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1.0 Background

For turbulent flow the evaluation of direct numerical simulations (DNS) where all scales are resolved and large-eddy simulations (LES) where only large-scales are resolved is difficult because the results are three-dimensional and transient. To simplify the analysis, the instantaneous flow field can be averaged in time for evaluation and comparison to experimental results.

The incompressible Navier-Stokes flow code HYDRA (Christon 1995) has been modified for calculation of time-average quantities for both DNS and LES. The following describes how time averages of the instantaneous quantities are generated during program execution (i.e., while generating the instantaneous quantities, instead of as a postprocessing operation). The calculations are performed during program execution to avoid storing values at each time step and thus to reduce storage requirements. The method used in calculating the time-average velocities, turbulent intensities, $\langle u''^2 \rangle$, $\langle v''^2 \rangle$, and $\langle w''^2 \rangle$, and turbulent shear, $\langle u''v'' \rangle$ are outlined. The brackets $\langle \rangle$ used here represent a time average. The described averaging methods were implemented in the HYDRA code for three-dimensional problem solutions. Also presented is a method for taking the time averages for a number of consecutive intervals and calculating the time average for the sum of the intervals. This method could be used for code restarts or further postprocessing of the time averages from consecutive intervals. This method was not used in the HYDRA implementation, but is included here for completeness. In HYDRA, the running sums needed for time averaging are simply written to the restart dump.

2.0 Time-Average Definitions

The instantaneous velocity field¹ can be decomposed into a mean velocity $\langle u \rangle$ (time, space, or ensemble averaged) and a fluctuating velocity, u'' , such that the instantaneous velocity u is defined as

$$u = \langle u \rangle + u''.$$

The averaging operation $\langle \rangle$ is defined such that $\langle u'' \rangle = 0$. If the turbulent flow field is stationary (i.e., statistically steady), averaging with respect to time can be used (Hinze 1975 or Tennekes and Lumley 1972). By definition,

1. We use the notation u , v , and w for the three velocity components in the x , y , and z directions.

$$\langle u(\underline{x}) \rangle = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} u(\underline{x}, t) dt.$$

For a finite value of the averaging interval T , we define the averaging operator by

$$\langle u(\underline{x}) \rangle = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} u(\underline{x}, t') dt'$$

with the condition that $T_1 \ll T \ll T_2$ (Hinze 1975) where the lower value T_1 is the time scale for the turbulence and the upper value T_2 is the period for slow variations in the flow field that we do not regard as part of the turbulence. If the averaging interval T meets the above condition, the average value should be independent of when you start the average. Thus, $\partial \langle u \rangle / \partial t$ is zero (or negligibly small) in the interval T and even zero over several T intervals (since $T \ll T_2$).

In the numerical simulation, the appropriate choice of the averaging interval T , such that $T_1 \ll T \ll T_2$, is not known until after the instantaneous velocities have been calculated over a relatively long time interval (i.e., for several T_2 intervals). However, to avoid storing the velocity field at every time step for postprocessing of the average quantities, we calculate the averages during program execution (i.e., while we calculate the instantaneous velocity). The following describes the method used to calculate time averages during program execution for a chosen *a priori* interval T . If this chosen interval does not produce a time invariant average value or one that varies only slightly over several T intervals, the values for interval T can be postprocessed. As described in the following sections, the averages for k intervals of time duration T can be used to generate values for an interval kT , where k is a positive integer.

If the chosen T is too small and the time average varies more than desired between adjacent intervals, the calculated time average is dependent on where the interval is started. The time-average velocity is then

$$\langle u(\underline{x}, t) \rangle = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} u(\underline{x}, t') dt'$$

where the value is defined at time t which is the midpoint for the interval T . Although the calculated time-average velocities, turbulent intensities, and shear are not the desired result, they can be postprocessed to obtain the time average for a longer time interval.

3.0 Time-Averaged Quantities

The methods used in calculating the time-averaged quantities are outlined in the following sections.

