Output from an ARGUS electromagnetic simulation of the TFTR Bay-M ICRH antenna at PPPL. Shown are two rows of Faraday bars, the current strap and drive port, the bumper tiles, and the vacuum electric field strength.

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1. **Abstract**

SAIC has undergone a three year research and development program in support of the DOE Office of Fusion Energy's (OFE) program in Ion Cyclotron Range of Frequencies (ICRF) heating of present, next generation, and future plasma fusion devices. The effort entailed advancing theoretical models and numerical simulation technology of ICRF physics and engineering issues associated predominately with, but not limited to, tokamak Ion Cyclotron Heating (ICH) and fast wave current drive (FWCD).

Ion cyclotron heating and current drive (ICH/CD) is a central element in all current and planned large fusion experiments. In recent years, the variety of uses for ICRF systems has expanded, and includes the following:

1. Heating sufficient to drive plasma to ignition.
   - Second-harmonic T heating (main heating mode).
   - He³ minority heating.
2. Second-harmonic He⁴ heating in H plasma (for non-activated phase).
3. Detailed equilibrium profile control minority heating.
   - Ion minority (He³) CD (for profile control on inside of plasma).
   - Ion minority (He³) CD (for profile control on outside of plasma).
4. Ion-ion hybrid regime majority ion heating.
5. Electron current drive.
6. Mode conversion to drive current.
7. Deuterium minority heating.
8. Sawtooth instability stabilization.
10. The generation of minority tails by ICRF to simulate D-T plasma particle physics in a deuterium plasma.

Optimization of ICRF antenna performance for either heating or current drive depends critically on the complex balance and interplay between the plasma physics and the electro-mechanical system requirements. For example, ITER IC rf designs call for an IC system frequency range from 20 MHz to 100 MHz. Additionally, antenna designs and operational modes that minimize impurity production and induced sheath formation may degrade current drive efficiency. Such effects have been observed in experiments involving π versus zero antenna phasing. Solutions to these kinds of technical problems – especially important for a reactor grade antenna where bandwidth and survivability become key issues – require an approach concurrently integrating theory, numerically-aided design through modeling and simulation, and experimental investigations.
SAIC's efforts in helping to formulate approaches to address the above issues have focused on two interdependent plasma physics areas: numerical modeling and simulation of ICRF antenna performance including antenna-plasma coupling, and sheath physics in the neighborhood of antennas with emphasis on Faraday shields and current strap sheaths. SAIC has achieved the main goals of the work, those being the following:

1. The development of numerical simulation technology, using ARGUS, to provide a predictive capability for evaluating antenna performance, including radiated field and power spectral distributions and plasma-antenna coupling;
2. An improved understanding of sheath physics phenomenology in the vicinity of ICRF addressing the effects of the magnetic field, two-dimensional physics, and sheath saturation;
3. To work towards the incorporation of the microscopic sheath effects into the macroscopic plasma/antenna performance predictions to be incorporated into the ARGUS numerical rf antenna model as well as to be used as a stand-alone analysis tool.

These goals have helped SAIC to formulate an approach to the development of a modeling and simulation environment in which the important technical questions affecting ICRF systems can be analyzed and evaluated. In particular, these goals work toward the inclusion of realistic macroscopic sheath contributions into the global ICRF ARGUS antenna simulations that reflect microscopic features of sheath dynamics on spatial scales too small to be resolved in a global 3-D simulation.

The report is organized as follows: Section 2 presents a brief overview of the status of ICH and current drive, and identifies some of the most important open problems amenable to theoretical modeling and simulation. Section 3 reviews SAIC's past work under the current OFE contract on ICRF antenna and rf sheath modeling. Additionally, this section details work done both by SAIC during the multi-year effort of this contract, as well as collaborations with members of the US fusion community. These members include ORNL, Lodestar Research Corp., PPPL, ITER US Home Team, GA, and UW. Section 4 provides a summary of the effort, giving an overall picture of the work accomplished. Section 4 also describes recommended future work and objectives in ICRF antenna modeling and simulation. For this, an organized and detailed presentation is given describing a plan that will fulfill the recommendations. Section 5 provides a list of references (some of which are reproduced in the Appendices). Finally, Appendices A-K
provide reprints of relevant papers and reports which include SAIC's past work in ICRF antenna modeling and simulation.
2. ICRF Application to Fusion Devices: Background

In this section, broad issues are discussed regarding ICRF applications and status in the fusion community. In particular, Sec. 2.1 discusses the current status of ICRF heating and current drive experiments, Sec. 2.2 identifies antenna and sheath-physics problem areas, and, finally, Sec. 2.3 reviews related ICRF theory efforts.

2.1. Status of ICRF Heating and Current Drive Experiments

Results of ICH experiments and/or analyses carried out at Culham (JET), Princeton (TFTR), GA (DIII-D), U. of Wisconsin (PHAEDRUS T), ORNL, Lodestar, SAIC, MIT (Alcator-C-Mod), CIT, ITER, and elsewhere over the past several years have shown significant improvement in ICRF antenna design, construction, and application to tokamak heating and current drive. In addition, our understanding of ICH has advanced to the point where good qualitative and quantitative comparisons have been obtained between theory, modeling and simulation, and experimental results. Some of the most relevant results, beginning with those reported at the 1992 Fifth Boulder workshop on “High power ICRF Antenna Design and Physics,” are briefly summarized below.

In general, Ion Cyclotron Heating (ICH) via the Fast Magnetosonic mode has been established as an efficient method for depositing tens of MW of auxiliary power into the plasma core over long pulses. In particular, ICH for both ignition and steady-state operation, as is possible for ITER, is an area of much interest. ICH has been successfully applied to H-mode, L-mode and supershot (TFTR) discharges. It has been combined with pellet refueling and neutral beam injection, and has been used for tokamak start-up without the main transformer. Antenna matching to the plasma impedance has been maintained using feedback loops for the rf frequency and current strap phase control. Minority ion and electron current drive has been demonstrated and utilized for current profile shaping and sawtooth suppression. Combining the application of FWCD with Lower Hybrid Current Drive (LHCD) demonstrated a synergetic efficiency enhancement in the observed current drive. Additionally, enhancement of alpha particle parameters and simulation of D-T plasma alpha particle physics in deuterium plasmas have been studied.

In the US, TFTR and DIII-D experiments have focused on ICH application to supershots, sawtooth stabilization and FWCD. Major emphasis has also been placed on impurity control. ORNL and the University of Wisconsin are optimizing Faraday screen designs to reduce induced plasma sheath formation and have studied screen-less operation.
In Europe, the JET team has demonstrated good continuous antenna coupling in a tokamak discharge using active feedback techniques for plasma positioning and automatic impedance matching. JET and TFTR have included tilted Faraday rods to better align with the total magnetic field used for normal plasma operation. JET results have also shown that impurity production can be minimized by the use of low Z materials (Be, B, B4C) on the plasma exposed surfaces, using angled Faraday screens, and phasing the antenna currents in the two straps 180° apart (π-phasing). Successful operation without Faraday screens has been reported in TEXTOR and has been attempted in DIII-D. Sheath formation has been suppressed in PHAEDRUS-T by applying BN coating on the side limiters and the antenna feeders.

Summarizing the results of recent achievements with respect to identifying areas of study that are suitable for theoretical modeling and simulation, two general problem areas emerge that SAIC has been actively pursuing: realistic antenna modeling and performance simulation, including realistic plasma-response; and sheath physics phenomena that affect antenna operation performance and impurity release. The following is a short overview of these two problem areas, with specific reference to SAIC’s work in each.

Realistic and technically useful ICRF modeling and simulation must accommodate the integrated three dimensional structure of the antenna including feeds, current straps, side walls, limiters, bumper tiles, internal support structures, Faraday screens, vessel walls, and the plasma. Existing 2-D codes and 3-D codes based on idealized antenna geometry and plasma profiles have been used in the past to address various aspects of antenna performance. SAIC’s 3-D electromagnetic simulation code, ARGUS, has been used to analyze ICRF antenna performance in vacuum scenarios, as well as in scenarios including a self-consistent plasma interface. ARGUS simulation results were compared against experimental results from benchtop mockups of the Bay-M and L antennas on TFTR, predicting correctly the effects of geometric details, such as the presence of side wall slits, on the vacuum field spectrum. Additionally, experimental benchmarks were made on TFTR Bay-M comparing 0° and π antenna phasings. A plasma loading benchmark was made on the ITER violin ICH antenna design comparing ARGUS results with those predicted using the MAntIS code. One notable achievement was the incorporation of ARGUS into the Lodestar ANSAT (Antenna Sheath Analysis Tool) code, where ARGUS provided the detailed field patterns of the antennas in the vicinity of the Faraday rods, to be input into the ANSAT model. In this case, ARGUS plays a core role in the ANSAT analysis suite. One of the other promising designs is that of the folded waveguide (FWG), which is considered for several ICH designs. ARGUS has been used
in both the frequency-domain eigenvalue mode as well as the single, driven-frequency mode to model the antenna design characteristics of the FWG.

In this contract, a single-frequency solver with self-consistent volumetric cold-plasma response (as opposed to the surface plasma-impedance model of the previous single, driven-frequency model) has been added to the ARGUS simulation code resulting in a brand new technology for ICRF antenna performance evaluation. Preliminary results obtained with the new simulation capability using model plasma profiles are very encouraging and show that realistic 3-D plasma/antenna coupling studies can, for the first time ever, now be carried out.

The study of macroscopic plasma sheaths induced by antenna launched rf is crucial to understanding impurity production and control. Predictions from 1-D theoretical models have been successfully used to optimize Faraday screen design. For 1-D sheath physics, analysis in the present contract has shown that the behavior of the sheath immersed in a tilted magnetic field dramatically increases the sheath potential. This contract has also shown that 2-D physics is very important for sheaths forming between adjacent Faraday bars (as compared with the unmagnetized sheath case), and must be studied further to properly account for loading and impurity effects. Presently, the Lodestar ANSAT code uses ARGUS-calculated fields as an input. ANSAT has prescribed sheath physics models and has been used to analyze the TFTR Bay-M antenna, providing intriguing results. The need to continually improve the sheath models in the ANSAT tool suite, and other models like it, is important for future low-cost efficient antenna design capabilities.

An improved understanding of sheath physics is also necessary to evaluate ICH performance without Faraday screens; this is an important issue currently under investigation for future reactor size devices. An understanding of sheath physics on the microscopic level is also required to improve our macroscopic modeling of ICH. Microscopic sheath behavior is needed to:

1. Model the overall wave propagation characteristics through the Faraday screen in terms of transmission, reflection, dissipation and mode conversion coefficients.
2. Prescribe the correct boundary conditions at plasma-wall interfaces, allowing the inclusion of sheath effects on large scale simulations without having to resolve the sheath spatial scales.
2.2. ICRF Antenna and Sheath-Physics Problem Areas

Since the 1992 Boulder workshop, a consensus has been shaped as to the most important problems facing ICH and current drive in tokamak plasmas. Among those problems discussed, the following specific areas are well suited for SAIC’s research program:

1. Development of a self consistent, 3-D simulation capability of the plasma/antenna coupling problem, including volumetric plasma response. The development of such a technology will help antenna designers optimize rf power spectra, coupling, and minimization of undesirable rf fields that cause sputtering, arcing, and possible convective cell formation. Effects of nearby objects such as limiters can also be evaluated.

2. Determination of the physics and dynamics of 1-D and 2-D rf-induced sheaths. The presence of non-aligned magnetic fields has been shown to significantly modify the 1-D sheath potential, as compared to the unmagnetized sheath limit. The understanding of two-dimensional effects is essential for the evaluation of rf induced sheaths and impurity production between the Faraday bars.

3. Development of a theoretical model that governs the macroscopic antenna Faraday screen performance, and accounts for wave attenuation, scattering, and possible harmonic generation driven by the sheath physics on the microscopic, sheath spatial scale.

4. Determination of appropriate techniques and boundary conditions that will allow a realistic treatment of sheath effects in large scale simulations without the need to resolve the sheath spatial scales.

5. Creation of a real antenna-design environment, capability, and technology, including a suite of antenna analysis tools that are user-friendly, transition to the US fusion community, and are supported.

2.3. Review of Related Theory Efforts

Until recently, theoretical modeling and simulation of ICRF heating and current drive phenomenology have generally been carried out by considering the total problem to be decomposable into two separately analyzed parts: one that concentrates on the absorption and other such effects that take place inside the plasma, \(^1-5\) and one that deals primarily with the electromagnetic coupling of the antenna to the plasma. \(^6-11\) The first kind of analysis, usually making use of full wave (2-D, toroidally axisymmetric) codes \(^1-4\) and ray-tracing
(also 2-D, but, in some cases treating the plasma as 3-D) codes, requires that the toroidal and poloidal antenna power spectra be known \textit{a priori}. Using such techniques and codes, it is impossible to model the effect of plasma loading on the antenna current distribution self-consistently. Moreover, the effect of inherently 3-D antenna structures such as the Faraday shields, which couples all spatial Fourier harmonics generated by the antenna geometry, had not been adequately treated. A series of two-dimensional\textsuperscript{6-8} and three-dimensional numerical codes\textsuperscript{9-11} have been applied to analyze the antenna plasma coupling problem. These codes calculate antenna power spectra and loading in the presence of plasma and are often used to provide power spectra information for the 2-D full wave and 3-D ray-tracing codes. Among the many approaches, Lehrman and Colestock\textsuperscript{8} and Theilhaber and Jacquino\textsuperscript{11} have treated the current flow in the antenna current straps in the presence of plasma self-consistently. However, Lehrman and Colestock's model does not consider Faraday shield effects, while Theilhaber and Jacquino's treatment is restricted to idealized antenna and Faraday shield geometries. Much valuable insight and useful information has been obtained from this work. Indeed, they have provided correct and qualitative answers to many of the tough design questions facing ICRF antenna design. However, as described subsequently in Secs. 3.1.3 and 3.1.5, there is now a capability for treating the plasma-antenna coupling problem, including the self-consistent plasma response, allowing many of the complexities of the real 3-D antenna geometry to be taken into account.

Sheath formation is one of the earliest problems considered in plasma physics and remains one of the most difficult plasma problems today.\textsuperscript{12-13} The rf-driven sheath formation, first studied in detail in unmagnetized plasmas,\textsuperscript{14-15} has been applied to tokamak antenna problems by Perkins,\textsuperscript{16} Myra and D'Ippolito,\textsuperscript{17-19} and others.\textsuperscript{20,21} Myra and D'Ippolito's work was the first to model the coupled system including high energy particle influx and antenna surface sputtering, and now is included in their ANSAT antenna analysis tool. This work resulted in qualitative agreement with the observed dependence in JET of the impurity influx on the antenna phasing and magnetic field angle, and has become very useful in providing input to the development of Faraday screen configurations designed to eliminate surface sputtering.

The 1-D models for \textit{rf} induced sheaths noted above assumed magnetized ions, but their results are independent of the magnetic tilt relative to the sheath electric field. A much larger sheath potential results under \textit{rf} drive when the magnetic field is nearly perpendicular to the electric, which is the situation for the metal surfaces facing the plasma. The ratio between DC and applied \textit{rf} voltage becomes a function of the inclination $\theta$ (the unmagnetized result is valid for parallel fields $\theta = 0$). A novel model created under the
present contract shows that the magnetized presheath ions become demagnetized inside the sheath (see Appendix J). The resulting increase in the sheath potential is proportional to the ratio of the effective ion masses between the presheath and the sheath (see Appendix K).

Two-dimensional effects become fundamental in rf sheaths when the sheath length, determined by the Faraday bar cross-section, is comparable to the sheath thickness determined by the edge plasma density. The two-dimensional structure of rf-driven sheaths around Faraday bars has been demonstrated in SAIC numerical simulations, exhibiting nested equipotentials and vortex flow patterns. Analytic estimates show that the 2-D scaling depends on both the sheath thickness-to-length ratio and the angle of the magnetic lines relative to the axis of the Faraday bar. It is also possible that in a 2-D vortex situation, the flux from the presheath cannot replenish the plasma flux into the wall until the density inside the sheath falls to levels much lower than those for the 1-D case. That can considerably weaken the sheath effects between FS bars, and needs long-time steady-state simulations to be verified.

Finally, it has been established that the periodic sheath arrangement on the FS bars acts as a “phased array” of oscillating dipoles, scattering the incoming fast mode into short wavelength electrostatic plasma modes. The amount of parasitic scattering into Bernstein modes has been evaluated, and is shown in Figure 2.1 (reprint of figures 7-10 from Ref. [26]). At low edge plasma density situations, a larger fraction of the incoming rf power is lost into scattering than into ion acceleration. Note, however, that the overall loss in that situation is rather small.
Figure 2.1. Parasitic scattering into Bernstein modes from Ref. [26].
3. Review of SAIC Accomplishments

This section reviews SAIC’s significant progress and accomplishments in the ICRF program within the contract. Section 3.1 presents the simulation technology developments as well as applications of the models to numerous test cases and antenna designs. In particular, the ARGUS code is discussed emphasizing the utility and match of the ARGUS code suite to the simulation of ICRF antenna designs. We discuss the present single-frequency solver with the plasma surface-impedance model that has been used rather extensively. Benchmarking results against other codes as well as experimental data is presented over a wide variety of ICRF antenna geometries, and is a result of SAIC’s collaboration with several fusion-community partners. The benchmarking is not only shown for the single-frequency solver built under this contract, but also with the previous ARGUS electromagnetic solvers. (Note that previous solvers, although having been present in the code, have undergone significant upgrades and maintenance during the present contract through other projects and funding sources, and present a significant benefit to the ICW program.) Details regarding the development and status of SAIC’s new, internal, cold plasma-response single-frequency model are included here, and presented is a brief tutorial of the issues concerning the need and utility of a model of this kind, as well as examples of test cases showing the application of this new electromagnetic solver. Section 3.2 reviews the sheath physics accomplishments made under the current contract. This part of the program represents a heavy weighting of both the theory of this complicated subject as well as simulations to help understand some of the delicate issues surrounding sheath physics in ICRF devices. In this section, the Mask code is presented, and results of simulations in both 1-D and 2-D are shown. Basic results from the 2-D simulations and theory are presented, modeling the area in the vicinity of the Faraday bars. In particular, results of 2-D, short-wavelength scattering by the Faraday screen are discussed along with issues regarding the dramatic effect of external magnetic field tilt angle on 1-D sheath physics.

3.1. Review of ARGUS Antenna Simulations

3.1.1. The ARGUS Code

ARGUS, the 3-D fully electromagnetic simulation code used in SAIC’s ICRF antenna studies, has been described in detail elsewhere.\textsuperscript{24,25} ARGUS is a general-purpose
simulation code designed to handle problems involving complex geometrical rf structures. Nonuniform grids are used in all three spatial directions to efficiently capture fine structural details. The code computes steady-state and time-dependent electrostatic fields as well as electromagnetic fields, where several methods of solution are available. These include a time-dependent initial-value field solver, a frequency-domain eigenmode solver, and a single-frequency solver. Additionally, ARGUS can perform PIC simulations in both a time-dependent and steady-state mode. The PIC modules include multiple particle species, the relativistic Lorentz equations of motion, phenomenological terms for elastic and inelastic scattering, and algorithms for the creation of particles by emission from material surfaces, injection onto the grid, and ionization. Imposition of boundary conditions is highly flexible: either metallic walls, walls of symmetry, periodic boundaries, waveguide port conditions describing ingoing and outgoing radiation, as well as outgoing (open) boundary conditions can be imposed. Internally, material structures can be defined on the grid as regions of specified material properties, e.g., perfect conductors, complex tensor dielectrics, complex tensor permeable materials, or tensor resistive materials.

3.1.2. ARGUS and ICRF Antenna Simulations

The key advantage of ARGUS is its ability to share realistic geometric representation among the disparate solvers mentioned above, thus permitting different aspects of a problem to be analyzed with the same code. Currently, five field solvers are applicable to the antenna modeling problem: an electrostatic field solver, an electromagnetic frequency-domain eigenmode solver, an electromagnetic time-domain solver, an electromagnetic single, driven-frequency solver with surface plasma response, and an electromagnetic single, driven-frequency solver with volumetric, internal plasma response. Additionally, a particle-in-cell module for self-consistent treatment of particle and field interaction can be imposed under steady-state or time-dependent regimes. Except for the two single-frequency solvers, all the other solvers were in place prior to the start of SAIC's antenna modeling program. Thus, SAIC's earliest tasks consisted of applying these solvers to ICRF antenna modeling specific requirements, e.g., adding appropriate diagnostics, linking to other codes such as the ORION full-wave code, etc.

The earliest experimental benchmarking of ARGUS applied to ICRF antenna configurations was carried out in collaboration with the University of Wisconsin. \cite{27-29} The experimentally determined vacuum characteristics of a model antenna geometrically similar to the TFTR design were compared with the results from an ARGUS simulation of the laboratory test apparatus. The agreement between the computed and measured results was
excellent. Figure 1 in Appendix C shows the numerical grid representation of the model antenna and the computed versus measured vacuum toroidal rf magnetic field. It was found that the currents flowing in the antenna strap and other parts of the antenna configuration (such as the backplane on which the test apparatus was mounted) should be calculated self-consistently for the actual antenna radiation pattern to be captured. In fact, because of the three-dimensionality of the experimental test set-up, certain features of the experimentally observed behavior were not understood until the simulation model results were studied.

Since these early investigations, our ability to capture more detailed antenna geometry, perform meaningful simulations, and present useful results for the analysis and evaluation of ICRF antenna performance has progressed and improved significantly. See Refs. [30] and [31] and Appendices D-F. Among the early antenna performance studies is the use of the ARGUS time-domain solver to analyze the current strap phase velocity and the self and mutual inductances of the ORNL TFTR Bay-L antenna. For the phase velocity, the time-of-flight along the current strap of an electromagnetic pulse excited at the antenna feed port is measured, thus simulating the experimental phase velocity measurement technique. These early ARGUS results indicate a phase velocity between 0.57c and 0.66c for the front section of the current strap, depending on the width of the test pulse. The measured value is around 0.66c, where the discrepancy and range of results can be accounted for by the following: actual dispersion unaccounted for in measurements; unintended numerical dispersion; poor resolution of the Faraday shield structure; or inappropriate application of the time-of-flight measurement (e.g., there is uncertainty concerning the appropriate path length of the current strap). The single-frequency solver, described in Sec. 3.1.3, has also been used for this purpose. For the inductances, a spatially and temporally constant current is driven on the straps; the inductances are deduced from the relation between the magnetic field energy and the currents, giving results that are in reasonable agreement with the experimentally obtained measurements. More details of these studies, along with those from other studies, are found in Appendix C. Previous studies of different model antennas and TFTR mockups, including Poynting flux, rf power spectral densities, and studies of geometrical effects on these quantities, among others, can be found in Appendices A-F.
3.1.3. ARGUS Single-Frequency Solver with Plasma Surface-Impedance Model

We have developed a single, driven-frequency solver for ARGUS that includes a self-consistent plasma response via a surface-impedance model. This development brings a new capability to the fusion community: realistic 3-D modeling of ICRH antennas with self-consistent strap-current and wall image-current distribution in the presence of plasma can now be simulated self-consistently.

Not all the details concerning the implementation of the single-frequency solver are given here. These will be reported on and published in appropriate journals. However, the essential details of the implementation are briefly described. The plasma surface-impedance single-frequency solver algorithm requires two levels of iteration: an inner loop and an outer loop. The inner loop solves the Helmholtz equation,

Eq. 3.1 \[ \nabla \times \nabla \times \mathbf{E} = (\omega/c)^2 \mathbf{E}, \]

in vacuum with complex internal boundaries, given the tangential components of the electric field at the boundaries of the computational domain. Here, \( \mathbf{E} \) is the electric field, \( \omega \) is the antenna frequency, and \( c \) is the speed of light in vacuum in MKS units. An iterative conjugate gradient (CG) solver and a generalized minimum residual (GMRES) solver have been developed for this loop.

The outer loop is where the plasma response is introduced. In this version of the single-frequency solver, the plasma response was modeled as a surface impedance, giving

Eq. 3.2 \[ \mathbf{E}_T(\omega, k_z) = \mathbf{Z}(\omega, k_z) \mathbf{B}_T(\omega, k_z), \]

where \( \mathbf{E}_T \) and \( \mathbf{B}_T \) denote the tangential electric and magnetic field components, \( \mathbf{Z} \) is the plasma impedance matrix, and \( k_z \) is the toroidal mode number. The elements in \( \mathbf{Z} \) can be evaluated from a variety of plasma models, i.e., a full wave code, a slab code, or analytical solutions. At each iteration, the new \( \mathbf{E}_T \) at the plasma-vacuum interface is evaluated from \( \mathbf{B}_T \) from the previous iteration. This new \( \mathbf{E}_T \) forms the boundary condition for the inner loop iterations. Fast Fourier transforms are used to communicate between real and Fourier space representation of \( \mathbf{E}_T \) and \( \mathbf{B}_T \). As a detail, we also move the components in \( \mathbf{B}_T \) that depend on \( \mathbf{E}_T \) to the left-hand side in the above equation for \( \mathbf{E}_T \) to enhance implicitness.
3.1.4. ARGUS Benchmark and Application Examples

In this section, a sample of benchmark simulations using ARGUS is presented, along with comparisons of numerical results with experimental results. These results represent a significant collaborative effort between the US fusion community and SAIC, especially with respect to ORNL, Lodestar Research Corp., PPPL, ITER US Home Team, GA, and UW. It should be noted that, for the following examples, ARGUS was the central tool of the analysis, but that some of the examples were executed by SAIC’s collaborators.

