*Residual Stress Measurement by
Successive Extension of a Slot:
A Literature Review*

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*Residual Stress Measurement by
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A Literature Review*

Michael B. Prime

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RESIDUAL STRESS MEASUREMENT BY SUCCESSIVE EXTENSION OF A SLOT: A LITERATURE REVIEW

Michael B. Prime

ABSTRACT

This report reviews the technical literature on techniques that employ successive extension of a slot and the resulting deformations to measure residual stress. Such techniques are known variously in the literature as the compliance or crack compliance method, the successive cracking method, the slotting method, and a fracture mechanics based approach. The report introduces the field and describes the basic aspects of these methods. The report then reviews all literature on the theoretical developments of the method. The theory portion first considers forward method solutions including fracture mechanics, finite element, analytical, and body force methods. Then it examines inverse solutions, including incremental inverses and series expansions. Next, the report reviews all experimental applications of slotting methods. Aspects reviewed include the specimen geometry and material, the details of making the slot, the method used to measure deformation, and the theoretical solutions used to solve for stress. Finally, the report makes a brief qualitative comparison between slotting methods and other residual stress measurement methods.

I. INTRODUCTION

This review is part of an internally funded research project at Los Alamos National Laboratory. The project aims to extend and further develop residual stress measurement techniques of the type described in this report. As part of the first year research effort, all available literature on both the theory and application of these techniques was assembled and reviewed. This report contains the result of that effort.

The remainder of Section I gives a brief background on residual stress and its measurement, defines the scope of the review, and outlines the organization of this report.

I.A. Residual Stress

Residual stresses play a critical role in failures due to fatigue, creep, wear, stress corrosion cracking, fracture, buckling, distortion, and more. Additionally, residual stresses often cause dimensional instability, such as distortion after heat treating or after machining a part with residual stress. Residual stresses are those present in a part free of external loads and are generated by virtually all manufacturing processes. They add to applied loads and are particularly insidious because they satisfy equilibrium and offer no external evidence of their existence.

Because of their major contribution to failures and almost universal presence, knowledge of residual stresses is crucial for any engineering structure where liberal safety factors are impractical. Ideally one would like to accurately predict residual stresses resulting from the various manufacturing operations. A great deal of research effort is focused on this task. However, the problem is very complex. Development of residual stress generally involves nonlinear material behavior and often involves material removal, phase transformations, and coupled mechanical and thermal problems. For the majority of problems, the current predictive capabilities are insufficient to give adequate knowledge of residual stresses. So, the ability to measure residual stress is critical for two purposes: (1) to minimize residual stress-related failures, and (2) to aid in developing predictive capabilities by verifying models.

One might prefer to measure residual stress nondestructively. However, there are crucial gaps in the capabilities of nondestructive techniques (Lu 1996). The two primary nondestructive techniques are x-ray diffraction (XRD) and neutron powder diffraction (NPD). XRD can measure residual stress in crystalline materials to a maximum depth of about 0.05 mm. Measuring to a greater depth requires layer removal, such as by etching, and negates the nondestructivity of XRD. NPD can measure residual stress depths of many millimeters but is generally constrained to measuring a volume no smaller than a cube 1 to 2 mm on a side. This constraint makes it difficult or impossible to resolve residual stress variations over distances of less than about 1 mm, which excludes the large range from 0.05 mm to 1 mm where nondestructive measurement is not feasible. Unfortunately, many of the most common manufacturing processes produce residual stresses that vary over this range: heat treating, machining, forging, cladding, and casting, for example. At

the same time, the primary contribution of residual stress to mechanical failures, such as from fatigue and fracture, can occur over this range.

Additionally, there are other limitations to the XRD and NPD methods:

- (1) Sensitivity to grain size and texturing effects. Crystalline structural anisotropy may render the strains to be ambiguous.
- (2) Difficulty measuring residual stresses in the presence of multiple material phases. Resolving strains for individual constituents in a multiphase material is complicated.
- (3) Complete inability to measure non-crystalline materials.
- (4) Difficulty measuring stresses on curved surfaces. The path length of the scattered beam is increased, which cannot be distinguished from strain, leading to errors.
- (5) A typical stress depth profile obtained using XRD or NPD methods may take a week to a month. The same stress profile using crack compliance could be accomplished within several days.

These limitations illustrate the need for residual stress measurement techniques that fill in the missing capabilities of current methods.

I.B. Scope of Review

This literature review is limited to a specific subset of the residual stress literature. The techniques reviewed here measure residual stress variation with depth by incrementally introducing a slot or cut into a part containing residual stress. Depth here refers to the direction of slot extension. Some measure of deformation, such as strain or displacement, is measured at each increment of depth. From these measurements, the residual stresses that originally existed in the part are calculated. Other similar methods, such as hole drilling, layer removal, and sectioning, are not considered here.

Every effort has been made to obtain relevant published literature. All available articles from technical journals are included. Articles from technical conferences are included only when the same work does not appear in a journal article and when the article provides unique information. Because of the difficulty in locating all possible conference articles, some have likely been inadvertently omitted.

A good source for information on the many other residual stress measurement methods is Lu (1996), although its coverage of the work reviewed here is very poor. Cheng and Finnie (1994c) review their contribution to the work covered in this report.

I.C. Organization of Report

Section II gives the background necessary to fully understand the report and its layout. A brief chronological history of the method is presented. These techniques can be broken down into several common aspects or components, which are defined and explained. These components are used in subsequent sections to organize the report. Some terminology and

a standard set of coordinate axes are defined. This provides a framework under which to compare diverse works.

Sections III and IV contain the bulk of the literature review. Section III reviews the theoretical developments; it is further subdivided into sections on the forward and inverse portions of the problem, as defined in Section II.B. Section IV reviews all experimental applications of the method. A paper that contains new and relevant information will be reviewed in each section. A given paper may appear in all of the sections or only one.

Section V is a brief qualitative comparison between the residual stress measurement methods reviewed here and other common techniques.

II. BACKGROUND & TERMINOLOGY

Techniques for measurement of residual stress using successive extension of a slot are known in the technical literature by several names: compliance method, fracture mechanics approach, successive cracking method, crack compliance method, slotting method, rectilinear groove method, etc. In this paper, the term originally coined by Cheng and Finnie (1985), the compliance method, is used as an inclusive term. The name came from the similarity of this technique to the compliance method for measuring crack length in a fatigue or fracture specimen (Saxena and Hudak 1978); a known load was applied to a cracked specimen, and the resulting strain was used to determine the crack length. In this paper, the crack length is known and the measured strain is used to calculate the residual stress.

II.A. Historical Overview

Although it does not meet the criteria of this review, Schwaighofer's (1964) early work deserves mention as the first to use a slot to measure residual stress. He machined two slots in a part containing residual stress and used a strain gauge centered between the slots to determine the surface residual stress. He recognized that the subsurface stress variation would affect the measurement but did not postulate the possibility of using successive slot extensions to measure the variation. The measurement of strain at incremental depths to measure residual stress variation with depth was introduced for use with hole drilling by Soete and VanCrombrugge (1954) and Kelsey (1956).

What is here termed the compliance method was originally introduced by Vaidyanathan and Finnie (1971). They measured residual stress in a butt-welded plate by introducing a hole in the plate and then extending a slot from the hole using a jeweler's saw. They measured the stress intensity factor, K_I , at each increment of slot length using a cumbersome photoelastic technique. They inverted a solution for K_I in a plate containing a crack to get a closed form solution for residual stress from the variation of K_I with slot length. The method appeared to successfully measure the residual stress, although there was little with which to compare the results.

This method saw minimal use in the following years. The experimental difficulty in performing the photoelasticity measurements likely discouraged others from applying Vaidyanathan and Finnie's idea. The original work also relied on a closed form solution for K_I for a crack subjected to arbitrary loading on the crack faces. Such solutions were not available for many practical configurations.

By the mid 1980s, technological advances stimulated new research using the compliance method. This new research is evidenced by publications from researchers in many countries: Cheng and Finnie (1985) from the United States, Ritchie and Leggatt (1987) from the Netherlands and the United Kingdom, Fett (1987) from Germany, Reid (1988a) from the United Kingdom, and Kang, Song, and Earmme (1989) from South Korea. Computational advances had made it both possible and relatively simple to solve the solid mechanics problems necessary for application of the compliance method for arbitrary

geometries. These computational techniques also allowed the residual stresses to be calculated from measured strains or displacements, instead of from K_I . This approach allowed the much more convenient and universally available strain gauges to replace photoelastic measurements. Since these early publications, the technique has seen many advances and new applications.

II.B. Components of Method

The compliance method can be broken down into several distinct components, common to all applications of the method. They include analytical, experimental, and application differences and are discussed below. Each choice will have an effect on the accuracy, precision, and ease of implementation of the compliance method. There are often tradeoffs between the advantages of the various possibilities. These choices and tradeoffs will be the focus of this literature review.

There are generally two components to the analytical portion of the compliance method: the forward and inverse solutions. The forward solution is the answer to the question "What are the strains (or displacements, or K_I s) that would be measured if one incrementally introduced a slot into a part with an arbitrary, known residual stress distribution?". This question can generally be answered using fracture mechanics solutions, the finite element method, or other numerical techniques. Often, several of these methods can give equally accurate solutions and the choice is made based on the effort involved. The inverse solution is then the answer to the question "What original residual stress distribution best matches the strains that were actually measured?". This question can be answered by using the forward solution and then solving for the average residual stress at each increment of depth sequentially, or by using more sophisticated inverse solutions such as the least squares series expansions. This choice will materially affect both the accuracy and the depth resolution of the measurement results and is an important decision.

