Title: ONLINE HIGH VOLTAGE POWER SUPPLY RIPPLE ESTIMATION AND FEEDFORWARD IN LEDA

Author(s): S. Kwon, A. Regan, Y-Ming Wang, T. Rohlev

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Title: Online high voltage power supply ripple estimation and feedforward in LEDA

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Online high voltage power supply ripple estimation and feedforward in LEDA
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Abstract—The Low Energy Demonstration Accelerator (LEDA) being constructed at Los Alamos National Laboratory will serve as the prototype for the low energy section of Acceleration Production of Tritium (APT) accelerator. This paper addresses the problem of LLRF control system for LEDA. We propose an estimator of the ripple and its time derivative and a control law which is based on PID control and adaptive feedforward of estimated ripple. The control law reduces the effect of the deterministic cathode ripple that is due to high voltage power supply and achieves tracking of desired set points.

I. INTRODUCTION

The low energy demonstration accelerator (LEDA) for the Accelerator Production of Tritium (APT) is being built at Los Alamos National Laboratory. The primary function of the low level RF (LLRF) control system of LEDA is to control RF fields in the accelerating cavity and maintain field stability within $\pm 1\%$ peak to peak amplitude error and $1^\circ$ peak to peak phase error[5].

This paper addresses the problem of LLRF control system attenuating the effect of ripple on the klystron cathode voltage that results from the high voltage power supply ripple. We propose a PID control law coupled with an adaptive feedforward of ripple estimate. The purpose of control is to reduce the effect of the deterministic cathode ripple that is due to harmonics of high voltage power supply[4] and to achieve tracking of desired set points. Low frequency ripple does not deteriorate current LLRF control system performance based on current PID control methodology. As frequency of the ripple increases, however, the effect of the ripple on the performance increases too. Simulation shows that $0.3\%$ high voltage power supply ripple yields $1.05^\circ$ peak to peak phase error at about $72kHz[3]$ and $1.0\%$ high voltage power supply ripple yields $1.07^\circ$ peak to peak phase error at about $20kHz[7]$. In order to suppress the high frequency ripple effect, the proposed controller makes use of a feedforward control coupled with a ripple estimator. The high voltage power supply ripple is coupled to the LEDA through a klystron. The effects of the ripple are on both the amplitude and the phase of a klystron. In [6],[7], the influences of the ripple are modeled by algebraic equations. A klystron is modeled by a nonlinear state space system. We, first, address two coordinate transformations of a klystron model. Based on new coordinates, we extract the ripple equation which is represented by algebraic equations of states of new coordinates. The ripple estimator proposed in this work is based on the algebraic equation and it estimates the ripple signal itself and the time derivative of the ripple as well. The estimate of ripple is feedforwarded to the current LLRF control system whose frame is a PID control. This simple addition of an adaptive feedforward greatly improves the closed loop system performance.

II. KLYSTRON MODEL

We consider a klystron model as shown in as shown in Figure 1.

It has two inputs, LLRF I and LLRF Q and two output HPRF I and HPRF Q. As intermediate outputs, Klystron has the normalized amplitude $N_{AMPLITUDE}$ and the normalized phase $N_{PHASE}$.

The first stage of a klystron are linear systems called FILTER AND AMPLIFIER. Let $u_1=LLRF_I$ and let $u_2=LLRF_Q$. FILTER AND AMPLIFIER is represented in state space as

$$\dot{x}_1 = -a_1 x_1 + a_1 u_1$$
$$\dot{x}_2 = -a_2 x_2 + a_2 u_2$$

where $a_1 = \frac{1}{2} \times 10^4$.

A klystron model has two look-up tables, called AMPLITUDE SATURATION and PHASE SATURATION. The input of the two look-up tables is given by

$$A = \frac{K_g}{10\sqrt{K_{P_m}} (0.01 R(t) + 1)^{1.25} \cdot \sqrt{x_1^2 + x_2^2}}$$

where $R(t)$ is the ripple, $K_g$ is the klystron gain, and $K_{P_m}$ is the maximum klystron power. $R(t)$, $K_g$, and $K_{P_m}$ are specified for a given klystron. For given $A$, the output of the look-up table AMPLITUDE SATURATION can be represented by

$$A_N = I_1(A)$$

and the output of the look-up table PHASE SATURATION can be represented by

$$\theta_N = I_2(A)$$

Table 1 and table 2 show data of look-up table AMPLITUDE SATURATION and data of look-up table PHASE SATURATION, respectively.
The normalized amplitude $N_{\text{Amplitude}}$, defined by $y^N_1$, and the normalized phase $N_{\text{Phase}}$, defined by $y^N_2$, of the klystron are expressed by

$$y^N_1 = A_N = f_1(A)$$

$$y^N_2 = \theta_N + \tan^{-1}\left(\frac{z^N_2}{x_1}\right) + 3 \cdot \frac{\pi}{180} \cdot R(t).$$

