pressure increases (decreases) at the bumps, $\xi_x < 0$ (dips, $\xi_x > 0$) of the sinusoidally perturbed surface. This can also be rephrased as a statement that the magnetic energy perturbation is positive, thereby providing a stabilizing effect (the perturbation of the gravitational energy, for the unstable perturbations, is negative).

Using relationship (6.16) and substituting solutions (6.6) and (6.8) into the boundary conditions (6.9) and (6.10) one finds two linear homogeneous equations for the constants A and B . From the condition that the determinant of this set of equations is zero, one obtains the following dispersion relation for the eigenfrequencies of the problem.

$$\left(\frac{\omega^2}{kg}\right)^2 - \frac{2k_y^2 h}{k} \frac{1 + \exp(-2kh)}{1 - \exp(-2kh)} \frac{\omega^2}{kg} - 1 + \frac{2k_y^2 h}{k} = 0. \quad (6.17)$$

Introducing an angle α between the magnetic field and the wave vector,

$$\cos \alpha = k_y / k, \quad (6.18)$$

one can present the roots of this dispersion relation as:

$$\frac{\omega^2}{kg} = kh \cos^2 \alpha \frac{1 + \exp(-2kh)}{1 - \exp(-2kh)} \pm \left\{ \left[kh \cos^2 \alpha \frac{1 + \exp(-2kh)}{1 - \exp(-2kh)} \right]^2 + 1 - 2kh \cos^2 \alpha \right\}^{1/2} \quad (6.19)$$

The sign "plus" corresponds to a stable root. The nature of a stable mode becomes particularly clear in the limit $k \rightarrow \infty$, where the eigenmode corresponding to this root gets strongly localized near the upper surface, with A/B in the equation (6.6) becoming of order of 1 (this means that, near the upper surface, the second term in the expression (6.6) is exponentially small compared to the first one). The stable mode is an analog of the gravity wave on the surface of a fluid (see, e.g., Landau and Lifshitz, 1987). At smaller k , the eigenfunction of this mode encompasses the whole layer but still is concentrated near the upper surface, where the gravity force is directed to the fluid, so that the stabilizing contribution dominates.

The second root corresponds to the mode that can be stable or unstable, depending on the wave number k . The mode is unstable at small k 's and stable at large k 's. The critical wave number k_0 at which the mode becomes stable, is:

$$k_0 = \frac{1}{2h \cos^2 \alpha}. \quad (6.20)$$

For $\alpha \sim 45^\circ$ the critical wave number is of the order of h^{-1} . Dependence of the critical wave number on the angle α is shown in Fig.VI.1. At large k 's, the magnetic energy perturbation (positive) overbalances the gravitational energy perturbation (negative, for a mode localized near the lower surface). This is the reason for improved stability at large k 's.

Fig. VI.1. Critical value of the wave number, k_0 , as a function of the angle between the magnetic field and the wave vector.

An exceptional role is played by the perturbations with $\alpha = \pi/2$. In this case, the system is unstable at all k 's. Perturbation with $\alpha = \pi/2$ is sometimes called the flute mode. Its remarkable feature is that it does not perturb the vacuum magnetic field and therefore the positive (stabilizing) contribution of the magnetic energy perturbation vanishes. In the cylindrical geometry this mode corresponds to axisymmetric perturbations, with no dependence on the azimuthal angle θ .

At small k 's the growth rate reduces to

$$\Gamma = \sqrt{kg \left[\sqrt{1 + \cos^4 \alpha} - \cos^2 \alpha \right]} \quad (6.21)$$

and becomes independent of the thickness of the layer. For these long-wavelength perturbations the layer can be considered just as a structureless infinitesimally thin gravitating sheet. The overall dependence of the growth-rate on the wave vector for several values of α is shown in Fig. VI.2.

Fig. VI.2. Growth rates for several values of the propagation angle α ; the values of α are $\pi/2$, $\pi/4$, and 0 (from the top to the bottom curve).

At $\alpha=0$ (a purely azimuthal mode in the cylindrical geometry) the growth rate is equal to $\sqrt{kg(\sqrt{2}-1)}$ (e.g., Harris, 1962; Kleev, Velikovich, 1990). At α close to $\pi/2$ (small k_y) the growth rate varies with k_y as

$$\Gamma \approx \sqrt{|k_z|g} \left[1 - \frac{k_y^2}{4k_z^2} \right]. \quad (6.22)$$

These results pertain to the constant density distributions. A broader class of density distributions of incompressible fluids has been studied by Munro (1988). If the lower surface of the fluid is free, then there exist modes with the growth rate $(kg)^{1/2}$. At large k they are strongly localized near the interface and have large growth rate. If, however, the transition is smooth enough, the growth rate is limited from above. Some further discussion of these modes can be found in Inogamov (1985), Bychkov, Liberman, Velikovich (1990), and Bud'ko et al. (1989).

B. Effects of compressibility

Various aspects of the R-T instability in compressible fluid have been discussed in Landau and Lifshitz (1987), Catto (1978), Parks (1983), Bernstein and Book (1983), Gonzales and Gratton (1988), Lezzi and Prosperitti (1989), Gratton, Gratton and Gonzales (1990), Budko and Liberman (1992), and Ryutov and Toor (1998). We discuss here the slab of a plasma whose

temperature in the unperturbed state is constant, supported from below by a uniform magnetic field in the geometry identical to the one shown in Fig. VI.3, under the assumption of the perfect plasma conductivity and vanishingly thin transition layer between the plasma and the magnetic field. Later, in Sec. VI.C, we address the issues related to the finite thickness of the transition layer.

Fig. VI.3. The Lagrange coordinates $\xi(z,t) \equiv (\xi_z(z,t); \xi_x(z,t))$ describing implosion of a thin shell.

If the composition of the plasma does not change in the vertical direction, then plasma density and pressure follow the exponential dependence,

$$p, \rho \propto \exp(-x/h) \quad (6.23)$$

with the scale-length

$$h = \frac{T}{\bar{m}g}, \quad (6.24)$$

and $\bar{m}$ being an average atomic weight (half a proton mass for the hydrogen plasma).

It turns out that the problem in the case of a sharp plasma-vacuum transition allows an exact solution (Parks, 1983, Bernstein and Book, 1983, Gratton, Gratton and Gonzales, 1990). A new element that emerges from the finite compressibility is the presence of propagating acoustic waves. When they propagate in a plasma with exponentially decreasing density (as is the case of a constant-temperature slab supported from below), their amplitude grows exponentially. Therefore, in the perturbation analysis one must allow for the presence of the solutions exponentially growing in the vertical direction and it would be incorrect to impose a constraint of

exponentially decreasing solution. Further details related to this issue can be found in a comprehensive analysis by Gratton, Gratton and Gonzales (1990).

We present dispersion relation for unstable modes in the form derived in Ryutov and Toor (1998). Using the same notation as in Sec. VI.A, one can write it as:

$$\omega^4 - k^2 g^2 + 2 \cos^2 \alpha k g \left[\omega^2 \left(2 \cos^2 \alpha \frac{k g h^2}{s^2} - 1 \right) + 2 k^2 g h (1 - k h \cos^2 \alpha) \right] = 0 \quad (6.25)$$

where

$$s^2 = \frac{\mathcal{P}}{\rho} = \frac{\gamma T}{\bar{m}} = \gamma g h \quad (6.26)$$

is the sound speed. Its unstable solution behaves very much in the same manner as the solution of Eq. (6.19): at small k 's, $kh \ll 1$, the growth rate is determined by the same expression as for the incompressible fluid, i.e. by Eq. (6.21), while at larger k 's it decreases and turns zero at some critical $k=k_0$, which is exactly the same as (6.20).

Formally, Eq. (6.25) has unstable solutions even at $k > k_0$. However, these solutions correspond to the modes whose amplitude grows in the vertical direction faster than $\exp(x/2h)$, so that the energy density diverges at large x . On this basis, Gratton, Gratton and Gonzales (1990) (correctly) consider these solutions as unphysical. In other papers, solutions are labeled as unphysical simply if their amplitude grows with the height. As it turns out (Ryutov, Toor, 1998), at $k > k_0$ the (formally) unstable solution automatically breaks a even more stringent more stringent of these two criteria [grows faster than $\exp(x/2h)$], and in this sense both criteria lead to the same result. What happens with perturbations at $k > k_0$, can be more clearly demonstrated by the analysis of the initial value problem based on the use of the Laplace transform: if one stirs the plasma near the lower boundary and creates there perturbations with spatial scales much smaller than $1/h$, then a part of the perturbation is radiated as acoustic waves in the upward direction, and a part stays near the boundary as a stable surface wave (Ryutov, 1997).

Let us discuss in some more detail the reason why, at small k 's, the scale-height h drops out from the dispersion relation so that all the information on the structure of the plasma slab disappears from the dispersion relation; the sound speed also drops out from it. First one notes that the sound speed s is related to the other parameters of the problem through the relationship (6.26). The sound propagation time over the distance $\sim 1/k$ is $1/ks$. Using Eq. (6.26), one can

easily show that for long enough perturbations with $k < k_0 \sim 1/h$, the instability e-folding time ($\sim 1/\sqrt{kg}$) becomes shorter than the sound propagation time ($1/ks$). Basically, this means that the parts of the slab that are separated by the distance exceeding $1/k_0$ cannot communicate to each other by acoustic signals propagating inside the slab; therefore, they evolve independently of each other.

From these considerations we see that, for the perturbations with a parallel scale-length exceeding the thickness of the layer, $k < 1/h$, one can neglect the interaction between different points along the surface of the imploding shell. The shell itself, for such perturbations, can be considered as a thin structureless surface, possessing some inertia (which is determined by the mass per unit area). This is a very important observation that helps one to make some clear predictions regarding the evolution of the perturbations with $k < 1/h$ (see Sec. VI.D).

These arguments also allow one to make some conclusions with regard to the stability of the wire array before the wire merging: if one considers the perturbations with the wavelengths much greater than the interwire distance, the fact that the array consists of separate wires does not manifest itself in any way, and expression (6.21) correctly describes the growth rate. Stability of some modes with the wavelengths comparable with the interwire distance has been studied by Felber and Rostoker (1981), and Samokhin (1988). The general analysis of the wire array stability is yet to be done. Of some help may be general expressions for the magnetic field for the wire arrays made of the wires of an arbitrary cross section (Waisman, 1979).

C. Smooth transition between the plasma and the magnetic field; local modes

The finiteness of the plasma resistivity smoothes the transition between the vacuum magnetic field and the plasma. Fast development of short-wavelength flute perturbations (for which the critical wave number is infinite) also smears out the transition. We will use notation h_l to denote the thickness of the transition; h_l can be smaller than or comparable to the total thickness of the shell h (Sec. III.G). For perturbations with wave numbers below $1/h_l$, one can use the results of the previous section. In this section, we consider perturbations with the wave number much greater than $1/h_l$, so-called local modes. The growth rate of these modes depends on the local value of the density gradient ρ' . As within the transition layer the characteristic value of the magnetic field strength is of the order of the vacuum magnetic field, these short-wavelength perturbations can be unstable only if they are of a flute type (otherwise, perturbation of the

magnetic energy becomes prohibitively high). The growth rate for the flute perturbations has been derived by Chen and Lykoudis (1972):

$$\Gamma^2 = \left(\frac{g^2}{a^2 + s^2} + \frac{g\rho'}{\rho} \right), \quad (6.27)$$

where $a^2 = 2p_m/\rho$ is the local value of the Alfvén speed. For $\rho' > 0$ the local modes are universally unstable. Taking into account rough estimates $\rho'/\rho \sim 1/h_1$ and $gh \sim a^2 + s^2$ that follow from the equilibrium condition, one can estimate the growth rate of the localized modes as

$$\Gamma^2 \sim g/h_1. \quad (6.28)$$

Note that the presence of the magnetic field within the shell does not change the conclusion made in Sec. VI.B regarding the properties of the long-wavelength perturbations: at $k \ll 1/h$, the instability e-folding time is shorter than the time needed for the Alfvén wave to propagate over the distance $1/k$. Therefore, even if the magnetic field penetrates into the shell, the long-wave perturbations behave as perturbations of a massive structureless infinitesimally thin sheet.

Another comment is related to the perturbations localized near the interface between the plasma and the magnetic field. The growth rate of the modes localized near this interface, at large k 's, does not reach a saturation, because the parameter ρ'/ρ near the interface becomes infinite. On the other hand, if the interface is smeared and the density decreases, say, exponentially, the growth rate at large k 's reaches a saturation (see also comment at the end of Sec. VI A).

D. More on the stability of a thin shell; effects of accretion

As has been shown in Sections VI.A and B, the analysis of perturbations with wavelengths exceeding the shell thickness can be carried out without account for the internal structure of the shell. This allows one to obtain a relatively simple description of the instability including effects of the cylindrical geometry of implosion (Harris, 1963), of the mass accretion effects (Gol'berg and Velikovich, 1992; DeGroot et al., 1997), and even get some insights into nonlinear phase of the problem (Ott, 1973; Bashilov, Pokrovskii, 1976; Manheimer, Colombant and Ott, 1984; Basko, 1994; Book, 1996). We derive the corresponding equations in the planar case. Later, in Sec. VI F, we discuss effects of a cylindrical geometry.

Consider a thin shell accelerated by a magnetic pressure p in the x direction (Fig. VI.3). The horizontal line at $x=0$ depicts the initial position of the shell. We will analyze only most

dangerous flute perturbations aligned with the magnetic field lines that are directed along the y axis. In other words, we consider perturbations that do not depend on the coordinate y . The motion occurs in the (x, z) plane. The magnetic pressure then is uniform even on the perturbed surface of the shell.

Following Ott, we denote by $\xi_x(z, t)$ and $\xi_z(z, t)$ the x and z displacements of a certain element of the shell (whose initial location on the shell surface was z). In other words, at this point we are using a Lagrangian description of the perturbations. These displacements are not assumed to be small: we are going to obtain a nonlinear set of equations. We will also take into account the possibility of mass accretion on the shell from the gas initially situated at the $x > 0$ half-space, assuming that the gas just sticks to the shell (strongly radiating plasma, see Sec. III.B and C). In the unperturbed motion one obviously has $\xi_z = 0$ and

$$\frac{\partial}{\partial t} \left(\sigma_0 \frac{\partial \xi_x^0}{\partial t} \right) = p, \quad \frac{\partial \sigma_0}{\partial t} = \rho \frac{\partial \xi_x^0}{\partial t}, \quad (6.29)$$

where ρ is the density of the cold resting gas swept by the shell, and σ_0 is the mass per unit area of the shell (varying with time because of the accretion).

Let us denote by Δz an initial distance between the two neighboring points at the surface (Fig. VI.3). One of them gets displaced to the point $(\xi_x(z, t), \xi_z(z, t))$, the other to the point $(\xi_x(z, t) + \Delta z \partial \xi_x(z, t) / \partial z, \xi_z(z, t) + \Delta z \partial \xi_z(z, t) / \partial z)$. Let the mass of this element of the surface be Δm . The change of the mass occurs because of the accretion. Using simple geometrical considerations (identical to the ones used by Ott (1973)), one finds:

$$\Delta \dot{m} = \rho \left(\frac{\partial \xi_z}{\partial z} \dot{\xi}_x - \frac{\partial \xi_x}{\partial z} \dot{\xi}_z \right) \Delta z, \quad (6.30)$$

or

$$\dot{\sigma} = \rho \left(\frac{\partial \xi_z}{\partial z} \dot{\xi}_x - \frac{\partial \xi_x}{\partial z} \dot{\xi}_z \right), \quad \sigma \equiv \frac{\Delta m}{\Delta z}, \quad (6.31)$$

where the dot designates a partial derivative over the time. Equations of motion can be obtained in the same way as in Ott (1973). They read as:

$$\begin{aligned}\frac{\partial}{\partial t}\left(\sigma \frac{\partial \xi_x}{\partial t}\right) &= p \frac{\partial \xi_x}{\partial z}; \\ \frac{\partial}{\partial t}\left(\sigma \frac{\partial \xi_z}{\partial t}\right) &= -p \frac{\partial \xi_x}{\partial z}.\end{aligned}\tag{6.32}$$

If the shell is being accelerated into the vacuum, then σ does not depend on time, and one does not need Eq. (6.31) for σ . In this case Eqs. (6.32) become the linear equations. This observation was made by Ott: the Lagrangian formulation of the problem in this case leads to linear equations describing even the finite-amplitude perturbations. Note that Bashilov and Pokrovskii (1976) generalized Ott's nonlinear solution to the cylindrical case. We will comment on the properties of the non-linear solution later and now will discuss small perturbations of the shell. To distinguish the perturbations, we will mark them by a symbol " δ ". We get:

$$\begin{aligned}\frac{\partial}{\partial t}\left(\sigma_0 \frac{\partial \delta \xi_x}{\partial t}\right) &= (p - \rho v^2) \frac{\partial \delta \xi_y}{\partial y} - \dot{v} \delta \sigma; \\ \frac{\partial}{\partial t}\left(\sigma_0 \frac{\partial \delta \xi_y}{\partial t}\right) &= -p \frac{\partial \delta \xi_x}{\partial y}; \\ \frac{\partial \delta \sigma}{\partial t} &= \rho v \frac{\partial \delta \xi_y}{\partial y},\end{aligned}\tag{6.33}$$

where $v = \partial \xi_x^0 / \partial t$.

If there is no material in front of the accelerated shell ($\rho=0$), then the set of equations (6.33) becomes particularly simple:

$$\begin{aligned}\frac{\partial^2 \delta \xi_x}{\partial t^2} &= g \frac{\partial \delta \xi_y}{\partial y}; \\ \frac{\partial^2 \delta \xi_y}{\partial t^2} &= -g \frac{\partial \delta \xi_x}{\partial y},\end{aligned}\tag{6.34}$$

Here $g \equiv p/\sigma_0$ is the effective gravity force. For the perturbations with $\exp(-ikz)$ dependence on the coordinates this equation yields a familiar expression for the growth rate (Cf. Eq. (6.21) with $\alpha = \pi/2$): $\Gamma = (kg)^{1/2}$.

An interesting point here is the surface density perturbation of the shell. The quantity σ that we have been using so far was the Lagrangian density: a mass that corresponds to a segment of the shell whose end points originated at the ends of the initial segment Δz , divided by this Δz

(see (6.31)). If there is no accretion, then the thus defined density is constant. But the real density of the shell, defined as a mass Δm occupying some segment $\Delta \ell$ on the surface of the shell, divided by this $\Delta \ell$, is changing. For small perturbations,

$$\Delta \ell \approx \left(1 + \frac{\partial \delta \xi_y}{\partial y} \right) \Delta y \quad (6.35)$$

and the perturbation $\delta \hat{\sigma}$ of the real density (we use the hat to distinguish it from the Lagrangian perturbation that, in the case of a zero accretion, is absent) can be presented as:

$$\delta \hat{\sigma} = -\sigma_0 \frac{\partial \delta \xi_y}{\partial y} \quad (6.36)$$

One can see from the first of the equations (6.34) that, for the exponentially growing perturbation, $\partial \delta \xi_y / \partial y = |k| \delta \xi_x$ and, therefore,

$$\delta \hat{\sigma} = -\sigma_0 |k| \delta \xi_x \quad (6.37)$$

We see that the surface density is redistributed over the shell in such a way that the density decreases on the tops of perturbations and increases on the bottoms (Fig. VI.4). This contains a hint on the process of a self-acceleration of the instability of the nonlinear stage: the areas of a lower (higher) density tend to move ahead (lag behind) faster than in a linear approximation. We will return to this issue in the Sec. VI.F.

Fig. VI.4. Surface density redistribution in the flute mode. The thickness of the solid line roughly corresponds to the density. The dashed line depicts the initial position of the shell.

Consider now a linear instability in a more general case where accretion of the material is substantial and one has to use a general set of equations (6.33). A characteristic time of changing the parameters of the system, in particular, the mass per unit area and the acceleration is

$$\tau \equiv \frac{\sigma}{\dot{\sigma}} = \frac{\sigma}{\rho v}. \quad (6.38)$$

As the e-folding time for short-wavelength perturbations decreases with k , at large enough k 's it becomes much shorter than τ . This happens at $k \gg \rho^2 v^2 / g \sigma^2$. The magnetic pressure is related to the ram pressure of the accreted material via $\rho v^2 \sim p$. Using this relationship, one finds that the limit of large growth rates corresponds to

$$k \gg \rho / \sigma, \quad (6.39)$$

in agreement with DeGroot et al. (1997). In some medium point in the acceleration process the r.h.s. of (6.39) is of the order of the inverse path made by the shell, in other words, the inverse pinch radius. In the case (6.39) one can consider all the unperturbed parameters entering the set (6.33) as constant. This yields the following expression for the instantaneous growth rate:

$$\Gamma = \left(\frac{k}{\sigma} \right)^{1/2} \left[p(p - \rho v^2) \right]^{1/4}. \quad (6.40)$$

This expression differs from the corresponding expression of DeGroot et al., 1997; in particular, Eq. (6.40) predicts that the growth rate approaches zero at p close to ρv^2 when the shell is moving without acceleration.

For the wavelengths that do not satisfy condition (6.39), the perturbation growth cannot be adequately defined in terms of the instantaneous growth rate; in this domain one has to solve a full set of differential equations (6.33). Qualitatively, in this domain the accretion still should lead to a decrease of the perturbation growth (Cf. Gol'berg and Velikovich, 1993). Cochran, Davis and Velikovich (1995) have shown by solving numerically a 2-D set of equations of radiative hydrodynamics that uniform gas-puffs are more stable with respect to axisymmetric Rayleigh-Taylor instability than annular gas-puffs; this is in agreement with a general trend predicted by Eq. (6.40).

E. The case of a detached shock wave

In the previous section we have been discussing stability of a system where the gas just sticks to the surface of the piston. As we have mentioned in Sections III.B and C such a model correctly reflects a situation where the gas collected by the piston is strongly radiating, so that the distance between the shock and the piston is negligible (an ultimate case of the snow-plow model). For the short current pulses typical for experiments on fast Z pinches and low initial

temperature of the matter, a strong shock will form that will propagate ahead of the piston. In this section, we discuss the situation of a weakly radiating plasma, where the plasma behind the shock remains hot, with the pressure at the surface of the magnetic piston equal to the magnetic pressure (Fig. VI.5). There exists a broad class of exact (self-similar) solutions describing plasma flow between the shock and the piston in the planar case, i.e., at the early stage of the implosion (see Krasheninnikova, 1955, and, in the context of Z pinches, Gol'berg and Velikovich, 1992). Stability of this solution with respect to Rayleigh-Taylor modes was discussed in Gol'berg and Velikovich (1992), who, in particular, formulated boundary conditions at the surface of the shock wave, and, later by DeGroot et al. (1997). We present here only (our own) qualitative discussion of the problem.

Denote by h the thickness of the layer between the shock and the piston. The modes with $k \gg 1/h$ localized near the piston do not differ in any significant way from the modes considered in Sec. VI.B. Among those, only perturbations close to flute modes are unstable. However, for these unstable modes the growth-rate is large, $\sim (kg)^{1/2}$. These perturbations will cause a gradual broadening of the transition between the plasma and the magnetic field.

Fig. VI.5. Density, pressure and velocity distribution in the plasma column in the case where strong shock is excited.

Consider now long-wavelength modes ($k \ll 1/h$). For these modes transition layer is "thin" and, according to the results of Sections VI.A through C, can be considered essentially as a structureless surface. A growth rate could be estimated again as $\sim (kg)^{1/2}$. In the $k \ll 1/h$ case, unlike in the opposite one, the modes with an arbitrary orientation of the wave vector are unstable. The mass of the layer gradually increases because of the addition of material swept by

the shock wave. At small k 's (small growth rates) the change of mass within one e-folding time may become considerable, and the concept of the instantaneous growth rate may break down (Gol'berg and Velikovich, 1993).

Note that the thickness of the layer h is relatively small even for a nonradiating plasma. Since the shock is strong, the density behind the shock is determined by Eq. (3.10). For a fully ionized gas with $\gamma=5/3$ the density is 4 times higher than the density before the shock and the thickness of the shocked material is, roughly speaking, 4 times less than the distance travelled by the shock. If one deals with a weakly ionized gas where a considerable fraction of energy delivered to the shocked gas is spent on the ionization, "the effective γ " becomes smaller than $5/3$ (Sec. IV.A) and the layer of the shocked material becomes even thinner.

At the convergence $\sim 3-4$ (for the gas with $\gamma=5/3$), the shock reaches the axis and returns to the piston, leaving behind a hot plasma with the pressure approximately equal to the magnetic pressure. The further compression of the hot material occurs in a quasi-static manner, almost adiabatically, with the plasma pressure equal to the magnetic pressure (Potter, 1978). This adiabatic compression (in the absence of radiative losses) may occur only if the current continues to grow. The plasma boundary, at this phase, moves with a deceleration and stops at the current maximum. The pinch at this point is very similar to the equilibrium Bennett pinch. Of course, the Rayleigh-Taylor instability ceases to exist. Among the hydrodynamic instabilities, only the ones driven by the curvature of the magnetic field lines remain. Their e-folding time is of the order of r/s where s is the sound speed and stability analysis of these modes goes beyond the scope of our survey. A discussion of this problem in a purely MHD approximation can be found in Kadomtsev (1965) and Bateman (1978); among possible equilibria there are equilibria stable with to the $m=0$ mode that have a diffuse pressure profile (the so called "Kadomtsev profile") shown in Fig. VI.6. Nonlinear evolution of the sausage instability for an incompressible fluid was studied by Book et al. (1976). Stabilizing effects produced by the axial shear flow have recently been discussed, with somewhat inconclusive results (Arber et al., 1995; Shumlak and Hartman, 1995). It should be remembered that the plasma column after the self-impact can be so hot that the MHD approximation breaks down and numerous non-MHD effects, in particular the ones caused by the large ion orbits, may become important. A general characterization of the parameter space for the stability problem of a Bennett-type pinch, with the identification of the sub-domains where various anomalies may surface, has been performed by Haines and Coppins (1991).

Fig. VI.6. The Kadomtsev pressure profile. Asymptotically, at large r , the pressure decreases as $r^{-10/3}$.

F. Effects of a cylindrical convergence

As we have seen, the most dangerous modes are flute modes with wave-numbers k of the order of the inverse shell thickness $1/h$. Until very late stages of the implosion, when the liner is already about to collapse on axis, the wavelengths of these modes are small compared to the liner radius ($kr \gg 1$). Therefore, their instantaneous growth rate should be adequately described by the planar model that was discussed in the previous three sections. Still, as we will shortly demonstrate, effects of the cylindrical convergence may become important even at this stage. There are two reasons for that.