3.1 Time-Averaged Velocities

The time-averaged velocity $\langle u \rangle$ for an interval T is defined as

$$\langle u(x, t) \rangle = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} u(x, t') dt'$$

and is the value at the interval midpoint t . The above integration can be approximated with the discrete summation

$$\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} u(x, t') dt' = \int_{t-\frac{T}{2}}^{t-\frac{T}{2}+\Delta t_1} u(x, t') dt' + \int_{t-\frac{T}{2}+\Delta t_1}^{t-\frac{T}{2}+\Delta t_1+\Delta t_2} u(x, t') dt' + \dots + \int_{t+\frac{T}{2}-\Delta t_n}^{t+\frac{T}{2}} u(x, t') dt'$$

$$\cong u\left(x, t - \frac{T}{2} + \frac{\Delta t_1}{2}\right) \Delta t_1 + u\left(x, t - \frac{T}{2} + \Delta t_1 + \frac{\Delta t_2}{2}\right) \Delta t_2 + \dots + u\left(x, t - \frac{T}{2} + \frac{\Delta t_n}{2}\right) \Delta t_n.$$

The above discrete summation (i.e., midpoint rule) is used for the time integration in the numerical calculation, and the averaging is performed using a constant time step to give

$$\langle u(x, t) \rangle \cong \frac{\sum_{i=1}^n u(x, t_i) \cdot (\Delta t)_i}{\sum_{i=1}^n (\Delta t)_i} = \frac{\sum_{i=1}^n u_i}{n}$$

where u_i is the value of u at time step i and at time t_i , n is the number of time steps, and $(\Delta t)_i$ is the integration step size which is constant in the existing code (i.e., $\sum (\Delta t)_i = n \Delta t = T$). This is the time-averaged velocity at the midpoint of the interval and is calculated for every node.

Because the summed values are at the midpoint of the interval according to the midpoint rule, the start of the averaging interval is $t - (T/2) = t_1 - (\Delta t_1/2)$ and the end of the averaging interval is $t + (T/2) = t_n + (\Delta t_n/2)$.

If a longer time interval for averaging is desired, say kT where k is an integer, the values for interval kT can be postprocessed using the following calculation. For time interval kT , the time-average velocity is

$$\begin{aligned}
\langle u(x, t) \rangle &= \frac{1}{kT} \int_{t-\frac{kT}{2}}^{t+\frac{kT}{2}} u(x, t') dt' \equiv \frac{\sum_{i=1}^{kn} u_i}{kn} = \frac{\sum_{i=1}^n u_i + \sum_{i=n+1}^{2n} u_i + \dots + \sum_{i=(k-1)n+1}^{kn} u_i}{kn} \\
&= \frac{1}{k} (\langle u \rangle_1 + \langle u \rangle_2 + \dots + \langle u \rangle_k) = \frac{\sum_{i=1}^{kn} \langle u \rangle_i}{k}
\end{aligned}$$

where $\langle u \rangle_i$ is the time-averaged velocity for the time interval $t + (i-1)T - \frac{T}{2}$ to $t + (i-1)T + \frac{T}{2}$.

3.2 Turbulent Intensities

The x-component of the velocity fluctuation u'' is defined as

$$u'' = u - \langle u \rangle$$

and the x-component of the turbulent intensity is

$$\langle u''^2 \rangle = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} u''^2 dt' = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} (u - \langle u \rangle)^2 dt'$$

If we chose T such that $T_1 \ll T \ll T_2$, we have

$$\langle \langle u \rangle \rangle = \langle u \rangle$$

and for the turbulence intensity

$$\begin{aligned}
\langle u''^2 \rangle &= \langle (u - \langle u \rangle)^2 \rangle = \langle u^2 - 2\langle u \rangle u + \langle u \rangle^2 \rangle \\
&= \langle u^2 \rangle - 2\langle u \rangle \langle u \rangle + \langle u \rangle^2 = \langle u^2 \rangle - \langle u \rangle^2.
\end{aligned}$$