**Time Domain Benchmark:** In the case depicted in Figure 3.1, one quarter of the complete structure, as represented by ARGUS, is shown of a mockup of the Bay-M antenna currently used on TFTR. This mockup was used by ORNL to measure the difference in the rf magnetic field structure caused by adding slits in the side walls enclosing the current straps (as shown in Figure 3.1). As a test, ARGUS was applied to two experimental cases - one with slits and one without. Comparison of the resulting rf magnetic field profiles (Figure 3.2) showed excellent agreement with measurements. The ARGUS simulations have, from the initial phases of our studies, given valuable insight to our understanding of the vacuum operation of ICRF antenna structures.

**Single-Frequency Benchmark with Plasma Interface:** Here, we describe the first results obtained from simulations using the single-frequency solver in conjunction with the cold plasma-response boundary condition model described in Sec. 3.1.3. This capability represents a breakthrough in fully 3-D electromagnetic antenna/plasma analysis and simulation. To demonstrate the feasibility of the numerical scheme, a simple uniform-density cold-plasma slab model with plane-wave solutions is used to calculate $Z$.

Figure 3.3 describes a test problem configured as a plasma interface at some standoff distance from a mockup of the DIII-D fast-wave antenna. For a case with toroidal field strength, $B=1$ T, edge plasma density, $n = 4x10^{19}$ m$^{-3}$; and the plasma positioned 2.5 cm in front of the Faraday shield, the antenna plasma loading impedance is found to be

$$Z = ( 3.55 - i 32 ) \, \Omega,$$
Figure 3.1. ARGUS representation of the Bay-M (TFTR designation) mockup antenna. By symmetry, only one quarter of the antenna needs to be modeled. The center septum is not shown.
where $\Omega$ is given in Ohms. Initial comparison of this result with the 2-D RANT code at ORNL with a single strap showed good agreement, although a thorough benchmarking program still needs to be carried out. There are fundamental differences between these 3-D results and previous 2-D calculations, and, as such, this plasma surface-impedance single-frequency solver in the ARGUS code provides us with the capability to determine and differentiate between 2 and 3-D effects during antenna design, analysis, and performance evaluation.

The antenna power spectral distribution over the full antenna face associated with the above simulation, as a function of distance from the bottom of the antenna, is shown in Figure 3.4. Again, these results are in agreement with expected values and, here, the major differences between 2 and 3-D simulations are clearly shown.

![Graph showing comparison of the toroidal component of the magnetic field strength (B-tor) measured at the midplane of the antenna. The slots shown in Figure 3.1 are removed for one case. The units are arbitrary.](image.png)
Figure 3.3. Cutaway view of a single strap of the mockup DIII-D ICRH antenna. The center septum is not shown.
Figure 3.4. Spectra of the toroidal magnetic field launched by the antenna shown in Figure 3.3 as a function of distance from the bottom of the antenna. This case has density, \( n = 4 \times 10^{19} \text{ m}^{-3} \), toroidal field, \( B = 1 \text{ T} \), and plasma positioned 2.5 cm in front of the Faraday shield.

**Vacuum Single-Frequency Benchmark:** A recent example of a vacuum-field benchmark for the ARGUS code is shown from its application to the mockup of the TFTR Bay-M antenna depicted in Figure 3.5. The effectiveness of ICRF heating and current drive in a tokamak is strongly dependent on the \( k_z \) spectrum of the launched wave. Hence, we have chosen to test the vacuum solver by comparing measured and calculated \( k_z \) spectra of an existing antenna. The toroidal magnetic fields, \( B_z \), at 1 cm in front of the Faraday shield was measured. The square of the Fourier transform of the field at the poloidal midplane is shown in Figure 3.6 for both the 0-0 and 0-180° phasing cases.

The computation reproduced the main features of the measured results. Namely, for the 0-180° case, the spectra peaked at \( \sim 9.5 \text{ m}^{-1} \), corresponding to the center-to-center strap separation of 0.33 m (half a wavelength). For the 0-0 phasing case, the spectra, perhaps unexpectedly, peaked at \( \sim 16 \text{ m}^{-1} \). This is caused by side walls and the septum that, despite being slotted, still carried significant image currents. The difference at \( k_l = 0 \)
is probably due to disparate boundary conditions (the computational domain is bounded; however, the measurements were made on a free standing antenna.). Calculations with reduced computational domain showed increased power at $k||=0$. Also, the model antenna ignored poloidal curvature to facilitate inclusion of plasma response in future calculations.

**Plasma Response Single-Frequency Benchmark with MAntIS:** We compared benchmark results from the surface plasma-response model with those from the MAntIS code. For this antenna-loading calculation, given a reasonable estimate of the current distribution in the current straps, these type of codes have been shown to give good estimates of antenna loading.\(^{23}\) For simplicity, the comparison is done on a very simple antenna structure, as shown in Figure 3.7. Only a bare current strap and a conducting back plane are modeled. The current strap front is 90 cm high and 7 cm wide. The radial feed section is 32 cm long. The elements of the plasma impedance matrix were derived from the parameters of the proposed TPX experiment at wave frequency of 40 MHz. Figure 3.8 shows the loading computed by the two codes at various distances between the current strap and the plasma. Despite great differences in the computation technique, excellent agreement is found in the loading results.
Figure 3.5. ARGUS representation of the TFTR Bay-M mockup. Vacuum solution, modeling only 1/4 of the structure due to symmetry.
* Experimental measurements and the above figures are provided by Drs. J. H. Rogers, R. Majeski, and S. Raman of PPPL.

Figure 3.6. Comparison of computed TFTR Bay-M mockup field solution with measurement. I-(a), $B_z$ for 0-180 phasing measured 1 cm from the Faraday shield at the midplane of a mockup of the Bay-M antenna and the field calculated by ARGUS. I-(b) The Fourier transform of the square of the fields. II-(a), $B_z$ for 0-0 phasing measured 1 cm from the Faraday shield at the midplane of a mockup of the Bay-M antenna and the field calculated by ARGUS. II-(b) The Fourier transform of the square of the fields.
Figure 3.7. Geometry of simple current strap antenna for comparison with MAntIS. This example of a case with surface plasma-response was for TPX at 40 MHz.
Figure 3.8. Plot of loading vs. plasma separation distance results of both ARGUS and MAntIS for the TPX 40 MHz example of Figure 3.7.
**ITER Application: Resonant & Single-Frequency Calculations:** One of the ARGUS applications involves the ITER IC heating rf antenna where we calculated the general ICH antenna electrical properties, such as characteristic impedance of the current straps, mutual coupling between antenna elements, and antenna loading over the frequency bands of interest. The analysis is carried out on ITER violin antenna designs as provided by the ITER JCT at Garching through the ORNL ICH system design team (see Figure 3.9). For this task, the target design is the 18 strap multi-frequency antenna designed to operate at the nominal resonant frequencies of 22, 44, and 66 MHz. For this reason, we applied the frequency-domain eigenmode solver in the ARGUS code to perform the vacuum field analysis. See Appendix D for a detailed description of this result. Different boundary conditions permit us to model the various phasings possible for an infinite array antenna, e.g., $180^\circ (0-\pi-0-\pi...)$, $90^\circ (0-0-\pi-\pi)$, $20^\circ (1 \text{ period extends 18 straps})$. The change in the dispersion characteristics of the antenna due to phasing variation gives information on the mutual coupling coefficients. The resonant frequencies versus the poloidal wavelength (i.e., antenna dispersion characteristics) are shown in Figure 3.10 for the different phasings.

For this example, in conjunction with the Eigenmode solver, we also applied the single-frequency solver to the ITER configuration in Figure 3.9. Here, the antenna is excited at a specified frequency by applying a driving voltage at the feed port. By driving the antenna at different frequencies and different phasing, we can analyze the field solution using a transmission line model and obtain relevant antenna electrical properties. In addition, the rf-electric field solutions were provided to D. A. D'Ippolito (Lodestar Research Corporation) to help assess rf sheath and impurity generation using ANSAT under ITER task D-90.

The resonant frequency of the strap was also checked by observing the port impedance while scanning frequency. The results in Figure 3.11 indicate that the port impedance changes from capacitive to inductive at around 55.5 MHz for boundary conditions that duplicate ($... 0, 0, \pi, \pi, 0, 0, \pi, \pi, ...$) phasing for an infinite array. The Eigenmode analysis for a geometry with no radial feed gives a resonant frequency of 56.019 MHz.
Figure 3.9. ARGUS rectilinear representation of the ITER "violin antenna". Poloidal cross sections of a single strap (of an array) are shown: (a) view showing the feed at the back of the strap; (b) different view showing the side slits.
Figure 3.10. Antenna resonant characteristics for the rectilinear antenna model shown in Fig. 1, but without the feed tap: (a) resonant frequency, \( n \), versus poloidal wavelength, \( \lambda \), and (b) inferred phase velocity in units of light speed, \( c \), versus \( \lambda \). Results are shown for four different toroidal array phasings: \( 180^\circ \) (2-strap periodicity), \( 90^\circ \) (4-strap periodicity), \( 20^\circ \) (18-strap periodicity), and \( 10^\circ \) (36-strap periodicity).
Folded Waveguide (FWG) Application Using the Eigenmode Frequency-Domain Solver: Another approach for IC heating is the folded waveguide (FWG), as show in Figure 3.12 for the PBX plasma at 58 MHz. This work was done by ORNL using ARGUS. Again, since these antennas are typically run at resonance, the ARGUS frequency-domain eigenmode solver was used. Figure 3.13 shows several views of magnetic fields associated with the 58 MHz resonant mode. Details regarding the FWG are found in Appendix E.
Figure 3.12. ARGUS representation of the geometrical configuration of the PBX folded waveguide operating at 58 MHz. This antenna operates at or near resonance, so the ARGUS frequency-domain eigenmode solver is used.
Vector Plots of The Magnetic Field On Either Side of the Front Faceplate

Magnetic Field Reverses Direction Between Adjacent Slots On Interior Side of the Faceplate

Magnetic Field Does Not Reverse Direction Between Adjacent Slots On Exterior Side of the Faceplate

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995

Figure 3.13. Vector plots of the magnetic field on either side of the front faceplate. These fields were calculated using the frequency-domain eigenmode solver in ARGUS.
Evaluation of Coupled Transmission Line Parameters: The ARGUS code was also used to create distributed-element, coupled transmission line models of ICRH antennas. This allows the ICRH antennas to be modeled as an intrinsic part of the total power transmission line, and permits fast calculation of antenna response to changes in frequency, phasing, and plasma loading. Real-time tuning, matching, and phasing of the ICRH system during plasma operation using rapid computer algorithms can be accomplished. In this model, ARGUS is used to calculate the inductance and acceptance matrices for the coupled transmission model. ARGUS field solutions for two frequencies and two phasings are required, which are then used to calculate voltage and current relations at two locations (the ground point and one point on the current strap). The resulting matrix equation is then inverted. Table 3.1 shows the results of this model applied to the original DIII-D FWCD antenna array. These results were compared to the calculations of the ORNL electro/magnetostatic codes, which have been benchmarked against measurements and routinely used as input to transmission line analyses. The agreement between the calculations is quite good and within the typical measurement accuracy for these parameters.

Table 3.1. Calculated Antenna Electrical Characteristics.

<table>
<thead>
<tr>
<th>Electrical Characteristics</th>
<th>ARGUS</th>
<th>ORNL codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strap Inductance</td>
<td>2.31e-7 H/m</td>
<td>2.26e-7 H/m</td>
</tr>
<tr>
<td>Strap Capacitance</td>
<td>1.25e-10 F/m</td>
<td>1.28e-10 F/m</td>
</tr>
<tr>
<td>Mutual Inductance</td>
<td>1.5%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Mutual Capacitance</td>
<td>0.03%</td>
<td>0</td>
</tr>
<tr>
<td>Characteristic Impedance</td>
<td>43.0 W</td>
<td>42.9 W</td>
</tr>
<tr>
<td>Phase Velocity</td>
<td>0.62 c</td>
<td>0.62 c</td>
</tr>
</tbody>
</table>
ANSAT/ARGUS Design Tool: One of the successes of ARGUS in the ICRF fusion program was through a collaborative effort with Lodestar Research Corporation, with the assistance of ORNL. Lodestar has been working on developing a set of sheath analysis tools for several years. In this collaboration, the Lodestar tools were integrated into a package called ANSAT (Antenna Sheath Analysis Tool) and linked with ARGUS. Figure 3.14 shows how ARGUS fits into this framework. One of the output from the ANSAT/ARGUS combination yields a contact point analysis, as shown in Figure 3.15. This figure shows how field lines connect to the Faraday screen, and identifies the existence of four classes of sheath formations. This result shows contours of voltage for a portion of the Faraday shield of the Bay-M mockup at 0-\(\pi\) phasing, with a 10° magnetic field alignment to the toroidal axis. A detailed description of the analysis is found in Appendix F. This effort has allowed sheath analysis of this type, for the first time ever, to be used with realistic antenna field geometries, which greatly increases the accuracy of ANSAT sheath physics analysis. This collaboration shows that ARGUS has transitioned to be an integral part of the US fusion community's ICRF antenna analysis capability.
Figure 3.14 Relationship of the ARGUS code to the ANSAT antenna sheath analysis post-processor.
Figure 3.15. The figure represents a sheath driving voltage evaluation. Contours of $V$ for a portion of the Faraday shield of the Bay-M mockup at $0-\pi$ phasing. (Assuming magnetic field aligned 10 degree from toroidal, toroidal curvature of screen ignored). Voltage estimate for 5 MW case with 80 KeV top to bottom voltage drop along the Faraday screen: points A-B ~ 3.2 KeV; C-D ~ 1.3 KeV; E-F ~ 0. KeV.

3.1.5. ARGUS RRNM Single-Frequency Solver with Volumetric, Cold Plasma-Response Model

Most recently, a new single, driven-frequency solver, called RRNM for Recursive Residual Norm Minimization, has been developed for ARGUS, which includes a self-consistent, volumetric, cold plasma-response model. This development allows a host of new rf plasma properties to be addressed. The plasma, which was treated only as a surface-impedance boundary condition in the solver described in Sec. 3.1.3, is now treated by an internal, volumetric, cold-plasma profile, and is self-consistent. Realistic plasma profiles can be imposed within the computational domain, allowing the effect of the plasma distribution, and its effect on the antenna properties, to be more accurately modeled.
Details regarding the implementation of the RRNM cold plasma-response single-frequency model can be found in the reprint in Appendix G, and will be reported on and published in appropriate journals. However, here, the basic model is described. At the outset, it should be noted that this model does not require the inner and outer iteration that the surface-impedance model requires, and should converge much more quickly as a result.

In the frequency domain and in the presence of a plasma density profile, Maxwell's equation can be written as

\[ \nabla \times \nabla \times \mathbf{E} = \left( \frac{\omega}{c} \right)^2 \left( 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)} \right) \mathbf{E}, \]

where \( \mathbf{E} \) is the rf electric field, \( \omega \) is the radian frequency, \( \omega_p^2 = 4\pi N_e e^2/m_e \) is the electron plasma frequency, and \( \nu \) is the electron collision frequency. The expression in Eq. 3.3 is valid without the effects of a static magnetic field. If a static magnetic field is present, then the governing equation is more complicated: the scalar dielectric constant on the right hand side of Eq. 3.3 becomes a dielectric tensor so that it is no longer a diagonal term when the finite difference version of the equation is derived.

Regardless of this complication, when Eq. 3.3 is solved by finite difference numerical techniques, using a rectangular 3-D mesh inside a physical domain of interest with appropriate boundary conditions, the problem is reduced to a matrix inversion problem; i.e., solving the following:

\[ A \mathbf{x} = \mathbf{b}, \]

where \( A \) is a sparse matrix, \( \mathbf{x} \) is the unknown vector formed by the components of the rf electric field at the grid points, and \( \mathbf{b} \) is a known vector deduced from the boundary conditions. The size of \( A \) depends on the number of grid points used. If the number of grid points in the \( x \), \( y \), and \( z \) direction are \( N_x \), \( N_y \), and \( N_z \), respectively, then \( A \) is a \( N \times N \) matrix with \( N = N_x \times N_y \times N_z \). Thus, for a 100x100x100 grid, \( A \) is of the size \( 10^6 \times 10^6 \), which is very large even for today's super computers.

To invert such a large matrix, a special numerical technique is required. Fortunately, most of the elements in \( A \) are zero, except the main diagonal and a few other sub-diagonals (otherwise even storing the matrix \( A \) in the computer will cause a problem). Therefore, for this type of matrix, we usually solve Eq. 3.4 by iteration methods. If \( A \) is positive definite, which is the case if \( \omega_p = 0 \), it is well known that the conjugate gradient
method (CGM) of Hestenes and Stiefel is easy to implement and has nice convergent properties. Hence CGM is employed in the field solver with the plasma surface-impedance model described in Sec. 3.1.3. However, when a non-zero volumetric \( \alpha \), is included in the model, the relevant \( A \) is no longer positive-definite, and CGM does not converge because the CGM algorithm is based on minimizing the energy norm \( (b-Ax)^T A (b-Ax) \), which is not necessary positive.

There are very few numerical methods that can solve Eq. 3.4 successfully when \( A \) is non-definite. Only one of them, the generalized minimal residual algorithm of Saad and Schultz (GMRES), is robust and well-behaved enough to be widely used. The GMRES algorithm approximates the exact solution, \( x \), by projecting it over the Krylov subspace \( K_k = \text{span}\{v_j; j=1,k\} \), where \( v_1=r_0/\|r_0\|, v_{j+1} = A v_j/\|Av_j\|, r_0 = b-Ax_0 \) and \( x_0 \) is the initial guess of the solution \( x \). The resulting residual is minimized over the Krylov space. Clearly if \( k \) is of order \( N \), the GMRES algorithm can offer no advantage compared with any direct numerical solution of Eq. 3.4, such as Gaussian elimination because the huge storage requirement of \( K_k \). In practice, \( k \) is fixed to be \( k_0 \), an integer much less than \( N \). When the dimension of \( K_k \) has reached \( k_0 \) and the desired accuracy of \( x \) has not been achieved, the GMRES algorithm is restarted with the most recent and much better approximated \( x \). This restart procedure is repeated until the desired accuracy of \( x \) is achieved. Very often, especially when physical problems require a large number of grid points, we cannot afford to use a large \( k_0 \) (the number of basis vectors in the Krylov space) because of storage limitations. In that case, the convergence rate of the GMRES algorithm can be extremely slow.

In addition to the ICRF case, the difficulties mentioned above are shared in the solution of problems in other areas as well, such as microwave plasma material processing. To deal with these, we have attempted to develop a new numerical technique to invert non-definite large sparse matrices. These efforts have produced fruitful results that have led to a new algorithm based on recursive residual norm minimization (RRNM). This new method is generally applicable to inversion of indefinite as well as positive definite matrices. On the one hand, it exhibits the advantages of CGM: it is simple and easy to implement and has minimal storage requirements. On the other hand, it improves on GMRES by implicitly retaining information from previous steps to efficiently minimize the residual with respect to the norm and the space spanned by the operator \( A \). The details of this new algorithm, as described by a paper submitted to Journal of Computational Physics, are included in Appendix G.

To compare the performance of these three algorithms, we conducted a series of tests to determine the time required to converge to a solution with the same accuracy,
starting from the same initial guess. The matrix $A$ we used in the test is a 900x900 matrix with the main diagonal elements equal to 4, and four other sub-diagonals with elements equal to -1. This matrix $A$ models the finite difference operator derived from the Laplacian operator, which is positive definite. To change the definiteness of the operator $A$, we reduce one of the diagonal elements from 4 to 0, gradually. In the example below, we arbitrarily select this to be the diagonal element, $A(N/4,N/4)$, and use it as the definiteness parameter. In Figure 3.16, we plotted the CPU times required for convergence to same accuracy from same initial guess for the three algorithms we have discussed: CGM, GMRES and RRNM, against the definiteness parameter. The maximum size of the Krylov space, $k_0$, being used in this test is 20. It is clear from the figure that, as the definiteness parameter varies from 4 to 1, the RRNM algorithm is the fastest of the three, roughly ten time faster than the CGM algorithm and two times faster than the GMRES algorithm. The CGM takes longer to converge as the definiteness parameter is reduced from 4 to 2.8, and fails to converge when the definiteness parameter is below 2.4. The GMRES algorithm also exhibits some erratic behavior when the definiteness parameter is between 2 and 3, otherwise being fairly insensitive to the definiteness parameter. The RRNM algorithm is fairly robust, and shows an insensitivity across the whole range.

We have implemented the RRNM algorithm to solve Eq. 3.3. To demonstrate the capability of the RRNM method, it is applied to a test case which is composed of a 60 cm x 60 cm x 80 cm waveguide, which is closed at the far end. See Figure 3.17. The figure shows a 3-D contour plot of the $E_z$ field inside a region. The number of grid cells used in the calculation is 60x60x80. A 47 MHz rf $TE_{10}$ waveguide mode is imposed as a boundary condition on the lower $z$ boundary surface (power input window). Horizontal metallic bars parallel to the $x$ direction, located one quarter of the total axial distance along the waveguide, model the Faraday shield bars in the ICRF problem. Inside the waveguide, a plasma density profile is imposed that begins at zero density at the mid point axially, and linearly increases to $10^{13}/cm^3$ at the shorted surface coinciding with the maximum $z$ face. A metallic boundary condition has been imposed on all but the drive face at $z=0$. This does not represent a realistic ICRF antenna scenario in tokamaks, but the sizes, drive frequency, and mode are comparable, except for the Faraday bar spacing, and should be sufficient for the purpose of demonstrating the capability of the new field solver. At 47 MHz, the $TE_{10}$ mode is below cut-off, and is evanescent into the waveguide. Therefore the $E_z$ field decays as it propagates inward from the power input window. The existence of the plasma profile half way down the waveguide further diminishes the $E_z$ field. The effect of the plasma profile on the electric field is evident if we compare Figure 3.17 with Figure 3.18, which is same case, except with the plasma profile absent. The existence of the metallic bars also
weakens the electric field downstream by partially reflecting the incoming wave although the wave propagates through due to the wave polarization. The difference between Figure 3.17 and Figure 3.18 is reduced because the rf fields are below cut-off. In order to dramatize the effects of the plasma profile, let us consider the case with a driving frequency of 470 MHz, where the vacuum wavelength for the rf field is comparable to the size of the domain of computation. At this frequency, very low wave attenuation is expected for the vacuum case. Including the presence of a plasma density profile, as in the case shown in Figure 3.17, (a linear ramp of the density from zero at the halfway point to $10^{13}$/cm$^3$ at the maximum z boundary), the electric field is shielded completely by the plasma, as shown in Figure 3.19. However, in the absence of the plasma profile, as in the case shown in Figure 3.18, the rf field now propagates into the cavity with very little (but observable) attenuation. See Figure 3.20.

