Single Phase Channel Flow Forced Convection Heat Transfer

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ABSTRACT

A review of our current knowledge of single phase forced convection channel flow of liquids ($Pr > 5$) is presented. Two basic channel geometries are considered, the circular tube and the rectangular duct. Both laminar flow and turbulent flow are covered. The review begins with a brief overview of the heat transfer behavior of Newtonian fluids followed by a more detailed presentation of the behavior of purely viscous and viscoelastic Non-Newtonian fluids. Recent developments dealing with aqueous solutions of high molecular weight polymers and aqueous solutions of surfactants are discussed. The review concludes by citing a number of challenging research opportunities.

NOMENCLATURE

a	geometric constant in Kozicki generalized Reynolds number, Eq-9
A	area of the rectangular duct cross section
b	geometric constant in Kozicki generalized Reynolds number, Eq-9
C_p	specific heat of fluid
c	exponent in Ellis model
D	diameter
D_h	hydraulic diameter, $4AP$
d	exponent in Cross model
e	exponent in Sutterby model
f	Fanning friction factor, $f = \tau_w / (\rho U^2/2)$
Gz	Graetz number, $m C_p / k x$
h	convective heat transfer coefficient, $q'' / (T_w - T_b)$
j_H	Colburn heat transfer factor, $Nu / (Re Pr^{1/3})$
k	thermal conductivity of fluid
L_{hy}	hydrodynamic entrance length
L_{hy}^*	dimensionless hydrodynamic entrance length, $L_{hy} / D_h Re$
L_{th}	thermal entrance length
L_{th}^*	dimensionless thermal entrance length, $L_{th} / (D_h Pe)$
m	mass flow rate

N_1	first normal stress difference
n	power law index in power law fluid model, $\tau = K\dot{\gamma}^n$
Nu	Nusselt number, hD_h / k
P	perimeter of the duct cross section
Pe	Peclet number, $\rho C_p U D_h / k$
Pr	Prandtl number, $\eta C_p / k$
Pr'	Prandtl number, based on Re' , Pe / Re'
Pr^*	Prandtl number, based on Re^* , Pe / Re^*
q''	heat flux per unit of heating area
Ra_q	Rayleigh number based on q''
Re	Reynolds number based on apparent or Newtonian viscosity, $\rho U D_h / \eta$
Re_J	Reynolds number introduced by Jones, $\rho U D_h / [\eta(a+b)]$
Re'	Metzner's generalized Reynold's number, Eq. [8]
Re^*	Kozicki generalized Reynolds number, Eq. [9]
St	Stanton number, Nu/Pe
T	temperature
U	mean velocity in axial direction
$wppm$	weight parts per million
Ws	Weissenberg number, $\lambda U / D_h$
x	axial rectilinear coordinate

Greek Symbols

α^*	aspect ratio
δ	phase shift angle
$\dot{\gamma}$	shear rate
η	viscosity of Newtonian fluid; apparent viscosity of non-Newtonian fluid at wall shear rate
λ	characteristic time of viscoelastic fluid
ρ	density of fluid
τ_w	wall shear stress
ω	angular frequency

Subscripts

b	bulk value
H	constant heat flux boundary condition

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w	wall
m	measured value
p	predicted value
T	constant wall temperature boundary condition
x	at axial location from the duct entrance
0	for very small (zero) shear rate
∞	for very large (infinite) shear rate

INTRODUCTION

The task of reviewing single phase forced convection heat transfer for channel flow is a difficult assignment. A measure of the challenge may be quantified by the following facts:

- There are now over 100 journals providing heat transfer information.
- The annual reviews of Professor Eckert and his Minnesota colleagues cite approximately 100 papers each year devoted to single phase channel flow.
- The 1987 Handbook of Single Phase Convective Heat Transfer edited by S. Kakac, R.K. Shah and W. Aung devotes 700 pages to single phase channel flow.

Against this huge repository of information, I have decided to stay within the general research area of non-Newtonian fluids, especially viscoelastic fluids, in laminar and turbulent flow through circular pipes and rectangular channels. Above and beyond the obvious fact that this area of research has occupied much of my time (and more importantly, of my graduate students' time,) of the past few decades, it remains a relatively unexplored field. Consequently, there are many opportunities, both experimental and analytical, for future research.

