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Multiple Shaker Random Vibration Control–An Update

David O. Smallwood
Sandia National Laboratories
Albuquerque, NM 87185-0553

BIOGRAPHY

David Smallwood received his BSME degree from New Mexico State University in 1962 and his MSME degree from New York University in 1964. He has worked for Sandia National Laboratories since 1967 and is currently a Distinguished Member of the Technical Staff in the Environments Engineering Group of the Mechanical and Thermal Environments Department at Sandia. He is a fellow of the IEST.

ABSTRACT

The theory of the control of multiple shakers driving a single test item is reviewed. Several improvements that have been introduced since the original papers on the subject will be discussed. The improvements include: 1) specification of the control spectra; 2) the control of non-square systems (the number of shakers does not have to be equal to the number of control points); 3) the connection between sine testing, waveform control, and random control; 4) improvements in feedback control; 5) overlap-add versus time domain randomization; and 6) reproduction of non-Gaussian waveforms.

KEYWORDS

Multiple shaker control, multiple axis vibration testing, vibration, testing, random, sine, waveform control, transient vibration.

The elements in the upper case matrices and vectors are all functions of frequency. The elements in the lower case matrices and vectors are all functions of time. In actual application the functions of frequency are defined at a discrete set of frequencies using the fast Fourier transform, FFT.

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INTRODUCTION

It has been over ten years since I have published a paper on the control of multiple shakers driving a single test item (Smallwood,1978, 1982a,1982b and Smallwood, Woodall, and Buksa, 1986). These systems are becoming more important as applications increase and the control systems become more robust. Improvements in hardware and algorithms have made the improved systems possible. Systems are now being used in the aerospace, defense, automotive, and seismic industries.

Since 1978 at least four commercial companies have marketed control systems for this purpose (for example, Hamma, en al, 1996, Chen and Wilson, 1998). The commercial companies marketing these systems rarely discuss the technical aspects of their systems in detail for competitive reasons. However, some of the improvements have been investigated independently by the author and can be discussed.

Testing of systems using multiple shaker control can be divided into several categories. First is the control of multiple independent test items on separate shakers using a single control system. This paper will not discuss this category. Testing of a single test item driven with multiple shakers will be discussed. This includes multiple axis control at a single control point and the control of multiple points in one or more axes. The test environment can then be divided into three broad categories: sine inputs, waveform control, and random. Some environments are a combination of these. Combined environments will not be discussed.

Waveform control involves testing where the desired waveforms are deterministic and predetermined. The waveforms might have a random character, but a particular realization is reproduced. The control system attempts to reproduce the reference waveforms at the control points on the test item. Transient or shock testing falls into this category. The road simulations commonly performed in the automobile industry

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also fall into this category, as does most seismic testing. This testing will be discussed first.

In a sense sine testing is a special case of waveform control discussed below. Sometimes the same control strategies are used, but in some cases a significantly different strategy is used. Sine testing will be discussed very briefly in this paper.

Random testing uses nondeterministic waveforms that are generated as the test progresses and are described in probabilistic terms. This form of testing will be discussed last.

NOTATIONAL NOTES

Before I start the discussion on control some notational items will be discussed.

Functions of time are defined in blocks at a discrete set of points using the inverse fast Fourier transform, FFT^{-1}, of a corresponding frequency domain description. Auto and cross-spectral densities are typically estimated using the Welch algorithm. This algorithm divides the time history into blocks, sometimes overlapping, multiplies the blocks by a window, and computes the FFT. Multiple blocks are averaged to arrive at estimates of the spectra.

Lowercase letters will be used to denote samples in time. Each element in a vector is a time history. The vector is in reality a matrix, time in one dimension and spatial location in the other dimension. But for convenience the time dimension is not included. The corresponding frequency domain description is denoted with the corresponding upper case letter. Similarly for convenience the frequency dimension is excluded. This avoids the requirement for 3-dimensional tensors later in the development. Matrices are denoted in bold.

An important change in the notation for the cross-spectrum is used. Bendat and Piersol (1986) define the cross-spectrum between x and y as

$$\Phi_{xy}^{B}(f)=\lim_{T\to\infty}\frac{2}{T}E\left[X^{*}(f)Y(f)\right], \quad (1)$$

where E[ ] is the expected value, T is the record length, the superscript * is the complex conjugate. This was the notation used in my previous papers.