The first is effect of convergence on the mass σ per unit area: one obviously has $\sigma = \sigma_0(r_0/r)$. This, in turn, may affect the sheath thickness, making it different from what it were in the case of the planar system with the same time-history of acceleration. Accordingly, the maximum growth rate changes compared to a planar case.

The second effect is the increase of the azimuthal component of the wave vector, $k_\theta = m/r$: as the mode number m does not change with time, k_θ scales as $1/r$ (we remind that, in our notations, $k_y \equiv k_\theta$). This causes a gradual decrease of the angle α between the wave vector and the direction of the magnetic field and may eventually lead to the stabilization of the mode by making the product

$$k \cos^2 \alpha \equiv \frac{k_y^2}{\sqrt{k_z^2 + k_y^2}} \quad (6.41)$$

less than $1/2h$ and, thereby, causing stabilization of the mode [see Eq. (6.20)]. Remember that, in a purely cylindrical system, k_z does not change with time. Consider, as an example, a perturbation in which initially one had $k_y = 0.3k_z$. Even at a modest convergence $C=3$, the r.h.s. of (6.41) increases, compared to its initial value, by a factor of 6, bringing the product $k \cos^2 \alpha$ closer to its critical value $1/2h$. This effect may considerably limit the domain in the (k_y, k_z) plane where the perturbations would experience an exponential growth until the late phase in the implosion. Fig. VI.7. illustrates this effect.

Fig. VI.7. The domain in the plane (k_y, k_z) where the perturbations with initial wave numbers k_y, k_z are stable from the very beginning ($C=1$) and become stable at the convergence $C=3$ and the convergence $C=4$; h is the instantaneous shell thickness. The stability domain is situated above the corresponding curves. Note how narrow is an interval of k_y contributing to the instability even at a modest convergence of $C=3$.

The growth of perturbations at the linear stage of instability is determined by the exponentiation factor (see, e.g., Lindl, 1995),

$$\xi(t) = \xi_0 \exp G(t) \quad (6.42)$$

where

$$G(t) \equiv \int_0^t \Gamma(t') dt', \quad (6.43)$$

and Γ is an instantaneous growth rate (6.3).

A linear approach breaks down as soon as the amplitude reaches certain level $\xi = \xi_{NL}$. A rough estimate for ξ_{NL} reads as $\xi_{NL} \sim 1/h$ for the fastest growing perturbations with $k \sim 1/h$, and $\xi_{NL} \sim 1/k$ for long-wavelength perturbations with $k \ll 1/h$. The transition to the non-linear stage occurs at the time instant determined from equation:

$$G(t) = \ln(\xi_{NL} / \xi_0). \quad (6.44)$$

As one expects that the initial perturbations are small, say, a couple of orders of magnitude less than ξ_{NL} , the logarithm is equal to 4-5, and weakly depends on both ξ_0 and ξ_{NL} .

For the long-wavelength perturbations, the growth rate does not depend on the structure of the shell and the function G has a universal dependence on time determined by the solution of Eq. (2.5). In this solution, the radius is a unique function of time and, therefore, G can be expressed in terms of the current value of the radius. For the current given by Eq. (2.8), and parameter Π (Eq. (2.6)) corresponding to the implosion at the point of the maximum current (i.e., $\Pi \approx 4$), the plot of $G/(kr_0)^{1/2}$ vs the convergence is presented in Fig. VI.8.

Fig. VI.8. The plot of amplification factor vs the convergence for the load imploding on axis at the time of the current maximum for purely axisymmetric perturbations.

However, for the most dangerous perturbations with $k \sim 1/h$ the function G depends also on the thickness h of the shell, which generally speaking, varies with time. A factor that acts towards reducing h is the growing acceleration [the scale-height is inversely proportional to g , see Eq. (6.22)]. Factors acting in the opposite direction are the radial convergence (that increases a mass per unit area), possible onset of the anomalous resistivity, and the growth of the temperature. So, the issue of the transition to the non-linear regime for fastest growing modes is more complicated. If one assumes that the thickness h remains constant during the implosion, one can use the plot of Fig. VI.8 to roughly find the transition point by assuming that $k \sim 1/h$ and imposing the condition that G is approximately equal to, say, 4. For the shell thickness equal to 0.1 of the initial liner radius (and, accordingly, $(kr_0)^{1/2} = 3$) the non-linear effects become important (i.e., G becomes equal to approximately 5) at the convergence equal to 4.

According to our previous discussion, we considered here only the modes with small azimuthal numbers m ; the modes with large m are stabilized at a moderate convergence because of the aforementioned effect of growing k_y ($k_y \sim 1/r$).

G. Nonlinear effects; turbulence and turbulent broadening of the shell

We start discussion from the local modes with $k > 1/h$. When the displacement ξ_x becomes greater than $1/k$, it changes the density gradient; this drives the instability and determines the growth rate - by the order of unity (Fig. VI.9). This signifies that the further development of the perturbations depends on their amplitude. In the case of short-scale perturbations, where the growth rate does not substantially depend on k , one can expect development of random motions with a broad spectrum of length-scales and the amplitude of order $1/k$. The characteristic time of turn-around of the vortices should be of the order of the inverse growth rate $1/\Gamma \sim (h/g)^{1/2}$. These random motions cause a kind of the diffusion evolution of the density profile with the diffusion coefficient $\Gamma/k^2 \propto 1/k^2$. One sees that the greatest contribution comes from the largest scale compatible with the local approximation ($1/k \sim h$). But this means that the diffusive approximation does not correctly describe the situation - characteristic step-size is of the order of the gradient length-scale. This is a fundamental difficulty of a non-linear theory of the R-T-instability. A diffusive description may become relevant if the mode with the largest scale is or one or another reason suppressed. We will not discuss here this rather artificial possibility.

Fig. VI.9. Variation of the density distribution caused by the local displacement of the fluid element. The average gradient within the segment ab becomes zero. The force g is acting towards the axis.

Dimensional arguments show that the broadening should occur according to the law:

$$h = \varepsilon g t^2 \quad (6.45)$$

where ε is some numerical factor. These arguments are very similar to the ones used in the theory of mixing at the interface of two semi-infinite fluids (Youngs, 1991). In this latter case ε is approximately equal to 0.07.

Let us now turn our attention to large-scale perturbations, with k much less than the inverse shell thickness. We consider only most dangerous flute-like (axisymmetric) perturbations. This type of motion can be properly described by Ott's equations [Eq. (6.32) with $\sigma = \text{const}$]. One can expect that, as there are no other scales in this formulation of the problem, the non-linearity turns on when the amplitude of the perturbations becomes of the order of $1/k$. This hypothesis can be easily checked on the basis of the exact solution of the problem obtained by Ott (1972). For a single-mode initial perturbation the time evolution of the shell is illustrated by Fig. VI.10 (Basko, 1994). Strong deformations from a sinusoid appear indeed at $\xi_x \sim 1/k$.

Fig. VI.10. Development of a non-linear mode of a thin shell. The time is measured in units of the inverse growth rate.

For a long-wavelength perturbation, entering the non-linear phase does not mean stabilization or slowing down. Quite the contrary, effects of the mass redistribution (Eq. (6.36)) cause an acceleration of the mode development. This, in particular, manifests itself in formation of the singular spikes within a finite time (Fig. VI.10). The time of the spike formation is equal to $2(kg)^{-1/2} \ln(2/k\xi_{x0})$. Although the Lagrangian description breaks down after formation of singularities, there are no reasons why the lighter parts of the shell should not continue their accelerated motion to the axis. So, there are no signs of a self-stabilization here. The picture gets more complex if one takes into account development of the multiple modes growing from initially random perturbations. The shorter wavelengths grow faster and reach their strongly non-linear stage earlier than the longer ones. Since the mode that has the strongest effect on distortion of the shell is a mode with a scale-length comparable to the shell thickness there exists a possibility that this mode will reach an amplitude several times greater than the instantaneous shell thickness.

We conclude this discussion on the note that, for not very thin shells, with the thickness of order of 0.1 of the initial radius, reaching the convergence $\sim 10-20$ seems feasible (Fig. VI.11). The thinner the shell, the faster do instabilities reach their non-linear phase. Whether at this non-linear stage the instability will cause a gradual broadening of the sheath and its mix with the magnetic field, or more coherent structures of the type shown in Fig. VI.12 will develop, causing disruptions of the current and violent destruction of well-defined shell, is an open

question. The numerical simulations and theoretical analyses seem to point in the direction of more violent scenarios (Fig. VI.13).

Fig. VI.11. View of 200-280 eV pinch radiation taken from 88° at Z. The radiation is created by the stagnation of a nested wire array on-axis. The diameter of the radiating zone is approximately 2 mm. Photo courtesy of C. Deeney at SNL.

Fig. VI.12. Nonlinear stage of the development of the R-T instability. A: flute (axisymmetric) mode (a vertical cross section); B: non-axisymmetric mode (a horizontal cross section).

Fig. VI.13. Numerical results showing the possibility of current disruption by the $m=0$ mode (a 40 mm diameter tungsten wire array on Z after impacting a 2.5 mm radius foam). Isodensity contours at the convergence $C \sim 10$. Collapse of wire array onto 5-mm diameter 14 mg/cc foam. Large radial extent features, 0.2 cm-0.8 cm, show the broadening of the imploding tungsten sheath. At $z=0.6$ cm there is tungsten penetrating into the foam. Tungsten is also calculated to wrap around the open end diagnostic aperture, $z=1$ cm. Simulation courtesy of D. Peterson, LANL.

H. More on the relationship between flute and non-flute modes

We will present the corresponding analysis for the case where the thickness of the current-carrying transition region is comparable with the overall thickness of the shell, $h_I \sim h$. As we know, one should distinguish between the flute modes, for which the wave-number is perpendicular to the magnetic field (an axisymmetric mode in the cylindrical geometry), and non-flute modes. As a representative example of the latter, we consider the mode propagating at an angle of 45° with respect to the magnetic field. The qualitative plot of the growth rates vs. the horizontal wave number k for these two modes is shown in Fig. VI.14. The curve for the flute mode lies above that for the non-flute mode. At small k 's, one can make an exact prediction of the growth rate (Eq. (6.21)) which, in this limit, is independent of the specifics of the density and magnetic field distribution inside the shell. The reason for this has been discussed in Sections VI.A and B.

Fig. VI.14. Overall sketch of the growth rate for the flute mode (upper curve) and the mode propagating at 45° to the magnetic field (lower curve) for a shell with a smooth density distribution that can be characterized by a single spatial scale h ; k is a tangential wave number. Shown in the figure is a *maximum* growth rate over all the modes with a given k . This comment becomes significant at large k , where localized (in x) modes form essentially a continuous spectrum, occupying the whole range of Γ , from the maximum one (shown in the figure) to zero.

At larger k 's, the non-flute mode becomes stable. A specific value of the critical wave number k_0 is determined by the details of the density and magnetic field distribution within the shell thickness. By the order of magnitude, $k_0 \sim 1/h$. For the flute mode the growth rate at large k 's reaches a saturation determined, by the order of magnitude, by Eq. (6.27). To qualitatively reflect this circumstance in Fig. VI.14, we "blurred" the part of the dispersion curve

corresponding to the local modes. Note that the maximum growth rate of the local modes is a well defined quantity: it corresponds to the maximum value of the r.h.s. of Eq. (6.27) over the thickness of the sheath (roughly speaking, it corresponds to the maximum value of ρ'/ρ).

This discussion underlines an exceptional character played by the flute (axisymmetric) mode in the dynamics of imploding liners. The prevalence of the flute mode becomes even more visible at the nonlinear stage of the instability. When the shell becomes strongly distorted with respect to its unperturbed state (Fig. VI.12a), the magnetic field at the tips of the finger remains the same (or even increases, if one takes into account effects of the cylindrical geometry) as in the unperturbed state, while the mass density in the fingers decreases. This causes a catastrophic self-acceleration of the break-up process and may cause the total disruption of the pinch.

This scenario of developing axisymmetric modes at their nonlinear stage, referring to the papers by Hussey et al. (1980), Kloc et al. (1982), and Roderick and Hussey (1986), has found confirmation in the modern numerical simulations of the axisymmetric perturbations (Peterson et al., 1996; Matuska et al., 1996). Fig. VI.13 depicts the isodensity contours with a clearly visible finger-like structure.

Nothing like this can happen for non-flute modes. The reason is a very different reaction of the magnetic field to strongly non-linear non-axisymmetric perturbations of the type shown in Fig. VI.12b: the magnetic field virtually does not penetrate to the "fingertips" in this case and is, on the contrary, increasing at the lagging parts of the liner surface. Therefore, one can expect an early non-linear saturation of the non-axisymmetric modes and their much weaker effect on the liner implosion. As we shall see (Sec. VIII E), one may even try to deliberately introduce non-axisymmetric perturbations to destroy too fast a growth of axisymmetric fingers.

The dominance of axisymmetric modes seems to be in agreement with experimental observations. Fig. VI. 11 shows an X-ray pinhole image of the z pinch near the point of a maximum compression in Saturn device. The features perpendicular to the pinch axis are most pronounced; this is as one should expect in the case of axisymmetric perturbations. Note that in cylindrical implosions driven by ablative force (Hsing et al., 1997) the high- m modes could be a dominant player in the dynamics of implosion, reaching a strongly non-linear stage and affecting the maximum convergence. This underlines once again the exceptional role of the magnetic drive in selecting the $m=0$ mode as the most dangerous one.

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VII. EFFECTS OF DISSIPATIVE PROCESSES

A. Viscosity

Effects of plasma viscosity are among the ones whose influence on the Rayleigh-Taylor instability has been studied in most detail, starting from Chandrasekhar's monograph (1961), where stability of a boundary between the two semi-infinite viscous fluids has been studied. A careful analysis of viscous effects in the case of a slab has been performed by Mikaelian (1995); see also references therein.

We will qualitatively discuss effect of viscosity for the smooth density distributions of the type shown in Fig. III.7 a, along the lines of Ryutov (1996). Let's first consider the local modes. If an element of the fluid is displaced by a small distance ξ in the vertical direction, it experiences the action of the buoyancy force produced by the difference of the densities inside this element and the surrounding substance, $\delta\rho \equiv \rho_i - \rho_e = -(\partial\rho/\partial x)\xi$. The force is equal to $\delta F = -Vg\delta\rho$, where V is the volume of the liquid element ($\sim 1/k^3$). The viscous force acting on the element of the size $\sim 1/k$ moving in the resting fluid is (Landau and Lifshitz, 1987) $\delta F_{\text{visc}} \approx -\rho\nu k^2 \dot{\xi}V$ where ν is the kinematic viscosity. In this way one arrives at the following equation of motion:

$$\ddot{\xi} = \frac{g}{\rho} \frac{\partial\rho}{\partial x} - \nu k^2 \dot{\xi}. \quad (7.1)$$

In the absence of the gravity force, this equation describes a viscous slowing down of the velocity perturbation with the time-scale

$$\tau_{\text{visc}} \sim 1/\nu k^2. \quad (7.2)$$

From (7.1), one gets the following expression for the growth rate:

$$\Gamma = \sqrt{\frac{g}{h}} \left(\sqrt{1 + \varepsilon_{\text{visc}} \frac{(kh)^4}{4}} - \sqrt{\varepsilon_{\text{visc}} \frac{(kh)^2}{2}} \right), \quad (7.3)$$

with $h^{-1} \equiv (1/\rho)\partial\rho/\partial x$, and

$$\varepsilon_{\text{visc}} \equiv \frac{\nu^2}{gh^3}. \quad (7.4)$$

The dimensionless parameter ϵ_{visc} characterizes the role of viscosity. For implosion of wire arrays, the parameter ϵ_{visc} is typically very small. Kinematic viscosity is related to the mean free path ℓ_{ii} of the plasma ions by the relationship $\nu \sim \ell_{ii} v_{Ti}$. Then, using estimate (6.26), relating the thickness of the shell and the gravity acceleration, one finds that

$$\epsilon_{visc} \sim \frac{A}{Z_{eff} + 1} \left(\frac{\ell_{ii}}{h} \right)^2. \quad (7.5)$$

We have taken into account a relationship:

$$\frac{\bar{m}}{m_i} = \frac{A}{Z_{eff} + 1}. \quad (7.6)$$

In the implosions of metal liners, the ion-ion mean-free-path is orders of magnitude smaller than the liner thickness. Therefore, in this situation ϵ_{visc} is universally small. Only modes with very short wavelengths are effected by the viscosity in this case,

$$kh > \epsilon_{visc}^{-1/4}. \quad (7.7)$$

Note that for these small-scale perturbations viscosity still does not provide a complete stabilization, it just reduces the growth-rate, which now becomes

$$\Gamma \approx \frac{1}{\epsilon_{visc}^{1/2} (kh)^2} \sqrt{\frac{g}{h}}. \quad (7.8)$$

In gas-puff systems, with lower density and higher temperature of the imploding plasma, one may reach conditions under which ϵ_{visc} becomes comparable with 1. Transition to the case $\epsilon_{visc} \sim 1$ may, in particular, occur in the implosion of two gaseous shells, where the outer shell hits the inner shell, causing a sudden increase of the temperature and, accordingly, of the viscosity (we remind that the Coulomb collision cross section decreases with the temperature). At even higher temperatures one may find that the ion gyro-radius becomes less than the ion-mean-free path (see Sec. IV). This causes reduction of the shear viscosity (Braginski, 1965) but the bulk viscosity remains high. The bulk viscosity shows itself up in a way similar to the thermal conductivity that we will describe shortly.

So far, we were discussing small-scale perturbations, with $kh > 1$. For the perturbations of a large scale $kh < 1$, viscosity becomes less significant, just because of a slower rate of the viscous dissipation at larger scales (see Eq. (7.2)).

B. Thermal conductivity and internal relaxation

Now we switch to the discussion of the effects of a thermal conductivity. The characteristic time for smoothing of temperature perturbations of a scale l/k is

$$t_{therm} \sim l/k^2 c, \quad (7.9)$$

where χ is thermal diffusivity. Usually, in the unmagnetized plasma, the thermal diffusivity is large compared to the kinematic viscosity (see Sec. IV).

In the model of the incompressible fluid, the temperature of the medium does not explicitly enter the governing equations. This shows that effects of the thermal conductivity may enter the problem of the Rayleigh-Taylor instability only via the finite compressibility of the matter. Therefore, we will base our discussion on Eq. (6.27) which takes compressibility effects into account. For simplicity, we consider only the case of an unmagnetized plasma, and set the Alfven velocity to be zero: $a=0$.

If thermal conductivity is very large, it maintains uniform temperature in the perturbations (isothermal perturbations). In particular, the sound speed that enters expression (6.27) for the growth rate becomes an isothermal sound speed. At a very low thermal diffusivity the sound speed is adiabatic. As the latter is greater than the former, the growth rate in the case of a very high thermal diffusivity - somewhat paradoxically - is higher than in the case of a no thermal diffusivity. At the intermediate thermal diffusivity the growth rate is intermediate.

The thermal conductivity affects the dissipation of the mechanical motion by creating a phase shift between the perturbation of the density of a certain liquid element and the pressure perturbation introduced by this change of the density. Then, an irreversible part appears in the PdV work. This circumstance can be taken into account in the phenomenological way, in the spirit of the approach described in Landau and Lifshitz (1987). In the particular problem under consideration, one can use the following line of reasoning (Ryutov, Toor, 1998): Assume that some element of the fluid experiences a sudden compression (rarefaction) which leads to the change of its density by some $\delta\rho$. The pressure variation is then given by the relationship $\delta p = \delta p_{ad} \equiv s_{ad}^2 \delta\rho$. If we wait for some time, without further changing $\delta\rho$, then the temperature inside the element we are considering comes to the equilibrium with a surrounding temperature, and the pressure perturbation gradually relaxes to the value $\delta p = \delta p_{iso} \equiv s_{iso}^2 \delta\rho$. This occurs within the time τ_{therm} determined by the rate of thermal conductivity (Eq. (7.9); for the sake of brevity,

we delete the subscript “therm” in the following equations). One can describe this transition by the following equation:

$$\frac{d}{dt}(\delta p - \delta p_{ad}) = -\frac{\delta p - \delta p_{iso}}{\tau}. \quad (7.10)$$

Equation (7.10) correctly describes phenomena occurring at short time-scales, when it predicts $\delta p \approx \delta p_{ad}$, and at long time-scales, when it predicts $\delta p \approx \delta p_{iso}$. It also qualitatively describes the relaxation processes that occur at the intermediate time-scales. For the process with the time dependence $\sim \exp(-i\omega t)$, it yields:

$$\delta p = \frac{\delta p_{iso} - i\omega\tau\delta p_{ad}}{1 - i\omega\tau} \equiv s_{iso}^2 F(\omega)\delta p; \quad F(\omega) = \frac{1 - i\omega\tau(s_{ad}/s_{iso})^2}{1 - i\omega\tau} \quad (7.11)$$

Fig. VII.1. Dispersion curves for the local Rayleigh-Taylor modes affected by the thermal conductivity. Note the presence of a mode of the oscillatory damping in a broad interval of relaxation times. Note also the presence of a weakly damped mode (the segment AB) at large τ . The parameters of the system: $s_{ad}^2/s_{iso}^2=1.25$; the growth rate is measured in the units of the “adiabatic” growth rate; the relaxation time is measured in the units of the inverse “adiabatic” growth rate.

With this relation between the density and pressure perturbations, the s^2 in the dispersion relation (6.27) is to be replaced by $s_{iso}^2 F(\omega)$. The τ dependence of the growth rate determined from this generalized dispersion relation is illustrated by Fig. VII.1. Effects of the bulk viscosity

enter the problem in the same way, via producing a phase shift between $\delta\rho$ and δp . These effects can be described by Eq. (7.10), with τ having a meaning of the viscous time determined by the bulk viscosity.

There may exist other relaxation processes in the system, for instance, establishing the ionization equilibrium (DeGroot, 1997). The effect of this process can also be described by Eq. (7.10) with τ now having the meaning of the time to reach the ionization equilibrium.

C. Resistivity

As has been pointed out by Hussey et al., (1995) and Hammer et al., (1996), effects of magnetic field penetration through the imploding shell may strongly influence R-T instability of flute (axisymmetric) modes. An expectation is, of course, that it will add some dissipation and thereby decrease the growth rate. However, as was shown by Hammer et al. (1996), there may exist modes that do not perturb the currents and therefore do not induce any additional dissipation. For the profile of the type of (3.23), such a mode is localized near the surface $x=0$ and has the growth rate unaffected by the resistivity. The presence of this mode is related to the singularity of ρ'/ρ as x goes to zero.

As soon as the density profile near the point $x=0$ gets smoothed by the development of the instability, the short-wavelength perturbations become affected by the finite resistivity. The expression for the growth-rate of these mode reads as (Hammer et al., 1996):

$$\Gamma = \sqrt{g \frac{\rho'}{\rho} + \left(\frac{D_M k^2}{2} \frac{s^2}{a^2} \right)^2} - \frac{D_M k^2}{2} \frac{s^2}{a^2}, \quad (7.12)$$

where D_M is a magnetic diffusivity (4.15), s and a are the sound velocity and Alfvén velocity, respectively. Note that in a strongly radiating plasma where the magnetic pressure is much greater than the plasma pressure (see discussion related to Eq. (3.23) in Sec. III G) and where, accordingly, $s \ll a$, the stabilizing effect of resistive losses contains a small parameter s^2/a^2 and is relatively insignificant.

D. Instabilities caused by a high resistivity

Somewhat surprisingly, the presence of resistive effects may cause an appearance of the instability on the negative slope of the density distribution (Fig. III.7c). We consider this problem in the case of a very high resistivity when the plasma pressure is negligible and the skin-depth greatly exceeds the thickness of the shell h (see Sec. III G). We assume that σ is constant and uniform within the shell and is zero outside the shell.

The unperturbed electric field directed from one electrode to another is maintained by the external source, it is assumed to be time-independent and, in case of a low resistivity, is independent of spatial coordinates, too (we are discussing the scales comparable to the shell thickness; one can show that the vortex part of the electric field is small as soon as the skin-depth is greater than the thickness of the shell).

Following the arguments of Sec. VI.D, we consider only perturbations independent on the coordinate y (in a cylindrical geometry these are axisymmetric perturbations with $m=0$): perturbations depending on y "turn on" restoring force associated with the curvature of the magnetic field lines and should be, therefore, more stable. One can choose the perturbed quantities in the form: $f(x)\exp(-i\omega t + ikz)$. For such perturbations, only the y -component of the magnetic field is perturbed,

$$\frac{\partial \delta B_y}{\partial x} = \mu \delta j_z, \quad ik \delta B_y = -\mu \delta j_x. \quad (7.13)$$

As outside the shell there are no current perturbations, the perturbation of magnetic field there is zero.

Linearized equations of motion, plus continuity equation, read:

$$\begin{aligned} -\omega^2 \rho \xi_x &= -(\partial / \partial x)(B_y \delta B_y / \mu) - g \delta \rho \\ -\omega^2 \rho \xi_z &= -ik(B_y \delta B_y / \mu) \\ \delta \rho + ik \rho \xi_z + (\partial / \partial x)(\rho \xi_x) &= 0 \end{aligned} \quad (7.14)$$

where $\xi_{x,z}$ are (small) displacements of the material elements of the shell with respect to their unperturbed positions.

For small shell conductivity, the vortex part of the electric field is negligibly small compared to the potential part. Accordingly, we write that

$$\delta \mathbf{j} = -\sigma \nabla \delta \varphi, \quad (7.15)$$

where $\delta \varphi$ is a perturbation of the electrostatic potential. We neglect here the $\mathbf{v} \times \mathbf{B}$ contribution. One can check that it is small at small σ . From the equation $\text{div} \delta \mathbf{j} = 0$ with $\sigma = \text{const}$ one finds that $\delta \varphi$ satisfies Laplace equation:

$$\nabla^2 \delta \varphi = (\partial^2 / \partial x^2 - k^2) \delta \varphi = 0, \quad (7.16)$$

The boundary condition for this equation is that the normal component of the current on the perturbed boundary of the shell should vanish:

$$(\delta j_x - ik j_0 \xi_x) \Big|_{x=0,h} = 0. \quad (7.17)$$

Both this boundary condition and equation $\text{div} \delta \mathbf{j} = 0$ are valid if the perturbation frequency is not too high, $\omega < \sigma / \varepsilon$ (with ε being an electric permittivity of the vacuum). The latter inequality is typically not very restrictive. For example, for a plasma with $T_e = 1$ eV σ is $2.6 \cdot 10^{13} \text{ s}^{-1}$. Condition (7.17) also guarantees the continuity of the magnetic field on the perturbed surface.

