Lattice Calculation for Lepton Capture from Vacuum-Pair Production in Relativistic Heavy-ion Collisions

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Abstract

The new colliding-beam heavy-ion accelerators, designed to investigate nuclear matter at high temperatures and densities, have motivated great interest during the last decade concerning possible new electromagnetic phenomena. One of the most interesting of these processes is the sparking of the vacuum to produce lepton-antilepton pairs. The process of electron capture from vacuum pair production is a principal beam-loss mechanism for highly charged relativistic ions in a storage ring, and thus must be considered in the design and the operation of these machines. In this paper, I present calculated impact-parameter-dependent probabilities and cross sections for muon capture from vacuum production in collisions of relativistic heavy ions by solving the time-dependent Dirac equation in unrestricted three-dimensional space using lattice-collocation techniques. Calculations are performed for muon-pair production with capture into the K -shell in collisions of $^{197}\text{Au} + ^{197}\text{Au}$ at collider kinetic energies of 2 GeV per nucleon.

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I. INTRODUCTION

The prospect of new colliding-beam accelerators capable of producing collisions of highly stripped high- Z ions, at fixed-target energies per nucleon up to 20 TeV or more, has motivated much interest in lepton-pair production from the QED vacuum. The time-dependent and essentially classical electromagnetic fields involved in such collisions contain large Fourier components which give rise to sizable lepton-pair production [2] in addition to many other exotic particles [14]. In particular, the process of electron-positron production with electron capture is a principal beam-loss mechanism for highly charged ions in a storage ring, and, thus, plays a central role in the design and operation of these machines [1,4]. In this process, the electron is created in a bound state of one of the participant heavy ions, thus changing the ion's charge state and causing it to be lost from the beam. This capture process is unique in atomic physics in that its cross section increases with the collision energy and that no real electrons need be present in the initial state.

There is a long and sometimes controversial history [1] concerning the use of perturbative methods in studying electromagnetic lepton-pair production; however, reliable perturbative calculations have been used as input into design models for the Relativistic Heavy-ion Collider (RHIC) [2,3]. These perturbative calculations for the electron-capture cross section are consistent with a 14-hour lifetime for beams of gold ions in a storage ring at RHIC [4]. However, applying perturbation theory to these processes at extreme energies and small impact parameters results in probabilities which violate unitarity [1,2]. This evidence, along with the initial nonperturbative studies [5] suggests that higher-order QED effects will be important for extreme relativistic collisions [15,1]. However, the magnitude of the nonperturbative enhancements has been a matter of some discussion [6]. Clearly, large nonperturbative effects in electron-pair production with capture will have important implications for RHIC.

The first measurement of the electron-capture process from pair production was performed at Lawrence Berkeley Laboratory's Bevalac accelerator [8]. Various perturbative and nonperturbative predictions for this cross section are within a factor of 2 or 3, but none

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reproduce these results accurately [8]. New capture experiments are planned in November, 1994 at CERN using the lead beams [7] and in 1996 at the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory using gold projectiles [9]. A very important feature of these planned experiments is that, taken together, they will enable extrapolation of the measured cross sections to RHIC energies, due to the moderate energy dependence of the capture cross section in this regime [6].

II. THEORETICAL APPROACH

We study the electromagnetic production of lepton pairs with capture in a reference frame in which one of the nuclei is at rest. The target nucleus and the lepton interact via the static Coulomb field, A_T^0 . The only time-dependent interaction, $A_P^\mu(t)$, arises from the classical motion of the projectile. We write the Dirac equation for a lepton described by a spinor $\phi(\vec{r}, t)$ coupled to an external, time-dependent electromagnetic field

$$[H_F + H_P(t)]\phi(\vec{r}, t) = i\frac{\partial}{\partial t}\phi(\vec{r}, t), \quad (2.1)$$

where the static Furry Hamiltonian, H_F , is given by

$$H_F = -i\vec{\alpha} \cdot \nabla + \beta - eA_T^0, \quad (2.2)$$

and the time-dependent interaction of the lepton with the projectile is

$$H_P(t) = e\vec{\alpha} \cdot \vec{A}_P(t) - eA_P^0(t). \quad (2.3)$$

In Ref. [10], the inclusive time-dependent probability for vacuum production of leptons with capture into a bound state, p , is determined by computing the expectation value of the lepton number operator, $\langle \hat{n}_p(t) \rangle$, for the bound state with respect to the time-evolved QED vacuum, $|\Phi_0(t)\rangle$, i.e.

$$\begin{aligned} P_p(t) &= \langle \Phi_0(t) | \hat{n}_p(t) | \Phi_0(t) \rangle \\ &= \sum_{r < F} |\langle \chi_p^{(+)} | \phi_r^{(-)}(t) \rangle|^2, \quad p > F, \end{aligned} \quad (2.4)$$

where $|\chi_p^{(+)}\rangle$ is a positive energy Furry eigenstate, and F denotes the Fermi surface of the initial QED vacuum state. One may use the time-reversal symmetry of the Dirac equation to obtain an expression where only one time-dependent solution of the Dirac equation is required

$$P_p(t) = \sum_{r < F} |\langle \chi_r^{(-)} | \phi_p^{(+)}(t) \rangle|^2, \quad p > F. \quad (2.5)$$