These examples demonstrating the robustness and speed of the RRNM model gives us much confidence in the ability of the new model to aid greatly in the simulation capability of ICRF designs. The inclusion of the plasma internal to the computational domain brings us to a new level of simulation technology and capability in the rf antenna modeling field.
Figure 3.16. Plot of the CPU time (on an IBM RS6000 workstation) required for convergence to the same level of accuracy from the same initial guess using the three algorithms: CGM, GMRES, and RRNM. This test problem solves Eq. 3.3. The number of basis vectors used in the GMRES algorithm is 20. The matrix $A$ is a 900x900 banded matrix with the main diagonal elements equal to 4, and four other sub-diagonals with elements equal to -1. This matrix $A$ models the finite difference operator derived from the Laplacian operator with a uniform grid, which is positive-definite. To change the definiteness of the operator $A$, we reduce one of the diagonal elements from 4 to 0 gradually. In this example, we arbitrarily select it to be the diagonal element $A(22,22)$ and use it as the definiteness parameter.
Figure 3.17. A 3-D slice contour plot of the $E_y$ field inside a region of size $60 \text{ cm} \times 60 \text{ cm} \times 80 \text{ cm}$ is shown here. The color scale is such that red stands for the highest intensity and blue the weakest. The driving rf electric field ($TE_{10}$) in the $y$ (vertical) direction with a frequency of $47$ MHz is imposed as a boundary condition on the left vertical boundary surface (power input window). Horizontal metallic bars (in the $x$ direction) parallel to the power input window are located axially about one quarter of the way down the waveguide to model the Faraday bars. Starting from the half-way point axially is a plasma density profile with a linear ramp up to a maximum density of $10^{13}/\text{cm}^3$ at the maximum $z$ surface. The boundary conditions used in all boundary surfaces except the power input window are metallic boundary conditions.
Figure 3.18. A 3-D slice contour plot of the $E_y$ field inside a region of size 60 cm x 60 cm x 80 cm similar to Figure 3.17 is shown here. The difference being the absence of a plasma.
Figure 3.19. A 3-D slice contour plot of the $E_y$ field inside a region of size 60 cm x 60 cm x 80 cm similar to Figure 3.17 is shown here. The difference being that the driving frequency is 470 MHz.
Figure 3.20. A 3-D slice contour plot of the $E_y$ field inside a region of size 60 cm x 60 cm x 80 cm similar to Figure 3.18 is shown here. The difference being that the driving frequency is 470 MHz.
3.2. Review of Theoretical and Numerical Simulations of Sheath Physics

SAIC's past work in analyzing the physics and dynamics of sheaths and sheath effects on ICRF antenna performance has concentrated on the 2-D nature of such sheaths. As discussed in Section 2, most previous work in this area, although providing very useful insight and qualitative understanding of sheath induced-antenna surface sputtering, was based on a 1-D assumption of the sheath structure. Using SAIC's 2-D particle simulation code, MASK, we have developed a picture of the 2-D nature of Faraday shield sheaths that is quite distinct from the 1-D picture. Highlighting SAIC's sheath research are the following findings: a) Magnetic line inclination is important to sheath scaling even for infinitely long, 1-D sheaths; b) Periodic sheath arrangement around the bars causes significant short wavelength scattering by the FS; c) Two-dimensional effects are very important around Faraday bars to the point of causing plasma depletion below the density threshold for sheath formation. Next, we briefly describe the MASK code and then we summarize the main findings of SAIC's sheath studies to date.

3.2.1. The MASK Code

MASK is a general 2-D PIC simulation code that has been developed for use in solving problems in plasma instability studies, microwave devices, accelerator physics, diode studies, intense beam propagation, and wave and antenna coupling. Although full electromagnetic simulation capability is also a feature of MASK, the electrostatic version has been adequate for sheath treatment. The code allows for multiple particle species, background dielectric and resistive elements, and complicated geometries. The particle dynamics are treated fully-relativistically where necessary, and permit the inclusion of both elastic and inelastic collisions. Charged particles can be initialized in equilibrium, injected, created by ionization, by secondary emission, or by field emission from user-specified surfaces.

3.2.2. Results From 2-D Sheath Studies

The results from SAIC's numerical simulations and theoretical analysis of rf driven sheaths around Faraday bars have been published. Appendices H-K contain reprints of these publications. They are summarized here as follows. The time-averaged sheath structure is generally two-dimensional, as shown in Figure 3.21 and its properties depend
on the sheath thickness-to-length ratio. The sheath length $d$ is determined by the bar cross-sectional dimension. The sheath thickness $A$ is determined by the ambient plasma parameters, including the inclination $\theta$ of the magnetic field, and the sheath length $d$ ($A$ is independent of $d$ in the 1-D theory). Ion flow is still determined by the time-averaged potentials, but it is not tied to the magnetic field lines. There is a critical angle $\theta_c$ below which cross-field drift dominates over field-aligned flow and an ion circulation vortex forms. The field structure is close to 1-D at a small thickness-to-length ratio $A/d \ll 1$, while the ion flow is nearly 1-D (field-aligned) at large angles, $\theta > \theta_c$. Fully 2-D theory applies for $A/d > 1$ and $\theta < \theta_c$ (the 1-D limit applies locally in the opposite case $A/d < 1$ and $\theta > \theta_c$). Since ions follow nearly equipotential surfaces and do not experience the full sheath potential difference, ion energization is caused in part by the oscillating voltage component, via parametric resonance. For typical large tokamak parameters, with an induced rf voltage amplitude of 500 V, an edge B-field of 1.5 T, and a rectified sheath voltage $\Phi_1=250$ V, the typical misalignment angle between the B-field and the bars $\theta = 10^\circ$-$15^\circ$ is below the computed critical angle $\theta_c=26^\circ$ and the cross field ion transport and vortex formation take place. The sheath is weakly 2-D, if its thickness $A$ remains less than $d/2$. If the 1-D scaling is used with $d = 0.5$ cm and $T_i = 16$ eV, one finds that the edge density must exceed $10^{10}$ cm$^{-3}$ to meet the condition $A \leq d/2$. For even lower edge densities, $n_o < 5 \times 10^9$ cm$^{-3}$, a fully 2-D situation arises.

Though an analytic assessment of 2-D effects is difficult, comparisons can be made between the 2-D numerical results and 1-D theoretical predictions. The numerically evaluated ion influxes at the Faraday bar generally exceed by an order of magnitude the 1-D theoretical predictions under the same edge plasma parameters. The influx increase is mainly attributed to the cross-field ion drift, which is much larger than the ion sound speed (for typical sheath values such as $T_e = 16$ eV and $E = 10^3$ V/cm) and enhances the flux from the "pre-sheath" (area labeled I in Fig. 13 of Appendix I) into the sheath. While the into-the-wall flux increases above the 1-D value, the closed circulation vortex seems to limit the flow from the main plasma (area labeled II in Fig. 13 of Appendix I) into the sheath below the 1-D levels. The outflux in the wall cannot be balanced until the plasma density between bars falls well below the density predicted by the 1-D model. That raises the intriguing possibility of plasma expulsion and reduction in the sputtering rate between the FS bars due to 2-D effects. Long time simulations of 2-D sheath formation until steady-state is reached are required to settle the issue.
Figure 3.21. Time averaged sheath equipotentials surrounding a square Faraday bar calculated with the MASK simulation code. The figure shows the inherent 2-D nature of the sheath. The computational domain is assumed periodic in the x-direction, indicating that other bars are to be found to the right and left of the bar shown.
3.2.3. Short-Wavelength Scattering by the FS

Another important sheath related effect is the rf scattering and short wavelength excitation by the Faraday screen. Figure 3.22 shows that the sheaths around the screen bars form a periodic sequence of dipoles oscillating at the rf frequency. They act as a short-period external driver, forcing electrostatic plasma responses of wavelength comparable to the screen period. If the rf frequency and the wavenumber of the screen match the dispersion relation for normal plasma modes, a resonant short wavelength excitation results. Mode conversion at the screen is thus an essential ingredient of the plasma-screen interaction. This phenomenon has been addressed in Refs. 26 and 38 (included as Appendices J and K). It has been shown that, at low edge densities, the power fraction converted into ion Bernstein modes can be of the same order as the power lost via ion acceleration in the sheaths. The angular distribution of the scattered wavenumbers peaks sharply in the directions corresponding to Bragg diffraction by a screen. Deposition of the scattered short wavelength energy near the screen via electron Landau damping can account for the observed edge plasma heating ICH. The scattering into the long wavelength, electromagnetic branch, is found small but nonzero. The present description improves on an earlier study of FS-plasma interaction during ion Bernstein wave heating (IBW) in that it includes the plasma sheath currents in the computation of the screen fringe fields that drive the main plasma. As a consequence the screen coupling to the ambient plasma is of order $\kappa_5 \Delta$ rather than $\kappa_5 \lambda_\Delta$, the sheath thickness $\Delta$ being much larger than the Debye length $\lambda_\Delta$ when the sheath potential is well above the plasma temperature. The coupling enhancement is partially offset by the small scattering cross-section of each screen bar; two dimensional effects from the bar cross-section geometry are included in the evaluation of the short wavelength scattering.
Figure 3.22 The figure shows that the sheaths around the screen bars form a periodic sequence of dipoles oscillating at the rf frequency.
3.2.4. Influence of the External Magnetic Field

Magnetic effects are important even for infinitely-long 1-D sheaths. Previous theoretical efforts\(^{36}\) sought to extend the applicability of the 1-D unmagnetized theory to magnetized plasmas, by assuming that the ion motion is constrained along the magnetic field. That treatment yields the same results as the unmagnetized theory; where the angle between the electric and the magnetic field does not seem to affect the scaling of the sheath properties. To check the validity of this, particle simulations of sheath formation in rf-driven magnetized plasmas were performed using the PIC code MASK.\(^{38-40}\) The numerical results in Figure 3.23, plotting the time-averaged potential between two parallel plates driven by an oscillating voltage, contradict the earlier conclusions: the sheath potential under given applied rf voltage depends strongly on the angle \(\theta\) between \(E\) and \(B\). The ratio between DC and applied rf voltage becomes a function of the inclination \(\theta\), shown in Figure 3.24. Both the simulations and a subsequent ion orbit analysis, do not support the assumption of \(B\)-aligned motion. Instead, a strong ion drift across the magnetic lines and along the (time-averaged) electric field \(E\) occurs, in addition to the \(E \times B\) drift parallel to the sheath. As a result, the magnetized presheath ions, whose motion is constrained along the magnetic lines, become demagnetized inside the sheath (see Appendix J) where cross-B transport becomes as important as parallel. Reduced 1-D equations for the ion motion across the sheath are derived from the exact 3-D dynamics, by introducing the effective ion mass \(m^*(\theta)\) to account for the influence of the magnetic field. The magnetized sheath scaling is parameterized by the sheath to presheath effective mass ratio, \(m^*/m_B\), where \(m_B\) stands for the fully magnetized presheath ions (see Appendix K). The effective mass change between sheath and presheath introduces magnetic effects into the sheath; when the sheath ions remain magnetized \(m^* = m_B\), magnetic effects drop out altogether.

The conclusion is that the properties of rf-driven sheaths in magnetized plasmas depend strongly on the angle between the magnetic and the rf field, as well as the relative strength of the magnetic field parameterized by \(\omega_{pi}^2/\Omega_i^2\). The induced DC sheath potential, for a given rf voltage, increases substantially as the angle between \(B\) and \(E\) increases; for nearly perpendicular fields, it reaches twice the unmagnetized sheath potential. Magnetic effects in sheath formation are ignorable only at small angles between \(E\) and \(B\). The magnetized 1-D theory is applicable to the sheaths formed at large surfaces of the antenna enclosure (limiters, FS frames, septum). The ion outflux to the metal surface, being always equal to the presheath flux, remains the same as in the earlier models that employ fully magnetized sheath ions. The per ion impact energy and sputtering yield are, however, substantially higher according to the present results.
Figure 3.23 Plot showing the time-averaged potential between two parallel plates driven by an oscillating voltage contradict earlier conclusions: the sheath potential under given applied rf voltage depends strongly on the angle $\Theta$ between $E$ and $B$. 
Figure 3.24 Plot showing how the ratio between DC and applied rf voltage becomes a function of the inclination \( \theta \).
4. Summary and Recommendations

This section discusses SAIC’s accomplishments during the contract and makes recommendations for future work objectives. Section 4.1 summarizes the progress that is reported in this report. Section 4.2 discusses recommendations for future work in the area of ICRF modeling and analysis. Here, we present an overview of an ARGUS-based modeling tool to be developed, including upgrades and planned extensions, as well as the new physics that should be captured in the model and how this effects the status of US ICRF modeling technology. Subsection 4.2.1 discusses specifics of an ARGUS tool suite, including features and capabilities. Subsection 4.2.2 addresses specific sheath physics objectives, and how results of those objectives would integrate into the ARGUS model.

4.1. Summary of Progress and Accomplishments

This section reviews SAIC’s significant progress and accomplishments in the ICRF program within the contract. In this report, we have presented the simulation technology developments as well as applications of the models to numerous test cases and antenna designs. In particular, the ARGUS code was discussed emphasizing the utility and match of the ARGUS code suite to the simulation of ICRF antenna designs. We discussed the present single-frequency solver with the plasma surface-impedance model that has been used rather extensively. Benchmarks with other codes as well as experimental data was presented over a wide variety of ICRF antenna geometries, which resulted from SAIC’s collaboration with several fusion-community partners. The benchmarking was not only shown for the single-frequency solver built under this contract, but also with the previous ARGUS electromagnetic solvers. In particular, we noted how collateral ARGUS development from other projects benefited the ICRF program. We discussed details regarding the development and status of SAIC’s new, internal, cold plasma-response single-frequency model, and presented a brief tutorial of the issues concerning the need and utility of a model of this kind, as well as examples of test cases showing the application of this new electromagnetic solver. Discussed also were the sheath physics accomplishments made under the current contract. This part of the program represented a heavy weighting of both the theory of this complicated subject as well as simulations to help understand some of the delicate issues surrounding sheath physics in ICRF devices. A discussion of the Mask code was presented, and results of simulations in both 1-D and 2-D were shown.
Basic results from the 2-D simulations and theory were presented modeling the area in the vicinity of the Faraday bars. In particular, results of 2-D, short-wavelength scattering by the Faraday screen were discussed along with issues regarding the dramatic effect of external magnetic field tilt angle on 1-D sheath physics.

4.2. Future Objectives and Recommendations

SAIC recommends a continued development of theoretical and numerical simulation technology applied to the analysis and evaluation of ICRF antenna performance. We suggest an extension the work in antenna simulation and sheath physics. This work should be aimed at developing a design tool, based on the ARGUS framework. This tool should have the capability to evaluate ICRF antenna performance that would account for all dominant geometrical complexities and electromagnetic details of the antenna structure (such as inductances, plasma loading for various plasma models, and general profiles), and incorporate the dominant sheath physics effects. Together with other design codes such as ANSAT, which use ARGUS-generated results, this technology would provide the US fusion program with an unprecedented analysis capability to design, analyze, and evaluate complex antenna structures for future fusion devices. This could greatly advance the ability of the US fusion community to have a team with the ability to completely design, build, and test an rf system.

The tool could ultimately have more general application than the ICRF antenna problem. It could also be useful for other types of wave-plasma interaction schemes in fusion devices, such as second-harmonic He³ heating in H plasma (for non-activated phase), detailed equilibrium profile control minority heating, ion minority (He³) CD (for profile control on inside or outside of plasma), ion-ion hybrid regime majority ion heating, electron current drive, mode conversion to drive current, deuterium minority heating, sawtooth instability stabilization, alpha particle parameter enhancement, etc. A prototype wave-plasma model for ARGUS (an independent effort to the ICRF contract) has already made an impact for microwave plasma production in plasma material processing applications. In addition, other fusion plasma configurations such as RFPs and stellarators could make use of such an rf modeling suite. This work would be primarily aimed at supporting ongoing antenna analysis efforts at major US fusion centers, as well as the ITER US Home Team. The primary goal of this suggested work, in addition to advancing the antenna analysis technology, would be to eventually transition the technology to the rf practitioners in these laboratories and universities. Next, in Subsections 4.2.1 and 4.2.2, the general research areas of these recommendations are described.
4.2.1. Antenna Modeling RF Design Tool

In accordance with the recommendations, the following tasks for the basic ARGUS simulation effort suggested are easily defined:

1. Complete the integration of the new single-frequency electromagnetic solver with cold plasma-response into the ARGUS framework. At present, a cold plasma dielectric function, in the absence of a static magnetic field, has been integrated in a 3-D single-frequency electromagnetic solver. A cold plasma dielectric tensor with a static magnetic field has been integrated in a 2-D cylindrical electromagnetic solver. A cold plasma dielectric tensor with a static magnetic field in a 3-D geometry remains to be completed. This cold magnetized plasma model would be sufficient only if the edge plasma is to be included in the simulation. In collaboration with ORNL, we would develop a general interface to generic plasma models that can be used to compute the plasma properties and profile to be input into ARGUS.

2. Include the macroscopic internal sheath physics model into ARGUS for use in the vicinity of the Faraday shield. This model would be derived from SAIC’s continued microscopic sheath physics analysis, as described in Sec. 4.2.2. Since the plasma-response is in the form of a complex dielectric tensor, this leads to a natural way to include the macroscopic sheath model in the tool.

3. Incorporate the warm plasma dielectric tensor into the frequency-domain electromagnetic solver, if the warm plasma in the interior of the tokamak is to be included in the simulation. This is not expected to present any significant difficulty, with the usual caveat that sufficient caution must be exercised to handle any new wave-plasma resonance introduced by the warm plasma dispersion. An additional interface to input plasma temperature profiles must be developed in collaboration with ORNL.

4. Address methods for better resolution of fine structures such as the Faraday bars. This plays an important role in the tool’s ability to properly address issues such as far-field antenna penetration by allowing a coarser resolution of the structure without losing accuracy, more accurate sheath modeling, and improved surface power dissipation modeling.

5. Include resistive-wall power dissipation for improved modeling of power loss effects.
6. Make the ARGUS tool more user-friendly, such that users can more easily build and analyze a new structure, or make changes to currently modeled structures more easily.

7. Transition and support of the tools use to US fusion labs and universities.

8. Include the plasma-response model into the frequency-domain eigenmode solver for analysis of resonant-antenna/plasma systems.

9. Port the tool to commonly-used UNIX workstations

10. Work toward building a US Team that would analyze, design, and build ICRF rf systems, such as for ITER. For example, a Concurrent Engineering Team could be assembled for this purpose. Along with these recommended improvements we would continue with benchmarking tests, develop appropriate diagnostics, provide links to other codes such as with ORION and ANSAT, and define the limits of applicability of the simulations to realistic situations.

For the long term, there should be continued support of the community, and the application of ARGUS to practical ICRF antenna design and development by collaborating with fusion laboratories and universities on applicable antenna problems as required. For the practical design problems, we should continue work with ORNL to integrate ARGUS into the ORNL antenna design, analysis, and evaluation system. It would be SAIC’s intent to continue the collaboration that we have had with the US fusion community. This should strengthen as ARGUS integrates into the design community. As needs arise, however, we would continue to adapt ARGUS to realistic antenna design requirements.

Finally, with appropriate modification to the particle-in-cell module, we may also be able to study the self-consistent interaction between test particle motion (e.g., high energy particles emerging from the plasma), antenna driven field, and sheath formation.

4.2.2. Numerical Simulations and Theory of Sheath Physics in 1-D and 2-D

One of the central issues in ICH is how much power is dissipated in the vicinity of the FS and where that dissipated power goes. The first question affects the overall efficiency of the ICH in two ways: directly (power loss) and indirectly (modification of the edge fields may affect propagation to the plasma core). The second question is related to important design considerations. For example, the fraction of the loss that goes into ion bombardment onto the FS determines whether there is need to have water-cooled FS. Additionally, for high-power ignition devices, damage estimates from this same cause must
be determined. The differences between 1-D and 2-D scaling affect another issue, namely deciding whether there is an advantage to operating with antenna limiters instead of FS bars. In this case, in essence, one chooses between the 1-D sheath scaling applicable to plasma limiters vs. 2-D scaling applicable to the FS bars.

The ANSAT tool designed to address such issues works in two steps. First it maps the induced rf voltages over the antenna bay surfaces using the field patterns obtained from ARGUS. Then it uses the result to compute the local DC sheath potentials and dissipated power, assuming the simple relation $V_{dc} = 0.43 V_{rf}$ obtained from 1-D unmagnetized sheath theory. We have already established in Secs. 3.2.2 through 3.2.4 that this simple relation between $V_{dc}$ and $V_{rf}$ needs to be amended to include magnetic field effects, 2-D effects, and losses from scattering into short wavelengths. Thus the goal of the recommended effort on sheaths is twofold: first, gain a thorough understanding of the sheath physics; and, second, apply SAIC’s models to improve the ANSAT/ARGUS design tool.

In the area of sheath physics, we recommend that the following tasks be undertaken:

1. Investigate the magnetic inclination effects on the sheath physics, retaining the 1-D sheath picture. Simulations using the MASK code modeling a parallel plate capacitor configuration with infinitely long plates would be undertaken over a wide parameter range relevant to ICH. A new relation $V_{dc} = 0.43 f(\theta, B) V_{rf}$, where $f(\theta, B)$ carries the magnetic field dependence, would be derived numerically and/or analytically and would be implemented into the ANSAT tool.

2. Continue the 2-D numerical simulations of sheaths around Faraday bars in order to determine the parameter range of importance for 2-D effects. The current MASK version, employing the cap-matrix approach for the bar structure, uses periodic conditions in both directions for the exterior boundaries. The ES solver of MASK would be upgraded either to CG, GMRES, or to the new solver developed for ARGUS, in order to handle more realistic boundary conditions in the direction perpendicular to FS.

3. Extend 2-D simulations long enough to achieve steady state. Preliminary results suggest that the 2-D sheath flow pattern expels most of the plasma from between the FS bars, leading to much lower density and dissipation rates on FS than the 1-D scaling. If indeed 2-D sheaths are weaker than 1-D, there may
be no advantage to operation without FS bars (and with antenna limiters), since, in essence, one replaces the weak 2-D sheath effects on the FS bars with strong 1-D sheath effects on plasma limiters.

4. Use microscopic sheath physics analysis to derive appropriate boundary conditions to treat sheath effects in large scale simulations, such as ARGUS, so the small sheath spatial scales would not have to be resolved. Prescription of the appropriate volumetric impedance from all dissipation channels, including short-wavelength scattering, would allow us to introduce sheath effects during ARGUS modeling using the plasma dielectric response.

5. Determine sheath saturation. The rf potential driving the sheath is usually given by the integral of $E \cdot dl$ over the length of the magnetic line connecting two surfaces. It has been suspected, but not yet proved, that a maximum connection length exists such that further increase of the distance between surfaces does not affect the sheath potential. It is important to investigate this issue for antenna bay structures with large connection lengths ($\sim 1$ m).
5. References


Appendix A  ICRF Impurity and Antenna Studies (IAEA-CN-53/E-III-1, Reference 27)
ICRF IMPURITY AND ANTENNA STUDIES*

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Abstract

ICRF IMPURITY AND ANTENNA STUDIES.

The paper summarizes recent work on three related topics: (i) Recent experiments and theory of metal impurity influx from the Faraday screens (FS) in JET are compared and show good agreement in both magnitude and scalings with key parameters. The theoretical model describes the acceleration of plasma ions by induced RF sheaths on the screen and the resulting sputtering of the screen material. It is shown that metal impurities can be reduced to negligible levels by proper antenna design and the use of a low atomic mass screen coating. (ii) Experiments on RF sheath formation at the FS of a fast wave ICRF antenna in Phaedrus are described. It is shown that the radial feeders of a typical two-strap fast wave antenna induce a large voltage on the FS with respect to an adjacent limiter. The magnitude of the voltage varies with strap phasing. Experiments with a low power single strap antenna mounted in the Phaedrus-B central cell demonstrate RF self-biases at the FS. (iii) A computational investigation of the vacuum properties of ICRF antennas is reported on. The numerical simulation code, ARGUS, is described briefly and results from a study of the PPPL TFTR ICRF antenna are given.

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1. IMPURITY PRODUCTION BY ICRF ANTENNAS IN JET

The release of impurities during ICRF heating can be attributed to two kinds of mechanism: global production caused by additional power flow in the scrape-off layer (SOL) and local release from the vicinity of the antenna. The latter process is particularly important in understanding the origin of metal impurities in ICRF experiments with good single pass absorption. Here we describe a comparison of recent experiments [1] and theory [2] on the metal impurity influx from the JET Faraday screens (FS), which are the major source of metal impurities in JET ICRF experiments. The main conclusions from this study are that (i) metal impurities can be reduced to a negligible level by proper antenna design, and (ii) an RF sheath based impurity model provides a framework for understanding the local impurity release and for designing optimal antenna systems in ICRF experiments.

Different antenna designs and FS coatings have been used on JET. Substantial local impurity influxes, seen in early operation, have been reduced to negligible levels in the latest series of experiments. The experimental and modelling results suggest that RF sheaths are responsible for the FS impurity release. These RF sheaths [3] are caused by RF flux linkage through circuits that include magnetic field lines in the plasma which intersect the conducting surfaces of the FS gaps and front face. The sheaths non-linearly rectify the induced RF voltage, and the rectified (dc) potential accelerates ions into the FS with sufficient energy (typically, keV) to cause physical sputtering of impurities. The impurity release model [2] couples SOL, RF sheath and sputtering physics self-consistently. The model includes sputtering by the majority species, one species of limiter impurity, and self-sputtering by the FS material.

The measured metal influx on JET depends on the antenna voltage, the edge density, the magnetic field angle, the phasing of adjacent current straps, the shape of the screen and the FS coating, as well as the extent of limiter conditioning and the impurity content of the SOL plasma. Many of the observed dependences are straightforward consequences of the model:

(a) For FS materials with high atomic mass, the model predicts a large increase of the metal influx at high voltage due to high energy self-sputtering and sputtering by heavy impurities. This prediction is in qualitative agreement with data [4] from a Cr-plated FS, which indicates an influx that is two to six times larger than that obtained in the recent Be evaporation experiments [1].

(b) In dipole phasing, the model predicts that sheath potentials on the front face of the Aₐ (V-shaped) FS are greatly reduced because of symmetry, which annihilates the net RF flux driving the sheaths, leaving only a smaller influx induced by gap sheaths, consistent with the observations.

(c) The calculated influx in monopole phasing increases with FS angle (relative to the equilibrium B field) because more RF flux is captured in the circuit containing the sheaths. This is in qualitative agreement with reversed field experiments on JET [5].
The model calculations suggest that self-sputtering contributed substantially to the observed influxes on JET with Cr and Ni screens.