There are also several experimental choices to be made when applying the compliance method. The first choice is how to introduce the crack or slot. Possibilities include saws, milling cutters, and electric discharge machining. The choice may depend on the particular application. For example, cutters tend to break when cutting into a compressive stress field. Then, one must decide how to measure the resulting deformations. Strain gauges may be used, and a decision must be made on where to place them. Other possible deformation measures are displacements measured with a clip gauge, moiré interferometry, or a micrometer.

There are many distinctions among the applications of the compliance method found in the literature. Materials tested include metals, polymers, and composites. Geometrical configurations for which solutions exist include surface and through thickness measurements, axisymmetric stresses in cylindrical bodies, compact tension specimens, ballised holes, and more.

II.C. Terminology and Definitions

Figure 1 defines a coordinate system for rectangular, or Cartesian, coordinates and some common terms. In the compliance method, a *slot* or *cut* is introduced into the part. A *crack* would be a *slot* of zero width. The slot starts from the *top face* or *surface* of the part and is extended in the *x*-direction towards the *back face*. The other two surfaces normal to the plane of the slot are the *edges*. The *y*-direction is normal to the slot. The normal stress component measured by such a slot is σ_y . Stress variation with *depth* means variation in the *x*-direction, and the *crack* or *slot depth* is called *a*. The *thickness* of the part is its dimension in the *x*-direction at the plane of the crack, *t*. The *remaining* or *uncracked ligament* is the intact portion of the part in the crack plane, given by $a < x < t$. The opening of the crack or slot at the *surface* is called the *crack opening displacement* or *COD*. For a slot of finite width, the *COD* is generally the total opening minus the undeformed slot width.

Figure 1. Coordinate system and terminology.

Figure 2 shows some additional definitions and terms for geometries involving cylindrical symmetry. The figure shows the two types of cuts made to measure residual stresses, both shown starting from the outer surface for convenience although they may start from the inner surface. Both cuts are extended incrementally in the radial direction, so they are identified by the other coordinate in the cut plane. A *circumferential cut* wraps around the entire part circumference. It releases *axial* stress, which is normal to the cut plane. An *axial cut* extends along the axial length of the part and releases the *hoop* or θ stress. Some researchers refer to this cut as a *radial cut* instead of axial.

Axial Cut

Circumferential Cut

Figure 2. Slots in cylindrical geometries.

III. REVIEW — THEORY

This review is divided into two portions: theory and applications. The theory section reviews papers that introduce or develop theoretical considerations for either the forward or inverse problems (see Section II.B). Since the forward and inverse components are quite distinct, the theory section is subdivided into these two categories. The application section examines experimental applications of the method. Considerations there include the material tested, the configuration tested, the details of cutting the slot, and the deformation measurement. A paper will be reviewed for each section relevant for that paper, which may be all three sections or only one. Within logical groupings, the literature is reviewed chronologically.

As discussed in Section II.B, calculation of residual stresses from measured deformations after introduction of a slot can generally be considered in two separate phases. The forward problem calculates what stresses would be measured for a given stress distribution. The inverse problem uses results from the forward solution to determine a stress distribution that “best” matches the experimentally measured deformations. This type of approach is generally necessary because a direct calculation of residual stresses from measured deformation is not possible. This review covers a few exceptions, which generally involve substantial approximations.

III.A. Forward Solutions

This section reviews forward solutions. Various researchers use fracture mechanics solutions, finite element methods, or other numerical methods to arrive at these solutions. A wide variety of geometries have been considered, leading to a convenient grouping distinguishing between solutions in Cartesian and cylindrical coordinates. The solutions are generally for linearly elastic, isotropic materials. Many solutions treat the slot as a mathematical crack. Others include the finite width of the machined slot.

Almost all of the work reviewed here makes use of a simplifying superposition principle to solve the forward problem. At first, the problem sounds daunting. When a slot or crack is introduced into a part containing residual stresses, some stresses are released and the general stress distribution is rearranged. At each increment of depth, the rearrangement is superimposed on the results from the previous step. How does one tractably calculate the deformations? A superposition principle (Bueckner 1958), originally developed for fracture mechanics, is employed. This principle, illustrated in Figure 3, says that the deformations can be calculated by considering the cracked body and loading the crack faces with the residual stresses that originally existed on this plane in the *uncracked* body. It is possible to make a forward solution without using this superposition principle. For example, Perl and Aroné (1994) use thermal loads to simulate residual stresses in a finite element model and then remove material to calculate deformations.

Figure 3. Superposition principle used to calculate deformations from releasing residual stresses.

III.A.1. Cartesian Coordinates

This section presents solutions for deformations due to stresses in Cartesian (x,y,z) coordinates. For near-surface stresses in polar coordinates, for example σ_θ near the surface of a cylinder, the stresses can be treated as Cartesian if the region examined is small compared to the radius of curvature. Figure 4 shows the various geometries referred to in this section. Solutions for surface stresses consider a crack or slot in the free surface of a semi-infinite body. This configuration is also often referred to as a single edge-notched

strip. Solutions for through-thickness geometries include the effect of the back face. Solutions are also reviewed for a crack in the interior of an infinite plate and for a crack starting from an interior hole.

Figure 4. Geometries for Cartesian coordinate forward solutions.

In the first appearance of the compliance method, Vaidyanathan and Finnie (1971) used a K_I solution for a crack in the interior of a plate. It was a weight function solution, which allows K_I to be calculated in integral form for arbitrary loading on the crack faces. A weight function K_I solution takes the form

$$K_I(a) = \int_0^a \sigma(y)h(y,a)dy,$$

where h is the weight function. Because they were measuring K_I directly, using photoelasticity, the solution calculated only K_I because of the release of residual stresses, rather than strains or displacements.

Fett (1987) calculated the crack opening displacement for a single edge-notched strip. He used an existing weight function K_I solution. A double integral equation related the stresses to displacements along the edge of the crack. Such equations generally require numerical solution. Kang et al. (1989) also calculated compliances for a single edge-notched strip using a different weight function solution for the same geometry. To get surface displacements, they used Castigliano's theorem and considered a virtual force at the location of strain measurement. They calculated strain energy due to the residual stress loading and the virtual force. The resulting double integral equation was differentiated with

respect to the virtual force to get displacements. It was differentiated a second time with respect to the location of the virtual force to get strain.

Cheng and Finnie (1988) claimed that previous weight function solutions are in error for the limiting a/t values less than 0.05 or near 1. They presented a solution, constructed from other solutions accurate for limited ranges of a/t , that is claimed to be accurate for all values of a/t . Cheng and Finnie (1990a) repeated this solution but instead for K_{II} , which could be used for measuring shear stresses using compliance. Then Cheng et al. (1992a) and Cheng and Finnie (1992c) applied both of these solutions to calculate displacements and strains.

Ritchie and Leggatt (1987) used the finite element method (FEM) to calculate compliances for a slot sawed through the thickness of a strip. The geometry modeled was a 2-D slot including the actual slot width and considering all deformations to be plane stress. Each increment of slot depth, from near surface to almost through the thickness, required a separate finite element calculation. They calculated strain on the top and bottom surfaces of the strip, as well as on the edge. Beghini and Bertini (1990) performed a similar calculation, also using 2-D plane stress finite elements. They calculated strains at various locations on the edge (see Figure 1) rather than on the top or bottom faces. Finnie et al. (1996) used FEM to calculate compliances for the case when the part width (in the z -direction) varied.

Reid (1988a) calculated a closed form approximation for the strains because of extending the notch in a compact tension specimen towards the back face. Although he derived the inverse solution directly (see Section III.B.3), it can be shown to be equivalent to the forward solution described here. He made use of a simple beam bending approximation, Figure 5. The uncracked ligament was conceptually separated from the cracked portion of the compact tension specimen. The stress distribution considered was used to calculate an equivalent force and moment to be applied to the beam. Simple beam theory was then used to calculate back face strains. No estimate was given for the error caused by this approximation.

Figure 5. Reid (1988a) approximation for compact tension specimen.

Cheng et al. (1991a) presented a solution for measuring near-surface residual stresses where the strains are measured very near the cut. They considered both normal stress loading, σ_y , and out-of-plane shear stress, τ_{yz} , using K_I and K_{III} solutions, respectively. They calculated displacements using Castigliano's theorem and then differentiated to get strain. The solution was claimed to be valid to a final depth of $a/t < 0.05$. They also estimated that the crack length in the out-of-plane direction, z , needed to only measure 8 times the final depth of cut for their 2-D solution to give accurate results for a 3-D experiment.

Cheng and Finnie (1993a) presented the first non-FEM solution for the compliances because of a slot of finite (non-zero) width. They considered a rectangular slot in a semi-infinite plane under loading due to both normal, σ_y , and in plane shear, τ_{xy} , loading from residual stresses. The calculations were performed using the body force method (Nisitani 1978), which makes use of the point force solution for an uncracked body. These point forces are applied along the prospective slot boundary, and their magnitudes are adjusted numerically to satisfy the appropriate boundary conditions. The calculations indicated that, for slots with a depth of less than 5 times the slot width, approximating the slot as a crack will result in significant errors. Cheng et al. (1994a) gave a simple correction for a slot with a semi-circular bottom, such as that produced using wire electric discharge machining.

