1998 SUMMER RESEARCH PROGRAM FOR HIGH SCHOOL JUNIORS

AT THE

UNIVERSITY OF ROCHESTER'S

LABORATORY FOR LASER ENERGETICS

STUDENT RESEARCH REPORTS

PROJECT COORDINATOR

Dr. R. Stephen Craxton

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Laboratory Report 300

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University of Rochester
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During the summer of 1998, 11 students from Rochester-area high schools participated in the Laboratory for Laser Energetics' Summer High School Research Program. The goal of this program is to excite a group of high school students about careers in the areas of science and technology by exposing them to research in a state-of-the-art environment. Too often, students are exposed to “research” only through classroom laboratories that have prescribed procedures and predictable results. In LLE’s summer program, the students experience all of the trials, tribulations, and rewards of scientific research. By participating in research in a real environment, the students often
become more excited about careers in science and technology. In addition, LLE gains from the contributions of the many highly talented students who are attracted to the program.

The students spent most of their time working on their individual research projects with members of LLE's technical staff. The projects were related to current research activities at LLE and covered a broad range of areas of interest including optics, spectroscopy, chemistry, diagnostic development, and materials science. The students, their high schools, their LLE supervisors and their project titles are listed in the table. Their written reports are collected in this volume.

The students attended weekly seminars on technical topics associated with LLE's research. Topics this year included lasers, fusion, holography, nonlinear optics, global warming, and scientific ethics. The students also received safety training, learned how to give scientific presentations, and were introduced to LLE's resources, especially the computational facilities.

The program culminated with the High School Student Summer Research Symposium on 26 August at which the students presented the results of their research to an audience that included parents, teachers, and members of LLE. Each student spoke for approximately ten minutes and answered questions. At the symposium an Inspirational Science Teacher award was presented to Mr. David Crane, a chemistry teacher at Greece Arcadia High School. This annual award honors a teacher, nominated by alumni of the LLE program, who has inspired outstanding students in the areas of science, mathematics, and technology.
<table>
<thead>
<tr>
<th>Student</th>
<th>High School</th>
<th>Supervisor</th>
<th>Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peter Grossman</td>
<td>Wilson Magnet</td>
<td>R. S. Craxton</td>
<td>Group Velocity Effects in Broadband Frequency Conversion on OMEGA</td>
</tr>
<tr>
<td>Joshua Hubregsen</td>
<td>Pittsford Sutherland</td>
<td>S. Jacobs</td>
<td>A Study of Material Removal During Magnetorheological Finishing (MRF)</td>
</tr>
<tr>
<td>Niharaj Jain</td>
<td>Pittsford Sutherland</td>
<td>M. Guardelben</td>
<td>Analyzing Algorithms for Nonlinear and Spatially Nonuniform Phase Shifts in the Liquid Crystal Point Diffraction Interferometer</td>
</tr>
<tr>
<td>Leslie Lai</td>
<td>Pittsford Mendon</td>
<td>M. Wittman</td>
<td>The Use of Design-of-Experiments Methodology to Optimize Polymer Capsule Fabrication</td>
</tr>
<tr>
<td>Irene Lippa</td>
<td>Byron-Bergen</td>
<td>K. Marshall</td>
<td>Synthesis and Analysis of Nickel Dithiolene Dyes in a Nematic Liquid Crystal Host</td>
</tr>
<tr>
<td>Michael Schubmehl</td>
<td>The Harley School</td>
<td>R. Epstein</td>
<td>An Analysis of the Uncertainty in Temperature and Density Estimates from Fitting Model Spectra to Data</td>
</tr>
<tr>
<td>Joshua Silbermann</td>
<td>Penfield</td>
<td>P. Jannimagi</td>
<td>Automated CCD Camera Characterization</td>
</tr>
<tr>
<td>Abigail Stern</td>
<td>The Harley School</td>
<td>J. Knauer</td>
<td>Design and Testing of a Compact X-Ray Diode</td>
</tr>
<tr>
<td>Amy Turner</td>
<td>Churchville-Chili</td>
<td>R. S. Craxton</td>
<td>Ray Tracing Through the Liquid Crystal Point Diffraction Interferometer</td>
</tr>
</tbody>
</table>
A total of 91 high school students have participated in the program since it began in 1989. The students this year were selected from approximately 60 applicants. Each applicant submitted an essay describing their interests in science, a copy of their transcript, and a letter of recommendation from a science or math teacher.