With matrices it is much more convenient to define the cross-spectrum as

$$\Phi_{xy}(f)=\lim_{T\to\infty}\frac{2}{T}E\left[X(f)Y^{*}(f)\right]. \quad (2)$$

Using this notation the cross-spectral density matrix of a column vector becomes

$$\Phi_{xx}(f)=\lim_{T\to\infty}\frac{2}{T}E\left[\mathbf{X}(f)\mathbf{X}'(f)\right], \quad (3)$$

where **X'** is the conjugate transpose of **X**.

The only difference is that

$$\Phi_{xy}^{B}(f)=\Phi_{xy}^{*}(f). \quad (4)$$

I will use the notation of Equations (2) and (3) in this paper.

WAVEFORM CONTROL

Multiple shaker waveform control was the first test procedure to be implemented successfully (Fisher, 1973, Fisher and Posehn, 1977, for example). The basic concept of waveform control is shown in Figure 1. The vector desired waveforms, $\{x(t)\}$, is defined. The waveforms usually transformed into the frequency domain, $\{X(\omega)\}$. If the waveform is too long to be conveniently described in the frequency domain, the waveforms are broken into blocks and overlap and add methods (Gold and Rader, 1969) are used to reproduce the waveforms.

Next the system must be identified. A matrix of frequency response functions, **H**, is identified. The elements of **H** can be identified by exciting the system one input at a time (other inputs zero) and measuring the response. The system can also be excited with a vector of independent or partially correlated inputs to identify the system frequency response matrix.

$$\mathbf{H}=\Phi_{rr}\Phi_{dd}^{-1} \quad (5)$$

where

Φ_{rr} = the cross-spectral density matrix of the return signals with the test item attached to the vibration system.

Φ_{dd} = the cross-spectral density matrix of the inputs.

It is assumed that the inverse of Φ_{dd} exists. This is why the inputs must be at least partially uncorrelated.

The cross-coupling matrix is then computed from the Moore-Penrose pseudo inverse

$$\mathbf{G} = pinv(\mathbf{H}) = \mathbf{H}^{-1}. \quad (6)$$

The original algorithms (Fisher, 1973; Smallwood, 1978; Smallwood, Woodall, and Buksa, 1986) required that **H** be square. The number of inputs had to be equal to the number of response points. Some provision was made for a rank deficient matrix (Smallwood, 1982). Some effort was made to control additional points (Paez, Smallwood, and Buksa, 1987). However, the use of the Moore-Penrose pseudo inverse is superior. If there are more response points than drive points, or if the system is ill-conditioned, the system is over-constrained and it will not be possible to match more than n autospectra, where n is the rank of the **G** matrix to be defined later.

If there are fewer response points than drive points, the system is under-constrained, and multiple solutions are possible. A desirable solution is one that will distribute the drives among the shakers in some form of an optimum fashion.

The drive $\{d\}$ needed to reproduce the waveforms can now be computed. The computed drives are applied to the system and the response is observed. The initial test is often done at reduced level. If the cross-spectral density of the drive matrix, Φ_{dd}, is of full rank, the drive and response information can be used to update the matrix, **H**. The inverse of **H** updates **G**. An error signal, the difference between the return and the desired waveforms, can be found. This error can be subtracted from the computed drive signals and the test repeated. The number of times this iteration can be repeated is limited. When the errors are reduced to noise at some frequencies that are not correlated with the drive, the noise will be amplified if the iteration is continued.

The biggest problem is nonlinearities in the system. The frequency response functions that identify the system are a linear least squares fit. The fit is usually acceptable for inputs near the excitation level used to measure the frequency response functions. Tests at a level lower than the full level are frequently run prior to the full-level test to iterate to an acceptable solution. If the system is significantly nonlinear, the full level test may not be acceptable. For those cases where multiple full-level tests are acceptable or required, this problem can be minimized.

