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REPORT

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Pseudo-MHD Ballooning Modes In Tokamak Plasmas

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Abstract

The MHD description of a plasma is extended to allow electrons to have both fluid-like and adiabatic-regime responses within an instability eigenmode. In the resultant “pseudo-MHD” model, magnetic field line bending is reduced in the adiabatic electron regime. This makes possible a new class of ballooning-type, long parallel extent, MHD-like instabilities in tokamak plasmas for $\alpha > s^2(2^{7/3}/9)(r_p/R_0)$ or $-d\sqrt{\beta}/dr > (2^{1/6}/3)(s/R_0q)$, which is well below the ideal-MHD stability boundary. The marginally stable pressure profile is similar in both magnitude and shape to that observed in ohmically heated tokamak plasmas.

Ohmically heated plasmas in high current ($I > 100$ kA), hot ($T_{e0} > 1$ keV) tokamaks are found [1], inexplicably, to typically attain a volume-average plasma pressure that is a small fraction of the poloidal (θ) magnetic field energy — $\bar{\beta}_\theta \equiv \bar{P}/[B_\theta^2(a)/2\mu_0] \sim 0.25 - 0.4$. Because β_θ is a key parameter in magnetohydrodynamics (MHD), this could indicate a transport equilibrium governed by MHD instabilities and turbulence. Also, plasma confinement in tokamaks often seems to scale with ideal MHD stability parameters such as the internal inductance ℓ_i [2]. However, for $\bar{\beta}_\theta < 1$ ideal MHD modes [3]

should be stable and resistive MHD ballooning modes [4] should grow only feebly and induce negligible transport.

A bispectral analysis of Beam-Emission-Spectroscopy (BES) fluctuation data [5] in the Tokamak Fusion Test Reactor [6] (TFTR) indicates that a significant source of fluctuations arises from rapidly growing ($\gamma \gtrsim 2 \times 10^4/s$) modes at low poloidal wavenumbers k_θ ($< 0.5/\text{cm}$). Also, striations observed via visible light imaging [7] in the plasma edge exhibit low k_θ ($\lesssim 1/\text{cm}$) coherent structures that exist for from 15 to 100 μsec . Such rapidly growing, long poloidal wavelength modes are not currently understood and, in particular, not obviously explained by drift-wave-type microinstabilities; rather, they seem to be MHD-like.

Regular MHD modes are stable or only weakly growing in low $\bar{\beta}_\theta$ ($\lesssim 1$) plasmas because the pressure gradient free energy accessible via magnetic field curvature effects is smaller than the perturbed magnetic field energy associated with the field line bending in the modes. The field line bending is due to the $\tilde{\mathbf{B}}_\perp$ caused by the $\tilde{\mathbf{J}}_\parallel$ induced (via an Ohm's law) by a nonzero $\nabla_\parallel \tilde{\phi}$ for ballooning modes localized on the large major radius, "bad" curvature side of a tokamak. Ballooning-type modes could be unstable if the pressure-gradient-term were enhanced (but two-fluid [8] and nonlinear analyses [4] usually indicate reductions), or the field-line-bending term were reduced. We explore the latter avenue. Specifically, within the instability eigenmode we allow electrons to be (on short parallel scale lengths) in the adiabatic

regime $[\omega, \nu_e \ll k_{\parallel} v_{Te}]$ in which $v_{Te} \equiv (2T_e/m_e)^{1/2}$ is the electron thermal speed]. This strongly reduces the $\tilde{J}_{\parallel}$ induced by $\nabla_{\parallel} \tilde{\phi}$, and hence the $\tilde{\mathbf{B}}_{\perp}$ and field-line-bending on short parallel scale lengths. It thus makes possible new, MHD-like instabilities that we call pseudo-MHD ballooning modes.

In this paper we give a derivation of the basic pseudo-MHD equations and ballooning instabilities. A more complete derivation that includes many small effects on these modes (coupling to sound waves, diamagnetic frequency, Landau damping, electron inertia, etc.) is presented in a companion paper [9]. For the pseudo-MHD model we assume the ions respond hydrodynamically (i.e., as a fluid, $\omega/k_{\parallel} \gg v_{Ti}$), but allow the electrons to respond either inertially as a fluid ($\omega \nu_e > k_{\parallel}^2 v_{Te}^2$) or approximately adiabatically ($\omega, \nu_e \ll k_{\parallel} v_{Te}$), depending on the relevant $k_{\parallel}$. Thus, the appropriate equations for the pseudo-MHD model are the electron density and momentum conservation equations, a total plasma momentum balance, and the quasineutral current continuity equation:

$$\partial n / \partial t + \nabla \cdot n \mathbf{V}_e = 0, \quad (1)$$

$$m_e n d\mathbf{V}_e / dt = -ne(\mathbf{E} + \mathbf{V}_e \times \mathbf{B}) - \nabla p_e - \nabla \cdot \underline{\underline{\Pi}}_e + ne\eta \mathbf{J}, \quad (2)$$

$$\rho_m d\mathbf{V} / dt = \mathbf{J} \times \mathbf{B} - \nabla P - \sum_s \nabla \cdot \underline{\underline{\Pi}}_s, \quad (3)$$

$$\nabla \cdot \mathbf{J} = (\mathbf{B} \cdot \nabla)(J_{\parallel}/B) + \nabla \cdot \mathbf{J}_{\perp} = 0, \quad (4)$$

in which n is the electron (or ion) density, $\mathbf{V}_e \simeq \mathbf{V}_{\perp e} - J_{\parallel} \hat{\mathbf{b}}/ne$ the electron flow velocity (the parallel ion flow velocity $\mathbf{V}_{\parallel i}$ is neglected here, but $\mathbf{V}_{\parallel i} \sim$

