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SPACE DEBRIS COLLISION AND PRODUCTION ANALYTIC ESTIMATES

Gregory H. Canavan

Analytic estimates provide useful estimates of collision rates, fragment production rates, and average collision masses and numbers in good agreement with analytic and numerical estimates for the principal quantities of interest.

Analytic estimates can provide useful estimates of collision rates, fragment production rates, and average collision masses and fragment numbers. The catalog collision rate can be evaluated with little approximation. The fragment production rate can be evaluated almost as accurately. Dividing the two gives about 80 fragments per collisions, weakly dependent on model parameters.¹

JSC has suggested calculating the rate of converting mass into fragments by collisions as an intermediate step.² That process can also be evaluated analytically, giving a value within about 5% of that quoted by JSC. The number of fragments per collision is about the same between the two models. The ratio of their fragment production rates is insensitive to the limits of the catalog and the prescription for which objects can cause fragmentation.

The fragment production rate calculation does not use fragment mass explicitly, but an average mass per collision can be inferred for its cumulative number of fragments. The predictions of the two models converge at large fragment production, but differ 20-30% for nominal values. The JSC calculation does not vary with fragment production exponent; the direct calculation of fragments per collision does because of the variation of production rates with fragment parameters. However, these variations do not effect the good agreement between analytic and numerical estimates for the principal quantities of interest.

Collision rate. The collision rate can be written as³

$$R_{\text{cat}} = (1/2W) \int_R^U dx \int_R^U dy n(x) n(y) \sigma V, \quad (1)$$

where x and y are the masses of the target and impactor, $n(x)$ and $n(y)$ are their densities, W is the volume of LEO which contains objects, V is the debris relative velocity, and $w = W/V \approx 1.4 \times 10^9 \text{ m}^2\text{-yr}$. The collision rate is restricted to catalogue particles by using $R \approx 0.6 \text{ kg}$ and $U \approx 6 \text{ tons}$, which are roughly the smallest and largest particle masses in the catalog, as the limits of integration. The cross section σ is roughly geometric; hence, $\sigma \approx K(x^{s/2} + y^{s/2})^2$, where $s \approx 1$ to 1.1 is an empirical, constant exponent.⁴ Assuming $x \gg y$ only involves $\approx 30\%$ errors, so the approximate form $\sigma \approx Kx^s$ is used below. Then Eq. (1) becomes

$$R_{\text{cat}} = (K/w) \int_R^U dx n(x) x^s \int_R^U dy n(y). \quad (2)$$

The second integral is the number of catalog objects, N . For $s = 1$, the first integral is the mass of catalog objects, M , and the catalog collision rate is

$$R_{cat} = (K/w)MN. \quad (3)$$

It is useful to explore the dependence of those integrals on the object distributions and parameters assumed. For a power law exponent c dependence of the cumulative number of objects on mass, the density varies as⁵

$$n(x) = NcR^c/x^{c+1}, \quad (4)$$

which for $U \gg R$, as above, integrates to $\int_R^U dx NcR^c/x^{c+1} = N$. For arbitrary s the first integral in Eq. (2) is

$$\int_R^U dx NcR^c/x^{c+1} x^s = NcR^c(U^{s-c} - R^{s-c})/(s-c) \approx NcR^c U^{s-c}/(s-c), \quad (5)$$

so the catalog collision rate, which is their product, is

$$R_{cat} = (K/w)N^2 R^c U^{s-c}/(s-c) = [c/(s-c)w](R/U)^c (KU^s)N^2. \quad (6)$$

Thus, the collision rate involves the total number of catalog objects but a cross section that corresponds to a smaller mass $[c/(s-c)](R/U)^c \approx [1/4/(3/4)](10^{-4})^{1/4} 6 \text{ ton} \approx 0.2 \text{ ton}$.

Fragment production rate. The number of fragments larger than mass z produced in a collision that fractures both objects' masses is approximately⁶

$$C(z) = A[(x+y)/z]^B, \quad (7)$$

where $B \approx 2/3$ is an empirical constant and $A = 1/B - 1$ by conservation of mass. The number of catalog fragments is $A[(x+y)/R]^B$; thus, the rate of production of catalog fragments in collisions between catalog objects of all sizes is

$$\begin{aligned} Q &\approx (1/2W) \int_R^U dx \int_{fx}^x dy n(x) n(y) \sigma V A [(x+y)/R]^B \\ &\approx (AK/wR^B) \int_R^U dx n(x) x^{s+B} \int_R^U dy n(y), \end{aligned} \quad (8)$$

where the limits of integration on the second integral have been replaced by fx and x , where $f \approx 0.001$ is the ratio of the smallest impactor to target mass that can produce complete fragmentation.⁷ Thus, the fragment production integral only involves a fraction of the masses included in the catalog collision rate. Substituting for $n(x)$ and $n(y)$ from Eq. (4) gives

$$\begin{aligned} Q(z) &\approx (AK/wR^B)(Nc)^2 \int_R^U dx c/x^{c+1} x^{s+B} [(fx)^{-c} - x^{-c}] \\ &\approx (AK/wR^B f^c)(Nc)^2 c \int_R^U dx x^{s+B-2c-1} \\ &\approx (AK/wR^B f^c)(Nc)^2 c U^{s+B-2c}/(s+B-2c) \\ &\approx [c/(s+B-2c)w f^c] A(U/R)^B (R/U)^{2c} (KU^s)N^2, \end{aligned} \quad (9)$$

which involves the total number of catalog objects and a number of fragments per collision corresponding to the largest objects, but a cross section smaller by a factor of

$$[c/(s+B-2c)](R/U)^{2c} \approx [1/4/(1/2+2/3)](10^{-4})^{1/2} \approx 2.1 \times 10^{-3} \quad (9a)$$

smaller than the largest catalog objects.

