MATHEMATICAL MODELLING OF DEFORMATION DURING HOT ROLLING

D. Jin, R.G. Stachowiak, I.V. Samarasekera and

J.K. Brimacombe

The Centre for Metallurgical Process Engineering
The University of British Columbia
Vancouver, B.C. Canada V6T 1Z4

ABSTRACT

The deformation that occurs in the roll bite during the hot rolling of steel, particularly the strain-rate and strain distribution, has been mathematically modelled using finite-element analysis. In this paper three different finite-element models are compared with one another and with industrial measurements. The first model is an Eulerian analysis based on the flow formulation method, while the second utilizes an Updated Lagrangian approach. The third model is based on a commercially available program DEFORM which also utilizes a Lagrangian reference frame. Model predictions of strain and strain-rate distribution, particularly near the surface of the slab, are strongly influenced by the treatment of friction at the boundary and the magnitude of the friction coefficient or shear factor. Roll forces predicted by the model have been compared with industrial rolling loads from a seven-stand hot-strip mill.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Dle

1.0 STATE-OF-THE-ART

Important attributes of hot-rolled steel sheet and strip are final dimensional tolerances and mechanical properties, both of which are strongly influenced by the thermal and deformation history during hot rolling. With the rapid growth of computer power over the past two decades, mathematical modelling has emerged as an effective tool for the analysis of the hot rolling process. The theoretical models of hot flat rolling, which were largely based on the assumption of homogeneous deformation, have been superseded by numerical analyses capable of accommodating an inhomogeneous strain field, and nonlinear material properties.

The finite-element method, in particular, has been applied extensively to the hot rolling process for steel to provide greater insight on the nature of deformation. Specifically, analyses based on this method permit a more accurate determination of the strain, strain-rate and temperature distribution in the roll bite. Several different formulations, based on the finite element method have been developed for hot and cold rolling. The principal difference in the methods stems from the Lagrangian or Eulerian frame of reference employed. The former is based on material coordinates, whereas the latter is for a coordinate system fixed in space. Two methods have been developed to model problems involving large displacements such as rolling. The first of these is the displacement method, which employs the Prandtl-Reuss equations to relate the deviatoric stress to the strain increment, in a Lagrangian reference frame. In the second technique, the flow-formulation method, an Eulerian frame of reference is utilized and the flow stress is related to the rate of deformation. Material behaviour is assumed to be rigid-plastic, elasto-plastic or visco-plastic, recognizing that with an Eulerian analysis elastic effects are generally omitted.

There are important differences between models for hot-rolling in the treatment of friction at the boundary. An excellent overview of the application of the finite-element analysis to hot-rolling of steel is provided in two recent reviews; and the reader is referred to these for details of the different methods [1,2].

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Although numerous approaches have been adopted by individual researchers to analyse the deformation field in the roll bite during hot rolling, a comparative study of the different methods has not been conducted, particularly with respect to their capability to predict strain and strain rates in the roll bite. In this paper, the predictions of three finite-element models are compared with one another and with industrial measurements. The sensitivity of model predictions to friction along the interface is examined, because of its influence on redundant shear and homogeneity of deformation. The first model is an Eulerian analysis based on the flow formulation method [3,4] while the second utilizes an Updated Lagrangian approach [5,6]. The third model is based on a commercially available program DEFORM[7,8], which utilizes a Lagrangian frame of reference. Roll forces predicted by the models are compared with measured values from a seven-stand finishing mill at USS Gary Works.

2.0 MODELLING OF HOT ROLLING

In the hot rolling of sheet and strip, a transfer bar generally 25~40 mm thick is reduced in successive rolling passes to a finished gauge of 2~8 mm. The width of the slab (1200~2000mm) is always significantly larger than the thickness and hence the plane strain assumption is valid over most of the transverse direction, except perhaps close to the edges. Thus calculations have been conducted for the two-dimensional plane through the roll bite zone at mid-width shown in Figure 1. Consequently deformation in the transverse direction has been neglected.