Since the Dirac equation is covariant under a gauge transformation of the electromagnetic potentials, the gauge may be chosen for convenience in any problem. The most familiar gauge used in problems with electric sources is the Lorentz gauge, defined by the condition $\partial_\mu A^\mu = 0$. The time-dependent electromagnetic interaction between the projectile and the lepton in the Lorentz gauge can be generated by a Lorentz-boost of the static Coulomb field. This results in

$$\begin{aligned} \bar{A}_P^0[r'(t)] &= \frac{-Z_P \alpha \gamma_f}{\sqrt{x^2 + (y - b)^2 + \gamma_f(z - \beta_f t)}} \\ \bar{A}_P^3[r'(t)] &= \beta_f \bar{A}_P^0[r'(t)] \\ \bar{A}_P^1 &= \bar{A}_P^2 = 0, \end{aligned} \quad (2.6)$$

where Z_P is the atomic number of the projectile, and α is the fine-structure constant. Here we assume that the projectile is moving in the z -direction, and that the reaction place is the y - z plane. The beam kinetic energy for a given frame of reference is $E_{\text{kin}} = m_0 c^2 (\gamma - 1)$, where γ denotes the Lorentz factor for the frame of interest. The Lorentz factors for the fixed target and collider frames are related by $\gamma_f = 2\gamma_c^2 - 1$.

The maximum of the scalar and vector components, which are proportional with proportionality constant $\beta_f \approx 1$, scale as γ_f and their half-width is inversely-proportional to γ_f . However, at extreme energies, the probability for capture at a finite impact parameter b is independent of the energy [11,6]. As can be anticipated from the form of the projectile Hamiltonian, this energy independence implies that large cancellations occur between the scalar and vector amplitudes which are troublesome for most approximate solutions [11].

In addition, the spiked nature of the Lorentz-gauge interaction is challenging for most numerical approaches [10,6]. One gauge whose interaction is relatively easy to represent on the lattice is the axial gauge, defined by requiring the z-component of the Lorentz-gauge interaction to vanish [10].

III. LATTICE REPRESENTATIONS

In lattice solutions to the Dirac equation, one forsakes a continuous representation of the quantum-state vector in favor of a representation on a discrete set of spatial lattice points. The Dirac Hamiltonian has a matrix representation of the same rank. The lattice representation of the electromagnetic interaction is diagonal with its values being simply the values of the interaction at a given lattice point. Kinetic energy operators have a lattice representation which may be banded, as in the finite-difference method, or full, as in lattice-collocation methods [12,13]. Given a lattice representation, one reduces the partial-differential equation with specified boundary conditions to a series of linear equations which may be solved using elimination or iterative techniques [16].

Reference [10] describes our approach to the lepton-capture problem in which we solve Eq. (2.1) in three-Cartesian dimensions using high-order lattice-collocation techniques. These methods have many advantages over finite-difference techniques, e.g. accuracy, stability [12], which make an unrestricted solution in three spatial dimensions possible. However, an important feature which all lattice approaches share is a minimum distance scale. Finite lattice spacing implies a maximum momentum content for solutions on the lattice, i.e. $p_{\max} = \pi/\Delta x$. Low order finite-difference representations of the momentum operator will accurately represent only momenta which are less than one-half the maximum lattice momentum. However, high-order lattice collocation representations exist which accurately represent most or all of the momenta allowed on the lattice [12,13].

Lattice calculations for the electron-capture process are more interesting and more difficult than the muon-capture problem. The fundamental difficulty is one of having a number

of natural length scales represented in the calculation. These length scales are the nuclear radius, the electron's Compton wavelength, the size of the atomic K-shell, and the width of the electromagnetic pulse associated with the projectile's interaction. As a consequence, we have presently deferred an attack on the electron-capture problem in favor of the muon-capture problem. The muon Compton wavelength and the heavy-ion nuclear radius are within a factor of three or four, reducing the number of length scales required, and, thus, easing the computational requirements for this problem.

IV. MUON-CAPTURE CALCULATIONS

Figure 1 shows the time-dependent capture probabilities for 2 GeV per nucleon gold collisions using the Fourier-collocation method with various lattice sizes in a box of volume $(40\lambda_\mu)^3$, where λ_μ is the muon Compton wavelength. These calculations demonstrate strong excitations near the distance of closest approach which later relax to an asymptotic value. The degree to which these calculations relax is strongly dependent on the lattice spacing, and smaller spacings are obviously needed to achieve fully relaxed, convergent calculations.