Lai et al. (1993) investigated measuring residual stresses near a hole in a plate. They considered the introduction of cracks symmetrically on opposite sides of the hole and calculated the resulting displacements. They used a fracture mechanics approach and the weight function for a crack emanating from a center hole.

Cheng & Finnie (1993b) presented a solution for measuring through thickness residual stresses near an attachment to a plate (see Figure 4). They gave a bracket welded to a nuclear reactor pressure vessel as an example of this configuration. The attachment could

be of arbitrary geometry and have elastic constants different from the plate. They combined a fracture mechanics solution for an edge-cracked strip with finite element calculations for the effect of the bracket. Strain at the back face was the quantity calculated. They performed calculations on trial configurations and found that the errors due to ignoring the presence of the attachment could exceed 10%.

Prime and Finnie (1996) presented a solution for compliances for a finite width slot in layered material. They considered a surface layer on a semi-infinite substrate with different elastic constants. The slot could penetrate into the substrate. They used the body force method, basically the same approach as Cheng and Finnie (1993a). They stated that the presence of the substrate significantly affected the compliances for a slot penetrating halfway through the layer where the elastic moduli differ by 50 percent or more.

III.A.2. Cylindrical Coordinates

This section examines forward solutions in cylindrical coordinates. It should be noted that, for measuring near-surface stresses in a cylindrical geometry, it is possible to use a surface stress solution in rectangular coordinates if the region considered is small compared to the radius of curvature. A word of caution is also in order. Some of the solutions presented are only accurate away from the immediate region of the cut. So they may not be applicable for data from a strain gauge near the slot.

Forward solutions are not given for radial residual stress, because there is no reasonable crack geometry to release these stresses. See Section III.B.3 for discussion of a method to calculate radial stresses from hoop stress measurements.

Cheng and Finnie (1985) calculated compliances in a thin-walled cylinder for the case of axisymmetric axial residual stresses. They considered a circumferential crack starting at the cylinder's inner surface. The calculated deformation was hoop strain on the outer surface. The authors used a K_I solution for arbitrary loading on the crack faces. They also mentioned the possibility of using strains measured on the inner surface for a crack starting from the outer surface. Cheng and Finnie (1987) revisited this solution and gave compliances in tabular form.

Cheng and Finnie (1986) calculated compliances in a thin-walled cylinder for the case of axisymmetric hoop residual stresses. They considered an axial crack, starting at the outer surface. The calculated deformation was hoop strain on the outer surface. They used a K_I solution for arbitrary loading on the crack faces. Cheng and Finnie (1990b) added corrections to the previous solution for the case of thick-walled cylinders. Kang and Seol (1996) also calculated compliances for a thick-walled cylinder. They used a weight function solution and calculated hoop strains on the outer surface.

Perl and Aroné (1994) calculated compliances for a thick-walled cylinder with a particular residual stress distribution. They considered an autofrettaged cylinder, one where sufficient internal pressure has been applied to cause yielding and subsequent residual stresses. The through-thickness distributions of both hoop and radial residual stresses are given in closed form if one uses the Von Mises yield criterion and assumes perfect plasticity. They

calculated compliances for an array of axial cuts extending from the cylinder inner surface using finite elements with the residual stresses simulated by a thermal load. The calculated deformation is hoop strain on the inner surface between adjacent cuts.

Schindler (1995) calculated compliances for a solid disk with an axial crack. He used a weight function solution and gave procedures to ensure accuracy for near-surface or very deep cracks, where weight function solutions may be in error. The solution gave surface hoop strains. Fett and Thun (1996b) also calculated compliances for a solid disk with an axial crack using a weight function. They gave more details of the solution than Schindler and also included a solution for a central, internal crack. Their solutions were formulated to give crack opening displacement.

III.B. Inverse Solutions

This section reviews the methodology used by various researchers to invert from measured deformations as a function of slot depth to the originally existing residual stresses. Most of these techniques make use of one of the forward solutions described in the previous section. The forward solution gives the deformations that would be measured for a given stress distribution. The inverse solution then gives a stress distribution that in some way results in the "best" correlation with the actual measurements.

III.B.1. Incremental Stress

Many compliance method investigators, beginning with Ritchie and Leggatt (1987), calculate residual stresses in a step-by-step manner. They calculate an equivalent stress for each increment of slot depth, based on the strain reading in that increment and the stresses from previous increments. This is the oldest and still most common method for calculating a stress profile, often used with layer removal methods and hole drilling. We will call this the *incremental stress* method.

In its simplest form, this technique suffers from many drawbacks. Compliance method researchers employ various techniques to counter these potential errors. To facilitate this review, a description of this technique in its simplest form is given. Then the major drawbacks are described. Following that, the variations on the incremental stress used by compliance researchers are described.

Incremental Stress - Basic Implementation

At each of the m increments of slot depth, the resulting deformation at some location is measured. Here strain is considered, although the measurement may be of displacement:

$$\epsilon_1, \epsilon_2, \dots, \epsilon_m .$$

Then the average stress in each increment is calculated:

$$\sigma_1, \sigma_2, \dots, \sigma_m .$$

One must realize that the strain measured at each increment of cut is a function not only of the stress in that increment but also of all previous increments:

$$\varepsilon_1 = f(\sigma_1), \varepsilon_2 = f(\sigma_1, \sigma_2), \dots, \varepsilon_m = f(\sigma_1, \sigma_2, \dots, \sigma_m).$$

The stress in the first increment is calculated by considering the forward solution for a uniform stress of 1 in the first increment. Using linear superposition, the actual magnitude of stress in the first increment is given:

$$\sigma_1 = \frac{\varepsilon_1}{\varepsilon(\sigma_1 = 1)}. \quad (1)$$

For subsequent increments, additional calculations are necessary. The strain after the second cut depends on the stresses in both of the first two increments. So the portion of strain due to the stress in the second increment only must be calculated by subtracting off the strain due to the first increment stress,

$$\varepsilon'_2(\sigma_2) = \varepsilon_2(\sigma_1, \sigma_2) - \varepsilon''_2(\sigma_1), \quad (2)$$

where ε_2 is the actual measured strain after the second cut, and ε''_2 is the strain that would be measured after the second cut due to the stress in the first increment only. Then the stress in the second increment can be calculated similarly, using the forward solution for a uniform stress of 1 in the second increment only:

$$\sigma_2 = \frac{\varepsilon'_2}{\varepsilon(\sigma_2 = 1)}. \quad (3)$$

This procedure is repeated for subsequent increments. The stress calculated in each increment will be a function of the calculated stress in all previous increments. Using this procedure, the calculated stress distribution **exactly** reproduces the measured strains.

As presented here in its simplest form, this inversion technique suffers from three main potential drawbacks:

1. Error Accumulation/Propagation

Because the stress calculated in each increment depends on the stresses calculated in each previous increment, errors will accumulate. The error in the stress measured in the first increment will add approximately linearly to that measured in the second. The stress in the third increment will then contain compound effects of the errors in the first two intervals, and so on. Unfortunately, the first increment is generally the most prone to errors because it has the lowest strain reading.

2. Measurement Error Intolerance

Because the number of known strains and unknown stresses are equal, the calculated stress distribution will exactly match the measurements. Because the

experimental measurements virtually always contain errors, this is generally not a desirable feature as it ensures errors in the stress distribution.

3. Error - Resolution Tradeoff

The two error types mentioned above can be reduced by taking larger increments of cut depth. However, this results in decreased spatial resolution of the stress distribution. So one must sacrifice accuracy for resolution or vice versa.

Incremental Stress Applied to Compliance

Ritchie and Leggatt (1987) combined the incremental stress approach with a least squares fit to minimize errors. Strains were measured at k distinct locations. A least squares fit was performed to give the m σ_i that best reproduced the $k \times m$ measured strains. Note that it would be possible to perform the least squares fit at each increment sequentially and use the results of previous increments in subsequent calculations. However, the authors combined all m increments into a single fit in order to minimize error propagation. Note also that to use this least squares approach, multiple deformation measurements at each increment of slot depth are necessary.

Kang et al. (1989) supplemented the incremental stress method with data smoothing. They fitted second order polynomials to successive sets of 7 data points using least squares. They also used stress increments of three times the cutting increment, $3\Delta a$, in reducing the data for their preferred compromise between resolution and accuracy. This also provided redundant data and allowed a least squares fit. Kang and Seol (1996) averaged the readings of two strain gauges on opposite sides of the crack and then used the 7-point polynomial smoothing technique.

Beghini and Bertini (1990) employed an approach that made use of one more unknown than the number of depth increments, a least squares fit, and additional constraints. They considered a residual stress distribution that varies linearly within each of the m increments. Considering continuity, this was characterized by a single stress value at $m + 1$ nodes. They also used readings from strain gauges at multiple locations. A least squares fit was then used to find the $m + 1$ nodal stresses that best reproduce the measurements. Because the authors measured residual stress through the thickness of a cross section, they added the constraints that the stress distribution satisfy force and moment equilibrium. Note that this approach, which gives a non-uniform stress in each increment, shares some characteristics with the series expansion approaches described below.