LLE plans to continue this program in future years. The program is strictly for students from Rochester-area high schools who have just completed their junior year. Applications are generally mailed out in February with an application deadline near the end of March. For more information about the program or an application form, please contact Dr. R. Stephen Craxton at LLE.

This program was supported by the U.S. Department of Energy Office of Inertial Confinement Fusion under Cooperative Agreement No. DE-FC03-92SF19460.
ANALYZING ALGORITHMS FOR NONLINEAR AND
SPATIALLY NONUNIFORM PHASE SHIFTS
IN THE LIQUID CRYSTAL POINT DIFFRACTION INTERFEROMETER

by

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Analyzing Algorithms for Nonlinear and Spatially Nonuniform Phase Shifts in the Liquid Crystal Point Diffraction Interferometer

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1998 Summer High School Academic Research Program (SHARP)

Abstract. Phase-shifting interferometry has many advantages, and the phase-shifting nature of the Liquid Crystal Point Diffraction Interferometer (LCPDI) promises to provide significant improvement over other current OMEGA wavefront sensors. However, while phase-shifting capabilities improve its accuracy as an interferometer, phase-shifting itself introduces errors.

Phase-shifting algorithms are designed to eliminate certain types of phase-shift errors, and it is important to choose an algorithm that is best suited for use with the LCPDI. Using polarization microscopy, we have observed a correlation between LC alignment around the microsphere and fringe behavior. After designing a procedure to compare phase-shifting algorithms, we were able to predict the accuracy of two particular algorithms through computer modeling of device-specific phase-shift errors.

1. Introduction

Scientists at the LLE are hard at work on improving the LCPDI\(^{(1)}\), a device with significantly greater capabilities than the current shearing interferometer used in OMEGA. This device not only adds phase-shifting capabilities in the measurement of aberrated wavefronts, but is compact, easy to align, robust, and cost efficient. Because of its phase-shifting capabilities, the LCPDI will be able to reduce the magnitude of noise and other errors in the detection of wavefront aberrations.

However, phase shifting can often introduce errors, and such phase-shift errors can reduce the accuracy of the LCPDI. It is for this reason that phase-shifting algorithms are so important. Phase-shifting algorithms, which calculate the phase distribution of fringe patterns, are designed to eliminate certain types of phase-shift errors. Such systematic phase-shift errors can be caused by linear miscalibration, nonlinear sensitivity, and spatial nonuniformity of the device. With many algorithms...
available for use with the LCPDI, scientists at the LLE must choose one which is most practical and effective. The advantages of phase-shifting capabilities can only be realized through the proper choice of a phase shifting algorithm.

2. The LCPDI

2.1 Background to interferometry

An interferometer is a device that splits a beam of light into two beams and causes them to interfere. The interference pattern created by the two beams can be captured by a camera and then analyzed to detect aberrations on the incident beam. The LCPDI does this through refraction. In the LCPDI, a glass microsphere is surrounded by a birefringent liquid crystal fluid in a 20μ gap between two glass plates. The microsphere has a different index of refraction from the liquid crystal molecules. As can be seen in Fig. 1, a portion of the incident beam passes through the sphere while the rest passes through the fluid around it. Because of the different indices of refraction, two different wavefronts will be created. The intensity in the resultant interference pattern can be given by the following equation, where $r$ ranges from 1 to the number of phase shifts used.