One obtains insight into the reasons for this difficulty in achieving convergence by binning the capture probabilities with respect to the continuum-state energies. Figure 2 contains a plot of $\Delta P_{\text{capt}}/\Delta E^{(-)}$ as a function of $E^{(-)}$, where $E^{(-)}$ is the energy of the Furry continuum states below the Fermi surface, showing the dominant contribution to the total capture probability coming from states with energies located at the maximum kinetic energy available on a one-dimensional lattice. As the lattice resolution increases, the peak's position is observed to move to correspondingly higher energies and to decrease in magnitude. This probability peak is an unphysical artifact of the lattice approximation, and its presence is the reason for the slow rate of convergence of the capture probability demonstrated in Fig. 1.

During the strong excitation observed near the distance of closest approach in the capture probabilities, high-energy components are excited in the time-dependent spinor. The lattice

approximation used in performing the calculation over-truncates the available momentum space such that physically required states are unavailable. As a result, the subsequent relaxation processes are distorted, and significant probability remains at relatively high continuum-state energies, shown in Fig. 2. The energy position of the peak is near the maximum kinetic energy for the longitudinal component of the kinetic-energy operator, $\sqrt{p_{\text{max}}^2 + 1}$, because the Lorentz-contracted nature of the electromagnetic interaction excites states with momenta predominantly in the longitudinal direction. We have tested this interpretation by computing the same collision processes using a lattice with exactly the same z-coordinate lattice as the 81^3 lattice, but with 29 lattice points used to discretize the transverse directions. The energy distributions of this and the previous calculations are very similar with the large unphysical peak located in the same position. A second test of this interpretation was performed by repeating the calculation using $29 \times 29 \times 161$ lattice points. For this lattice, the peak position in energy has roughly doubled as expected, and is no longer the dominant feature in the spectrum. These tests demonstrate the important point that convergence with respect to the lattice size is reached for the transverse dimensions using a modest number of points and long before the longitudinal dimension converges.

Clearly, this convergence problem can be solved by increasing the number and density of lattice points in use. These larger calculations are under production using the Intel Paragon located at Oak Ridge National Laboratory. Presently, we report an approximation to the converged muon-capture probability. Since the unphysical probability peak dominates the energy distribution, but will vanish with increasing density of lattice points, a reasonable approximation is obtained by simply removing the unphysical peak from the energy distribution. Integration under the remainder of the curve provides an estimate of the converged capture probability. Results for various lattices are given in Table I. Note that the trend is for the asymptotic probability to relax as the size of the lattice increases. However, the estimated converged probabilities, computed with the unphysical peak artificially removed, all give the same value within 25%.

To calculate the inclusive muon-capture cross section for peripheral collisions, one should

integrate the geometrically weighted probability for muon capture from a grazing impact parameter outward; i.e.,

$$\sigma_{\text{capt}} = 2\pi \int_{b=b_{\text{graz}}}^{\infty} b P_{\text{capt}}(b) db, \quad (4.1)$$

where σ_{capt} , in this case, is the cross section for capture into the muonic K-shell, and $b_{\text{graz}} = 7.35\lambda_{\mu}$ is the grazing impact parameter for gold ions. As described above, we approximate the impact-parameter dependence of the muon-capture probabilities using lattices with 67^3 points. Integrating the capture probabilities for impact parameters from $b_{\text{graz}} = 7.353\lambda_{\mu}$ to $b = 32\lambda_{\mu}$ gives a muon-capture cross section of $\sigma_{\text{capt}} = 90\mu b$. As smaller probabilities resulting from the larger impact parameters are sensitive to the finite-box size, one may be led to weigh these larger impact parameters less in fitting the exponential function. Following this idea results in a muon-capture cross section of $\sigma_{\text{capt}} = 74\mu b$, which differs from the first results by about 20%.

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FIG. 1. Depicted are time-dependent capture probabilities during a 2 GeV per nucleon collision of $^{197}\text{Au} + ^{197}\text{Au}$ at $b = 1\lambda_\mu$ using the axial gauge for various lattice sizes. Convergence tests are shown with each calculation performed using N^3 lattice points.

FIG. 2. Energy-binned capture probabilities for the three largest calculations shown in Fig. 1.

TABLE I. Depicted are the nonconvergent, asymptotic capture probabilities, P_{capt} , calculated for a 2 GeV per nucleon collision of $^{197}\text{Au} + ^{197}\text{Au}$ at $b = 1\lambda_\mu$ using the axial gauge, computed using various lattice sizes. The quantity, P_{cut} , is the estimate made for the convergent capture probability by removing the contribution from the unphysical probability peak in the energy distribution in each of the calculations.

Lattice Size	P_{capt}	P_{cut}
$(57)^3$	2.07×10^{-4}	1.5×10^{-5}
$(67)^3$	1.19×10^{-4}	1.4×10^{-5}
$(81)^3$	6.10×10^{-5}	1.5×10^{-5}
$29 \times 29 \times 81$	6.32×10^{-5}	1.8×10^{-5}
$29 \times 29 \times 161$	1.89×10^{-5}	1.8×10^{-5}

Fig. 1

Fig. 2