EVALUATION OF WAVE DISPERSION,
MODE-CONVERSION, AND DAMPING FOR

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ECRH With Exact Relativistic Corrections

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ABSTRACT

The complex dispersion functions of Eq. (3) in Ref. 1 have recently been computed accurately and reliably over their entire range of parameters, without recourse to the usual "slightly-relativistic" approximation, which may have difficulty for oblique incidence.² In the future, the local dispersion properties of ECRF waves will be reevaluated for parameters of interest to ECRF conditions in several existing and proposed fusion experiments, with particular emphasis on the damping and mode-conversion of both ordinary and extraordinary waves to electrostatic waves near the upper hybrid and cyclotron frequencies.

DISCUSSION

At frequencies around the electron cyclotron resonance, both the ordinary and the extraordinary waves may couple to electrostatic waves, in a manner that is similar to the strong coupling between the fast magnetosonic wave and the Bernstein wave near the Ion Cyclotron Resonance. While many highly detailed and successful analytical and computational investigations have been performed of this lower frequency coupling phenomenon, similar techniques have not yet been applied to the higher Electron Cyclotron Range of Frequencies (ECRF) with much success. Hence the exact details of the ECRF wave behavior near an absorption layer are still under considerable investigation. Two principal issues complicate the study of the field structure near an electron cyclotron resonance, the importance of relativistic effects and the simultaneous existence of two cold-plasma propagating waves, the ordinary and extraordinary branches, rather than the single magnetosonic wave for ICRF.

Because of the low rest mass energy of electrons, the plasma's dispersion of waves near the cyclotron resonance is affected by relativistic effects. These effects are noticeable at electron temperatures of current experiments, and will be important for higher temperatures expected in future experiments.³ The addition of relativistic effects into the standard linear wave dispersion theory has been studied analytically in Reference 1, as well as many others, with the net result of introducing a great deal more mathematical complexity into the analysis. One of the major discoveries of the work in Reference 1 is the substantial modification of the resonant dispersion due to relativistic plasma response. For an equilibrium plasma, this is described by the complex integral of Equation 3 in Reference 1, namely

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$$F_q\left(\frac{\mu n \omega_b}{\omega}, \frac{k_{\parallel} c}{\omega}, \mu\right) \equiv -i \int_0^{\infty} \frac{dt \exp\left\{\mu - \mu \left[(1-it)^2 + \frac{k_{\parallel}^2 t^2 c^2}{\omega^2}\right]^{1/2} - in\mu \frac{\omega_b}{\omega} t\right\}}{\left[(1-it)^2 + \frac{k_{\parallel}^2 t^2 c^2}{\omega^2}\right]^{q/2}}.$$

Unlike the case for a nonrelativistic plasma, this dispersion function cannot be easily evaluated or approximated over the entire range of its three parameters. Consequently, various different approximations are utilized in various different parameter regimes. However, the accuracy of analysis based on these approximations is always subject to their careful application only in their respective regions of validity. Since no single approximation is valid across an entire plasma column, and in particular, since few approximations are reliable near resonance at oblique incidence ($k_{\parallel} \neq 0$), it is difficult to arrive at confident conclusions which achieve a good measure of universal acceptance. For this reason, we have concluded that as a first step to investigating ECRF wave properties near resonance, the above function needs to be numerically evaluated in a reliable and exact manner, over its entire range of parameters, such that its use may be taken for granted in our future analyses of wave dispersion, mode-conversion, and absorption. In this paper, we report on the successful completion of this goal, and the methods employed.

With this stage of the work completed, it is our future goal to implement the above dispersion function into several codes which have been developed previously for the analysis of ICRF in tokamaks. These programs will be used to investigate the dispersion characteristics of both ordinary and extraordinary wave propagation, mode-conversion, and absorption near the EC resonance. An improved understanding of the EC wave damping and emission mechanisms may aid in the current use of EC emission for diagnostics purposes, in addition to providing an improved understanding of ECRF heating of fusion plasmas.