III.B.2. Series Expansion

The second common approach is to solve for the stress variation expressed as a series expansion. This approach was pioneered for residual stress measurement method by Schajer (1981) for hole drilling and Popelar et al. (1982) for a sectioning method. It was first applied to the compliance method by Cheng and Finnie (1985) and Fett (1987). As in the previous section, a general description of the approach will be followed by descriptions of its application to compliance problems.

Series Expansion - Basic Implementation

It is first assumed that the unknown stress variation as a function of depth can be expressed as a series expansion,

$$\sigma_y(x) = \sum_{i=1}^n A_i P_i(x) = [P]\{A\}, \quad (4)$$

where the P_i are some functional series, such as polynomials, x is generally normalized by the final cut depth, and the A_i represent unknown coefficients to be solved for. Using a forward solution, the strains that *would* be measured for each term in the series are calculated. Here, this is called the compliance function C . Using superposition, the strains given by the series expansion can now be written as

$$\varepsilon_y(x) = \sum_{i=1}^n A_i C_i(x) = [C]\{A\}. \quad (5)$$

A least squares fit is now performed to minimize the error between the strains given by Eq. 5 and the m strain measurements. This gives the A_i and can be written in matrix form as

$$\{A\} = ([C]^T [C])^{-1} [C]^T \{\varepsilon_{measured}\}. \quad (6)$$

Now the stress distribution is given by Eq. 4.

As with the incremental stress method, there are several potential drawbacks to this method.

1. Convergence

It is generally hoped that as the order of the expansion, n , increases, the result will converge to a solution. But after some point, the fit will diverge. This necessitates some method for selecting an appropriate fit order.

2. Ability of Expansion to Fit Actual Distribution

The accuracy of the solution is dependent on the ability of the chosen series expansion to fit the actual distribution within a convergent number of terms in the series. If the actual stresses vary too rapidly over the fit range or in some other way are not expressible in terms of the chosen expansion, errors will result.

3. Endpoint Stability

Series expansions of this type using polynomials often exhibit instability near the endpoints of the fitted range. This can often be observed by comparing results for successive even and odd expansions, n and $n+1$ in Eq. 4. So the stresses given by this technique may be less accurate at the surface and the final cut depth.

Series Expansion Applied to Compliance

Cheng and Finnie (1985) used LeGendre polynomials to express axisymmetric axial residual stresses through the thickness of a cylinder. By selecting LeGendre polynomials and excluding the 0^{th} order (uniform stress) term, the resulting stresses are guaranteed to satisfy axial equilibrium. They used three terms in the series to fit strain data from eight cut depths and several strain gauges all at once. They noted that calculations using a power series instead of LeGendre polynomials resulted in a less convergent series fit. Cheng and Finnie (1990b) noted that, for measuring through-thickness longitudinal stresses in a plate, excluding the 0^{th} and 1^{st} order terms guarantees the satisfaction of stress and moment equilibrium. Cheng et al. (1992d, 1993b) successfully used such a series with $n = 9$. Cheng and Finnie (1993b) averaged between successive orders of the series expansion (i.e., 8th and 9th), presumably to minimize endpoint instability.

Fett (1987) used a power series expansion of 5th degree to express stresses partially through a bar. Cheng et al. (1991a) also used a power series expansion for measuring stresses near the surface. Fett and Thun (1996b) used the symmetry when measuring axisymmetric hoop stresses through the thickness of a solid disk and used only even terms in a power series expansion.

Fett (1987) suggested the use of a Fourier expansion instead of polynomials if the stress distribution is expected to have a discontinuity. Gremaud et al. (1992) used two separate polynomial expansions to get stresses in a clad layer and the underlying substrate, there being an allowable discontinuity across the interface.

For a stress field that cannot be accurately fit with continuous polynomials, Gremaud et al. (1994) proposed a spline-based alternative. They used a series of "overlapping piecewise functions." They divided the region of stress variation into a set of overlapping intervals. Then the stress in each interval was expressed as a linearly or quadratically varying series expansion using a least squares fit. The intervals were fit sequentially, with the effect from each previous interval considered, as with the incremental stress method. An averaging procedure gave the stresses at the region of overlap between successive intervals. This technique is more computationally intensive than a continuous series expansion but promises to combine the best features of the series expansion and incremental stress methods.

III.B.3. Miscellaneous Inverse Methods

Vaidyanathan and Finnie (1971) were able to use a closed form inverse because of their unique experimental methods and choice of specimen configuration. First, they used a photoelastic coating to directly measure K_I , rather than strain or displacement, as a saw cut was extended. Second, they considered a slot interior to a plate. For a crack of length $2a$ cut into a residual stress field symmetric about the center of the crack, $x = 0$, an integral equation for the stress intensity factor is available:

$$K_I = 2\sqrt{\frac{a}{\pi}} \int_0^a \frac{\sigma_y(x)}{\sqrt{a^2 - x^2}} dx, \quad (7)$$

which can be solved for the stresses

$$\sigma_y(a) = \frac{1}{\sqrt{\pi}} \frac{d}{da} \int_0^a \frac{K_I(x)\sqrt{x}}{\sqrt{a^2 - x^2}} dx. \quad (8)$$

Similar equations were given for shear stress in terms of K_{II} and for an asymmetrically loaded crack. Presumably Eq. 8 was solved numerically from measurements of K_I at the discrete cut lengths. This closed form inverse required direct measurement of K_I , here using a cumbersome photoelastic technique, and was limited to a crack internal to a plate.

Joerms (1987) used an approximate finite element method to solve for the stresses from displacement measurements after introducing a radial sawcut into a railroad wheel. In this case, the stresses varied in the out-of-plane direction, z , as well as in the depth direction, which required a different solution strategy. He interpolated measured displacements to get displacement values throughout the surface of the cut. Then he modeled the final state of the wheel after the sawcut and forced the displacements back to the uncut state. He claimed that the resulting stress distribution may not be the actual one, but it could exist and satisfies the boundary conditions.

Reid (1988a) used a beam bending approximation to calculate residual stresses in a compact tension specimen from strains measured during extension of the notch. As discussed in Section III.A.1 and illustrated in Figure 5, he calculated the equivalent force and moment due to releasing residual stresses and applied them to the uncracked ligament. This calculation ignores any effects due to the notch presence. The result was a closed form integral equation for residual stresses in terms of the measured strains,

$$\sigma_y(x) = E \left[\frac{x}{2} \frac{d\varepsilon}{dx} - 3x \int_t^T \frac{1}{t} \frac{d\varepsilon}{dt} dt - \varepsilon(t) \right], \quad (9)$$

where, unlike the convention used in Figure 5, x is measured from the back face, T is the location of the notch tip before the cut is introduced, and t is a dummy variable for x . To implement this solution, the measured strains were fitted with splined polynomials. Then these functions for strains were used in Eq. 9.

Read (1989) measured the J -integral due to residual stresses for a semi-elliptical surface crack of successive depths. Strain gauges were placed along a contour around the crack. By approximating some terms in the contour integral and then numerically integrating, J was given as a function of depth. He estimated the uncertainty to be about $\pm 20\%$.

Cheng et al. (1992b) presented a solution for predicting axisymmetric axial and radial stresses from measuring hoop stress, in both plane stress and plane strain, using compliance. Residual hoop stresses were measured in a long thick-walled cylinder and a

thin ring cut from the cylinder. Using equilibrium and elasticity relations, the radial and axial stress distributions were derived from these measurements.

Perl and Aroné (1994) inverted from measured strains to the autofrettage level in a cylinder. Their forward solution, Section III.A.2, indicated that the strains measured when the cut was from 7.5% to 15% through the thickness of the cylinder wall were especially indicative of autofrettage level. A simple formula was given to calculate autofrettage level from these strains. It was then possible to get a through-thickness distribution of residual hoop and radial stresses using the predicted distributions given by autofrettage calculations. The calculations assumed perfectly plastic materials and Von Mises yielding.

Orkisz and Skrzat (1996) outlined a technique to reconstruct residual stresses from measurements taken during successive extension of a radial slot in a railroad wheel. For such a part, both geometry and residual stresses vary in the out-of-plane, z , direction as well as the depth direction, x , increasing complexity significantly. They proposed to use strain measurements, crack opening displacements, and Moiré interferometry. The inverse solution for residual stress was to be accomplished using constrained optimization methods. The method seeks a solution that does the following:

- (1) Satisfies equilibrium and boundary conditions.
- (2) Satisfies inequality constraints that the resulting measurement predictions (strain, displacement, etc.) are within some error bounds of the actual measurements .
- (3) Minimizes some functional. They propose a functional including "smoothness" as calculated by integrating the local curvature throughout the volume. A second component of the functional is a measure of the closeness of the predictions to measurements.

This is applied first to a solution for the plastic zone, the region in the wheel where irreversible strains occurred. From the plastic zone solution, a solution in the surrounding elastic zone is similarly obtained. They outlined the measurement requirements to reconstruct different components of residual stress in different regions.

IV. REVIEW — APPLICATIONS

This section reviews all known experimental applications of the compliance method. "Computational" or simulated experiments are excluded.