Another troublesome problem is notches (zeros) in the frequency response functions. The equalization is basically a deconvolution problem. These problems are notoriously unstable. Unfortunately, two zeros almost always exist in the frequency response functions: one at zero frequency and one at infinite frequency. Great care must be taken at very low and very high frequencies and at other frequencies where the frequency response function matrix is ill conditioned.

Waveform control is basically an open loop control algorithm. The method assumes the system frequency response functions do not change during a test. The method will not correct for changes in the system response during the application of a waveform. The waveform can be corrected from one iteration to the next if multiple reproductions of the waveform are allowed.

SINE CONTROL

A sine test is a deterministic test. The control waveforms can be specified before the test. One method for sine control is to compute the waveforms and store them. The test then proceeds as a waveform control test. This method will allow for the correction of some distortion and nonlinear response through the iteration techniques described in the section on waveform control.

Another technique is to derive a feedback control method that mimics an analog sine control system. The amplitude of the response is measured, compared with the desired amplitude, and a correction to the drive is made. This method will not be discussed here in any more detail.

RANDOM VIBRATION TESTING

The reference spectrum for a multiple input random vibration test is defined by the auto spectra of the m responses, the ordinary coherence between pairs of responses, and the phase relationship between pairs of responses. This is an improvement over the earlier method of specifying the cross-spectral density matrix. Defining the reference in this manner removes the burden from the user of

determining if the cross-spectrum is realizable and improves insight into the specification.

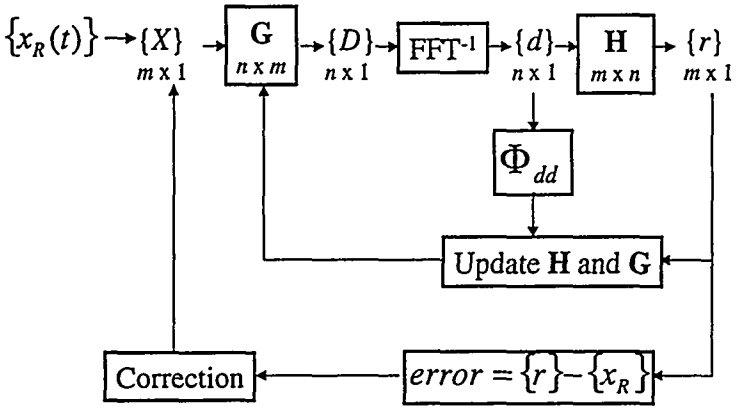

Figure 1 Block diagram of a multiple shaker control algorithm for waveform control.

If the number of responses exceeds the number of drive signals, the responses are typically partitioned into the primary responses equal to the number of drive signals with the remainder of the responses as secondary responses. The reference or desired elements of the cross-spectral density matrix of the primary responses are then computed from

$$\Phi_{jk}^{RR} = \gamma_{jk}^2 \Phi_{jj}^{RR} \Phi_{kk}^{RR} \exp(i\phi_{jk}) \qquad (7)$$
$$j = 1:n, k = 1:n$$

Φ_{jk}^{RR} = the reference cross-spectral density between the jth and kth points

γ_{jk}^2 = the ordinary coherence between the jth and kth points

Φ_{jj}^{RR} = the auto spectrum of the jth reference point

Φ_{kk}^{RR} = the auto spectrum of the kth reference point

ϕ_{jk} = the phase between the jth and kth points

$i = \sqrt{-1}$.

The control algorithm for random control is outlined in Figure 2. The first step in the control algorithm is to factor the reference cross-spectral density matrix (Dodds and Robson, 1975) using a Cholesky factorization with zeros:

$$\Phi_{RR} = \mathbf{RR'} \qquad (8)$$

where **R** is a lower triangular matrix, and **R'** is the complex conjugate transpose of **R.** The factorization with zeros means that if a zero or a negative diagonal element is found during the factorization, the entire row and column of the factorization is set to zero.

Alternately any decomposition of the form

$$\Phi_{RR} = \mathbf{RWR'} \qquad (9)$$

can be used, where **W** is a real positive diagonal matrix. Cholesky factorization, with ones on the diagonal, or singular value decomposition are examples (Smallwood and Paez, 1991) that can be used. The diagonal elements in **W** will define the auto spectrum of a vector of independent noise sources.