$$I_{tot,r} = I_{ref,r} + I_{obj,r} + 2\sqrt{I_{obj,r}} \times I_{ref,r} \cos(\phi + \alpha_r) \quad (1)$$

$\phi = \text{Phase difference between object and reference wavefronts}$

$\alpha_r = r\text{th phase shift}$

2.2 Phase shifting

One of the greatest features of the LCPDI is its ability to shift the phase of the object wavefront in relation to the reference wavefront. The LCPDI has this ability because of the birefringence of the liquid crystal molecules, as seen in Fig. 2. These molecules have two distinct indices of refraction, $n_o$ and $n_e$. The molecules also have dielectric anisotropy, so they have the ability to rotate as voltage is applied (Fig. 3). Consequently, an incident beam of light will see a changing index of refraction as voltage is applied.

Each time the phase is shifted, the
Fig. 4 Four simulated interference patterns where the object beam is shifted 90° increments. According to Eq (1), the intensity field, represented by the patterns above, is dependent upon the relative phases and the intensities of the reference and object beams. The bright areas represent areas where the object and reference beams constructively interfere, while the dark areas represent the areas where they destructively interfere, as viewed by the camera.

fringes created by the two interfering beams will shift (fig. 4). This shift represents the dependence upon \( \alpha \) in Eq (1).

2.3 Combining data into a phase plot

Once the interference patterns have been obtained, they must be analyzed to obtain information about the object beam wavefront. An algorithm can calculate the phase of the object beam wavefront by combining the data from each of the interference patterns. Eq (2) is a general algorithm that varies depending on the number \( m \) of phase shifts used and the amplitudes \( b_r \) and \( a_r \). With the 4 interference patterns in Fig. 4, \( m=4 \). The amplitudes \( b_r \) and \( a_r \) are derived algebraically to give the following 4-sample algorithm:

\[
\phi = \arctan \left( \frac{\sum_{r=1}^{m} b_r I_r}{\sum_{r=1}^{m} a_r I_r} \right)
\]

In the work reported here, a 5-sample and 6-sample algorithm were used and their relative immunity to phase-shift errors caused by LC misalignment was investigated.

3. Phase-shift errors

3.1 Liquid crystal misalignment

As explained in section 2.2, the LCPDI has phase shifting capabilities because of the nature of the liquid crystal (LC) molecules. Not only do they contain two different indices of refraction (birefringence), but they also have the ability to rotate when voltage is applied (dielectric anisotropy). When an incoming beam sees a changed index of refraction, the object beam's phase will shift.
However, microscopy has revealed that there is a misalignment of the LC molecules around the microsphere as voltage is applied. Fig. 5 shows how the LCPDI device was examined. Basically, vertically polarized light was passed through the device. The light was then horizontally polarized, and viewed through a microscope. Although the light was polarized horizontally and vertically, some light passed through to the microscope because the liquid crystal molecules in the device changed the polarization of the light. The degree to which the liquid crystal molecules changed the polarization of the light depended on the refractive indices of the molecules, and therefore, the alignment. Hence, by viewing the amount of light that passed through at different locations on the device, we were able to understand the alignment of the molecules. Fig. 6 displays images of a device as viewed with this procedure. It is very evident that there is some misalignment around the microsphere (the clear circle in the middle of the images). Also noticeable is that the misalignment changes with voltage. As the voltage increases from V1 to V2, the alignment around the sphere becomes increasingly bipolar. However, as the voltage continues to increase from V2 to V3, the bipolar alignment becomes less apparent.

