Pulse Rocket Performance

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This note derives some relationships between the various parameters describing the nuclear pulse rocket, and gives numerical examples of performance attainable under the assumption of a fixed chamber temperature.

1. Introduction

The pulse rocket to be analysed here is a device to be designed for optimum performance in taking a fixed payload of about 50 tons from earth orbit through a round trip to Mars. The vehicle consists of the payload, which includes crew, shielding, life support equipment, controls, etc.; of some additional hardware such as exhaust nozzle, pumps, and the like, whose mass really depends on other design variables but will be treated as part of the fixed mass here; of propellant material to be expelled for momentum transfer; of propellant tankage; of a large and strong chamber or pressure shell in which discrete charges of propellant are raised to high temperature; and of a supply of nuclear energy source devices, one of which is to be introduced into the chamber with each propellant charge. This nuclear material could take the form of fairly conventional nuclear weapon-like devices of very small yield, or might be entirely different.

The analysis given here emphasizes the effect of varying the proportion of propellant material added to each charge. Maximum energy per unit mass, maximum exhaust velocity, and hence maximum propulsion efficiency, for a fixed value of energy per charge, are attained by not adding any propellant material to that necessary for the nuclear energy production. The latter can reasonably be supposed to consist mainly of high explosive material. Unfortunately this would also result in very

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high chamber temperatures, so that the addition of a supplementary propellant material of low atomic weight will probably be necessary. If one is limited to a given chamber temperature, then the highest specific impulse is attained by adding large amounts of hydrogen. At least some hydrogen or other material in addition to the nuclear device appears to be needed for pressure vessel and nozzle cooling.

In order to be able to use simple analytical relationships here, the hot gas in the pressure vessel is treated as a gamma-law gas, and the effect of varying amounts of dissociation on the energy-pressure relationship is ignored. The effect of pressure vessel size and gas temperature on the weight of the vessel is thus omitted entirely. This relationship is illustrated in a parallel study described by T. Stubbs.¹

2. Analysis

The system to be analysed will be described in terms of the following variables:

M_o	-----	Total vehicle mass at start of mission,
M_f	-----	Fixed mass: payload plus nozzle, pump, controls, structure, etc.,
M_s	-----	Mass of pressure vessel,
M_c	-----	Total mass of nuclear devices,
M_p	-----	Mass of supplementary propellant, such as hydrogen, water, methane, etc.,
μ	-----	Ratio of initial to final total mass of vehicle,
β	-----	Propellant tankage mass/propellant mass,

1. T. Stubbs, ADN No. 36.

Δv	-----	Overall velocity increment for mission,
v_{ex}	-----	Exhaust velocity,
g	-----	Acceleration due to gravity (980 cm · sec ⁻¹)
I_{sp}	-----	Specific impulse,
H	-----	Initial enthalpy per unit mass of propellant mixture in pressure vessel,
Y	-----	Energy yield of each nuclear device,
N	-----	Number of charges used throughout mission,
ϵ	-----	Mass of a single complete nuclear device,
ω	-----	Ratio of pressure vessel mass to energy yield per charge,
χ	-----	Mass fraction of supplementary propellant (e.g. hydrogen) in the total propellant mixture,
p	-----	Initial steady pressure in the pressure vessel, after decay of shocks, etc.
T	-----	Initial temperature of propellant mixture,
M	-----	Representative atomic weight of material comprising the nuclear device.

These variables are related by a number of equations, which

follow:

The initial total mass is the sum of its component parts:

$$1) \quad M_o = M_f + (1 + \beta) M_p + M_s + M_c .$$

The initial to final total mass ratio is given by the familiar rocket equation:

$$2) \quad \frac{M_o}{M_o - M_p - M_c} = \mu = e^{\frac{\Delta v}{gI_{sp}}} .$$

The ideal exhaust velocity is equal to the square root of twice the specific enthalpy in the chamber; as an estimate of the average effective value allowing for pressure decay during a pulse, for incomplete expansion, and for incomplete recovery of dissociation energy, we will use 70% of that value. This reduction is believed to be about right, perhaps a little on the pessimistic side. Separate studies will produce a better number.