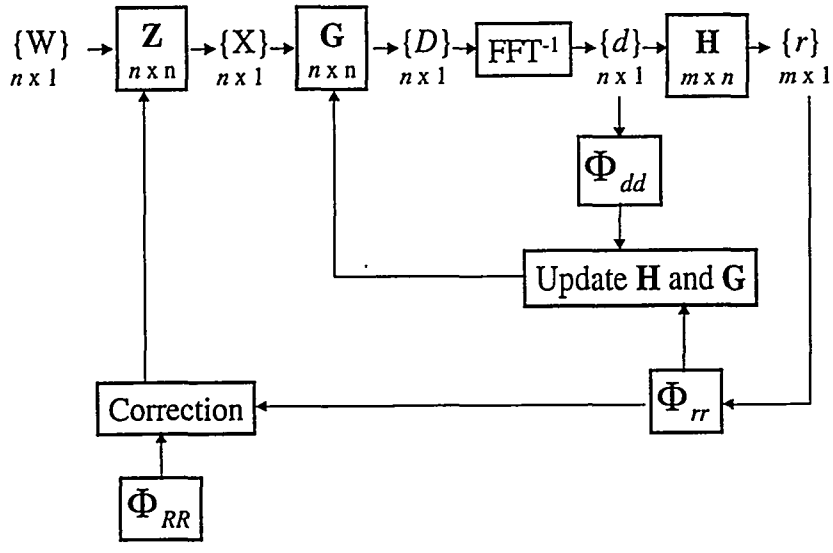

Figure 2 Block diagram for a multiple shaker random control algorithm.

If Cholesky decomposition is used, **W** is the identity matrix, and the independent sources will have unity spectral amplitude.

The order of the elements in the reference spectrum is the same as the order of the return measurements. The measurements are placed in a descending order of importance. The autospectrum of the first element will be matched first, then the autospectrum of the second, etc. The cross-spectrum between pairs will be matched as possible. Preference will be given to the autospectra, as will be explained later.

Set the initial value of the coupling matrix, **Z**, to

$$\mathbf{Z}_0 = \mathbf{R}. \tag{10}$$

The second step in the control algorithm is to identify the system under test. Drive the system with n partially or fully uncorrelated drives to measure the system frequency response function matrix just as for the waveform control algorithm.

The third step in the control algorithm starts the test with our initial estimates of the coupling matrix, **Z**, and the system cross-coupling matrix, **G**. {**W**} is a vector of independent noise sources in the frequency domain with uniform random phase. The auto-spectra are determined by the diagonal elements of **W**. The noise sources are coupled through a coupling matrix, **Z**, to give a vector of coupled sources, **X**. The coupled sources are equalized with the cross-coupling matrix, **G**, giving the drives in the frequency domain, **D**. The drives are transformed to the time domain, **d**, using high-speed overlap and add convolution. The drives are then applied to the system, resulting in a set of responses, **r**. The cross-spectral density matrix of the responses, Φ_{rr}, is then estimated. The response cross-spectral density can be a partition of the complete cross-spectral density or a linear combination of responses, but it needs to be the same size as the reference spectra. The response cross-spectral density matrix is used with the reference cross-spectral density matrix, Φ_{RR}, to estimate a correction to the coupling matrix, **Z**, completing the loop. An algorithm for the correction derived by the author will be presented later in a slightly modified form.

If the loop cannot be completed during the time for one block of data to be output to the shakers, time domain randomization can be used to generate

additional frames of output data, as described by Smallwood (1983). When time domain randomization is used, the vector of noise sources, W, is written as a diagonal matrix of independent noise sources, **W**. The drives become a full matrix of drives, **D** and **d**. Random circular shifts of the columns of the drive matrix, **d**, adding of rows, windowing, overlap, and add generate new drive signals while the loop is being completed.

If the loop-time is shorter than the time to output a block of drive signals, time domain randomization can be avoided. A block of data is a sequence of data points N long, where N is the size of the FFT used. The time to output a block of data is the product of the block size and the sample interval. Hence lower bandwidth tests (larger sample interval) are easier to control without time domain randomization.