The configuration of the application drives many of the experimental choices, such as type of cutting, type and location of deformation measurement, and forward and inverse solutions used. For this reason, applications in Cartesian and cylindrical coordinates are considered separately in this review. An effort is made to quantify relevant experimental details, such as the increment of cutting depth. Sometimes the values reported here are given explicitly in the paper; other times they are estimated from graphs and figures.

The vast majority of the applications are to metals. Exceptions are Fett's (1987, 1996a, 1996b) applications to PMMA (Plexiglas) and PVC (polyvinyl chloride) and Hermann's (1995) application to metal matrix composites.

Table 1 chronologically lists all of the experimental applications of the compliance method reviewed here. For each application, the table gives the theoretical approach used, the details of making the slot, the deformation measurement and calculated stress component, and the material and geometry tested. A key to the abbreviations is given after the table.

Table 1. Experimental Applications of the Compliance Method*

Authors	Solution		Slotting $a = \text{depth}, t = \text{thickness (mm)}$				Component		Specimen	
	frwd	inv	tool	a_t	da	a/t	meas.	calc	material	configuration
Vaidyanathan & Finnie (71)	K_I	CF	saw	50	?	NA	K_I	σ_y	Al	butt-welded plates
Cheng & Finnie (85)	K_I	SE-L	MC	2	0.25	0.6	ϵ_0 -BF	σ_{ax}	steel, 304SS	circ.-welded thin cylinder
Cheng & Finnie (86)	K_I	SE-L	MC	42	1.6	1	ϵ_0 -S	σ_0	Al, 7050	quenched thin cylinder
Cheng & Finnie (87)	K_I	SE-L	MC	23	0.8-1.6	0.8	ϵ_0 -BF	σ_{ax}	steel, 304SS	butt-welded thin cylinders
Fett (87)	K_I -wf	SE-P	saw	33	0.6-3.0	0.65	COD-E	σ_y	PMMA	single-edge notched strip
Joerms (87)	NA	FEM	saw	?	?	0.5	COD-S,E	σ_0	steel	railroad wheel
Ritchie & Leggatt (87)	FEM	IS-Q	MC	25	1.66	1	ϵ_y -S,E,BF	σ_y	steel	4-point bent beam
" " specimen 2	"	"	"	"	"	"	ϵ_{ax} -E,BF	σ_{ax}	steel	section from welded cylinders
Reid (88)	NA	CF	WEDM	28	0.25-0.5	1	ϵ_y -BF	σ_y	steel	compact tension specimen
Kang et al. (89)	K_I -wf	IS	saw	80	2	0.6	COD, ϵ_y -S	σ_y	steel	butt-welded plates
Read (89)	NA	J	MC	14	1	0.28	ϵ_y -S,BF	J	steel	welded plate
Beghini & Bertini (90)	FEM	IS-Q	saw	75	1	1	ϵ_y -E	σ_y	steel	welded plates
Cheng & Finnie (91b)	K_I	SE-L	?	25	?	1	ϵ_0 -S	σ_0	4335V steel	water-quenched thick cylinder
Cheng et al. (92d)	K_I	SE-L	WEDM	19	?	1	ϵ_y -BF	σ_y	steel, 304SS	4-point bent beam
Gremaud (92), Cheng (94b)	BF	SE-?	WEDM	1.0	0.025	0.05	ϵ_y -S	σ_y	stellite, steel	laser clad layer
Lai et al. (93,95)	K_I -wf	?	saw	14	NA	2	COD-E	σ_y	steel	ballised hole in plate
Cheng et al. (94a)	BF	SE-P	WEDM	0.8	0.025-0.05	0.05	ϵ_y -S	σ_y	steel, 304SS	4-point bent beam
Cheng et al. (94b)	BF	SE-S	WEDM	0.65	0.013	0.02	ϵ_y -S	σ_y	Ti-6Al-4V	shot peened
Hermann (94)	NA	CF	WEDM	12	?	1	ϵ_y -BF	σ_y	7017T651 Al	compact tension
Schindler et al. (94,95)	K_I	SE,IS	WEDM	140	1.3-15	1	ϵ_0 -BF	σ_0	tool steel	quenched solid cylinder
Perl & Aroné (94)	FEM	CF	saw	39.3	1.7-5.6	0.7	ϵ_0 -S,BF	σ_0 +	steel	autofrettaged thick cylinder
Hermann (95)	NA	CF	WEDM	12	?	1	ϵ_y -BF	σ_y	Al-Li + SiC	compact tension MMC
Fett (96b)	K_I -wf	SE-P	saw	90	5	0.9	COD-E	σ_0	PVC	solid disks
Kang & Seol (96)	K_I -wf	IS	saw	18.6	0.8	1	ϵ_0 -S	σ_0	steel	water-quenched thick ring
Finnie et al. (96)	FEM	SE-L	WEDM	52	1.0	1	ϵ_y -S,BF	σ_y	stellite, steel	laser-clad layer
Galatolo & Laciotti (97)	?	?	MC	70	?	?	ϵ	σ_y	AL, 2219	arc-welded plates

*See Key on next page

Table 1. - Key

All Categories		Cutting Tool	
NA	not applicable	saw	hand or machine straight edge saw
?	applicable but unspecified	MC	milling cutter using circular saw blade
		WEDM	wire electric discharge machine
Forward Solution		Measured Deformation Component	
K_I	mode I stress intensity factor solution	$\epsilon_y, \epsilon_{ax}, \epsilon_\theta$	normal, axial, hoop strain
K_I -wf	using Buckner's weight function	COD	crack opening displacement
FEM	finite element method		
BF	body force method	-S	top surface (see Figure 1)
		-BF	back face "
		-E	edge "
Inverse Solution		Calculated Stress Component	
CF	closed form	$\sigma_y, \sigma_{ax}, \sigma_\theta$	normal, axial, hoop stress
SE	series expansion	J	J-integral
	-L LeGendre polynomials		
	-P power series		
	-S splined (piecewise)		
IS	Incremental Stress		
	-Q + least squares fit		
J	J-Integral		
		Material/Specimen	
		PMMA	Plexiglas
		PVC	polyvinyl chloride
		MMC	metal matrix composite

IV.A. Cartesian Coordinates

This section reviews applications where residual stresses are measured in Cartesian coordinates, also known as rectangular or (x,y,z) coordinates. It is further divided into measurements of through-thickness or near-surface stresses.

IV.A.1. Through-Thickness Stresses

This section reviews compliance method applications for measurement of residual stresses through the complete, or a substantial portion, of the part thickness. This approach is distinguished from near-surface stress measurements in that the effect of the free surface back face must be considered in the analysis, i.e., the forward solution. Strain measurements are often made on the back face instead of, or in addition to, on the top face near the slot. Since through-thickness residual stresses must satisfy force and moment equilibrium, this can serve as a constraint in the solution process or a check on the validity of results.

Fett (1987, 1996a) measured residual stress through about 65% of the thickness of a PMMA beam. A 50 mm thick beam was heated on one face to 120°C and held at room temperature on the opposite face. After 15 minutes the specimen was water quenched. Two indentations were made on one edge of the beam to allow COD measurements to be made with a microscope to a precision of about $\pm 2 \mu\text{m}$. A hand saw was used to make a cut towards the back face in increments ranging from 0.6 mm to 3 mm. The COD readings were taking 20 to 30 minutes after completing the cut, to allow cooling to equilibrium. A similar test on a stress-free specimen indicated that the sawing induced negligible stresses and would not effect the results. The residual stresses were calculated using a K_I forward

solution and a power series expansion inverse. The results indicated that a 3rd order power series did not adequately describe the stresses or accurately reproduce the measured strains. The 4th and 5th order expansions were similar and accurately reproduced the measured strains, indicating convergence in the expansion.

Ritchie and Leggatt (1987) measured residual stress through the thickness of a cold bent beam. The 25 mm thick beam was made of BS 4360 Grade 50D structural steel. The beams were carefully stress relieved before bending in a four point fixture. A 2.4 mm thick saw was used to cut a slot in 1.66 mm increments through the beam thickness. Multiple gauges measured strains on the top surface, edge, and back face. A FEM solution, including the finite width of the slot, was the forward solution. The inverse solution was incremental stress combined with a least squares fit for the multiple strain readings. The results agreed well with a prediction based on an elastic perfectly plastic material and the measured yield stress. The results from using only back face strain data agreed well with those using all the strain gauges. Tests using the same procedure on a stress-relieved part indicated that errors due to cutting induced stresses would not exceed ± 20 MPa.

Reid (1988a) measured residual stress through the remaining ligament of a steel compact tension specimen. A standard geometry 25 mm thick compact tension specimen of mild steel was compressively preloaded to produce residual stresses. Reid was the first compliance method researcher to use wire electric discharge machining (wire EDM) to machine the slot. A 0.25 mm diameter wire made the cut in 0.25 to 0.5 mm increments from the crack tip to the back face. A gauge on the back face measured the released strains. A closed form inverse was used to solve for the stresses as a function of depth. The solution is approximate, applying the equivalent force and moment caused by residual stress release to the uncracked ligament and using simple beam bending relations. To perform the inverse, the measured strains as a function of slot depth were fit with two 6th order splined polynomials. Reid noted that the calculated stresses satisfy equilibrium in the cross section to within 5%. Reid et al. (1988b) compared the results in this specimen with neutron diffraction results and used the results to predict crack growth rates in a fatigue test.