In the numerical calculation, we use the discrete summation for the time integration of u^2 and thus the expression for calculating $\langle u''^2 \rangle$ is

$$\langle u''^2 \rangle \equiv \frac{\sum_{i=1}^n u_i^2}{n} - \langle u \rangle^2.$$

If a longer time interval for averaging is desired, say kT where k is an integer, the intermediate values for interval kT can be postprocessed using the following calculation. For time interval kT , the turbulent intensity is

$$\langle u''^2 \rangle = \frac{1}{kT} \int_{t-\frac{kT}{2}}^{t+\frac{kT}{2}} u''^2 dt'.$$

Thus, we have

$$\langle u''^2 \rangle \equiv \frac{\sum_{i=1}^{kn} u_i^2}{kn} - \langle u \rangle^2$$

where

$$\frac{\sum_{i=1}^{kn} u_i^2}{kn} = \frac{1}{k} \left(\frac{\sum_{i=1}^n u_i^2}{n} + \frac{\sum_{i=n+1}^{2n} u_i^2}{n} + \dots + \frac{\sum_{i=(k-1)n+1}^{kn} u_i^2}{n} \right)$$

and

$$\langle u \rangle^2 \equiv \frac{1}{k^2} (\langle u \rangle_1 + \langle u \rangle_2 + \dots + \langle u \rangle_k)^2.$$

There is no need to save for postprocessing the sum of the squared velocity because we can calculate it from the output average velocity and turbulent intensity as follows:

$$\begin{aligned} \frac{\sum_{i=1}^n u_i^2}{n} &= \langle u''^2 \rangle_1 + \langle u \rangle_1^2, \\ \frac{\sum_{i=n+1}^n u_i^2}{n} &= \langle u''^2 \rangle_2 + \langle u \rangle_2^2, \text{ etc.} \end{aligned}$$

Thus, the turbulent intensity for the interval $t - \frac{kT}{2}$ to $t + \frac{kT}{2}$ can be postprocessed from the k intermediate intervals using the following expression

$$\langle u''^2 \rangle \equiv \frac{1}{k} \langle u''^2 \rangle_1 + \langle u \rangle_1^2 + \langle u''^2 \rangle_2 + \langle u \rangle_2^2$$

$$+ \dots + \langle u''^2 \rangle_k + \langle u \rangle_k^2 - \frac{1}{k^2} (\langle u \rangle_1 + \langle u \rangle_2 + \dots + \langle u \rangle_k)^2$$

or in summation notation,

$$\langle u''^2 \rangle \equiv \frac{1}{k} \left(\sum_{i=1}^k \langle u''^2 \rangle_i + \sum_{i=1}^k \langle u \rangle_i^2 - \frac{1}{k} \left(\sum_{i=1}^k \langle u \rangle_i \right)^2 \right).$$

3.3 Turbulent Shear

The turbulent shear is defined as

$$\langle u''v'' \rangle = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} u''v'' dt' = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} (u - \langle u \rangle)(v - \langle v \rangle) dt'$$

and in the numerical calculation, we have

$$\langle u''v'' \rangle \equiv \frac{\sum_{i=1}^n u_i v_i}{n} - \langle u \rangle \langle v \rangle.$$

If a longer time interval for averaging is desired, say kT where k is an integer, the values for interval kT can be postprocessed using the following calculation. For the time interval kT

$$\langle u''v'' \rangle = \frac{1}{kT} \int_{t-\frac{kT}{2}}^{t+\frac{kT}{2}} u''v'' dt' \equiv \frac{\sum_{i=1}^{kn} u_i v_i}{kn} - \langle u \rangle \langle v \rangle$$

where

$$\sum_{i=1}^{kn} u_i v_i = \frac{1}{k} \left(\frac{\sum_{i=1}^n u_i v_i}{n} + \frac{\sum_{i=n+1}^{2n} u_i v_i}{n} + \dots + \frac{\sum_{i=(k-1)n+1}^{kn} u_i v_i}{n} \right)$$

and

$$\langle u \rangle \langle v \rangle \equiv \frac{1}{k^2} (\langle u \rangle_1 + \langle u \rangle_2 + \dots + \langle u \rangle_k) (\langle v \rangle_1 + \langle v \rangle_2 + \dots + \langle v \rangle_k)$$