The paper will begin with a brief overview of Newtonian fluids and will be limited to flows through circular pipes and rectangular channels and will be restricted to relatively high Prandtl number fluids ($Pr > 5$). Purely viscous non-Newtonian fluids will then be treated and the review will finish with an overview of our current knowledge of the fluid mechanical and heat transfer behavior of viscoelastic fluids. In conclusion, a number of challenging research opportunities in the study of viscoelastic fluid mechanics and heat transfer are identified.

NEWTONIAN FLUIDS

The forced convection behavior of single-phase Newtonian fluids in laminar channel flow is well understood. In general, the dimensionless hydrodynamic entrance length $L_{h,y}/D_h$ is proportional to the Reynolds number $\rho U D_h / \eta$ while the dimensionless thermal entrance length is proportional to the product of the Reynolds number and the Prandtl number. Thus,

for a high Prandtl number fluid in laminar flow the thermal entrance region is generally much longer than the hydrodynamic entrance length.

In the case of a circular tube $L_{h,y}^*$ and $L_{t,h}^*$ are of the order of 0.05 to 0.06, whereas $L_{h,y}^*$ for a rectangular channel is of the order of 0.08 to 0.09 for aspect ratios ranging from 0.2 to 1. The corresponding thermal entrance length $L_{t,h}^*$ for fully developed hydrodynamic conditions ranges from 0.05 to 0.07.

For fully established constant property flow in circular and rectangular channels the friction factor may be given in terms of a generalized Reynolds number, Re_J , introduced by Jones [1]:

$$f = 16/Re_J \quad [1]$$

Here

$$Re_J = \rho U D_h / [\eta(a+b)] \quad [2]$$

For the circular tube, the values of a and b are 0.25 and 0.75 respectively and Re_J reduces to the conventional Reynolds number. For the rectangular channel, representative values of a and b are given in Table 1 as a function of the aspect ratio.

Table 1

Aspect Ratio	a	b
1.0	0.2121	0.6771
0.8	0.2155	0.6831
0.6	0.2297	0.7065
0.4	0.2659	0.7571
0.2	0.3475	0.8444
0.0	0.5000	1.0000

The applicability of the Reynolds number introduced by Jones is not restricted to laminar flow of Newtonian fluids but is also useful for predicting the friction factors of turbulent channel flows of Newtonian fluids as well.

It is well known that the forced convection behavior of Newtonian fluids in laminar flow through channels is very sensitive to the thermal boundary conditions and to the channel geometry. For example, the Nusselt number for fully established hydrodynamic and thermal conditions (in the absence of viscous dissipation and heat sources) in a circular tube is 4.36 for the constant heat flux boundary condition and 3.66 for the constant wall temperature condition.

In the thermal entrance region of a circular pipe for fully established laminar flow conditions the following equations are applicable [2]:

$$Nu_{xH} = 1.41 Gz^{1/3} \quad [3a]$$

$$Nu_{xT} = 1.16 Gz^{1/3} \quad [3b]$$

In the case of a square duct heated on all four walls the fully established Nusselt values are as follows: $Nu_T = 2.98$, $Nu_{H1} = 3.61$, $Nu_{H2} = 3.09$ [2]. Here the subscripts have the following meaning:

- T Constant wall temperature.
- H1 Constant wall heat flux axially and constant temperature peripherally.
- H2 Constant wall heat flux axially and peripherally.

The availability of sophisticated numerical methods coupled with modern computers makes the solution of forced convection laminar channel flow problems a relatively straightforward task. The citing of some typical constant property laminar flow results provides a background for comparison with non-Newtonian fluids.