Equations (7.15)-(7.17) allow one to express the magnetic field perturbation in terms of the displacements of two boundaries. The result is:

$$\delta B_y = \frac{B_0}{\delta(e^{kh} - e^{-kh})} \left\{ [\xi_x(h) - \xi_x(0)e^{-kh}] e^{kx} + [\xi_x(0)e^{kh} - \xi_x(h)] e^{-kx} \right\}. \quad (7.18)$$

Note that the magnetic field gets perturbed only if the boundaries are perturbed. This is a consequence of our model where conductivity is uniform.

Eliminating ξ_z and $\delta \rho$ from the set (7.14), one easily obtains a single equation that relates ξ_x and δB_y :

$$\frac{\partial}{\partial x}(\rho \xi_x) + \frac{\omega^2}{g}(\rho \xi_x) = \frac{1}{g} \frac{\partial}{\partial x} \left(\frac{B_y \delta B_y}{\mu} \right) + \frac{k^2}{\omega^2} \left(\frac{B_y \delta B_y}{\mu} \right). \quad (7.19)$$

Equations (7.18) and (7.19) form a closed set. We will analyze only those solutions which are localized inside the layer and do not perturb the layer's boundaries. Such solutions describe the evolution of the initial perturbations produced by some stirring of the fluid in a localized region

between the boundaries. For these the perturbation of the magnetic field is zero and one finds that they satisfy equation:

$$\frac{\partial}{\partial x}(\rho \xi_x) + \frac{\omega^2}{g}(\rho \xi_x) = 0. \quad (7.20)$$

Considered as an equation for the function $\rho \xi_x$, this equation has constant coefficients. For the solution with the x-dependence of the form $\exp(ikx)$ one finds dispersion relation

$$\omega^2 = -iqg \quad (7.21)$$

Dispersion relation (7.21) is valid only for high enough q 's,

$$q \gg 1/h. \quad (7.22)$$

This constraint stems from the assumption that the boundaries are not perturbed. Dispersion relation (7.21) predicts a universal instability with a high growth rate, $(|q|g/2)^{1/2}$. The growth rate formally increases with q . However, the most dangerous perturbations that can cause a gross disruption of the implosion are probably the ones with the smallest possible q 's still compatible with inequality (7.22). The growth rate for them is still quite high, $\sim (g/h)^{1/2}$.

The instability which we have just discussed can be called a "coalescence" or "coagulation" instability. It will cause the appearance of the regions with high and low density within the shell. Stabilization may come from the effects of a finite plasma pressure and heat conduction.

So far, we were discussing the instability of the purely flute mode, i.e., the only mode that remains unstable at large wave-numbers in the case of a perfectly conducting plasma (see Sections VI.A through C). The short-wavelength modes that have a cross-field (azimuthal) component of the wave vector are stabilized by virtue of the restoring force produced by the curved magnetic field. This field is frozen into the plasma and follows its displacements. It is clear that the high plasma resistivity decouples plasma displacement and the magnetic field. A complete analysis of this problem goes beyond the scope of our survey. Here we restrict ourselves to a notion that in a case of plasma pressure comparable to the magnetic pressure and skin-depth comparable to the whole thickness of the layer (Alfven velocity comparable to the sound velocity), the resistive uncoupling becomes significant at $D_M > sh$, where s is the sound velocity. The point that we are making is that the finite resistivity may lead not only to stabilizing but also to destabilizing effects.

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VIII. POSSIBLE WAYS OF MITIGATING THE RAYLEIGH-TAYLOR INSTABILITY

In the previous two sections we discussed in some depth the physics of the Rayleigh-Taylor instability. One important (and already mentioned) difference between the stability of an imploding liner and the stability of a steady-state object (like a plasma in devices with magnetic confinement) is that implosion takes a finite time, while a steady-state plasma configuration is supposed to last essentially forever. Therefore, if some instability is present in the steady-state system, the perturbations certainly reach a nonlinear stage, which is independent of the initial perturbations. The saturated turbulence then exists as long as does the plasma formation (sustained by the external particle and energy sources). In the imploding systems, on the other hand, the exponentiation factor G introduced by Eq. (6.43) is finite and, sometimes, not very large, in the range of 5. Under such circumstances, one can hope to reduce the deleterious role of instability by making more perfect initial states, with the relative root-mean-square perturbations less than 10^{-2} . And, conversely, if the growth rate can be reduced by, say, 20% ($\Delta G/G \sim 0.2$) the requirements for the symmetry of the initial state could be reduced by a factor of 2.

This is one possible line of defense: creating more ideal initial states and looking for the means to reduce the linear growth rate. If this approach fails, and the instability reaches a nonlinear stage, one can try to prevent the most disastrous scenarios, associated with a self-accelerating growth of the "bubbles" and gross violations of the cylindrical symmetry of the liner (Fig. VI.12 a). In the further discussion, we mention various effects that may influence the linear and nonlinear stages of the problem. We emphasize that the original stability analyses that we are referring to were often not directly related to fast pinches.

Mitigation methods discussed below do not provide an ultimate solution in making the instability effect insignificant. Moreover, many of them introduce serious complications into the experimental setting. Still, we present a more-or-less complete set of existing suggestions with the hope that they may help in finding an efficient solution. One more general conclusion that can be drawn, with respect to the problem under consideration, is that very little can be done to affect the linear stability of long-wavelength perturbations of an "empty" thin shell ($\lambda > h$, where h is the shell thickness). This is because the linear behavior of these perturbations is described by Eq. (6.21); which does not contain any free external parameters.

A. Magnetic shear

As was mentioned in Sections VI.A and B, the most dangerous modes are axisymmetric modes that do not create any ripple in the magnetic field lines, maintaining their circular (straight in the planar geometry) shape. These modes remain unstable at high wave numbers $k \sim 1/h$ and have a large growth rate $\sim (g/h)^{1/2}$. Conversely, the modes with a finite azimuthal component of the wave number become stable if the wave number is high enough. It is well known from the theory of magnetically confined plasmas (Bateman, 1980; Freidberg, 1982) that one may reach reduction of the growth rate or even stabilization of flute modes by creating a magnetic shear. That is by creating a situation where the magnetic field vector, remaining normal to the gravity force, would change direction over the depth of the transition layer. In the z-pinch geometry this would require introducing the axial magnetic field B_z , possibly, varying over the thickness of the shell.

From the outset, for the reasons discussed in the last two paragraphs of Sec. VI.B, one can conclude that the magnetic shear will have no effect on the stability of long-wavelength modes, $\lambda \gg h$. One can, however, hope that the growth rate of the modes with $\lambda \sim h$ (and these are the most dangerous modes) will be reduced. Indeed, applying a general approach based on the energy principle and ascending to an early paper by Suydam (1958), Gratton, Gratton, and Gonzalez (1988) have shown that, for the local modes ($\lambda < h$), the presence of the shear leads to the appearance of a stabilizing contribution to the expression (6.27) for the growth rate:

$$\Gamma^2 = \frac{g^2}{a^2 + s^2} + \frac{g\rho'}{\rho} - \frac{a^2\zeta^2}{4}. \quad (8.1)$$

The parameter ζ is the x derivative of the angle formed by the magnetic field with the direction of the wave vector at the point where the magnetic field is normal to the wave vector. If the z component of the magnetic field is zero, then shear is zero too. If the z component of the magnetic field is comparable with the azimuthal component, then $\zeta \sim 1/h$. In this case the magnetic shear can considerably reduce the growth rate of the local modes and can even completely stabilize them. In other words, the shear stabilization has some promise for local modes.

As we have already mentioned, the shear does not stabilize the large-scale modes with $\lambda \gg h$. Still, the presence of the axial magnetic field enclosed by the liner may have some effect on the stability. Using the approach ascending to Harris (1962), one can show that, if the magnetic field on the outer side of the shell has components B_{ye} and B_{ze} and on the inner side of the shell B_{zi}

(there is no y component of the magnetic field inside the shell if there is no axial current inside the shell), the dispersion relation for the $\lambda \gg h$ modes [an analog of Eq. (6.21)] becomes (see Bud'ko et al., 1990):

$$\omega^4 - 2kg\omega^2 \frac{B_{ye}^2 \cos^2 \alpha + (B_{ze}^2 + B_{zi}^2) \sin^2 \alpha}{B_{ye}^2 + B_{ze}^2 - B_{zi}^2} - k^2 g^2 = 0, \quad (8.2)$$

where α is defined according to Eq. (6.18), and the acceleration is

$$g = \frac{B_{ye}^2 + B_{ze}^2 - B_{zi}^2}{\mu\sigma}. \quad (8.3)$$

Dispersion relation (8.2) universally has an unstable root that scales as $\sqrt{kg}$ (we assume that the acceleration is directed inward, i.e. $B_{ye}^2 + B_{ze}^2 - B_{zi}^2 > 0$). On the other hand, if $B_{zi} > B_{ze}$, the growth rate for the $\alpha=0$ mode is reduced compared to (6.21).

One should remember that it is undesirable to have an axial magnetic field enclosed by the shell, because then part of the energy of the imploding liner would be spent on the compression of this magnetic field. The other difficulty with imposing the axial magnetic field is that, in the time-frame of the implosion, the axial magnetic field remains frozen into the conducting electrodes. Therefore, strong distortions of the cylindrical symmetry of the type shown in Fig. III.3 are inevitable.

B. Rotation

In Sec. III.D we have already mentioned the possibility of using rotation of the shell for stabilization purposes (see Book, Winsor, 1974; Barcion, Book, Cooper, 1974; Turchi et al., 1976). The stabilizing effect comes from the centrifugal force that is directed oppositely to the effective gravity force near the stagnation point. In the early works, it was supposed that rotation will be set in by mechanical means.

The concept of centrifugal stabilization has been recently reconsidered by Rostoker, Peterson, and Tashiri (1995), who suggested using a cusp magnetic field to create an azimuthal torque that would appear because of the interaction of the z component of the current and the r component of the magnetic field. Hammer and Ryutov (1996) suggested using an ablative torque by producing a left-right asymmetric structure at the surface of the shell. Ryutov (1996), in conjunction with implosions of wire arrays, suggested using left-right asymmetric coatings on the

surfaces of wires (Fig. VIII.1): an ablation of the coating early in the pulse would produce a torque acting on the wires and imparting an angular momentum to them.

Fig. VIII.1. Left-right asymmetric coating. Shown in thicker lines are the areas coated by the material with a lower sublimation energy, which will ablate early in the pulse. the direction of the ablation flow is shown by arrows. Diameters of the wires are grossly exaggerated. Only part of the array is shown. Current flows into the paper.

To have an appreciable effect on the instability, the rotation should change the radial acceleration by an order of 1. This, in turn, means that, near the point of maximum compression, the rotation energy should be comparable to the total implosion energy (see Sec. III.D). This is the energy penalty associated with this method of stabilization.

Stability analysis of the imploding rotating liner compressing the axial magnetic field was carried out by Barcillon, Book, and Cooper (1974). The critical point in this system is the turning point of the liner. They concluded that it is difficult to reach a strong stabilizing effect, especially in the case of a thin liner. Velikovich and Davis (1995) studied stability of a steady-state configuration, $r=\text{const}$, where the centrifugal force is exactly balanced by the magnetic pressure (in the Z-pinch geometry). The stabilizing effect in this case was relatively modest. Although these results are somewhat discouraging, it is probably worth considering the stability of the Z pinch under realistic assumptions with regard to the time-history of the pinch radius $r(t)$.

C. Velocity shear

In principle, one can introduce not the solid-body rotation but an azimuthal shear flow. The possible stabilizing effect of shear flow on the Rayleigh-Taylor instability was mentioned as early as 1960 in a famed monograph by Chandrasekhar; in conjunction with fast Z pinches it was discussed by Hammer and Ryutov (1996). To qualitatively understand the role of the shear, consider a slab geometry (Fig. VIII.2), with the slab of an incompressible fluid supported from below by a massless fluid. Let the unperturbed flow velocity be directed along the axis y and be linearly dependent on x : $v_y = u(x/h)$.

Fig. VIII.2. The slab geometry of shear flow.

The shear flow will have the strongest effect on the perturbations with the wave vector directed along the direction of flow. For such perturbations, it will lead to their stretching in the y direction, as shown in Fig. VIII.3, and thereby to strongly changing their eigenfunction.

Fig. VIII.3. Stretching of the perturbations by the shear flow. a) without shear flow. b) with shear flow.

One can hope that stretching of the “fingers” typical for the Rayleigh-Taylor instability will reduce their growth. The stretching of the perturbation occurs over a time period of the order h/u , independent of the scale of the perturbation, while the growth rate of the gravity-driven perturbations is of the order $(kg)^{1/2}$. Obviously, the shear flow can have a significant effect on development of the perturbations if condition $(h/u)(kg)^{1/2} < 1$ holds. For the most dangerous perturbations with $k \sim 1/h$, one can rewrite this condition in terms of a so-called Froude number,

$$Fr \equiv \frac{u^2}{gh} > 1. \quad (8.4)$$

Taking as an example $h=0.1$ cm, and $g=3 \cdot 10^{15}$ cm²/s, one finds that the velocity of the shear flow should be rather high, greater than 10^7 cm/s. It is difficult to produce such a velocity directly. One can, however, expect an enhancement of the azimuthal velocity during the implosion because of the conservation of the angular momentum of a cylindrical shell. (For the “canonical” wire array implosions, the viscous damping of the shear flow is insignificant; see Sec. IV.D.2). If the initial velocity of shear flow is 10^6 cm/s, the desired value will be reached at the convergence equal to 10. One can speculate that the actual time required for the perturbations to grow from the initial small level to a nonlinear phase constitutes at least several e-folding times $(kg)^{-1/2}$. Accordingly, one could hope that, in the r.h.s. of inequality (8.4), one should write, instead of 1, some small number. Whether this essentially nonlinear effect is present requires further analysis.

Fig. VIII.4 (adapted from Hammer and Ryutov, 1996) illustrates the effect of a shear flow on the linear growth rate. Indeed, one can see that the most dangerous perturbations are stabilized.

The growth rate of a Kelvin-Helmholtz instability driven by the presence of a shear flow is less than the growth rate of the Rayleigh-Taylor modes.

Fig. VIII.4. Dispersion relation for a slab of the gravitating fluid in the presence of linear shear flow as in Fig. VIII.3 (after Hammer, Ryutov, 1996). The growth rate Γ is in units of v/ℓ . The slab thickness is ℓ , the gravity acceleration is such that $v^2/12g\ell$. At small wavenumbers k , the instability is of Kelvin-Helmholtz type. When the flow is absent, the growth rate is $(kg)^{1/2}$ for all k 's (dashed line).

Unfortunately, shear flow does not have any effect on perturbations with the wave vector perpendicular to the direction of the flow (in the geometry of Fig. VIII.2, these are the perturbations with $k_y=0$). In other words, if the shear flow is in the azimuthal direction (the differential rotation), it does not affect azimuthally-symmetric perturbations ($m=0$). To stabilize these perturbations, one has to generate the axial shear flow, with v_z varying with x . This type of shear motion cannot be enhanced by the angular momentum conservation; therefore, requirements stemming from inequality (8.4) become quite stringent.

To generate the shear flow, one can use the target consisting of two nested liners (Hammer, and Ryutov, 1996). If the left-right asymmetric features are embedded into one of these liners, then the ablation will cause its rotation; when the two liners collide, a differential rotation emerges. If up-down asymmetric features are created, then the axial shear flow is formed.

The axial shear flow may have a stabilizing influence on quasi-equilibrium Z pinches that can be formed near the stagnation point. This stabilization mechanism for equilibrium Z pinches was discussed by Shumlak and Hartman (1995). Their conclusion was that, if the initial profile is not

very far from the marginally stable “Kadomtsev profile,” then even a weak velocity shear can produce considerable stabilization.

D. Hourglass effect

Douglas, Deeney, and Roderick (1997) have discovered in numerical simulations that, by making the initial surface of a uniform-fill Z pinch concave (Fig. VIII.5), one can suppress the growth of Rayleigh-Taylor perturbations. Because of the characteristic shape of the sheath, one can call this effect an “hourglass effect.” Douglas, Deeney, and Roderick considered implosions of high-Z (strongly radiating) gas-puffs (Ne, Xe), where the transition layer between the magnetic piston and the shock front is thin (Cf. Sec. III.B). For a sufficiently large initial curvature, the stabilizing effect is quite strong. The authors attribute this effect to either the advection of perturbations to the electrodes (there is tangential flow along the curved surface, in the direction of electrodes) or the presence of the axial shear flow. In principle, one could discriminate between these two possibilities by changing the sign of the curvature (making the surface convex instead of concave): the flow would have changed its sign, and would advect perturbations to the equatorial plane, while the effect of shear should have remained the same as for the concave surface.

Fig. VIII.5. An effect of a curved surface of a uniform-fill krypton Z pinch (from Douglas, Deeney, and Roderick, 1997). Shown are density isocontours at the same time into the implosion for (a) a straight cylinder; (b) a 1.0 mm circular arc, (c) a 2.5 mm circular arc, and (d) a 5.0 mm circular arc.

The curved sheath is typical for implosions in the other pulsed system, the plasma focus. In facilities of this type, an annular discharge initiated at a large distance from the axis (Fig. VIII.6) eventually converges on axis. The sheath in the intermediate stages of the discharge has considerable curvature. It would be interesting to reproduce this type of configuration for the set of parameters typical for the present-day fast pinch experiments, both for the gas-puffs and the wire arrays.

Fig. VIII.6. Typical geometry of a plasma focus discharge. C is a cathode. Shown in thin lines are successive positions of the current sheath. Increasing time is denoted by 1-5.

E. Deliberate violation of the azimuthal symmetry

As already mentioned in Sec. VI, the most dangerous mode shows a trend to a strongly nonlinear development, with the finger-like structures penetrating deep to the axis of the device (Fig. VI.13). Self-acceleration of the “fingertips” occurs because the mass density at the “fingertips” decreases, while the driving azimuthal magnetic field freely penetrates to this area through disk-like slots; this is a characteristic feature of axisymmetric perturbations. M. Derzon, T. Nash, and D. Ryutov (1997) suggested reducing the growth rate of the axisymmetric perturbations by deliberate introduction of a periodic azimuthal asymmetry, as shown in Fig. VIII.7, with a large-enough amplitude and a large-enough mode number m . The idea is to destroy the azimuthal coherence of the finger-like structure and to create conditions for short-circuiting (crow-barring) the disk-like slots in many (m) points over the azimuth.

Fig. VIII.7. A corrugated wire array. The azimuthal mode number in this case is $m=6$. The thickness of the line in the panel (a) corresponds to the local surface density of the liner material. The surface of a perfectly conducting liner coincides with one of the field lines. The adjacent field line is shown in dashed line. Panel (b) shows a part of the initial array that can produce a structure similar to the one shown in panel (a): the array is assembled of the wires of two different diameters, d_1 and d_2 .

For this closure of the slots to occur at a moderately nonlinear stage of the growth of axisymmetric perturbations, when the peak-to-valley distance becomes of the order of their axial period λ , one has to produce the azimuthal perturbations for which amplitude, ξ , satisfies, roughly speaking, a condition:

$$\xi > \lambda/2. \quad (8.5)$$

One can conceive of several ways of creating azimuthal perturbations in a controlled way. One way is by assembling the wire array of wires of azimuthally varying thickness (i.e., of varying mass), similar to what has been shown in Fig. VII.7b: the heavier wires will lag behind the lighter ones, thereby creating a corrugated structure. Another possibility is to use wires of different materials (and, accordingly, of different mass). Still one more possibility is to use a kind of "imprinting" produced by the discrete structure of the return current structure (which typically consists of ~ 10 separate posts; the gaps between the posts provide a necessary diagnostic access). At the early stage of the discharge, when the separation of the current sheath from the posts is comparable to the inter-post distance, the radial driving force is varying over the azimuth, giving rise to formation of the corrugated structure. This corrugated structure has been observed experimentally [M. Derzon, T. Nash, and D. Ryutov (1997)].

In the wire array implosions, the axial period of the most dangerous modes is in the range of 1-1.5 mm. Therefore, according to (8.5), a relatively modest amplitude of corrugation (0.5-0.75 mm) could be sufficient to produce considerable stabilizing effect. The mode number of the corrugation should be made large enough to provide many short-circuit channels over the radius. This is limited from above by the constraint that, at a given amplitude, the high-mode-number perturbation become nonlinearly stabilized by the effect of expulsion of the magnetic field from the tips of azimuthal perturbations (Fig. II.4b). The upper limit on the mode number set by this constraint is:

$$m < r/2\xi \sim r/\lambda. \quad (8.6)$$

For the typical set of parameters of a wire array implosion, the optimum mode number is ~ 15 .

F. Accretion

As mentioned in Secs. III and VI, in the implosions of gas-puff loads one can, in principle, create such initial density distribution that the sheath will converge on axis without any acceleration. In the clearest form, this idea was expressed by Hammer (1995) and, later, by Hammer et al. (1996), and Velikovich, Cochran, and Davis (1996). The energy penalty is associated with the radiative losses of the accreted material. The stabilizing effects are caused merely by the absence of acceleration. The interface between the magnetic field and the plasma remains stable with respect to exponentially growing modes, even for very short wavelengths (shorter than the distance between the shock and the interface). Another source of stabilization is related to the presence of a detached shock, as discussed by Gol'berg and Velikovich (1993): because the shock front itself is stable with respect to the ripple perturbations, it extends its stabilizing influence over the whole area between the shock and magnetic piston.

One can think of a discrete version of the scheme proposed by Hammer et al. (1996) where, instead of a continuous density distribution, one would create a set of nested wire arrays, with the masses approximately following the desired density distribution. This would mean that, on the outside, there would be the lightest arrays, and that their mass would gradually grow towards inside. Whether the improved stability will outweigh the added complexity is a question that can eventually be answered only experimentally.

G. Enhanced thermal dissipation

Attempts have been made (Ryutov, 1996; Ryutov, Toor, 1998) to increase the rate of viscous and thermal dissipation by replacing a uniform medium with a finely structured medium, with a scale a of pre-existing nonuniformities that are small compared to the scale λ of the most dangerous perturbations. In the case of wire arrays, this could be done by replacing uniform wires with bundles of interwoven finer wires, or by alternating the composition of the wires in the array. The hope is that the presence of the fine structures will introduce small-scale motions (and temperature variations) overlaid on the “averaged” motions (and temperature variations) on the larger scale of the most dangerous perturbations. Because the dissipation rate by both viscosity and thermal conductivity scales as $1/a^2$, one could expect that the growth rate of the instability will be substantially reduced. The effect of enhanced dissipation is certainly present, but in the examples studied so far it causes only a relatively small decrease of the linear growth rate (see also Sec. VII.B). A new element that emerges in the picture is the appearance of the modes of oscillatory damping (in addition to the “standard” Rayleigh-Taylor modes). One can expect that the presence of these modes may favorably affect the nonlinear stage of the instability. However, this issue has not been studied thus far.

H. Finite Larmor radius (FLR) effects

As pointed out in Sec. II, the ion gyroradius is small compared to the typical sheath thickness during the run-in phase. At this stage, the ion component can be well described by fluid equations. The situation changes dramatically after the impact, when the ion thermal energy increases by orders of magnitude. Arber et al. (1995) studied stability of an equilibrium pinch with the ion Larmor radius comparable to the pinch radius. They conclude that even if the $m=0$ mode is almost stable by virtue of reaching the Kadomtsev profile, the $m=1$ mode still remains strongly unstable even with FLR effects taken into account. The growth rate is reduced by a factor of a few with respect to an ideal MHD, but this is insufficient to have a long-lasting steady-state equilibrium. Scheffel et al. (1997) have shown that the effects of a finite electron temperature have a destabilizing effect on the finite Larmor radius plasma.

Isichenko, Kulyabin, and Yan'kov (1989) considered the pinch column with a transition layer between the magnetic field and the pinch interior much thinner than the ion gyroradius (so that the ion motion in the pinch interior is unmagnetized). They have found that the growth of short wavelength perturbations, with $\lambda < r$, reaches a saturation $\sim v_T/r$, i.e., becomes slower than in the MHD approximation (where it is $\sim (v_T/r)(r/\lambda)^{1/2}$). However, the most disruptive mode with $\lambda \sim r$

remains essentially as unstable as in the MHD approximation. Therefore, the effects of a large Larmor radius do not provide sufficient stabilization in the imploded state.

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IX. NON-MHD PHENOMENA

A. Microturbulence and anomalous resistivity

A potential source of microinstabilities is the relative motion of electrons and ions. The velocity u of this motion is directly related to the current density:

$$j = enu \quad (9.1)$$

where n is the electron density. In the case of implosions of thin shells (like the ones formed in wire array implosions), assuming that the current occupies the whole shell thickness, one can express u in terms of the total pinch current I :

$$u(\text{cm/s}) \sim 10^4 \frac{AI(\text{MA})}{Z_{\text{eff}} \hat{m}(\text{mg/cm})}. \quad (9.2)$$

The assumption that the current flows through the whole thickness of the shell is quite plausible for the run-in phase of strongly radiating liners (Sec. III.G).

In Z pinches, the current is directed across the magnetic field; the electron temperature is comparable to or lower than the ion temperature. Under such circumstances, the most probable instability that can lead to the appearance of anomalous resistivity is the lower hybrid instability, described by Krall and Liewer (1971), Liewer and Krall (1973), and Davidson and Gladd (1975). The early studies of this instability have been summarized in the survey by Davidson and Krall (1977). More recent results, specifically addressing the issues of nonlinear stabilization, can be found in analytical studies by Drake, Huba, and Gladd (1983) and by Drake, Guzdar, and Huba (1983) and in numerical analysis by Brackbill et al. (1984). Possible effects of this instability on the Pease-Braginski current in a fiber pinch was studied by Robson (1991) and Chittenden (1995).

The "natural" frequency of the lower-hybrid oscillations is (see, for example, Davidson and Krall, 1977):

$$\omega_{LH} = \omega_{ce} \sqrt{\frac{Z_{\text{eff}} m_e}{A m_p}} \quad (9.3)$$

where m_p is the proton mass. In "practical" units,

$$\omega_{LH}(s^{-1}) = 4.5 \cdot 10^9 B(T) \sqrt{\frac{Z_{eff}}{A}}. \quad (9.4)$$

Expression (9.3) for the lower-hybrid frequency pertains to the situation $\omega_{pe} > \omega_{ce}$, which is typical for the Z pinches (ω_{pe} is the electron plasma frequency). Indeed, if rewritten in "practical" units, this condition reads as:

$$n_e(cm^{-3}) > 10^{13} [B(T)]^2 \quad (9.5)$$

and, as one can see, is satisfied with a large margin in the implosions of wire arrays and most of the gas-puffs.