$\mathbf{V}_{\parallel e}$ is treated in [9]), m_e the electron mass, e the electron charge, $\mathbf{B}$ the vector magnetic field, $p_e \equiv n T_e$ the electron pressure, $P \equiv n(T_e + T_i)$ the total isotropic plasma pressure, Π_s the stress tensor (anisotropic part of pressure tensor) of species s ($= e, i$ for electrons, ions), $\eta \equiv m_e \nu_e / n e^2$ the plasma electrical resistivity, $\rho_m \equiv n(m_i + m_e) \simeq n m_i$ the plasma mass density, $\mathbf{V} \equiv (m_i \mathbf{V}_i + m_e \mathbf{V}_e) / (m_i + m_e) \simeq \mathbf{V}_i$ the total plasma flow velocity, and the $\parallel$ and $\perp$ subscripts on the current indicate components relative to the magnetic field direction [$J_{\parallel} \equiv \hat{\mathbf{b}} \cdot \mathbf{J}$, $\mathbf{J}_{\perp} \equiv -\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \times \mathbf{J})$ in which $\hat{\mathbf{b}} \equiv \mathbf{B}/B$ is a local unit vector along $\mathbf{B}$]. For simplicity here, we assume isothermal, plasma species, i.e., $T_e = \text{constant}$ and $T_i = \text{constant}$, although we allow $T_e \neq T_i$. The fluid through adiabatic response of the electrons is represented by the combination of the $\hat{\mathbf{b}} \cdot \nabla p_e$ and $\hat{\mathbf{b}} \cdot \nabla \cdot \underline{\underline{\Pi}}_{\parallel e}$ terms in the complete parallel electron momentum balance, Eq. (2), using a kinetically derived [10] closure relation, and includes electron Landau damping [9]. Coupling to compressional Alfvén waves is removed by assuming that radial force balance [$\nabla_{\perp}(P + B^2/2\mu_0) \simeq 0$] is maintained by the perturbations, i.e., that $\tilde{B}_{\parallel}/B_0 \simeq -\mu_0 \tilde{P}/B_0 \equiv -(\beta_0/2)(\tilde{P}/P_0)$, which is negligible for the $\bar{\beta}_{\theta} \lesssim 1$ plasmas of interest here.

Note that while Eqs. (1)–(4) are superficially similar to the usual MHD equations, there are some important differences. Specifically, in ideal MHD Eq. (1) is replaced by $\partial \rho_m / \partial t + \nabla \cdot \rho_m \mathbf{V} = 0$, and in resistive MHD Eq. (2) is replaced by the resistive Ohm’s law $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J}$. The additional terms

(particularly $\hat{\mathbf{b}} \cdot \nabla p_e$) in the parallel component of Ohm's law, Eq. (2), coupled with the parallel compressibility $[\nabla_{\parallel} J_{\parallel}]$ in the electron density equation will lead to the pseudo-MHD modifications of the usual MHD modes. Finally, in ideal MHD one uses the MHD fluid isentropic (or “adiabatic”) equation of state $(d/dt)(\ln P/\rho_m^{\Gamma}) = 0$. In pseudo MHD, while the ions have an isentropic equation of state, the electrons do not.

We now proceed to derive the conditions for linear pseudo-MHD instabilities from Eqs. (1)–(4). First, we take the cross product of Eqs. (2) and (3) with $\mathbf{B}$ to obtain the lowest order (in a small gyroradius, low frequency ordering) perpendicular components of the electron flow and current:

$$\mathbf{V}_{\perp e} \simeq (1/B^2)\mathbf{B} \times [\nabla\phi - (T_e/en)\nabla n] \equiv \mathbf{V}_E + \mathbf{V}_{de}, \quad (5)$$

$$\mathbf{J}_{\perp} \simeq [(T_e + T_i)/B^2]\mathbf{B} \times \nabla n + (1/B^2)\mathbf{B} \times \rho_m d\mathbf{V}/dt \equiv \mathbf{J}_d + \mathbf{J}_p, \quad (6)$$