In contrast with the laminar flow case, our knowledge of forced convection under turbulent channel flow conditions rests largely on empirical foundations. Excellent reviews on this topic may be found in the paper by Shah and Johnson [3] and in the Handbook of Single Phase Convective Heat Transfer [4]. The hydrodynamic entrance region in a circular tube is reported to be of the order of 15 to 30 pipe diameters at a Reynolds number of 4×10^5 , whereas the thermal entrance region (for fully developed hydrodynamic conditions) is of the order of 5 to 10 pipe diameters [4]. An estimate of the corresponding entrance lengths for turbulent flow in a rectangular channel is given in the above referenced handbook: L_{hy}/D_h is of the order of 15 hydraulic diameters for a flat duct, increasing to 90 hydraulic diameters for a square channel. The corresponding thermal entrance lengths, L_{th} , under fully developed hydrodynamic conditions are of the order of 10 diameters for high Prandtl number liquids.

The expression for the friction factor for fully established turbulent flow in a circular tube which is due to Filonenko [5] is reported to give the best agreement with experimental data [3]:

$$f = [1.58 \ln Re - 3.28]^{-2} \quad [4]$$

The same expression may be used for a rectangular duct if the above-cited Jones Reynolds number, Re_J , is used in place of Re [1]. A simpler expression is the so-called Blasius formulation which is within a few percent of the Filonenko equation

$$f = 0.0791 (Re_J)^{-0.25} \quad [5]$$

The recommended [3] fully established heat transfer for turbulent pipe flow is given by the following formulation due to Gnielinski [6]

$$Nu = \frac{(f/2) (Re - 1000) Pr}{1 + 12.7 (f/2)^{1/2} (Pr^{2/3} - 1)} \quad [6]$$

Here either Eq. 4 or Eq. 5 is recommended for the friction factor. For liquids the Gnielinski equation may be used for either the constant wall temperature or the constant heat flux boundary condition. For fully established turbulent flow in a rectangular channel subject to the T or H1 boundary condition imposed on all four walls the equation may be used with the characteristic dimension being the hydraulic diameter. The same formulation may be used to estimate (within $\pm 20\%$) the turbulent heat transfer performance in the case where some of the rectangular duct walls are adiabatic

Purely Viscous Non-Newtonian Fluids

A number of constitutive equations have been advanced to describe the behavior of shear-thinning non-Newtonian fluids. Some of the more common models are given in Table 2 [7].

Since the simplest and most widely used formulation is the power law (Oswald-deWaele) model it will be used here to bring out some of the major features of non-

Table 2

Model	Constitutive Equation
Power Law (Oswald-de Waele)	$\eta = K \dot{\gamma}^{n-1}$
Ellis	$\eta = \frac{\eta_0}{1 + (\tau / \tau_{1/2})^{c-1}}$
Cross	$\eta = \frac{\eta_0}{1 + (\dot{\gamma} / \dot{\gamma}_{1/2})^d}$
Modified Power Law	$\eta = \frac{\eta_0}{1 + (\eta_0 / K) \dot{\gamma}^{1-n}}$
Sutterby	$\eta = \eta_0 \left[\frac{\sinh^{-1} \lambda \dot{\gamma}}{\lambda \dot{\gamma}} \right]^o$
Powell - Eyring	$\eta = \eta_\infty + \frac{(\eta_0 - \eta_\infty) \sinh^{-1} \lambda \dot{\gamma}}{\lambda \dot{\gamma}}$

Newtonian purely viscous behavior. For laminar flow of power law fluids in circular tubes the dimensionless hydrodynamic entrance length L_{hy}^+ decreases from 0.05 to 0.02 as the power law exponent decreases from 1.00 to 0.25. For laminar flow in rectangular channels the hydrodynamic entrance length is a function of the aspect ratio α^* and the power law index n as shown on Figure 1 [8]. The thermal entrance length depends on

Figure 1. Dimensionless hydrodynamic entrance length for laminar flow of a power law fluid in a rectangular duct as a function of aspect ratio and power law exponent.

Figure 2. Dimensionless thermal entrance length for fully established laminar flow of a power law fluid in a rectangular duct as a function of aspect ratio and power law exponent; T and H boundary conditions on all walls.

Figure 1 [8]. The thermal entrance length depends on the aspect ratio, the power law exponent and the thermal boundary conditions. Representative values of L_{th}^+ are shown on Figure 2 for the case where all bounding walls are heated [8].