In the analysis of the lower-hybrid instability, usually only perturbations with $k_{||}=0$ are considered. In order for such perturbations to cause electron scattering and contribute to the anomalous resistivity, their transverse scale-length should be comparable to or shorter than the electron gyroradius ρ_e . Otherwise, because the perturbations are slow, $\omega_{LH} \ll \omega_{ce}$, the electron magnetic momentum will be conserved, and the electrons will experience only adiabatic (reversible) variations of their velocity, thereby making impossible the appearance of anomalous resistivity.

The other current-driven microinstability is the ion acoustic instability, which typically has a higher threshold in terms of the relative velocity of electrons and ions. Extensive studies of this instability are summarized in the surveys by Vedenov and Ryutov (1973) and Galeev and Sagdeev (1976). In a singly charged plasma this instability can be present only if the electron temperature is much higher than the ion temperature, $T_e \gg T_i$; at $T_e \sim T_i$ the ion sound speed is comparable to the thermal velocity of the ions, and acoustic waves experience a strong ion Landau damping. However, in a plasma with $Z_{eff} \gg 1$, this instability can be excited even at $T_i > T_e$. Indeed, the sound speed in a plasma with high-Z ions is equal to

$$\sqrt{\frac{Z_{eff} T_e + T_i}{m_i}}, \quad (9.6)$$

while the ion thermal speed is $\sqrt{2T_i/m_i}$. Imposing a constraint that the sound speed exceed the ion thermal speed by a factor of 2, one finds the condition of weakly damped ion acoustic waves in a high-Z plasma:

$$T_e > 7T_i / Z_{eff}. \quad (9.7)$$

One sees that, at $Z_{eff} \gg 1$, the weakly damped ion acoustic modes can exist even at $T_i > T_e$. The critical current velocity for the onset of the ion acoustic instability under such conditions is several ion thermal velocities,

$$u_{crit} = \zeta v_{Ti}, \quad (9.8)$$

with ζ equal to 2-4. In the case of substances containing considerable amounts of hydrogen (for example, agar), the hydrogen ions, because of their high thermal velocities, can considerably increase the Landau damping and push the critical velocity to higher levels.

The effect of microinstability on plasma resistivity is traditionally described in terms of effective electron scattering frequency ν_{eff} that should be added to the electron-ion collision frequency $1/\tau_{ei}$ in the expression for the plasma resistivity,

$$\eta = \frac{m_e(\nu_{eff} + 1/\tau_{ei})}{ne^2}. \quad (9.9)$$

An estimate that is commonly used for the effective collision frequency produced by the lower-hybrid instability is (Davidson and Gladd, 1975; Drake et al., 1984):

$$\nu_{eff} = \zeta_1 \omega_{LH} \left(\frac{u}{v_{Ti}} \right)^2, \quad (9.10)$$

where ζ_1 is a numerical factor of the order of 1.

At low plasma density and high pinch currents, when u reaches the threshold of the ion-acoustic instability, this latter instability becomes dominant because it usually results in a higher effective collision frequency approaching the ion plasma frequency. Under condition (9.5), the ion plasma frequency is much higher than the lower hybrid frequency. Therefore, if the threshold for the excitation of the ion acoustic perturbations is reached at all, this instability takes over in establishing the anomalous resistivity. Upon reaching the instability threshold, $u = u_{crit}$, the effective scattering turns on so sharply that, in most cases of interest for Z pinches, it just keeps the relative velocity at the threshold level, so that the current density is

$$j = enu_{crit} \quad (9.11)$$

with u_{crit} as in (9.8). Equations (9.9)-(9.11) can serve as a basis for the analysis of the effect of anomalous resistivity on the properties of Z pinches.

Microfluctuations produced by the plasma current, in addition to the anomalous resistivity, may cause acceleration of a part of the plasma ions to suprathermal energies (see Sec. IX.B). The anomalous resistivity, if present, affects the skin depth and, therefore, the Rayleigh-Taylor instability. The heating rate of the electrons during the implosion phase may grow considerably and therefore lead to enhancement of the radiation losses compared to classical estimates. In the case of quasiequilibrium pinches, the anomalous resistivity affects the Pease-Braginski current (Robson, 1991). Our discussion provides only a general framework for the analysis of the corresponding processes. We will not try to consider a completely self-consistent picture. As we shall see, in the fast Z-pinch environment the current velocity u is typically smaller than or comparable to the ion thermal velocity. Therefore, we concentrate our attention on the discussion of the lower hybrid instability.

1. Run-in phase

To be specific, we discuss implosion of a wire array; later in this subsection we also mention gas-puffs. The velocity u of the relative motion of the electron and ion fluids is typically comparable to the ion thermal velocity. For a set of characteristic parameters of the run-in phase of tungsten wire array implosions ($A \sim 180$, $T \sim 40$ eV, $Z_{eff} \sim 7$, $I = 10$ MA, $\hat{m} \sim 3$ mg/cm), one finds that $u \sim 10^6$ cm/s, roughly equal to v_{Ti} . For the "canonical" lower-hybrid instability, according to the paper by Davidson and Gladd (1975), the growth rate at $u < v_{Ti}$ scales as

$$(u/v_{Ti})^2 \omega_{LH}. \quad (9.12)$$

In the aforementioned numerical example, it is of the order of $\omega_{LH} \sim 1.5 \cdot 10^{11} \text{ s}^{-1}$, i.e., the e-folding time for this instability is orders of magnitude shorter than the duration of the run-in phase. Accordingly, the instability reaches its nonlinear saturation.

One should, however, remember that the Coulomb collision times are very short,

$$\tau_{ei}(s) \sim \frac{5 \cdot 10^4 [T_e(\text{eV})]^{3/2}}{Z_{eff}^2 n_i(\text{cm}^{-3})}, \tau_{ii}(s) \sim \frac{2 \cdot 10^6 A^{1/2} [T_i(\text{eV})]^{3/2}}{Z_{eff}^4 n_i(\text{cm}^{-3})}, \quad (9.13)$$

for the electron-ion and ion-ion collisions, respectively, with A being the atomic weight of the ion. For the set of parameters chosen above, and $n_i \sim 10^{19} \text{ cm}^{-3}$, one finds that both the products $\omega_{LH} \tau_{ei}$ and $\omega_{LH} \tau_{ii}$ are considerably less than 1. In other words, with respect to the lower-hybrid modes, the plasma is strongly collisional. By itself, this does not necessarily mean that the instability is totally impossible. This rather means that the theory presented in the

aforementioned references should be reworked to include totally hydrodynamic description of both electrons and ions, in terms of the shifted Maxwellians. It also makes it very difficult for the instability to affect the classical estimate (4.14) for the plasma resistivity (because the anomalous collision frequency has to compete with classical collision frequency, which is very high).

In the gas-puff implosions with a smaller particle density the role of instabilities can be more important. However, the classical electron-ion collision time remains shorter than the expected anomalous collision time. Therefore, again, the current penetration should be determined by a classical resistivity and/or by gross hydrodynamic instabilities. The anomalous resistivity can possibly play some role in the behavior of a very low-density halo plasma that may surround the main discharge.

The effect of anomalous resistivity can, in principle, be used to reduce the current rise time in a wire array. The idea is to surround it by another, lower-density shell where the current would be sharply terminated by development of the anomalous resistivity (Branitski et al., 1996). Experimentally, these authors studied implosions of gas-puffs on a thin agar cylinder in the Angara-5 facility; Xe, Ar, and C_3H_8 (propane) were used as working gases. The annular jet had the radius $r_0 = 1.6$ cm and the mass 0.07 mg/cm; the maximum current was ~ 3 MA and the half-width of the current pulse was ~ 100 ns. The inner cylinder was made of agar, its radius was $r_i = 0.5$ -1 mm, and a mass 0.05-0.07 mg/cm. The switching of the current indeed occurred, but it didn't have a sharp front. The authors concluded that the current disruption in the outer shell was probably caused by the Rayleigh-Taylor instability, although other factors may have also contributed.

Baksht et al. (1997) also studied a multiwire array surrounded by an outer gas shell. The dimensions were larger than in the previous work: the diameter of the gas jet was 8 cm and the diameter of the wire array was 3 cm, with wires 20 micrometer in diameter (68 microgram/cm). Reproducibility of the current switching was not very good. To improve the symmetry of implosions, the authors are going to introduce preliminary ionization of the gaseous shell (Cf., Sec. V. of this paper). Note that another way of switching the current (not based on the phenomenon of anomalous resistance) is to use a light external wire array imploding on the smaller-diameter heavier wire array: the current through the inner array will be small until the outer array reaches it (Davis, Gondarenko, Velikovich, 1997).

2. The stagnation phase

In the stagnation phase, the temperatures of both plasma species are much higher, with the ion temperature considerably exceeding the electron temperature. On the other hand, the relative velocity u , according to (9.3), only decreases because of the increased Z_{eff} . Therefore, the ratio u/v_{Ti} should drop by a factor ~ 100 compared to the run-in phase. This decrease more than compensates for the growth of the magnetic field caused by a reduced pinch radius, therefore the growth rate (9.8) decreases to, roughly speaking, 10^8 s^{-1} . The corresponding e-folding time is long compared to the duration of the stagnation phase, and this instability can hardly have a significant effect on the plasma resistivity during this phase.

It is interesting to note that, for the stagnation phase, the ratios of the relative velocity u to the ion thermal velocity is equal to the ratio of the ion gyroradius to the pinch radius and can be expressed in terms of very few input parameters. One can show that, assuming that the current flows uniformly over the cross section of the stagnated plasma,

$$\frac{\rho_i}{r_{\min}} \sim \frac{u}{v_{Ti}} \sim \frac{A}{Z_{eff} \sqrt{\ln C}} \sqrt{\frac{m_p}{r_{op} \hat{m}}} \sim 10^{-3} \frac{A}{Z_{eff} \sqrt{\hat{m}(mg/cm)}}, \quad (9.14)$$

where $r_{op} = 1.6 \cdot 10^{-16} \text{ cm}$ is a classical radius of the proton, and C is the convergence (1.1). The estimate (9.14) corresponds to the ion temperature before the equilibration with electrons began. That is the ion thermal velocity approximately equals to the liner velocity just before the on-axis collapse.

Of course, if the current is concentrated in a thin shell, or flows through the low-density plasma halo, the role of microinstabilities may become more important. The other place where anomalous resistivity may become important is the neck (Fig. II.5) formed as a result of the development of a sausage mode (we discuss this latter situation in Sec. IX.B). A discussion of the effects of anomalous resistance in the neck and some further references on that issue can be found in Sasorov (1992).

Further study of the effects produced by anomalous resistivity in the equilibrium pinch can be found in Chittenden (1995). In the equilibrium pinches, where plasma density is typically lower than in fast pinches, the instability can become quite important because of a higher ratio u/v_{Ti} and much lower frequency of the Coulomb collisions.

B. Generation of suprathermal particles and particle beams

As mentioned in sections II and VI, development of the Rayleigh-Taylor instability may lead to a situation where one or several constrictions of the type shown in Fig. II.5 will be formed. It is commonly recognized that the formation of high-energy particles in Z pinches is related to formation of constrictions. A summary of the experimental results and earlier theories can be found in a paper by Vikhrev (1986).

One can distinguish three mechanisms that lead to the formation of high-energy particles in the constrictions. First, there is a direct acceleration mechanism related to the generation of a high inductive voltage during the current break-up. This mechanism was the first to be suggested to explain the generation of the neutrons in the experiments of early 1950s (see Vikhrev, 1986, for references to experimental works). Second, there is a mechanism related to compressional heating of the substance situated in the neck, accompanied by ejection of the hot material from the ends of the constriction (Fig. II.5b). Third, if a microturbulence is excited in the constriction (because of a high current density), there may occur a stochastic acceleration of the tails of the ion distribution function leading to generation of the high-energy ions. It is quite conceivable that all three mechanisms for the formation of fast ions may act simultaneously. This is what makes the analysis of experiments on the generation of fast ions so difficult.

The chain of events that leads to inductive acceleration is as follows. After a constriction develops, its impedance grows, and the current through the neck diminishes, causing the generation of a large inductive voltage. Spatial and temporal evolution of electric fields that can be generated in such an event have been analysed in great detail by Trubnikov (see a summary of these results in Trubnikov, 1986).

Some qualitative conclusions that can be drawn from Trubnikov's analysis are as follows. Consider the motion of an individual ion in the time-varying electric field perpendicular to the azimuthal magnetic field. If the ion gyroradius is small compared to the neck radius a , and the electric field varies at the time scale exceeding ω^{-1}_{Bi} , the electric field does not accelerate ions but rather causes their drift motion, with the velocity proportional to the electric field strength E . This is an adiabatic process in the sense that the drift velocity grows when electric field (in the location of the ion) grows and decreases when electric field decreases (either because of the temporal variation of the electric field, or because the ions leave the zone of a strong electric field near the neck). Therefore, to ensure an efficient nonadiabatic energy transfer to the ions, one has to assume that electric field varies on a time-scale short compared to the ion gyro-period. The

other possibility is the ion acceleration near the axis of the discharge where the magnetic field is small. (An interesting question is what sets the lower limit for the pinch radius in the constriction. One can speculate that this is the ion gyroradius.) An inductive electric field generated by the decreasing current is directed towards the cathode. Accordingly, the ions should be accelerated predominantly in this direction. An analysis of the experimental measurements at the Angara-5 facility from the viewpoint of their compatibility with the beam source of the neutron radiation is presented by Imshennik (1992).

One can note in passing that it is conceivable that the currents possibly present on galactic scales, form pinch-like structures, and development of the sausage instability gives rise to the generation of high-energy cosmic rays. This viewpoint was presented by Trubnikov (1990). The arguments pointing out at the presence of the currents up to 10^{19} A in the galactic environment were presented by Peratt (1986, 1990). Fig. IX.1 depicts an object near the center of our galaxy where the presence of filaments may be a reflection of the pinch-effect (Yusef-Zadeh, Morris, and Chance, 1984).

Fig. IX.1. The image of filaments near the center of our Galaxy obtained at the wavelength 20 cm (Yusef-Zadeh et al., 1984). According to Trubnikov (1990), they may be pinches.

The second mechanism, most completely presented in the paper by Vikhrev (1986), attributes the generation of fast particles to an adiabatic compression of the plasma in the neck. This process is conceived as a gradual compression of a plasma along the sequence of Bennett equilibria, with a gradual decrease of the linear particle density $\mathcal{N}$ (the number of particles per the unit length of the pinch) in the constriction by virtue of the axial ion losses through the ends. The Bennett equilibrium condition,

$$T = \frac{\mu I^2}{4\pi N}, \quad (9.15)$$

shows that the plasma temperature in the constriction grows. If the constriction is short, with the radius r comparable with its length ℓ , the hot plasma would escape very rapidly through the ends, and no significant amount of hot particles would be formed. However, if the length of the constriction is large, $\ell \gg r$, then the number of hot particles would increase, and conditions for generation of a considerable amount of neutrons (in case of deuterium or deuterium-tritium plasma) would be reached.

If the ion-ion collision frequency is high enough, then the ion distribution is almost isotropic, and so should be the neutron radiation generated in the constriction. Still, if some hot ions do escape along the axis, an anisotropy of the distribution function may appear. The situation is illustrated by Fig. IX.2, which depicts two ion trajectories originating on the axis. If the initial ion velocity is slightly tilted to the axis and directed towards the cathode, the magnetic field produces a focusing force, and the ion rapidly moves along the axis towards the cathode. On the other hand, if the initial velocity is directed toward the anode, the magnetic force is defocusing, and the ion gets involved in a gyromotion with a slow drift towards the anode. Therefore, some anisotropy of the neutron radiation may be present in this case too.

Fig. IX.2. Two ion trajectories originating in the same point "O" on the axis and forming initially the same small angle with the axis. If the initial velocity is directed towards the cathode, the ion trajectory remains in a close vicinity of the axis; if the initial velocity has an opposite direction, the ion trajectory acquires a peculiar character, with a much slower drift towards the anode.

The third mechanism can be efficient in a plasma of a relatively low density, where the plasma resistance is dominated by microfluctuations (Sec. IX.A). The ion scattering on microfluctuations usually leads to formation of a high-energy tail of the ion distribution function (see surveys by Vedenov and Ryutov (1975) and Galeev and Sagdeev (1976)). The evolution of the high-energy tail of the ion distribution function is governed by a Fokker-Planck-type equation,

$$\frac{\partial f}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} v^2 D(v) \frac{\partial f}{\partial v}, \quad (9.16)$$

with the diffusion coefficient proportional to the energy density of fluctuations. The distribution function in (9.16) is normalized according to the relationship

$$dn = 4\pi f(v) v^2 dv, \quad (9.17)$$

where dn is the number of ions per interval dv of the ion velocities. For simplicity, we present Eq. (9.16) for the case of an isotropic spectrum of the fluctuations. Equation (9.16) describes the diffusive broadening of the high-energy tail of the ion distribution. The majority of the ions remains in the ion core dominated by the Coulomb collisions. Despite the small number of ions in the tail, they may be responsible for nuclear reactions with a high energy threshold and may thereby be used for diagnostics purposes (to identify regimes where suprathermal particles are present).

Consider in more detail the ion acceleration in the ion acoustic turbulence in a high- Z plasma. Assuming approximately equal temperatures of the electrons and ions, one finds that the oscillations excited by this instability near its threshold (9.8) have the wave-number

$$k \sim \sqrt{Z_{eff}} \frac{\omega_{pe}}{v_{Te}}. \quad (9.18)$$

Based on the standard equations of the quasilinear theory, one finds that the diffusion coefficient in (9.16) is directly related to the effective electron collision frequency ν_{eff} that enters the expression for anomalous resistivity:

$$D \sim v^2 \nu_{eff} Z^2 \sqrt{\frac{m_e}{m_i}} \left(\frac{T}{m_i v^2} \right)^{5/2}. \quad (9.19)$$

The characteristic energy W^* of the tail ions at the time t after the onset of the anomalous resistivity will be

$$W^* \sim T(Z^2 v_{eff} t)^{2/5} (m_e / m_i)^{1/5}. \quad (9.20)$$

Equation (9.20) directly links the ion energy with the anomalous resistivity (9.9) and provides thereby a phenomenological link between the two phenomena: the anomalous resistivity and the formation of the ion tail.

The maximum energy is limited by the time of the existence of the turbulent state or by the residence time of the ion within the neck. An absolute upper limit is set by the condition that the ion gyroradius becomes comparable to the neck size. This yields the following estimate for the maximum ion velocity v_i :

$$\frac{v_i m_i}{ZeB} \sim a \quad (9.21)$$

where a is the neck radius. As $B \sim \mu I / 2\pi a$, one finds that

$$\frac{v_i}{c} \sim \frac{Z}{A} \frac{I(\text{MA})}{30}. \quad (9.22)$$

At a current ~ 20 MA, the protons, in principle, can be accelerated to subrelativistic energies (Eq. (9.22) corresponds to nonrelativistic energies; at higher currents it breaks down). Of course, this is an estimate from above. Still, it shows the significance of the pinch current in providing conditions for generating high-energy particles.

So far we have been discussing the generation of fast ions. Fast pinch discharges are often accompanied also by bursts of hard x rays, pointing to the presence of high-energy electrons. As a curiosity, one can mention the generation of multi-MeV electrons in high-altitude lightnings (Fishman et al., 1994). Formation of electron beams is strongly suppressed by the presence of the transverse magnetic field of the pinch. It is hard to expect formation of electric field exceeding cB — this would require a complete current breakup within the time of the order of r/c . At $E < cB$, on the other hand, electrons cannot accelerate; they experience a slow nonrelativistic drift motion, with a velocity $E/B \ll c$. For this reason, the models that attribute the formation of high-energy electrons by the mechanism of a local adiabatic compression are of some interest. They predict the formation of hot (possibly relativistic) electrons near the necks (Vikhrev, 1986). These areas then could serve as sources of hard x rays. It is interesting to note that electron beams were detected in so called X pinches (a discharge through two or more crossed wires), where they were generated near the intersection point (Ivanenkov et al., 1996).

Formation of beams of runaway electrons is possible near the pinch axis where the magnetic field is weak and the condition $E > cB$ can be satisfied. At a given density and a given electric field strength, there exists a group of electrons experiencing runaway. To be involved in a runaway process, the electron should have high-enough initial energy so that, before the first scattering, it doubles its energy. This condition, if expressed in terms of the effective collision frequency, reads as:

$$\frac{eEv}{v_{\text{eff}}(v)} > \frac{m_e v^2}{2}. \quad (9.23)$$

We have explicitly included the dependence of the effective collision frequency on the electron velocity. Electrostatic fluctuations of the lower-hybrid or ion acoustic type give rise to the dependence of $1/v^3$ type in v_{eff} , very much like for classical Coulomb collisions. If the classical collisions are important, v_{eff} should include them too. From Ohm's law, one finds that

$$u = \frac{eE}{m_e v_{\text{eff}}(v_{Te})}. \quad (9.24)$$

Because the current is carried by the main body of electron distribution, the collision frequency that enters this equation corresponds to "thermal" electrons. For the $1/v^3$ dependence of the collision frequency, one finds from (9.23) and (9.24) that the critical energy above which the runaway process begins is (Benford, 1978):

$$w_{\text{crit}} = T \frac{v_{Te}}{u}. \quad (9.25)$$

The drift velocity u usually does not exceed a few ion thermal velocities. Therefore, only a small fraction of the total electron population can be involved in a runaway process. The spectrum of the runaway electrons can be found in Benford (1978). Phenomena discussed in this section can be strongly affected by the presence of even weak axial magnetic field. In particular, the neck formation can be stopped because the axial field would grow inversely proportional to the square of the neck radius, while the azimuthal field would grow inversely proportional to the first power of the radius. On the other hand, axial magnetic field considerably broadens the zone where the runaway electrons can be accelerated.

In the implosions of hollow shells (e.g., the wire arrays), favorable conditions for runaway formation may be met inside the shell, where there is no magnetic field and the plasma density is

low. The axial electric field may be present inside the shell if the skin depth exceeds the thickness of the shell (see Sec. III.G). The beam of accelerated electrons will be formed then much earlier than the on-axis collapse occurs. An early appearance of the beam (detected by the x-ray radiation from the anode) can serve as an indicator of a great skin depth.

C. The Hall effect

When the electron gyrofrequency becomes greater than the electron collision frequency, the Lorenz force in the electron momentum equation becomes dominant over the electron-ion friction term. In a uniform plasma, the electron momentum equation should be written as:

$$m_e v_{ei} \mathbf{u} = -e\mathbf{E} - e(\mathbf{u} + \mathbf{v}) \times \mathbf{B} . \quad (9.26)$$

Here the electron-ion collision frequency includes, generally speaking, both the Coulomb collisions and anomalous scattering; $\mathbf{v}$ is the ion velocity (which almost coincides with the velocity of the center of mass); $\mathbf{u}$ is a relative electron-ion velocity. If $|\mathbf{u}|$ is greater than the characteristic velocity of the ion motion, then equation (9.26) shows that the magnetic field is convected together with electron fluid (not with a plasma as a whole). In the limit of a low collision frequency, the magnetic field is frozen into electron fluid. The possibility appears then that the magnetic field will be redistributed at a time-scale that is short compared to the time-scale of the ion motion. Because the ion density within this short time-scale remains constant, only those electron displacements are allowed which do not perturb electron density. This type of motion is described by so-called "electron magnetohydrodynamics" (EMHD), Gordeev, Kingsep, and Rudakov (1994), or "Hall magnetohydrodynamics" (Sudan, 1979). The latter name is related to the analogy of the last term in (9.26) to a Hall term in the theory of current flow in solid conductors. A general survey of EMHD has been published by Gordeev, Kingsep, and Rudakov (1994). A set of criteria defining the parameter domain where the effects of EMHD are important has been summarized in a more recent paper by Kingsep and Rudakov (1996). Dissipative phenomena in EMHD were discussed by Sevast'yanov (1993). Roughly speaking, the effects of EMHD become important if the following two conditions are satisfied:

$$\omega_{ce} > \nu_{ei}; \quad u > s, v_A \quad (9.27)$$

where s and v_A are the sound and the Alfven velocities, respectively.

The motions of electron fluid become particularly interesting when the plasma density is nonuniform. If the plasma density varies in the z direction, the skin-effect becomes dependent on the direction of the current (see Gordeev, Kingsep and Rudakov, 1994). Fast axisymmetric striations can be self-generated (Rudakov, Sevast'yanov, 1996). The streamlines of the current become wavy, with the axial wavelength of the order of the shell thickness. This happens within a time that is short compared to the time within which the ions would react to the corresponding force (the ion background is assumed to be steady).

An interesting and so far unsolved issue is that of the influence of these fast phenomena on the quality of the shell implosion and the development of slower instabilities involving the ion motion (in particular, the Rayleigh-Taylor instability). In this respect, one should note that the current perturbations discussed by Rudakov and Sevast'yanov propagate along the axis with a velocity approximately equal to u ; therefore, although the instantaneous force acting on the ions is strongly z -dependent, an averaging that occurs because of the travelling nature of these perturbations should make the average force z -independent and possible seed for $m=0$ hydrodynamic instability decreases.

An important issue is the electron flow at the boundary between the electrode and the plasma: at the electrode surface the tangential component of the electric field vanishes, and the electron flow becomes almost parallel to the wall. Weak collisions gradually shift the electrons in the direction of the wall. Because this (axial) motion is slow (at weak collisions) the resistivity of the transition layer becomes anomalously high. More details on these issues can be found in the aforementioned survey by Gordeev, Kingsep, and Rudakov. A 2-D effect in electron magnetohydrodynamics may lead to increased plasma resistance (Esaulov, Sasorov, 1997).

