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STRESS INTENSITY MAGNIFICATION FACTORS FOR FULLY CIRCUMFERENTIAL CRACKS IN VALVE BODIES (THICK CYLINDERS)

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ABSTRACT:

The stress intensity solutions presented herein were obtained using an energy method in conjunction with a two-dimensional finite element program in order to explicitly account for curvature effect for fully circumferential cracks. The magnification factors for a specific crack depth were calculated by successively loading the crack surface by a uniform, linear, quadratic, and a cubic loading distribution. The magnification factors can be used to calculate the stress intensity factors by superposition method.

The functions for each load condition in terms of radius to thickness ratio (R/t) and a fractional distance in terms of crack depth to thickness ratio (a/t) were developed. The validity of these function is $R/t = 1.5$ to 10.0 and for $0.0125 \leq a/t \leq 0.8125$. The functions agree to within 1% of the finite elements solutions for most magnification factors.

KEYWORDS: cylinders, curvature constraint, continuous circumferential cracks, finite element, stress, energy, magnification factors, stress intensity factors

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INTRODUCTION

Fatigue crack growth and brittle fracture evaluations are often carried out using stress intensity factor solutions developed for crack-like surface defects in flat plates. The stress intensity factor solutions are expressed in terms of membrane and bending stress intensity magnification factors calculated for plates that are subject to no geometric constraints. The lack of geometric constraint maximizes the crack opening at the crack tip and assures that the calculated stress intensity factors are conservative.

However, stress intensity factors for cracks in geometrically complex structures are often calculated by explicit finite element methods. The magnitude of stress intensity factor depends upon the location, shape, and orientation of the crack as, well as upon the geometry of the component. In real structures the application of flat plate stress intensity factor solution to structures with significant wall curvature, such as valves, results in overly conservative crack growth predictions. The curvature produces a less compliant structure than a flat plate in the presence of a crack and, due to reduced opening of the crack-tip, the resulting stress intensity factor is less than that of a flat plate.

The effect of curvature constraint on stress intensity factor is maximized when continuous cracks are considered. In this case curvature inhibits bending deformation in the uncracked ligament; no such constraint exists for the flat plate case. When the crack configuration is that of a part-through semielliptical surface crack, an inherent restraining effect due to the adjacent uncracked section inhibits bending deformation. Therefore, separate constraint mechanisms exist for semielliptical part-through and continuous surface cracks in terms of bending deformation. In this paper, magnification factors are developed for a fully circumferential crack for numerous values of curvature (R/t) and relative crack depth (a/t), where R is the inside radius of the cylinder, t is the cylindrical wall thickness, and a is the crack depth.

Several analytical procedures are available in the literature to calculate elastic stress intensity factors for complex structures using finite element analysis. For conditions of plane stress or plane strain, the potential energy (G) of the structure released as a result of crack extension can be related to the stress intensity factor (K_I), Reference (1). The advantage of using the energy-based finite element procedure is that it does not utilize any special treatment of the mechanical response in the immediate vicinity of the crack tip.

The energy method in conjunction with a two-dimensional elastic finite element program was used to expeditiously evaluate the curvature effect for fully circumferential cracks in thick-walled and thin-walled cylinders. The results were verified using other published numerical solutions for internally pressurized cylinders with continuous cracks.

STRESS INTENSITY FACTORS K_I

Calculation of Strain Energy Release Rate, G

The 2-dimensional elastic finite element program was used in conjunction with the automated evaluation of stress intensity factors for continuous cracks in planar or axisymmetric structures, to calculate the change in potential energy due to a unit increase in crack length for plane strain and plane stress conditions. The energy release rate method for calculating stress intensity factors is described as:

$$G \delta a = \int_a^{a+\delta a} \sigma_1 U_1 dx \quad (1)$$

where σ = stress distribution ahead of crack length a ,

U_1 = crack surface displacement.

G is related to K_I by Reference (1):

$$K^2 = E'G \quad (2)$$

where E' is $\begin{cases} E \text{ for plane stress, and} \\ E/(1 - \nu^2) \text{ for plane strain} \end{cases}$

Procedure to Determine K_I

The procedure used to calculate the stress intensity factor solution consists of applying the superposition principle in the loading of the finite element model and in expressing the stress in terms of the coefficients of a third order polynomial representing the stress profile perpendicular to the section of the structure.

