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EFFECT OF IMPACTOR AREA ON COLLISION RATE ESTIMATES

Gregory H. Canavan

Analytic and numerical estimates provide an assessment of the effect of impactor area on space debris collision rates, which is sufficiently small and insensitive to parameters of interest that it could be neglected or corrected.

Analytic estimates can provide assessments of collision and fragment production rates, but it is difficult to assess the effect of impactor area on those rates analytically. This note uses a combination of analytic and numerical calculations to bound the effect of impactor area on the rate of catastrophic collisions. The ratio of the exact and approximate rates is independent of the mass of the target objects and varies only with the object distribution power law exponent and mass-area scaling exponent. Numerical integration shows that the variation with the latter is negligible and with the former about 10%. The overall error is sufficiently small and varies so little with parameters of interest that it could be neglected or inserted as a correction.

The rate of catastrophic collisions between a debris objects of mass x and impactors of mass y can be written as

$$R_{\text{cat}} = (1/W) \int_R^U dx \int_{fx}^x dy n(x) n(y) \sigma V, \quad (1)$$

where $n(x)$ and $n(y)$ are their number densities, W is the volume of LEO that contains objects, V is their relative velocity, and $w = W/V$.¹ The collision rate is restricted to catalogue target particles by taking $R \approx 0.6$ kg and $U \approx 6$ tons, which are roughly the smallest and largest particle masses in the catalog, to be the limits of integration.² The limits of integration of the impactor y are taken to be fx and x , where $f \approx 0.001$ is the smallest impactor to target mass ratio that can produce complete fragmentation.³ The cross section σ is roughly geometric; hence,

$$\sigma \approx K(x^{s/2} + y^{s/2})^2, \quad (2)$$

where $s \approx 1$ to 1.1 and $K \approx 0.017 \text{ m}^2/\text{kg}$ are empirical constants.⁴

Calculations. For a power law dependence of the cumulative number of objects on mass, with exponent c , the cumulative object density varies as $N(m) = N(R/m)^c$, where N is the total number of objects with mass greater than R . Thus, the differential density varies as

$$n(m) = NcR^c/m^{c+1}, \quad (3)$$

It is possible to approximate R_{cat} analytically, if it is assumed that $x \gg y$ throughout, so that $\sigma \approx Kx^s$, and Eq. (1) becomes

$$R_{\text{cat}} = (K/w) \int_R^U dx n(x) x^s \int_{fx}^x dy n(y). \quad (4)$$

For the usual limits on the second integral over y , R to U , it would yields the number of catalog objects, N . For $s = 1$, the first integral would provide the total; mass of catalog objects, M , so their product would give the approximate catalog collision rate

$$R_{cat} = (K/w)MN. \quad (5)$$

For the calculation of interest here, s is not unity, so the first integral does not give the mass. That is a small correction. A larger effect is the restriction of the limits of integration for the impactor integral to fx to x . The variation of σ with y is a smaller effect.

Kernel. The goal is to explore the error involved in approximating $\sigma \approx K(x^{s/2} + y^{s/2})^2$, by Kx^s . In doing so, the integral over the target mass, x , masks the magnitude of the approximation, which primarily affects the integral over the projectile mass y . For that reason, the discussion below concentrates on the kernels of the integral

$$I_{full}(x) = \int_{fx}^x dy n(y) (x^{s/2} + y^{s/2})^2, \quad (6)$$

in which the full expression for $\sigma \approx K(x^{s/2} + y^{s/2})^2$ is retained, and on

$$I_{appr}(x) = \int_{fx}^x dy n(y) x^s, \quad (7)$$

which retains only $\sigma \approx Kx^s$. The latter is a useful vehicle for examining the sensitivities to the approximations made in both integrals.

Analytics. Equation (7) can be solved approximately by taking x^s outside the integral and substituting from Eq. (3)

$$I_{appr}(x) = x^s \int_{fx}^x dy NcR^c/m^{c+1} \approx NRCx^s[(fx)^{-c} - x^{-c}] \approx N(R/f)^c x^{s-c}, \quad (8)$$

in which the approximation in going from the first to the second form of the integral is that the definition of f is imprecise. In going from the second to the third it is the neglect of 1 relative to $1/f^c \approx 5.6$. That is sloppy, but the alternative is carrying along a number of expressions like $1/f^c - 1$, which defeats the purpose of an analytic solution.

Change of variables. On the substitution of variables $z = y/fx$, I_{full} becomes

$$\begin{aligned} I_{full}(x) &= \int_{fx}^x d(fxz) NcR^c/(fxz)^{c+1} x^s (1 + (fz)^{s/2})^2 \\ &= NcR^c(fx)^{-c} x^s \int_{fx}^x dz /z^{c+1} (1 + (fz)^{s/2})^2, \end{aligned} \quad (9)$$

which is the same as $I_{appr}(x)$ apart from the $(fz)^{s/2}$ in the parentheses. Thus, both terms have an integrand that decreases rapidly as $1/z^{c+1}$, $I_{full}(x)$ has an additional term that is important at the upper limit of the integral, where $fz \approx 1$. The ratio of the two integrals is

$$I_{full}(x) / I_{appr}(x) = \int_1^{1/f} dz /z^{c+1} (1 + (fz)^{s/2})^2 / \int_1^{1/f} dz /z^{c+1}, \quad (10)$$

which is independent of x . Thus, it is only necessary to perform the integrals once for each value of c and s . The attached figure shows $I_{full}(x) / I_{appr}(x)$ as a function of c for values of s from 1 to 1.2. For $s = 1$, as c varies from 0.15 to 0.35, the ratio falls from 1.57 to 1.34, or about 17%. The curves fall very slightly as s increases. At the nominal $s = 1$ to 1.1 and $c = 0.25$, the ratio is about 1.4, so that the collision rate from the detailed calculation is about 40% above that from the approximate expression. The difference between the two expressions is sufficiently small and varies so little for parameters of interest that it could be neglected or inserted as a correction.

Summary and conclusions. Analytic estimates can be used to provide an assessments of collision and fragment production rates. However, it is difficult to treat the effect of impactor

area on those rates. This note uses a combination of analytic and numerical calculations to bound the effect of impactor area on the rate of catastrophic collisions. The analytic discussion indicates the types of approximations implicit in both treatments. A change of variables shows that the ratio of the full and approximate rates is independent of the mass of the target objects and varies only with the distribution exponent c and the mass-area scaling exponent s . A numerical integration shows that the variation with s is negligible, that with c is about 10%, and that the overall error is on the order of 30-40%, which is sufficiently small and varies so little for parameters of interest that it could be neglected or inserted as a correction.

References

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rel coll rrt vs c; s area

Relative collision rate versus power law exponent.