D. Spontaneous generation of the magnetic field

In implosions of uniform gas loads, where a shock wave propagates in front of the magnetic piston, there exists a zone of highly ionized plasma behind the shock but before the piston where the magnetic field is zero, at least in the ideal case where no stray early-time breakdowns occurred at the beginning of the shot. It turns out that, if the system does not possess a perfect symmetry, high magnetic field can be spontaneously generated in this zone. The mechanism we are referring to was identified a long time ago in conjunction with experiments on laser plasma heating and is associated with non-collinearity of gradients of electron temperature and density (Stamper et al., 1971). When ∇n and ∇T aren't parallel, an electromotive force is generated in

plasma that drives the current and produces magnetic field. This term should be added to the standard induction equation, which acquires the form (Stamper et al., 1971):

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{v} \times \mathbf{B} - \nabla \times (D_m \nabla \times \mathbf{B}) + \frac{\nabla n \times \nabla T}{en} . \quad (9.28)$$

The first term in the r.h.s. describes the convection of the magnetic field with a plasma flow (line-tying), the second term describes joule dissipation (in the case of a uniform magnetic diffusivity it reduces to a diffusion term $D_m \nabla^2 \mathbf{B}$), and the last term is the thermo-electromotive force. Non-collinearity of ∇n and ∇T in the problem under consideration may emerge from the waviness of the piston caused, in turn, by the Rayleigh-Taylor instability.

In the absence of convection and ohmic losses, the magnetic field would grow (formally) indefinitely. More precisely, it would be enhanced to such a value that the electrons would get magnetized and the equation (9.28) would break down. Let us for a while neglect the ohmic losses (this is equivalent to a statement that the skin-depth is much smaller than the spatial scale of the perturbations). Then the maximum magnetic field is determined from the balancing of the first and the third terms in the r.h.s. of Eq. (9.28). Estimating the velocity of a plasma flow as a sound velocity s , we find that, by the order of magnitude, the magnetic field will have the amplitude

$$B \sim \frac{T}{es\lambda} \quad (9.29)$$

where λ is a spatial scale of nonuniformities. It will vary (randomly, if the perturbations are random) at the scale λ . The magnetic field (9.29) corresponds to the condition that the ion gyroradius evaluated with this magnetic field is of the order of $\lambda/Z_{eff}^{1/2}$. Assuming that one deals with an Ar plasma with the temperature of 100 eV and $Z_{eff} \sim 10$, one finds that for $\lambda \sim 0.5$ mm, the magnetic field is $B \sim 10$ T. Under typical Z-pinch conditions, this magnetic field does not lead to magnetization of electrons (Eq. (2.13)). However, this field will be compressed at later stages of the implosion, when the shock wave had already converged on axis. This happens if the resistive dissipation time is longer than the compression time. In the opposite case, the magnitude of the magnetic field will be considerably decreased. The estimates for the conditions of a particular experiment can be made on the basis of Eq. (9.28).

Although this stochastic field is usually small compared to the external magnetic field, it may play a significant role after the quasiequilibrium configuration is formed (Sec. III) and necks

develop. In particular, this stochastic field may prevent the runaway electrons from being freely accelerated along the axis of the column (Sec. IX.B).

The other situation where this mechanism for magnetic field generation may play a role is in the case of a blow-off plasma filling the dynamic hohlraum. One can expect that this plasma will be strongly nonuniform, and that the conditions for the appearance of the thermal electromotive force will be satisfied. The presence of the random magnetic field will affect transport properties of the blow-off plasma.

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X. APPLICATIONS OF THE FAST Z PINCHES

A. Radiation sources

1. Hard x rays

One of the traditional applications of fast Z pinches lies in generating flashes of intense kiloelectron-volt radiation, with the energy of the quanta in the range up to 10 keV. For such applications, one can use a wire array made of some high-Z material (say, nickel), or an annular gas-puff of gases like Ar or Kr. If the parameters of the pinch are properly chosen, a plasma with an electron temperature of several hundred electron-volts to over 1 keV can be formed, and the excitation of the L or K shells becomes feasible. A survey of the studies in this area prior to 1988 was published by Pereira and Davis (1988).

A rough optimization of the Z-pinch parameters for the highest yield in the desired K or L transition can be made based on the following arguments: The efficiency of converting the magnetic energy into the kinetic energy of the imploding shell is determined by the condition $\Pi = \Pi_{opt}$, where Π is a dimensionless parameter defined by Eq. (2.6). As is clear from Eq. (2.6), for a given pulse-power generator, i.e., for given values of I_{max} and τ , the product $\hat{m} r_0^2$ must be kept constant. This means that the pinch mass $\hat{m}$ and the initial pinch radius r_0 can be varied only subject to the constraint

$$\hat{m} r_0^2 = const. \quad (10.1)$$

The kinetic energy per one ion scales as the implosion velocity square, i.e., as $(r_0/\tau)^2$. The electrons acquire their energy from the ions, and the electron temperature therefore correlates with $(r_0/\tau)^2$. According to Eq. (10.1), the heavier liners have smaller initial radii, i.e. the electron temperature at stagnation decreases with increased mass of the liner. Eventually the electron temperature becomes insufficient to excite a certain K or L transition. Conversely, for the liners with less mass (larger initial radii), the kinetic energy per ion grows. At first sight, this means that the lighter liners are better as the sources of kilo-electronvolt x rays. However, if one goes too far in this direction, the radiation yield starts to decrease because the radiation power per unit volume scales as the density squared and, at small masses, decreases. Because of this, the pinch plasma begins to expand and cool down before any substantial fraction of the thermal energy gets

converted into the radiation (see Sec. IV.B). The slower energy exchange between electrons and ions at higher temperatures also acts in this same direction (Sec. IV.C). These two opposite trends — decrease of the electron temperature at high masses, and decrease of the radiation power at low masses — determine the optimum mass of the imploding liner.

More detailed discussion of these issues, together with supporting experimental information, can be found in Pereira and Davis (1988), Thornhill, Whitney and Davis (1990), Deeney et al. (1993, 1994), and Whitney et al. (1990, 1994). In the recent experiments at the Z facility (Deeney et al., 1997c), the energy radiated was over four hundred kilojoules and the peak power nominally 85 TW (Fig. X.1).

Fig. X.1. Comparison of the measured current and x-ray power pulse (solid curves) with the calculated (dashed curve) current for a 120-wire, 17-mm-dia., 450- $\mu\text{m}/\text{cm}$ array (shot 2224). The calculation does not reproduce the late-time behavior. The calculated x-ray power is shifted 1 ns. The timebase has $t=0$ equal to the start of the equivalent voltage waveform used to drive the circuit model; this is 47 ns before the extrapolated zero of the current pulse. From Deeney et al. (1997c)

In real life, although a considerable fraction of the kinetic energy can be converted to radiation upon stagnation, the plasma column still has sufficient pressure to somewhat expand and to be compressed again by the magnetic pressure. This process probably explains the presence of a longer, lower-amplitude, and longer wavelength radiation pulse that follows the main peak (Peterson, 1997). Another phenomenon that may effect the final radiation yield is a short-circuiting of the transmission lines later in the pulse (Giuliani et al., 1990).

The stability of the Z-pinch implosion is important for efficient x-radiation generation; it sets the minimum effective size to which the pinch can be compressed. A more stable implosion allows one to increase the initial pinch radius and to reduce the mass while still having high density at stagnation (sufficient to radiate the thermal energy before the pinch rebounds).

To obtain harder x radiation, in the range of tens of kiloelectronvolts, one could use an alternative approach based on the adiabatic compression of hydrogen plasma seeded with heavy impurities; the plasma temperature could possibly be made as high as 10 keV, allowing excitation of the K-lines of such elements as Xe. We discuss this possibility in some more detail at the end of Sec. X.D.1.

2. Black-body radiation

Fast Z pinches with high atomic number materials are also used as a source of a thermal radiation with the temperature from tens of electronvolts to ~200 eV (and, in future, over 300 eV). If the implosion occurs in the center of a closed cavity (sometimes called a "hohlraum"), the radiation from the pinch, after several reflections from the walls, becomes almost black-body radiation (Matzen, 1997). The wall (and the radiation) temperature can be roughly evaluated from the equation:

$$P_{rad} = (1 - \alpha)A\sigma T^4, \quad (10.2)$$

where P_{rad} is the power radiated by the pinch, A is the surface area of the cavity, and α is the albedo of its walls. In the experiments at the Saturn facility at Sandia, the radiation temperatures were in the range of 80-90 eV (Matzen, 1997). In the experiments at the PBFA-Z facility, temperatures in the range of 120-140 eV have been obtained (Porter, 1997).

B. Studies of material properties under extreme conditions

Thermal radiation generated by the just-described method can be used to drive shock waves in various materials. By studying the shock velocity, one can gain information about the equations of state of the materials under study. A typical geometry of such an experiment is shown in Fig. X.2. Thermal radiation causes ablation of the material from the inner side of the sample, and the ablation pressure drives a shock whose velocity can be measured by measuring the time of the shock arrival at the outer side of the sample. To be sure that the radiation spectrum is indeed

close to the black-body spectrum (and, therefore, that the drive can be characterized by a single function $T(t)$), small cavities can be attached to the main one in such a way (Fig. X.3) that the samples are protected from direct irradiation by the pinch, and only the radiation from the walls of the cavity hits the surface of the sample (Matzen, 1997). Several configurations have been proposed that increase the uniformity of the radiation on the sample under study.

Fig. X.2. Experimental arrangements used in the studies of the shock wave propagation (from Olson, 1997): a) the average shock velocity can be measured from comparing the shock breakout times at two steps; b) continuous measurements of the shock velocity can be made in a wedge sample.

The hohlraum technique has been successfully used to study propagation of shocks in the materials which will be used in ICF capsules (Olson et al., 1997) and to study equations of state of metals in the pressure range of ~ 3 Mbar (Branitski, Fortov et al., 1996). The same technique is broadly used in physics research with laser-driven hohlraums where it has reached a high degree of sophistication. A general survey of this approach can be found in Rosen (1996). It has been successfully used for studies of the structure of shock-compressed materials (Kalantar, Chandler, et al., 1997), of hydrodynamic Rayleigh-Taylor and Richtmyer-Meshkov instabilities with controlled initial perturbations (Remington, Budil, et al., 1997), of the effects of a material strength on hydrodynamic phenomena (Kalantar, Remington et al., 1997, Remington, Budil et al., 1997), and for astrophysical simulations (Remington et al., 1997). An advantage of the Z-pinch-driven hohlraums is a larger size and considerably higher total radiation output (centimeters vs. millimeters, and hundreds of kilojoules vs. tens of kilojoules). This, for example, allows one to use thicker hydrodynamic packages, minimizing problems with radiation preheat.

Fig. X.3. A side-on view (above) and an end-on view (below) of a configuration with secondary hohlraums attached to the the primary pinch hohlraum eliminates the effect of a direct irradiation of the sample by the pinch plasma. From Matzen (1997).

Gasilov et al. (1995) suggested generating multimegabar shocks in a central rod (~ 1 cm long) hit by an imploding liner. For yet larger experimental volumes, other techniques, based on magnetic compression driven by chemical explosives, are feasible (Hawke et al., 1972).

C. Generation of high magnetic fields

The use of the imploding cylindrical shell for generating high magnetic fields had been suggested many years ago (Fowler et al., 1960; Sakharov et al., 1965). High fields are generated by compressing an initial modest axial field by an imploding, conducting cylindrical shell. Experiments with explosively driven systems have been reported in the 1960s (Fowler et al., 1960; Sakharov et al., 1965), reaching as high magnetic fields as 20 MG. Implosions of metal shells in the Z-pinch geometry were studied by Alikhanov et al. (1981); a maximum magnetic field of 3.5 MG was obtained in the volume of a few tens of cubic centimeters.

We present a qualitative consideration of the magnetic field compression in the Z-pinch setting, assuming that the thickness of the conducting shell is negligibly small and that the shell has a high conductivity. The condition of the conservation of the axial magnetic flux enclosed by this plasma shell is:

$$B_z r^2 = B_{z0} r_0^2, \quad (10.3)$$

where B_{z0} is the initial axial magnetic field. When the convergence ratio is high enough, the final axial magnetic field can be considerably greater than the initial one. Here we neglect the edge effects of the type discussed in Sec. III.F. This is correct if the length of the pinch is greater than its radius.

The compression of the axial magnetic field can be analyzed in a particularly straightforward fashion in a reference case of a constant pinch current. Then, the energy conservation law shows that at the point of the maximum compression, where the liner is at rest and its kinetic energy is zero, the following relationship holds:

$$\pi r^2 \frac{B_z^2}{2\mu} - \pi r_0^2 \frac{B_{z0}^2}{2\mu} = \pi r^2 \frac{B_\phi^2}{2\mu} \left(2 \ln \frac{r_0}{r} \right), \quad (10.4)$$

where B_ϕ is the azimuthal magnetic field at the initial radius of the liner. At a convergence $C \gg 1$, one can neglect the second term in the l.h.s. of this equation. One then finds that, at the stagnation point, the (axial) magnetic field inside the liner is related to the (azimuthal) magnetic field of the pinch current by the equation

$$B_z = B_\phi \sqrt{2 \ln C}. \quad (10.5)$$

Taking $C=20$, one finds that the magnetic field inside the liner can be made approximately 2.5 times higher than the azimuthal Z-pinch field at the point of the maximum compression. In other words, this scheme leads to relatively modest enhancement of the internal field compared to the external field that would be reached at the same convergence ratio. Still, this factor is non-negligible, especially because it is topologically more convenient to use the magnetic field inside the shell for the studies of interaction of super-high fields with matter.

From Eqs. (10.3) and (10.5) one can see that the initial axial magnetic field required to reach this state is

$$B_{z0} = B_{\phi0} \frac{\sqrt{2 \ln C}}{C} \quad (10.6)$$

where $B_{\phi0}$ is the initial azimuthal magnetic field. Taking, as an example, $B_{\phi0} = 0.5$ MG, and a modest convergence $C=20$, one finds that the initial axial magnetic field should be ~ 60 kG, and the final axial magnetic field will be 25 MG. Accounting for the finite thickness of the imploding shell leads to somewhat smaller enhancement factors, because part of the implosion energy is spent on the plasma heating and compression. These and other pertinent effects have been discussed, e.g., by Felber, Liberman, and Velikovich (1985).

In experiments carried out during the last decade, annular gas-puffs have been used to produce conducting imploding shells (Wessel et al., 1986; Felber, Malley et al., 1988; Felber, Wessel et al., 1988). Magnetic fields in the range of 40 MG were reported in Felber, Malley et al. (1988). In summary, an axial implosion of the seed magnetic field is a proven way of reaching an axial magnetic field ~ 3 times higher than the azimuthal magnetic field at the stagnation point.

D. Controlled thermonuclear fusion (CTF)

There are two significantly different ways for using fast Z-pinchs for reaching CTF. The first is based on the direct shock and/or adiabatic heating of an imploding DT plasma (possibly nested inside a liner made of a heavier material). The second is based on generating high-temperature black-body radiation by colliding the liner with some inner shell; the black-body radiation then drives a spherical pellet in very much the same fashion as in the indirectly driven laser fusion systems (for a survey of those see Lindl, 1995). We discuss these two schemes in the next two sub-sections.

As was mentioned in the Introduction, quasi-equilibrium pinchs (not the fast pinchs that are the subject of this survey) have also been considered as a potential candidate for fusion reactors, but we will not discuss that approach here. Surveys of quasistatic pinchs in conjunction with fusion applications have been published by Haines (1982), Dangor (1986), and Robson (1993).

A well-known difficulty with using fast Z pinchs in a future commercial fusion reactor is related to the considerable neutron and thermomechanical damage that would be suffered by the pulse power generator if the pinch is not separated from the generator by a large-enough distance. A possible solution to this problem was suggested by Robson (1989), who envisaged using two

long liquid lithium jets that would serve as electrodes for the Z-pinch. Drake et al. (1996) considered another technique that could possibly be used if the energy to be delivered to the Z pinch is less than ~ 1 MJ per pulse. For this case one could conceive of dropping miniature diode assemblies, consisting of fusion targets and the necessary circuitry, to the reactor chamber and energizing them by a charged particle beam or even by a fast projectile (in this latter case, the assembly would have to carry a seed magnetic field that would be compressed by a fast projectile and drive the Z-pinch circuit).

The whole issue of stand-off energy sources has not been explored in any detail. It would therefore be premature to write off the Z pinches as a prototype of a fusion reactor solely on the basis of the absence of the proven solutions for the power supply problem. In addition, even if such solutions are not found, the fast Z pinches can still be very useful for the fusion program; they could serve for a relatively inexpensive demonstration of the fusion ignition in a variety of pulsed power fusion concepts.

1. Plasma heating by implosion

First consider implosions of thin shells made of a DT mixture. Cryogenic DT fiber arrays can serve as such shells. The lifetime τ of the hot plasma column formed at the stagnation point will be of the order of r_{min}/v_{Ti} where r_{min} is the radius of the column (related to the initial radius of the shell by Eq. (1.1)), and v_{Ti} is the ion thermal velocity corresponding to the temperature ~ 10 keV (i.e., $v_{Ti} \sim 10^8$ cm/s). The Lawson criterion reads as

$$n(\text{cm}^{-3})\tau(\text{s}) > Q \cdot 10^{14} \quad (10.7)$$

where n is the DT plasma density and Q is the ratio of the fusion energy to the thermal energy of the imploded plasma (the gain factor). This condition can be rewritten as $W(\text{J/cm}) > 10^{31} Q^2/n(1/\text{cm}^3)$, where W is the energy per a unit length of the plasma column. Even with a relatively modest assumption with regard to the required gain, $Q \sim 10$, this condition means that one has to reach a density level of $\sim 10^{25} \text{ cm}^{-3}$ to keep the energy of the plasma below 10^8 J/cm. According to Eq. (3.2), to reach this energy, one would have to generate unrealistically high pinch currents, $\sim 10^9$ A. On the other hand, increasing the density in the imploded state above 10^{25} cm^{-3} would require attaining unrealistically high convergences. A possibility of improving the implosion stability and, thereby, the axial convergence was analyzed by Golberg, Liberman and Velikovich (1990), with a conclusion that a break-even requires a radial convergence ~ 30 , with the energy release in the range of hundreds of megajoules/cm.

These observations point out the desirability of using a heavier shell to confine the DT plasma near the stagnation point. In this case, however, another difficulty surfaces: high electron thermal conductivity in the fusion plasma. Because the thermal capacity of the heavy shell is much greater than that of the plasma it confines, there will be heat losses from the DT plasma to the confining shell. It turns out that this heat-loss mechanism leads to the approximately same energy limitations as discussed above and, for this reason, does not make this fusion concept more realistic.

Potentially, very high densities can be achieved in 3-D implosions with a linear convergence of ~ 30 (typical for laser-driven fusion). The whole concept then becomes similar to that of laser fusion, with the only difference being that the implosion of the capsule is driven by magnetic pressure. In principle, almost spherically symmetric implosions are feasible in the magnetic compression scheme, despite the fact that the magnetic pressure cannot be made spherically symmetric. This had been demonstrated in a multi-megajoule explosively driven experiment described by Mokhov et al. (1979). More recently, quasispherical implosions in the Z-pinch geometry have been experimentally studied by Degnan et al. (1995). A linear convergence ratio of $\sim 6-7$ has been reached. It remains, however, unclear whether the very fast, high-convergence implosions needed to ignite the fuel in the center of the capsule are actually feasible. This issue requires further analysis.

One more school of thought (e.g., Yan'kov, 1991) pursues the detonation wave approach, in which the nuclear burn wave would be ignited in some point of a cylindrical column and would propagate along the axis. The detonation could be ignited in a "neck" that could be deliberately produced at a certain axial location. Linhart et al. (1993) consider even the possibility of detonating a column of a pure deuterium (not a DT mixture!) by imploding a short section filled with DT. These schemes would give rise to very large energy release per pulse, in the range of 1 GJ (approximately equivalent to 250 kg of high explosives). Strong heating of a CH fiber plasma in the zone of a deliberately created constriction was observed by Aranchuk et al. (1997). Numerical simulations by Lindemuth (1990) have shown a spontaneous formation of hot spots resulting from the development of a sausage instability in cryogenic deuterium fibers.

Very high densities of DT fuel can be reached in the so-called staged pinch (Rahman et al., 1995), where a liner would implode onto a DT fiber situated near the liner axis and carrying an initial axial current. Compression of the azimuthal magnetic field has an interesting feature; the rate of its growth becomes very high when the outer liner comes close to the inner fiber. This is a

result of the fact that, when the gap between the fiber and the liner is much greater than the fiber radius, the fiber inductance depends on the gap only logarithmically, whereas with gaps smaller than the fiber radius, this dependence changes to linear. Therefore, the current in the fiber experiences a very sharp rise, compresses the fiber and ignites the fusion reactions. Numerical examples presented in Rahman et al. (1995), show that the fiber density may reach the values $\sim 10^{25} \text{ cm}^{-3}$ even at a relatively modest current of 2 MA in the imploding liner with the initial radius of 2 cm. However, this optimistic conclusion is based on the assumption that the inner surface of the liner remains cylindrical with the accuracy of a few micrometers at the time of the maximum compression. This does not seem easily achievable.

Probably, the most straightforward approach, if not to ignition then to break-even, could be based on adiabatic compression of magnetized plasma. It has been understood for many years that, to suppress heat losses from the fusion plasma to the walls of the imploding liner, one can use a relatively weak magnetic field, such that its pressure is small compared to the plasma pressure, i.e., the condition

$$\beta \equiv \frac{2\mu p}{B^2} \gg 1 \quad (10.8)$$

is satisfied. This condition is, in fact, almost mandatory because, if the inequality (10.8) reverses its sign, the liner implosion becomes inefficient: the liner works predominantly against the magnetic pressure, and the liner energy is converted predominantly to the energy of the compressed magnetic field (not the thermal energy of a plasma).

As was pointed out by Drake et al. (1996), a 3-D implosion of the liner is preferable compared to a purely cylindrical implosion. This can be understood in the following way: the z -component of the magnetic field inside the shell scales as the square instantaneous convergence C^2 ; in other words, the magnetic pressure scales as C^4 :

$$p_M = p_{M0} C^4, \quad (10.9)$$

where p_{M0} is the magnetic pressure at the beginning of the implosion. The pressure p of a fully ionized hydrogen plasma scales as a volume to the power $(-5/3)$, or

$$p = p_0 C^{10/3}. \quad (10.10)$$

Clearly, the magnetic pressure grows faster than the plasma pressure, and, at the high convergence ratios that are of interest in this problem, becomes greater than the plasma pressure.

On the other hand, for a 3-D implosion, the scaling for the magnetic pressure remains unchanged (Eq. (10.9)), while the plasma pressure now scales as C^5 and grows faster than the magnetic pressure.

Plasma confinement under condition (10.8) was discussed many years ago by Budker and his coworkers (see Alikhanov et al., 1966; Budker, 1973). In the context of a laser heated plasma it was discussed by Pashinin and Prokhorov (1971). Since then, it had been studied in a lot of detail theoretically and numerically (e.g., Vekshtein, 1987). The general conclusion was that, because the plasma pressure is almost constant over the radius, the plasma density becomes very high near the cold walls. Magnetic field gets convected to this high-density region from the plasma core, and a "cushion" of a very high magnetic field is formed near the walls. At moderate plasma betas (see Eq. (10.8) for the definition), below 10-20, the resulting confinement of the hot plasma core proves to be quite satisfactory: the confinement time exceeds tens of the Bohm confinement time. At high densities typical of the system under consideration, this confinement time does not substantially limit the plasma gain (see Drake et al., 1996, for more details). This concept is sometimes referred to as magnetized target fusion (MTF), Lindemuth and Kirkpatrick, 1983. A high-energy (many tens of megajoules) variant of this system is the MAGO device under study at Los Alamos National Laboratory and the All-Russian Scientific Research Institute of Experimental Physics (Lindemuth et al., 1996).

Among magnetic configurations that could be imploded are the field-reversed configuration (FRC), the spheromak, and the diffuse Z-pinch. Figure X.4 shows an FRC nested inside the liner. To make the implosion 3-dimensional, it was suggested to vary the liner mass density over the length, with a maximum density near the equatorial plane. The liner would be then squeezed near the ends faster than near the equator, and the FRC would be compressed both axially and radially. What is yet to be proven in this approach is the formation of a FRC suitable for the subsequent compression. The experience here is limited to relatively large (tens of centimeters in diameter) FRCs with a plasma with the density $\sim 10^{15} \text{ cm}^{-3}$. Preliminary scaling analysis (Ryutov, 1997) shows that creation of a much smaller (1-2 cm diameter) FRC with a plasma density $\sim 10^{18} \text{ cm}^{-3}$ and temperature $\sim 100 \text{ eV}$ is feasible.

Fig. X.4. A field reversed configuration nested inside the liner with axially varying thickness of the walls.

In Drake et al. (1996) this scheme has been analyzed for relatively slow ($1\text{--}2\ \mu\text{s}$) implosions of heavy (at least a few grams) liners that could be driven by relatively simple condenser banks. The conclusion was drawn that a 10-keV DT plasma under break-even conditions can be formed at as low plasma energy content as $\sim 100\ \text{kJ}$. No analyses have been made with regard to potentialities of this scheme with much lighter liners and faster drivers, such as the ones used in the PBFA-Z facility.

Note that, if seeded with heavier impurities, this plasma could serve as a high-power source of hard x rays. If the atomic number of the impurities is chosen in such a way that they are completely stripped at 10 keV, one can generate a smooth bremsstrahlung spectrum, corresponding to the temperature $\sim 10\ \text{keV}$. If the impurities are heavy (like Xe), then a considerable fraction of energy could be radiated in K-shell lines with the energies $\sim 20\ \text{keV}$ and higher (Toor, Ryutov, 1997).

2. Generation of a black-body radiation to drive a fusion capsule

A very different way of using Z-pinchs for fusion (Smirnov, 1991; Matzen, 1997; and also earlier unpublished reports from both Sandia and Troitsk) is based on the scheme resembling indirect-drive laser inertial confinement fusion (ICF); for a survey of the latter, see Lindl, 1995. When an imploding liner collides with an inner shell situated near the axis, the impact energy is converted into the thermal energy in both shells. If the imploding plasma shell is sufficiently thick, it will trap the radiation produced by the stagnation (Fig. X.5). In analogy with the terminology used in laser fusion, the interior of the shell filled with (almost) black-body radiation

is called a "hohlraum." To emphasize the fact that the walls of the hohlraum continue to implode after the impact between the two shells, a term "imploding liner hohlraum" or "dynamic hohlraum" is often used (Matzen, 1997; Brownell, 1997). A spherically symmetric capsule filled with DT fuel is situated in the center of the dynamic hohlraum. The thermal radiation causes the ablation and implosion of the surface layers of the capsule. We will not discuss here issues of capsule design and capsule implosion physics, but focus on some issues related to Z-pinch-driven hohlraums.

Fig. X.5. A schematic of the dynamic hohlraum experiment.