An estimate of a non-singular drive cross-spectral density can be used with the response cross-spectral density to update our estimate of the system frequency response matrix, **H**, and hence our estimate of the cross-coupling equalization matrix, **G**. If the drive cross-spectral density cross-spectral density matrix is singular, this correction cannot be made.

A vector the length of the number of frequency lines can be kept to flag the frequencies at which the cross-coupling matrix, **G**, should not be updated.

It might be possible to use a Bendat Type 1 system (Bendat, 1998, Page 24, Fig. 2.4) to correct for some on the nonlinearities of the system.

The response of the system will be nominally Gaussian distributed. A Bendat Type 1 System or a zero memory nonlinear (ZMNL) transformation (Bendat, 1998, Smallwood, 1997) can be inserted between $\{X\}$ and **G** to generate non-Gaussian responses. Time domain randomization cannot be used if non-Gaussian responses are desired.

This method is a closed loop control system. If the system frequency response functions change during the test, the algorithm will attempt to correct the drives by changing the **Z** matrix to compensate.

RANDOM CORRECTION ALGORITHM

Correction to the coupling matrix, Z, can be made with the following guidelines.

1) Correction of the power spectral density, the diagonal terms in Φ_{rr}, will be given top priority. Priority of these terms will be in the response channel order i.e., the first channel will have highest priority, the second channel next, etc.
2) A cross-spectral density term will be increased only if the affected power spectral densities are not above their respective references. As in Step 1 the cross-spectrum will be corrected in the order of the response channels.
3) The correction will be computed in two forms. One form will be used if the affected power spectra are equal or below their respective reference. The second form will be used when the affected power spectra are above their respective references.

The first form of the correction is found by solving the following equation for the correction, Δ.

$$\Delta \mathbf{Z}^{'} + \mathbf{Z}\Delta^{'} = \Phi_{RR} - \Phi_{rr} = \mathbf{E}. \qquad (11)$$

The solution for Δ is

$$\Delta_{11} = \frac{E_{11}}{2Z_{11}} \qquad (12)$$

$$\Delta_{j1} = \frac{E_{j1} - \Delta_{11} Z_{j1}}{Z_{11}} \quad j = 2, N$$

$$\Delta_{jj} = \frac{E_{jj} - \sum_{i=1}^{j-1} (\Delta_{ji} Z_{ji}^{*} + \Delta_{ji}^{*} Z_{ji})}{2Z_{jj}} \quad j = 2, N$$

$$\Delta_{jk} = \frac{E_{jk} - \Delta_{kk} Z_{jk} - \sum_{i=1}^{k-1} (\Delta_{ki}^{*} Z_{ji} + \Delta_{ji} Z_{ki}^{*})}{Z_{jj}}$$

$$j = k+1, N.$$

This form corrects both the amplitude and phase. The second form of the correction is found from the equation

$$\Delta_{jk}^{alternate} = \frac{Z_{jk}^{old} (\Phi_{kk}^{RR} - \Phi_{kk}^{rr})}{2j\Phi_{kk}^{RR}}. \qquad (13)$$

This form corrects the amplitude but does not correct the phase. Several minor variations are required to handle the ill-conditioned problem. The algorithm is given in the form of a MATLAB function in Appendix A.

This algorithm differs from the original algorithm in three minor, but important, details.

In the original algorithm if Z_{ii} grew large, the corrections became progressively smaller because of the division by Z_{ii}. This could lead to Z_{ii} becoming stuck at an unrealistically high level. This was corrected by rescaling the diagonal corrections when the correction becomes small compared to the value of Z_{ii}.

The corrections in the *i*th column can also become very large if the corresponding diagonal element becomes small. To prevent this, the corrections are scaled by the maximum of Z_{11} and Z_{ii}. Because of the structure of the problem, Z_{11} should never become small.

The number of nonzero rows and columns in **Z** should not exceed the rank, as determined by a tolerance, of the system identification matrix, **H**. The rank is loosely defined as the number of singular values larger than a tolerance.

A small detail was added to prevent lower triangular elements of **Z** from becoming identical to zero. Because of multiplication by elements this can lead to terms being stuck at zero.