The impurity release model also shows good quantitative agreement with recent Be evaporation experiments [1], where the FS was coated with Be (Fig. 1). The Be influx from one screen was measured spectroscopically, along a line of sight intercepting the screen, by means of the Be (4573 Å) line intensity. These measurements were obtained for monopole phasing with C limiters and with Be limiters. An absolute calibration of the Be influx $\phi_{\text{Be}}^{\text{FS}}$ for the C limiter case was obtained from erosion rate estimates [1] ($(1-2.5) \times 10^{19}$ atoms·MW⁻¹·s⁻¹) and from Be concentration measurements ($(2-4) \times 10^{19}$ atoms·MW⁻¹·s⁻¹). The spectroscopic Be influx data is illustrated in Fig. 1 for the normalization $2 \times 10^{19}$ atoms·MW⁻¹·s⁻¹, which represents a good compromise between these two experimental methods, with a power level of 1 MW per antenna corresponding to an antenna line voltage of 18 kV. The experimental data illustrate the important voltage and (indirectly) density dependences of the impurity influx.

The theoretical model [2] and the spectroscopic data both yield the neutral impurity influx at the FS before screening of the impurities by ionization occurs. The solid curves in Fig. 1 are the result of detailed calculations in which we have used the data and model together to infer relationships among the antenna voltage, the edge density and the SOL diffusion coefficient. The inference from this procedure is that the voltage dependence of the experimental influx is primarily due to the increase in

![Graph](image)

**FIG. 1.** Be influx (measured by the intensity $I_{\text{Be}}$ at the FS) versus antenna line voltage $V_1$ for the two cases of C (•) and Be-belt (○) limiters with monopole phasing. The data are normalized to the estimated flux $\phi_{\text{Be}}^{\text{FS}} = 2 \times 10^{19}$ atoms·MW⁻¹·s⁻¹ and the data for the C limiter case are corrected for erosion decay. The solid lines represent the theoretical model calculation.
edge density with RF power (which has been determined from probe data) and to the increase in the local diffusion coefficient with RF voltage (assumed here to be linear). The sputtering yield of Be is insensitive to energy (and hence to antenna voltage) in the range corresponding to Fig. 1. The lower influx with Be limiters was modelled by assuming a reduced rise of edge density with RF power and a reduced fraction of limiter material available to sputter the screen (motivated by the observed gettering of O and pumping of D by the Be limiters).

2. ICRF ANTENNA STUDIES IN THE PHAEDRUS PROGRAMME AND NUMERICAL MODELLING OF TFTR ANTENNAS

2.1. Phaedrus studies

RF sheaths at the Faraday shield are now suspected to be responsible for impurity production during ICRF heating experiments. Attention first focused on sheaths in the gaps between FS elements [3]. It has recently been shown for JET that more important sheaths occur along field lines which traverse the face of the JET FS (see the preceding section of this paper and Ref. [2]). Here, we discuss an additional mechanism of sheath generation. We show that the azimuthal component of the B-dot fields generated by the radial feeders of a fast wave antenna can generate inductive RF voltages, and therefore rectified potentials (self-biases) between the FS and the nearest limiter.

The mechanism is illustrated in Fig. 2(a). An inductive voltage is generated along the path ABCD, owing to the radial feeder fields. The net flux through ABCD is clearly a function of antenna strap phasing. An in-air demonstration of this mechanism has been performed by using the two-strap antenna constructed for ICRF experiments in the Phaedrus-T tokamak. A capacitive probe is used as the voltage sensing element, and a conducting path is chosen to mimic the FS–wall–limiter geometry illustrated in Fig. 2(a). Results for three antenna phasings are shown in Fig. 2(b). Note that the inductive voltage is reduced by a factor of four as phasing is varied from 0° to 180°. The voltage peaks at the ends of the antenna straps, where the radial feeders are located. This antenna–limiter geometry is similar to the TFTR Bay M antenna–RF limiter geometry and is a possible explanation for the discharge phenomena observed on the Bay M antenna during in-phase operation of that antenna [6].

We have also measured the plasma floating potential near the FS of a low power (≤ 3 kW) single strap model antenna using the central cell of Phaedrus-B ($n_e(0) \leq 5 \times 10^{12} \text{ cm}^{-3}$; $T_i$, $T_e \sim 20–40 \text{ eV}$) as a source plasma. The antenna operates at $2.7 \Omega_{ci}$ or 3.5 MHz. The FS bars of the model antenna are tilted 3° with respect to the static magnetic field to allow probe measurements on field lines which terminate on the bars (see Fig. 3(a)). The shield is in the limiter shadow; field lines originating on the FS terminate on the limiter. In Fig. 3(b), we show a plot of the increase in
floating potential when the model antenna is excited, as a function of probe position as the probe is scanned across the FS gap region. The peak increase in floating potential occurs when the probe is located on a field line which terminates at the centre of the FS. Only small changes in the floating potential are seen when the probe is on a field line which transits the FS gap without terminating on the shield structure.

The increase in floating potential produced by the RF scales linearly with the antenna strap current. The (grounded) FS collects electrons during antenna operation from the biased scrape-off layer plasma, as measured with a current transformer. A frequency spectrum of the collected current shows strong non-linear generation of harmonics, with the harmonic at $2\omega_{RF} < 3$ dB down from the fundamental. Strong harmonic generation has also been seen in FS experiments and capacitive probe measurements in the SOL of TEXTOR [7] during ICRF heating.
2.2. TFTR modelling

The successful application of RF heating to future fusion plasmas generated in CIT and ITER size devices will require power levels that are much higher than those in use today and, consequently, the design of suitable antennas will require the ability to predict the antenna performance accurately. We report here on progress made in developing a modelling and simulation capability for the vacuum characteristics of ICRF antennas that will play a role in future antenna design and evaluation.

The SAIC ARGUS system of codes [8] used for the ICRF antenna studies presented here is modular and contains both electromagnetic and electrostatic field solvers in the frequency and time domain, a PIC module and a fluid plasma module.
that self-consistently contribute to the field solver, as well as many other features that model realistic physics of devices such as secondary emission from surfaces. ARGUS allows fairly arbitrary and complex geometrical structures to be simulated and, by means of block decomposition techniques, large data structures resulting from finely gridded computer models can be accommodated.

Experimental benchmarking of ARGUS applied to ICRF antenna configurations has been carried out in collaboration with the University of Wisconsin. The experimentally determined vacuum characteristics of a model antenna geometrically similar to the TFTR design were compared with the results from an ARGUS simulation of the laboratory test apparatus. The agreement between the computed and measured results was excellent. It was found that the currents flowing in the antenna strap and other parts of the antenna configuration (such as the backplane on which the test apparatus was mounted) should be calculated self-consistently for the actual antenna radiation pattern to be captured. In fact, because of the finite three-dimensionality of the experimental test set-up, certain features of the experimentally observed results could not be understood without the simulation model results.
FIG. 5. (a) Toroidal RF field ($B_{z}^{\text{RF}}$) profile in front of the current strap at midplane (0–$\pi$ phasing); (b) toroidal RF field ($B_{z}^{\text{RF}}$) profile in front of the Faraday shield at midplane (0–$\pi$ phasing).
More recently, ARGUS has been used to model the full PPPL antenna design for TFTR. Figure 4 shows a view of a slice through the upper right quadrant of 1/4 of the double strap antenna. Shown are the current strap, the power lead and the bottom row of Faraday bars. Also shown are the end and side walls. Not shown, but included in the model, is the centre septum. Simulations of this antenna model have shown a strong similarity to the results of the UW model antenna. The double strap antenna has been simulated for both symmetric and antisymmetric current phasing. Noticeable differences between the two phasings are present in the simulation results, but it is not known at present how these affect antenna performance. An example of the results from the present study is shown in Fig. 5. Figure 5(a) shows the toroidal RF magnetic field measured in the toroidal direction at the front of the strap at the midplane for antisymmetric phasing; Fig. 5(b) shows the same information measured in front of the FS. The effect of the actual antenna geometry is clearly displayed in the curves shown in Fig. 5. Other information available from the simulations includes all RF electric and magnetic fields generated, strap impedances, coupling coefficients, power spectra as a function of toroidal and poloidal wavenumber on the same surface, strap current distribution both along and around the strap, and all these quantities as functions of input parameters such as port voltage and frequency. ARGUS can model most types of antennas including those used for IBW heating.

REFERENCES

Appendix B  Numerical Modeling, Analysis, and Evaluation of ICRF Antennae (EPS-Berlin, Reference 29)
NUMERICAL MODELING, ANALYSIS, AND EVALUATION OF ICRH ANTENNAE

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Introduction. Most analyses of ICRH antennae and rf plasma heating studies are carried out using combinations of one- and two-dimensional plasma-antenna models. These studies (see e.g. ref. 1) have provided valuable insights to antennae designers and have also been successful in analyzing the results of current ICRH experiments. In spite of the usefulness of today's simulation capabilities, reactor antennae will require more robustness and fidelity in the model predictions if survivable antennae are to be designed and built. To this end, SAIC is developing a three-dimensional simulation capability of rf antennae that will allow rf performance (with and without plasma) and structural and thermal analyses to be carried out on the same common geometrical representation.

Simulation Codes. SAIC's MASK (2-D) and ARGUS (3-D) system of codes is fully electromagnetic and consists of modules that couple fluid and PIC solvers to treat complex geometrical situations involving variable media. External boundary conditions ranging from simple metallic walls or periodic boundaries to boundary conditions describing ingoing or outgoing radiation can be imposed on the simulation. Internal material regions (structures) can be specified on the grid, either as perfect conductors or as regions of specified complex tensor material properties. The two codes can treat a wide range of complex geometry and plasma/beam interactions. The results of ARGUS ICRH benchmark comparisons with experimental data from vacuum antenna measurements have been reported earlier2-3.

3-D Antenna Simulation. The numerical grid resulting from application of the ARGUS geometry package on the double strap TFTR Bay M antenna is shown in Figure 1. Dimensions and details of the antenna were obtained from engineering drawings supplied by the Princeton Plasma Physics Laboratory. The figure shows a cutaway of a quadrant of the antenna (symmetry used to reduce computing requirements) with current straps, Faraday rods, end plates and center septum. Power leads to the antenna are simulated through a gap in the back plate fed with an applied voltage at the actual frequency. ARGUS calculates all rf electric and magnetic field components, current distribution in straps, Faraday rods and walls. Derived quantities such as strap current phase velocity and mutual inductances (multiple straps) can be calculated from the ARGUS
output. In this paper, we present results from an analysis of the near field power radiated away in vacuum from the antenna in the radial direction. Figure 2a shows the Poynting flux and 2b the poloidal electric field component (in arbitrary units) for the PI phasing of the current straps. Figures 2c and 2d show the results for the in phase case. The figure shows the full antenna radiation pattern obtained by applying appropriate phase boundary conditions across the symmetry planes. Since the toroidal rf magnetic field is roughly uniform along the poloidal direction, the Poynting flux distribution is similar to the poloidal electric field. The peaks or lobes in the electric field distribution are due to the spatial gaps between the last Faraday rod and the antenna end plane. Simulations of other antennae lacking the large gap show significantly reduced lobes. A Fourier analysis of the poloidal electric field in Figure 2 shows that most of the radiated power is in the main toroidal wave numbers with a small amount of energy spread over poloidal wave numbers.

2-D rf-Sheath Formation. Sheath formation around the Faraday screen bars during intense rf heating is a well known effect. Analytical and numerical efforts to model rf sheaths in magnetized plasmas have, to date, been focused on one-dimensional models. The rf-driven sheath theory in unmagnetized plasmas leads to a result similar to Langmuir-Childs law for dc discharge sheaths. The sheath width is $\Delta_s = 2\lambda_p (eV/kT)^{3/4}$, replacing the dc potential with the time-averaged, rectified rf potential $\langle v \rangle$. The theory, valid for $\lambda_p \ll \Delta_s$, assumes an electron density boundary, oscillating with the rf and ions responding to the averaged sheath potential. It has been argued that the model remains valid in magnetized plasmas with magnetic field lines intercepting the conducting surface.

The main limitation of 1-D models in magnetized plasmas is that transport occurs only parallel to the magnetic field. If the plasma-boundary interface is along the X-direction, with the sheath across the Y-direction, no electrostatic perturbations with finite $k_y$ can exist in 1-D. The canonical momentum $P_y$ and consequently the $Y_{GC}$ position of the guiding center are exact invariants, prohibiting cross-field transport. In reality, however, the opposite $E_yX_B^z$ drifts among electrons and ions excite strong electrostatic perturbation of finite $E_y$, $k_y$. This destroys the $Y_{GC}$ invariance and introduces considerable cross-field transport.

A two dimensional simulation of rf sheath formation in a magnetized plasma has been undertaken, using the PIC module in the SAIC MASK code. The simulation plane $xy$ is a cross-section of one screen bar, $x$ corresponding to the radial and $y$ to the poloidal direction. The fields are purely electrostatic. Presently, the driving rf field is modeled by an alternating voltage between the top and the bottom metal plates. In a new code version, the capacitor field with the rod inside is stored as the externally applied field, while the plasma sees only the metal bar boundaries in the center, and periodic boundary conditions in both $x$ and $y$. 
Plots of equipotential surfaces for typical runs A and B are shown in Figure 3a and 3b respectively. In both cases we use $m_i/m_e = 180$ with real electron mass, and $T_e = T_i = 16$ eV. Case A corresponds to high density $n_0 = 3.75 \times 10^{10} \text{cm}^{-3}$, $B_{zo} = 0.5$ T, and magnetic field lines tilted 15° relative to the toroidal direction, $B_{yo}/B_{zo} = 0.25$. Case B corresponds to low density $n_0 = 1.60 \times 10^9 \text{cm}^{-3}$, $B_{zo} = 0.3$ T, and $B_{yo}/B_{zo} = 0.25$. The applied external voltage amplitude is 500 V in both cases. The parameters chosen mimic well the experimental situation. Thus we have in both cases $\Delta_s > \rho_i > \lambda_D$ and $\Omega_i < \omega_{rf} < \omega_{\text{DI}}$ with $\Omega_i \sim \omega_{\text{pe}}$.

A sheath with strong potential gradients localized near the bar surface, as well as at the top and bottom boundaries, is formed in Case A. In contrast, no strong sheath formation can be seen in Case B, although the plasma thickness should be enough, according to the 1-D result to, to rectify the applied potential. Instead, a potential "cell" with dimension $\Delta_x \sim \Delta_y$ is formed in the presheath area.

We speculate from the results in A and B that when the predicted 1-D thickness of the sheath becomes comparable to the bar cross-section, anomalous ion transport becomes important. This is consistent with the scaling $E_x/E_y \sim \Delta_y/\Delta_x$ and the fact that cross-transport is induced by $E_y$. Thus 2-D effects dominate when the sheath thickness is comparable to its length. Ion transport is enhanced to the point where the steady state condition $J_e \approx J_i$ is maintained without a strong potential barrier to slow down electrons, i.e. without electron depletion. In the high density Case A, however, $\Delta_y \ll \Delta_x$ and the simulation results seem to be in agreement with the 1-D theory.

In cases of strong sheath formation, strong ion energization localized inside the sheath has been observed. The ion energies exceeded the sheath averaged potential $\langle V \rangle$ by a factor of 3. A stochastic acceleration theory for magnetized ions has been proposed to account for the effect. Additionally, considerable electron heating is observed to be uniform throughout the simulation area.

**Conclusions.** The utility of ARGUS for modeling future reactor configurations lies in the ability to use a common geometry data base for rf performance, and for structural and thermal analyses. A plasma module has recently been added to the ARGUS antenna simulator, allowing antenna plasma loading to be evaluated. The antenna simulations will be applied to the PBX experiment and other future devices such as ITER. The 2-D rf sheath formation simulations will continue to examine different Faraday rod cross-sections, allowing optimum Faraday rod designs to be evaluated under realistic operating conditions.

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Appendix C  Numerical Modeling of the TFTR ICRF Antennas
(AIP Conf., Reference 30)
NUMERICAL MODELLING OF THE TFTR ICRH ANTENNAS

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ABSTRACT

A general purpose 3D electromagnetic field solver code, ARGUS, is being used to analyze the TFTR Antennas. To date, the vacuum radiation patterns produced by the bay M and L antennas have been obtained and reported. Recent work has concentrated on antenna performance comparison and understanding the role of geometry on performance (e.g., the impact of end-effects on current 2D models). Additional diagnostics such as evaluation of phase velocity and strap inductance are being implemented to enhance our understanding and to better compare with measurements.

INTRODUCTION

To date, the most useful theoretical analyses of ICRH antennae and rf plasma heating have been carried out using combinations of one- and two-dimensional plasma-antenna models. These studies (see e.g. Ref. 1) have provided valuable insights to antenna designers and have also been successful in analyzing the results of current ICRH experiments. In order to design, analyze and evaluate antennae proposed for future fusion reactors—where conditions will be extreme—we will require more robustness and fidelity in the model predictions. To this end, SAIC is developing a fully 3-D simulation capability that allows detailed modelling of realistic ICRF antennas. Our research thus far has concentrated on vacuum operations. The goals are to validate the simulation modules in place (through comparison with experiments), to learn to better interpret our results, and to investigate features of the vacuum field solutions which may directly impact antennae performance with plasma. This paper presents the present status of simulation efforts associated with the ICRF antennas on the PPPL TFTR experiment: namely, the Bay L and M antennas (early results have been presented elsewhere2-4).

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THE ARGUS SIMULATION CODE

The ARGUS code\textsuperscript{5-6} is a general-purpose three-dimensional simulation code designed to handle problems involving complex geometric structures. Nonuniform grids are used in all three dimension to efficiently capture fine structural details. Presently, the code computes electrostatic and electromagnetic fields, the eigenmodes of rf structures, and can perform PIC simulation in either a time-dependent mode or a steady-state mode. The PIC modules include multiple particle species, the Lorentz equations of motion, phenomenological terms for elastic and inelastic scattering, and algorithms for the creation of particles by emission from material surfaces, injection onto the grid, and ionization. Imposition of the boundary condition is highly flexible: either metallic walls, periodic boundaries, or port conditions describing in-going or outgoing radiation can be imposed. Internally, material structures can be defined on the grid as regions of specified material properties (e.g., perfectly conducting, dielectric constant, resistivity, or permeability).

ARGUS has been successfully applied to and benchmarked with a broad spectrum of problems. The electrostatic solvers have been used as stand-alone codes to analyze the dc component of the fields in a cross-field amplifier tube. They have also been used with the PIC module to carry out steady-state simulations of electron guns, depressed electron collectors, ion guns, and low-energy electrostatic ion accelerators. The electromagnetic modules in ARGUS have been used to carry out cold-test calculations for microwave coupled-cavity tubes and accelerator structures. Calculations with lossy materials have been carried out to model severs in a microwave amplifier and high-order-mode dampers in a gyroklystron amplifier circuit.

CODE VALIDATION STUDIES

Initial attempts at validating ARGUS for application to ICRF antennas were concentrated on the University of Wisconsin model antenna\textsuperscript{2-3}. Figure 1(a) shows a numerical grid representation of half the antenna structure as configured on the test-stand. In Figure 1(b), we compare the computed versus measured vacuum toroidal rf B field taken above the strap surface as a function of toroidal position. The agreement is excellent. Notably, this work also revealed that current flowing in the support structures (such as the backplane) can significantly affect the antenna radiation pattern, thus reaffirming the value of full 3D simulations.
Fig. 1. Comparison between ARGUS and experiment: (a) numerical grid representation of half the University of Wisconsin antenna; (b) computed versus measured vacuum toroidal magnetic field, $B_z$, taken above the strap surface as a function of toroidal, $z$, position.

**TFTR ANTENNA SIMULATION**

Recently, we have used ARGUS to model the ICRH antennas at TFTR. The numerical grid representation of the double strap Bay M antenna is shown in Figure 2. Only the upper right quadrant is modeled; symmetry conditions are used to reduce computing requirements. The figure shows a cutaway of the antenna with current straps, Faraday rods, end plates and center septum. Not shown, but included in the model, are the side walls. Power leads to the antenna are simulated through a gap in the back plate fed with an applied voltage at the actual frequency (typically driven at 47Mhz). ARGUS calculates all rf electric and magnetic field components, current distribution in straps, Faraday rods and walls. Derived quantities such as strap current phase velocity and mutual inductances (multiple straps) can be calculated from the ARGUS output.

Fig. 2. Numerical grid representation of the TFTR bay-L antenna.
We now present results from an analysis of the near field power radiated away in vacuum from the antenna. In particular, we evaluate the Poynting flux through a vertical flat plane separated by two different distances from the front of the current strap (~10.5 cm and 12 cm from the center short). Figure 3 compares their temporal evolution for the 0-phasing (two straps driven in phase) case. Note that the Poynting flux rapidly decrease with distance away from the antenna face. This indicates that spatial Fourier analysis of the power spectra on a surface will sensitively depend on the shape of the surface; great care is required when interpreting those results.

![Figure 3](image.png)

**Fig. 3.** Comparison of Poynting flux through a vertical flat plane at (a) approximately 10.5 cm and (b) approximately 12 cm in front of the model bay-L current strap. The distances are measured from the center short.

From the ARGUS computed field solutions, we can indirectly evaluate the current phase velocity and strap inductances. The phase velocity is obtained by fitting a cosine function to the current distribution along the strap. Figure 4 shows the strap current distribution for both zero- and π-mode operations along the curved front section of the strap starting from the center short. The rapid fall-off of the current at the right side of the curves is due to the

![Figure 4](image.png)

**Fig. 4.** Current distributions along the front strap of the bay-L antenna. The amplitude in amps do not represent realistic current levels.
change in direction of the strap current; there, the vertical front section is
connected to the horizontal feed. The small scale fluctuations are due to
grid discretization. From this current distribution, the phase velocity is
estimated to be 0.57c, where c is the speed of light in vacuum.

For the strap inductance evaluations, we use the relationship
between strap current and total magnetic energy, \( W \), where

\[
W = 0.5 \, L \left( I_1^2 + I_2^2 \right) + M \, I_1 \, I_2 .
\]  

Here, \( I_1 \) and \( I_2 \) are the currents flowing the two straps respectively, \( L \) is
the self-inductance and \( M \) is the mutual-inductance. Using both zero-
mode and \( \pi \)-mode results, we obtain two linearly dependent equations
from which we solve for the inductances. For the model Bay L antenna,
the results are \( L = 2.4 \times 10^{-7} \) henry, and \( M = 4.9 \times 10^{-9} \) henry. There is,
however, one caveat: the current is nonuniform along the strap. For
these calculations, we opted to use the current value at the backplane.
Future calculations of the inductances will require running the
simulation in a steady state such that the current distribution will be
uniform.

CONCLUSION

The utility of ARGUS for modeling future reactor configurations
lies in the ability to use a common geometry data base for rf
performance, and for structural and thermal analyses. While we have
demonstrated some of its existing capability here, significant
diagnostic/code upgrades and further validation studies are underway.
Briefly, ARGUS results will be benchmarked against measurements
done on mockup DIII-D and TFTR antennas at ORNL. An idealized
antenna model is being used to elucidate the role of end-effects on
current 2D models. A frequency domain solver is being implemented
that will significantly reduce the computational time, and facilitate the
addition of the plasma simulation module. And for diagnostics, we are
installing the capability to evaluate the \( k_{11} \) power spectrum on a flux
surface.

ACKNOWLEDGMENT

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ITER RF DESIGN FINAL REPORT

Date: May 13, 1995
Task Number: D-89
Task Title: ITER R.F. Design
Authors: Y. L. Ho (Science Applications International Corporation) and P. M. Ryan (ORNL, U.S. HT contact on task D-89)

1. Introduction

The purpose of this task is to calculate general ICH antenna electrical properties such as characteristic impedance of the current straps, mutual coupling between antenna elements, and antenna loading over the frequency bands of interest. The analysis is carried out on ITER antenna designs as provided by the ITER JCT at Garching through the ORNL ICH system design team. For this task, the target design is the 18 strap multi-frequency antenna designed to operate at the nominal resonant frequencies of 22, 44, and 66 MHz. This report describes the results of our analysis for an antenna operating in vacuum. Plasma loading calculations are still ongoing and will be reported later.

As the ICH antenna currently envisioned for ITER is significantly different from conventional designs, it is recognized that analysis techniques will need to evolve as our understanding of the antenna improves. For example, given that the antenna operates near resonances, we've found it useful to utilize the Eigenmode solver in the ARGUS code package to perform the vacuum field analysis [1]. The solver finds Eigenmodes in a computational domain given metal, symmetric, or periodic boundary condition with arbitrary phase shift. The different boundary conditions permit us to model the various phasings possible for an infinite array antenna, e.g., 180° (0-π-0-π...), 90° (0-0-π-π), 20° (1 period extends 18 straps). The change in the dispersion characteristics of the antenna due to phasing variation gives information on the mutual coupling coefficients.