The radiation temperatures required for ignition of an indirect-drive DT capsule are in the range of 250 eV for designs similar to ICF capsules driven by lasers (Lindl, 1995). The temperatures presently reached in the Z-pinch experiments are in the range of 130-160 eV (e.g., Nash et al., 1997b). These ICF ignition and high yield capsules also require precise pulse shaping and a high degree of radiation symmetry.

The temporal dependence of the radiation flux can be controlled by adjusting the shape of the inner and/or the outer shells. For example, the configuration shown in Fig. X.6a will produce a relatively long "pedestal" caused by the interaction of the ends of the two shells, followed by a sharp pulse produced during the impact of the central, almost parallel, parts of the shell. The minimum attainable duration of the impact (and, accordingly, the maximum possible radiation flux) is determined by the thickness of two shells at the time of their impact. In the overall

context of pulse shaping and radiation symmetry, the importance of eliminating gross hydrodynamic instabilities becomes quite clear.

Fig. X.6. Various configurations of the dynamic hohlraum. a) A configuration with a shaped inner shell. A hyperboloid of revolution shape can be made from straight wires, by tilting them by the same angle with respect to the axis of revolution; b) A quasispherical implosion. This type of the implosion can be generated also with the initially cylindrical wire array with the axially varying linear mass density. The axial variation of the mass can be reached by a controlled surface deposition technique; the substrate will be an initially uniform wire array.

For these ICF capsules to reach ignition, the radiation field at the location of the capsule ablating surface should be spherically symmetric to within an accuracy of $\sim 1\%$ in the lower azimuthal modes. Because the Z-pinch geometry does not possess this symmetry, the size of the capsule should be a small fraction of the size of the radius of the dynamic hohlraum. One method of isolating the radiation source from the capsule is to fill the dynamic hohlraum with a low atomic number, low density material that creates a large plasma pressure but is relatively thin to the radiation produced by the stagnation of the imploding plasma. A more radical solution of this problem is an overall spherical symmetrization of the implosion, as shown in Fig. X.6b. Although the problem of the irradiation symmetry is difficult, it does not look insurmountable. Detailed studies of laser-driven hohlraums have shown that one can reach quite satisfactory results by a proper shaping of the hohlraum, by using optimally placed passive screens, and by reducing the pellet diameter to approximately one-third of the diameter of the hohlraum.

An attractive feature of Z-pinch-driven dynamic hohlraums is the relatively low cost of the pulsed-power generator and the high total impact energies (in the range of a few hundred kilojoules to a few megajoules) available in the existing devices like PBFA-Z, Saturn, or Angara-5.

In addition to radiation temperature, pulse shaping, and symmetry, several other issues should be considered in the design of the dynamic hohlraums. We mention some of them here without attempting serious analysis. (1) The ablation surface of the capsule must be isolated from the imploding liners and the shocks that they generate. For example, if the hohlraum is filled with a foam, the shock wave excited in the foam by the impact with the liner must not interact with the ablation surface of the capsule before the capsule implodes. In the case of an empty hohlraum, the low-density ejecta can be of some concern if they reach the capsule before the ablation process is well established. (2) Some low-density blow-off plasma will almost certainly be present inside of an empty hohlraum during the early phase of the Z-pinch implosion (its source can be the radiation preheat during the run-in phase or inductive splitting of the drive current). By itself, because of its low density, it will probably have no significant effect on the pellet. However, if the axial electric field penetrates through the liner, it could generate particle beams in this low-density plasma, which might then cause considerable preheat of the pellet and violate its spherical symmetry. (3) A magnetic field can be generated inside the plasma filling the hohlraum that may affect pellet performance.

All these issues are in a relatively early stage of assessment. On the other hand, none of them seems to pose insurmountable problems for the dynamic hohlraum concept in general. In particular, one could eliminate most of them altogether by using geometries of the type shown in Fig. X.7, where the Z-pinch implosions occurs at the ends of the main hohlraum.

Fig. X.7. A dynamic hohlraum with two Z-pinch radiation sources situated at the ends. From Matzen, 1997.

E. Other possible applications

In some modes of operation, especially if the neck formation could be triggered in a controlled way, Z pinches could serve as sources of high-energy, high-intensity beams of charged particles, in particular protons and deuterons. Such beams could then be used for the generation of various short-lived isotopes. The proton-rich isotopes required for positron emission tomography for medical purposes could possibly be produced (see Dawson, 1993, for a discussion of a different method of producing these isotopes). The beam could also be used for measuring the nuclear cross sections for very short-lived isotopes.

Rudakov et al. (1991) and Kingsep et al. (1997) discussed a possibility of creating a very-high-power flux to the electrodes by an adiabatic compression of plasma by an imploding liner. The heat losses to the liner would be suppressed by an axial magnetic field. The amplification of the flux to the electrodes would occur because of a very strong dependence of the electron thermal diffusivity on the electron temperature ($\chi_e \propto T_e^{5/2}$, see Sec. IV.D.4).

The fact that a Z pinch produces high-intensity radiation with a spectrum that is at least crudely controllable can be used for generating population inversion in various active media. Porter et al. (1992a,b) have successfully used the radiation of a sodium wire array to pump the Ne gas cell and create the population inversion for the transitions with the wavelength of 11 Å.

Z pinches have already been used for collecting information that could be of interest in astrophysics. Very encouraging results have been achieved in the studies of the opacities of the iron plasma (Springer, 1997). Interesting possibilities exist for simulating various high-energy-density astrophysical phenomena — for example, the formation of high-energy intergalactic jets. For this purpose, deliberate creation of jets of the type shown in Fig. III.4 could be made, and their propagation could be detected through the gas or plasma filling the space beyond the anode surface.

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XI. SUMMARY AND A GLANCE TO THE FUTURE

The fast Z pinch is a fascinating object, whose behavior is determined by a variety of processes of magnetohydrodynamics, radiative transport, atomic physics, plasma microinstabilities, and beam physics. A particular "shot" is formed by a chain of inseparable stages, from the current initiation and fast early-time instabilities, through the run-in phase where hydrodynamic instabilities distort and broaden the imploding shell, to a final on-axis stagnation, accompanied by a burst of intense radiation, by the possible formation of a transitional quasi-equilibrium configuration, and, sometimes, by disruption of the plasma column and the generation of fast particles. The Z pinch is to a high degree a self-organized object, where a change of a single input parameter may trigger a long chain of tightly interwoven processes occurring on various temporal and spatial scales and leading to an outcome very different from simple "mechanistic" predictions.

We believe that all pieces of physics that are important for the Z-pinch performance have been identified in this survey. Theory and simulations correctly describe many sides of this phenomenon. In particular, the gross dynamics of implosions of wire arrays is nicely predicted by 1-D hydrodynamic simulations, which provide a correct value of the time of a pinch collapse on axis (or on the inner cylinder). On the other hand, it is still difficult to predict, based on first principles, the temporal evolution of the thickness of the shell and the experimentally observed shape of the radiation pulse (although, by playing with a few adjustable parameters, one can reach a reasonable agreement).

In addition, experimental information on the development of hydrodynamic perturbations during the run-in phase is relatively sparse. More generally, one can say that, although the key physics phenomena have probably already been identified, their sometimes subtle interplay still requires a much better understanding.

One area where experimental information is almost nonexistent is the direct detection of a microturbulence that may be responsible for the anomalous resistance and other effects. Any measurements of this kind are particularly difficult at large facilities, where the huge energy release in the diode region requires heavy shielding and forces one to move the diagnostics equipment to large distances from the pinch area. In such a situation, indirect information can probably be used to detect anomalous plasma resistance. If it is actually present, then one can expect a considerable axial electric field to exist inside the empty imploding shell, leading to

generation of electron beams early in the pulse. Another way of making indirect measurements is related to the possibility of changing the composition of the pinch material. For instance, varying the relative amount of a light (say, deuterium) component may considerably affect microturbulence and may interfere with the phenomena of electron magnetohydrodynamics. Effects of electron magnetohydrodynamics can be controlled to certain extent by a weak axial magnetic field that would lift the restriction associated with the current flow across the field lines in the outer part of the pinch. The present survey contains information and references necessary for the planning of such dedicated experiments, which seem to be quite important. They would allow one to define the parameter domain where fast Z pinches are governed by standard MHD equations, and would establish the significance (or insignificance) of the anomalies outside that domain. Smaller university-scale facilities where one can study specific phenomena in a more benign environment can also be of great help.

Previous advances in fast-Z-pinch physics were reached in direct correlation with progress in pulsed power technology: the higher pinch current has always led to a considerable increase in the maximum kinetic energy of the imploding liner and maximum radiation power. A good recent example is the progress made in a transition from the Saturn facility to PBFA-Z facility. The current pulsewidth was increased over a factor of 2 (from ~ 50 to ~ 110 ns) and the current was increased by a factor of ~ 2.5 (from approximately 7 to approximately 18 MA), resulting in an increase in the radiated energy of a factor of ~ 5 and an X-ray power increase of a factor of ~ 3 (see Matzen, 1997). Therefore, it is interesting to discuss what one can expect from a further increase of the pinch current if new facilities become available. In the discussion that follows, we assume that the current can be presented as

$$I = I_{\max} f(t/\tau), \quad (11.1)$$

where f is some given bell-shaped function with a maximum equal to 1; in other words, we assume that the shape of the current waveform does not change, only scaling factors over the horizontal (τ) and vertical ($I_{\max}$) axes do (τ has a meaning of pulse-width). For discharges with similar current waveform, the optimum set of parameters is related by the equation (2.6):

$$\frac{\mu I_{\max}^2 \tau^2}{4\pi \hat{m} r_0^2} = \Pi = \text{const} \quad (11.2)$$

Provided the parameter Π is kept constant, the time-histories of the pinch radius are similar for similar current wave-forms (i.e., for the same function f in (11.1)). One can expect that, if the shell thickness h is determined by hydrodynamic instabilities, then the thicknesses of

two shells with the same value of the parameter Π will also have similar time-histories, i.e., the shell thickness will be proportional to r_0 times some function $g(t/\tau)$, identical for two systems with similar current wave-forms. This would also mean that, in two implosions with the same function f and the same Π , the attainable convergence C_{max} will be the same.

The velocity of the shell scales as r_0/τ ,

$$v \sim r_0/\tau. \quad (11.3)$$

According to (11.2), this means that the kinetic energy of the shell (per unit length) at the instant of self-impact scales as I_{max}^2 :

$$W_{kin} \propto I_{max}^2 \quad (11.4)$$

Remarkably, the radius, mass, and implosion time do not enter this relationship. The maximum power Q (per unit length) that can be released in the stagnation is of the order of

$$Q \sim W_{kin}(v/h) \quad (11.5)$$

where h in this equation is a shell thickness at the instant of the stagnation. For similar implosions, h scales as r_0 . Therefore, according to (11.3),

$$Q \propto I_{max}^2 / \tau \quad (11.6)$$

The initial radius of the pinch and the mass $\hat{m}$ do not enter this equation (provided the parameter Π is kept constant). Equations (11.4) and (11.6) provide a rationale for increasing the current in the generators used to feed the pinch discharge; both the maximum attainable implosion energy and the maximum power scale as I_{max}^2 . They also show that the maximum power is inversely proportional to the current pulse-width.

Some additional constraints on the parameters of the systems with a higher current may stem from the observation that, at higher currents, some of the applicability conditions of the hydrodynamic description of the system may break down. In particular, at higher currents one may enter the parameter domain where the relative velocity of the electrons and ions would considerably exceed the ion thermal velocity, triggering the onset of anomalous resistivity and increasing the Ohmic losses at the implosion phase. The relative velocity u of electrons and ions scales (for the liners made of the same material) as $I_0/\hat{m}$ (see Eq. (9.2)). The plasma temperature during the implosion of liners of heavy materials is not sensitive to the other parameters of the system and is in the range of 30-40 eV (i.e., the ion thermal speed is essentially constant). For a

tungsten liner with $T=40$ eV and $Z_{eff}=6$, one finds that the constraint $u < 4v_{Ti}$ (Eq. (9.8)) can be rewritten as:

$$\frac{I_{max}(MA)}{\hat{m}(mg/cm)} < 10 \quad (11.7)$$

The electron magnetization parameter $\omega_{Ce}\tau_{ei}$ (see Sec. II.G) at a constant temperature (characteristic for the run-in phase), and for liners made of the same material, scales as $I_0 r_0 / \hat{m}$. As we have seen in Sec. IX.A, in the implosions of the tungsten wire arrays at the PBFA-Z facility ($I_{max}=20$ MA, $\hat{m}=2$ mg/cm, $r_0=2$ cm) the magnetization parameter is ~ 0.5 . Accordingly, the condition that this parameter remains less than 1 can be presented as:

$$\frac{I_{max}(MA)r_0(cm)}{\hat{m}(mg/cm)} < 40 \quad (11.8)$$

It is not obvious that violation of the conditions (11.7) and (11.8) should necessarily lead to any catastrophic consequences. Still, to remain in the domain where a relatively simple hydrodynamic description is valid and where successful experiments at the existing devices PBFA-Z and Saturn have been carried out, it is probably reasonable, in the planning of the future experiments, to take into account constraints (11.7)-(11.8). Figure XI.1 shows the split of the parameter domain by the constraints (11.7)-(11.8) for $I_{max}=20$ MA, $\tau=100$ ns (Fig. XI.1a) and $I_{max}=60$ MA, $\tau=150$ ns (Fig. XI.1b).

Fig. XI.1. Parameter space for fast Z-pinches: a) $I_{\max}=20$ MA, $\tau=100$ ns; b) $I_{\max}=60$ MA, $\tau=150$ ns. Shaded area represents the domain where conditions (11.7)-(11.9) are satisfied. The bold line corresponds to Eq. (11.2) with $[I_{\max}(\text{MA})]^2 [\tau(\text{ns})]^2 / \dot{m}(\text{mg/cm}) [r_0(\text{cm})]^2 = 5 \cdot 10^5$ (a typical value for the current experiments).

At higher currents, constraints on the dimensions of the diode assembly may become important. If the magnitude of the surface current in the magnetically insulated transmission line (MITL) is too high, an explosion of the skin layer in the MITL may occur within the pulse duration, resulting in a great increase of Joule heating losses in the line. The surface current in the MITL scales as the current divided by the diode radius. To keep this current below its critical value, one would have to increase the diode radius proportionally to the current. The increase of the radius of the return current conductor should be accompanied by a proportional increase of the initial pinch radius (to maintain the parasitic inductances at a low level). Therefore, one concludes that the parameter $I_{\max}/r_0$ should remain below some critical level. Taking this value from the current experiment at the PBFA-Z facility, we obtain one more constraint on the parameters of the experiment with a higher current:

$$\frac{I_{\max}(\text{MA})}{r_0(\text{cm})} < 15. \quad (11.9)$$

The inspection of Fig. XI.1 reveals that there is a broad area in the parameter space where the pinch with a current several times higher than the presently-reached level of 20 MA can operate with a high efficiency characteristic of the existing experiments. Taking as an example the liner mass 10 mg/cm, the optimum radius is approximately 4 cm. With the assumed pulse-width of

150 ns, this would give implosion velocity only 30% higher than in the current experiments. This is beneficial in the sense that collisional relaxation times will remain short, and no further deviations from the local thermodynamic equilibrium than in current experiments will occur. At the same time, the total liner energy will increase by a factor of 9, and the power will increase by a factor of 6.

Reaching a higher current may be interesting not only for providing a means for generating higher radiation power or a higher temperature of the dynamic hohlraum but also for a range of problems of more general interest. In particular, it is worth noting that the presently achieved current is only several times less than the so-called proton Alfvén current,

$$I_{pA} \equiv \frac{2\pi m_p c}{e\mu} = 30 \text{ MA} \quad (11.10)$$

At the current exceeding I_{pA} , the gyroradius of a subrelativistic proton becomes smaller than the radius of the current channel. The attainment of this current may bring about some new interesting phenomena in the generation of high-energy ion beams at the stagnation phase (Sec. IX.B). This may be of great value for improving our understanding of the mechanism for the generation of cosmic rays.

To conclude: during the past decade, the physics of fast Z pinches has made significant progress, both in terms of pinch parameters attained in experiments at large facilities and in the identification of key physics issues governing pinch phenomenon. In the coming years, one can expect further progress related to: (1) development of diagnostic instrumentation; (2) dedicated experiments at smaller, university-scale facilities; (3) advances in computer simulations, and (4) development of schemes of mitigation of the most dangerous instabilities. Fast Z pinches will continue to play an important role as the sources of kilo-electronvolt radiation, as drivers for fusion-related experiments, and as sources of information on material properties at extreme conditions. With the development of better means of control of the neck formation at the point of a maximum compression, new possibilities can open for generating high-current beams of heavy ions. Fast Z pinches may also provide important insights into the mechanisms of astrophysical phenomena.

REFERENCES

- Afonin, V.I. (1995). "Ionization-thermal instability in a multicharged-ion plasma of Z-pinch", *Plasma Physics Reports*, 21, 250.
- Aivazov, I.K., V.D. Vikharev, G.S. Volkov, L.B. Nikandrov, V.P. Smirnov, V. Ya. Tsarfin (1988). "Formation of axial foreplasma channel in the initial stage of the compression of a multiwire system by megampere currents (experimental)", *Sov. J. Plasma Phys.*, 14, 110.
- Alikhanov, S.G., V.G. Belan, G.I. Budker, A.I. Ivanchenko, G.N. Kichigin (1967). "Creation of Megagauss fields by the method of magnetodynamic cumulation." *Sov. At. Energy*, 22, 1307.
- Alikhanov, S.G., V.P. Bakhtin, V.M. Brusnikin, I.S. Glushkov, R.Kh. Kurtmullaev, A.L. Lunin, A.D. Muzychenko, V.P. Novikov, V.V. Pichugin, V.N. Semenov, G.E. Smolkin, E.G. Utyugov, I. Ya. Shipuk. "Investigation of model thermonuclear systems with liners." *Plasma Physics and Controlled Nuclear Fusion Research, Proc. of the 6th Intern. Conf.*, Vienna, IAEA, 1977, v. 3, p. 517(1977)
- Alikhanov, S.G., V.P. Bakhtin, V.I. Vasil'ev, I.S. Glushkov, A.D. Muzychenko, V.P. Novikov, A.G. Oleinik, D.A. Toporkov (1981). "Review of fast Z-pinch liner programme." *Plasma Physics and Controlled Nuclear Fusion Research, Proc. of the 8th Intern. Conf.*, Vienna, IAEA, 1981, v. 2, p. 699.
- Alikhanov, S.G., V.I. Vasil'ev, V.I. Konov, E. Ya. Koshelev, K.N. Sidel'nikov, D.A. Toporkova (1984). "Formation of micropinches in a high-current linear Z-pinch with pulsed gas injection," *Sov. J. Plasma Phys.*, 10, 1051.
- Al'bikov, V.A., E.P. Velikhov, A.N. Veretennikov et al. (1990) "Angara-5-1 experimental complex," *Soviet Atomic. Energy*, 68, #1, 29.
- Antolak, A.J., A. Demiris, M.S. Derzon, L.W. Hrubesh, H. Louis, W.F. McNamara, D.H. Morse, J.F. Poco (1997). "Fabrication and characterization of Z-pinch foam targets". *Rev. Sci. instrum.*, 68, 858.
- Aranchuk, L.E., S.L. Bogolyubski, O.V. Tel'kovskaya (1985). "Energy balance of a high-current discharge in an exploding-wire plasma," *Sov. Phys. Techn. Phys.*, 30, 1312.
- Aranchuk, L.E., S.L. Bogolyubskii, G.S. Volkov, V.D. Korolev, Yu.V. Koba, V.I. Liksonov, A.A. Lukin, L.B. Nikandrov, O.V. Tel'kovskaya, M.V. Tulupov, A.S. Chernenko, V. Ya. Tsarfin, V.V. Yan'kov (1986). "Radiatively cooled Z-pinch produced by an exploding copper wire". *Sov. J. Plasma Phys.*, 12, 765.
- Aranchuk, L.E., S.A. Dan'ko, A.V. Kopchikov, V.D. Korolev, A.S. Chuvatin, V.V. Yan'kov (1997). "Experimental study of a plasma in the constriction of a fast Z-pinch". *Plasma Physics Reports*, 23, 194.
- Aranson, I., B. Meerson, P. Sasorov. "Nonlinear radiative-condensation instability and pattern formation: One-dimensional dynamics." *Phys. Rev*, E47, 4337.

- Arber, T.D., P.G.F. Russel, M. Coppins, J. Scheffel (1995). "Linear stability of the high temperature, dense Z-pinch". *Phys. Rev. Lett.*, 74, 2698.
- Atchinson, W.L., R.J. Fael, V. Morgan, R.E. Reynovsky. "Examination of instability growth in solid liner surfaces using comparisons of two dimensional MHD calculations and measured data". Paper C204, Conf. on High-Density Z Pinches, Vancouver, Canada, May 1997.
- Averkiev, V.V., A.N. Dolgov, V.K. Lyapidevskii, A.S. Savelov, G. Kh. Shalakhutdinov (1992). "Multichannel detection of X-ray dynamics of micropinch discharge." *Sov. J. Plasma Phys.*, 18, 374.
- Bailey, J., Y. Ettinger, A. Fisher, N. Rostoker (1982). "Gas-puff Z-pinches with D_2 and D_2 -Ar mixtures," *Appl. Phys. Lett.*, 40, 460.
- Baker, W.L., M.C. Clark, J.H. Degnan, G.F. Kiuttu, C.R. McClenahan, R.E. Reinovsky (1978). "Electromagnetic implosion generation of high-energy-density plasma", *J. Appl. Phys.*, 49, 4694.
- Baker, L. (1983). "Compressible Rayleigh-Taylor instability." *Phys. Fluids*, 26, 950.
- Baker, L., Freeman, J.R. (1981). "Heuristic model of the nonlinear Rayleigh-Taylor instability". *J. Appl. Phys.*, 52.
- Baksht, R.B., I.M. Datsko, A.A. Kim, A.Yu. Labetski, S.V. Loginov, V.I. Oreshkin, A. G. Russkikh, A.V. Fedyunin, A.V. Shishlov (1995). "Rayleigh-Taylor instability and K-radiation yield in the implosion of the Gas-Puffs." *Plasma Physics Reports*, 21, 907.
- Baksht, R.B., A.Yu. Labetski, S.V. Loginov, V.I. Oreshkin, A.V. Fedyunin, A.V. Shishlov (1997). "Z-pinch of a multiwire liner surrounded by an outer gas sheath." *Plasma Physics Reports*, 23, 135.
- Baksht, R.B., A.G. Russkikh, A.A. Chagin (1997). "Study of the effect of preionization on current sharing between sheaths of a double-cascade gas-puff." *Plasma Physics Reports*, 23, 175.
- Baksht, R.B., A.G. Russkikh, A.V. Fedyunin (1995). "Photoionization of the gas shell of an Ar liner." *Tech. Phys.*, 40, 1224.
- Baksht, R.B., A.L. Velikovich, B.A. Kablambaev, M.A. Liberman, A.V. Luchinski, N.A. Ratakhin (1987) "Compression of plasma sheaths containing frozen-in magnetic fields." *Sov. Phys. Techn. Phys.*, 32, 145.
- Baldwin, D. E., Ryutov, D. D. (1995). "Spinning laser targets." *Comments on Plasma Physics and Controlled Fusion*, 17, 1.
- Barak, G., N Rostoker (1982). "Semihydrodynamic model for ion separation in a fast pinch." *Appl. Phys. Lett.*, 41, 918.

- Barcilon, A., Book, D.L., Cooper, A.L. (1974). "Hydrodynamic stability of a rotating liner". *Phys. Fluids*, 17, 1707.
- Bartnik, A., G.V. Ivanenkov, L. Karpinski et S.A. Pikuz, T.A. Shelkovenko (1990). "Investigation of hot plasma produced by the electric explosion of glass fibers." *Sov. J. Plasma Phys.*, 16, 857.
- Bartnik, A., G.V. Ivanenkov, L. Karpinski, A.R. Mingaleev, S.A. Pikuz, V.M. Romanova, W. Stepniewski, T.A. Shelkovenko, K. Jach (1994). "Dynamics of a plasma formed by exploding metal wires and insulator fibers." *Quant. Electron.*, 24, 169.
- Bashilov, Yu. A., S.V. Pokrovskii (1976). "Taylor stability of the thin cylindrical shell." *JETP Lett.*, 23, 421.
- Basko, M.M. (1994). "Rayleigh-Taylor eigenmodes of a thin layer in the nonlinear regime." *Phys. Plasmas*, 1, 1270.
- Bateman, G. (1980). "MHD instabilities," MIT Press, Cambridge, MA.
- Beg, F.N., A. E. Dangor, P. Lee, M. Tatarakis, S.L. Niffikeer, M.G. Haines (1997). "Optical and X-ray observations of carbon and aluminum fibre Z-pinch plasmas." *Plasma Phys. Contr. Fusion*. 39, 1.
- Benford, G (1978). "Runaway electron model for x-ray emission in pinched discharges." *Appl. Phys. Lett.*, 33, 983.
- Bennett, W.H. (1934). "Magnetically self-focussing streams". *Phys. Rev.*, 45, 890.
- Bernstein, I.B., D.L. Book (1983). "Effect of compressibility on the Rayleigh-Taylor instability." 26, 453.
- Bishop, A. (1958). "Project Sherwood (The US Program in Controlled Fusion)." Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, USA.
- Bobrova, N.A., Razinkova, T.L., Sasorov, V.P. (1992). "Electrical explosion of frozen deuterium filaments." *Sov. J. Plasma Phys.*, 18, 269.
- Book, D.L.(1996). "Suppression of the Rayleigh-Taylor instability through accretion." *Phys. Plasmas*, 3, 354.
- Book, D.L., N.K. Winsor (1974). "Rotational stabilization of a metallic liner." *Phys. Fluids*, 17, 662.
- Book, D.L., E. Ott, M. Lampe (1976). "Nonlinear evolution of the sausage instability." *Phys. Fluids*, 19, 1982.
- Brackbill, J.U., D.W. Forslund, K.B. Quest, D. Winske (1990) "Nonlinear evolution of the lower-hybrid drift instability." *Phys. Fluids*, 27, 2682.
- Braginski, S.I. (1958). "The behavior of a completely ionized plasma in a strong magnetic field." *Sov. Phys. JETP*, 6, 494.