where we have written $\mathbf{E} = -\nabla\phi - \hat{\mathbf{b}}\partial A_{\parallel}/\partial t$ and in which we have identified the $\mathbf{E} \times \mathbf{B}$ ($\mathbf{V}_E$) and electron diamagnetic ($\mathbf{V}_{de}$) flows, and the diamagnetic ($\mathbf{J}_d$) and polarization ($\mathbf{J}_p$) currents. The lowest order perpendicular component of the total flow velocity $\mathbf{V}$ is simply the $\mathbf{E} \times \mathbf{B}$ flow velocity as in ideal MHD: $\mathbf{V}_{\perp} \simeq \mathbf{V}_E$. Next, we substitute these perpendicular flow and current components into Eqs. (1), (4) to obtain more explicit forms of the electron density and current continuity equations:

$$\partial n/\partial t = -\mathbf{V}_E \cdot \nabla n + 2n\kappa \cdot \mathbf{V}_{\perp e} + (\mathbf{B} \cdot \nabla)(J_{\parallel}/eB), \quad (7)$$

$$-(\rho_m/B^2)(d/dt)\nabla_{\perp}^2\phi - 2[(T_e + T_i)/B^2]\kappa \cdot \mathbf{B} \times \nabla n + (\mathbf{B} \cdot \nabla)(J_{\parallel}/B) = 0, \quad (8)$$

in which we have used $\nabla \cdot \mathbf{B} = 0$ and the approximation (for $\beta \ll 1$) $\nabla \cdot (1/B^2)\mathbf{B} \times \nabla f \simeq -2\kappa \cdot (\mathbf{B} \times \nabla f)/B^2$ where $\kappa \equiv (\hat{\mathbf{b}} \cdot \nabla)\hat{\mathbf{b}} \simeq \nabla_{\perp} \ln B$ is the curvature vector of the magnetic field. Note that the electron density conservation equation includes temporal evolution effects due to advection ($\mathbf{V}_E \cdot \nabla n$), and the perpendicular ($\kappa \cdot \mathbf{V}_{\perp e}$) and parallel $[(\mathbf{B} \cdot \nabla)(J_{\parallel}/eB)]$ compressibilities.

Next, we consider the parallel component of the electron momentum balance (the parallel Ohm's law), Eq. (2), which in linearized form (perturbed quantities are indicated by tildes over them and equilibrium quantities by zero subscripts) can be written as

$$\begin{aligned} i\omega m_e \tilde{J}_{\parallel} = & -n_0 e(-\hat{\mathbf{b}} \cdot \nabla \tilde{\phi} - \partial \tilde{A}_{\parallel} / \partial t) - T_{e0}(\hat{\mathbf{b}} \cdot \nabla) \tilde{n} \\ & -[n_0 \hat{\mathbf{b}} \cdot \nabla \tilde{T}_e + \hat{\mathbf{b}} \cdot \nabla \cdot \tilde{\Pi}_{\parallel e}] + n_0 e \eta \tilde{J}_{\parallel}. \end{aligned} \quad (9)$$

In the usual resistive MHD limit the electrons are assumed to be fluidlike ($\omega \nu_e \gg k_{\parallel}^2 v_{Te}^2$) and electron inertia is neglected. Then, using the parallel component of Ampere's law, $\mu_0 \tilde{J}_{\parallel} = -\nabla^2 \tilde{A}_{\parallel}$, the perturbed parallel Ohm's law reduces for wavelike perturbations to simply $\tilde{J}_{\parallel} = -ik_{\parallel} \tilde{\phi} / \eta_{fl}$ in which the effective resistivity is $\eta_{fl} = -i\omega \mu_0 / k^2 + \eta$. In the adiabatic electron regime ($\omega, \nu_e \ll k_{\parallel} v_{Te}$) the parallel electron flow compressibility effect $[T_{e0} \hat{\mathbf{b}} \cdot \nabla \tilde{n} \rightarrow T_{e0}(\hat{\mathbf{b}} \cdot \nabla)^2(\tilde{J}_{\parallel} / (-i\omega e))]$ dominates over the electron inertia and Landau damping effects. Then, using the fact that (neglecting perpendicular compressibility for now) the perturbed form of Eq. (7) in a sheared-slab model becomes $\tilde{n}/n_0 = (\omega_{*e}/\omega)(e\tilde{\phi}/T_e) - (k_{\parallel}/\omega)(\tilde{J}_{\parallel}/en_0)$ in which $\omega_{*e} \equiv \mathbf{k} \cdot \mathbf{V}_{de}$

is the electron diamagnetic flow frequency, we find the parallel Ohm's law can be written to lowest order as $\tilde{J}_{\parallel} = -ik_{\parallel}\tilde{\phi}(1 - \omega_{*e}/\omega)/\eta_{ad}$ in which the effective resistivity is

$$\eta_{ad} \simeq -i\omega\mu_0/k^2 + (m_e/n_0e^2) \left[\nu_e + (k_{\parallel}^2 v_{Te}^2/2)/(-i\omega) \right]. \quad (10)$$