In conjunction with the Eigenmode solver, we also utilized the single frequency solver. Here, the antenna is excited at a specified frequency by applying a driving voltage at the feed port. By driving the antenna at different frequencies and different phasing, we can analyze the field solution using a transmission line model and obtain relevant antenna electrical properties. In addition, the rf-electric field solutions were provided to D. A. D’Ippolito (Lodestar Research Corporation) to help assess rf sheath and impurity generation under ITER task D-90.

As is customary for numerical analysis of this nature, solution checks are necessary. The most typical are tests of solution convergence with respect to grid
resolution and computational domain size. Here, we also check the effect of ignoring the antenna width variation and possible effect of the feed tap position.

For the plasma loading calculation, we performed loading versus distance analysis for the 90° phasing case at 22 MHz. The scope of this work is limited for two reasons. First, at higher frequency and with plasma present, the system is always near some resonances which makes the operator to be inverted singular. Presently, we have no way of dealing with such resonances. Second, the novelty of this antenna means that our understanding of its behavior using a non-idealized model is limited. As a result, rather than a complete loading analysis, the focus of this work is to provide some information on the loading characteristics for comparison with idealized model. In the process, we provide additional insight into the antenna's behavior.

2. Eigenmode Analysis

The Eigenmode analysis is performed on a rectilinear antenna model. Two poloidal cross-sectional views of the antenna, as represented by ARGUS, are shown in Figure 1. The rectilinear model is chosen for two reasons: to be able to model an infinite array via symmetry and phase shifted periodic boundary conditions, and to match the plasma surface defined on a plane. The second approximation is easily justified since the poloidal curvature of the design antenna is small. The effect of the first approximation will be discussed in the section on solution checks.

Figure 1(a) also shows a port in the back of the antenna through which the antenna is tapped and excited. At the frequencies in question and without plasma absorption or antenna Ohmic losses, the antenna should operate in resonance with negligible current in the tap. Thus, the Eigenmode analysis is performed on a structure with the tap removed. Possible effect of the tap on the resonance characteristics of the system is also discussed in the Solution Check section. The basic dimensions of the antenna is shown in Fig. 2.

In this analysis, the 3 lowest frequency Eigenmodes (corresponding to poloidal wavelengths of 7m, 3.5m, and 2.33m for the fields around the straps) are derived for each possible array phasing. Only one strap is modeled. To model a large array, suitable boundary conditions are applied at the symmetry plane separating the antennas to simulate the different array phasings possible. Four cases are considered:

a) 180° (π) phasing between straps, modeled by anti-symmetric boundary conditions (i.e., "electric walls").
b) 0°, 0°, 180°, 180° phasing (to approximate a 90° phasing case), bounded by one symmetric (i.e., "magnetic walls") and one anti-symmetric boundary.
c) 20° phasing between straps modeled by forcing a 20° phase difference between the complex field solution at the boundaries.
d) 10° phasing between straps.
Fig. 1. ARGUS rectilinear representation of the ITER 'violin antenna'. Poloidal cross sections of a single strap (of an array) are shown: (a) view showing the feed at the back of the strap; (b) different view showing the side slits.
3. Single Frequency Solver with Transmission Line Model Analysis

For this work, ARGUS solves the finite-differenced 3D vacuum Helmholtz equation \( \nabla \times (\nabla \times \mathbf{E}) = (\omega/c)^2 \mathbf{E} \) in the region around the antenna. The antenna
Fig. 3. Antenna resonant characteristics for the rectilinear antenna model shown in Fig. 1, but without the feed tap: (a) resonant frequency, $v$, versus poloidal wavelength, $\lambda$, and (b) inferred phase velocity in units of light speed, $c$, versus $\lambda$. Results are shown for 4 different toroidal array phasings: $180^\circ$ (2-strap periodicity), $90^\circ$ (4-strap periodicity), $20^\circ$ (18-strap periodicity), and $10^\circ$ (36-strap periodicity).
was driven at fixed frequencies to obtain additional information about characteristic parameters and to compare with the results of the Eigenmode analysis.

At 50 MHz, with metallic-symmetric boundary conditions on the toroidal boundaries, analysis of the currents and voltages along the strap gives a characteristic impedance of 27 ohms and a phase velocity of 0.64 c. The inductive coupling can be calculated by applying alternate boundary conditions to change the phase relationship between adjacent straps. The coupling coefficient calculated by this method is 5.6%, which is in general agreement with that calculated by Eigenmode analysis of the case with no radial feed.

The resonant frequency of the strap was also checked by observing the port impedance while scanning frequency. Fig. 4 indicates that the port impedance changes from capacitive to inductive at around 55.5 MHz for boundary conditions that duplicate (... 0, 0, \pi, \pi, 0, 0, \pi, \pi, ...) phasing for an infinite array. The Eigenmode analysis for a geometry with no radial feed gives a resonant frequency of 56.019 MHz.

![Graph of port impedance vs frequency](image)

Fig. 4. Port impedance vs frequency for (... 0, 0, \pi, \pi, 0, 0, \pi, \pi, ...) phasing.
4. Solution Checks

4.1 Grid resolution

Convergence tests are performed on the 180° toroidal phasing case to verify the adequacy of the grid resolution and the computational domain size (to model an antenna in a vacuum, the boundaries are located far from the antenna). The 180° case is chosen since it has the shortest toroidal wavelength and is thus the most sensitive to grid resolution problems.

In general, the solutions are well converged. For example, doubling the toroidal resolution raises the resonant frequencies by a maximum of 0.78%. The shortest poloidal wavelength mode, 2.33m in this case, demands the highest resolution. For the longest wavelength case, 7m, frequency increase is about 0.6%. Radial resolution and the computational domain size are both more than adequate.

Since ARGUS constructs structures out of rectangular blocks, an early concern is our ability to resolve the cylindrical surface of the Faraday bars. This influences the capacitance, and hence the phase velocity of the antenna. Poloidal grid resolution is the determining factor here as the radial resolution is already very high (to resolve the 0.016m spacing between the Faraday bars and the current strap). To illustrate, in Fig. 5, we compare the cross-sectional view of a Faraday bar as represented using the standard versus the double poloidal resolution grid. Analysis shows that the high resolution cases have higher resonant frequencies: from 0.74% for the 7m poloidal wavelength case to 0.875% for the 2.33m case.

Adding the various sources of errors, our conclusion is that the rectilinear model solutions presented here are accurate to within 2%. In general, increasing the resolution tends to increase the resonant frequency, i.e., increase the phase velocity.

Fig. 5. Comparison of Faraday bar shape at two poloidal resolutions. The left side of each bar faces the current strap and determines the capacitances.
4.2 Effect of the tap

The effect of the tap (c.f. Fig.1a) through which the power can be supplied to the antenna to compensate for losses should not affect the transmission line characteristics of the long section of the current strap (the section above the tap). This is verified by the agreement in the Eigenmode (without tap) and the single frequency solver (with tap) analysis of the key electrical characteristics. However, the tap may significantly perturb the field near the tap and, as a result, alter the resonant frequency of the antenna structure. The actual effect will depend on the size and location of the tap.

As an example test, we compare an Eigenmode analysis with no radial tap to an Eigenmode analysis with a small cross-section radial tap. The tap is placed 0.75m from the cavity bottom. It is .025m thick in the poloidal direction and .048m thick in the toroidal direction (the smallest size our grid resolution can accommodate). Again, the 180° toroidal array phasing case is used for comparison. To do the Eigenmode analysis, it is necessary to ground the tap, hence the configuration is not identical to the driven case. Nevertheless, our result showed that the tap can significantly alter the field in its vicinity: resonant frequency increased by as much as 14% for the 7m poloidal wavelength case (c.f., Fig. 6). The increase is smaller for the 2.33m wavelength case, ~3.8%. This behavior is probably caused by an introduction of a mix mode that by itself would resonate at high frequency. The "pollution" of the regular strap modes thus raise the resonant frequencies of the lower modes. A comparison of the radial electric field, $E_r$, pattern for the lowest frequency Eigenmodes is shown in Fig. 7.

4.3 Wedge shape vs. rectilinear shape

The largest approximation assumed by the rectilinear model is the neglect of poloidal tapering in the width of the antenna. In an actual torus, the major radius of the vessel wall varies poloidally. To check the effect of ignoring this tapering, we analyze a wedge-shaped antenna. The cavity width of this antenna is tapered from 0.5m wide at the top (same as the rectilinear model) to 0.4m at the bottom (c.f., Fig. 8). The strap width is tapered from 0.24m to 0.192m. In this case, the only comparison case possible is the 180° phasing case. Result of the Eigenmode analysis is shown in Fig. 9. As expected, the reduction in the toroidal cavity width increased the resonant frequency of the antenna (by ~6%). It can be assumed from this result that a proper rectilinear model of the array antenna should use a width equal to the average of the actual antenna.
Fig. 6. Comparison of resonant frequencies for an antenna with a feed tap versus one without. Results shown are for the 180° phasing case.

Fig. 7. Contour plots of $E_r$ for cases with and without tap. Poloidal cross-sections are shown.
Fig. 8. One strap of the ITER antenna with 20% taper to reflect major radius change in a torus. Faraday shield not shown.
Fig. 9. Comparison of resonant frequencies for the reference rectilinear antenna versus the wedge shaped antenna. Results shown are for the 180° phasing case without feed taps.

5. Plasma Loading Calculation

The plasma loading calculation also utilizes the single frequency solver. The difference from the vacuum calculation is that at the plasma surface, the vacuum field solution is iteratively matched to the plasma surface impedance described by a 2 x 2 complex impedance matrix at each toroidal and poloidal Fourier mode. The elements of this matrix are calculated by the Orion full-wave code [2].

The impedance matching procedure requires that the antenna field be known over the entire plasma surface. It is, however, computationally unfeasible to model an entire 18 strap array in ARGUS with realism. Our approach is to use a semi-infinite array model. In this approach, only one strap is modeled. At the plasma surface, the field is repeated 18 times. Depending on antenna phasing, appropriate reflective symmetry operations are applied to the fields. A filtering procedure is applied at the ends to force the fields to fall to zero smoothly.

To compute the loading, we use the relation $R = S/I^2$ where $R$ is the loading, $S$ is the Poynting flux into the plasma, and $I$ is the strap current. The problem with this formula is that the strap current is non-uniform. This problem is usually not
serious problem if the system supported one single poloidal mode, then $R$ can just be calculated for that mode, and we can use either the average or peak value of $I^2$ along the strap. It is only necessary to make sure that the design calculation for the electrical system is based on the same formula for $R$.

In the multi-frequency antenna system operating at 22 MHz, at least two modes are supported. This is shown in Fig. 10 where we plot the current along the strap for a $90^\circ$ (0-0-$\pi$-$\pi$) phasing case with a 14 cm distance from the Faraday shield face to the separatrix. We used a representative ITER plasma density profile with a 5 cm scrape-off layer beyond the separatrix. The result clearly show that the primary mode in the system is supported by the short section of the strap fed by the feed (current at the feed post is .374225). Considering this part alone, the system resembles a traditional ICRH antenna. There is, however, a weaker mode supported by the long section. As intended for the multi-frequency antenna, this mode should dominate since its longer wavelength would couple better to the plasma.

![Graph](image)

**Fig. 10.** Current distribution along the current strap for a $90^\circ$ (0-0-$\pi$-$\pi$) phasing case with a 14 cm distance from the Faraday shield face to the separatrix.

Figure 11 shows the loading of the $90^\circ$ phasing case at 3 different distances to the plasma. For this calculation, we used the current value at the top end of the current strap (e.g., right limit of Fig. 10).
6. Summary

In summary, the vacuum characteristics of the 18 strap multi-frequency antenna has been analyzed using the ARGUS Eigenmode solver and the single frequency solver. Resonant frequencies in the system are found to be $\sim 28$ MHz at the fundamental, $55$ MHz at the second harmonic, and $80$ MHz for the 3rd harmonic. There are variations in the resonances depending on strap phasing from which we can infer the mutual inductive coupling. It is between $6.5\%$ at the fundamental and $8.5\%$ at the second harmonic. These results are consistent with the single frequency solver solution analyzed with a transmission line model.

Loading calculations are performed for the $22$ MHz, $90^\circ$ phasing case at 3 different distances to the plasma. In general, the loadings are very low, probably due to our observation that the primary mode in the system appear to have short poloidal wavelength.

7. Reference


Appendix E Modeling of the Folded Waveguide Antenna for TFTR/PBX-M
Modeling of the Folded Waveguide Antenna For TFTR/PBX-M

Abstract

A high power, 12-vane folded waveguide antenna has been designed for fast wave and ion Bernstein wave launching into the PBX plasma at a frequency of 58 MHz. The square cross section of the waveguide allows coupling to either the fast or slow waves to be achieved by physically rotating the launcher. Calculations of the plasma loading due to power coupling to the fast wave, the launched wave spectrum in the presence of plasma, and the power handling capability of the launcher are made with the RANT3D code[1]. Eigenmode analysis, internal field distributions, and the power coupling to the launcher itself is performed with the ARGUS code[2]. Response of various tuning and matching schemes are evaluated for anticipated load changes with a transmission line analysis. Comparisons of calculations with measurements will be made where possible.


P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Modeling of Vacuum Fields of Launchers

- The vacuum fields are calculated with the ARGUS 3-D Electromagnetic Code (SAIC).
- The fundamental resonant frequency and associated fields for the Folded Waveguide (FWG) are calculated with the eigenmode solver.
- The fields for an equivalent loop antenna are calculated with the driven frequency solver.
Vector Plots of The Magnetic Field On Either Side of the Front Faceplate

Magnetic Field Reverses Direction Between Adjacent Slots On Interior Side of the Faceplate

Magnetic Field Does Not Reverse Direction Between Adjacent Slots On Exterior Side of the Faceplate

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Toroidal B-Field At the Inner Surface Of The Polarizing Face Plate

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Toroidal B-Field At the Front Surface Of The Polarizing Face Plate

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
ARGUS Model of the Quarter-Wavelength Folded Waveguide

Front of Folded Waveguide, Showing the Polarizing Face Plate and Cutaway of the Vanes

Interior of the Folded Waveguide from rear, showing the power feed mechanism.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Field Comparison of the FWG Launcher to an Equivalent Loop Antenna

- Replace FWG vanes with a 14 cm wide, 25 cm high current strap.
- Front surface of the strap is 3.4 cm from front surface of the faceplace (acting as a Faraday shield)
- Strap is fed out-of-phase at the ends, creating a virtual ground at the poloidal midplane.
- Driven frequency is 56.56 MHz.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Vector Plots of B-field Through the Slots of the FWG Faceplate

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Vector Plots of B-field Through the Slots of the Loop Faceplate

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
The Poloidal Ripple in $B_{\text{toroidal}}$ Falls Off Rapidly With Radial Distance for FWG

FWG: $B_{\text{toroidal}}$ is more square in the toroidal direction and less square in the poloidal direction.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Radial Falloff of $B_{\text{toroidal}}$ For Loop Antenna

Loop Antenna: $B_{\text{toroidal}}$ is more square in the poloidal direction and less square in the toroidal direction.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Relative Maximum Values of Electric Field Components at the FWG Faceplate

- $E_{\text{poloidal}} = 1.00$
- $E_{\text{radial}} = 0.25$
- $E_{\text{toroidal}} = 0.04$

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Relative Maximum Values of Electric Field Components Near the Loop Faceplate (FS)

- $E_{\text{poloidal}} = 1.00$
- $E_{\text{radial}} = 0.65$
- $E_{\text{toroidal}} = 0.05$

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Comparison of the Electric Fields at Faceplate For FWG and Loop (Radial position 3.9 mm in front of the faceplate)

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
$B_{\text{poloidal}}$ Falls Off Rapidly in Radial (Axial) Direction for $\lambda/4$ Folded Waveguide.

The cutoff frequency in the evanescent region of the waveguide is 530.8 MHz.

The e-folding decay length for the fundamental mode at 56.56 MHz is 9.43 cm.

The fields calculated by ARGUS have a decay length of approximately 9.05 cm.

The rapid falloff of field strength permits the extent of the coupling region to be short.

This rapid falloff also requires the power coupling loop to be in close proximity to the vanes.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Poloidal B-field In the Coupling Region of the Quarter Wave FWG, With and Without Coupling Loops.

- The eigenmode solution with the loop present imposes zero voltage at the input port (metallic b.c.)
- The net poloidal flux enclosed by the loop must therefore be zero.
- The total poloidal flux over the loop area for the unperturbed fields gives the open loop port voltage.
- The self-inductance of the loop may be found from the port impedance when driven off resonance.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Examples of Matching to Folded Waveguide

Changing loads can be compensated by changing frequency and tap point (turns ratio).
Tap point cannot be varied rapidly (if at all)

Resonant frequency doesn’t change with resistive load but reflection coefficient does.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Examples of Matching to Folded Waveguide

- Conditioning FWG
  - May need to condition at different frequency.
  - Not much coupled power needed because of high Q.
  - Match provided to transmitter while prematched line can handle VSWR ~5.

- Load change variations
  - Note that long prematch lines are also HIGH Q devices (sensitive to Δf).
  - FWG can respond to changing loads provided BOTH frequency and plasma position are variable (same as conventional launchers with long prematch lines).

Changes in reactive load shift the resonant frequency.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Folded Waveguide Operation Under Vacuum Conditioning and Plasma Load Excursions

- **Vacuum Conditioning**
  - \( Q/Q_0 = 6.7 \) for nominal load on PBX/TFTR FWG (ratio of ~5 expected for ITER)
  - Power needed for conditioning FWG to full fields is \( P_{\text{full}}Q/Q_0 \)
  - When matched at FWG for nominal load, the VSWR in the prematched line is 6.9 during unloaded conditioning.
  - The maximum voltage on the prematched line for unloaded FWG conditioning to full fields is therefore \( V_{\text{SWR}}Q/Q_0 \) times the line voltage for full power into nominal load (1.03 for this case).

- **Load Excursions**
  - Changes in the plasma load by factors of 2 will result in a reflection coefficient of 0.34 (VSWR of 2). A rapid match at the transmitter can be re-established with frequency shift tuning in conjunction with plasma position control.
  - If plasma position control is unavailable, the reflected power will need to be shunted away from transmitter with hybrid coupler circuit.
  - The transmission lines need to handle the voltages associated with the higher VSWR during load excursions.

P. M. Ryan, APS-DPP Conf, Louisville, KY, 8 November 1995
Appendix F  ANSAT, an Antenna Sheath Analysis Tool
ANSAT, an Antenna Sheath Analysis Tool

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A computer code, ANSAT, (Antenna Sheath Analysis Tool) has been developed which allows an analysis of rf sheaths on the plasma facing regions of Ion Cyclotron Range of Frequency (ICRF) antennas. The code calculates the contact points of the tokamak magnetic field lines on the surface of the antenna Faraday Screen and nearby limiters for realistic three dimensional magnetic flux surface and antenna geometries, determines the rf voltage that can drive sheaths at the contact points, and assess the resulting sheath power dissipation, rf-driven sputtering, and rf-induced convective cells (edge profile modification). It is shown that ANSAT is a useful tool for antenna design, because it can assess the strengths and weaknesses of a given design with respect to critical voltage handling and edge plasma interaction issues. Additionally, example are presented where ANSAT has been useful in the analysis and interpretation of ICRF experiments.
Introduction

It is by now well established that Ion Cyclotron Range of Frequency (ICRF) systems can heat fusion plasmas for a wide range of physics heating scenarios, and efforts are well under way to employ ICRF systems for driving steady state current in tokamaks. Experience has shown that while ICRF systems can be made robust and effective, attention must be paid to certain critical ICRF - edge interaction issues. Experiments have shown that under adverse conditions, ICRF systems can lead to increased impurity injection from the antennas and/or their limiters into the plasma, the formation of "hot spots" and arcs on the antenna surface, excessive power dissipation at the edge, nonlinear loading (i.e. loading that is a function of power), and modifications of the scrape off layer (SOL) plasma. Many of these issues, while present in ICRF heating configurations, are expected to be even more critical for extended current drive arrays, which fill a substantial fraction of the SOL volume, and for which antisymmetric (dipole, or 0-π) phasing cannot be employed.

The experience gained over the years with high power ICRF has lead to many techniques for controlling undesirable ICRF - edge interactions. These include the design of low voltage (high loading) antennas, low Z wall coatings, operation in antisymmetric phasing, alignment of Faraday screen (FS) bars with the magnetic field, and good matching of the flux surface to the FS surface. There is, by now, considerable literature which shows the relationship of rf sheaths on the antenna surface to the experimental observations. In the past, this analysis has been done mostly "after the fact", i.e. rf sheath theory has been used to help interpret experimental observations. It has been useful as an intuitive guide for new antenna designs, but the geometrical complexity of real antenna systems has often made a direct application of the theory to a new antenna difficult. In this paper, we describe ANSAT, a code which we believe will remove this deficiency, and enable antenna designers to assess the implications of rf sheaths for a given design beforehand, and thereby make any necessary design modifications prior to construction and installation of the antenna.

In addition, developing the ANSAT code has enable us to collect together the past physics on this problem (which up until now has been scattered over many publications and many non interacting codes) into a integrated unit. The design of the code is highly modular, and is able to incorporate a wide variety of rf sheath and edge physics, based upon both our own work, and in the future, hopefully that of many other authors.

The present paper summarizes work to date as a result of our Phase I effort. In Sec. II we describe the basic ANSAT algorithm, and show the relation of ANSAT to the electromagnetic (EM) field code ARGUS. In Sec. III we present examples of the antenna
physics and design issues address by ANSAT and show sample ANSAT output for a mock up of the TFTR (Tokamak Fusion Test Reactor) Bay-M antenna. These issues include calculation of the sheath driving voltages, convective cells patterns, sputtering, sheath power dissipation, the effect of the edge field line mappings, FS misalignment, error fields and antenna installation errors, the effect of bumper limiters, and finally of the edge plasma itself on the rf sheaths. In Sec. IV, we discuss the successes and limitations of the techniques presented so far, and discuss possible future developments.

II. The ANSAT algorithm

The existing ICRF - edge interaction codes that we are aware of use simplified treatments of the antenna geometry to obtain rough estimates of the rf driven sheath voltages and their effect on the edge plasma. Our goal in this work was to increase the level of geometrical sophistication with which the rf sheaths are treated (i.e. the dependence on the actual hardware design) by integrating existing sheath and edge physics codes with modern EM three dimensional (3D) field codes that include the full geometry of the antenna. The examples presented here have been done in conjunction with the EM field code ARGUS,\footnote{ARGUS is widely used in the tokamak community.} which is extremely well suited to this purpose. ARGUS calculates the 3D field patterns given a structured representation the antenna (including FS, antenna box with sidewall slots, if any, and bumper limiters). ANSAT is then employed as a post processor on the ARGUS output files, which for present purposes may be regarded as consisting of the three components of the EM field in 3D space, and a structure file. In principle, ANSAT could also post process the output from other antenna EM field codes. The communication between codes takes place by means of "hdf" files, a widely employed, public domain standard. The relationship of ANSAT to ARGUS is illustrated in Fig. 1. The preprocessor, is part of the ARGUS code. The figure also illustrates that test stand measurements on the TFTR Bay-M mock up antenna have shown excellent agreement with ARGUS calculations,\footnote{Test stand measurements are crucial in validating the models.} giving us confidence in the validity of the ARGUS results. Comparison of ANSAT results with ICRF edge physics experiments is in progress,\footnote{Experimental validation is essential for confidence in the models.} and together with past sheath theory results that have shown good agreement,\footnote{Past results provide a benchmark for new models.} will provide a high level confidence in trusting ANSAT as a design tool.

The first step in the ANSAT algorithm is to create a 3D grid of points in the vicinity of the antenna FS, embedding all plasma facing components. The structure file provided by ARGUS determines whether a given point lies inside or outside the structure of the antenna. Figure 2 shows the level of detail at which ARGUS is able to represent the structure of the TFTR Bay-M mock-up antenna. Each point that is not already inside the structure is considered to be an initial point on a field line mapping, where distance along
each equilibrium field line of the tokamak is parametrized by \( s \). The mapping will be discussed in more detail shortly. A rootfinding procedure is employed to locate the two contact points, \( s_a \) and \( s_b \), that the field line makes with the antenna/limiter structure. The "V" shape of FS, as well as the toroidal curvature and rotational transform of the field line are retained in this procedure. The parallel component of the rf electric field can then be integrated along the magnetic field between contact points to determine the rf sheath driving voltage,

\[
V = \frac{s_b}{s_a} \int ds \mathbf{E}_\parallel .
\]  

Normally a fraction of this voltage, approximately 0.6 V (where \( V \) is the 0-peak voltage), appears as a rectified dc voltage across sheaths near each contact point. Consequently, we assign \( V \) as the rf sheath driving voltage for each of the two contact points. This procedure is repeated for all initial points in the 3D grid to obtain a database of voltages \( V(x_i, y_i, z_i) \) and field line lengths for the corresponding contact points \( i = 1, N_c \) where \( N_c \) is typically a few tens of thousands.