A standard set of conditions was chosen as for the preliminary comparison of the three mathematical models and consisted of applying 45 pct. reduction in a single pass to a 20 mm thick slab, with an initial temperature of 900°C. The tangential roll speed was constant at 943 mm/s. The following constitutive equation, experimentally determined on a Gleeble 1500 Thermomechanical simulator for a DQSK steel, was employed in the model.

$$\sigma = \frac{1}{\alpha} \sinh^{-1} \left[\left(\frac{\dot{\epsilon} \exp\left(\frac{Q}{RT}\right)}{A} \right)^{\frac{1}{n}} \right] \quad (1)$$

where T is the temperature in °K; $\alpha=0.013 \text{ MPa}^{-1}$;
 $Q=293 \text{ kJ/mol}$; $n=0.47$; and $\ln A=25.6$.

As mentioned in the previous section, three finite-element models - Eulerian, Updated Lagrangian and DEFORM - were employed to compute the strain, strain-rate and temperature distributions in the roll-bite for the conditions described above. Details of the mathematical formulation have been described previously [3-8]. In all three models heat transfer and deformation are coupled, through the heat of deformation. A value of 50 kW/m^2 has been employed as the heat-transfer coefficient at the roll-steel interface in all of the calculations. The material behaviour was assumed to be visco-plastic for both DEFORM and the Eulerian model while the Updated Lagrangian model simulates visco-elastoplastic behaviour, although elastic effects are likely to be small in hot rolling.

An important difference between the three models is the treatment of friction at the roll-slab interface. In DEFORM, a velocity-dependent frictional stress has been employed to approximate constant frictional stress conditions [9]. The boundary condition at the roll-slab interface is expressed as:

$$f_s = -mk \left(\frac{2}{\pi} \tan^{-1} \frac{u_s}{u_0} \right) \quad (2)$$

where f_s is the frictional shear stress; m is the interface shear factor; k is the shear yield stress of the slab; and u_s is the relative sliding velocity of the roll and the slab. The term u_0 is included for numerical purposes only with a ratio of u_s/u_0 larger than 10 recommended to approximate the condition of constant frictional stress at the interface. In the Eulerian finite-element model[3,4] friction is simulated through a set of narrow interface elements along the arc of contact. At the roll/slab interface, the interface element nodes are assigned a

velocity equal to the roll velocity, which is essentially a no-slip condition. However within the interface elements

$$\tau = \mu p \quad (3)$$

where τ is the shear stress within the element; μ is the coefficient of friction, and p is the normal pressure at the interface.

The Updated Lagrangian model incorporates Coulomb friction at the interface. On the entry side where the tangential slab velocity is less than the roll velocity,

$$v_n = 0, f_t = \mu f_n \quad (4)$$

Over the region where sticking occurs,

$$v_n = 0, v_t = \omega R \quad (5)$$

and on the exit side,

$$v_n = 0, f_t = \mu f_n \quad (6)$$

where f_t and f_n are the tangential and normal nodal forces respectively of boundary elements; v_n and v_t are the normal and tangential velocities of boundary nodes; μ is the coefficient of friction and ωR is the peripheral roll speed.

3.0 APPLICATION OF DEFORM TO HOT ROLLING

3.1 Model Predictions and Sensitivity Analysis

The DEFORM model was utilized to simulate the standard conditions specified earlier. An interface shear factor of 1.0, which represents sticking friction conditions, was assumed. In the code, four-node isoparametric elements were employed to discretize the domain. Predicted strain rates for the case of 3000 elements are shown in Figure 2. It is evident that effective strain rate is high near the entrance and at the exit from the roll bite, with a dead zone associated with the neutral point closer to the roll bite exit. The maximum effective strain rate is approximately 115 s^{-1} which is an order of magnitude greater than the nominal strain rate for this pass. The corresponding normal pressure

distribution along the arc of contact is shown in Figure 3, which shows a friction-hill type pressure distribution, with a maximum pressure at the neutral point. Although a shear factor of 1 was assumed, the model does not simulate true sticking friction conditions because the relative velocity between the roll and slab is not employed as a boundary condition. Consequently regions of backward and forward slip are present on either side of the neutral point. By virtue of the boundary condition specified in Eq.(2), high shear is induced in the slab close to the entrance and near the exit because the relative velocity between the slab and the rolls is a maximum at these locations.