The origin of this misalignment behavior is likely the interaction of surface-induced anchoring forces on the cell walls and microsphere with electric-field-induced torque. For example, in thinner LCPDI cells, where surface-anchoring forces have a greater effect in the bulk of the fluid, bipolar alignment is generally less pronounced. Fig. 7 shows a device with a 10μ distance between the glass plates, whereas the device in Fig. 6 is about 20μ thick. The region around the microsphere of Fig. 7 showed remarkably good alignment symmetry. Other spheres within this device, however, showed somewhat greater bipolar alignment than this microsphere although not as severe as in Fig. 6. Regardless of these differences, higher electric field strength overcomes competing surface anchoring forces in both devices. This is evident in the decrease of bipolar alignment at voltage=V3 with both devices.
3.2 Affect of LC misalignment on fringe behavior

Through further study into fringe behavior, we have discovered a correlation between LC alignment changes and fringe behavior as voltage is applied. The LC misalignment seen in Figs. 6 and 7 appears to be the cause of irregularity in fringe patterns as voltage is applied. This error can be seen in the fringe patterns in Fig. 8. At a phase shift of 0, which corresponds to V1 in Fig. 6, there is little noticeable error. However, as voltage is applied and the phase is shifted by π radians, error becomes evident. This error is manifested in a brightness on the sides of the fringes closest to the center of the interference pattern. However, as the phase is shifted to 2π radians, this brightness seems to decrease. This fringe behavior would be expected from the decrease in LC misalignment from V2 to V3 in Figs. 7 and 8. However, some phase shift error clearly does exist in going from 0 to 2π radians, in that the pattern at 2π phase shift is different from that at 0 phase shift. Theoretically, since 2π is a full wavelength, a shift of 2π should result in the same exact pattern.

3.2 Mathematical representation of phase-shift error

The phase-shift errors seen in the fringe patterns above can be represented with a general mathematical equation as an infinite series, as stated in the paper by Hibino et. al.(1) Where m is the number of samples used in the algorithm, each phase shift value, α_r, can be given as a function of the phase-shift parameter, α_or, as

\[
\alpha_r = \alpha_{or}[1 + \epsilon(\alpha_{or})]
\]

\[
\alpha_r = \alpha_{or}\left[1 + \epsilon_1 + \epsilon_2\left(\frac{\alpha_{or}}{\pi}\right) + \epsilon_3\left(\frac{\alpha_{or}}{\pi}\right)^2 + \ldots + \epsilon_p\left(\frac{\alpha_{or}}{\pi}\right)^{p-1}\right]
\]

for \( r=1,2,\ldots,m \)

where \( p \) (\( p \leq m-1 \)) is the maximum order of the nonlinearity, \( \epsilon_q \) (\( 1 \leq q \leq p \)) are the error coefficients, which can be spatially dependant, and \( \alpha_{or} \) is the unperturbed phase shift.

4. Reduction of phase shift errors with algorithms
4.1 Derivation of algorithms

Many algorithms have been derived in works such as the paper by Hibino et. al.\textsuperscript{(2)} to calculate phase distribution based on fringe data. Algorithms are derived in such a way as to eliminate certain types of error. Phase errors caused by specific orders of the non-linear series (Eq 4) can be reduced or eliminated in the derivation of the sampling amplitudes, \( a_r \) and \( b_r \) in Eq (2), and the number of samples, \( m \) in Eq (2). Hibino describes a method for deriving appropriate algorithms that will not be discussed here.

We decided to test two types of algorithms given in the paper by Hibino et. al.-one which uses 5 samples and phase shift increments of \( \pi/2 \) radians (Eq 5) and another which uses 6 samples and phase shift increments of \( \pi/3 \) radians (Eq 6). The 5-bucket algorithm was specifically designed to eliminate errors caused by spatially nonuniform phase shifts with linear miscalibration \((p = 1)\). The 6-bucket algorithm was specifically designed to eliminate spatially nonuniform errors up to the second order \((p = 2)\).