In addition, for given $y^N_1$ and $y^N_2$, HPRF.I and HPRF.Q are given by

$$\text{HPRF.I} = 10 \sqrt{K_{P_m}} \cdot y^N_1 \cdot \cos(y^N_2)$$

$$\text{HPRF.Q} = 10 \sqrt{K_{P_m}} \cdot y^N_1 \cdot \sin(y^N_2).$$

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Table 1. AMPLITUDE SATURATION Data

$$y^N_1 = \sum_{i=1}^{N} c_i e^{f_i w(t)} \sqrt{x_1^2 + x_2^2}$$

$$y^N_2 = \sum_{i=1}^{N} d_i e^{f_i w(t)} \sqrt{x_1^2 + x_2^2} + \tan^{-1}\left(\frac{z^N_2}{x_1}\right) + 3 \cdot \frac{\pi}{180} \cdot R(t).$$

where

$$w(t) = \frac{K_g}{10 \sqrt{K_{P_m}}} (0.01 R(t) + 1)^{1.26}.$$ (14)

The normalized amplitude $y^N_1$ and the normalized phase $y^N_2$ of the klystron are

$$y^N_1 = \sum_{i=1}^{N} c_i e^{f_i w(t)} \sqrt{x_1^2 + x_2^2}$$

$$y^N_2 = \sum_{i=1}^{N} d_i e^{f_i w(t)} \sqrt{x_1^2 + x_2^2} + \tan^{-1}\left(\frac{z^N_2}{x_1}\right) + 3 \cdot \frac{\pi}{180} \cdot R(t).$$

II-A. The Klystron in z-coordinate

Consider the normalized amplitude $y^N_1$ and the normalized phase $y^N_2$ as given in (15) and (16). Let

$$z_1 = \sqrt{x_1^2 + x_2^2}$$

$$z_2 = \tan^{-1}\left(\frac{z^N_2}{x_1}\right).$$ (17)

(18)

We consider a transformation from x-coordinate to z-coordinate. In z-coordinate, the state equations (1) and (2) are reduced to

$$\dot{z}_1 = -a_1 z_1 + a_1 \cos(z_2) u_1 + a_1 \sin(z_2) u_2$$

$$\dot{z}_2 = -a_1 \sin(z_2) u_1 + a_1 \cos(z_2) u_2.$$ (19)

(20)

Also, the curve fitting equations (10) and (11) are reduced to

$$A_N = \sum_{i=1}^{N} c_i e^{f_i w(t)} z_1$$

$$\theta_N = \sum_{i=1}^{N} d_i e^{f_i w(t)} z_1.$$ (21)

(22)

The normalized amplitude $y^N_1$ and the normalized phase $y^N_2$ are represented by

$$y^N_1 = \sum_{i=1}^{N} c_i e^{f_i w(t) z_1}$$

$$y^N_2 = \sum_{i=1}^{N} d_i e^{f_i w(t) z_1} + z_2 + 3 \cdot \frac{\pi}{180} \cdot R(t).$$ (23)

(24)
Note that the exponents of the first term of (23) are the same as the exponents of the first term of (24). Also, note that the phase $y_2$ is linear with respect to $z_2$.

II-B. THE KLYSTRON IN $\mathcal{Z}$-COORDINATE

Consider the Klystron equation in $z$-coordinate given in previous section.