Figure 4, showing the strain distribution through the thickness at the roll exit, illustrates that a total of 3000 elements was required to achieve convergence. The effective strain is close to the nominal strain at the center of the slab, but increases significantly towards the surface, because of the induced shear stresses. The model-predicted strain distribution exhibits a subsurface peak when a large number of elements were employed. For a small number of elements the model underpredicts the extent of strain inhomogeneity. Figure 5 underscores the influence of the shear factor and work piece thickness on the strain distribution. The higher values of strain in Figure 5 corresponds to an initial workpiece thickness of 24mm, a reduction of 34pct. and an entry temperature of 1000°C, which represents conditions typical of the early finishing passes of a hot-strip mill. The second simulation was conducted for an initial workpiece thickness of 11mm, with an entry temperature of 950°C and a reduction of 22 pct., which is more typical of middle passes on a finishing mill. Not surprisingly, the maximum effective shear strain in the workpiece, increases with increasing thickness and higher values of m . The results suggest that strain inhomogeneity is probably small or insignificant in the later passes of a hot-strip mill.

3.2 Comparison of Model Predictions with Micro-deformation Measurements

To assess the accuracy of the DEFORM model for the hot rolling of steel, the predicted distortion field has been compared with micro-deformation measurements from a laboratory rolling study[10]. In the latter work, a grid of

squares was inscribed onto the surface of a steel specimen which was inserted into a machined slot in a C-Mn steel specimen. The sample was partially rolled, and the deformation of the inscribed grid was measured to characterize the strain field. The laboratory experiment was conducted on a single-stand rolling mill with 360mm diameter rolls. The 20mm thick specimen was heated to 900°C and rolled in a single pass to 11 mm. To compare DEFORM predictions with the experimental results, the FLOWNET post-processing tool was used to track the deformation of an identical grid during rolling. Results of the DEFORM simulation which were conducted with a shear factor of 0.85 compare well with the measurements, shown in Figure 6. It is interesting that the distortion of the experimental grid suggests that the strain is a maximum at the surface, whereas the DEFORM model predicts a subsurface peak. Intuitively, it is difficult to envisage why the strain should be a maximum below the surface since redundant shear within the material will be highest at the interface.

4.0 COMPARISON OF DIFFERENT MODELS

An Eulerian finite-element model based on the flow formulation method [3,4] and an Updated Lagrangian model [7,8] have also been employed to calculate the strain field for the standard case specified in Section 2.0. Figures 7 and 8 show strain-rate contours in the deformation zone predicted by the two models. Eight-node isoparametric elements were employed in both cases. The results, shown in Figure 7, were obtained with the Eulerian code, and are very similar to the predictions of DEFORM. The strain rate is high at the surface of the workpiece close to the entrance and exit of the roll bite. The Eulerian model predicts a local maximum of 53 s^{-1} at the exit, while DEFORM predictions are close to 65 s^{-1} . At the entrance, DEFORM predicts a lower maximum strain rate of 105 s^{-1} , compared to 181 s^{-1} from the Eulerian model. In contrast, the results of the Updated Lagrangian model shown in Figure 8, reveal a different pattern. Although the strain rate in the workpiece is high near the surface at the entrance to the roll bite, $\sim 130 \text{ s}^{-1}$, it is low at the exit. Furthermore the region of high shear extends over a longer distance in Figure 8, compared to the other two models where the deformation is localized.

The absence of a high strain rate zone at the exit in Figure 8 is a consequence of the boundary condition employed in the Updated Lagrangian model. In the latter formulation, true Coulomb friction is simulated by imposing a constraint on the ratio of normal and tangential nodal forces along the boundary. In the Eulerian model, the no-slip condition imposed on boundary nodes gives rise to high shear in the boundary friction elements, which is reflected in the strain rate pattern.

