

Title: NO STRINGS ATTACHED POTENTIAL VS. INTERACTION ENERGY IN QCD

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No Strings Attached Potential vs. Interaction Energy in QCD*

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Abstract

In infrared-stable fixed-point field theories, the interaction energy of a test particle is proportional to the non-relativistic (heavy source) coordinate-space potential derived from the field strength produced by that source. This is no longer true in ultraviolet-stable fixed-point field theories (UVSFPFT) as they may not have a finite infrared fixed point. This leads to the possibility that UVSFPFTs may have quite conventional field strength distributions despite the unusual spatial dependence expected for the interaction energy.

1 Introduction

In infrared-stable fixed-point field theories, such as quantum electrodynamics (QED), the quantum field theoretic renormalization induced scale dependence of the effective coupling constant, $\alpha(\mu^2) \equiv (g^2/4\pi)$, is innocuous at

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large distance scales, because $\alpha(\mu^2)$ tends to a constant (1/137.036 for QED) as μ , the momentum scale at which the renormalization is defined, tends to zero[1]. As a result, the interaction energy (V) between a test particle and a heavy (non-relativistic) source may be described as the product of the coordinate-space potential produced by the source multiplied by the charge ‘ g ’ of the test particle. The (static) potential, ϕ , in turn is just the convolution of the source charge distribution with the Green’s function for the boson which is the ‘force carrier’. It is therefore common to write V in the interaction Hamiltonian as $g \times \phi$ in a coordinate space representation.

This orthodoxy has led to an unnecessary confusion in the extension of such descriptions to the case of ultraviolet-stable fixed-point field theories (UVSFPFTs), such as Quantum ChromoDynamics (QCD) which are widely believed to lead to confining forces. In QCD, in particular, the interaction Hamiltonian for a heavy source is frequently written as a potential energy, and so, (incorrectly, I claim,) a (field strength) potential which rises linearly with increasing modulus of the distance from the source. This is true despite more sophisticated momentum space analyses[2] which show that this is a property of the interaction energy alone. The inexplicability of the spatial variation of the field energy then leads to strange physical pictures wherein the (field strength) potential of a heavy source is not spherically symmetric, and indeed is undefined until a test particle is provided which then defines a direction for a so-called ‘string’ tying the heavy source to the test particle.

My purpose here is to point out that a much more conventional view of the field distribution may still be tenable in such cases, provided one retains the distinction between the field strength distribution (field strength potential) of the force carrier and the interaction energy (potential energy).

To this end, I employ the description of Goldhaber and Goldman[3], which identifies the force carrier of color confinement as a Lorentz scalar (effective) boson, a composite of two or more gluons. Ref.([3]) begins with a perturbative approach to make the discussion more specific. In perturbation theory the coupling due to gluon exchange between two quarks which are off mass shell by some characteristic amount ΔM may be estimated at small four momentum transfer squared q^2 by focusing on the most singular part of the QCD coupling.

For exchange of the two-gluon color singlet combination, the quark-quark potential in momentum space should be

$$\tilde{V}(q^2) \approx \frac{(g_S(q^2))^2 q^4}{(\Delta M)^2 q^4}, \quad (1a)$$

where $(g_S(q^2))^2$ is $4\pi\alpha_s(q^2)$, the factor of q^4 in the numerator comes from the integration over (small) loop momentum, the q^4 in the denominator comes from the two gluon propagators, and $(\Delta M)^2$ from the quark propagators. We assume the correctness of the Richardson[4] ansatz for the leading behavior of α_s ,

$$\alpha_s(q^2) = \frac{12\pi}{(33 - 2n_f)\ln(1 + q^2/\Lambda^2)}, \quad (2)$$

where Λ is of order the QCD scale but not necessarily equal to $\Lambda_{\overline{MS}}$, and the coefficient is determined by the one-loop β -function for QCD which depends on the number of light quark flavors, n_f . (For a recent confirmation that $\alpha_s(q^2)$ at least diverges for $q^2 \rightarrow 0$, see [5].) This implies a pole in α_s at $q^2 = 0$, giving a double pole in $\tilde{V}(q^2)$ and hence by Fourier transformation a linearly rising potential $V(r)$ in coordinate space.