Define

$$\begin{align*}
\bar{z}_1 &= (0.01R(t) + 1)^{1.26}z_1 \\
\bar{z}_2 &= z_2 + 3 \cdot \frac{\pi}{180} \cdot R(t).
\end{align*}$$

(25)  
(26)

The Klystron is expressed in $\bar{z}$-coordinate by

$$\begin{align*}
\dot{\bar{z}}_1 &= -\bar{z}_1 \bar{z}_1 + b_{211}(\bar{z}, R(t), \bar{R}(t))u_1 + b_{212}(\bar{z}, R(t), \bar{R}(t))u_2 \\
&\quad + E_{21}(\bar{R}(t), \bar{R}(t)) \\
\dot{\bar{z}}_2 &= b_{221}(\bar{z}, R(t), \bar{R}(t))u_1 + b_{222}(\bar{z}, R(t), \bar{R}(t))u_2 \\
&\quad + E_{22}(\bar{R}(t), \bar{R}(t)) \\
y_1 &= \sum_{i=1}^{N} c_i e^{-f_i \Delta T_1} \\
y_2 &= \sum_{i=1}^{N} d_i e^{-f_i \Delta T_1} + \bar{z}_2.
\end{align*}$$

(27)  
(28)  
(29)  
(30)

where

$$\begin{align*}
b_{i1} &= a_1 - 0.0125(0.01R(t) + 1)^{-1}\bar{R}(t) \\
b_{i21}(\bar{z}, R(t), \bar{R}(t)) &= a_1(0.01R(t) + 1)^{1.26}\cos(\bar{z}_2 - 3 \cdot \frac{\pi}{180} \cdot R(t)) \\
b_{i22}(\bar{z}, R(t), \bar{R}(t)) &= a_1(0.01R(t) + 1)^{1.26}\sin(\bar{z}_2 - 3 \cdot \frac{\pi}{180} \cdot R(t)) \\
E_{i1}(\bar{R}(t), \bar{R}(t)) &= 0 \\
E_{i2}(\bar{R}(t), \bar{R}(t)) &= 3 \cdot \frac{\pi}{180} \bar{R}(t) \\
M &= \frac{K_m}{10 \sqrt{K_p}}.
\end{align*}$$

Note that $\bar{R}(\bar{z}, R(t), \bar{R}(t))$ is invertible for any nonzero $\bar{z}_1$. In $\bar{z}$-coordinate, state equations are dependent upon the ripple $R(t)$ but the output equations are independent upon the ripple $R(t)$.

III. THE RF CAVITY

Figure 2 shows the RF cavity model.

RF cavity has four inputs, HPRF_I, HPRF_Q, BEAM_I, and BEAM_Q, two outputs, CAV.FLD_I and CAV.FLD_Q. Let $u_1=HPRF_I$, $u_2=HPRF_Q$, $u_3=BEAM_I$, $u_4=BEAM_Q$, and let $y_1=CAV.FLD.I$, $y_2=CAV.FLD.Q$. Then, the RF cavity can be expressed in the state space form.

$$\begin{align*}
\dot{z} &= Az + Bu \\
y &= Cz.
\end{align*}$$

(31)  
(32)

System matrices $A$, $B$, $C$ of RF cavity are given in [3].

Also, FLD.I and FLD.Q of the RF cavity Field Sample System are given by

$$\begin{align*}
FLD.I &= FA \cdot \cos(GD) \cdot y_1 - FA \cdot \sin(GD) \cdot y_2 \\
FLD.Q &= FA \cdot \sin(GD) \cdot y_1 + FA \cdot \cos(GD) \cdot y_2
\end{align*}$$

(33)  
(34)

and FLD.AMP and FLD.PHS of the field Sample System are given by

$$\begin{align*}
FLD.AMP &= \sqrt{FLD.I^2 + FLD.Q^2} \\
FLD.PHS &= \tan^{-1} \left( \frac{FLD.Q}{FLD.I} \right)
\end{align*}$$

where

$$\begin{align*}
FA &= 0.00037809 \\
GD &= \frac{\pi}{180} \cdot (-0.039455).
\end{align*}$$

The RF cavity as given in (31), (32) is Hurwitz stable and is inverse stable as well.

IV. RIPPLE ESTIMATION

The purpose of the low level RF control(LLRF) system is to maintain the field stability within ±1.0% amplitude and 1.0° phase. The present LLRF control system is based on PID control[4],[5]. As has been investigated in [3],[7], the high voltage power supply ripple has influence on the LLRF PID control system. Low frequency ripple does not deteriorate the performance of the closed loop system seriously but high frequency ripple deteriorates the performance of the closed loop system seriously.

For the remedy to the poor performance of the PID controlled LLRF control system due to the high frequency high voltage power supply ripple, we propose a feedforward control of the estimate of the ripple. The feedforward improves the performance significantly[1],[6].

In this section, we address the ripple estimator which estimates the ripple $R(t)$ and its time derivative $\frac{dR(t)}{dt}$.