Superposition Principle

The superposition method is illustrated in Figure 1. The stress intensity factor K_I for the crack in Section A-A of the structure, subjected to a remote loading (F , M) represented by a force F and a moment M , is equal to the

stress intensity factor K_I for the same crack in Section A-A of the structure, where the crack surface is subjected to a stress profile $\sigma(x)$ identical to the stress profile developed perpendicularly to the uncracked Section A-A by the remote loading (F, M).

Figure 1: Superposition Principle

The stress profile $\sigma(x)$ perpendicular to Section A-A of the structure during a transient in the absence of a flaw, can be fitted, by least squares regression, to a third degree polynomial as:

$$\sigma = A_0 + A_1 x + A_2 x^2 + A_3 x^3 \quad (3)$$

where A_0, A_1, A_2, A_3 are coefficients of the polynomial representing the stress profile $\sigma(x)$ in the uncracked Section A-A.

If a continuous flaw is now assumed to be present in Section A-A during the transient considered, the stress intensity factor is normally expressed as:

$$K_I = \sqrt{\pi a} [A_0 G_0 + A_1 G_1 + A_2 G_2 + A_3 G_3] \quad (4)$$

where: a = the crack depth

$G_0, G_1, G_2,$ and G_3 = magnification factors corresponding to the geometry and the crack depth.

In the above equation, the magnification factors G_0 , G_1 , G_2 , and G_3 are functions of the crack geometry and are independent of the magnitude of loading. Therefore, the magnification factors can be calculated using any arbitrary stress profile applied to the crack surface. In this paper, the magnification factors for a specific crack depth were calculated by successively loading the crack surface as follows:

- (1) a uniform loading distribution ($\sigma = A_0$)
- (2) a linear loading distribution ($\sigma = A_1 x_1$)
- (3) a quadratic loading distribution ($\sigma = A_2 x^2$)
- (4) a cubic loading distribution ($\sigma = A_3 x^3$)

The above procedure is then applied to many combinations of crack depth and wall curvature to determine the variation of the magnification factors G_0 , G_1 , G_2 , and G_3 with the crack depth to thickness ratio (a/t) for various radius to thickness ratios (R/t).

The various stress distributions applied to the crack surface are shown in Figure 2 and the magnification factors are calculated as:

Figure 2: Magnification Factors Determination

$$G_0 = \frac{K_{I(0)}}{A_0 \sqrt{\pi a}} \quad (\text{uniform loading}) \quad (5)$$

$$G_1 = \frac{K_{I(1)}}{A_1 \sqrt{\pi a}} \quad (\text{linear loading}) \quad (6)$$

$$G_2 = \frac{K_{I(2)}}{A_2 \sqrt{\pi a}} \quad (\text{quadratic loading}) \quad (7)$$

$$G_3 = \frac{K_{I(3)}}{A_3 \sqrt{\pi a}} \quad (\text{cubic loading}). \quad (8)$$

In this paper stress intensity factors were calculated using the strain energy release rate method described above.

FULLY CIRCUMFERENTIAL INSIDE SURFACE CRACK IN CYLINDERS

Geometry

A typical circumferential cracked cylinder in tension is shown in Figure 3. The symbols R_i and R_o represent the inside and outside radius of the cylinder, respectively, t is the cylinder thickness ($R_o - R_i$), and a is the crack size.

The thickness of the cylinder was kept constant and the inside radius, R_i , was varied to determine various R_i/t ratios. The height of the cylinder was kept ten times the thickness to ensure the infinite nature of the cylinder.

Finite Element Mesh

Grid points and two free bodies (SA and SB) and the finite element mesh of a fully circumferential part-through crack in a cylinder are shown in Figure 4. By symmetry only one half of the specimen was modeled. The boundary conditions are zero surface tractions everywhere but on grid points 3 to 4 where the condition that displacement in the normal direction be zero is imposed. The model was loaded by specifying and applying load between grid point 2 and 3 on the crack surface. The enlarged finite element mesh near the crack is also shown in Figure 5.