My point, however, is that the result above is the interaction energy, and cannot be simply translated into the field strength distribution. The latter is found, in momentum space, from the product of the source current, in this case, one of the two quarks, and the propagator for the (effective) boson that is being exchanged. Since we are looking at an exchange channel with 0^+ quantum numbers for that (scalar) boson, we first rewrite Eq.(1a) in terms of that, as

$$\tilde{V}(q^2) \approx \frac{(\alpha_s(q^2))^2}{(q^2 - m^2)}, \quad (1b)$$

where m is the effective mass (at low q^2) for the scalar boson, and we have ignored the normalization (Z) factor for its pole strength, as the pole may not even actually exist. All that is needed is that the propagator is approximately constant for $q^2 \approx 0$. The potential of Eq.(1b) has the same properties as Eq.(1a) if we identify M with m , that is, as some sort of minimal off-shellness required for the quarks for the whole picture to apply.

Returning to the question of the field strength (potential), we find

$$\tilde{\phi}(q^2) \approx \frac{(\alpha_s(q^2))}{(q^2 - m^2)}, \quad (3)$$

where ϕ is the field strength of the scalar boson. This has entirely different properties from V when Eq.(2) is applied as is appropriate for a UVSFPFT. There is now only a single pole (from the one power of α_s) at $q^2 = 0$. Correspondingly, the Fourier transform to coordinate space produces a potential $\phi(r)$ that varies only Coulombically at large r , that is

$$\phi(r) \approx \frac{12\pi\Lambda^2}{(33 - 2n_f)m^2} \frac{1}{r}. \quad (4)$$

in the static limit.

There is nothing exceptional about this field strength distribution: The integral of the energy density associated with it is logarithmically divergent in the infrared just as for the electromagnetic Coulomb problem. There is a Gauss theorem and a conserved, finite total charge enclosed, although the nature of that charge is not apparent from these considerations. (See Ref.([3]) for some conjectures on this point.)

Indeed, the only problem remaining is to understand why, phenomenologically, there does not seem to be any van der Waals potential associated with dipole fluctuations of color singlet systems. This remains to be investigated, but some possibilities are immediately apparent. One is the nature of the type of charge carried by the sources[3]. A second is the effective nature of the composite boson – below some minimal dipole separation, its coupling to the charges may be significantly reduced. A third is that, again, the fluctuations must be considered first in the field strength, not the interaction energy, which is secondary from this point of view. The rapid falloff with r

of conventional van der Waals (r^{-7} when retardation effects are taken into account) will thus lead to a more rapid falloff with r than in calculations which have not distinguished between the potential and the potential energy (although still not as rapid as the Yukawa potential, of course). Finally, the energy excitation cost of the dipole formation further suppresses the fluctuations which produce the van der Waals potential, and these are much more severe in the case of a confining interaction energy than in the conventional case.

I conclude that there is no well founded reason to reject the notion that the potential produced by a (heavy) quark, or other isolated (static) color source is of a Coulombic scalar form, while the interaction energy of such a source with a test particle nonetheless grows linearly with the separation between the source and the test particle.

Acknowledgments

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References

- [1] M. Gell-Mann and F. E. Low, *Phys. Rev.* **95** (1954) 1300; P. Ramond, *Field Theory: A Modern Primer*, (Addison-Wesley, Redwood City, California) 1989.

- [2] W. Buchmüller and S. H. H. Tye, *Phys. Rev. D* **24** (1981) 132; A. K. Grant, J. L. Rosner, and E. Rynes, *Phys. Rev. D* **47** (1993) 1981.
- [3] A. S. Goldhaber and T. Goldman, *Phys. Lett. B* **344** (1995) 319.
- [4] J. L. Richardson, *Phys. Lett.* **B82** (1979) 272.
- [5] A. Hauck, L. von Smekal and R. Alkofer, “The Infrared Behaviour of α_S from Mandelstam’s Approximation to the Gluon Dyson-Schwinger Equation”, hep-ph/9604430; ANL-PHY-8386-TH-96.