The parallel compressibility effects dominate over the ideal MHD effects (the first term) in η_{ad} when

$$1 \ll (kc/\omega_{pe})^2 (k_{\parallel}^2 v_{Te}^2/2\omega^2) = (k\rho_s)^2 (k_{\parallel}^2 c_A^2/\omega^2) \quad (11)$$

in which $\omega_{pe} \equiv (n_0e^2/m_e\epsilon_0)^{1/2}$ is the electron plasma frequency, $\rho_s \equiv c_s/\omega_{ci}$ sound gyroradius, $c_s = (T_e/m_i)^{1/2}$ sound speed, $\omega_{ci} = eB/m_i$ ion gyrofrequency and $c_A = B/(\mu_0 n_0 m_i)^{1/2}$ Alfvén speed. When (11) is satisfied, the electron response becomes adiabatic in the presence of $\tilde{A}_{\parallel}$ effects and the field line bending is reduced. In particular, we anticipate that condition (11) will apply to short-scale (along $\mathbf{B}$), adiabatic phenomena, and significantly reduce the field-line-bending stabilization effects, while normal fluidlike descriptions will apply over longer scale lengths where $k_{\parallel}$ is smaller.

Next, we explore pseudo-MHD ballooning modes in a toroidal tokamak plasma taking into account these parallel electron compressibility effects. We use the ballooning-mode formalism [3] to represent toroidal mode number n perturbations in a circular cross-section tokamak in the form

$$\tilde{\phi}(\mathbf{x}, t) = \sum_{\ell} \hat{\phi}(\theta + 2\pi\ell) \exp[inq'x(\theta + 2\pi\ell) - in\zeta - i\omega t] \quad (12)$$

in which $q' \equiv dq/dr$ and ζ is the toroidal (axisymmetry) angle. For such perturbations and an approximate ($\beta \lesssim \epsilon^2$, $\epsilon \equiv r/R_0 \ll 1$) magnetic field, $\mathbf{B} \simeq B_0(R_0/R) [\hat{\zeta} + (r/R_0 q)\hat{\theta}] = B_\zeta \hat{\zeta} + B_\theta \hat{\theta} \equiv B \hat{\mathbf{b}}$, $R_0/R \simeq 1 - (r/R_0) \cos \theta$, in which r, R_0 are the minor, major radii of a magnetic flux surface, $q(r) \equiv r B_\zeta / R_0 B_\theta$ is the “safety factor,” $\hat{\theta} \cdot \nabla \rightarrow ik_\theta \equiv im/r$, $\hat{\mathbf{r}} \cdot \nabla \rightarrow ik_r = ik_\theta s \theta$, $\nabla_\perp^2 \rightarrow -k_\perp^2 = -k_\theta^2(1 + s^2 \theta^2)$ and $\hat{\mathbf{b}} \times \nabla \hat{\phi} \cdot \kappa = -ik_\theta \hat{\phi} \tilde{\kappa}(\theta)/R_0$, $\nabla_\parallel \equiv (R_0 q/B) (\mathbf{B} \cdot \nabla \theta) (d/d\theta) \simeq d/d\theta$ in which $s \equiv (r/q)(dq/dr)$ is the standard magnetic shear parameter and

$$\tilde{\kappa}(\theta) = \cos \theta + s \theta \sin \theta \quad (13)$$

is a normalized curvature function that includes both normal ($\cos \theta$) and geodesic ($\sin \theta$) curvature effects. Using these standard ballooning mode formalism results in the linearized forms of Eqs. (7-9), we find that, neglecting electron inertia and Landau damping effects, the governing equations can be written in dimensionless form as (assuming $\omega_{*e}/\omega \ll 1$ for simplicity)

$$\hat{N} = (\omega_{*e}/\omega) \hat{\Phi} - (\omega_{De}/\omega) \tilde{\kappa}(\theta) (\hat{\Phi} - \hat{N}) - (\omega_\parallel^2/\omega^2) \nabla_\parallel \hat{J}, \quad (14)$$

$$[1 + i(\nu_R/\omega)(1 + s^2 \theta^2)] \hat{J} = - (1 + s^2 \theta^2) \nabla_\parallel (\hat{\Phi} - \hat{N}), \quad (15)$$

$$-(\omega^2/\omega_A^2)(1 + s^2 \theta^2) \hat{\Phi} - \alpha \tilde{\kappa}(\theta) (\omega/\omega_{*e}) \hat{N} + \nabla_\parallel \hat{J} = 0, \quad (16)$$

where $\hat{N} \equiv \tilde{n}/n_0$, $\hat{\Phi} \equiv e\tilde{\phi}/T_e$, $\hat{J} \equiv (-i\omega)(eR_0 q \mu_0/k_\theta^2 T_e) \tilde{J}_\parallel$, $\nu_R \equiv k_\theta^2 \eta/\mu_0$, $\omega_{*e} \equiv -k_\theta(T_e/eB_0)(d \ln n_0/dr)$, $\omega_{De} = -2k_\theta(T_e/eB_0)/R_0$, $\omega_A \equiv c_A/R_0 q$, $\alpha \equiv -R_0 q^2 d\beta/dr (> 0)$, $\beta \equiv n_0(T_e + T_i)/(B^2/2\mu_0)$ and

$$\omega_\parallel^2 \equiv \frac{k_\theta^2 T_e}{n_0 e^2 \mu_0 R_0^2 q^2} = \left(k_\theta \frac{c}{\omega_{pe}} \right)^2 \left(\frac{v_{Te}^2}{2R_0^2 q^2} \right) = k_\theta^2 \rho_s^2 \omega_A^2 \quad (17)$$

is the characteristic frequency for the parallel compressibility effects. Note that condition (11) corresponds to $1 \ll (k^2/k_\theta^2) (\omega_\parallel^2/\omega^2) = (1+s^2\theta^2) (\omega_\parallel^2/\omega^2)$. When compressibility effects are neglected ($\omega_\parallel \rightarrow 0$), ideal [3, 11] and resistive [4, 11] MHD ballooning instabilities can be derived [9] from Eqs. (14) - (16).