For fully established laminar flow of a power law fluid in a circular tube Metzner and Reed [14] demonstrated that the friction factor could be given by the following formulation:

$$f = 16 / Re' \quad [7]$$

where

$$Re' = \rho U^{2-n} D_h^n / (K [(3n+1)/4 n]^{n-1} 8^{n-1}) \quad [8]$$

It is clear that Re' reduces to the conventional Reynolds number when $n = 1$.

Turning next to the case of a power law fluid in established laminar flow in a rectangular duct Kozicki et al [15] introduced a generalized Reynolds number defined as follows:

$$Re^* = \rho U^{2-n} D_h^n / 8^{n-1} [b + (a/n)]^n K \quad [9]$$

This yields the following familiar expression for the friction factor,

$$f = 16 / Re^* \quad [10]$$

Representative values of a and b are given in Table 1 [8].

An examination of the Kosicki Reynolds Number reveals that the Reynolds numbers introduced by Jones [1] and by Metzner and Reed [14] are special cases of Re^* .

For fully developed hydrodynamic and thermal conditions the Nusselt number for power law fluids in laminar pipe flow is a function of the power law exponent and of the thermal boundary conditions. Figure 3 presents the results for the constant wall temperature and the constant heat flux boundary conditions. It is interesting to note that the influence of n is relatively weak (less than 10 percent) over the range of n from 0.5 to 1.0 which covers many of the purely viscous fluids. In the thermal entrance region, for fully developed hydrodynamic conditions, the following relations due to Bird [11] apply:

$$Nu_{th} = 1.41 (3n+1/4n)^{1/3} Gz^{1/3} \quad [11a]$$

subject to $Gz > 25\pi$

$$Nu = 1.16 (3n+1/4n)^{1/3} Gz^{1/3} \quad [11b]$$

subject to $Gz > 33\pi$

Figure 3. Fully developed Nusselt numbers for laminar pipe flow of a power law fluid as a function of the power law exponent for T and H boundary conditions.

For fully developed conditions the Nusselt number for power law fluids in laminar flow through rectangular channels is a function of the power law exponent, the aspect ratio and the thermal boundary conditions. In the case of the H1 boundary condition imposed on all four walls the fully developed Nusselt numbers are given on Figure 4. Again, for a fixed aspect ratio the influence of n is less than 10 percent over the range of n from 0.5 to 1.0.

In the case of turbulent channel flow of purely viscous power law fluids the hydrodynamic and thermal entrance lengths can be taken as the same as the corresponding values for a Newtonian fluid. The fully established friction factor for turbulent pipe flow may be obtained from the Dodge-Metzner Equation [19]:

$$\frac{1}{\sqrt{f}} = \frac{4.0}{n^{0.75}} \log_{10} [Re' f^{1-(n/2)}] - 0.4/n^{1.2} \quad [12]$$

An explicit equation giving good agreement with Equation 12 was recommended by Yoo [20]

$$f = 0.079 n^{0.675} (Re')^{-0.25} \quad [13]$$

In the case of fully established turbulent flow in a rectangular channel either equation is recommended with Re' being replaced by the Kozicki Reynolds Number Re^* .

Figure 4. Fully developed Nusselt number for laminar flow of a power law fluid in a rectangular duct as a function of the aspect ratio and the power law index; H1 boundary condition on all bounding walls.

Metzner and Friend [21] recommend the use of the following equation for predicting the fully established heat transfer performance of purely viscous fluids flowing turbulently in circular pipes.

$$St = \frac{f/2}{[1.2 + 11.8 \sqrt{f/2} (Pr - 1) Pr^{0.3}]} \quad [14]$$

However based on a comparison of the measured and predicted values of the heat transfer performance in circular tubes and rectangular channels the Gnielinski Equation 6 was found to be as good (or as bad) as the Metzner-Friend equation. Therefore Eq. 6 is recommended for predicting the fully established heat transfer behavior of purely viscous fluids in turbulent flow through circular pipes and rectangular ducts.