EXAMPLE OF THE CORRECTION ALGORITHM WITH AN ILL-CONDITIONED SYSTEM

Consider an extreme case for a three-input/three-output case. Let the test item be rigid and the frequency response function relating the input to response be unity.

$$\mathbf{H}=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} \quad (14)$$

Assume we desire the responses to be independent. The desired spectrum is the identity matrix

$$\mathbf{\Phi}_{RR}=\begin{bmatrix}1 & & \\ & 1 & \\ & & 1\end{bmatrix}. \quad (15)$$

The decomposition of the desired spectrum yields

$$\mathbf{R}=\mathbf{\Phi}_{RR}. \quad (16)$$

We cannot achieve this result. The system has a single independent input. The rank of the matrix **H** is one. To force the responses to be independent the rank of the matrix **H** must be three. The best we can do is generate inputs such that the cross-spectral density matrix of the return signals is

$$\mathbf{\Phi}_{rr}=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}. \quad (17)$$

The autospectra will be as desired, but the cross-spectra will be much different than originally desired. We cannot make the responses independent.

The pseudo inverse of the system matrix, **H**, is

$$\mathbf{G}=\begin{bmatrix}1/9 & 1/9 & 1/9\\ 1/9 & 1/9 & 1/9\\ 1/9 & 1/9 & 1/9\end{bmatrix}. \quad (18)$$

The initial estimate of the coupling matrix will be

$$\mathbf{Z}_0=\begin{bmatrix}1 & & \\ & 1 & \\ & & 1\end{bmatrix}. \quad (19)$$

The initial return spectrum will be

$$\mathbf{\Phi}_{rr}=\begin{bmatrix}1/3 & 1/3 & 1/3\\ 1/3 & 1/3 & 1/3\\ 1/3 & 1/3 & 1/3\end{bmatrix} \quad (20)$$

All the autospectra are third of the desired values.

However, with a convergence factor of one, after 60 iterations (control loops) and 200 averages to

estimate the return cross-spectral density, the coupling matrix has changed to something close to

$$\mathbf{Z} = \begin{bmatrix} 3.0 & 0 & 0 \\ .000 & .000 & 0 \\ .000 & .000 & .000 \end{bmatrix}. \quad (21)$$

The cross-spectral density matrix of the drive becomes

$$\Phi_{dd} = \begin{bmatrix} .11 & .11 & .11 \\ .11 & .11 & .11 \\ .11 & .11 & .11 \end{bmatrix}. \quad (22)$$

The drive is shared equally among the three drive signals. The return cross-spectral density converges to something close to

$$\Phi_{rr} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad (23)$$

which is close to the accepted values.