Kang et al. (1989) measured residual stresses across a butt-welded plate. Two 10 mm thick steel plates were butt-welded together using gas metal arc welding. Each plate was 50 mm long in the direction normal to the weld line. Shaping and grinding removed 2 mm from each face to eliminate the weld bead geometry, leaving a 6 mm thick plate. A crack was introduced from one edge of the plate in 2 mm increments using a 0.6 mm thick hand saw. A second saw with a sharp tip was used before each strain measurement to best approximate a mathematical crack. At 50 mm depth, the slot passed through the weld line and continued an additional 30 mm. Two top surface gauges, located 50 mm on either side of the slot, measured strains. An extensometer also measured COD. The authors used a weight function K_I solution as the forward solution. The inverse solution was incremental stress using 6 mm increments, three times the cutting increment. The strain measurements on either side of the slot were averaged and then a smoothing technique was used before inversion. The results using strains and those using COD agreed fairly well. The results were compared with hole drilling results.

Read (1989) measured the J -integral due to residual stress near a weld in a steel plate. To prepare specimens, a 5 cm thick A-387 Grade 22 steel plate was cut with troughs 5 cm wide, 3.2 cm deep, and 20 cm long. The troughs were filled by welding. Two plates were tested as welded, and two were heat treated first. The slot was cut perpendicular to the weld using a 7.5 cm diameter circular saw. This resulted in a crack with a circular front and with a length that varied as the depth increased. The slot was cut in 1 mm depth increments to a final depth of 14 mm. Readings from multiple strain gauges were used to calculate the J -integral.

Beghini and Bertini (1990) measured residual stress fields in steel plates with various weld geometries. They used ferritic steel plates, 20 mm thick, 500 mm long, and about 72 mm wide. A laser created welds in the long (500 mm) direction. In some specimens the weld was through the 20 mm plate thickness in the center of the 72 mm width. In other specimens, the weld line was made on the centerline of one or both of the 20 mm thick edges and only penetrated part way through the 72 mm plate width. After welding, 4 mm was milled from each face, leaving 12 mm thickness. A saw was used to introduce a slot in 1 mm increments from one edge. After reaching half-width, about 36 mm, cutting was started from the other edge and continued until separation. Five gauges measured strain on the plate face along the length of the cut. An FEM forward solution was used. An enhanced incremental stress inverse was used, giving linear stress variation in each increment and supplemented by a least squares fit and constraints to ensure force and moment equilibrium. The results were used to map the residual stresses in compact tension specimens cut from the welded plates.

Cheng et al. (1992d) measured residual stresses through the thickness of a beam having a known residual stress distribution. The beam was made of stress-relieved 304 stainless steel and had a 19 mm square cross section. The beam was cold bent in a four-point bend fixture. Strains measured on the top and bottom surfaces during bending allowed the computation of the residual stress distribution from stress strain curves measured during the actual bending. A slot was introduced using wire EDM, and strain was measured with a gauge on the back face directly opposite the cut. The authors used a K_I forward solution and a LeGendre series expansion inverse. They found close agreement between expansions with orders 7, 8, and 9. The results agreed very closely with the known distribution, in spite of the low magnitude of the residual stress distribution, which was less than 100 MPa throughout.

Cheng and Finnie (1993b) measured stress through the thickness of a plate at the toe of a welded attachment. A 50.8 mm wide bracket was welded to a 166 mm thick A533-B low carbon steel plate. Wire EDM was used to machine a 0.33 mm wide cut in increments ranging from 4 mm to 10 mm through the 166 mm thickness. Three gauges on the back face measured the released strain. The authors used a K_I forward solution for an edge-notched strip combined with a numerical procedure to include the effect of the attachment. The inverse solution was a LeGendre series expansion with the uniform and linear terms set to zero to ensure force and moment equilibrium. They averaged the 8th and 9th order

expansion results to reduce the endpoint instability that is characteristic of polynomial expansions. Cheng and Finnie (1991b) briefly reported results on a similar specimen.

Hermann (1994) measured residual stress in the uncracked ligament of an aluminum compact tension specimen. Ten mm thick 7017-T651 aluminum compact tension specimens were compressively preloaded to three different load levels. A slot was incrementally extended using wire EDM from the tip of the pre-existing notch to the back face, about 12 mm. A gauge measured the released strain at the back face. The measured strains as a function of depth were fit with a 5th order polynomial and then inverted to residual stresses using the approximate closed form inverse of Reid (1988a). The results clearly demonstrated the effect of the preload magnitude. Hermann (1995) repeated these test on specimens of 8090 Al-Li reinforced with 17 vol% silicon carbide (SiC). The results again compared various preloads and also looked at specimens without the SiC reinforcement.

Finnie et al. (1996) measured residual stress through the thickness of laser-cladded parts. Stellite F was cladded in layers 12 to 20 mm thick to a 40 mm thick substrate of 304 stainless steel. In some tests the substrate was preheated in an attempt to reduce tensile residual stresses. Wire EDM incrementally introduced a slot in 1 mm increments starting from the cladded surface, using a 0.25 mm diameter wire. A gauge on the back face measured strain. In the first test from a specimen prepared without preheating the substrate, a crack spontaneously propagated during machining of the slot. In subsequent tests on the non-preheated specimens, the strain gauge was placed on the cladded surface and the slot started from the other face. The authors used a FEM forward solution to account for the layer and substrate having different thicknesses in the out-of-plane (z) direction. They used a series inverse with LeGendre polynomials and separate series in the layer and substrate. Their results agreed well with a computer simulation.

IV.A.2. Near-Surface Stresses

This section reviews compliance method applications for measurement of stresses near the surface. This means that the slot penetrates a small enough fraction of the full thickness that the part can be considered semi-infinite for analysis purposes. An alternate definition is measuring residual stresses variation within about 1 mm depth of the free surface. For such measurements, strain or displacements must be measured on the free surface and close to the slot. It may be important to make a narrow slot and control its depth precisely. Because of the proximity of the gauge to the machined slot, stresses introduced during cutting can affect the results. Although these analyses use Cartesian coordinate solutions, near-surface stresses in curved parts can often be measured with sufficient accuracy using these solutions if the depth is small compared to the radius of curvature.

Gremaud et al. (1992) measured stresses through the thickness of a laser-cladded layer and into its substrate. A 0.58 mm layer of Stellite 6 was cladded to a carbon steel substrate using a fast axial flow CO₂ laser. Careful mechanical grinding and subsequent sanding prepared the surface for strain gauging and removed about 40 μ m. Gauges measured strain on the top surface, close to the cut. The slot was introduced in 25 μ m increments using wire EDM and a 51 μ m diameter molybdenum wire. The authors used a body force method

forward solution for edge notches of finite width. The inverse solution was a series expansion, apparently using separate series in the layer and substrate. The results are qualitatively compared with x-ray results and agree well except in one regard. The compliance method measured a small region of compressive stress in the substrate steel just below the interface that was not measured by x-rays. Subsequent metallographic analysis revealed a martensitic transformation in this region. Such a phase change is accompanied by a volume expansion that generally produces compressive residual stress. This result gives strong evidence of the compliance method's ability to resolve variation with depth. Cheng et al. (1994b) revisited this test and reduced the same data using a piecewise series expansion to improve the results.

Cheng et al. (1994a) measured near-surface stresses in a cold bent steel beam that had an accurately known residual stress distribution. A beam of 304 stainless steel was bent in a four-point fixture. Strains measured on the top and bottom surfaces during bending allowed the computation of the residual stress distribution from stress strain curves measured during the actual bending. A slot was introduced using wire EDM and a 51 μm diameter molybdenum wire. This slot was extended in 25 μm increments to a depth of about 0.4 mm and then in increments of 50 μm to the final depth of 0.8 mm. One cut was made on the tensile stress side of the beam with the EDM machine in "finishing mode," which involves more gentle, but slower, machining. The stress distribution agreed quite well with the known one and agreed almost exactly when the slight EDM correction was applied. A second cut was made on the compressive stress side with the EDM machine in "roughing mode." The results agreed with the known distribution after the EDM correction. With the roughing mode cut, the correction was fairly substantial, but this was considered to be partially due to the low magnitude of the residual stresses. Cheng et al. (1992d) and Gremaud et al. (1992) reported results from very similar tests on specimens prepared in this same manner.

Cheng et al. (1994b) measured the near-surface residual stress distribution on a shot-peened titanium part. Peened specimens were 43 mm thick and made of Ti-6Al-4V. The slot was made in 13 μm and then 25 μm increments using a 25 μm wire EDM to a final depth of 0.65 mm. A small surface gauge, placed as close as possible to the cut, measured the released strains. Tests on a stress-relieved specimen indicated that the cutting had no effect on the strain measurements. A body force method forward solution that included the finite width of the cut was used. The inverse solution was a piecewise series expansion, with three quadratic functions over the first 14 data points and linear functions over the next 5 intervals. Results were presented for two tests on the same specimen and agreed well. The compressive peening stresses existed in the first 100 μm of depth and appeared to be well resolved by this method. The results compared favorably with x-ray results.

IV.A.3. Interior Stresses

This section reviews the infrequent compliance method applications for measuring residual stresses interior to a plate or other structure. In these cases, the slot is not started from an exterior free surface. Instead, it is usually started by drilling a hole in the interior or by using a pre-existing hole. The forward solution generally considers the free surfaces (other than the hole) to be at infinity and not to affect the stresses relieved by making the slot. For

such a geometry, the deformations must be measured on the edge rather than on a face, by our definitions in Figure 1.