The recommended procedure is as follows:

- (1) Determine the friction factor from Eq. 12 or Eq. 13
- (2) Convert Re^* to Re using the following relation, $Re = Re^* (a + bn) / n$
- (3) Use the Gnielinski Equation 6 to predict the Nusselt number.

Figure 5. Predicted values of j_H versus measured values of j_H for fully established turbulent flow of power law fluids in circular pipes and rectangular channels using recommended procedure.

As shown on Figure 5 this procedure should yield results which are within 20 percent of the actual values, with the predicted values being 10 - 20 percent high.

Viscoelastic Fluids

The pioneering work of Mysels [22] and Toms [23] revealed that the addition of small amounts of a high molecular weight polymer to a Newtonian fluid in turbulent pipe flow resulted in a dramatic decrease in pressure drop. Evidence of this behavior is seen in Figure 6 which reveals that the presence of 10 parts per million by weight of polyacrylamide in water results in a 40 percent reduction in pressure drop and a comparable decrease in pumping power [17]. As a result of these

Figure 6. Influence of addition of 10 wppm polyacrylamide on pressure drop of water in fully established turbulent pipe flow. [24]

observations considerable attention has been paid to the fluid mechanical behavior of aqueous solutions of high molecular weight polymers in turbulent pipe flow. Using available empirical data, pipe line systems involving the use of polymer additives have been designed. Examples include the Trans-Alaskan oil pipeline and central city sewer systems [25]. Nevertheless, our understanding of the physics of these flows is still limited.

In contrast with the turbulent pipe flow case where there have been hundred of studies of the fluid mechanical behavior (see ref. 26 for example), relatively few studies on laminar flow have been reported. Furthermore, studies of the heat transfer performance of such fluids are also very limited. In this regard, the research contributions of Metzner [27]; Mizushima and Usui [28]; Matthys [29, 30]; and Ghajar [31, 32] deserve special notice.

In describing these fluids special physical property measurements are needed. Above and beyond the usual properties such as density, specific heat, and thermal conductivity, which are relatively unaffected by the presence of small amounts of the polymer, measurements of the apparent viscosity and the first and second normal stress differences as a function of the shear rate $\dot{\gamma}$ are needed to characterize the fluid. Some recent studies [33] also suggest that extensional viscosity measurements are essential. Furthermore, oscillatory viscometric measurements may be required to quantify the elasticity of the fluid. Examples of such measurements are shown on Figures 7 to 9. Depending on the flow and heat transfer situation being considered all of these properties and more may be important. In most cases it remains unclear as to which basic rheological properties are needed to adequately characterize a viscoelastic fluid. A related problem arises from the fact that many applications of drag reducing fluids involve such small quantities of polymer additives (10 to 50wppm) that it is practically impossible

Figure 7. Apparent viscosity as a function of shear rate for various concentrations of polyacrylamide in Chicago tap water. [24]

Figure 8. First normal stress difference as a function of shear rate for 1000 wppm of polyacrylamide in Chicago tap water. [34]

Figure 9. Oscillatory phase shift of 1000 wppm aqueous Carbopol solution. [35]

to detect any difference between the drag-reducing fluid and the solvent.

Turning to the fluid mechanics and heat transfer behavior of aqueous solutions of high molecular weight polymers in flow through circular pipes and rectangular channels there are several recent review papers which describe our present state of knowledge [8, 24, 29, 30, 36]. A brief summary of some of the main feature of the behavior of such fluids is given here. For more details the reader is referred to the cited reviews.

Under fully developed laminar pipe flow conditions there is no mechanism for the elasticity to play a role and the viscoelastic fluid behaves as a purely viscous fluid. Measurements of the fully established friction factor and the local Nusselt number for aqueous polymer solutions confirm this statement as shown on Figures 10 and 11. Excellent agreement with the power law purely viscous predictions is evident.

Figure 10. Fully established friction factors for laminar flow of viscoelastic fluids as a function of Re' . Power law prediction: $f = 16 / Re'$.

Figure 11. Measured Nusselt numbers for established laminar pipe flow of viscoelastic fluids compared with analytical prediction for power law fluid.