We first consider equations as given in (17) and (18).

$$\begin{align*}
z_1 &= \sqrt{x_1^2 + x_2^2} \\
z_2 &= \tan^{-1}(\frac{x_2}{x_1})
\end{align*}$$

(35)  
(36)

where $x_1$ and $x_2$ satisfy

$$\begin{align*}
z_1 &= -a_1 z_1 + a_1 u_1 \\
z_2 &= -a_2 z_2 + a_2 u_2
\end{align*}$$

(37)  
(38)

and $u_1=LLRF.I$, $u_2=LLRF.Q$. Given LLRF.I and LLRF.Q, we can obtain $z_1$ and $z_2$ by solving differential equations (37), (38) and algebraic equations (35), (36).

Second, we consider equations given by (8) and (9).

$$\begin{align*}
HPRF.I &= 10 \sqrt{K_{pm}} \cdot y_1 \cdot \cos(y_2) \\
HPRF.Q &= 10 \sqrt{K_{pm}} \cdot y_1 \cdot \sin(y_2).
\end{align*}$$

(39)  
(40)

From (39) and (40), for given HPRF.I and HPRF.Q, we obtain the normalized amplitude $y_1^*$ and the normalized phase $y_2^*$ of the klystron by solving algebraic equations.

$$\begin{align*}
y_1^* &= \frac{1}{10 \sqrt{K_{pm}}} \sqrt{HPRF.I^2 + HPRF.Q^2} \\
y_2^* &= \tan^{-1} \left( HPRF.Q \div HPRF.I \right).
\end{align*}$$

(41)  
(42)
Third, we consider the klystron model as given in Figure 1. In Figure 1, the normalized amplitude of the klystron is the output of the look-up table AMPLITUDE SATURATION. The input of the look-up table AMPLITUDE SATURATION is given by

$$ A = \frac{K_g}{10\sqrt{K_p}} (0.01R(t) + 1)^{1.25} \cdot z_1 $$

(43)

in z-coordinate, or

$$ A = \frac{K_g}{10\sqrt{K_p}} z_1 $$

(44)

in \( \xi \)-coordinate. Also, there exists a region of \((A, y_f)\) pairs where there is an inverse look-up table of the look-up table AMPLITUDE SATURATION. This region can be extracted from data given in Table 1 and Table 2. As in the case of AMPLITUDE SATURATION, we obtain the curve fitting equation for the inverse look-up table for AMPLITUDE SATURATION. Since, in controller design, we make use of the output equations (22) and (24) or (29) and (30) which are based on the curve fitting equation, in order to obtain the curve fitting equation for the inverse look-up table for AMPLITUDE SATURATION within the region of invertibility, we use the output equation as given in (29). Based on the generated data pairs from (29) where the selected data of \( y_f^k \) and \( z_1 \) guarantee invertibility, we obtain the curve fitting equation as follows.

$$ z_1 = \sum_{i=1}^{N} c_i e^{-j_i y_f^k} $$

(45)

where \( N = 7 \), coefficients \( f_i, \ i = 1, 2, \ldots, N \) are given and the coefficients \( c_i, \ i = 1, 2, \ldots, N \) are obtained by applying the optimization toolbox of Matlab/Simulink. Table 5 gives the data of the coefficients of the curve fitting. The nonlinear least square algorithm in Matlab/Simulink guarantees 1% accuracy of the curve fitting.

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Table 5. Coefficients of Curve fitting equation for Inverse AMPLITUDE SATURATION

The estimate of the ripple \( \hat{R}(t) \) and the estimate of the time derivative \( \frac{d\hat{R}(t)}{dt} \) of the ripple \( R(t) \) are obtained by considering the klystron system both in z-coordinate and \( \xi \)-coordinate. The relation between z-coordinate and \( \xi \)-coordinate is given by

$$ z_1 = (0.01R(t) + 1)^{1.25} \cdot z_2 $$

(46)

$$ \xi = z_2 + 3 \cdot \frac{\pi}{180} \cdot R(t) $$

(47)

Whenever \( z_1 \) and \( z_2 \) are obtained from (37) and (38), then we can obtain \( z_1 \) by using (35), and also whenever \( y_f^k \) is obtained from (41), we can obtain \( z_1 \) by using (45). For given \( z_1 \) and \( \xi \), we can obtain the estimate \( \hat{R}(t) \) of the ripple \( R(t) \) by solving algebraic equation (46).

$$ \hat{R}(t) = 100 (z_1(t) \frac{z_1(t)}{z_1(t)}^{0.8} - 1.0). $$

(48)

Also, the estimate \( \hat{R}(t) \) of time derivative \( \frac{d\hat{R}(t)}{dt} \) of the ripple \( R(t) \) is obtained by differentiation of \( \hat{R}(t) \).