A critical factor determining the severity of antenna sheaths is the relative geometry of the antenna surface and the magnetic field. Typically, Faraday shields for multiple strap antennas consist of several straight segments toroidally which are joined together at an angle. For the two strap Bay-M antenna, this results in a simple "V-shaped" geometry, as shown in Fig. 3 a), with "V" angle given by \( \alpha \). Figure 3 a) also shows the toroidal curvature of the magnetic field and typical contact points.

The Bay-M mock-up treats the Faraday screen as straight in the toroidal direction. Because the angle \( \alpha \) is typically very small (2 - 3°), its departure from zero is unimportant in the EM calculation in ARGUS, but still critical in determining the contact points in ANSAT. Therefore, we define a transformation acting on the 3D space which maps the bent Faraday screen into one which is flat. The magnetic field is then specified in ANSAT in the new transformed coordinates, as shown in Fig. 2 b). Additional linear transformations of the space allow for the study of the effect of antenna installation errors (especially skew) and magnetic field errors and ripple on the voltage patterns and hot spots that develop on the antenna surface.

Having produced the database of field line contact points, field line lengths and rf sheath driving voltages, other modules within ANSAT employ the database to carry out the desired analysis and diagnostic displays. A flowchart structure of the entire ANSAT suite of codes as presently existing is shown in Fig. 4. Gray arrows (e.g. the near zone plasma module which will be discussed in Sec. III) are optional. After the main database is set up,
one can optionally extract subsets from the database to analyze specific classes of sheaths (e.g. those making one contact with the FS surface, and one contact with the bumper limiters), or to zoom in on particular regions of the antenna.

The various analysis routines provide extensive diagnostics of the sheaths, and subsidiary physics calculations to determine their effect on edge plasma interaction an antenna behavior. The plasma facing analysis produces contour plots of rf sheath driving voltage#13 looking at the screen face on, as well as voltage histograms and voltage-length scatter plots, all to be discussed subsequently. The contact point analysis provides a diagnostic for visualizing the location of a given subset of sheaths (selected, for example, by a voltage range) by superimposing the corresponding field lines on a view of the antenna. It is thus an important tool for iterative design improvements. The convective cell analysis show the rf driven flow patterns in the radial-poloidal plane, and gives estimates of the rf-driven transport enhancement in the SOL.#14 Coupled to the convective cell analysis, which helps to determine the plasma density on the FS surface, are the impurity influx (sputtering) analysis,#10 the rf sheath power dissipation,#7 and the calculation of SOL modifications, especially of the edge density profile, and the penetration of the rf E\| into the SOL.

All of the above tools work for arbitrary phasings (including \(\pi/2\) or current drive phasing) of the rf antennas. In the following, in order to conserve space, we present examples for 0 and \(\pi\) phasing, since the contrast in sheath formation is largest for these two most disparate cases. Results for \(\pi/2\) phasing are generally in between the 0 and \(\pi\) cases.

Sample output from the contact point analysis, which illustrates the importance of geometry, is presented in Figs. 5 and 6. In Fig. 5 we show a portion of the FS and a few field lines terminating at their contact points with the structure. TFTR has a complicated structure of gaps and sidewall slots resulting in different classes of field line connections and rf sheaths.#18 Shown in the figure are gap (G), screen-screen (SS) or front face, screen-limiter short (SL\(_s\)), and screen-limiter long (SL\(_l\)) sheaths. Gap sheaths connect adjacent bars of the FS due to the poloidal motion of a field line in the case where the FS bars are not perfectly aligned with the tokamak magnetic field. Front face sheaths occur due to the "V" shape of the screen and the toroidal curvature of the field line, and are symmetric (or nearly so) with respect to the midplane of the antenna. Screen limiter sheaths result when a field line connects the front face of the FS to a limiter, either the short way (near side of the screen to adjacent limiter) or the long way. For this example the voltages of the indicated sheaths are 0.22, 0.08, 0.47 and 0.38 for the sheaths in the figure labeled G, SS, SL\(_s\), and SL\(_l\) respectively. The units quoted here are code voltage units; to obtain sheath driving voltages in kV for the examples in this paper, multiply by 3.08
$P(MW)^{1/2}$ where $P$ is the antenna power. Other parameters for this TFTR Bay-M example are: $\pi$ phasing, $\theta = 3^\circ$ (FS - B field misalignment angle), $R_c = 3.6$ m (toroidal radius of curvature of B field), and $\alpha = 2.6^\circ$ ("V" angle of FS). Unless stated, these parameters are assumed throughout paper. Note that the voltage of the symmetric SS sheath is small because of the cancellation that occurs in antisymmetric ($\pi$) phasing.

In Fig. 6 we show another kind of output produced by the contact point analysis, the L-r scatter plots which show how the flux surface geometry interacts with the screen. The L-r scatter plots are useful in determining what classes of sheaths are produced by a given geometry [e.g. the "V" angle $\alpha$, magnetic field tilt $\theta$ (edge rotational transform), and toroidal field radius of curvature $R$]. The x-axis in Fig. 6 gives the radial location of the field line in flux variables (i.e. the $r$ that the given field line has when it is mapped to some reference toroidal location, taken here to be geometrical center of the antenna). The y-axis in Fig. 6 gives the length of the corresponding field line between its contacts points with the structure. This example is for $\theta = 6^\circ$ relative to the B field, and shows five types of sheaths: SL long, SL short, front face sheaths that hop two and three bars (denoted 2B and 3B), and one bar (denoted 1B or gap) sheaths. For smaller misalignment angles, the 3B and eventually the 2B sheaths disappear.

While it is clear that the database of contact points and rf sheath driving voltages is useful for identify hot spots on the FS and suggesting modifications in the connection geometry (e.g. addition of bumper limiters) to eliminate classes of, say high voltage, sheaths, it is not immediately obvious how the database can be employed to construct integrations of sheath related quantities over the surface of the antenna. An important example is the calculation of impurity sputtering or power dissipation. Typically, such integrations have the form

$$I = \int dA \cdot b f(V) g(r_f)$$  (2)

where $dA$ is an area element on the antenna, $b$ is a unit vector along the magnetic field, $f(V)$ is a quantity that depends on the sheath voltage (e.g. the sputtering yield) and $g(r_f)$ is a weighting factor that depends on the radial flux surface variable $r_f$ (e.g. the plasma density). Physically, the $dA \cdot b$ factor arises from the flux of electrons or ions along the magnetic field line into the surface. The integral $I$ is not correctly given by a straightforward sum of all the points in the database, because of the $dA \cdot b$ factor, which is furthermore awkward to calculate directly for a fully 3D structure. (Poor numerical resolution of the structure on the grid could cause large errors in the direction of $dA$.) Fortunately, a different mathematical algorithm comes to the rescue.

To understand the algorithm, consider a flux tube surrounding a given field line. Let the area of the flux tube, projected normal to $b$ be $dA$. The flux tube intersects the
structure at a contact point, "wetting" an area on the surface of antenna which will be much larger than dA when the angle of contact is near glancing. Importantly, dA*b is independent of the angle of contact or the shape of the surface, but depends only on the flux tube. The summing over all flux tube, weighting each by their normal area, and by the f(V) g(r1) factors gives the correct answer. What remains is to from the database an average that corresponds to an average over equal area flux tubes.

To this end, we denote an average over the Nc points in the database by <Q> where for any quantity Q in the database

\[ <Q> = \frac{1}{N_c} \sum Q_i \]  

(3)

Many initial points in our 3D grid can trace out the same, or very similar field lines (flux tubes). Thus <Q> double (or multiple) counts some flux tubes. The crucial point is that the normally projected area of a flux tube is equal to its volume divided by its length, and that the degree of multiple counting in <Q> is proportional to the flux tube volume (hence its length) for a uniform 3D grid of points i. With these observations, it is straightforward to show that an average over flux tubes of a quantity Q, denoted {Q} is given by

\[ \{Q\} = \frac{<Q/L>}{<1/L>} \]  

(4)

where L is the field line length between contact points, taken from the database. It also follows that a numerical approximation to I is given by

\[ I = A_s \{f(V) g(r1)\} \]  

(5)

where for a given 3D grid

\[ A_s = \frac{N_s V_g}{\rho_g (L)} \]  

(6)

and Ns is the number of points in the database striking (the desired portion of) the antenna structure, Vg is the volume of the grid, and \( \rho_g \) is the (uniform) density of points on the grid.

Thus the main ANSAT database, combined with the above algorithm for flux tube weightings, provides the basis for calculating all sheath related diagnostics that are presently envisioned. In the section which follows, we will present many more examples of ANSAT output, categorized by the underlying physics and design issues which each one addresses.

III. Physics and design issues, implementation and applications

1) Sheath driving voltages
The rf sheath driving voltages are fundamental to the rest of the sheath analysis, and we have found that examining their values and distribution on the FS is generally a good place to begin the ANSAT analysis for a new design. The plasma facing analysis routines produce three types of diagnostic plots for this purpose. The first is a voltage-length (V-L) scatter plot. For each point in the database, a point is drawn in the V-L plane, with field line length between contact points along the x-axis and sheath driving voltage along the y-axis. An example of such a plot is given in Fig. 7 a) for TFTR Bay-M in π phasing for a misalignment angle of θ = 3°. Three main groups of sheaths are present. The gap and front face sheaths (SS) are more or less merged together at the lower left of the plot for the 3° case, SL short are in the middle, and SL long are at the far right. This diagnostic makes it evident that the SL sheaths have the highest voltages, and that in sheer numbers, the SL short sheaths dominate the SL long sheaths. A band of points connecting the SL short and SL long groups will also be noticed. Using the database subset feature of ANSAT to select these points, and then running the contact point analysis to produce figures similar to Fig. 5, we were able to identify this class of sheaths with field lines that strike the top or bottom of the antenna frame, connecting it to a far away rf limiter.

Figure 7 b) shows a second useful diagnostic, which is a histogram of N(V), the number of sheaths (i.e. the number of points in the database) having voltage in a range dV about V. For dipole phasing, in code voltage units, the maximum voltage obtained in π phasing is about 0.8. Note the spike near V = 0.2 - 0.3, corresponding to the numerous gap and front face sheaths. In Fig. 7 c), we show the distribution of high voltage contours on the face of the FS. Contours with a voltage value exceed V = 0.5 in code units are filled in solid. This diagnostic illustrates the important fact that the TFTR Bay-M antenna has substantial sheath voltages, even in π phasing, especially near the corners of the antenna. Similar plots with voltage mapped to color palettes can show many details of the sheath voltage distribution. One example is given in Fig. 7 d), which shows an enlargement of the poloidal region of the FS between 0.22 and 0.35 m. The highest voltages in the figure are in red, and the lowest ones in blue. Note that the region near the center of the FS has low voltages, both because it is recessed radially due to the "V" (giving short contact lengths), and because of anti-symmetric cancellation in π phasing for these sheaths (see following paragraphs). The 3° slope of the B field (poloidal and toroidal scales are different in this figure) is evident from the green contours near the center of the FS. (See Fig. 5 for the B field lines). Note also that the highest voltage (red) screen-limiter sheaths connect to the lower side of the bars on the left, and to the upper side of the bars on the right. Finally, for this section of the FS, the top left corner is the hottest, as also seen from Fig. 7 c).
The existence of high sheath voltages in the antenna corners in \( \pi \) phasing is an example of something that our earlier geometrically simple models could not predict. Ideally in \( \pi \) phasing, \( E_{\parallel} \) should be anti-symmetric about the midplane of the antenna, and therefore all symmetric sheaths should have \( V = 0 \). Sheaths which are not symmetric about the antenna midplane are difficult to treat in simple models. In Fig. 8 we verified the odd parity of \( E_{\parallel} \) for a field line passing through the geometrical center of the antenna, but the figure also shows that for off-center field lines, especially those near the corners, large asymmetric \( E_{\parallel} \)'s exist. Further examination of the ARGUS output shows that the edges and corners contain high values of induced charge.

Figures 9 a) - c) correspond to Figs. 7 a) - c) except that they are for Bay-M in 0 rather than \( \pi \) phasing. These plots show that the sheath voltages are sensitive to phasing, as expected. Each point in Fig. 9 a) has the same \( L \) value as in Fig. 7 a) (because the geometry was not changed), but the corresponding voltages are considerably higher. This is particularly obvious for the SL long sheaths at far right. The \( N(V) \) voltage histogram tells the same story, now showing some voltages well beyond the \( V = 0.8 \) cutoff of the \( \pi \) phasing case. The high voltage contours of Fig. 9 c) (again for \( V > 0.5 \)) indicate a much larger area of high voltage sheaths, and relatively less concentration towards the corners.

The plasma facing analysis output shown in this subsection is useful to get a quick first look at what sheaths are the worst, what the absolute sheath voltages typically are for a given antenna power, and to compare different phasings. The plasma facing analysis is also fundamental to the sputtering and power dissipation analysis to be discussed later.

2) Convective cells

For the convective cell analysis, we employ a subset of the full database, using only initial points which lie in a given radial-poloidal plane, typically taken at the toroidal symmetry plane of the antenna. The idea is that each field line is charged up to a given (rectified dc) voltage due to the sheaths, and these voltages vary with both radial and poloidal position. Consequently, the resulting radial and poloidal electric fields drive \( \mathbf{E} \times \mathbf{B} \) convection which can enhance the transport and modify plasma profiles in the SOL. The self-consistent modification of the SOL density profile is complicated because the plasma density near the FS also determines the extent to which rf sheaths can penetrate the SOL. (This point will also be discussed under the subsection entitled plasma.) At present ANSAT does the convective cell analysis at two levels. The most detailed output is available for a given set of ARGUS produced fields, in this example for the fields corresponding to a run of ARGUS in vacuum. Self-consistency is then implemented for a simplified representation of
these fields. A fully self-consistent 2D model of SOL modifications is part of our plans for future (Phase II) work.

First, examining the detailed vacuum output, Figs. 10 a) and b) show respectively the poloidal variation of sheath driving voltage $V$, and a contour plot of $V$ in the radial-poloidal plane, both for $\pi$ phasing. Fig. 10 a) clearly shows poloidal variation on two spatial scales. The trace with the most structure is for a "flux surface" that is located just behind the front face of the FS, and it show the fine scale variations associated with the individual screen bars. The other trace is on a "flux surface" that is located well in front of the FS. It shows the remnants of a ripple at the periodicity of the sidewall slots, and an overall variation of the scale of the entire antenna, emphasizing again the effect of the high voltage corners and edges of the antenna. Again, the FS / B-field misalignment angle $\theta$ was taken to be 3°. The voltage contours of Fig. 10 b) may also be interpreted as streamlines or flow contours for the resulting $E \times B$ plasma motion. They indicate a radial convective cell width of about 3 mm for this example. The front face of the FS is at $\Delta r = 0.08$ m in this plot and the convective cells extend to about $\Delta r = 0.11$. This figure employs numerical smoothing, both to compensate for loss of resolution at the finest scales, and to simulate the smoothing effect of the convective cell equation\(^{14}\) which viscously damps modes on short scale lengths.

For comparison, Figs. 11 a) and b) show the same diagnostic plots as Figs. 10, but for the case of 0 phasing. As expected the highest voltages are somewhat higher, and the convective cell pattern is more pronounced. Note also, that the variation of $V$ in Fig. 11 a) on the long scale (over the whole antenna) is considerably different than for the dipole case.

The convective cell module can also output radial plots of the poloidal electric field, and its poloidal rms average $\langle |E_p|^2 \rangle \propto D$ which in convective cell theory is proportional to the rf induced enhancement of the diffusion coefficient, which in turn determines the SOL profiles. A zero order attempt at self-consistency in this calculation is carried out in our SOL modifications module. In this module, the two characteristic radial points are considered, $r_{FS}$ and $r_{lim}$ where $r_{lim}$ is a limiter tangency surface at which the private SOL of the antennas joins the last closed flux surface or the main SOL of the tokamak. The following two simultaneous equations are then solved:

$$n_e(r_{FS}) = n_e(r_{lim}) \exp[-\Delta r/\lambda_n], \quad (7a)$$
$$D(r_{lim}) = D(r_{FS}) \exp[-\Delta r/L_{sw}], \quad (7b)$$

where $\Delta r = r_{lim} - r_{FS}$, $\lambda_n$ is the density gradient scale length as determined from the convective cell equation, and $L_{sw}$ is the slow wave (i.e. $E_b$) scale length as determined from the evanescent slow wave dispersion relation. Table 1 illustrates the type of results available from this module. More sophisticated implementation of these ideas is a topic for...
future work, which will verify whether the qualitative conclusions of the simple calculation remain robust in a fully 2D self-consistent SOL model.

3) Sputtering and impurity influx

The calculation of sputtering rates proceeds from the main database of contact points and sheath voltages, using the formulation of surface area integrals presented in Eqs. (2) - (6). It is straightforward, using this technique, to obtain the net influx of impurities sputtered from the antenna for a given antenna power (which provides the normalization of code voltage units), by employing the available sputtering rate coefficients $Y(V)$ for the specified antenna material and SOL plasma atoms.$^{10,11}$ Contour plots of the damage patterns on the antenna surface can also be made in much the same way as for the sheath voltage patterns shown in Figs. 7 c) and 9 c).

To illustrate an additional, often useful feature of ANSAT, we present the results of a sample impurity sputtering calculation using the "group analysis" routines. These routines enable the sheaths to be sorted into physically meaningful categories, so the contributions of each category to the total sputtered influx can be determined. For groups, in the 3° misalignment case that we now consider, we pick the 1 bar (gap and front face), SL short and SL long classes of sheaths. The analysis yields the mean length, sheath voltage and normally projected flux tube area for each class, as indicated in Table 2. Sputtering rate calculations then give the corresponding contributions to the impurity influx. This analysis makes it evident that 1 bar and SL short sheaths dominate the influx, even though SL long sheaths have the highest voltages. It is partly a consequence of the saturation of the sputtering yield curves for these material at high voltage. For these calculations, the results of the SOL modifications module for the density at the FS (see Table 1) were employed. Preliminary comparisons of the calculated titanium influx with that deduced from spectroscopic observations on TFTR show encouraging agreement.$^{22}$

4) Sheath power dissipation

Conveniently, the integral over the antenna yielding the total sheath power dissipation$^7$ also falls under the general form of Eq. (2) with $f(V) \propto V$ and $g(r_f) \propto n_e c_s$ where $c_s$ is the sound speed. Proceeding, as with the sputtering calculation, it is instructive to examine the group analysis. Results are also presented in Table 2. For 2 MW of total rf power, the sheath dissipated power for this case ($\theta = 3^\circ$) is 54 kW. As is the case for the impurity influx, the sheath power is dominated by the SL short group. While the present example shows that sheath power dissipation can (and in an desirable situation should) be a negligible fraction of total rf power, this is not always the case. Sheath power dissipation
(≈ \(V\)) scales with \(\sin \theta\) at constant density for many of the sheaths, and exhibits an even stronger scaling when the self-consistent effect of convective cells is taken into account. Thus when the misalignment angle is large (e.g. the \(\theta = 20^\circ\) case to be discussed in subsection 5, which is relevant to reversed field operation and/or strong D shaping with non aligned screens) the sheath power can be substantial, reaching 20% of the launched power in some extreme but experimentally relevant cases that we have examined. The small volume into which the sheath power is deposited (≈ \(AL\) from Table 2) makes it all the more of a concern.

The fact that the sheath power dissipation is proportional to \(\nu\), while the Poynting flux transmitted power into the plasma is proportional to \(|\nu|^2\) has an interesting and perhaps useful consequence, which could be exploited by ARGUS/ANSAT in the future to benchmark sheath power dissipation in ICRF experiments. The scalings imply that at sufficiently small \(\nu\), sheath power dissipation will dominate Poynting power. (At these low powers, the effects of convective cells can usually be ignored.) It can be shown that this crossover typically happens at a power \(P_x\) on the order of a few 10's of kW, and that it results in an apparent plasma loading curve \(R(P)\) which increases at powers \(P < P_x\). In the future, we hope to employ ARGUS/ANSAT to calculate and compare the \(R(P)\) loading curve with experimental data, preferably for more than one antenna phasing. The largest uncertainty in the calculation arises from the uncertainty in \(n_e\) at the FS. There is some hope that an experiment fit could be employed to gain insight into the value and variation of this parameter.

5) Field line mapping: FS misalignment, error fields, skewed installation

The ANSAT method described in Sec. II for following field lines and determining their contact points with the antenna structure, is both flexible and general in its ability to describe a wide variety of realistic magnetic field line mappings and antenna geometries. Before proceeding with some illustrative results, we first give the various parametrizations and transformations in more mathematical detail.

In the physical coordinate system of Fig. 2 a) the magnetic field of the tokamak is assumed to have toroidal curvature and a local pitch angle with respect to the toroidal direction (given by the rotational transform). In general, the toroidal radius of curvature \(R\) has contributions from both its mean value (the major radius of the given field line) and error fields due to magnetic ripple, which are toroidally periodic with wavenumber \(k_z = n / R_a\) where \(n\) is the number of toroidal field coils. Consequently

\[
R^{-1} = R_a^{-1} [1 + n (\delta B/B) \sin(k_z z)],
\] (8)
where $R_\Delta$ is the average radius of curvature, $z$ is a locally toroidal spatial coordinate and $\delta B/B$ measures the size of the field ripple. The field line angle with respect to $z$ in the poloidal ($y$) and toroidal ($z$) plane is denoted $\theta$, so that
\[
dy/dz = \tan \theta.
\] (9)

Denoting the "V" angle of the FS by $\alpha$, and applying the transformation of Fig. 2 b) (which makes the screen flat) the magnetic field of Eqs. (8) and (9) is given by the parametrization
\[
x(s) = x_0 + z^2/2R - \alpha lzl - z_0^2/2R + \alpha lz_0l
\]
\[
y(s) = y_0 + s \sin \theta
\]
\[
z(s) = z_0 + s \cos \theta
\] (10)

where $s$ parametrizes distance along the field line from the initial point $(x_0, y_0, z_0)$, coordinates are in the ARGUS/ANSAT system, the triplet $(x, y, z)$ corresponds to locally Cartesian variables that are (radial, poloidal, toroidal) respectively, and $z = 0$ is the toroidal symmetry plane of the antenna. Various nonessential small angle approximations have been made to simplify Eqs. (10) for presentation. Note that the radial and poloidal labels $x_0$ and $y_0$ (or, one could use $x_0$ and $z_0$ when $\theta \neq 0$) can be thought of as "flux variables", since they serve to label a given field line.

The above magnetic field line mapping is localized to a single subroutine in the ANSAT code, so it is straightforward to generalize the mapping to include additional error field terms or antenna installation errors. For example, we have implemented antenna installation skew by applying a rotation matrix to Eqs. (10) which causes one of the corners of the antenna to stick out farther into the SOL than the other three. Results will be discussed subsequently, in connection with Fig. 13.

First, we demonstrate the simple, but important effect of varying the angle $\theta$ which, in the case of TFTR Bay-M with its toroidally directed FS bars, measures the misalignment angle between the FS bars and the magnetic field. Figure 12 shows the V-L scatter plot for the case of $\theta = 20^\circ$ in 0 phasing, and is to be contrasted with Fig. 9 a) for $\theta = 3^\circ$ but otherwise the same parameters. The additional classes of 2 - 8 bar sheaths for the 20$^\circ$ case are evident here. At these large values of $\theta$ the maximum voltages are seen to be quite high. We note that this case is of more than pedagogical interest; aligned screens operated in reversed field operation typically have $\theta > 15^\circ$, especially in experiments with strong plasma "D" shaping.\textsuperscript{7}

Figure 13 shows contours of the contours power dissipation (proportional to $V_{180}$) that result from a run with installation skew error, as discussed above. The magnitude of the skew error for this case was chosen to result in a deviation of $\pm 2$ mm from true, for the antenna corners, with the lower right corner sticking farthest out into the SOL. The higher
concentration of density weighted sheaths at this point is evident from the figure. Other diagnostics reveal that the magnitude of typical sheath voltages is not changed much by the skew, as expected, although of course some of the field line lengths are modified.

6) Bumper limiters

The present code is ideally suited to testing the effects of various hardware modifications of the performance of an antenna with respect to sheaths and edge interactions. As an example, in this subsection we focus on the effect that antenna bumper limiters have on the voltages and distribution of sheaths.

Up until now, all the examples have been for the TFTR Bay-M mock-up antenna, without bumper tiles. We have added bumper tiles to the ARGUS input files, and compared the sheath analyses for the two cases. To be effective, it is only necessary that the bumper tiles protrude far enough radially to intercept field lines that would otherwise strike the FS directly, connecting the screen to a far away limiting surface. Figures 14 a) and b) show the field line, FS and bumper limiter geometry as viewed in the toroidal-radial plane (looking down on the antenna), in both the ANSAT a) and physical b) coordinate systems. In both cases, the radial scale is greatly magnified for clarity. It is evident that the 2 mm bumpers shown in the figure should be adequate to break up many of the SL sheaths.