Turning next to the rectangular channel, the measured friction factor for the established laminar flow of a viscoelastic aqueous polymer solution in this geometry is also found to be in agreement with the purely viscous prediction as shown on Figure 12. Against this background it might be anticipated that the heat transfer behavior of a viscoelastic fluid in established laminar flow through a rectangular channel could be predicted by the purely viscous power law formulation. However this hypothesis turns out to be incorrect and the measured local Nusselt number turned out to be two to three times the purely viscous prediction as demonstrated on Figure 13 [35]. This observed higher heat transfer performance which has also been found for aqueous polyacrylamide solutions [35] is ascribed to a secondary flow which comes about as a result of the

Figure 12. Fully established friction factors for laminar flow of viscoelastic fluids in a square channel as a function of Re^* . Power law prediction: $f = 16 / Re^*$. [37]

Figure 13. Measured Nusselt numbers for established laminar flow of viscoelastic aqueous Carbopol solutions in 2:1 rectangular duct as a function of Graetz number.

normal stress differences acting on the fluid boundaries of the non-circular channel. Analytical predictions of Gao [39] and Payvar [40] support this explanation. Furthermore these analytical studies reveal that such secondary flows are maximum for a square duct and decrease with decreasing aspect ratio.

The turbulent channel flow of aqueous solutions of high molecular weight polymers, particularly turbulent pipe flow, has received considerable attention. It has been reported that the fully established friction factor and dimensionless heat transfer coefficient j_H are functions of the Reynolds number based on the apparent viscosity evaluated at the wall shear rate and of a new dimensionless group called the Weissenberg number, Ws

$$Ws = \lambda U / D_h \quad [15]$$

Here λ is a characteristic time, a measure of the time required for a viscoelastic fluid to respond to a change in stress. Recognizing that the characteristic time is difficult to measure an empirical approach has been adopted based on the Powell-Eyring Newtonian model [34, 41]:

$$\eta = \eta_- + (\eta_0 - \eta_-) \frac{\sinh^{-1} \lambda \dot{\gamma}}{\lambda \dot{\gamma}} \quad [16]$$

If the viscosity is measured as a function of shear rate then it is possible to determine the value of λ giving the best fit to the experimental data. This value is used to determine Ws . The Weissenberg number is influenced by the following factors: (1) the chemistry of the polymer; (2) the concentration of the polymer; (3) the chemistry of the solvent; and (4) the state of degradation of the polymer. Typical results for the fully established friction factor and dimensionless heat transfer j_H factor are shown on Figures 14 and 15 [42, 43]. A few observations relative to these figures are in order. Generally speaking, at a fixed Reynolds number the fully developed friction factor and the j_H factor decrease with increasing Weissenberg number until asymptotic values are reached. The asymptotic friction factor is reached at a critical Weissenberg number of approximately 10; beyond this value the friction factor is a function of Reynolds number only. Corresponding behavior is found for j_H with the critical Weissenberg number being of the order of 100 to 200. Relative to Newtonian values the heat transfer reduction is generally greater than the friction factor reduction for a fixed Weissenberg number.

The entrance lengths are of the order of 100 hydraulic diameters for L_{hy} and 500 for L_{th} under conditions corresponding to critical Weissenberg values. These long entrance lengths reflect the fact that the eddy diffusivities of momentum and of heat decrease with increasing values of the Weissenberg number. Furthermore, under conditions corresponding to critical Weissenberg numbers the eddy diffusivity of heat is an order of magnitude lower than the eddy diffusivity of momentum [24].

Estimates of the friction factor and heat transfer performance of viscoelastic fluids in rectangular channels may be obtained by substituting the hydraulic diameter for the diameter if circular tube results are available for the viscoelastic fluid being considered [8]. A note of caution is advised in applying these results to large scale flow systems as pointed out by Matthys [29] since the available data are largely limited to small scale test facilities.

Figure 14. Fully established friction factors for aqueous polyacrylamide solutions in turbulent pipe flow as a function of the Weissenberg and Reynolds numbers.

Figure 15. Fully established dimensionless heat transfer, j_H for aqueous polyacrylamide solutions in turbulent pipe flow as a function of the Weissenberg and Reynolds numbers.