- Braginski, S. I. (1965) In: *Reviews of Plasma Physics* (Consultants Bureau, NY), v.1 p.205.
- Branitskii, A.V., V.D. Vikharev, S.V. Zakharov, A.G. Kasimov, V.P. Smirnov, V.Ya. Tsarfin (1991). "Initial stage of liner compression in the Angara-5-1 device". *Sov. J. Plasma Phys.*, 17, 311.
- Branitskii, A.V., V.D. Vikharev, A.K. Kasimov, A.A. Rupasov, G.S. Sarkisov, V.P. Smirnov, V.Ya. Tsarfin, A.S. Shikanov (1992 a). "Magnetic field diagnostics using the Faraday effect in high-current discharges in the Angara-5-1 device". *Sov. J. Plasma Phys.*, 18, 129.
- Branitskii, A.V., V.D. Vikharev, A.G. Kasimov, S.L. Nedoseev, A.A. Rupasov, G.S. Sarkisov, V.P. Smirnov, V.Ya. Tsarfin, A.S. Shikanov (1992 b). "Measurement of magnetic fields in a high-current Z-pinch in Angara-5-1 using Faraday rotation". *Sov. J. Plasma Phys.*, 18, 588.
- Branitskii, A.V., S.A. Dan'ko, A.V. Gerusov et al (20 co-authors) (1996). "Penetration of azimuthal magnetic flux to the interior of the unstable liner." *Plasma Physics Reports*, 22, 277.
- Branitski, A.V., V.E. Fortov, K.N. Danilenko, K.S. Dyabilin, E.V. Grabovski, O. Yu. Vorob'ev, M.E. Lebedev, V.P. Smirnov, A.E. Zakharov, I.V. Persiantsev (1996). "Excitation of intense shock waves by soft x-radiation." *Proc. 11th Intern. Conf. on High Power Particle Beams*, Prague, Czech Rep., June 1997, v.1, p.292. Czech Ac. Sci., K. Jungwirth and J. Ullschmied, Editors.
- Breizman, B.N., D.D. Ryutov (1974). "Powerful relativistic electron beams in a plasma and in a vacuum (theory)." *Nucl. Fusion*, 14, 873.
- Brownell, J.H. "Radiation environments generated by plasma Z-pinch stagnation on central targets." *Bull. Am. Phys. Soc.*, 42, 1947.
- Budker, G.I. (1973). "Fusion in a dense plasma." *Proc. 6th Europ. Conf. Contr. Fusion and Plasma Phys.*, Moscow, USSR, July 30—Aug. 4, 1973. v.2, p. 136.
- Bud'ko, A.B., M.A. Liberman, F.F. Kamenets (1990). "Self-similar dynamics of dense Z-pinch." *Plasma Phys. Contr. Fusion*, 32, 309.
- Bud'ko, A.B., M.A. Liberman (1992). "Stabilization of the Rayleigh-Taylor instability by convection in smooth density gradient: Wentzel-Kramers-Brillouin analysis." *Phys. Fluids*, B4 3499.
- Bud'ko, A.B., M.A. Liberman, A.L. Velikovich, F.S. Felber (1990). "Suppression of Rayleigh-Taylor and bulk convective instabilities in imploding plasma liners and pinches." *Phys. Fluids.*, B2, 1159.
- Bud'ko, A.B., A.L. Velikovich, M.A. Liberman, F.S. Felber (1989). "Development of Rayleigh-Taylor and bulk convective instabilities in the dynamics of plasma liners and oinches." *Sov. Phys. JETP*, 69, 76.

- Bychkov, V.V., Golberg, S.M., Liberman, M.A. (1994). "Self-consistent model of the Rayleigh-Taylor instability in ablatively accelerated laser plasma." *Phys. Plasmas*, 1, 2976.
- Bychkov, V.V., M.A.Liberman, A.L. Velikovich (1990). "Comment on 'Analytic solutions for Rayleigh-Taylor growth rates in smooth density gradients'." *Phys. Rev. A*, 42, 5031.
- Camarcet, N., J. Delvaux, B. Etlicher, D. Mosher, G. Raboisson, A. Perronnet (1985). "Electrical pulsed power generators of the 1 TW class." *Laser and Particle Beams*, 3, 415.
- Catto, P.J.(1978). "Model of the Rayleigh-Taylor stability of an ablating fluid." *Phys. Fluids*, 21, 30.
- Chandrasekhar, S. (1961). "Hydrodynamic and hydromagnetic stability." Clarendon Press.
- Chen, C.J., P.S. Lykoudis (1972). "Velocity oscillations in solar plage regions." *Solar Phys.*, 25, 380.
- Chittenden J.P. (1995). "The effect of lower hybrid instabilities on plasma confinement in fiber Z-pinch." *Phys. Plasmas*, 2, 1242.
- Chittenden, J.P., M.G. Haines (1990). "Processes terminating radiative collapse in a hydrogen fiber Z-pinch." *Phys. Fluids*, B2, 1889.
- Chittenden, J.P., R. Aliaga Rossel, S.V. Lebedev, I.H. Mitchell, M. Tatarakis, A.R. Bell, M.G. Haines (1997). "Two-dimensional magneto-hydrodynamic modeling of carbon fiber Z-pinch experiments." *Phys. Plasmas*, 4, 4309.
- Choi, P., C. Dumitrescu-Zoita (1997). "On the radiative collapse phenomenon." In: "Dense Z-pinch," AIP Conference Proceedings, v. 409, p. 51.
- Chukbar, K.V. (1993 a). "Hot current-carrying shell of a strongly emitting liner." *Plasma Physics Reports*, 19, 783.
- Chukbar, K.V. (1993 b). "Boundary effects on the magnetic field dynamics in thin films." *Plasma Physics Reports*, 19, 785.
- Clark, R.W., J.Davis, F.L. Cochran (1986). "Dynamics of imploding neon gas-puff plasmas," *Phys. Fluids*, 29, 1971.
- Clark, R.W., R. Richardson, J. Brannon, M. Wilkinson, J. Katzenstein (1982). "The dynamics of imploding Argon plasma," *J.Appl. Phys.*, 53, 5552.
- Cochran, F.L., Davis, J., Velikovich, A.L. (1995). "Stability and radiative performance of structured Z-pinch loads imploded on high-current pulsed power generators", *Phys.Plasmas*, 2, 1.
- Coppins, M., I.D. Culverwell, M.G. Haines (1988). "Time dependent Z-pinch equilibria." *Phys. Fluids*, 31, 2688.
- Dangor, A.E. (1986). "High density Z-pinch." *Plasma Phys. Contr. Fusion*, 28, 1931.

- Davidson, R.C., N.T. Gladd (1975). "Anomalous transport properties associated with the lower-hybrid-drift instability." *Phys. Fluids*, 18, 1327.
- Davidson, R.C., N.A. Krall (1977). "Anomalous transport in high-temperature plasmas with applications to solenoidal fusion systems." *Nuclear Fusion*, 17, 1313.
- Davis, J., N.A. Gondarenko, A.L. Velikovich (1997). "Fast commutation of high current in double wire array Z-pinch loads." *Appl. Phys. Lett.*, 70, 170.
- Dawson, J.M. (1993). "Non-energy applications for fusion". *Proc. of the 2nd Wisconsin Symposium on ^3He and Fusion Power*, p. 127, University of Wisconsin, WSCAR-TR-AR3-9307-3, J. Santarius, compiler.
- Deeney, C., P.D. LePell, F.L. Cochran, M.C. Coulter, K.G. Whitney, J. Davis (1993). "Argon gas puff implosion experiments and two dimensional modelling." *Phys. Fluids*, B5, 992.
- Deeney, C., P.D. LePell, B.H. Failor, J.S. Meachum, S. Wong, J.W. Thornhill, K.G. Whitney, M.C. Coulter (1994). "Radius and current scaling of argon K-shell radiation", *J. Appl. Phys.*, 75, 2781.
- Deeney C., J. McGurn, D. Noack, J.L. Porter, R.B. Spielman, J.F. Seaman, D.O. Jobe, M.F. Vargas, T. Gilliland, M.R. Douglas, M.K. Matzen. (1997a). "Streaked laser shadowgraphy of tungsten wire array implosions on the Saturn generator." *Rev. Sci. Instrum.*, 68, 653.
- Deeney, C., T.J. Nash, M.R. Douglas, R.B. Spielman, G.A. Chandler, D. Fehl, J.W. Thornhill, J. Davis, R.C. Clark, J.P. Apruzese, K.G. Whitney (1997b). "Scaling of above 4 keV x-ray sources to 100 kJ yields from the "Z" accelerator." *Bull. Am. Phys. Soc.*, 42, 1878.
- Deeney, C., T.J. Nash, R.B. Spielman, J.F. Seaman, G.C. Chandler, K.W. Struve, J.L. Porter, W.A. Stygar, J.S. McGurn, D.O. Jobe, T.L. Gilliland, J.A. Torres, M.F. Vargas, L.E. Ruggles, S. Breeze, R.C. Mock, M.R. Douglas, D. Fehl, D.H. McDaniel, M.K. Matzen, D.L. Peterson, W. Matuska, N.F. Roderick, J.J. MacFarlane, *Phys. Rev. E, Part B*, Vol. 56, No. 5-B, 5945 (1997c)
- Degnan, J.H., et al. (25co-authors) (1995). "Electromagnetic implosion of spherical liner". *Phys. Rev. Lett.*, 74, 98.
- DeGroot, J.S., C. Deeney, T.W.L. Sanford, R.B. Spielman, K.G. Estabrook, J.H. Hammer, A. Toor (1997). "High velocity implosions on PBFA Z." In: "Dense Z-pinches," AIP Conference Proceedings, v. 409, p. 157.
- DeGroot, J.S., A. Toor, S.M. Goldberg, M.A. Liberman (1997). "Growth of the Rayleigh-Taylor instability in an imploding Z-pinch". *Phys. Plasmas*, 4, 737.
- Derzon, M.S., G.A. Chandler, R.J. Dukart, D.J. Johnson, R.J. Leeper, M.K. Matzen, E.J. McGuire, T.A. Mehlhorn, A.R. Moats, R.E. Olson, C.L. Ruiz (1996). "Li-beam-heated hohlraum experiments at Particle Beam Fusion Accelerator II", *Phys. Rev. Lett.*, 76, 435.

- Derzon, M.S., T.J. Nash, G.O. Allhouse et al. (18 co-authors) (1997). "Current initiation in low-density foam Z-pinch plasmas". Rev. Sci. Instr., 68, 848.
- Derzon, M.S., et al. (15 co-authors). "Dynamic hohlraum results from Saturn and PBFA-Z experiments." Bull. Am. Phys. Soc., 42, 1994.
- Derzon, M.S., T.J. Nash, D.D. Ryutov (1997). "Mitigation of 2D Rayleigh-Taylor instability in an imploding wire array with the introduction of 3D features." Bull. Am. Phys. Soc., 42, 2069.
- Douglas, M.R., C. Deeney, N.F. Roderick (1997). "Effect of sheath curvature on Rayleigh-Taylor Mitigation in high-velocity uniform fill, Z-pinch implosions". Phys. Rev. Lett., 78, 4577.
- Drake, J.F., P.N. Guzdar, J.D. Huba (1983). "Saturation of the lower-hybrid-drift instability by mode coupling". Phys. Fluids, 26, 601.
- Drake, J.F., J.D. Huba, N.T. Gladd (1983). "'Stabilization' of the lower-hybrid-drift instability in finite-beta plasma". Phys. Fluids, 26, 2247.
- Drake, J.F., P.N. Guzdar, A.B. Hassam, J.D. Huba (1984). "", Phys. Fluids, 27, 1148.
- Drake, R.P., J.H. Hammer, C.W. Hartman, L.J. Perkins, D.D. Ryutov (1996). "Submegajoule liner implosion of a closed field line configuration." Fusion Technology, 30, 310.
- Dunning, M.J., S.W. Haan (1995). "Analysis of weakly nonlinear three-dimensional Rayleigh-Taylor instability growth", Phys. Plasmas, 2, 1669.
- Eidman, K., T. Kishimoto, P. Herrman et al., (1986). "Absolute soft x-ray measurements with a transmission grating spectrometer." Laser and particle Beams, 4, 325.
- Esaulov, A.A., P.V. Sasorov (1997). "Electric resistance and electron viscosity of plasma." Plasma Physics Reports, 23, 1997.
- Felber, F.S. (1982). "Self-similar oscillations of a Z-pinch". Phys. Fluids, 25, 643.
- Felber, F.S., M.A. Liberman, A.L. Velikovich (1985). "Methods for producing ultrahigh magnetic fields," Appl. Phys. Lett., 46, 1042.
- Felber, F.S., M.M. Malley, F.J. Wessel, M.K. Matzen, M.A. Palmer, R.B. Spielman, M.A. Liberman, A.L. Velikovich (1988). "Compression of ultrahigh magnetic fields in a gas-puff Z pinch." Phys. Fluids, 31, 2053.
- Felber, F.S., N. Rostoker (1981). "Kink and displacement instabilities in imploding wire arrays", Phys. Fluids, 24, 1049.
- Felber, F.S., F.J. Wessel, N.C. Wild, H.U. Rahman, A. Fisher, C.M. Fowler, M.A. Liberman, A.L. Velikovich (1988). "Ultrahigh magnetic fields produced in a gas-puff Z-pinch." J. Appl. Phys., 64, 3831.

- Fishman, G.J., P.N. Bhat, R. Mallozzi, J.M. Horack, T. Koshut, C. Kouveliotou, G.N. Pendleton, C.A. Meegan, R.B. Wilson, W.C. Paciesas, S.J. Goodman, S.J. Christian (1984). "Discovery of intense gamma-ray flashes of atmospheric origin," *Science*, 264, 1313.
- Fowler, C.M., W.B. Garn, R.S. Caird (1960). "Production of very high magnetic fields by implosion." *J. Appl. Phys.*, 31, 588.
- Freidberg, J.P (1982). "Ideal magnetohydrodynamic theory of magnetic fusion systems." *Rev. Mod. Phys.*, 54, 801.
- Galeev, A.A., R.Z. Sagdeev (1979). "Nonlinear plasma theory." In: "Reviews of Plasma Physics" (M.A. Leontovich, Ed.). Consultants Bureau, N.-Y., v.7, p.1.
- Gasilov, V.A. S.V. Zakharov, A. Yu. Krukowskii, K.V. Skorovarov (1995). "Generation of intense radiation fluxes and megabar shock waves using compressing liners." *Plasma Physics Reports*, 21, 399.
- Gasque, A.M., P. Grua, P. Romary, J.M. Sajer, D. Friart, P. Zrhnter, L. Jacquet, J. Pernet, S. Dan'ko, Y. Gorbunin, Y.G. Kalinin, A. Kingsep (1996). *Proc. of the 11th Intern. Conf. on High-Power Particle Beam*, Prague, *Inst. Plasma Phys*, v. 1, p.550.
- Giuliani, J.L., Jr., J.E. Rogerson, C. Deeney, T. Nash, R.R. Prasad, M. Krishan (1990). "Aluminum wire array implosions: Comparison of experiments and simulation." *J. Quant. Spectrosc. Radiat. Transfer*, 44, 471.
- Glimm, J., X.L. Li (1988). "Validation of the Sharp-Wheeler bubble merger model from experimental and computational data." *Phys. Fluids*, 31, 2077.
- Golberg S.M., M.A. Liberman, A.L. Velikovich (1990). "Plasma compression, heating and fusion in megagauss Z- θ pinch systems." *Plasma Physics and Controlled Fusion*, 32, 319.
- Gol'berg, S.L., Velikovich, A.L. (1993). "Suppression of Rayleigh-Taylor instability by the snowplow mechanism." *Phys. Fluids*, B5, 1164.
- Goncharov, V.N., R. Betti, R.L. McCrory (1997). "Self-consistent stability analysis of ablation fronts with small Froude numbers." *Phys. Plasmas*, 3, 4665.
- Gonzalez A.G., J. Gratton (1990). "Compressibility effects on the gravitational instability of a plasma-vacuum interface." *Plasma Physics and Controlled Fusion*, 32, 3.
- Gordeev, A.V. (1987). "Dynamics of a fast liner with several ion species." *Sov. J. Plasma Phys.*, 13, 713.
- Gordeev, A.V., A.S. Kingsep, L.I. Rudakov (1994). "Electron magnetohydrodynamics." *Physics reports*, 243, 215.
- Gratton, J., F.T. Gratton, A.G. Gonzalez (1988). "Convective instability of internal modes in accelerated compressible plasmas." *Plasma Physics and Controlled Fusion*, 30, 435.
- Grigor'ev, S.F., Zakharov, S.V. (1987). "Magnetohydrodynamics of an intensely emitting liner plasma." *Sov. Tech. Phys. Lett.*, 13, 254.

- Guderley, G. (1942). "Starke kugelige und zylindrische Verdichtungsstösse in der Nähe des Kugelmittelpunktes bzw der Zylinderachse." *Luftfahrtforschung*, 19, 301.
- Haan, S.W. (1991). "Weakly nonlinear hydrodynamic instabilities in inertial fusion." *Phys. Fluids*, B3, 2349.
- Haan, S.W. (1991). "Hydrodynamic instabilities on ICF capsules". LLNL report UCRL-JC-107592, June 1991.
- Haines, M.G. (1960). "The effect of electrodes in a linear pinch discharge." *Proc. Roy. Soc.*, 77, 643.
- Haines, M.G. (1982). "The physics of the dense Z-pinch in theory and in experiment with application to fusion reactor." *Physica Scripta*, T2/2, 380.
- Haines, M.G. (1989). "An analytic model of radiative collapse of a Z-pinch." *Plasma Phys. Contr. Fus.*, 31, 759.
- Haines, M.G., M. Coppins (1991). "Universal diagram for regimes of Z-pinch stability." *Phys. Rev. Lett.*, 66, 1462.
- Haines, M.G., A.E. Dangor, M. Coppins, P. Choi, I.H. Mitchell, J.P. Chittenden, J.M. Bayley, R.F. Aliaga Rossel, T.D. Arber, F. Beg, A.R. Bell, J. Scheffel, G. Decker, P. Russel, J.F. Worley (1996). "Fiber Z-pinch experiments and calculations in the finite Larmor radius regime." *Laser and Particle Beams*, 14, 261.
- Haines M.G. (1997). "An overview of the DZP project at Imperial College." In: "Dense Z-pinchs," AIP Conference Proceedings, v. 409, p. 27.
- Hammel, B.A., J.D. Kilkenny, D. Munro, B.A. Remington, H.N. Kornblum, T.S. Perry, D.W. Phillion, R.J. Wallace (1994). "X-ray radiographic imaging of hydrodynamic phenomena in radiation-driven materials - Shock propagation, material compression, and shear flow." *Phys Plasmas*, 1, 1662.
- Hammel, J.E. (1989). "The high density Z-pinch experiment." In: "HDZP," AIP Conference Proceedings, v. 195, p. 303.
- Hammer, J.H. (1995). "2D MHD simulations of radiating Z-pinch dynamics with LASNEX." *Bull. Am. Phys. Soc.*, 40, 1863.
- Hammer, J.H., J.L. Eddleman, P.T. Springer, M. Tabak, A. Toor, C. Deeney, R. Humphries, T.J. Nash, T.W.L. Sanford, R.B. Spielman, J.S. DeGroot (1996). "2D radiation-magnetohydrodynamic simulations of SATURN imploding Z-pinchs." *Phys. Plasmas*, 3, 2063.
- Hammer, J.H., Ryutov, D.D. (1996). "Generation of rotation and shear flow in an imploding liner" *Proc. of 11th Int. Conf. on High-Power Particle Beams*, Prague, June 1996, v.1, p. 178.

- Han, S.J., B.R. Suydam (1982). "Hydrodynamic instabilities in an imploding cylindrical plasma shell." *Phys. Rev.*, A26, 926.
- Harris, E.G. (1962). "Rayleigh-Taylor instabilities of a collapsing cylindrical shell in a magnetic field". *Phys. Fluids*, 5, 1057.
- Hawke, R.S., D.E. Duerre, J.G. Huebel, H. Klapper, D.I. Syeinberg (1972). "Method of isentropically compressing materials to several megabars." *Appl. Phys.*, 43, 2734.
- Hsing, W.W., C.W. Barnes, J.B. Beck, N.M. Hoffman, D. Galmiche, A. Richard, J. Edwards, P. Graham, S. Rothman, B. Thomas (1997). "Rayleigh-Taylor evolution in ablatively driven cylindrical implosions". *Phys. Plasmas*, 4, 1832.
- Huba, J.D. (1994). *NRL Plasma Formulary*, NRL/PU/6790-94-265.
- Hussey, T.W., Roderick, N.F., Kloc, D.A. (1980). "Scaling of (MHD) instabilities in imploding plasma liners", *J. Appl. Phys.*, 51, 1452.
- Hussey, T.W., Roderick, N.F. (1981). "Diffusion of magnetic field into an expanding plasma shell", *Phys. Fluids*, 24, 1384.
- Hussey, T.W., M.K. Matzen, N.F. Roderick (1986). "Large-scale-length nonuniformities in gas puff implosions." *J. Appl. Phys.*, 59, 2677.
- Hussey, T.W., N.F. Roderick, U. Schumlak, R.B. Spielman, C. Deeney (1995). "A heuristic model for the nonlinear Rayleigh-Taylor instability in fast Z-pinch". *Phys. Plasmas*, 2, 2055.
- Imshennik, V.S., "Nonhydrodynamic model of the plasma focus and the neutron yield in the pinch regime in Angara 5-1", *Sov. J. Plasma Phys.*, 18, 349 (1992).
- Imshennik, V.S., Neudachin V.V. (1987). "Azimuthal instability of a cylindrical radiative Z-pinch; estimate of the corresponding growth rate". *Sov. J. Plasma Phys.*, 13, 707.
- Imshennik, V.S., Neudachin V.V. (1988). "Numerical solution of the problem of the azimuthal instability of a cylindrical Z-pinch". *Sov. J. Plasma Phys.*, 14, 393.
- Inogamov, N.A. (1985). "Hydrodynamic instability of the ablation front in the presence of ablation acceleration of a layer." *PMTF*, 5, 110.
- Inogamov, N.A., A.V. Chekhlov (1993). "Multiplicity and uniqueness in the Rayleigh-Taylor instability theory: possible steady state solutions and selection from them", *Physics Doklady (Former Sov. Phys. Doklady)*, 38, 32.
- Isichenko, M.B., K.L. Kulyabin, V.V. Yan'kov (1989). "Kinetic theory of sausage instability of Z pinch." *Sov. J. Plasma Phys.*, 15, 617.
- Ivanenkov, G.V., A.R. Mingaleev, S.A. Pikuz, V.M. Romanova, T.A. Shelkovenko (1996). "Experimental study of X-pinch dynamics". *Plasma Physics Reports*, 22, 363.

- Kadomtsev, B.B. (1966). "Hydromagnetic stability of a plasma." In: *Reviews of Plasma Physics* (Consultants Bureau, NY), v.2, p. 153.
- Kalantar, D., D. Hammer (1993). "Observation of stable dense core within an unstable coronal plasma in wire-initiated dense Z-pinch experiments." *Phys. Rev. Lett.*, 71, 3806.
- Kalantar, D. H., E.A. Chandler, J.D. Colvin, R. Lee, B.A. Remington, S.V. Weber, L.G. Wiley, B.H. Failor, A. Hauer, J.S. Wark, M. Meyers, G. Ravichandran (1997). "Transient x-ray diffraction used to diagnose shock compressed solids on the Nova laser." *Bull. Am. Phys. Soc.*, 42, 2068.
- Kalantar, D.H., B.A. Remington, E.A. Chandler, J.D. Colvin, S.V. Weber, L.G. Wiley (1997). "Solid state material strength effects in hydrodynamic instability experiments on the Nova laser." *Bull. Am. Phys. Soc.*, 42, 1953.
- Katzenstein, J. (1981). "Optimum coupling of imploding loads to pulse generators," *J. Appl. Phys.*, 52, 676.
- Kilkenny, J.D., S.G. Glendinning, S.W. Haan, B.A. Hammel, J.D. Lindl, D. Munro, B.A. Remington, S.V. Weber (1994). "A review of the ablative stabilization of the Rayleigh-Taylor instability in regimes relevant to inertial confinement fusion," *Phys. Plasmas*, 1, 1379.
- Kikoin I.K. (Ed) (1976). "The tables of physical quantities." Moscow, Atomizdat (in Russian).
- Kingsep, A.S., Rudakov, L.I. (1995). "On a modification of MHD-models for Z-pinch", *Plasma Physics Reports*, 21, 576.
- Kingsep, A.S., V.I. Kosarev, A.I. Lobanov, A.A. Sevast'yanov (1997). "Numerical simulations of plasma compression by a light liner." *Plasma Physics Reports*, 23, 953.
- Kleev, A.I., Velikovich, A.L. (1990). "Filamentation instabilities of dynamic Z-pinchs and theta pinchs". *Plasma Phys. and Contr. Fus.*, 32, 763.
- Kloc, D., Roderick N.F., Hussey, T.W. (1982). "A simple model for plasma temperature in imploded plasma liners." *J. Appl. Phys.*, 53, 6706.
- Kolb, A.S. "Magnetic compression of plasma." *Rev. Mod. Phys.*, 32, 74 (1960)
- Krall, N.A., P.C. Liewer (1971). "Low-frequency instabilities in magnetic pulses" *Phys. Rev.*, A4, 2094.
- Kull, H.J. (1991). "Theory of the Rayleigh-Taylor Instability". *Physics Reports*, 206, 197.
- Landau, L.D., E.M. Lifshitz (1977). "Quantum Mechanics. Nonrelativistic Theory." NY, Pergamon Press.
- Landau, L.D., E.M. Lifshitz (1980). "Statistical Physics." NY, Pergamon Press.
- Landau, L.D., E.M. Lifshitz (1987). "Fluid Mechanics." NY, Pergamon Press.