That this happens, is most evident from the V-L scatter plots shown in Figs. 14 c) and d), which compare the no-bumper and bumper cases, respectively, in 0 phasing. In this case, we have run ANSAT on a portion of the screen extending from 0.22 to 0.35 m poloidally, so that the fine structure of the many individual classes of sheaths remains distinct. (Poloidal variations of sheath voltage along the antenna tend to merge the fine structure together when the run is done on the full antenna. This, of course, is normally what one would do, however, for pedagogical reasons, we display the fine structure here.)

Figure 10 c) shows six distinct types of sheaths, each identified from the contact point analysis described earlier. They are: GF (gap-frame), Gi (gap sheaths, to "interior" bars, i.e. those between sidewall slots), Ge (gap sheaths, to "exterior" bars, i.e. those hoping across sidewall slots), SS (front face, i.e. screen-screen), SLs (screen-limiter short), and SLl (screen-limiter long). With the addition of bumper tiles, Fig. 10 d) shows that there is an overall reduction in both field line length and sheath voltage. Many of the field line connections have been broken up into pieces because of the bumpers (e.g. SLs → BSs + BL). Here the nomenclature is as before, with the additional classes BSs (bumper-screen short), BSl (bumper-screen long), BL (bumper-limiter). The net effect of the bumpers for this example was to reduce the average sheath voltage from 0.205 to 0.142 (in code voltage units), and the average field line length from 0.205 m to 0.142 m, both which act to reduce...
detrimental ICRF edge interactions in ways that are quantifiable using the other code modules, as previously described.

7) Effect of plasma on the sheaths

Up until now, the ANSAT results presented here have been for the case of vacuum antenna near fields, viz. the input fields to the ANSAT code were the EM fields calculated from an ARGUS vacuum run. Of course, the effect of the plasma on the impurity sputtering and power dissipation is subsequently included by ANSAT, but in this subsection we wish to focus on the effect of plasma on the EM fields themselves and hence on the voltages that driving the sheaths. While the presence of plasma is not expected to introduce major qualitative changes, there are two physical effects that require quantitative investigation. They are: a) the role of plasma in loading the antenna, and hence changing both the overall antenna voltage and also its distribution on the Faraday screen; and b) the fact that plasma in the near zone causes the slow wave (SW) to be evanescent radially on a shorter scale than would otherwise occur in a vacuum. The latter effect means that the $E_{||}$ fields responsible for sheath generation would not penetrate as far radially, e.g. from the Faraday screen out towards antenna bumper tiles or other limiters.

a) Plasma loading

To model the effect of plasma on the antenna loading, ARGUS runs were done using a surface plasma impedance matrix obtained from the RANTED code. An iterative technique is used to match the vacuum solution to the plasma surface impedance at the plasma-vacuum interface, which for these runs is taken to be 2 cm in front of the FS. Figure 15 shows voltage histograms for the cases of a) a vacuum run, b) a plasma run and c) a vacuum run with a close-in conducting wall. For all three runs, the same antenna geometry with bumper tiles (i.e. that of Fig. 14) was employed, and the phasing was 0-0.

First comparing cases a) and b), the most obvious differences are that a high voltage tail develops in the plasma case and that there appears to be less fine structure in the histogram below $V = 0.8$. Not obvious from the figure, is the fact that there is a slight reduction (less than 10%) in the voltages of the three dominant histogram peaks below $V = 0.8$. We have tentatively identified the physics which we believe is responsible for these differences between the plasma and the vacuum runs. The plasma, which acts to reflect portions of the spectrum, squeezes the flux coming out through the Faraday screen, and forces more of it to exit via the sidewall slots. This i) increases $|B|^2$ in front of the screen (a fact that has been verified with ARGUS diagnostics), ii) reduces the voltages of the FS sheaths (the gap and front-face sheath spikes below 0.8 in the figure), and iii) increases the

- 15 -
voltage of the sidewall slot driven sheaths, namely the long field line connections (bumper and screen - limiter sheaths above 0.8 in the figure). There is less fine structure in the histogram for the plasma case because a "smooth" boundary condition has effectively been imposed close to where we measure the sheath voltages.

These and other intuitive considerations suggest that it may be possible to model the effect of the plasma impedance on antenna sheaths using a much less sophisticated plasma impedance matrix than would be required for accurate antenna loading calculations. The simplest such model one could imagine is that of replacing the plasma by a perfectly conducting wall. The case illustrated in Fig. 15 c) is for a close-in conducting wall 2 cm in front of the FS. Comparison with a) and b) shows that it produces the same qualitative effects (lower voltage of the histogram peaks below 0.8, smoother structure below 0.8 and a high voltage tail) as the plasma, but more exaggerated. The high voltage tail in the close-in wall case actually extends out to \( V = 2.3 \), far beyond the right edge of the figure. While it is evident that a perfectly conducting wall so close to the FS is too severe a correction to model the effect of plasma, the present comparisons suggest that a very simple impedance model, or perhaps even a conducting wall at an appropriate distance further away, might be adequate for the purpose of modeling antenna sheaths.

As another illustration of how we have used the ANSAT tools to analyze the antenna sheaths, we set about to determine the location sheaths which cause the high voltage tail in the histogram of Fig. 15 c), the case with plasma. A V-L scatter plot (not shown) had already indicated that the high voltage sheaths \( (V > 0.8) \) result from the longer field line connections \( (L > 0.7) \). To determine the location of these sheaths we employed the sheath group and contact point analyses (see the flowchart of Fig. 4). The group analysis first enabled the selection of a subset of the full database which contained only those sheaths with \( V > 0.8 \). Then, using the contact point routine, we plotted the corresponding field lines in Fig. 15 d). The figure makes it evident that these high voltage sheaths occur for field lines that hit the top and bottom bumper limiters, and the high voltages may be related to the presence of the top and bottom feeders in the TFTR Bay-M design.

b) Plasma in the near zone

To answer the question about the effect of plasma in the near zone on the calculated sheath voltages, we have incorporated a plasma module into ANSAT itself. The module inputs a set of fields from ARGUS, and applies the plasma-induced corrections to the SW component of the EM fields, before proceeding with the rest of the sheath calculations and analysis. The self-consistent response of the antenna currents to the near zone plasma is not treated. Only the important effect of the plasma-induced evanescence of \( E_4 \) can be
obtained by this method. The ARGUS input could, in principle, be either the result of a vacuum run or a run with plasma in the far zone. If it were possible to run ARGUS with plasma in direct contact with the Faraday screen, the ANSAT module described here would not be required. At present, ARGUS cannot be run in such a manner. For clarity, in the discussion which follows, we shall refer to the input fields for this calculation as vacuum fields.

The algorithm for the ANSAT plasma module is as follows. The total vacuum $E$ field is first read in on a reference toroidal-poloidal plane where we imagine the plasma filled region to begin (e.g. a plane just tangent to the Faraday screen surface). This data is then Fourier decomposed to produce $E_q$ where $q$ is the two dimensional projection of the wavenumber $k$ in the toroidal, poloidal plane. The parallel projection, $E_{||} = E_q \cdot b$ is then computed and used to obtain the amplitude of the SW component of the field,

$$E_{sw} = E_{||} / b \cdot e_{sw}$$  \hspace{1cm} (11)

where the unit vector in the direction of the slow wave electric field polarization is

$$e_{sw} = \frac{P n_{\perp} + b}{|P n_{\perp} + b|}$$  \hspace{1cm} (12)

and

$$P = \frac{n_{||}}{n_{||}^2 - e_{\perp}}.$$  \hspace{1cm} (13)

The SW field components are projected radially forward through the plasma by the relation

$$E_{sw}(r) = E_{sw}(r_0) \exp(-\kappa (r-r_0))$$  \hspace{1cm} (14)

where $\kappa = -i k_r > 0$, with $k_r$ the radial component of the SW wavenumber, as determined from the SW dispersion relation

$$n_{||}^2 \varepsilon_{||} + \varepsilon_{\perp} (n_{\perp}^2 - \varepsilon_{\perp}) = 0,$$  \hspace{1cm} (15)

for each given toroidal and poloidal Fourier mode. Since we are interested in the plasma induced decay of $E_{||}$ (because it is the component that gives rise to the rf sheaths), it is not necessary to consider plasma induced modifications of the FW (which do not contribute to $E_{||}$). In any case, the radial scales of interest here are short, on the order of a few mm to a few cm, and on these scales the FW is relatively unaffected by low density plasma. Once $E_{sw}(r)$ has been formed, it is added to the (unmodified) FW component to reconstruct the total vector field $E_q(r)$. Finally, the latter is inverse Fourier transformed in the poloidal and toroidal variables to give $E(r)$ in the plasma region, $r > r_0$.

The SW dispersion relation implies that the scale length $\kappa^{-1}$ for radial evanescence of $E_{||}$ is controlled by three effects:

$$\kappa^2 - k_p^2 + \omega_p c^2 + k_{||}^2 (m/m_e),$$
where \( k_p \) is the wavenumber in the poloidal direction (e.g. the rippled structure of the fields due to the individual Faraday screen bars). The \( k_p \) term is essentially a vacuum effect while the remaining two terms are plasma effects. The above relation gives the correct order of magnitude for the last term when \( \omega_p/\Omega_i \gg 1 \) and \( \omega \sim \Omega_i \).

Output from a sample run of the ANSAT code using the above described plasma module is shown in Fig. 16, for the case of 0-0 phasing, and a plasma density of \( n_e = 10^{12} \text{ cm}^{-3} \) that is radially constant to the right of the point marked FS. Other parameters employed were \( \theta = 3^\circ, B = 4.5 \text{ T} \) and \( f = 47 \text{ MHz} \). The radial decay of the sheath potential is quite evident in the plasma case, and is due primarily to the skin depth term in this example, since we have chosen a rather high density for purposes of illustration.

IV. Discussion and Conclusions

The existing demonstration version of ANSAT is by no means without significant limitations, and these suggest areas for future developments, which we now discuss. The topics fall loosely into three categories: 1) programming: code efficiency and interface, 2) additional modules and capabilities, and 3) improved plasma physics.

1) Programming: Code Efficiency and Interface

The existing codes comprising the ANSAT suite were written to optimize flexibility (through modularity) and minimize the human time required for code development during Phase I. While these choices tend to result in a code architecture that is generally considered good programming practice, there are instances within ANSAT where a heavy price was paid in terms of numerical efficiency and hence code speed. Most of the modules run on a Cray-2 in a few minutes, so coding efficiency is a small issue for these. However, the module that creates the main database can take more than an hour of processor time on a Cray-2 for a high resolution run. There is reason to believe that a factor of 2 - 4 in code speedup is achievable with reasonable coding effort, and without causing undue degradation in the structure of the coding.

Other coding work which we envision would improve the human interface, which now consists of typing input parameters into UNIX shell scripts, and examine the possibility of porting the code to platforms other than the Cray, especially to UNIX workstations. At present, three ANSAT modules run in Mathematica,\(^{26}\) and still need to be translated into FORTRAN and integrated into the suite in a more seamless way than is presently the case.

2) Additional Modules and Capabilities

The modular structure of ANSAT makes it particularly easy to add additional capabilities to the code. The presently existing modules have been chosen because they illustrate physics and antenna design concepts which we believe to be important, but by no
means exclusively so. In the future, we wish to include a wider range of modules in the
code, in the hopes that ANSAT could become a kind of test bed for bench marking and
comparing a variety of ICRF edge physics theories which each other and with experiment.
Some examples of additional modules which would expand the present capabilities are: the
calculation of scattered power,\textsuperscript{27} the calculation of Fermi acceleration of electrons and
electron heating in the vicinity of the screen,\textsuperscript{28,29} and a Monte-Carlo calculation of the
rf-sheath induced self-sputtering of antenna materials.

In addition to new physics capabilities, several new technical capabilities would be
helpful. At present, ANSAT can handle a limited class of antenna symmetries. For
example, if ARGUS is run in a symmetric phasing for the right half of a symmetric antenna,
ANSAT can perform the necessary symmetry transformations before calculating the sheath
voltages for sheaths that span the entire reflected domain. With the increasing importance
of ICRF current drive, however, many modern antenna designs employ complex antenna
arrays with multiple straps, and multiple Faraday screens. It would be desirable to have
more sophisticated symmetry tools at our disposal, which properly transform not only the
EM fields, but also the Faraday screens and other hardware, in the (curved) toroidal
direction. Note that the tokamak magnetic field may have different (or no) symmetries with
respect to the antenna modules, and a proper account of these differences is critical in the
determination of the contact points for long (and likely high-voltage) sheaths that span
many antenna array modules. In addition, the ability to specify a relative phase of antennas
bay-bay would be useful in modeling existing antennas such as on TFTR.

3) improved plasma physics

By far the most complicated and substantial issues that arise in thinking about
ANSAT improvements involve the physics of the plasma in front of the antenna. While the
existing plasma capabilities of the code move a step in the right direction, there are many
ways in which this physics could be improved. First, it would be preferable if plasma right
next to the FS could be included self-consistently in the EM field calculation, either directly
in ARGUS, or possibly indirectly by iterating ARGUS with the ANSAT near field module.
Regarding the ARGUS plasma loading runs, the encouraging results of Fig. 15 b) and c)
indicate that, for the purpose of sheath calculations, it may be possible to employ a
simplified model of the plasma impedance, perhaps even using a close-in conducting wall
at an appropriate distance from the FS. Additional efforts in this direction seem worthwhile
pursuing.

Several improvements in our present treatment of convective cell SOL profile
modifications are also possible, and probably necessary to obtain reliable estimates of the
sputtered impurity influx and sheath power dissipation. The present ANSAT plasma module
assumes a constant density of the SOL plasma immediately in front of the FS, in order to use the simple exponential decay expressed by Eq. (14). This can and should be generalized to a density profile which has spatial variation, either by parametrizing a profile which yields to an analytical generalization of the exponential, or by solving the 2nd order ODE for $E_{\parallel}(r)$ numerically. Similarly, the SOL modifications module could easily be modified to solve for the $n_e(r)$ profile in front of the FS by solving the diffusion equation in the SOL (again a 2nd order ODE). Recall, that the diffusion coefficient can be calculated from convective cell theory. Finally, it would then be possible to iterate the above $n_e(r)$ and $E_{\parallel}(r)$ to a self-consistent set of profiles, akin to the physics given in Eqs. (7) for a two point model.

**Conclusion**

As the above paragraphs indicate, there are many possible areas for improvement and extension of the ANSAT concept. However, even in its present form, we have found ANSAT to be a useful tool in the interpretation of ICRF edge physics data, and are beginning to successfully employ it in antenna design studies.

As demonstrated in Sec. III, the ARGUS/ANSAT approach to antenna sheath analysis has already provided several new insights, which were previously unavailable from geometrically simplified descriptions of the antenna structure and its EM near fields. A prime example of this is the large residual sheaths that can occur in anti-symmetric phasings (such as 0-\pi) due to induced charge effects at the corners of the antenna. The effect of specific hardware features, such as sidewall slots and antenna bumper tiles are evident, as is the effect of "mismatches" in the hardware to the magnetic flux geometry, such as the FS - B field misalignment angle $\theta$, the antenna "V" angle $\alpha$ relative to the toroidal B field curvature, and antenna installation skew errors.

The ANSAT modules seem useful in the diagnosis and correction of potential sheath problems and should be able to assist antenna designers in optimizing antenna performance by appropriate trade-offs between antenna loading, well treated by other existing codes, and the antenna edge interactions considered here.

**Acknowledgments**

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References

#1. see for example Proceedings of the Tenth APS Topical Conference on Applications of Radio-Frequency Power to Plasmas, Boston, MA (AIP, New York, 1994), pp 3 - 80.


#21. TFTR test stand - ARGUS agreement.


#24. Sheath R_l as a diagnostic.

#25. RANT3D


Figure Captions

1. Relationship of ANSAT to ARGUS, and of both to the engineering design, test stand, and the experimental stages of antenna development and operation.

2. ARGUS representation of the TFTR Bay-M mock-up antenna structure

3. Faraday screen "V-shaped" geometry, and ANSAT transformation. In a) a "V" shaped Faraday screen is shown contacting a magnetic field line with toroidal curvature. Two of the contact points are A and B. In b) the same geometry is shown in a transformed coordinate system, where the screen in flat.

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6. L-r scatter plot showing interaction of flux surface geometry with antenna. The y-axis is the length of each field line; the x-axis is the radial flux coordinate. Five types of sheaths (contacts) are evident: screen-limiter long (SL₄), screen-limiter short (SL₃), front face sheaths that hop two and three bars (2B and 3B), and one bar (1B) or gap sheaths. The Faraday screen tangency surface (FSTS) is indicated by the arrow at the top of the figure.

7. Plasma facing analysis plots for π phasing. Shown are a) the V-L scatter plot for sheath contacts, b) the N(V) voltage histogram, and c) filled high voltage contours on the face of the FS (V > 0.4). Voltage is given in code voltage units; see text for the conversion to kV at a given power level. In d) an enlargement of the poloidal region of the FS between 0.22 and 0.35 m. is shown. The highest voltages in the figure are in red, and the lowest ones in blue.

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10. Convective cell analysis for π phasing. Shown in the top figure is V vs poloidal position for flux lines at two different radii (rf = 0.219 and rf = 0.226 m). Note the poloidal variation occurs on two scales, that of the whole antenna (most evident for
the smoother of the two curves at \( \tau_f = 0.226 \), and that of the FS bars and side-wall slots (most evident for the other curve). In the bottom figure, the convective cell flow (V) contours are given for an expanded view near the poloidal center of the antenna. The FS tangency surface is approximately at \( \tau_f = 0.22 \) near the center of the figure.

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12. V-L scatter plot for \( \theta = 20^\circ \) and 0 phasing. This run was done on a small poloidal portion of the full screen. Note that for this large a misalignment angle, front face sheaths up to 8-Bar are visible (i.e. field lines hopping across eight gaps between contacts). Comparision with Fig. 9 a) (the same case for \( \theta = 3^\circ \)) shows that much larger voltages are generated in the present case.

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14. Effect of bumper limiters on sheaths. In a) we show the geometry of 2 mm bumpers in the toroidal-radial plane (ANSAT space) while b) shows the same geometry for physical space. In c), the V-L scatter plot with no bumpers and 0 phasing is given. This is to be compared with the run in d) for the case with the 2 mm bumpers. Note that the bumpers break up many of the long field line connections, and result in somewhat lower sheath voltages.

15. \( N(V) \) histograms for the cases of a) a vacuum run, b) a plasma run and c) a vacuum run with a close-in conducting wall. The plasma run employs a surface impedance matrix boundary condition in ARGUS.\(^19\) The antenna phasing for these runs was 0-0. Note the high voltage tail induced in cases b) and c). In d), the field lines and their contact points are shown for the high voltage tail (sheaths with \( V > 0.8 \)) of case b).

16. Radial variation of the sheath potential for a vacuum run and for a run invoking the ANSAT near field plasma module. The skin-depth-induced decay of the evanescent SW, and hence of the sheaths, is evident.
Tables

<table>
<thead>
<tr>
<th>Phasing</th>
<th>D_{cc}/D_B</th>
<th>D_{total} (m^2/s)</th>
<th>n_e(FS) (10^{12} \text{ cm}^{-3})</th>
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</thead>
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<tr>
<td>\pi</td>
<td>0.1</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>\pi/2</td>
<td>1.2</td>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td>0</td>
<td>9.2</td>
<td>7.1</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Table 1. Results of preliminary SOL modifications module for calculating self-consistent density at the Faraday screen. The module employs a two point model to describe the evanescent slow wave penetration into the SOL plasma, and the rf driven convective cell enhancement of SOL transport. \( D_{cc}/D_B \) is the ratio of the convective cell portion of the diffusion coefficient to Bohm, \( D_{total} = D_{cc} + D_B \), and \( n_e(FS) \) is the self-consistent density at the Faraday screen.

<table>
<thead>
<tr>
<th>Group</th>
<th>( L ) (cm)</th>
<th>( V ) (V)</th>
<th>( A ) (cm^2)</th>
<th>( \Delta r ) (cm)</th>
<th>influx ((10^{19} \text{ atoms/s}))</th>
<th>P (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bar</td>
<td>14.1</td>
<td>469</td>
<td>116</td>
<td>0.4</td>
<td>1.4</td>
<td>12.</td>
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<tr>
<td>SL short</td>
<td>10.9</td>
<td>1986</td>
<td>80</td>
<td>0.3</td>
<td>3.0</td>
<td>38.</td>
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<tr>
<td>SL long</td>
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<td>2757</td>
<td>3</td>
<td>0.0</td>
<td>0.5</td>
<td>4.</td>
</tr>
</tbody>
</table>

Table 2. Group analysis for TFTR Bay-M sheaths, for 0 phasing at 3° misalignment. The (flux tube area weighted) average field line length between contacts \( L \), sheath voltage \( V \), total perpendicularly projected flux tube are \( A \), average distance \( \Delta r \) from the sheaths to the FSTS (Faraday screen tangency surface), Ti impurity influx, and sheath power dissipation \( P \) are given. The influx and sheath power analyses (done for TFTR running Bays L and M) is for 2 antennas powered at 1 MW each, with Ti coated screens and a dominant impurity of C in the SOL. See the main text for further discussion of \( P \).
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ANSAT FLOW CHART

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Phasor tracking analysis plots for 0 phase. Phasor are same as in Fig. 7. Note
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Appendix G  A Recursive Residual Norm Minimization Algorithm for Solving Large Sparse Non-Symmetric Indefinite Linear Systems
A Recursive Residual Norm Minimization Algorithm for Solving Large Sparse Non-Symmetric Indefinite Linear Systems

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October 1, 1995

Abstract

A new algorithm based on minimizing the Euclidean norm of the residual with a recursive relation is proposed for iterative solution of large sparse linear systems. The new method is generally applicable to indefinite and positive definite matrix. It exhibits the advantages of both the conjugate gradient method (CG) and generalized minimal residual (GMRES) technique of Saad and Schultz. The Recursive Residual Nörm (RRN) algorithm requires minimum storage like the Conjugate Gradient method, but it improves on GMRES by implicitly retaining information from previous steps to efficiently minimize the residual with respect to the norm and the space spanned by the operator. We were motivated to develop the RRN algorithm by the problem of microwave energy deposition in a plasma process reactor. We found that the CG method failed completely when the plasma density approached cut-off (when the operator was no longer positive definite). In that case while GMRES converged it required impractically large basis sets. For the test problem of a 3D frequency domain electromagnetic field solver in a closed space with specified boundary conditions, we have found that RRN converges rapidly and is as efficient as the CG method.
1. INTRODUCTION

The classical conjugate gradient algorithm (CG hereafter) of Hestenes and Stiefel [1] is one of the most widely used and effective iterative methods for solving large sparse symmetric positive definite (SPD) linear system of equations. It can be used in combination with preconditioning technique [2] to achieve excellent convergence for a variety of problems. However, the classical CG method often fails for non-symmetric indefinite linear systems because the basic algorithm relies on the minimization of the energy norm of the residual, and there is no guarantee that this minimum of the energy norm exists. Many generalizations of the classical CG method have been published over the years, for example: Generalized Conjugate Gradients (Axelsson [3]), Orthomin (Vinsome [4]), Orthodir (Young and Jea [5]), Bi-Conjugate Gradients (Fletcher [6]), and CGS (Sonneveld [7]). Among these, an efficient and popular way to solve the non-SPD linear systems, \[ Ax = b, \] (1)
is a generalized minimal residual algorithm (GMRES) proposed by Saad and Schultz [8]. This algorithm approximates the exact solution by projecting it over the Krylov subspace \( K_k = \text{span}\{v_1, Av_1, ..., A^{k-1}v_1\} \), where \( A \) is the \( N \times N \) matrix operator, \( b \) is a known \( N \) dimensional vector, \( v_1 = r_0 / ||r_0|| \), \( r_0 = b - Ax_0 \) with \( x_0 \) being the initial guess of the solution \( x \) of dimension \( N \) and \( ||r_0|| \) is the norm of \( r_0 \). The GMRES algorithm is a robust procedure to obtain a converged solution for \( x \) by minimizing the norm of the residual at each iterative step of increasing \( k \) until the desired accuracy is achieved. Clearly, if the maximum value of \( k \) is of the order of \( N \), there is no advantage the GMRES algorithm has over any direct method like Gaussian elimination, because the basis vectors of \( K_k \) has to be stored in memory during the iterative procedure. In practice, \( k \) is a fixed integer, \( k_0 \), which is big but still much less than \( N \). When the dimension of the Krylov subspace \( K_k \) has reached \( k_0 \), decision has to be made. If the desired accuracy for \( x \) has been reached, the procedure is stopped and the algorithm is convergent. If the desired accuracy for \( x \) has not been achieved, the algorithm is restarted with the most recently obtained and much better approximated \( x \) as the initializing \( x_0 \). This restarting procedure
is repeated until the desired accuracy of $x$ is achieved. In general, there is an optimal $k_0$ which results the fastest overall convergent rate. The optimal size of the number $k_0$ is problem dependent. The restarting procedure has the disadvantage, compared to CG, that new set of basis vector, and hence the new trial solutions, are no longer orthogonal to those from previous iterations. This can result in a slow rate of convergence.