The effective strain contours obtained with the Eulerian and Updated Lagrangian models are shown in Figures 9 and 10 respectively. The predicted strain is a maximum at the surface for the Updated Lagrangian model and is higher than that for the Eulerian model which shows a subsurface maximum like DEFORM. The extended shear zone, shown in Figure 8 is responsible for the higher maximum strains in Figure 10. The through-thickness strain profiles at the exit of the roll bite computed with the three models are compared in Figure 11. There is good agreement over most of the thickness, and the predicted strains at the centre approach the nominal strain for the pass. Deviations between the models are limited to the surface region, where the boundary condition has a strong influence. The similarity between the strain fields predicted by DEFORM and the Eulerian model is readily apparent; and the maximum is seen to occur below the surface. The strain is a maximum at the surface with the Updated Lagrangian model, which is in better qualitative agreement with the experimental micro-deformation measurements in Figure 6. A major drawback of the Updated Lagrangian approach is the long computational time compared to the other two models. Computational times on a Silicon Graphics work station are presented in Table I which shows that although convergence is obtained with fewer eight-node isoparametric elements, the Updated Lagrangian method takes nearly 24 hours. By comparison the Eulerian analysis is computationally efficient and yields relatively accurate strain predictions over most of the thickness, except near the surface. Model predictions are also sensitive to the shear factor, m , and the friction coefficient, μ . The values employed for these variables are given in Table I, which lie in the range considered typical for hot rolling.

5.0 COMPARISON WITH INDUSTRIAL ROLLING LOADS

The three models have been employed to simulate industrial rolling conditions; and predicted roll forces have been compared with measurements from a seven-stand finishing mill at USS Gary Works. A plain carbon steel coil (DQSK), was chosen for the comparison.

The rolling schedule, strip speeds, and measured roll forces are compared with model predictions in Table II and Figure 12. It is evident that the best agreement between model predictions and mill measurements were obtained with the Updated Lagrangian model, where the differences are within 10 pct. for all stands. The Eulerian model also gives comparable agreement for the same friction coefficient of 0.25. When the friction coefficient is increased to 0.35 the deviation between predicted and measured rolling loads approaches 20 pct. for several stands. The DEFORM model tends to under-predict the rolling loads for nearly all the passes. Although the values employed for the shear factor, m , and the friction coefficient, μ , lie in the range cited as typical for hot rolling, model predictions are very sensitive to these variables. More precise friction data is clearly necessary for a thorough evaluation of the capability of the different models.

6.0 CONCLUSIONS

The capability of three different finite-element models to predict the strain and strain-rate distribution in the roll bite zone during hot rolling has been analysed. The first model is based on the Eulerian flow formulation method, the second utilizes the Updated Lagrangian approach, while the third is a commercially available code, DEFORM. Model predictions have been compared against one another, and with roll forces measured on an industrial hot-strip mill. The following conclusions have been drawn from this study.

- [1] The commercial program, DEFORM is highly mesh sensitive because it employs four-node isoparametric elements, and requires 3000 elements to converge. The other two models are based on eight-node

isoparametric elements and yield a convergent solution with less than 300 elements.

- [2] The strain-rate distribution predicted by DEFORM and the Eulerian finite-element model are very similar. There are two zones of high strain rate at the surface of the strip, near the entrance and exit of the roll bite. The Updated Lagrangian model predictions show only one such zone, located near the entrance.
- [3] The strain through the thickness is non-uniform and increases towards the surface, particularly near the exit from the roll. The through-thickness strain profiles predicted by the three models, are almost identical over the central region of the strip and differ close to the surface because of differences in the treatment of friction at the interface.
- [4] The strain in-homogeneity increases with increasing friction and strip thickness.
- [5] The predicted roll forces have been compared with measured values from the seven-stand hot-strip mill at USS Gary Works. Model predictions are sensitive to the friction coefficient, and good agreement was obtained between industrial measurements and predictions from the Eulerian and Updated Lagrangian model for a friction coefficient of 0.25. The DEFORM code utilizes a friction factor, and a value of 0.85, was found to be suitable for the prediction of roll forces.
- [6] The Eulerian model was computationally efficient and required 20 minutes on a Silicon Graphics work station compared to 2 hours for DEFORM and 24 hours for the Updated Lagrangian models. With the exception of a zone very close to the surface (~1mm) the strain predictions from the Eulerian model are in good agreement with the other two models.