Applied Loads

The magnification factors G_0 , G_1 , G_2 , and G_3 in Equation 20 are functions of the crack geometry and the cylinder curvature and are independent of the magnitude of loading. Therefore, the magnification factors can be determined using any arbitrary loading profile applied to the crack surface. The magnitude of load in this example was one ksi. The magnification factors (G_0 , G_1 , G_2 , and G_3) relative to a given crack depth, a , were determined by successively loading the crack surface to uniform ($\sigma = A_0$), linear ($\sigma = A_1x$), quadratic ($\sigma = A_2x^2$), and cubic ($\sigma = A_3x^3$) stress profiles.

These stress distributions were applied as piecewise linear over the 39 elements.

The various magnification factors as functions of a/t and R/t with the crack depth to thickness ratio (a/t) were established. Figure 3 shows the application of four loading conditions applied to the surface of the crack and equations used in each case for the corresponding magnification factor.

Boundary Conditions

By symmetry only one half of the structure was modeled. The boundary conditions are zero surface tractions everywhere but on grid points 3 to 4 where the conditions where displacement in the normal direction be zero are imposed. The crack surface was successively loaded as described above.

Mesh Sensitivity Evaluation

In order to determine the adequacy of the finite element mesh, elastic solutions were obtained to investigate the effects of several parameters (i.e. number of elements in the mesh along the thickness of the model, radial spacing of the elements, and the length of the specimens in the analytical model) on the computed stress intensity factor along the crack length. In order to achieve an optimum finite element mesh, several finite element models were developed. The final stress intensity factors for the circumferential crack model are based on three free bodies. There were forty (40) equally spaced elements along the thickness direction including 39 elements over the cracked area. The first free body (SA) has ten (10) elements in the length direction with an element size ratio of one to ten. Similarly the second free body SB has ten elements with an element size ratio of twenty. The largest element adjoining to the first free body (SA). The element ratios between 10 to 50 did not change the results as shown in Figure 6.

Figure 3: Fully Circumferential Crack in a Cylinder

Figure 4: Grid Points and Two Free Bodies (SA and SB) for Fully Circumferential Crack

Figure 5: A Two-Dimensional Finite Element Model Having a Fully Circumferential Crack

Figure 6: Finite Element Ratio Study

Determination of Magnification Factors

The magnification factors were calculated using the energy method. As discussed in previous sections the available energy (G) for unit crack extension is related to stress intensity factors (K_I). For a specified load condition, the stress intensity factor K_I for two-dimensional analysis was calculated. The input to computer code was modified to accept a linear, non-linear, and cubic load distribution at the crack surface. The stress intensity factors were obtained for crack depths up to 80 percent of the wall thickness. The magnification factors (G_0 , G_1 , G_2 , and G_3) were calculated using Equation 12.

Verification of Fully Circumferential Magnification Factors for $R/t = 10$

The magnification factors for a circumferential inside surface crack in a cylinder developed in this report, using the energy method (Toor) are compared with the Buchalet and Bamford, Reference (2) and Zahoor's, Reference (3) finite element solutions for $R/t=10$. The comparison is shown in Figures 7-10.

Magnification Factor G_0 --The magnification factor G_0 is due to a uniform stress distribution over the crack depth. Figure 7 compares three solutions. It is observed in this figure that References (2) and (3) solutions cross over at $a/t = 0.5$. The energy method magnification factors agree with Buchalet solution up to $a/t = 0.3$ but are consistently above Buchalet and Zahoor's solution up to $a/t = 0.8$. For

deeper crack $a/t = 0.8$, the energy solutions are 15 percent higher than the other two solutions.

Figure 7: Fully Circumferential R/T = 10
Verification of Magnification Factor, G_0

Magnification Factor G_1 --The magnification factor G_1 is associated with linear stress distribution over the crack depth. Figure 8 compares three solutions (Toor, Buchalet, and Zahoor). Again, the energy method solution results are slightly higher than the other two solutions.

Figure 8: Fully Circumferential Crack R/T = 10
Verification of Magnification Factor, G_1

Magnification Factor G_2 --The magnification factor G_2 is associated with quadric stress distribution over the crack depth. Three solutions are compared in Figure 9. It is observed from this figure that energy method values of the magnification factor are higher than the other two solutions.