For our test problem the solution for the frequency domain electromagnetic field within a microwave plasma process reactor in three dimensional geometry, requires $k_0$ of order 100 for $N$ of the size $20 \times 20 \times 30$. Even $k_0$ is much less than $N$ in this test problem, it is still too big to be practical for real application. Therefore, a new algorithm with much lower memory requirement and faster convergence is essential. In the following, we describe the new algorithm combining the advantages of both the CG and GMRES methods while eliminating their shortcomings.

2. THE NEW ALGORITHM

For any linear system described by Eq. (1), we define the residual, $r_k$, at each iterative step $k$ as:

$$r_k = b - Ax_k.$$  

(2)

In both the CG and the GMRES scheme, each $k$-th approximate solution $x_k$ is projected to the basis of $K_k$

$$x_k = x_0 - \sum_{i=1}^{k} c_{k,i} A^{i-1} r_0.$$  

(3)

Therefore we have,

$$r_k = [1 + \sum_{i=1}^{k} c_{k,i} A^i] r_0 = P_k(A) r_0.$$  

(4)

Here $P_k(A)$ is a polynomial in $A$ of order $k$. Obviously, we have $P_k(0) = 1$. In the GMRES algorithm, the $k$ coefficients, $c_{k,i}$, in Eq. (4) are obtained by solving a least square problem
to minimize the Euclidean norm of $r_k$. In the CG algorithm, Eq. (4) is rewritten [9] in a form similar to the Lanczos recursion [10]:

$$r_{k+1} = -\alpha A r_k + \beta r_k + \gamma r_{k-1},$$

or equivalently

$$x_{k+1} = \alpha r_k + \beta x_k + \gamma x_{k-1},$$

where $\alpha$, $\beta$ and $\gamma$ are determined by conditions that derived from minimizing the energy norm $E_k = r_k^T A^{-1} r_k$ and $P_k(0) = 1$. It can be shown [9] that $E_k$ is minimized if and only if $r_k$ is orthogonal to all previous residuals, i.e. $r_k^T r_j = 0$, for $j = 1, ..., k - 1$. Thus, the algorithm reduces to the familiar CG procedure of Hestenes and Stiefel [1]. Comparing with GMRES, the CG algorithm needs to store at most 6 vectors: $r_{k+1}$, $r_k$, $r_{k-1}$, $x_{k+1}$, $x_k$, and $x_{k-1}$ at each iterative step, because the coefficients $c_{k,i}$ are formal combination of the three independent parameters $\alpha$, $\beta$ and $\gamma$. While Eq.(5) is more restrictive than Eq.(4) as the coefficients $c_{k,i}$ in GMRES are independent at each step, it guarantees the preservation of information from previous iterations. The simplicity of the CG algorithm is obtained by minimizing the energy norm $E_k$, which exists for positive definite systems, from the Lanczos recursion form. Enlightened by this derivation of the CG algorithm, we can generalize it by minimizing the norm of $r_k$, just as in GMRES. Unlike GMRES, we use the Lanczos recursion form, Eq. (5), for minimization instead of the more general form in Eq. (4). The differences between the CG, GMRES and the new recursive residual norm minimization (RRN) algorithm are shown explicitly in Table 1. The new algorithm has the advantage that it is (i) simple to implement; (ii) efficient in the use of storage; and (iii) converges reliably for indefinite sparse linear systems.

To derive the RRN algorithm, we obtain the Euclidean norm of $r_k$ using Eq.(5) then minimize it with respect to $\alpha$ and $\gamma$, keeping in mind that $\beta + \gamma = 1$. The result is the following new algorithm for indefinite systems:

**Step 1:** Choose initial guess $x_0$, find $r_0 = b - A x_0$, then evaluate $\alpha_1 = r_0^T A r_0 / (A r_0)^T A r_0$ and $x_1 = \alpha_1 r_0 + x_0$. 


Step 2 Iteration for $k \geq 1$: first evaluate $I_0 = (Ar_k)^T Ar_k$, $I_1 = (r_k)^T r_k$, $I_2 = r_{k-1}^T r_{k-1}$, $I_{01} = (Ar_k)^T r_k$, $I_{02} = (Ar_k)^T r_{k-1}$, and $I_{12} = r_k^T r_{k-1}$, then obtain

$$D = I_0(I_1 + I_2 - 2I_{12}) - (I_{01} - I_{02})^2,$$

$$\alpha_k = D^{-1}[I_{02}(I_1 + I_2 - 2I_{12}) + (I_2 - I_{12})(I_{01} - I_{02})],$$

$$\gamma_k = (I_{01} - I_{02})^{-1}(\alpha_k I_0 - I_{02}),$$

and

$$x_{k+1} = \alpha_k r_k + \gamma_k x_k + (1 - \gamma_k)x_{k-1}.$$

Step 3: Repeat Step 2 until convergence criterion is satisfied.

This new algorithm is slightly more complicated than the CG method. However, its application is much more general than the CG method because at each step in the iteration it minimizes a positive definite quantity $r_k$ regardless whether $A$ is positive definite or not. RRN’s demand on memory storage is much less than that for GMRES because the residual at the next step $k + 1$ is expressed as a linear combination of $Ar_k$, $r_k$, and $r_{k-1}$ (Eq. (5)) instead of the more complicated form in Eq. (4) where all the basis vectors of the Krylov subspace $K_k$ have to be stored in memory. Moreover, the new algorithm is computationally more efficient than solving the least square problem at each step $k$. In the following, we demonstrate the new algorithm on a challenging test problem and find that the new method is faster than the CG method for positive definite systems, and produces converged solutions much faster than GMRES for indefinite systems.

3. TEST PROBLEM: A FREQUENCY DOMAIN FIELD SOLVER

As a test problem for the new algorithm, we consider the frequency domain Maxwell equation in the presence of a plasma.

$$\left(\frac{\omega^2}{c^2} \left[ 1 - \frac{\omega_p^2}{\omega(\omega + iv)} \right] - \nabla \times \nabla \times \right) \vec{E} = 0,$$  \hspace{1cm} (6)
where $\vec{E}$ is the electric field vector, $\omega_p^2 = 4\pi N_e e^2/m_e$ is the electron plasma frequency squared, $N_e$ is the electron density, $m_e$ is the electron mass, $\omega$ is the frequency of the electromagnetic field drive, and $\nu$ is the electron collision frequency.

Equation (6) describes the electromagnetic fields inside a microwave or induction plasma process reactor commonly used in etching and deposition of electronic materials. Microwave field of a fixed frequency is introduced into the reactor from waveguide through a dielectric window, or the RF field is generated by currents in an antenna set inside the reactor vessel. Discharge occurs inside the reactor at location around where the electric field peaks. As the plasma density profile grows in the reactor, the electromagnetic field also changes. At each instant in time during this process, the numerical problem is to determine the electromagnetic field profile inside the reactor vessel that is consistent both with the plasma density and the boundary conditions.

In the limit of $N_e = 0$ (in vacuum), Equation (6) reduces to the Helmholtz equation

$$\left( \nabla^2 + \frac{\omega^2}{c^2} \right) \vec{E} = 0. \quad (7)$$

The Helmholtz equation is a definite system if the driving frequency is below cut-off (i.e. the vacuum wavelength of the driving field longer than the characteristic length of the computational domain). Both the CG and the GMRES methods converge to appropriate physical solutions in this case for both Dirichlet and Neumann boundary conditions. When the driving frequency is above cut-off (i.e. the vacuum wavelength of the driving field shorter than the characteristic length of the computational domain), the Helmholtz equation is an indefinite system, and the CG method still works despite the fact that the Helmholtz equation is no longer a definite system, because indefiniteness is not sufficient to cause CG method to fail.

In the presence of plasma ($N_e \neq 0$), Eq (6) is the appropriate equation to be solved for the electric field in frequency domain with proper boundary conditions. The resulting equation is an indefinite linear system. However, the CG method diverges for this indefinite linear system and only the GMRES method converges to a physical solution.
In a 3-dimensional rectangular computational domain, in order to facilitate the application of the boundary conditions on the components $E_x$, $E_y$, and $E_z$ of the electric field so that any parallel component of $\vec{E}$ to a metallic wall is zero, it is convenient to arrange $E_x$, $E_y$, and $E_z$ in a staggered manner [11]. A two-dimensional version of the staggered field components in a rectangular grid is shown in Fig. 1. Then, the $\nabla \times \nabla \times$ operator in Eq. (6) can be written in the following finite difference form:

\begin{align*}
\nabla \times \nabla \times \vec{E} |_x &= d_x^{-1} d_y^{-1} [E_y(i, j + 1, k) - E_y(i, j, k) - E_y(i - 1, j + 1, k) + E_y(i - 1, j, k)] \\
&\quad + d_x^{-1} d_z^{-1} [E_z(i, j, k + 1) - E_z(i, j, k) - E_z(i - 1, j, k + 1) + E_z(i - 1, j, k)] \\
&\quad - d_y^{-2} [E_x(i, j, k + 1) - 2E_x(i, j, k) + E_x(i, j, k - 1)] \\
&\quad - d_z^{-2} [E_x(i, j, k) - 2E_x(i, j, k + 1) + E_x(i, j, k + 1)], \quad (8a)
\end{align*}

\begin{align*}
\nabla \times \nabla \times \vec{E} |_y &= d_x^{-1} d_y^{-1} [E_x(i + 1, j, k) - E_x(i, j, k) - E_x(i + 1, j - 1, k) + E_x(i + 1, j - 1, k)] \\
&\quad + d_y^{-1} d_z^{-1} [E_z(i, j, k + 1) - E_z(i, j, k) - E_z(i, j - 1, k + 1) + E_z(i, j - 1, k)] \\
&\quad - d_z^{-2} [E_y(i + 1, j, k) - 2E_y(i, j, k) + E_y(i - 1, j, k)] \\
&\quad - d_z^{-2} [E_y(i, j, k + 1) - 2E_y(i, j, k) + E_y(i, j, k - 1)], \quad (8b)
\end{align*}

\begin{align*}
\nabla \times \nabla \times \vec{E} |_z &= d_x^{-1} d_y^{-1} [E_x(i + 1, j, k) - E_x(i, j, k) - E_x(i + 1, j - 1, k) + E_x(i + 1, j - 1, k)] \\
&\quad + d_y^{-1} d_z^{-1} [E_y(i, j + 1, k) - E_y(i, j, k) - E_y(i, j + 1, k - 1) + E_y(i, j, k - 1)] \\
&\quad - d_z^{-2} [E_z(i + 1, j, k) - 2E_z(i, j, k) + E_z(i - 1, j, k)] \\
&\quad - d_y^{-2} [E_z(i, j, k + 1) - 2E_z(i, j, k) + E_z(i, j, k - 1)]. \quad (8c)
\end{align*}

In Eqs. (8), the indexes $i, j, k$ of $E_x$, $E_y$, and $E_z$ refer to the $x, y$, and $z$ directions, and $d_x, d_y, d_z$ are the grid spacings in the $x, y$, and $z$ directions. In the following test of the new algorithm, we solve Eq.(6) in a rectangular box with a grid of the size $20 \times 20 \times 30$. The boundary condition is Dirichlet such that all but one wall in the lower $z$ direction are metallic (with all parallel field components zero), and a given drive-field in the $E_x$ and $E_y$ components are imposed in the lower $x - y$ plane. Since Eq.(6) is complex, $E_x$, $E_y$, and $E_z$ are complex vectors, which means the resulting real finite difference system is of the order $N = 2 \times 3 \times 20 \times 20 \times 30 = 72,000$ and the total number of elements in the operator
\( A \) is of the order \( N^2 = 72000^2 \approx 5.2 \times 10^9 \). For such large linear system, using any of the direct methods such as Gaussian elimination or Gauss-Jordan algorithm, which requires an order of \( N^3 \) operations, is completely out of the question.

In the following, we compare the convergence between the GMRES and our new algorithm for a case of Dirichlet boundary condition with a given elliptical shaped Gaussian plasma density distribution at the center of the domain of calculation. The peak electron density \( (N_e)_0 \) is fixed at \( 2.0 \times 10^{11} \text{cm}^{-3} \). We choose the driving frequency to be \( 4.0 \times 10^9 \text{sec}^{-1} \) so that the peak electron plasma frequency to driving wave frequency ratio \( \left( \frac{\omega_{pe}}{\omega} \right) \) is 1.005, a case when the solution differs significantly from the vacuum solution but not to the extreme as the cases of much larger \( \frac{\omega_{pe}}{\omega} \) where the electromagnetic field is highly shielded from the plasma. This case of \( \frac{\omega_{pe}}{\omega} \approx 1.0 \) is numerically more difficult to converge.

The vacuum wavelength of the field is \( 7.5 \text{cm} \). The dimension of the rectangular domain is chosen to be \( L_x = 10 \text{cm}, L_y = 10 \text{cm}, \) and \( L_z = 15 \text{cm} \) so that the driving frequency is above cut-off and the grid spacing is sufficiently small compared with the vacuum wavelength. The boundary condition is such that the driving \( E_x \) and \( E_y \) (both located on the z-grid) field is fixed at the lower x-y boundary plane. All the other five boundary planes is treated as metallic so that components of electric field parallel to the planes are zero. The drive fields at the \( z = 0 \) plane are those corresponding to the TE10 wave-guide mode, i.e. \( E_x = 0 \) and \( E_y = \sin(\pi x/L_x) \). The geometry of our computational domain and the driving field at the boundary are shown in Fig. 2.

The CG method of Hestenes and Stiefel[1] diverges for this particular test case. Applying GMRES algorithm with 99 basis vectors and an initial guess with \( E_x, E_y \) and \( E_z \) all equal to zero, we get an converged solution after about 3000 iterations. The iteration is proceeded in such a way that the Krylov basis vectors are added one by one to the maximum number \( (k_0 = 99) \) to span an approximated solution that minimizes the residual \( r_k \) as defined in Eq. (2). This constitutes the inner iteration loop. Then the most recently obtained approximated solution is used as the initial guess to restart the inner iteration. After 30 times of restarting the inner iterations, or a total of 2970 inner iterations, the
resulting residual becomes 6 orders of magnitude smaller than the initial residual. Defining the relative error at each cumulative inner iteration $k$ as $\varepsilon_k = \|r_k\|/\|r_0\|$ we plot the relative error as function of $k$ in Figure 3 to characterize the rate of convergence of the GMRES algorithm for our test problem. The total CPU time required for the whole computation is 432 minutes in an IBM- RS6000 workstation. If we increase the number of basis vectors of the Krylov subspace to 179 and start the iterating process with same initial guess we get a solution converged to same degree of accuracy after about 2000 iterations. But the total CPU time required for the computation is 446.3 minutes, slightly more than the case with 99 Krylov basis vectors. The reason is even the number of iterations for converging to same accuracy is smaller but the average amount of work of each iteration is more because a bigger matrix has to be inverted. The relative error for the case with 179 Krylov basis vectors as function of iteration is shown in Fig. 4. Similar plot is obtained for same GMRES computation with 12 Krylov basis vectors as shown in Fig. 5. However for the case with 12 Krylov basis vectors, it took 659 minutes in CPU time and about 16,000 iterations to achieve an relative error of $3.0 \times 10^{-5}$. We can observe from this set of runs that there is an optimal number of Krylov basis vectors to achieve a desired degree of accuracy for a minimum amount of computational time. This optimal number of Krylov basis vectors is problem dependent and cannot be determined a priori.

In contrast, applying the new algorithm outlined in Section 2 we achieve the same accuracy in about 2000 iterations using 81 minutes CPU time in the same IBM- RS6000 workstation. The rate of convergence for this algorithm with the same initial guess is shown in Figure 6. The new algorithm requires the storage of six field vectors of size $N = 72,000$ compared with 100 field vectors of same size in the GMRES algorithm used in Fig. 3. The new algorithm is also much faster than the GMRES because there are only three coefficients to be determined in each iteration whereas in the GMRES algorithm matrices of sizes up to $k_0 \times k_0$ have to be inverted.

As an example of a converged electric field solution in the presence of a plasma density profile, the 3-D contour of the electric field, $\|E\|^2 = \|E_x\|^2 + \|E_y\|^2 + \|E_z\|^2$, of the converged solution is shown in Fig. 7. All parameters are the same as stated previously,
except the frequency is 2.45 GHz, as is frequently used in microwave plasma experiments. In Fig. 7, a 3-dimensional view of the rectangular computation domain is presented. The contours of $\|E\|^2$ in two vertical (x-y) planes at the end faces of the rectangular domain and one horizontal (x-z) plane are shown in grey scale where the brighter color represents higher electric field. The vertical plane in the lower left corner is the window through which external power source corresponds to a TM11 mode is fed into the rectangular cavity, and the vertical plane in the upper right corner is the midplane of the system. The system is symmetrical with respect to the midplane. The electric field has a low value inside an ellipsoidal region around the center of the midplane where the electron density peaks with a maximum value of $2.0 \times 10^{11}$. This is in contrast with the vacuum solution of the same problem (i.e. without the plasma), where the electric field peaks on the midplane. The existence of the shady region of low electric field around the midplane is the manifestation of the shielding effects of the plasma density profile above cut-off. The peak electric field is being pushed by the plasma from the midplane towards the window along both sides of the center line. Both the GMRES and the new algorithm give the same result.

4. CONCLUSION

In this paper, we proposed a new algorithm based on minimizing the norm of residual based on the Lanczos recursion (RRN) to solve a large sparse non-SPD (symmetric positive definite) linear system iteratively. This method combines the advantages of the conjugate gradient method and GMRES. A 3-dimensional frequency domain electromagnetic field solver is developed based on this new method for applications in a microwave plasma processing reactor. This algorithm is not much more complicated than that of the conjugate gradient method of Hestenes and Stiefel, but has the advantage of producing convergent solutions both when the operator is positive definite and indefinite. Unlike the GMRES algorithm, which requires large numbers of memory for storing the basis vectors of the relevant Krylov subspace, this new method requires the storing of only a few previous iterations of the solution. As shown in the test problem, the new method is only slightly
slower than the conjugate gradient method for positive definite system, and produces convergent solutions much faster than GMRES for non-SPD linear systems.

From the series of runs presented in this paper, the computational time required for a given accuracy depends on the number of Krylov basis vectors being used. If the number of Krylov basis vectors is chosen too small, the memory storage requirement is relaxed but the computational time is much longer to achieve the same accuracy. One the other hand, if the number of Krylov basis vectors is chosen too large, both the memory storage requirement and the computational time cost more for the same problem. There exists an optimal number of Krylov basis vectors for each individual problem so that both the memory storage requirement and the computational time will be minimized. However, even with this optimal number of Krylov basis vectors the GMRES algorithm is still too slow in convergence and requires to much memory storage compared with the new algorithm presented here. In the test case we considered in this paper, the new algorithm is more than 5 times faster than GMRES, and more than 16 times less memory required.

The new algorithm may be generalized by extending Eq.(5) to include more iterative terms on the right hand side. It can also be used in combination with preconditioning techniques [12] for further acceleration of convergence.

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REFERENCES


Table Captions:

Table 1 shows the differences between the CG, GMRES methods and the new algorithm (RRN) in how the residual is expanded and which quantity is minimized.
Figure Captions:

Figure 1: A staggered $E_x$ and $E_y$ components of the electric field in a rectangular grid is shown. The $E_x$ component is cell-centered in the x direction and grid-centered in the y and z direction. The $E_y$ component is cell-centered in the y direction and grid-centered in the x and z direction. Similarly, the $E_z$ component (not shown) is cell-centered in the z direction and grid-centered in the x and y direction.

Figure 2: Schematic view of the geometry of the computational domain and the Dirichlet boundary condition used in the test problem.

Figure 3: The relative error $\varepsilon_k = \|r_k\|/\|r_0\|$ is plotted as function of the iteration number $k$ in solving the test problem by the GMRES algorithm with 99 Krylov basis vectors, where $\|r_k\|$ is the norm of the residual defined in Eq. (2). The CPU time shown is that on an IBM-RS6000 workstation.

Figure 4: The relative error $\varepsilon_k$ is plotted as function of the iteration number $k$ in solving the same test problem by the GMRES algorithm with 179 Krylov basis vectors.

Figure 5: The relative error $\varepsilon_k$ is plotted as function of the iteration number $k$ in solving the same test problem by the GMRES algorithm with 12 Krylov basis vectors.

Figure 6: The relative error $\varepsilon_k$ is plotted as function of the iteration number $k$ in solving the same test problem by the new algorithm as described in Section 2.

Figure 7: A 3D view of contours of the electric field, $\|E\|^2 = \|E_x\|^2 + \|E_y\|^2 + \|E_z\|^2$, of the converged solution obtained from both the GMRES and the new algorithm are shown in two vertical (x-y) planes at the end faces of the rectangular domain and one horizontal (x-z) plane. The contours are shown in grey scale where the brighter color represents higher electric field. External power corresponds to a TM11 mode is coupled into the rectangular computational domain with a size of $10cm \times 10cm \times 7.5cm$ through the vertical plane in the lower left corner. The driving frequency is $2.45 \times 10^9 sec^{-1}$. 
<table>
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<td>Euclidean norm</td>
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Table 1
Fig. 2
Convergence of GMRES with 99 Krylov basis vectors

Relative error

1

10^{-6}

10^{-5}

10^{-4}

10^{-3}

10^{-2}

10^{-1}

1

Total CPU time 432 minutes

Iterations

1

10

100

1000
Convergence of GMRES with 179 Krylov basis vectors

Fig. 4

Convergence of GMRES with 179 Krylov basis vectors

Relative error

Total CPU time 446.3 minutes

Iterations

1

10

100

1000

1

0.1

0.01

0.001

0.0001

10^{-5}

10^{-6}
Convergence of GMRES with 12 Krylov basis vectors

Fig. 5

Relative error vs. iterations graph showing the convergence rate.
Relative error

Convergence of new algorithm

Fig. 6
3D view of Electric Field contours
REFERENCES


9. Taking the presheath ion velocity greater or equal to the ion sound speed is a necessary condition that the unmagnetized sheath equations admit solutions. A different requirement may apply to magnetized sheaths.


FIGURE CAPTIONS

Fig. 1 Illustration of the charge distribution across the sheath cross-section, according to the "moving plate capacitor" model.

Fig. 2 Geometry with magnetically aligned coordinates, and ion motion in the plane perpendicular to $B_0$. Dashed line is a typical orbit in a uniform electric field. Solid line corresponds to unstable ion drift caused by the sheath electric gradient $dE_0/dx$. Motion along $B_0$ not shown.

Fig. 3 (a) Ratio of the effective mass to the fully magnetized $m^*/m_B$ plotted vs. $\theta$ for different values of $\omega_i^2/\Omega_i^2$. (b) Ratio of the of the effective to the actual mass $m^*/m_i$ for same parameters as in (a).

Fig. 4 Ratio between the cross-B and along-B contributions to transport along the electric field, plotted vs. $\theta$ for various $\omega_i^2/\Omega_i^2$.

Fig. 5 Particle simulation results. Profiles of the time-averaged sheath potential $V_0(x) = \overline{V(x,t)}$ across the capacitor plates for various angles $\theta$ between the electric and magnetic field. The driving RF amplitude is 500 V and $\omega_i^2/\Omega_i^2 = 0.76$ in the main plasma.

Fig. 6 (a) Ratio of the total DC sheath potential to the applied RF amplitude vs. angle $\theta$. (b) Ratio of the total DC sheath potential to the unmagnetized limit $V_{dc}(\theta)/V_{dc}(0)$ vs. angle $\theta$. Diamons are from the numerical simulation results in Fig. 5. Solid line is formula (24) for best fitting sheath charge density $\omega_i^2/\Omega_i^2 = 0.45$.

Fig. 7 Illustration of the instantaneous potential $V_i(t) = V(0,t) - V(\Delta, t)$ across each sheath vs time (solid lines); the sum of the two sheath potentials at each moment equals the RF driving potential (dashed). (a) maximum sheath potential equals the RF amplitude (b) maximum sheath potential exceeds RF due to sheath biasing.

Fig. 8 Profiles of the instantaneous sheath potential $V(x,t)$ taken at the moment of minimum $V_{rf}(t) = -500V$ across the capacitor plates, for various angles $\theta$ and same parameters.
as in Fig. 5.

Fig. 9 Orbits from numerical integration of Eqs. (41), following six particles starting at the origin with $0.01 < v_{⊥0} < 1$ and $0.005 < v_{⊥0} < 2$. (a) Orbits in velocity space $v_∥, v_⊥$ after 5 ion cyclotron periods (b) Orbits in velocity space $v_∥, v_⊥$ after 7.5 ion cyclotron periods. The diamonds, marking the final locations, are aligned along a line with slope given by Eq. (52). Here $\theta = 82.5^o$, $\omega_i^2/\Omega^2 = 0.5$, length is in units of $\lambda_d$, velocity in units of $v_o = \omega_i \lambda_d$ and $eE_p/(mv_o^2/\lambda_d) = 0.02$. 
Fig. 3
$\frac{\omega_i^2}{\Omega_i^2}$

Fig. 4
Vrf = 500 V

Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9