- Lazier, S.E., T.L. Barber, M.S. Derzon, J.W. Kellogg (1997). "Time and space resolved measurements of visible-light and soft x-ray emission from foam z-pinch plasmas and implosions." *Rev. Sci. Instr.*, 68, 660.
- Leeper, R.J., G.A. Chandler, G.W. Cooper et al. (1997). "Target diagnostic system for the national ignition facility." *Rev. Sci. Instr.*, 68, 871.
- Lezzi, A.M., A. Prosperetti (1989). "Rayleigh-Taylor instability for adiabatically stratified fluids." *Phys Fluids*, A1, 1784.
- Lindemuth, I.R., R.C. Kirkpatrick (1983). "Parameter space for magnetized fuel targets in inertial confinement fusion." *Nucl. Fusion*, 23, 263.
- Lindemuth, I.R. (1990). "Two-dimensional fiber ablation in the solid-deuterium Z pinch." *Phys. Rev. Lett.*, 65, 179.
- Lindemuth, I.R., C.A. Eckdahl, R. Kirkpatrick et al. (1996). "Magnetic compression/magnetized target fusion (MAGO/MTF): a marriage of inertial and magnetic confinement" *Plasma Physics and Controlled Nuclear Fusion Research, Proc. of the 8th Intern. Conf., Vienna, IAEA*, v.2, p.723.
- Lindl, J. (1995) "Development of the indirect-drive approach to inertial confinement fusion and the target physics basis for ignition and gain". *Phys. Plasmas*, 2, 3933.
- Lozansky E.D., O.B. Firsov (1975). "Teoriya iskry" ("The theory of a spark"). Moscow, Atomizdat (in Russian).
- MacFarlane, J.J., P. Wang, T.J. Nash, M.S. Derzon, G.A. Allhouse, C. Deeney (1997). "Analysis of x-ray spectra obtained in foam Z-pinch experiment." *Rev. Sci. Instr.*, 68, 1069.
- Manheimer, W., D. Colombant, E. Ott. (1984). "Three-Dimensional, Nonlinear Evolution of the Rayleigh-Taylor Instability of a Thin Layer". *Phys. Fluids*, 27, 2164.
- Matuska, W., R.L. Bowers, J.H. Brownell, H. Lee, C.M. Lund, D.L. Peterson, N.F. Roderick. "Two dimensional modeling of the x-radiation output from perturbed Z pinches". *Phys. Plasmas*, 3, 1415 (1996).
- Matzen, M.K. (1997). "Z pinches as intense x-ray sources for high energy density physics applications". *Phys. Plasmas*, 4, 1519.
- Maxon, S., J.H. Hammer, J.L. Eddleman, M. Tabak, G.B. Zimmerman, W.E. Alley, K.G. Estabrook, J.A. Harte, T.J. Nash, T.W.L. Sanford, J.S. De Groot (1996). "Magnetohydrodynamic solution for a Z-pinch showing the production of a hot spot". *Phys. Plasmas*, 3, 1737.
- Meek, J.M., J.D. Craggs (1978). "Electric breakdown of gases." John Wiley and Sons, N.-Y.
- Meierovich, B.E. (1986). "Toward the realization of electromagnetic collapse," *Sov. Phys. Usp.* 29, 506.

- Meierovich, B.E., Sukhorukov, S.T. (1991). "Neutron yield in experiments with Z-pinchs in frozen deuterium filaments". *Sov. J. Plasma Phys.*, 17, 598.
- **Meshkov, E.E. (1969). "Instability of the interface of two gases accelerated by a shock wave", *Izv. AN SSSR, Mekhanika Zhidkosti I Gaza*, v.4, #5 151-157 (English translation:...)
- Mikaelian, K.O. "Rayleigh-Taylor Instability in Finite-Thickness Fluids with Viscosity and Surface Tension". *Phys. Rev. E* 54, 3676 (1996).
- Miyamoto, T. (1984). "Analysis of high-density Z-pinchs by a snowplow energy equation", *Nucl. Fusion*, 24, 337.
- Mokhov, V.N., V.K. Chernyshev, V.B. Yakubov, M.S. Protasov, V.M. Danov, E.I. Zharinov (1979). "Possible solution of the controlled thermonuclear fusion problem based on magnetogasdynamic energy storage." *Sov. Phys. Dokl.*, 24, 557.
- Mosher, D., D. Colombant (1992). "Pinch spot formation in high atomic number discharges", *Phys. Rev. Lett.*, 68, 2600.
- Mosher, D., S.J. Stephanakis, K. Hain, C.M. Dozier, F.C. Young (1975). "Electrical characteristics of high energy-density exploded-wire plasmas," *Ann. NY Acad. Sci.*, 251, 632.
- Mulser, P. (1987). "Reduction of Rayleigh-Taylor growth due to viscosity effects." *Laser and Particle Beams*, 6, part 1, 119.
- Munro, D.H. (1988). "Analytic solution for Rayleigh-Taylor growth rates in smooth density gradients", *Phys. Rev. A*, 38, 1433.
- Muron, D.J., M.J. Hurst, M.S. Derzon (1997). "Development of a visible framing camera diagnostics for the study of current initiation in z-pinch plasmas". *Rev. Sci. Instrum.*, 68, 656.
- Nash, T.J., M.S. Derzon, G. Allhouse, C. Deeney, D. Jobe, J. Seaman, T. Gilliland, J. McGurn (1997a). "Spectroscopic diagnosis of foam Z-pinch plasmas on SATURN". *Rev. Sci. Instrum.*, 68, 1083.
- Nash, T.J., M.S. Derzon, G. Allhouse, C. Deeney, J. Seaman, J. McGurn, D. Jobe, T. Gilliland, J.J. MacFarlane, P. Wang, D.L. Peterson (1997b). "Dynamic hohlraum experiments on SATURN." In: "Dense Z-pinchs," *AIP Conference Proceedings*, v. 409, p. 175.
- Nedoseev, S.L. (1991). "Dense Z-pinch investigation at Angara-5-1." IN: "Physics of Alternative Magnetic Confinement Schemes," *Proc. of the Intern. School of Plasma Phys. "Piero Caldirola"*, Editrici Compsitori, Bologna, 1991.
- Neudatchin, V.V., P.V. Sasorov (1991). "Magnetohydrodynamic instabilities of radiating Z-pinchs," 31, 1053.
- Newcomb, W.A. (1983). "Compressibility effect on instability growth rate." *Phys. Fluids*, 26, 3246.

- Olson, R.E., J.L. Porter, G.A. Chandler et al. (20 co-authors) (1997). "Inertial confinement fusion physics experiments on Saturn and Nova". Phys. Plasmas, 4, 1818.
- Ott, E. (1972). "Nonlinear Evolution of the Rayleigh-Taylor instability of a Thin Layer." Phys. Rev. Lett. 29, 1429.
- Pashinin, P.P., A.M. Prokhorov (1971). "High-temperature high-density plasma from a special gas target heated by a laser." Sov. Phys. JETP, 60, 1630.
- Parks, D.E. (1983). "Rayleigh-Taylor instabilities of an accelerating, compressible, perfectly conducting plane layer." Phys. Fluids, 26, 448.
- Pease, R.S. (1956). "Equilibrium condition of a pinched gas discharge cooled by bremsstrahlung radiation". Proc. Roy. Soc., 70, 11.
- Pereira, N.R. (1990). "A simple derivation of the Pease-Braginski current." Phys. Fluids, B2, 677.
- Pereira, N.R., J. Davis (1988). "X rays from Z-pinchs on relativistic electron-beam generators". J. Appl. Phys., 64, R1.
- Pereira, N.R., N. Rostoker, J.S. Pearlman (1984). "Z-pinch instability with distributed current." J. Appl. Phys., 55, 704.
- Peratt, A.L. (1986). Evolution of the Plasma Universe: I. Double Radion Galaxies, Quasars, and Extragalactic Jets." IEEE Trans. Plasma Sci., PS-14, 639.
- Peratt, A.L. (1990). "The evidence for electrical currents in cosmic plasma." IEEE Trans. Plasma Sci., PS-18, 26.
- Periera, N.R., Davis, J., and Pulsifer, P.E., Ed. of "Dense Z-Pinchs," AIP, Conference Proceedings # 409, 1997
- Peterson, D.L., R.L. Bowers, J.H. Brownell, A.E. Greene, K.D. McLenithan, T.A. Oliphant, N.F. Roderick, A.J. Scannapieco (1996). "Two-dimensional modeling of magnetically driven Rayleigh-Taylor instabilities in cylindrical Z-pinchs". Phys. Plasmas, 3, 368.
- Peterson, D.L., R.L. Bowers, J.H. Brownell, C. Lund, W. Matuska, K. McLenithan, H. Oona, C. Deeney, M. Derzon, R.B. Spielman, T.J. Nash, G. Chandler, R.C. Mock, T.W. L. Sanford, M.K. Matzen, N. Roderick (1997). "Application of 2-D simulations to hollow Z-pinch implosions." In: "Dense Z-pinchs," AIP Conference Proceedings, v. 409. p. 201(1997)
- Pikuz S.A., T.A. Shelkovenko, V.M. Romanova, G.S. Sarkisov, D.A. Hammer, D.F. Acton (1997). Studies of X-pinch plasma fine structure using high resolution optical and imaging spectroscopy methods." In: "Dense Z-pinchs," AIP Conference Proceedings, v. 409, p. 429.
- Plesset, M.S., D.-Y. Hsieh (1964). "General analysis of the stability of superposed fluids." Phys. Fluids, 7, 1099.

- Porter, J. L., R.B. Spielman, M.K. Matzen, E.J. McGuire, L.E. Ruggles, M.F. Vargas (1992a). "Demonstration of population inversion by resonant photopumping in a neon gas cell irradiated by a sodium z pinch" *Phys. Rev. Lett.*, 68, 796
- Porter, J. L., R.B. Spielman, M.F. Vargas, M.K. Matzen (1992). "Development of a sodium z-pinch load for use on the Saturn accelerator", *Rev. Sci. Instrum.*, 63, 5703 (1992b).
- Porter, J.L. (1997). "Z-pinch driven hohlraums as a radiation source for high energy density physics experiments." *Bull. Am. Phys. Soc.*, 42, 1948.
- Potter, D. (1978). "The formation of high-density Z-pinch". *Nuclear Fusion*, 18, 813.
- Raizer, Yu.P. (1991). "Gas Discharge Physics." Springer Verlag, Berlin-Heidelberg.
- Rahman, H.U., P. Amendt, N. Rostoker (1985). *Phys. Fluids*, 28, 1528.
- Rahman, H.U., F.J. Wessel, N. Rostoker (1995). "Staged Z pinch." *Phys. Rev. Lett.*, 74, 714.
- Rayleigh, Lord (1900). "Investigation of the character of equilibrium of an incompressible heavy fluid of variable density", *Scientific Papers*, Cambridge, England.
- Remington, B.A., J. Kane, R.P. Drake et al. (1997). "Supernova hydrodynamics experiments on the Nova laser." *Phys. Plasmas*, 4, 1994.
- Remington, B.A., K.S. Budil, S.G. Glendinning, D.H. Kalantar, R. Wallace, M.M. Marinak, S.V. Weber, S.W. Haan, J. Colvin, P. Miller, T. Peyser, D. Farley, A. Hauer, C. Cherfils, A. Richard, D. Galmiche, J. Kane, D. Arnett. (1997). "Hydrodynamic instability experiments using the Nova laser," *Bull. Am. Phys. Soc.*, 42, 1840.
- Robson, A.E. (1991). "Lower-hybrid-drift instability and radiative collapse of a dense Z-pinch". *Phys. Fluids*, B3, 1461.
- Roderick, N.F., T.W. Hussey (1984). "A model for the saturation of the Rayleigh-Taylor instability," *J. Appl. Phys.*, 56, 1387.
- Roderick, N.F. (1986). "Diffusion of the azimuthal magnetic fields into a cylindrical plasma", *J. Appl. Phys.*, 60, 1269.
- Roderick, N.F., T.W. Hussey (1986). "Magnetic diffusion smoothing with application to the hydromagnetic Rayleigh-Taylor instability," *J. Appl. Phys.*, 59, 662.
- Rosen, M.D. (1996). "The science applications of the high-energy density plasmas created on the Nova laser." *Phys. Plasmas*, 3, 1803.
- Rosenau, P., R.A. Nebel, H.R. Lewis (1988). "Analysis of Z-pinch dynamics with application to fibers." *Phys. Fluids*, B1, 1233.
- Rostoker, N., Peterson, G.G., Tahsiri, H. (1995). "Spin Control of the Rayleigh-Taylor Instability in a High-Density Pinch". *Comments on Plasma Phys. and Contr. Fus.*, 16, 129.

- Rudakov, L.I., K.A. Baigarin, Y.G. Kalinin, V.D. Korolev, M.A. Kumachov (1991). "Pulsed plasma-based x-ray source and new x-ray optics", *Phys Fluids*, B3, 2414.
- Rudakov, L.I., A.A. Sevastyanov (1996). "Fast dynamics of the current in a liner accelerated by the magnetic field of the current." *Plasma Physics Reports*, 22, 1095.
- Ruden, E., H.U. Rahman, A. Fisher, N. Rostoker (1987). "Stability enhancement of a low initial density gas-puff Z-pinch by E-beam preionization," *J. Appl. Phys.*, 61, 1311.
- Ryutov, D.D. (1996). "Rayleigh-Taylor instability in a finely structured medium". *Phys. Plasmas*, 4, 4376.
- Ryutov D.D. (1996) - unpublished.
- Ryutov, D.D. (1997). "A user facility for research on fusion systems with dense plasmas." Inv. talk at 2nd Symp. "Current trends in international fusion research," Wash. DC, March 1997, p.44.
- Ryutov D.D., R.Z. Sagdeev (1970). "Quasi-gasdynamical description of a hot electron cloud in a cold plasma". *Sov. Phys. JETP*, 31, 396.
- Ryutov, D.D., A. Toor (1998). "Enhanced thermal dissipation and the Rayleigh-Taylor instability in emulsion-like media". *Phys. Plasmas*, 5, 22.
- Sakharov, A.D., R.Z. Lyudaev, E.N. Smirnov, Yu. I. Plyushchev, A.I. Pavlovskii, V.K. Chernyshev, E.A. Feoktistova, E.I. Zharinov, Yu. A. Zysin (1965). "Magnetic cumulation." *Sov. Phys. Doklady*, 10, 1045.
- Samokhin, A.A. (1988). "Stability of symmetrical compression of a cylindrical liner modeling a system of wires." *Journ. Appl. Mech. Techn. Phys.*, 29, 243.
- Sanford, T.W.L., G.O. Allhouse, B.M. Marder et al. (22 co-authors) (1996). "Improved symmetry greatly increases X-Ray power from wire-array Z-pinches". *Phys. Rev. Lett.*, 77, 5063.
- Sanford T.W.L., T.J. Nash, R.C. Mock et al. (13 co-authors) (1997). "Time-dependent electron temperature diagnostics for high-power, aluminum z-pinch plasma". *Rev. Sci. Instrum.*, 68, 852.
- Sanford, T.W.L., G.O. Allhouse, R.C. Mock, R.B. Spielman, K.W. Struve, J.H. Hammer, J.S. De Groot, K.G. Whitney, J.P. Apruzese (1997). "Dynamics of a high-power aluminum-wire array Z-pinch implosion." *Phys Plasmas*, 4, 2188.
- Sarkisov, G.S., Etlicher, B. (1995). "Measurement of the electron temperature distribution in a Z-pinch by the laser absorption method". *JETP Lett.*, 61, 797.
- Sarkisov, G.S., Shikanov, A.S., Etlicher, B., Attelan, S., Rouille, C. (1995). "Observation of current-loss effect at the neck of a Z-pinch formed in the explosion of a wire". *JETP Lett.*, 61, 484.

- Sarkisov, G.S., Etlicher, B., Attelan, S., Rouille, C. (1995). "Detection of a "cold core" in a Z-pinch formed in a wire explosion". JETP Lett., 61, 555.
- Sasorov, P.V. (1991). "Turbulent corona in a dense Z-pinch". Sov. J. Plasma Phys., 17, 874.
- Sasorov, P.V. (1992). "Electrical resistance of constriction in Z-pinch". Sov. J. Plasma Phys., 18, 143.
- Scheffel, J., T.D. Arber, M. Coppins, P.G.F. Russel (1997). "Kinetic stability of the finite electron temperature Z-pinch", Plasma Phys. Contr. Fus., 39, 559.
- Sethian, J.D., A.E. Robson, K.A. Gerber, A.W. DeSilva (1987). "Enhanced stability and neutron production in a dense Z-pinch plasma formed from a frozen deuterium fiber". Phys. Rev. Lett., 59, 892.
- Sevast'yanov, A.A. (1993). "Effect of dissipative current pinching in electron magnetohydrodynamics". Plasma Physics Reports, 19, 227.
- Sharp, D.H. (1984). "An overview of Rayleigh-Taylor instability." Physica, 12D, 3.
- Shumlak, U., Hartman, C. (1995) "Shear flow stabilization of the $m=1$ kink mode in Z-pinch". Phys. Rev. Lett., 75, 3285.
- Shvarts, D., U. Alon, D. Oter, R.L. McCrory, C.P. Verdon (1995). "Nonlinear evolution of multimode Rayleigh-Taylor instability in two and three dimensions" Phys. Plasmas, 2, 2465.
- Skowronek, M., P. Romeas (1985). "Experimental study of the current penetration in a dense Z-pinch," J.App. Phys., 57, 2519.
- Smirnov, B.M. (1975). "Fizika slaboionizovannogo gaza" ("Physics of weakly ionized gases"). Moscow, Atomizdat (in Russian).
- Smirnov, V.P. (1991). "Fast liners for inertial fusion", Plasma Physics and Controlled Fusion, 33, 1697.
- Smith, R.S., W.O. Doggett (1985). "Experimental characterization of a puff-gas z-pinch plasma prior to implosion." Appl. Phys. Lett., 46, 1128.
- Spielman, R.B., D.L. Hanson, M.A. Palmer, M.K. Matzen, T.W. Hussey, J.M. Peek (1985a). "Efficient x-ray production from ultrafast gas-puff Z pinches." J. Appl. Phys., 57, 830.
- Spielman, R.B., Matzen, M.K., M.A. Palmer, P.B. Rand, T.W. Hussey, D.H. McDaniel (1985b). "Z-pinch implosions onto extremely low-density foam cylinders". Appl. Phys. Lett., 47, 229.
- Spielman, R.B., P. Corcoran, J. Fockler, et al. (1989). Proc. 7th IEEE Pulsed Power Conf., Monterey, p. 445.
- Spielman, R.B., S.F. Breeze, C. Deeney et al. (1996). "PBFA-Z: A 20 MA Z-pinch driver for plasma radiation sources", Proc. of the 11th International Conf. on High-Power Particle

- Beams, Prague, Czech Rep., 1996, edited by K. Jungwirth and J. Ullschmied (Ac. Sci. Czech Rep.).
- Spielman R.B., C. Deeney, G.A. Chandler et al. (1997). "PBFA Z: a 60 TW/5MJ Z-pinch driver." In: "Dense Z-pinches," AIP Conference Proceedings, v. 409, p. 101.
- Spielman, R.B., et al (1998). "Tungsten wire-array z-pinch experiments at 200 TW and 2 MJ", Phys. Plas. 5, 2105.
- Springer, P. T., K.L. Wong, C.A. Iglecias et al. (15 co-authors) (1997). "Laboratory measurements of opacity for stellar envelopes." J. Quant. Spectrosc. Radiat. Transfer, **58**, 927.
- Stallings, C, K. Childers, I. Roth, R. Schneider (1979). "Imploding Argon plasma experiments," Appl. Phys. Lett., **35**, 524.
- Stamper, J.A., K. Papadopoulos, R.N. Sudan, S.O. Dean, E.A. McLean (1971). "Spontaneous Magnetic fields in laser produced plasmas." Phys. Rev. Lett, **26**, 1012.
- K.W. Struve, T.H. Martin, R.B. Spielman, W.A. Stygar, P.A. Corcoran, J.W. Douglas (1997). "Circuit-code modeling of the PBFA-Z for Z-pinch experiments." 11th IEEE Int. Pulsed Power Conf., Baltimore, MD, June 29-July 2, 1997.
- Stygar, W.A., R.B. Spielman, H.C. Ives, W.B.S. Moore, J.F. Sieman, A.W. Sharpe, T.C. Wagoner, T.L. Guilleland, R.S. Broyles, J.A. Mills, T.A. Dinwoodie, J.S. Slopek, K.W. Struve, P.G. Reynolds (1997). "D-dot and B-dot monitors for Z-vacuum section power-flow measurements." 11 IEEE Int. Pulsed Power Conf., Baltimore, MD, June 29-July 2, 1997.
- Taylor, G. (1950). "The instability of liquid surfaces when accelerated in a direction perpendicular to their planes. I," Proc. Roy. Soc. (London), A201, 192.
- Thornhill J.W., K.G. Whitney, J. Davis (1990). "Transition from I^4 to I^2 scaling of K-shell emission in aluminum array implosions." J. Quant. Spectrosc. Radiat. Transfer, **44**, 251.
- Thornhill, J.W, J. Davis, J.P. Apruzese, K.G. Whitney, F.L. Cochran. "Decade Quad load performance." In: "Dense Z-pinches," AIP Conference Proceedings, v. 409, p. 193.
- Tonks, L. (1937). Trans. Electrochem. Soc., **72**, 167.
- Tonks, L. (1939). "Theory of magnetic effects in the plasma of an arc". Phys. Rev., **56**, 360.
- Toor, A., D. Ryutov (1997). "Generation of hard X-rays in a dense magnetized plasma adiabatically compressed by an imploding liner." Bull. Am. Phys. Soc., **42**, 2053.
- Trubnikov, B.A. (1986). "Particle acceleration and neutron production at the necks of plasma pinches". Sov. J. Plasma Phys., **12**, 271.
- Trubnikov, B.A. (1990). On the possible generation of cosmic rays in plasma pinches". Sov. Phys. Usp., **33**, 1061.

- Trubnikov, B.A., S.K. Zhdanov, V.P. Vlasov (1995). "Pinch mechanism of cosmic gamma-ray burst generation." *Plasma Physics Reports*, 21, 730.
- Turchi, P.J., W.L. Baker (1973). "Generation of high-energy plasmas by electromagnetic implosions," *J. Appl. Phys.*, 44, 4936.
- Turchi, P.J., A.L. Cooper, R. Ford, D.J. Jenkins (1976). "Rotational stabilization of an imploding liquid cylinder". *Phys. Rev. Lett.*, 36, 1546.
- Uman, M.A. (1986). "All about lightning." Dover Publications, 1986.
- Vedenov, A.A., D.D. Ryutov (1975). "Quasilinear effects in two-stream instabilities." In: *Reviews of Plasma Physics* (Consultants Bureau, NY), M.A. Leontovich, Editor, v.6, p. 1.
- Vekshtein, G.E. (1990). "Magneto-thermal processes in a dense plasma." In: *Reviews of Plasma Physics* (Consultants Bureau, NY), M.A. Leontovich, Editor, v.15, p. 1.
- Vekshtein, G.E., D.D. Ryutov, R.Z. Sagdeev (1971). "Asymptotic solution in the problem of the anomalous plasma resistance". *JETP Lett.*, 14, 212.
- Velikovich, A.L. (1991). "Rayleigh-Taylor instability of a plasma-vacuum interface in the limit of a large larmor radius." *Sov. Phys. Techn. Phys.*, 36, 210.
- Velikovich, A.L., G. Dimonte (1996). "Nonlinear perturbation theory of the incompressible Richtmyer-Meshkov instability." *Phys. Rev. Lett.*, 76, 3112.
- Velikovich, A.L., F.L. Cochran, J. Davis (1996). "Suppression of Rayleigh-Taylor instability in Z-pinch loads with tailored density profile". *Phys. Rev. Lett.*, 77, 853.
- Velikovich, A.L., Davis, J. (1995). "Implosions, equilibria, and stability of rotating, radiating Z-pinch plasmas." *Phys. Plasmas*, 2, 4513.
- Veretennikov, V.A., M.O. Koshevoi, N.F. Panferov, V.V. Pikalov, A.A. Rupasov, O.G. Semenov, A.S. Shikanov (1992). "Emission tomography of a micropinch discharge plasma." *Sov. J. Plasma Phys.*, 18, 131.
- Vikhrev, V.V. (1986). "Mechanism for neutron production in Z-pinches." *Sov. J. Plasma Phys.*, 12, 262.
- Vikhrev, V.V., E.O. Baronova. "Electron beam generation in Z-pinch discharges." *Proc 1996 Conf. on Plasma Phys.*, Nagoya, 1996.
- Vikhrev, V.V., S.I. Braginski (1986). "Dynamics of the Z-pinch." In: *Reviews of Plasma Physics*, edited by M.A. Leontovich, Consultants Bureau, New York, v. 10, p. 425.
- Vikhrev, V.V., Dobryakov, A.V., Zabaidullin, O.Z. (1995). "Simulation of an imploding Z-pinch during intense thermonuclear heat release." *Plasma Physics Reports*, 22, 96.
- Vlasov, V.P., B.A. Trubnikov, S.K. Zhdanov (1997). "Model of cosmic ray generation." *JETP*, 84, 211.

- Volkov, G.S., Utyugov, E.G., Frolov, I.N. (1993). "Compactness with which fast liners can be accelerated in Angara-5-1." Plasma Physics Reports, 19, 579.
- Waisman, E.M. (1979). "The magnetostatic field of a periodic cylindrical array of perfect conductors of arbitrary x-y cross section," J. Appl. Phys., 50, 23.
- Wessel, F.J., F.S. Felber, N.C. Wild, H.U. Rahman, A. Fisher, E. Ruden (1986). "Generation of high magnetic fields using a gas-puff Z-pinch", Appl. Phys. Lett., 47, 1116.
- Whitham, G.B. (1974). *"Linear and Nonlinear Waves"*, John Wiley & Sons, New York.
- Whitney, K.G., J.W. Thornhill, J.P. Apruzeze, J. Davis (1990). "Basic considerations for scaling Z-pinch x-ray emission with atomic number". J. Appl. Phys., 67, 1725.
- Whitney, K.G., J.W. Thornhill, J.L. Giuliani, Jr., J. Davis, L.A. Miles, E.E. Nolting, V.L. Kenyon, W.A. Speicer, J.A. Draper, C.R. Parsons, P. Dang, R.B. Spielman, T.J. Nash, J.S. McGurn, L.E. Ruggles, C. Deeney, R.R. Prasad, L. Warren (1994). "Optimization of K-shell emission in aluminum Z-pinch implosions: Theory versus experiment". Phys Rev E50, 2166.
- Wouchuk, J.G., A.R. Piriz (1995). "Growth rate reduction of the Rayleigh-Taylor instability by ablative convection," Phys. Plasmas, 2, 493.
- Yan'kov, V.V. (1991). "Z-pinches", Sov. J. Plasma Phys., 17, 305.
- Youngs, D.L. (1991). "Three-dimensional numerical simulation on turbulent mixing by Rayleigh-Taylor instability". Phys. Fluids, A3, 1312.
- Yusef-Zadeh, F., M. Morris, D. Chance (1984). "Large, highly organized radio structures near the galactic centre." Nature, 310, 557.
- Zabaidullin, O.Z., V.V. Vikhrev (1996). "Joule heating of a current layer due to plasma density inhomogeneities". Phys. Plasmas, 3, 2248.
- Zel'dovich, Ya.B., Yu.P. Raizer (1967). "Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena." Academic Press, NY and London.

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