$$3) \quad gI_{sp} = v_{ex} \approx 0.7 \sqrt{2H} .$$

We will assume that the supplementary propellant is hydrogen. In the pressure-temperature range of interest to us, the specific enthalpy is equal approximately to 1.2 times the energy per unit mass,

$$H \approx 1.2 \frac{Y' N}{M_p' + M_c'} ,$$

or putting Y and M_p , M_c into convenient units of tons TNT equiv. and tons mass, with H in c.g.s. units,

$$4) \quad H = 5.29 \times 10^{10} \frac{YN}{M_p + M_c} .$$

The total mass of nuclear devices is N times the mass per unit,

$$5) \quad M_c = Nc .$$

The pressure vessel mass is assumed to vary in constant proportion to the energy yield per charge^{2,1}:

$$6) \quad M_s = \phi Y$$

We have now six equations and eight unknowns describing the system. The unknowns are M_o , M_p , M_s , M_c , I_{sp} , H , Y , and N . Specification of any two of these will then serve to define the system completely. Let us now reduce the system of equations to a single one relating M_o to I_{sp} and M_p . By combining (3) and (4), we obtain

$$7) \quad \frac{YN}{M_p + M_c} = 19.3 (10^{-6} g I_{sp})^2,$$

and then with (5) and (6) we have

$$8) \quad M_s = 19.3 (10^{-6} g I_{sp})^2 \frac{\epsilon \omega (M_p + M_c)}{M_c}.$$

Rearranging (2) gives

$$9) \quad M_c = \frac{\mu-1}{\mu} M_o - M_p,$$

and finally, substituting (8) and (9) into (1) leads to

$$10) \quad M_o = \mu \left[M_f + \beta M_p + \frac{19.3 (10^{-6} g I_{sp})^2 \epsilon \omega}{1 - \frac{\mu M_p}{(\mu-1) M_o}} \right].$$

In Section 1, the statement was made that minimizing the value of M_p , for fixed Y , would give maximum propulsion efficiency (i.e. minimum M_o) by producing maximum I_{sp} . Now we can see from inspection of equation (10) that zero M_p gives minimum M_o even for a fixed value of I_{sp} . This comes

about through elimination of tankage weight and minimization of the pressure vessel weight (fixed I_{sp} and low mass implies low yield, hence low vessel weight).

Setting $M_p = 0$ in equation (10) gives:

$$11) \quad M_o = e^{\frac{\Delta v}{g I_{sp}}} \left[M_f + 19.3 (10^{-6} g I_{sp})^2 \epsilon \omega \right],$$

and to minimize M_o , we differentiate with respect to I_{sp} and set the result equal to zero, giving

$$12) \quad 0 = \frac{\Delta v}{g} \left[\frac{M_f}{I_{sp}^2} + 19.3 \times 10^{-12} g^2 \epsilon \omega \right] - 38.6 \times 10^{-12} g^2 \epsilon \omega I_{sp}.$$

Now take

$$\Delta v = 60,000 \text{ ft} \cdot \text{sec}^{-1}$$

$$M_f = 62.5 \text{ tons}$$

$$\epsilon = 0.025 \text{ tons (50 lb.)}$$

$$\omega = 21.3 \text{ tons/ton TNT equiv.}$$

and obtain

$$I_{sp} = 2177 \text{ sec.}$$

$$M_o = 260 \text{ tons (520,000 lb.)}.$$

This system, using no propellant mass except that of the nuclear devices themselves, is an unrealistic one, since clearly at least some additional material will be pumped through the pressure shell as a coolant, and also since no attention has been paid to temperature. Reference to equation (3) gives $H = 1.1 \times 10^5 \text{ cal} \cdot \text{g}^{-1}$, and if we assume that the nuclear device is a weapon-like implosion system with most of the mass being TNT, with average (dissociated) atomic weight ~ 10 , then the temperature is in the neighborhood of $200,000^\circ \text{K}$.

Being now satisfied that a substantial amount of low-atomic-weight additive is necessary, we will look for a more reasonable estimate of performance. Let us assume that the chamber temperature must be limited to some fixed maximum value T . We will then study performance as a function of the one remaining degree of freedom in the system.

The contents of the rocket chamber at the beginning of each pulse cycle will consist of material with a mass fraction (χ) of low-atomic-weight supplementary propellant, and a mass fraction ($\chi-1$) of residue from the nuclear device, which we will treat as a monatomic gas of atomic weight M . The enthalpy of the device residue will therefore be $\sim \frac{5}{2} RT \sim 5T$ cal $\cdot$ mole $^{-1}$ or $5T/M$ cal $\cdot$ g $^{-1}$. The enthalpy (cal $\cdot$ g $^{-1}$) of the supplementary propellant will be denoted by h_p . For gI_{sp} in c.g.s. units, we can write, again with a 70% recovery factor,

$$gI_{sp} = 0.7 (2H)^{1/2}$$

$$= 6.40 \times 10^3 \left[\chi \sqrt{h_p} + (1-\chi) \sqrt{\frac{5T}{M}} \right]$$

whence

$$13) \quad \chi = \frac{1.56 \times 10^{-4} g I_{sp} - \sqrt{\frac{5T}{M}}}{\sqrt{h_p} - \sqrt{\frac{5T}{M}}}$$