Fragments per collision. Dividing the fragment production rate of Eq. (9) by the collision rate of Eq. (6) gives the number of fragments per collision

$$Q(z)/R_{cat} \approx [(s-c)/(s+B-2c)] A/f^c (U/R)^{B-c}, \quad (10)$$

which is shown for $s = 1$ in Fig. 1, as the dependence on s is algebraic and weak in the interval of interest. Observations of objects in space give $c \approx 1/4$, and laboratory railgun and field Atlas experiments give $B \approx 2/3$, for which $Q(z)/R_{cat} \approx 80$ fragments per collision. For $c = 1/4$, the number would increase to about 140 by $B = 0.85$, after which it would decline. At 0.85, decreasing c would increase $Q(z)/R_{cat}$ to ≈ 160 at $c = 0.15$, although debris objects do not have that scaling. Thus, the number of 80 fragments/collision is typical of experimental conditions. Other conditions could lead to about a factor of two more fragments, but do not correspond to observed conditions.

Fragment mass. JSC has suggested calculating the rate of converting mass into fragments by collisions as an intermediate step.⁸ In the notation of Eq. (1), that can be written

$$\begin{aligned} J &= (1/w) \int_R^U dx \int_{f_x}^x dy n(x) n(y) K(x+y)(x+y) \\ &\approx (K/w) \int_R^U dx N c R^c / x^{c+1} x^2 \int_{f_x}^x dy N c R^c / y^{c+1} \\ &\approx (K(NR^c)^2 / w f^c) \int_R^U dx c / x^{c+1} x^{2-c} \approx (K(NR^c)^2 / w f^c) U^{2-2c} c / (2-2c). \end{aligned} \quad (11)$$

The fragment mass production rate J can be divided by the catalog collision rate R_{cat} to give an estimate of the mass per collision, which is

$$J / R_{cat} = R^c U^{1-c} / 2 f^c, \quad (12)$$

which is about $(0.6 \text{ kg})^{1/4} (6 \text{ ton})^{3/4} / 2 (.001)^{1/4} \approx 1690 \text{ kg}$, approximately the value quoted by JSC. Figure 2 shows J / R_{cat} as a function of B for the fragments per collision of Eqs. (10) and (12). The JSC value is about 10% higher for $B < 0.85$, then the two converge as they both fall sharply. This agreement is characteristic of the two models at other values of c and s as well.

Fragment production ratios. Although Fig. 2 shows that the fragments per collision is about the same between the two models, it is instructive to divide their fragment production rates to find the ratio

$$A(J/R)^B / Q/R = [(1+B-2c)/2B(1-c)] (fU/R)^{c(1-B)}, \quad (13)$$

which is about 1.2 for nominal conditions, as seen above. Moreover, the exponent of the large term fU/R , which is $c(1-B) \approx 0.08$ is so small that this result is insensitive to the limits of the catalog and the prescription for which objects can cause fragmentation. The strongest variation is to B , which is known experimentally.

Alternative mass per collision. Although the calculation of the fragment production rate does not use mass explicitly, one can be inferred from its rate. Equation (7) for the cumulative fragments C can be inverted to give

$$\text{massfrag/coll} = R[C(R)/A]^{1/B} \approx R[Q(z)/R_{cat} A]^{1/B}, \quad (14)$$

which is shown in Fig. 3. The top curves are from Eq. (11); the bottom curves from Eq. (13). The two sets converge at large B ; they move apart 20-30% for the nominal B . The JSC calculation does not vary with fragment production exponent B at the level of average mass; the mass inferred from the number of fragments per collision because $Q(z)/R_{cat}$ varies as $(U/R)^{B-c} / (s+B-$

2c), so that $\text{mass}_{\text{frag/coll}}$ varies as $(U/R)^{1-c/B} / [(s+B-2c)(1/B-1)]^{1/B}$. However, as shown above, these disparate variations of mass are not reflected in the number of fragments per collision, which is the quantity of interest.

Summary and conclusions. This note derives useful analytic estimates for collision rates, fragment production rates, and average collision masses and fragment numbers. The catalog collision rate can be evaluated almost without approximation. The fragment production rate can be evaluated almost as accurately to derive a rate that involves the total number of catalog objects and all the largest objects, but a cross section smaller by a factor of about a factor of 500 than that of the largest catalog objects. Dividing the two gives the number of fragments per collisions, which is about 80 for experimental conditions. Other conditions could lead to about a factor of two more fragments.

JSC has suggested calculating the rate of converting mass into fragments by collisions as an intermediate step. It can also be evaluated analytically, giving a value within about 5% of that quoted by JSC. Although the number of fragments per collision is about the same between the two models, it is instructive to divide their fragment production rates, insensitive to the limits of the catalog and the prescription for which objects can cause fragmentation.

Although the fragment production rate does not use mass, it can be inferred by inverting the equation for the cumulative number of fragments. The predictions of the two models converge at large fragment production, but move apart 20-30% for nominal values. The JSC calculation does not vary with fragment production exponent B at the level of average mass; the mass inferred from the number of fragments per collision does because of the variation of the production rate with fragment parameters. However, these disparate variations of mass are not reflected in the number of fragments per collision, which is the quantity of interest. Thus, there is good agreement between analytic and numerical estimates for the principal quantities of interest.

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Q/Rcat vs B; c

Fragments per collision versus production and power law exponents.

frag/col vs B; JSC & LA

Fig. 2 fragments per collision bs production exponent.

avg mass vs B; JSC & LA

Fig. 3. Average mass per collision.