There is no need to save the sums of the velocity products (i.e., $\sum u_i v_i$), because we can calculate them from the output turbulent shear as follows

$$\frac{\sum_{i=1}^n u_i v_i}{n} = \langle u'' v'' \rangle_1 + \langle u \rangle_1 \langle v \rangle_1,$$

$$\frac{\sum_{i=n+1}^{2n} u_i v_i}{n} = \langle u'' v'' \rangle_2 + \langle u \rangle_2 \langle v \rangle_2, \text{ etc.}$$

Thus, the turbulent shear for the interval $t - \frac{kT}{2}$ to $t + \frac{kT}{2}$ can be postprocessed from the k intermediate intervals using the following expression

$$\begin{aligned} \langle u'' v'' \rangle &\equiv \frac{1}{k} \langle u'' v'' \rangle_1 + \langle u \rangle_1 \langle v \rangle_1 + \langle u'' v'' \rangle_2 + \langle u \rangle_2 \langle v \rangle_2 \\ &\quad + \dots + \langle u'' v'' \rangle_k + \langle u \rangle_k \langle v \rangle_k \\ &\quad - \frac{1}{k^2} (\langle u \rangle_1 + \langle u \rangle_2 + \dots + \langle u \rangle_k) (\langle v \rangle_1 + \langle v \rangle_2 + \dots + \langle v \rangle_k) \end{aligned}$$

or in summation notation,

$$\begin{aligned} \langle u'' v'' \rangle &\equiv \frac{1}{k} \left(\sum_{i=1}^k \langle u'' v'' \rangle_i + \sum_{i=1}^k \langle u \rangle_i \langle v \rangle_i \right. \\ &\quad \left. - \frac{1}{k} \left(\sum_{i=1}^k \langle u \rangle_i \right) \left(\sum_{i=1}^k \langle v \rangle_i \right) \right). \end{aligned}$$

3.4 Turbulent Kinetic Energy

The turbulent kinetic energy is defined as

$$K = \frac{1}{2} (\langle u''^2 \rangle + \langle v''^2 \rangle + \langle w''^2 \rangle).$$

This quantity does not have to be calculated during calculation of the instantaneous velocity, but was done in the HYDRA code for convenience.

If a larger time interval is desired, say kT , the value is calculated using the above expression with the intensity values at the intermediate intervals.

4.0 Implementation in HYDRA Code

The HYDRA code was modified to calculate time averages as described in the previous section. Table 1 shows a summary of the storage requirements and calculated time-averaged quantities.

TABLE 1. Summary of Storage and Output Requirements

For each node and each time average interval:

Running Sum from $t - \frac{T}{2}$ to $t + \frac{T}{2}$	Output
$\sum_i u_i, \sum_i v_i, \sum_i w_i$	$\langle u \rangle, \langle v \rangle, \langle w \rangle$
$\sum_i u_i^2, \sum_i v_i^2, \sum_i w_i^2$	$\langle u''^2 \rangle, \langle v''^2 \rangle, \langle w''^2 \rangle, k$
$\sum_i u_i v_i$	$\langle u'' v'' \rangle$

To turn on time averaging in the HYDRA code, the control card *itavg* is set to 1 (default is 0). The time average for multiple intervals is calculated by writing the running sums to the restart dump and saving the total number of steps in the parameter *itavg*.

The time average velocities $\langle u \rangle$, $\langle v \rangle$, and $\langle w \rangle$ are written to the time history file instead of the instantaneous quantities when *itavg* = 1. The plot files were modified to include two states for each time output so that all the desired time-average quantities could be output. This was required because the number of nodal variables per state is limited in a plot file due to the database limitations of the postprocessor. Table 2 is a description of the plot output and corresponding labels in the GRIZ (Speck and Dovey 1996) postprocessing routine.