In order to develop an approximate solution of Eqs. (14-16), we use a multiple-length-scale expansion technique [4, 11]. Specifically, we assume

$$\hat{\Phi} = \bar{\Phi}(z) + \tilde{\Phi}(z)\tilde{\kappa}(\theta), \quad \hat{J} = \bar{J}(z) + \tilde{J}(z)\nabla_\parallel\tilde{\kappa}(\theta), \quad (18)$$

with $\hat{N}$ expanded similarly to $\hat{\Phi}$. Here z is a long scale poloidal angle variable along $\mathbf{B}$ while θ is the corresponding short scale ($-\pi < \theta < \pi$) variable, both in the ballooning mode representation. Substituting expansions (18) into Eqs. (14-16) and subtracting the θ -average [$\langle f(\theta, z) \rangle \equiv (1/2\pi) \int_{z-\pi}^{z+\pi} d\theta f(\theta, z) \equiv \bar{f}(z)$] of the equations from the original equations, we obtain to lowest order the following short-scale and long-scale equations:

$$\begin{aligned} \tilde{N} &= (\omega_{*e}/\omega)\tilde{\Phi} - (\omega_{De}/\omega)(\tilde{\Phi} - \tilde{N}) + (\omega_\parallel^2/\omega^2)\tilde{J}, \\ [1 + i(\nu_R/\omega)(1 + s^2z^2)] \tilde{J} &= - (1 + s^2z^2)(\tilde{\Phi} - \tilde{N}), \\ - (\omega^2/\omega_A^2)(1 + s^2z^2)\tilde{\Phi} - \alpha(\omega/\omega_{*e})\tilde{N} - \tilde{J} &= 0; \end{aligned} \quad (19)$$

$$\begin{aligned} \bar{N} &= (\omega_{*e}/\omega)\bar{\Phi} - (\omega_{De}/2\omega)(\bar{\Phi} - \bar{N}) - (\omega_\parallel^2/\omega^2)(d\bar{J}/dz), \\ [1 + i(\nu_R/\omega)(1 + s^2z^2)] \bar{J} &= - (1 + s^2z^2)(d/dz)(\bar{\Phi} - \bar{N}), \\ - (\omega^2/\omega_A^2)(1 + s^2z^2)\bar{\Phi} - (\alpha/2)(\omega/\omega_{*e})(1 + s^2z^2)\bar{N} + (d\bar{J}/dz) &= 0. \end{aligned} \quad (20)$$

In obtaining these equations, we have used the facts that $\nabla_{\parallel}^2 \tilde{\kappa} \simeq -\tilde{\kappa}$, $\langle \tilde{\kappa}^2 \rangle \simeq (1 + s^2 z^2)/2$. The average normal magnetic curvature $\bar{\kappa}$ is higher order in ϵ and has been neglected.

For the pseudo-MHD modes we explore the effects of finite parallel electron compressibility, i.e., $\omega_{\parallel} \neq 0$. In particular, we go to large z ($s^2 z^2 \gg 1$) where we assume $s^2 z^2 (\omega_{\parallel}^2 / \omega^2) \gg 1$, but allow $(\nu_R / \omega)(1 + s^2 z^2) \sim 1$. Then, assuming $1 \gg \omega_{*e} / \omega$ we obtain from Eqs. (19) a nearly adiabatic ($\tilde{N} \simeq \tilde{\Phi}$) short scale electron response with $\tilde{J}$ limited by $\omega_{\parallel}$:

$$\tilde{J} \simeq \frac{\omega^2}{\omega_{\parallel}^2} \left[\tilde{\Phi} + \frac{\omega_{De}}{\omega} (\tilde{\Phi} - \tilde{N}) \right], \quad \tilde{N} \simeq - \frac{\alpha (\omega_{\parallel}^2 / \omega \omega_{*e}) \tilde{N} + (\omega_{De} / \omega) (\tilde{\Phi} - \tilde{N})}{1 + (\omega_{\parallel}^2 / \omega_A^2) (s^2 z^2)}. \quad (21)$$

The small, but crucial, short-scale potential variation $\tilde{\Phi}$ is driven, via the short-scale current continuity equation, by the long-scale pressure-gradient free energy ($\sim \alpha \tilde{N}$) made accessible through the geodesic curvature. Parallel compressibility limits the $\tilde{J}$ induced by $\tilde{\Phi}$ and hence leads to a much larger $\tilde{N}$ due to $\tilde{N}$ than is obtained in resistive ballooning modes.