Vaidyanathan and Finnie (1971) measured residual stresses in the interior of a plate made by butt welding two aluminum plates together. The plates, 6.35 mm thick 6061-T6 aluminum, were joined using electron beam welding. Using the coordinates of Figure 4, one plate would be defined by $x > 0$ and the other by $x < 0$, with the weld line running along the y -axis. A 1.6 mm diameter hole was drilled 50 mm in the x -direction from the weld centerline. A slot was then extended from the hole in the x -direction towards the weld using a 0.15 mm thick "jeweler's saw," a hand saw. The slot was extended to and through the weld. K_I was measured at the slot tip at each increment using a photoelastic coating. The authors used a K_I forward solution and a closed form inverse to get $\sigma_y(x)$.

Lai et al. (1993) measured stresses in the vicinity of a ballised hole in a steel plate. In ballising, a ball is forced through a slightly smaller hole, producing compressive residual stress and an improved surface finish. In this work, 10 mm thick plates of medium carbon steel were ballised with a 19.046 mm diameter tungsten carbide ball. Several specimens were prepared with varying amounts of interference between the ball and hole. Radial direction saw cuts were introduced at opposite edges of the hole in 2 mm increments to lengths of 14 mm each. Two points separated by 60 mm on opposite sides of the hole and on a line perpendicular to the slot were used to measure displacements. A traveling microscope measured the displacements to $\pm 0.5 \mu\text{m}$ precision. A weight function K_I forward solution was used. The inverse solution was not specified but appeared to be an incremental stress procedure. The results showed the compressive stresses near the hole changing to balancing tensile stresses farther away. They also tested a plate that was cut in half through the center of the hole after ballising to simulate cracks joining the hole from a free surface in the plate or from another hole. In this case the stresses near the hole were tensile. Oh et al. (1993) discussed these results further and compared them to a theoretical prediction. Lai and Siew (1995) repeated these tests with different specimens in which the hole was finished by ballising, wet blasting, and shot peening. They correlated the residual stress results with fatigue life.

Fett and Thun (1996b) measured hoop stress in a solid PVC cylinder. In some of the tests, the slot was extended from the center of the disk and the COD was measured at the center. Unlike the other applications in this section, the free surface (outer radius) effects were included in the analysis. This work is further discussed in IV.B.2.

Galatolo and Lanciotti (1997) measured residual stress in welded plates. Plasma arc welding was used to create a weld along the centerline of 7 mm thick aluminum 2219-T851 plates. Few of the testing details are given. A central cut was extended progressively so as to simulate the growth of a crack across the weld bead. Measured strains were used to calculate residual stresses as a function of the crack length. The method of solving for stresses is not stated. The tests indicated that the stresses for a crack perpendicular to the weld bead were much larger than for a crack parallel to the weld.

IV.B. Cylindrical Coordinates

This section reviews measurements of hoop, also known as circumferential, and axial residual stresses in cylindrical, or r, θ, z coordinates. No experimental applications of measuring radial stress with the compliance method are available, because of the difficulty of releasing the stress component by a slot. Cheng et al. (1992b) present a method to deduce radial and axial residual stresses from hoop stresses measured in a long cylinder and in a thin ring cut from the cylinder (see Section III.A.2).

IV.B.1. Axial Stress

In this section, compliance method applications for measuring residual axial stress in cylindrical geometries are considered. This involves introducing a circumferential notch in order to release the stresses. The slot may proceed from the outside surface in, or vice versa. Either released hoop or axial deformations may be measured on the cutting surface or the back face. Some investigators measured stresses in a thin ring cut from a long cylinder.

Cheng and Finnie (1985) measured axisymmetric axial residual stresses in a circumferentially welded thin-walled cylinder. The cylinder was 3.3 mm thick, 66 mm in diameter, and made of 304 stainless steel. An electron beam made a single-pass weld around the circumference of the cylinder in the middle of the cylinder's 71 mm length. The heating conditions were such that the weld penetrated to the inner wall. A single cylinder was used to simulate a butt weld between two cylinders while eliminating alignment errors. A circumferential slot was made using a 0.23 mm thick milling cutter. The cut started at the inner surface and progressed in 0.25 mm depth increments toward the outer surface until the cutter broke about 62% of the way through the thickness. Three strips of 10 strain gauges each were mounted on the outer surface of the cylinder, separated by 120° around the circumference. On each strip the gauges measured hoop strain with the first gauge on the weld centerline and each successive gauge 2 mm further away from the weld. For data reduction, the strains from the gauges closest to the weld were not used because these strains were too sensitive to the distance from the weld. For the strains used, the average of the three gauges at the same distance from the weld was used. A K_1 forward solution and a LeGendre series expansion inverse up to order 3 were used to get $\sigma_{ax}(r)$. The use of a single-pass weld allowed a theoretical prediction of the welding residual stresses, which agreed well with the measurements.

Cheng and Finnie (1987) measured axisymmetric axial residual stresses in a multi-pass circumferentially welded cylinder. The specimen tested was two cylinders of 304 stainless steel butt welded together using 22 weld passes. The cylinders were 16.5 mm thick with a mean diameter of 307 mm. At each of four locations around the cylinder circumference, three strain gauges oriented to measure hoop strain were placed 57 mm to 70 mm axially away from the weld centerline. These distances were chosen so as to locate the gauges in the region where the strain measurements were expected to vary minimally with distance from the weld. A circumferential cut was made from the inner wall along the weld centerline using a 0.8 mm thick milling cutter. The cut proceeded in 0.8 mm increments and then 1.6 mm increments toward the outer wall, apparently proceeding through about 80% of the wall thickness. The average strain readings of the three gauges at each location,

fit with a sixth order polynomial, were used in calculating stress. A K_1 forward solution and a series expansion inverse were used to solve for the stresses. The expansion used a fourth order LeGendre series, excluding the 0th order term to ensure force equilibrium. The results obtained at the four circumferential locations agreed fairly well and agreed qualitatively with other available data.

Ritchie and Leggatt (1987) measured residual axial stresses in a section cut from butt-welded cylinders. Two 25.4 mm thick steel cylinders with 761 mm outer diameters were butt-welded together using submerged arc welding. To avoid having to circumferentially slot such a large cylinder, strips including the weld sections were removed for subsequent slotting. Twenty strain gauges were placed on each section before removal to allow calculation of stresses due to removing the section. A slot was introduced in 1.66 mm increments using a 2.4 mm wide milling cutter through the weld. The back face and an edge were outfitted with 5 strain gauges each to measure released strains. A FEM solution including the finite width of the slot was the forward solution. The inverse solution was incremental stress combined with a least squares fit for the multiple strain readings.

IV.B.2. Hoop Stress

Cheng and Finnie (1986) measured axisymmetric residual hoop stresses through the thickness of a quenched thin cylinder. They used two water-quenched 7050 aluminum cylinders 42 mm thick with mean diameters of 378 mm. An axial cut was made using a 0.8 mm wide milling cutter, starting at the outer surface and progressing in 1.6 mm increments completely through the thickness. Gauges measured hoop strains at 15°, 30°, 90°, and 120° circumferentially from the cut on the outer surface. The measured strains were fit with a 6th order polynomial before solving for stresses. A K_1 forward solution and 4th order LeGendre series expansion inverse were used to get $\sigma_\theta(r)$ through the thickness. Results were compared with results from x-rays and hole drilling. Agreement with x-rays was fair and with hole drilling measurement at both surfaces was very good.

Joerms (1987) measured residual hoop stresses in a steel railroad car wheel. The wheel geometry varied in the out-of-plane (z) direction, unlike most of the situations considered in this review. A saw cut was made radially inward from the outer surface of the wheel about half way to the center. COD was measured at the outer surface at each increment. Additionally COD was measured on the edge of the wheel along the full length of the cut. These measurements were used to generate a displacement map on the entire surface of the cut. These displacements were applied to a finite element mesh of the railroad wheel, to model forcing it back to its undeformed geometry. The resulting stress distribution is considered to be a possible, though not necessarily unique, solution for the original residual stresses.

Cheng and Finnie (1991b) measured residual hoop stress through the thickness of a quenched, thick-walled cylinder. A 2.5 cm thick, 8.45 cm outer diameter 4335V steel cylinder was quenched in water from a temperature of 1160 °K. No details of the cutting tool, cut width, or cutting increments were given. A K_1 based forward solution was used. A LeGendre polynomial series inverse was used, with the 0th order term set to zero to ensure equilibrium. A fifth order series was found to sufficiently represent the stress field. The

results were compared with a FEM calculation and x-ray surface measurements, and agreed very well with both.

Perl and Aroné (1994) measured autofrettage levels (see III.A.2) in a thick-walled cylinder. The cylinder was a steel gun barrel, with inner radius 52.5 mm and outer radius 113.0 mm. Seven cuts, equally spaced along the circumference, were extended from the inner surface to the outer surface using a band saw. Gauges measured hoop strain on the inner surface between cuts. The cuts were extended in increments starting at 1.7 mm and increasing to 5 mm. They were able to calculate the autofrettage level from the measured strain. The autofrettage level corresponds to a certain residual stress distribution.