TABLE 2. 3D HYDRA Time Average Variables, and Corresponding GRIZ Labels

	HYDRA	GRIZ
1st state	$\langle u \rangle$	x-velocity
	$\langle v \rangle$	y-velocity
	$\langle w \rangle$	z-velocity
	vorticity with time average velocities:	
	ω_x	x-vorticity
	ω_y	y-vorticity
	ω_z	z-vorticity
2nd state	$\langle u''^2 \rangle$	x-velocity
	$\langle v''^2 \rangle$	y-velocity
	$\langle w''^2 \rangle$	z-velocity
	$\langle u''v'' \rangle$	x-vorticity
	$\langle u''v'' \rangle$	y-vorticity
	K	z-vorticity

See the Appendix for a detailed description of the coding changes.

References

Hinze, J., 1975, *Turbulence*, McGraw-Hill.

Tennekes, H. and Lumley, J.L., 1972, *A First Course in Turbulence*, MIT Press.

Speck, D. E. and Dovey, D. J., 1996, "GRIZ Finite Element Analysis Results Visualization for Unstructured Grids User Manual," LLNL, UCRL-MA-115696 Rev. 1.

Christon, M.A., 1995, "HYDRA, A Finite Element Computational Fluid Dynamics Code User Manual," LLNL, UCRL-MA-121344.

Appendix

Details of Coding Changes

The following is a list of modified files with a detailed description of the code changes.

STORAGE	<ul style="list-style-type: none"> pointer 88 identified for control variable <i>itavg</i> and summation storage <i>tavg</i> (nnp, ntasms) <i>itavg</i> = 0 no time averaging = 1 time average Note: <i>itavg</i> stores the stop count for restart (i.e., total steps).
pointers.h	<ul style="list-style-type: none"> <i>tavg</i> added to common <i>ptrs</i> and dimension and pointers set.
chkpnt.F	<ul style="list-style-type: none"> subroutine <i>ckpnt</i>: parameter <i>ntasms</i> = 7, included <i>tavg</i> and <i>itavg</i> = <i>icntl</i> (88) in restart. subroutine <i>rstart</i>: reads <i>tavg</i> if part of dump file and allocates memory, total number of steps set from <i>itavg</i>.
drv3d.F	<ul style="list-style-type: none"> subroutine <i>fe3d</i>: keeps sums for time averages, changes for plotting, writes total number of steps to <i>itavg</i>, changed <i>ndim</i> to <i>ndof</i> in <i>vort3d</i> calls for <i>u</i> and <i>tavg</i>, added <i>tavg</i> ascii output, added second state plot when <i>itavg</i> $\neq$ 0, <i><u></i>, <i><v></i>, <i><w></i> written to time history when <i>itavg</i> $\neq$ 0.
fem3d.F	<ul style="list-style-type: none"> subroutine <i>vort3d</i>: <i>ndof</i> is now only dimension for <i>u</i> in call, changed dimension of <i>vor</i> and <i>b</i> from <i>ndof</i> to <i>ndim</i> so could use routine for time averages (note: <i>ndim</i> wasn't used). subroutine <i>tasums</i>: new subroutine for calculating sums for time averages. subroutine <i>tavgdo</i>: new subroutine calculates time averages from sums. subroutine <i>tavgun</i>: undoes time average to get back sums.
hydra.F	<ul style="list-style-type: none"> subroutine <i>allocm</i>: added memory allocations for time averages and since called on starting run (not restart), zeroed storage.
io.F	<ul style="list-style-type: none"> subroutine <i>ekoctl</i>: added time averages. subroutine <i>getctl</i>: added time averages. subroutine <i>getprm</i>: added time averages. subroutine <i>modprm</i>: added time averages and expanded memory if starting run (<i>icntl</i> (88) = 0). subroutine <i>tdbst</i>: generalized for possibility of second state dump at same time (<i>itr</i> = 999).