ACKNOWLEDGEMENTS

The authors are grateful to the American Iron and Steel Institute and the United States Department of Energy for financial support of this work. The assistance of C. Mariani and A. Kmiec of USS Gary Works in obtaining mill data is gratefully acknowledged.

REFERENCES

1. P. Montmitonnet and P. Buessler, "A Review on Theoretical Analysis of Rolling in Europe", ISIJ International, Vol. 31, 1991, No. 6, pp. 525-538.
2. P. Hartley, C.E.N. Sturgess, C. Liu and G.W. Rowe, "Experimental and Theoretical Studies of Work Piece Deformation, Stress and Strain During Flat Rolling", International Materials Reviews, 1989, Vol. 34, No. 1, pp. 19-34.
3. A. Kumar, I.V. Samarasekera and E.B. Hawbolt, "Roll-Bite Deformation During the Hot Rolling of Steel Strip", Journal of Materials Processing Technology, Vol. 30, 1992, pp. 91-114.
4. O.C. Zienkiewicz, P.C. Jain and E. Onate, "Flow of Solids During Forming and Extrusion: Some Aspects of Numerical Solutions", Int. J. of Solids and Structures, Vol. 14, 1978, pp. 15-38.
5. S.I. Oh, W.T. Wu, J.P. Wang, A. Vedanayagam, "Capabilities and Applications of FEM Code DEFORM: The Perspective of the Developer", J. Mater. Processing Technology, Vol. 27, 1991, pp. 25-42.
6. S. Kobayashi, S.I. Oh, T. Altan, "Metal Forming and the Finite-Element Method", First Edition, 1989, Oxford University Press, Oxford, U.K., pp. 111-130.
7. H.D. Hibbity, P.V. Marcal and J.R. Rice, "A Finite-Element Formulation for Problems of Large Strain and Large Displacement", Int. J. Solids Structures, Vol. B (1970), pp. 1069-1089.
8. D.Q. Jin, K. Kato and H. Iseki, "Finite-Element Analysis of Forming Property in Plastic Flow Joining", J. Japan Soc. Tech. Plast., Vol. 34, No. 394.
9. S. Kobayashi, S.I. Oh, T. Altan, "Metal Forming and the Finite-Element Method", First Edition, 1989, Oxford University Press, Oxford, U.K., p. 119.

10. L. Lachmann and G. Zouhar, "Investigation of Strain Inhomogeneities During Hot Flat Rolling - Influences on the Microstructure Evolution", Steel Research, No. 10, 1991, pp. 447-452.

TABLE I. Features of the Finite-Element Models

Name	Mat. Behaviour	Shear Factor or Coefficient of Friction	Number of Elements	Element Type	Computational Time
DEFORM	Visco-plastic	$m = 0.85$	3000	four-node isoparametric	2 hours
Eulerian	Visco-plastic	$\mu = 0.25$	200	eight-node isoparametric	20 minutes
Updated Lagrangian	Viscoelastic-plastic	$\mu = 0.25$	105	eight-node isoparametric	24 hours