In the same limits the average or long scale equations yield, to lowest order [assuming $(\omega_{\parallel}^2 / \omega^2) (d\tilde{J}/dz) < \tilde{N}$, $(\omega_{*e} / \omega) \tilde{\Phi}$, which requires $s^2 \omega_{\parallel}^2 / \omega^2 \ll 1$], regular MHD-like advective density and parallel Ohm's law responses:

$$\tilde{N} \simeq \frac{\omega_{*e}}{\omega} \tilde{\Phi}, \quad \tilde{J} \simeq - \frac{s^2 z^2}{1 + i(\nu_R / \omega) s^2 z^2} \frac{d\tilde{\Phi}}{dz}. \quad (22)$$

Using Eqs. (21, 22) in the average current-continuity equation, we then

finally obtain (for $s^2 z^2 \gg 1$, $\nu_R \omega / \omega_{\parallel}^2 \ll 1$) for $\bar{\Phi}(z)$:

$$\frac{d}{dz} \frac{s^2 z^2}{1 + i(\nu_R/\omega)s^2 z^2} \frac{d\bar{\Phi}}{dz} + s^2 z^2 \left[\frac{\omega^2}{\omega_A^2} - \frac{(\alpha^2/2)(\omega_{\parallel}^2/\omega\omega_{*e}) - \alpha\epsilon_n}{1 + (\omega_{\parallel}^2/\omega_A^2)s^2 z^2} \right] \bar{\Phi} = 0 \quad (23)$$

in which $\epsilon_n \equiv \omega_{De}/2\omega_{*e} = r_n/R_0$, $1/r_n \equiv d \ln n_0(r)/dr (< 0)$. Note that the effective $k_{\parallel}$ in the field-line-bending (first) term can be small since it only involves the long-scale variation in $\bar{\Phi}$ — the short-scale length effects were eliminated by the adiabatic-regime response there. The combination of parallel electron compressibility and geodesic curvature have yielded a large (for $s^2 z^2 \gg 1$), destabilizing (for $\omega\omega_{*e} < 0$) pressure-gradient-driven (α^2) term; the $\alpha\epsilon_n$ term represents the perpendicular electron compressibility effects which strongly stabilize the much weaker resistive MHD ballooning modes [4], but will only provide a minimum $k_{\theta}\rho_s$ for the more robust pseudo-MHD instabilities we will obtain.

The differential equation in Eq. (23) has a potential well structure for $1 \ll s^2 z^2 < \omega_A^2/\omega_{\parallel}^2 = (k_{\theta}\rho_s)^{-2}$. To obtain estimates of its localized eigenmodes ($\bar{\Phi} \rightarrow 0$ as $|z| \rightarrow \infty$) most simply and accurately, we multiply by $\bar{\Phi}^*$ and integrate over z from $-\infty$ to ∞ to obtain the quadratic form

$$\int_{-\infty}^{\infty} dz \left\{ -\frac{\omega_A^2 s^2 z^2}{1 + i(\nu_R/\omega)s^2 z^2} \left| \frac{d\bar{\Phi}}{dz} \right|^2 + s^2 z^2 \left(\omega^2 + \frac{\gamma_p^3/\omega - \omega_1^2}{1 + (k_{\theta}\rho_s)^2 s^2 z^2} \right) |\bar{\Phi}|^2 \right\} = 0 \quad (24)$$

in which we have defined (for $\omega_{*e} < 0$) a pseudo-MHD growth rate parameter

$$\gamma_p = \left[\frac{\alpha^2 \omega_{\parallel}^2 \omega_A^2}{2 |\omega_{*e}|} \right]^{1/3} = \omega_A \alpha^{1/2} \left(\frac{r_p}{2R_0} \right)^{1/6} |k_{\theta}\rho_s|^{1/3} = c_s \left[\frac{2 |k_{\theta}\rho_s|}{r_p R_0^2} \right]^{1/3}, \quad (25)$$

and the perpendicular compressibility frequency $\omega_{\perp}^2 \equiv -\alpha\epsilon_n\omega_A^2 = 2c_s^2/R_0^2$.

Asymptotically (for z outside the well turning points, but $szk_{\theta}\rho_s < 1$), the solutions of Eq. (23) behave as $e^{-z^2/2\sigma^2}$. Thus, we assume a test function of the form $\bar{\Phi}(z) = \Phi_0 e^{-z^2/2\sigma^2}$ in Eq. (24) and obtain (neglecting ν_R effects)

$$-(3/2)(\omega_A^2/\sigma^2) + \omega^2 + (\gamma_p^3/\omega - \omega_{\perp}^2)[1 - (3/2)\sigma^2 s^2 (k_{\theta}\rho_s)^2 + \dots] = 0. \quad (26)$$