EVALUATION OF RELATIVISTIC DISPERSION

As a first step to an exact computation of the above integral, we have chosen to renormalize the integral and arguments, so as to put it in a form which reduces to a commonly recognizable entity, in the nonrelativistic limit. As a common reference point, we use the nonrelativistic plasma dispersion function for an equilibrium plasma, described by Fried and Conte,⁴ also featured in the NRL Plasma Formulary. Thus, without approximation, the complex integral from Ref. 1 may be rewritten as

$$F_q\left(\frac{\mu n \omega_b}{\omega}, \frac{k_{\parallel} c}{\omega}, \mu\right) = \frac{-1}{\left|\frac{k_{\parallel} c}{\omega}\right| \sqrt{2\mu}} Z_q(\nu, \lambda, \delta),$$

where the arguments to the renormalized relativistic dispersion function, $Z_q(\nu, \lambda, \delta)$, are related to the physical quantities according to

$$\nu = \frac{n \Omega}{|k_{\parallel}| v_{th}}, \quad \lambda = \frac{\omega}{|k_{\parallel}| v_{th}}, \quad \delta = \frac{1}{\mu} = \frac{1}{2} \frac{v_{th}^2}{c^2} = \frac{T}{mc^2},$$

and the relativistic dispersion function is the complex integral

$$Z_q(\nu, \lambda, \delta) \equiv -i \int_0^\infty \frac{dt \exp\left\{\frac{1}{\delta}[1 - u(\lambda, \delta; t)] - i \nu t\right\}}{u^q(\lambda, \delta; t)}$$

$$u(\lambda, \delta; t) = \sqrt{(1 - i \lambda \delta t)^2 + \frac{1}{2} \delta t^2}$$

Here ω and $k_{\parallel}$ are the (angular) frequency and parallel wavenumber of an rf excitation, Ω is the local (angular) cyclotron frequency due to the applied magnetic field, n is the cyclotron harmonic number, T is the equilibrium temperature of the plasma, m is the mass of the electron mass, and c is the speed of light. The value of q is related to the harmonic number, n , typically a value such as $n+1/2$ or $n+5/2$. In the nonrelativistic limit, i.e. $\delta \rightarrow 0$, $Z_q(\nu, \lambda, \delta)$ approaches the nonrelativistic dispersion function described by Fried and Conte:

$$Z_q(\nu, \lambda, \delta=0) = Z_{FC}(\lambda - \nu) .$$

The method used to perform the complex integration is to distort the path of integration to approach the direction of steepest descent and least oscillation, and then to use integration packages from popular subroutine libraries such as IMSL or NAG, thus providing dial-an-accuracy capability. In doing so, particular care must be made not to cross the branch cut which lies on the imaginary axis between the roots of the $u(\lambda, \delta; t)$ function. These roots also change position with respect to the origin depending on the values of λ and δ . In addition, because the parameter q usually contains a half power, an additional branch cut runs from the lower root of u to $-i\infty$. Figure 1 shows the two possible configurations of the branch cut, and the distortion to the integration path used to evaluate the integral for each of these cases.

Figure 1. Integration paths in the complex t -plane for the relativistic dispersion function.

Figure 2 shows the relativistic dispersion function vs. the parameter ν ($\sim$ position near resonance) for fixed λ and δ (frequency and temperature) at two

different values of the relativistic parameter δ . The figures show the flat zero value of the imaginary dispersion, i.e. absorption, for phase velocities above the speed of light. Also notice that as the relativistic parameter increases, the size of the resonance region increases, physically spreading out the resonance region.

Figure 2. Relativistic dispersion function vs. v (position) parameter.

STUDY OF THE EXACT RELATIVISTIC DISPERSION RELATION

Work has commenced on the effort to utilize the dispersion function above in an analysis of ECRF wave behavior near the cyclotron resonance. The dispersion function has been placed into the full 3×3 plasma dielectric tensor described by Equation 4 in Reference 1, which has in turn been inserted into the electromagnetic wave equations. Initial results show a promising ability to resolve the dispersion relation for the natural modes of the plasma on a fine scale. Figure 3 shows the general dispersion roots, $k_{\perp}^2$, vs. position near an EC resonance. The resonant extraordinary wave / Bernstein wave is seen to couple to the ordinary wave. The skewed shape of the relativistic dispersion function is also visible in the imaginary parts of the roots. In the future a wave solver will be run to determine coupling coefficients for all of the wave branches.

Figure 3. Solution of Dispersion Relation and Magnified coupling point on Ordinary Wave Branch.

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