TABLE II. Comparison of Industrial Rolling Loads with Predictions

Stand No.	Reduction %	Strip Speed (m/s)	Measured Roll Force (tonnes)	DEFORM m = 0.85 (tonnes)	Updated Lagrangian $\mu = 0.25$ (tonnes)	Eulerian	
						$\mu = 0.25$	$\mu = 0.35$
1	32.7	1.30	1481.4	1310.4 (-11.5%)	1400.0 (-5.4%)	1440.0 (-2.76%)	1543.0 (+4.0%)
2	34.1	1.97	1413.4	1452.0 (+2.7%)	1425.0 (+0.85%)	1495.0 (+5.8%)	1051.0 (+16.8%)
3	29.4	2.78	1309.0	1250.0 (-4.5%)	1328.0 (+1.45%)	1296.0 (-1.0%)	1435.0 (+9.6%)
4	21.8	3.54	1114.9	910.0 (-18.3%)	1199.5 (+7.6%)	1007.2 (-9.6%)	1127.0 (+1.16%)
5	21.5	4.53	917.2	885.0 (-3.5%)	954.0 (+4.0%)	995.0 (-8.5%)	1102.0 (+20.17%)
6	19.5	5.61	867.3	825.0 (-4.8%)	856.0 (-1.26%)	944.0 (+8.8%)	1044.0 (+20%)
7	9.7	6.23	544.3	420.0 (-23%)	602.3 (+10.66%)	570.0 (+4.7%)	614.0 (+12.8%)

LIST OF FIGURES

- Figure 1. Schematic Diagram of Roll-Bite Zone.
- Figure 2. DEFORM predictions of effective strain-rate distribution in the deformation zone of a 20 mm thick slab for 45 pct. Reduction at 900°C.
- Figure 3. DEFORM predictions of pressure distribution along the roll/slab interface for 45 pct. Reduction of a 20 mm slab at 900°.
- Figure 4. The influence of mesh density on DEFORM predictions of strain distribution at the exit from roll bite.
- Figure 5. The effect of shear factor and slab or strip thickness on DEFORM predictions of strain distribution at the exit from the roll bite.
- Figure 6. Comparison of DEFORM predictions of distortion of an inscribed grid with experimental measurements.
- Figure 7. Eulerian finite-element model predictions of effective strain-rate distribution in the deformation zone off a 20 mm thick slab for 45 pct. Reduction at 900°.
- Figure 8. Updated Lagrangian finite-element model predictions of effective strain-rate distribution in the deformation zone of a 20 mm thick slab for 45 pct. Reduction at 900°.
- Figure 9. Eulerian finite-element model predictions of effective strain distribution in the deformation zone of a 20 mm thick slab for 45 pct. Reduction at 900°.
- Figure 10. Updated Lagrangian finite-element model predictions of effective strain distribution in the deformation zone of a 20 mm thick slab for 45 pct. Reduction at 900°.

Figure 11. Comparison of strain distribution predicted by three finite-element models at the exit from roll bite in a 20 mm thick slab reduced 45 pct.

Figure 12. Comparison of industrial roll forces measured on a seven-stand hot-strip mill with predictions from three finite-element models.

Figure 1. Schematic Diagram of Roll-Bite Zone.

Figure 2. DEFORM predictions of effective strain-rate distribution in the deformation zone of a 20 mm thick slab for 45 pct. Reduction at 900°C.

Figure 3. DEFORM predictions of pressure distribution along the roll/slab interface for 45 pct. Reduction of a 20 mm slab at 900°.

Figure 4. The influence of mesh density on DEFORM predictions of strain distribution at the exit from roll bite.

Figure 5. The effect of shear factor and slab or strip thickness on DEFORM predictions of strain distribution at the exit from the roll bite.

Figure 6. Comparison of DEFORM predictions of distortion of an inscribed grid with experimental measurements.

Figure 7. Eulerian finite-element model predictions of effective strain-rate distribution in the deformation zone off a 20 mm thick slab for 45 pct. Reduction at 900°.

Figure 8. Updated Lagrangian finite-element model predictions of effective strain-rate distribution in the deformation zone of a 20 mm thick slab for 45 pct. Reduction at 900°.

Figure 9. Eulerian finite-element model predictions of effective strain distribution in the deformation zone of a 20 mm thick slab for 45 pct. Reduction at 900°.

Figure 10. Updated Lagrangian finite-element model predictions of effective strain distribution in the deformation zone of a 20 mm thick slab for 45 pct. Reduction at 90°.

Figure 11. Comparison of strain distribution predicted by three finite-element models at the exit from roll bite in a 20 mm thick slab reduced 45 pct.

Figure 12. Comparison of industrial roll forces measured on a seven-stand hot-strip mill with predictions from three finite-element models.