The variational parameter σ is obtained by setting the differential of this equation with respect to σ^2 to zero; for localized eigenmodes ($\text{Re } \sigma > 0$) this yields (neglecting $\omega_{\perp}$)

$$\sigma_p^2 = (\omega\omega_A^2/\gamma_p^3)^{1/2}/(sk_{\theta}\rho_s) \simeq \omega_A/(\gamma_p sk_{\theta}\rho_s) \gg 1. \quad (27)$$

Substituting this result into Eq. (26) and neglecting $\sigma_p^2 s^2 (k_{\theta}\rho_s)^2$ compared to unity we obtain

$$\omega[\omega^2 - 3\omega_A^2/2\sigma_p^2 - \omega_{\perp}^2] = -\gamma_p^3. \quad (28)$$

This basic pseudo-MHD mode dispersion relation is cubic in ω and admits unstable ($\text{Im } \omega > 0$) roots of the form (asymptotically)

$$\omega = e^{i\pi/3}\gamma_p = (1 + i\sqrt{3})\gamma_p/2 \quad (29)$$

for $\gamma_p^2 > (4/27)^{1/3}(\omega_{\perp}^2 + 3\omega_A^2/2\sigma_p^2)$. The unstable pseudo-MHD ballooning modes have growth rates of $(\sqrt{3}/2)\gamma_p$ and positive real frequencies $(\gamma_p/2)$ which indicates they propagate in the ion diamagnetic flow direction ($\omega_{*e} <$

0). Taking account of the various conditions used in the derivation we find that these new modes are unstable in the narrow poloidal wavenumber range

$$(8/27)^{1/2}(r_p/R_0) < k_\theta \rho_s < \alpha^{3/4}(r_p/2R_0)^{1/4}/s^{3/2} \quad (30)$$

in which the lower limit results from the condition $\gamma_p^2 > (4/27)^{1/3}\omega_\perp^2$ and the upper limit results from the condition $s^2\omega_\parallel^2/\omega^2 < 1$ which is required for fluidlike or hydrodynamic responses on the long ($z \sim \sigma_p \gg 1$) scale length. A range of unstable k_θ exists for

$$\alpha > s^2(2^{7/3}/9)(r_p/R_0), \text{ or } -d\sqrt{\beta}/dr > (2^{1/6}/3)(s/R_0q). \quad (31)$$

Note that since usually $r_p/R_0 \ll 1$, the pseudo-MHD modes are unstable at much lower α (or β) than ideal MHD ballooning modes whose instability condition [3, 11] for $s \gtrsim 1$ is $\alpha \gtrsim 0.6s$. Inclusion [9] of electron and ion diamagnetic flow effects and the coupling to ion sound waves does not significantly change Eq. (28) and hence Eq. (29) — because under the conditions given by (30) and (31) we have $\gamma_p \gg \omega_{*e}$, $\omega_s \equiv c_s/R_0q$.

Plasma resistivity influences pseudo-MHD ballooning modes primarily through its reduction of the field-line bending term [first one in Eqs. (23, 24)] and is characterized by the parameter

$$C_R \equiv \frac{\nu_R}{\gamma_p} \sigma_p^2 s^2 = \frac{(\eta/\mu_0)}{(\omega_A \rho_s^2)(\alpha/s)} \left(\frac{k_\theta \rho_s}{r_p/2R_0} \right)^{1/3} \geq \frac{(\eta/\mu_0)(27/32)^{1/2}}{(\omega_A \rho_s^2)(sr_p/2R_0)}. \quad (32)$$

Resistivity effects are important for $C_R > 1$, in which case the axial extent of the mode is reduced ($\sigma_p \rightarrow \sigma_p/C_R^{1/6}$) and the condition for a hydrodynamic

response on the long scale is changed from $s^2\omega_{\parallel}^2/\omega^2 \ll 1$ to $\omega_{\parallel}^2/(\omega\nu_R\sigma_p^2) \ll 1$. Then, the upper $k_{\theta}\rho_s$ limit in (30) is increased by a factor of $C_R^{3/4}$; however, diamagnetic flow effects are now important for the higher k_{θ} unstable modes with $k_{\theta}\rho_s \gtrsim \sqrt{2}(r_p/R_0)(r_n/r_p)^{3/2}$. Thus, when $C_R \gg 1$ there would be regular (ω_{*i} direction) pseudo-MHD ballooning modes in the $k_{\theta}\rho_s$ range indicated in (30), and, for $C_R > (27/4)^{2/3} \simeq 3.6$, unstable higher $k_{\theta}\rho_s$ drift-wave-type instabilities, which can propagate in the electron diamagnetic flow direction if $T_i \gg T_e$ [9].