From equation (9),

$$\chi = \frac{M_p}{M_p + M_c} = \frac{\mu M_p}{(\mu-1) M_o}$$

$$14) \quad M_p = \frac{\chi(\mu-1)}{\mu} M_o$$

and substituting (14) into (10) gives

$$\begin{aligned}
 15) \quad M_o &= \mu \left[M_f + \frac{\beta \chi (\mu - 1)}{\mu} M_o + \frac{19.3 (10^{-6} g I_{sp})^2 \epsilon \omega}{1 - \chi} \right] \\
 &= \mu \frac{M_f + \frac{19.3 (10^{-6} g I_{sp})^2 \epsilon \omega}{1 - \chi}}{1 - \beta \chi (\mu - 1)}
 \end{aligned}$$

Other equations necessary to determine all parameters of interest either have been given above or else can be derived in obvious ways. To summarize:

$$\begin{aligned}
 16) \quad M_p &= \chi \left(1 - \frac{1}{\mu} \right) M_o \\
 M_c &= \left(\frac{1}{\chi} - 1 \right) M_p \\
 N &= \frac{M_c}{\epsilon} \\
 Y &= 19.3 (10^{-6} g I_{sp})^2 \frac{M_p + M_c}{N} \\
 M_s &= \omega Y
 \end{aligned}$$

Pressure does not appear explicitly in the equations above, but it does enter through equation (13) in determining the value of h_p for a given value of T . Decreasing the pressure increases the degree of ionization and the enthalpy, producing improved performance, although this benefit is offset somewhat by an increased pressure vessel weight due to the additional moles of gas contained. The effect of varying chamber pressure on I_{sp} for pure hydrogen at 6000°K is indicated below:

$$T = 6000^\circ K$$

p	I_{sp}
1000 atm	1290 sec
316	1440
100	1610

This improvement at lower pressures results from increased degree of dissociation, with an increase in enthalpy per unit weight due both to the additional particles and the energy of dissociation. As mentioned above, the overall effect of varying pressure will be illustrated elsewhere¹. To obtain numerical examples here, a value for pressure of 316 atm. has been chosen.

For 316 atm pressure, then, and a chamber temperature of 6000°K and an M value of 12, values of the system parameters have been calculated for several different values of I_{sp} , using the same values for other parameters as given after equation 12, and tabulated values of h_p . In addition, an estimated cost of \$0.05 million per nuclear device and an estimated freight charge of \$0.40 million per ton to put M_o into earth orbit have been used to estimate total cost of the system as it would stand at the beginning of its mission:

$$T = 6000^{\circ}\text{K}$$

$$p = 316 \text{ atm.}$$

$$M = 12$$

I_{sp} (sec)	1000	1100	1200	1300	1400
χ	.61	.70	.79	.88	.97
M_o (tons)	683	663	720	970	2785
M_c (tons)	228	165	122	91	70
M_p (tons)	351	377	447	648	1990
M_s (tons)	25	39	67	136	571
Y (tons TNT)	1.2	1.8	3.1	6.4	26.8
N	9120	6600	4860	3628	2790
Cost (mil. \$)	729	595	531	569	1252

Some prominent characteristics of the system are visible at once. The total mass has a minimum value, for a fixed 6000° chamber temperature, at an I_{sp} value of slightly under 1100 sec, and a hydrogen mass fraction of around 0.7. For mass fractions of hydrogen approaching unity, the energy per pulse becomes very large to heat the large amount of gas, and consequently the pressure vessel mass becomes high, leading to high total mass. For small proportions of hydrogen, the primary effect on mass is through a decreasing I_{sp} and increasing μ , leading to high total mass.

The number of pulses necessary to carry out a mission decreases with increasing specific impulse, so that if an overall figure of merit is taken to be dependent both on total mass and on the number of charges expended, the latter quantity tends to shift the optimum specific impulse and hydrogen fraction toward higher values. As an example, costs figured on the basis described above are shown in the table to have a minimum value at about 1200 sec. I_{sp} . A higher relative cost per nuclear device would shift the position of the minimum slowly to the right, but clearly it will be in the neighborhood of 1200 sec.

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