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APPENDIX A
CORRECTION ALGORITHM

```
function [Znew,D,Dp] = ...
correction(Y,C,Z,e,tol,rankH)
```

```
% function [Znew,D,Dp] =
% correction(Y,C,Z,e,tol,rankH)
% function to calculate the correction for a
% multiple input control system
% INPUT:
% Y = the matrix of the desired cross-spectral
% density matrix
% C = the matrix of the return control cross
% spectral density matrix
% The cross-spectral density matrix between x
% and y is defined as E[XY']
% This is different from Bendat and previous
% papers which defined cross-spectrum as
% E[conj(X)*Y.']
% The current notation is more compact in
% Matlab
% The only difference is that the new cross
% spectrum is the conjugate of the old cross
% spectrum
% Z = a lower triangular matrix, The original Z
%  is equal to ZZ'=Y,
%     Z will be changed in response to the
% realizability of the return spectrum
% Generally the attempt is to keep the auto
% spectra of C equal to the autospectra of Y.
% Higher precedence is given to the lower
% indexes of Y. After the autospectrum is met
% the cross-spectra
% are matched. Again precedence is given to
% the lower order indexes.
% e = a convergence factor
% tol = optional tolerance factor
%       If a diagonal element of
% Znew<tol*abs(max(max(Z))) the magnitude
of all elements
%       in that column will be set to the same
% value. default: tol=.0001
% rankH = the rank of the system identification
% matrix to a tolerance tol
%         optional, default rankH=min(size(Y))
% OUTPUT:
% Znew = the corrected Z
% D = the matrix correction to Z, Z(new) =
% Z(old) + D
% Dp = an alternate correction to Z used
% sometimes
%
% Ref: Smallwood, D. O.,1999, "Multiple
% Shaker Random Vibration Control – An
% Update," P. of IEST

% David Smallwood, Sandia National
% Laboratories, Albuquerque NM 87185
% Version 1.0 1/5/99

if nargin<6, rankH = min(size(Y));, end
if isempty(rankH), rankH=min(size(Y));, end
if nargin<5, tol=.0001;, end
if isempty(tol), tol=.0001;, end
scale = abs(max(max(Y)));
minZ = tol*scale;
[r,c] = size(Y);
if rankH>r, error('rankH> size of Y'), end
if rankH<1, error('rankH<1'), end
if tol>1, warning('tol>1'), end
E = Y-C;   % The error spectrum
N = min(rankH,r); % This will be the number of
% nonzero rows & columns in Znew
D = zeros(size(Y));
Znew = D;
Dp = D;
[re,ce]=size(e);
if re<r
  e=e*ones(size(Y));
end
if N<r
  Z(N+1:r,:)=0;
  Z(:,N+1:r)=0;
end
% Find the correction
  % first column of D
  D(1,1) = real(E(1,1)/(2*Z(1,1)));
if N>1
  for k=2:N
    D(k,1) = (E(k,1)-D(1,1)*Z(k,1))/(Z(1,1));
    if abs(E(k,k))<.5*Y(k,k)
      Dp(k,1) = Z(k,1)*E(1,1)/(2*k*Y(1,1));
    else
      Dp(k,1) = sign(E(k,k))*.5*Z(k,1);
    end
  end
  % Columns 2:N
  for j=2:N
    D(j,j) = E(j,j);
    for i=1:j-1
      D(j,j) = D(j,j) - ...
(conj(D(j,i))*Z(j,i)+conj(Z(j,i))*D(j,i));
    end
    % I don't want to blow up if Z(j,j) gets small
    D(j,j) = ...
real(D(j,j)/(2*max(abs(Z(1,1)),Z(j,j))));
    if abs(D(j,j))>1e6
       warning('D(j,j) too large')
    end
    if j<N
      for k=j+1:N
        D(k,j) = E(k,j) - D(j,j)*Z(k,j);
        for i=1:j-1
          D(k,j) = D(k,j) - ...
(conj(D(j,i))*Z(k,i)+conj(Z(j,i))*D(k,i));
        end
        D(k,j) = D(k,j)/max(Z(1,1),Z(j,j));
```

```
        if abs(E(k,k))<.5*Y(k,k)
          Dp(k,j) = Z(k,j)*E(k,k)/(2*j*Y(k,k));
        else
          Dp(k,j) = sign(E(k,k))*.5*Z(k,j);
        end
      end % end k=j+1:N
    end % end if j<N
  end % end for k=2:N
end  % end N>1
% make sure diagonal corrections are small
for i=1:N
    test=max(abs(Z(i,i)),abs(Z(1,1)));
    if abs(D(i,i))>.5*test
      D(i,i)=D(i,i)*.5*test/abs(D(i,i));
    end
end
% correct the Z matrix
Znew = Z + e.*D;
  % I don't want to get stuck at zero
  if N>1
    for i=2:N
      if Znew(i,i)<=0
        Znew(i,i)=minZ;
      end
    end
  end
if Znew(1,1)<0, error('Z(1,1)<0'), end
% Check to see if alternate form should be
used
for i=1:N
  for k=i+1:N
    if Z(k,i)~=0 & E(k,k)<0 &
abs(Znew(k,i))>abs(Z(k,i))
      % The cross term is not zero,
      % The return autospectrum is too large
      % And cross-spectra is increasing use
        % alternate form
         Znew(k,i) = Z(k,i) + e(k,i).*Dp(k,i);
    end
    if Znew(k,i)==0
      Znew(k,i) = minZ;
    end
  end
end
% If Znew is reduced to zero I want entire
column to be zero
for k=2:N
  if Znew(k,k)<minZ
    for j=k+1:N

Znew(j,k)=Znew(j,k)*.01*minZ/abs(Znew(j,k));
    end
  end
end
```