In order to illustrate the typical parameters of pseudo-MHD ballooning instabilities, we evaluate them for parameters typical of those where BES fluctuations were analyzed [5] on TFTR: $B_T \simeq 4.8$ T, $I \simeq 1.6$ MA, $R_0 \simeq 2.45$ m, $a \simeq 0.8$ m, $q_a \simeq 3.7$, $q(r/a) \simeq 2.5$, $s \sim 1.4$, $T_e \sim T_i \sim 1$ keV, $n_e \simeq 3 \times 10^{19}$ m $^{-3}$, deuterium ions, $Z_{eff} \sim 2$, $r_n \sim 0.24$ m, $r_p \sim 0.12$ m. (Here, we use that the fact [9] that the temperature gradients contribute to the pressure gradient in the determination of r_p .) These parameters yield $\nu_e \sim 10^5$ /s, $\eta/\mu_0 \simeq 0.1$ m 2 /s, $\beta \simeq 0.001$, $\alpha \simeq 0.13$, $c_s \simeq 3 \times 10^5$ m/s (using the more precise [9] $c_s = \sqrt{(T_e + T_i)/m_i}$), $\rho_s \simeq 0.13$ cm, $\omega_{\perp} \simeq 1.7 \times 10^5$ /s, $c_A \simeq 1.4 \times 10^7$ m/s, $\omega_A \simeq 2.2 \times 10^6$ /s, $C_R \simeq 0.7$, and indicate unstable modes for $0.027 < k_{\theta}\rho_s < 0.051$, which implies poloidal mode numbers $m \equiv rk_{\theta}$ in the range $12 < m < 22$. For a typical $k_{\theta}\rho_s \simeq 0.04$ ($m \simeq 17$) we obtain $\omega_{\parallel} = 8.8 \times 10^4$ /s, $\gamma_p \simeq 0.066 \omega_A \simeq 1.5 \times 10^5$ /s and $\sigma_p \simeq 16$, which indicates an extension of the mode along a field line (to the eigenmode half-amplitude

point) of $16R_0q \simeq 100$ m, or about 7 toroidal transits of TFTR. (For these same parameters $-\omega_{*e} \simeq \omega_{*i} \simeq 2.7 \times 10^4/\text{s}$ and $\omega_s \equiv c_s/R_0q = 5 \times 10^4/\text{s}$, all of which are small compared to γ_p and hence negligible.)

Thus, pseudo-MHD ballooning instabilities have substantial, MHD-like growth rates ($1/\gamma \sim 17/\omega_A \sim 8 \mu\text{sec}$) in the very low $k_\theta \rho_s$ range (< 0.05) where the BES bispectral analysis [5] infers strongly unstable modes that are a source of the fluctuations whose amplitude peaks at higher $k_\theta \rho_s$ (~ 0.1). Further, since C_R scales as $T_e^{-5/2}$, we would expect a transition to resistive pseudo-MHD modes for lower values of T_e , and the appearance of drift-wave-type modes for $T_e < 1 \text{ keV}/(3.6/0.7)^{2/5} \sim 0.5 \text{ keV}$. Experimentally, an abrupt transition from “core ion mode” fluctuations to edge ion and fast electron (drift-wave-type?) modes was observed in BES fluctuations [12] for $r/a > 0.88$ or $T_e \lesssim 0.5 \text{ keV}$. Thus, pseudo-MHD ballooning modes seem to exhibit a number of characteristics that are quite similar to the properties of fluctuations observed in TFTR by BES [5, 12]. Also, the resistive version of the pseudo-MHD ballooning modes could be responsible for the striations observed [7] in the edge of tokamak plasmas.

Since pseudo-MHD ballooning modes have large growth rates and scale lengths, when unstable they could cause substantial anomalous transport. Hence, the conditions for marginal stability are of interest. Integrating the second criterion in (31) assuming $\beta(a) = 0$, we find the β or pressure profile

at marginal stability is

$$\beta_m(r) = \frac{2^{1/3}}{9R_0^2} \left[\int_r^a dr' \frac{s(r')}{q(r')} \right]^2 = \frac{2^{1/3}a^2}{9R_0^2q_a^2} \left[q_a \int_{q(r)}^{q_a} dq \frac{r(q)/a}{q^2} \right]^2. \quad (33)$$

For relevant $q(r)$ profiles, this produces pressure profiles similar in magnitude and shape to those observed [1] in hot ($T_{e0} \gg$ critical $T_e \sim 0.5$ keV where $C_R = 1$) ohmic tokamaks. Taking the (cylindrical) volume average of $\beta_m(r)$ for $q(r) = q_0 + \Delta q(r/a)^n$ yields $\bar{\beta}_{\theta m} \simeq 0.1, 0.2$ for $n = 2, 3$, respectively. Therefore $\bar{\beta}_{\theta m}$ is apparently slightly below the $\bar{\beta}_\theta \sim 0.25 - 0.4$ typically observed [1] in hot ohmic tokamaks. Also, since $\bar{\beta}_{\theta m}$ increases with the q profile index n and the magnetic shear s over the larger volume outer half of the plasma [see (31), (33)], it increases with the peakedness of the current profile and hence the plasma internal inductance ℓ_i . This trend is also in accord with experimental observations [2] of the empirical scaling of plasma stored energy.

Thus, we conjecture that pseudo-MHD ballooning modes may be responsible for the fluctuations and anomalous transport that lead to the transport equilibrium in ohmically heated tokamak plasmas, and could cause the observed degradation in plasma energy confinement [13] with increasing auxiliary heating.

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