Figure 9: Fully Circumferential Crack $R/T = 10$
Verification of Magnification Factor, G_2

Magnification Factor G_3 --The magnification factor G_3 is due to a cubic form of stress distribution over the crack depth. The comparison of the solution is displayed in Figure 10. It is again observed that Toor and Buchalet solution agree well over a/t range from 0 to 0.8. Zahoor's solutions are consistently lower bound and for deeper crack the difference between Zahoor and Toor solution increases.

Figure 10: Fully Circumferential Crack $R/T = 10$
Verification of Magnification Factor, G_3

Figure 11.2: Fully Circumferential Crack
Magnification Factors for R/T = 10

Functional Form of the Magnification Factors--The SOLO statistical package, "version 4" Reference (5) was used to fit the model to the finite element generated magnification factors for $R/t = 1.5, 2, 2.5, 3, 5, 10$. For each R/t four magnification factors (G_0 , G_1 , G_2 , and G_3) were available from the finite element results in a tabular form. The non-linear least square fit procedure produced the following function for various types of loading.

Uniform Loading--The magnification factor associated with uniform loading is defined as G_0 . The functional form of G_0 from the non-linear least square fit is as follows:

$$G_0 = a_0 (1 - a/t)^{b_0} \exp[c_0 (a/t)] \quad (9)$$

where:

$$a_0 = 1.078 (R/t)^{0.003} \exp[-0.009 (R/t)],$$

$$b_0 = -0.648 (R/t)^{-0.357} \exp[-0.106 (R/t)] \text{ and}$$

$$c_0 = 3.739 - 4.502 (R/t)^{-0.175} \exp[-0.013 (R/t)].$$

Linear Loading--The magnification factor associated with linear loading is defined as G_1 . The functional form of G_1 is as follows:

$$G_1 = a_1 (a/t)^{b_1} \exp[c_1 (a/t)] \quad (10)$$

where:

$$a_1 = 0.248 (R/t)^{0.259},$$

$$b_1 = 0.594 (R/t)^{0.163} \text{ and}$$

$$c_1 = 1.465 (R/t)^{-0.090} \exp[0.010 (R/t)].$$

Quadratic Loading--The magnification factor associated with quadratic loading is defined as G_2 . The functional form of G_2 is as follows:

$$G_2 = a_2 (a/t)^{b_2} \exp[c_2 (a/t)] \quad (11)$$

where:

$$a_2 = 0.115 (R/t)^{0.285} \exp[0.020 (R/t)],$$

$$b_2 = 1.259 (R/t)^{0.095} \exp[0.009 (R/t)] \text{ and}$$

$$c_2 = 1.984 (R/t)^{-0.096} \exp[-0.012 (R/t)].$$

Cubic Loading--The magnification factor associated with cubic loading is defined as G_3 . The functional form of G_3 is as follows:

$$G_3 = a_3 (a/t)^{b_3} \exp[c_3 (a/t)] \quad (12)$$

where:

$$a_3 = 0.063 (R/t)^{0.276} \exp[0.042 (R/t)],$$

$$b_3 = 1.945 (R/t)^{0.063} \exp[0.011 (R/t)] \text{ and}$$

$$c_3 = 2.393 (R/t)^{-0.060} \exp[-0.031 (R/t)].$$

Note: The functional form generated magnification factors are shown in figure 11 for $R/t=10$.

Conclusions

The magnification factors developed for fully circumferential part-through crack for $R/t = 10$ are compared with finite element results of two known solutions in the literature (References (8) and (9)). It is concluded that magnification factors developed using the energy method agree reasonably well for $a/t = 0.3$, and are conservative relative to the two other solutions for a/t from 0.3 to 0.8 for a cylinder having $R/t = 10$. The finite element generated magnification factors for each load distribution on the crack face in tabular form were utilized to develop

the functional form of these magnification factors. The statistically generated functions and the tabular values agree within 1%. The advantage of these functions is the convenience of incorporating them in any crack growth computer program within the given applicability. This eliminates the cumbersome interpolation of the functions for a specific R/t and a/t values. The validity of these functions is $1.5 \leq R/t \leq 10$ for $0.0125 \leq a/t \leq .8125$.

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