Five-sample algorithm:

\[
\phi = \arctan \left[ \frac{2(I_2 - I_4)}{2I_3 - I_5 - I_1} \right] \quad (5)
\]

Six-sample algorithm:

\[
\phi = \arctan \left[ \frac{\sqrt{3}(5I_1 - 6I_2 - 17I_3 + 17I_4 + 6I_5 - 5I_6)}{I_1 - 26I_2 + 25I_3 + 25I_4 - 26I_5 + I_6} \right] \quad (6)
\]

4.2 Developing a procedure to compare algorithms

Our first step in comparing the two algorithms was to design a method to test them and then to confirm the accuracy of our results. We did this by writing a PV-WAVE procedure that would test the algorithms on simulated fringe data. We first tested the algorithms on simulated fringes with first and second degree spatially uniform phase-shift errors. The phase shift was given by the following equation, which added the two error terms to the ideal phase shift:

\[
\alpha_r = \alpha_{or} \left[ 1 + \epsilon_1 + \epsilon_2 \left( \frac{\alpha_{or}}{\pi} \right) \right] \quad (7)
\]

Arbitrary values were substituted for the two error terms, \( \epsilon_1 \) and \( \epsilon_2 \). From the resultant phase plot, a reference plot (created by using error-free simulated fringes) was subtracted. The program then calculated the peak-to-valley difference between these two plots. This peak-to-valley difference represents the residual error after the algorithms are tested on the simulated data. The residual error is given as

\[
\Delta \phi = \phi_{perturbed} - \phi_{ideal} \quad (8)
\]

Our results from this test (Table 1) showed that there was a smaller residual error when the 6-bucket
algorithm was used than when the 5-bucket algorithm was used. This was justifiable when a second order error term was added, because the 6-bucket algorithm was designed to handle such errors while the five-bucket was not. However, the 6-bucket algorithm also seemed to work better when only first order error was introduced. These results matched exactly the results given by those who derived the algorithm, given in Table 3 of Ref (2).

4.3 Modeling device-specific errors

Given the high accuracy of our procedure for testing algorithms, we next sought to model the phase-shift errors seen in the laboratory. Our modeled error term is a general functional form of the error seen in the laboratory. This term is given as

$$\varepsilon(\alpha_r) = e^{-A\alpha_r} \times e^f(x,y)$$

$$e^f(x,y) = H\left(1 - e^{-|Bx+Cy|}\right)\left(e^{-\left(Dx^2+Ey^2\right)^{1/2}}\right) \sin\left(\arctan\left(\frac{Kx}{My}\right)\right)$$

Table 1 Residual errors due to linear and quadratic spatially uniform phase shift errors. Results confirmed on right.

<table>
<thead>
<tr>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>Five-Sample</th>
<th>Six-Sample</th>
<th>Six-Sample$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 0</td>
<td>0.0020</td>
<td>0.00005</td>
<td></td>
<td>0.00005</td>
</tr>
<tr>
<td>0.1 0.2</td>
<td>0.0265</td>
<td>0.0015</td>
<td></td>
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</tr>
<tr>
<td>0.1 0.2</td>
<td>0.0260</td>
<td>0.0025</td>
<td></td>
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</tr>
<tr>
<td>0.1 0.4</td>
<td>0.0610</td>
<td>0.0060</td>
<td></td>
<td>0.0060</td>
</tr>
<tr>
<td>0.1 0.4</td>
<td>0.0595</td>
<td>0.0050</td>
<td></td>
<td>0.0050</td>
</tr>
</tbody>
</table>

where $A-M$ are amplitudes. Eq (9) displays the error term as a function of the phase-shift parameter and of space. The spatial dependence is expanded into the function in Eq (10), where a complex algebraic equation models the general form of the nonuniformity seen around the center of the interference patterns.