V. FEEDFORWARD CONTROL OF THE ESTIMATE \( \hat{R}(t) \)

The work in [7] shows that the AMPLITUDE DEPENDENCE of the high voltage power supply ripple is not seriously effective to the performance of tracking set points of the LLRF control system. The main effect of the high voltage power supply ripple is due to PHASE DEPENDENCE. In this section, we propose a feedforward control of the estimate \( \hat{R}(t) \). The feedforward control improves the tracking performance of the field phase significantly.

We consider the output equations as given in (23), (24) and (39), (40). The amplitude and the phase of the high power RF(HPRF) amplifier are

$$ HPRF.AMP = \sqrt{HPRF.J^2 + HPRF.Q^2} $$

(49)

$$ HPRF.PHS = \tan^{-1}\frac{HPRF.Q}{HPRF.J}. $$

(50)

In (24), the normalised phase \( y_f^k \) is affected by the PHASE DEPENDENCE term

$$ -3 \cdot \frac{\pi}{180} \cdot R(t). $$

(52)

When \( R(t) \) is estimated within a satisfactory accuracy, a feedforward control loop from the estimate \( \hat{R}(t) \) to \( y_f^k \) can attenuate the effect the high voltage power supply ripple \( R(t) \) to the phase of the klystron and so the phase of field.

Define a feedforward loop gain from the estimate \( \hat{R}(t) \) as

$$ R(t) = \frac{3 \cdot \pi}{180} \cdot \hat{R}(t). $$

(51)

and consider \( HPRF.AMP, HPRF.PHS \) defined by

$$ HPRF.AMP = HPRF.AMP $$

(53)

$$ HPRF.PHS = HPRF.PHS - 3 \cdot \frac{\pi}{180} \cdot \hat{R}(t). $$

(54)

Plugging (24), (42), and (50) into (54), we obtain

$$ HPRF.PHS = y_f^k - 3 \cdot \frac{\pi}{180} \cdot \hat{R}(t) $$

$$ = \sum_{i=1}^{N} d_i e^{-f_i u(t)} z_1 + z_2 + 3 \cdot \frac{\pi}{180} \cdot R(t) - 3 \cdot \frac{\pi}{180} \cdot \hat{R}(t) $$

$$ = \sum_{i=1}^{N} d_i e^{-f_i u(t)} z_1 + z_2 + 3 \cdot \frac{\pi}{180} (R(t) - \hat{R}(t)). $$

(55)

The estimate \( \hat{R}(t) \) as given in (48) is defined by the solution of an algebraic equation (46). \( z_1 \) is given by (35) whose variables are the state of the exactly same system of FILTER AND AMPLIFIER of a klystron. \( \xi \) is given by the curve fitting equation (59) of the lookup table of inverse AMPLITUDE SATURATION.

Of course, we have to consider the AMPLITUDE DEPENDENCE represented by (0.01R(t) + 1)^{1.25}(1% ripple). However, this amount does not have an influence on the phase of the field significantly[8,7].

From (33) and (54), we reconstruct \( M_{HPRF.J} \) and \( M_{HPRF.Q} \) which drive the RF cavity.

$$ M_{HPRF.J} = HPRF.AMP \cdot \cos(HPRF.PHS) $$

(56)

$$ M_{HPRF.Q} = HPRF.AMP \cdot \sin(HPRF.PHS). $$

(57)

Figure 3-4 shows the simulation results of the proposed LLRF Control System with adaptive feedforward of the estimate \( \hat{R}(t) \). The ripple is

$$ R(t) = \sin(2\pi f t), \quad f = 40 kHz.$$
The peak-to-peak amplitude error is 0.864% and the peak-to-peak phase error at steady state is 0.0474°. In order to compare the performance, we also simulate the closed loop system without the feedforward of the estimate $\hat{R}(t)$ (Figure 5). The peak-to-peak amplitude error is 0.922% and the peak-to-peak phase error at steady state is 2.075°. The peak-to-peak amplitude error is not much reduced with feedforward but the peak-to-peak phase error is greatly reduced (438% improvement).

REFERENCES