Anomalous Behavior of Aqueous Polyacrylic Acid Solutions

As noted in an earlier paper [36] an exception to the generally observed drag reduction in turbulent channel flow of aqueous polymer solutions occurs in the case of aqueous solutions of polyacrylic acid (Carbopol from B.F. Goodrich Co.). Rheological measurements taken on an oscillatory viscometer shown on Figure 9 clearly demonstrate that such solutions are viscoelastic. This is also supported by the laminar flow behavior shown on Figure 13. Nevertheless, the pressure drop and heat transfer behavior of neutralized aqueous Carbopol solutions in turbulent pipe flow reveals little reduction in either of these quantities. Rather, these solutions behave like clay slurries and they have been often identified as purely viscous non-Newtonian fluids. To document this anomalous behavior the measured dimensionless friction factors are shown on Figure 16 where it may be seen that both the Carbopol and clay slurries are in good agreement with the modified Dodge-Metzner equation:

Figure 16. Fully developed friction factors for clay slurries and aqueous Carbopol solutions in turbulent flow through rectangular channels.

$$1/\sqrt{f} = \frac{4.0}{n^{0.75}} \log_{10} [Re^* (f)^{1-(n/2)}] - \frac{0.4}{n^{1.2}} \quad [17]$$

The turbulent flow heat transfer behavior of Carbopol solutions is also found to be in good agreement with the results found for clay slurries.

Recent Developments

The addition of high molecular weight polymers to water generally results in decreases in pressure drop and heat transfer under turbulent flow conditions with the heat transfer reduction being greater than the reduction in pressure drop. Furthermore, aqueous polymer solutions tend to degrade as the molecular bonds are ruptured by the shearing stresses encountered under flow conditions and this degradation causes the fluid to gradually lose its drag reducing capability. The degradation process is irreversible for aqueous polymer solutions.

These characteristics - low heat transfer and degradation - are unfavorable in many practical applications. Recent studies [44-48] have revealed that the use of relatively low molecular weight surfactant additives may counter these problems. While these aqueous surfactant solutions do degrade under severe shearing stresses they do reconstitute under conditions of low shearing stress. Therefore, by clever design of the flow system, these special properties of surfactant solutions may result in low pumping power without a serious heat transfer penalty.

The basic mechanics of drag reduction in turbulent channel flow of viscoelastic fluids, both aqueous polymer and aqueous surfactant solutions, remains unresolved. In this regard the role of extensional

viscosity in drag reduction in turbulent channel flow has been debated for some years. The technique recently reported by Vißmann and Bewersdorff [33] may resolve this issue. These investigators using a rotating viscometer connected in series with an orifice measured a relative extensional viscosity as a function of shear rate and pressure drop across the orifice. They reported a substantial increase of extensional viscosity for aqueous polyacrylamide solution under shearing conditions. If these investigators are correct in their hypothesis that drag reduction results from extensional viscosity then viscoelastic Carbopol solutions which are not drag reducing should show an extensional viscosity close to that of water.

Concluding Remarks

Our understanding of the behavior of viscoelastic aqueous polymer and aqueous surfactant solutions under both laminar and turbulent channel flow condition is quite limited. To improve the situation the following research challenges are noted:

- Characterization of aqueous polymer and aqueous surfactant solutions, including new approaches to the evaluation of the characteristic time.
- Development of new instrumentation and new techniques to measure rheological properties. An example is the approach reported by Vißmann and Bewersdorff [33] for the measurement of elongational viscosity. Such measurements should be extended to aqueous Carbopol solutions.
- Studies of the flow and heat transfer behavior of viscoelastic fluids in large diameter systems.
- Studies of the laminar flow and heat transfer behavior of aqueous surfactant solutions in non-circular channels.
- Studies of heat transfer enhancement in turbulent channel flow of viscoelastic fluids.
- Studies of friction and heat transfer behavior of viscoelastic fluids in flow over external surfaces.
- Formulation of analytical models that are simple enough to predict heat transfer behavior but are sufficiently complex to capture the physics.

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