Our perturbation term has a mathematical relation to Eq (4), where the error can be explained in a series of error terms of increasing order. This perturbation term can be expanded into an infinite series, where $p$ increases ad infinitum. This relation is made by Eq (11), where the exponential function is expanded into an infinite series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

Our modeled error term seems to be similar to the general form of the error seen in experimental interference pattern (Fig. 9). The simulated fringes have a brightness on the sides of the fringes closest to the center of the interferogram. Also evident is the voltage dependence of the simulated fringes. As the phase is increased from $0\pi$ to $3\pi/2$, the magnitude of the error also increases. As the phase continues to increase, from $3\pi/2$ to $2\pi$, the error decreases. This not only is similar to the error seen in the experimental fringes, but it also demonstrates the correlation between the LC misalignment and fringe behavior. However, it is important to note that any change in fringe contrast due to the
Fig. 9 Modeled fringes show phase shift errors similar to experimental fringes (Note: see Fig. 10 at end of paper for enlarged image).

Table 2. Residual phase error from simulated fringe data (units are 2π radians)

<table>
<thead>
<tr>
<th></th>
<th>Five-Sample</th>
<th>Six-Sample</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>.121</td>
<td>.064</td>
</tr>
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</table>

5. Expediency in choosing an algorithm

Scientists at the LLE have many algorithms to choose from for use with the LCPDI. Algorithms have been derived to eliminate many different errors. Generally, it is accepted that an algorithm with more samples will be more effective in eliminating phase shift errors. However, LLE scientists must be
think of expediency in choosing the proper algorithm. Using algorithms with many samples can be impractical for many reasons.

One reason that the number of samples in an algorithm should be minimized is the time factor. LC molecules in the device take time to rotate as voltage is applied. This means that there must be a pause each time the phase is shifted. The more samples that are taken, the longer it will take. With the many iterations that will occur in the testing of the device, it is preferable to minimize this time factor.

The number of samples in an algorithm should also be kept low due to the calibration of the voltage applied. Calibration is done manually at this point, and it is difficult to calibrate phase steps of less than $\pi/3$ rad. As the phase-shift interval decreases, phase-shift errors of high magnitude can be introduced in the calibration alone.

Lastly, because of the limited range of the refractive indices of the LC molecules (approx. 1.5 to 1.7), it is difficult to achieve a total phase shift of more than $2\pi$. With a large number of samples, the total phase shift will increase past what the device is capable of shifting.

6. Possible future work

Scientists continue to work on improving the LCPDI device here at the LLE. What follows is a summary of areas where the research can progress in order to improve the accuracy of the LCPDI.

In comparing the 5- and 6-bucket algorithms, most of the tests were simulated. What should follow is a comparison on experimental fringe data. Such a comparison would not only give a realistic comparison of the two algorithms, but it can also be used to test the accuracy of our simulated perturbation term.

Only two algorithms were compared in detail. Further research should follow with other N-bucket algorithms. The comparisons of these algorithms should follow the procedure used to compare the 5- and 6-bucket algorithms.

Further study should be performed on the alignment of LC molecules in the device. At this point, we possess only a basic understanding of this misalignment. With a better understanding, better measures can be taken to prevent such misalignment and fringe symmetry can be improved.

7. Conclusion

Because of its relatively low cost, immunity to vibration, self-referencing nature, ease of use, and phase-shifting capabilities, the LCPDI has proved to be a very promising interferometer. However, to maximize the benefits of the LCPDI, a proper phase-shifting algorithm must be used. Such an algorithm must be suited to eliminate device-specific errors, but must at the same time be practical.

Through microscopy, we have shown that LC misalignment exists in the device. Through exploration into the behavior of fringes, we discovered a correlation between device-specific errors and the LC misalignment. After developing a procedure to compare algorithms, we were able to predict the performance of two specific algorithms with the LCPDI.
8. References


9. Acknowledgments

First and foremost, I would like to thank Mark Guardalben. Day in and day out, he made himself available to me. Without his countless hours of guidance, this past summer would not have been such a success. I would next like to thank Ryan Brecker, Devon Battaglia, and Dan Bouk, whose experience proved to be very helpful. I would also like to thank all of the other program participants, who made this experience lively and exciting. Lastly, I would like to thank Dr. R.S. Craxton and the Laboratory of Laser Energetics for providing me with this remarkable opportunity.