Schindler et al. (1994, 1995) measured residual hoop stress through the thickness of a large solid cylinder. The quenched and tempered tool steel cylinder was 140 mm in diameter, and stresses were measured in a disk 6.4 mm wide that was cut from the center of a long piece. Cuts were made radially inward using wire EDM in increments that started at 1.3 mm and increased to about 15 mm. Strain was measured using a gauge opposite the cut. In Schindler et al. (1994) a K_I solution that interpolated between exact solutions for very shallow and very deep cuts was used as the forward solution. A LeGendre polynomial series expansion inverse was used, with the first two terms set to zero to ensure equilibrium. The solution was found to converge for a 4th order series. The results compared favorably with results from compliance measurements made after drilling a hole in the disk. In Schindler (1995) the same data was used. A K_I weight function solution was constructed to be accurate for all ranges of cut depth. An incremental stress inverse was used, and the results agreed well with the previous results.

Kang and Seol (1996) measured residual hoop stress through the thickness of a water-quenched steel ring. The medium carbon steel ring was 4 mm thick and had an outer diameter of 62 mm and an inner diameter of 42 mm.¹ A saw made cuts in 0.8 mm increments from the outer surface radially through the thickness of the ring. Just before completion of the final cut, the slot faces closed at the outer surface. Gauges located 90 degrees circumferentially on both sides of the cut measured the hoop strains on the outer surface. The average of these two gauges was used in data reduction and was smoothed using a polynomial technique. A weight function K_I forward solution was used. An incremental stress inverse was used with 2.4 mm steps, three times the cut increment. The results compared favorably with results from a sectioning method. The resulting distribution also appeared to satisfy equilibrium requirements, even though it was not constrained to do so.

Fett and Thun (1996b) measured residual hoop stress in a solid PVC cylinder. Several disks 5 to 10 mm thick were cut from a 100 mm diameter PVC cylinder. In some of the specimens, a saw introduced a 0.5 mm wide edge cut in 5 mm increments along a diameter starting from the outer radius. An optical method gave crack opening displacements near the outer surface. In other specimens an internal cut was extended

¹ These dimensions are given as radii, not diameters, in the paper, but they must be diameters to be consistent with other dimensions in the paper.

symmetrically from the center of the disk, and the crack opening was measured at the center. Weight function K_I forward solutions were used. Power series expansion inverses were used with symmetry about the disk center enforced for the internal crack tests only. Results from the two types of tests agreed well. The edge crack tests appeared to provide better results for near-surface stresses, and the internally cracked tests worked better for deeper subsurface stresses.

IV.C. Cutting Methods

Several different techniques have been used to introduce the slots in the compliance method. Some are easier to implement than others, and some have effects on the measurements. This section briefly discusses the relevant issues for each.

Saws are commonly used to make the cuts. Here, saws are taken to mean a saw with a straight edge, as compared to circular-shaped saws. Such saws include band saws, jig saws, and hand saws. Saws as thin as 0.15 mm (Vaidyanathan and Finnie 1971) have been used. When a slot is cut into a compressive stress field, the slot may close up and pinch the saw. Kang et al. (1989) used a saw with a tapered profile, narrower at the back, to prevent this. Kang et al. (1989) also used two different saws, with the second one used to sharpen the slot profile and better approximate a crack. Kang and Seol (1996) had a slot close on itself when trying to cut through the thickness of a ring, but only when the remaining ligament was very small.

Milling cutters also are used to machine slots. A milling cutter uses a circular, rotating saw blade to cut the slot. Using the table adjustments on the mill can make alignment of the cut in subsequent passes and the cut depth more precise than for saws. Cutters as thin as 0.23 mm (Cheng and Finnie 1985) and as thick as 2.4 mm (Ritchie and Leggatt 1987) have been used. Thin cutters may break during cutting, and restarting the cut after a break is often impossible (Cheng and Finnie 1985, 1987).

Mechanically machining the slot, using a saw or milling cutter, is likely to introduce residual stresses. Several investigators have measured the stresses induced by cutting on a stress-free specimen and found that the machining induced only small errors in measured residual stresses, for the proper cutting parameters (Fett 1987, Ritchie and Leggatt 1987).

Reid (1988a) was the first to use wire EDM to make the slot for residual stress measurement. In wire EDM, the wire is electrically charged with respect to the workpiece. As the wire approaches the workpiece, a spark jumps the gap and locally melts and removes material. A control loop advances the wire as material is removed. The cutting occurs in a dielectric fluid, usually deionized water, and the wire never actually contacts the piece. Using this technique, a much finer slot may be cut than with conventional machining. Cheng et al. (1994b) used a 25 μm diameter wire for two tests on a titanium alloy, which made slots approximately 32 and 42 μm wide.² Wire EDM can only be used on electrically conductive material. For hard materials (e.g., martensitic steel), EDM cuts much more easily than conventional machining. Although wire EDM generally introduces

² Private communication, Weili Cheng, February 1997.

less stress than conventional machining, it can still affect measurements for very near-surface stresses when the strain gauge is very near the slot.

Cheng et al. (1994a) thoroughly investigated the use of wire EDM for near-surface residual stress measurement. They mentioned that cutting conditions and material properties have a large effect on the possibility of introducing residual stresses during cutting. In general, larger thermal expansion coefficients and lower thermal conductivities will increase the stresses introduced. Also, cutting in "roughing" mode introduces more stress than cutting in "finishing" mode. They developed and experimentally verified a technique to correct for the stresses introduced by wire EDM cutting during residual stress measurements. They performed tests on a 304 stainless-steel beam in both roughing and finishing mode. Corrections were successful in both cases and minimal for the finishing mode test.

V. COMPARISONS WITH OTHER METHODS

This section qualitatively compares the compliance method to the most common measurement techniques: x-ray diffraction, neutron diffraction, hole drilling, and layer removal. These methods are compared with regard to destructivity, sensitivity, depth profiling, accuracy, measuring time, and stress components measured. Lu (1996) contains a more detailed comparison between measurement techniques. Although more methods are considered than in this comparison, Lu does not include the compliance method.

These comparisons reveal that there is no single best method for measuring residual stress. The selection of a method should consider the particular application and the strengths and weaknesses of all methods.

V.A. Destructivity

The only method considered here that can nondestructively measure residual stress variation with depth is neutron diffraction. Neutron diffraction can measure to depths of tens of millimeters but can only resolve stresses in regions approximately 1 mm³ or larger. X-ray diffraction can measure only very near-surface stresses nondestructively. Depth profiling requires etching away layers at the spot to be measured. Hole drilling and compliance are considered semi-destructive methods. They both require local material removal, with compliance generally removing more material. Layer removal is a destructive method in which layers are removed from the entire surface of a part. A complete residual stress profile by removing layers will destroy the part.

V.B. Sensitivity

Sensitivity refers to the level of data measurement for low residual stress levels. The mechanical methods measure strain due to releasing residual stress, and the reading can be low for low stresses. Layer removal generally has the lowest sensitivity, but this sensitivity depends greatly on the part thickness. Hole drilling has increased sensitivity, and compliance has even greater sensitivity. Cheng et al. (1991a) explicitly compared the sensitivity of compliance to hole drilling and showed compliance to be superior. One drawback of sensitivity is that it generally also corresponds to a greater propensity to yielding at high stress levels, which leads to errors.

The diffraction methods measure crystal lattice spacing. Because this spacing is only slightly changed by residual stress levels, diffraction methods can easily measure the spacing corresponding to low stresses. For example, 1 angstrom is about as easy to measure as 1.1 angstrom. So there is no sensitivity difficulty that is directly analogous to that for mechanical methods. However, low stresses cause other difficulties. An explanation of these difficulties is beyond the scope of this discussion.

V.C. Depth Profiling

Figure 6 shows the approximate depth ranges over which the various methods are able to resolve residual stress. The depth range shown for x-rays is for non-destructive measurements. Profiles up to 1 mm depth are commonly made with x-rays by

electrochemically etching away material. The figure illustrates the need to consider two important factors when choosing a residual stress measurement method for a particular part: (1) the depth of residual stresses that was generated in manufacturing the part of interest, and (2) the depth to which residual stresses will contribute to the potential failure mechanism.

Figure 6. Depth ranges of measurement techniques compared with typical generated profiles and failure mechanisms. Similar to figure in Leggatt et al. (1996).

V.D. Accuracy

A wide variety of factors influence the accuracy of all measurement methods. No single method can be considered the most accurate. The x-ray and neutron diffraction methods are based on properties of the crystalline microstructure. The presence of multiple phases and texture (preferred orientation) can reduce accuracy in stress measurements. Because it measures a surface spot with little depth penetration, the x-ray method can give unexpected results because of the presence of a local surface effect, such as slight oxidation. Large grains can also increase errors using x-rays because the spot examined may contain too few grains to provide a sufficient statistical sample. The mechanical methods, hole-drilling, compliance, and layer removal, function completely independently of microstructure. However, they are susceptible to errors from stresses induced during the material removal process. If the stresses removed are excessively large, yielding can occur which increases errors. Because the analysis for hole drilling assumes a hole located in the center of the strain rosette, hole misalignment results in increased errors.

V.E. Time Required for Measurements

The time to perform diffraction measurements depends on factors such as the material being examined and the sampling volume. Generally, neutron diffraction takes longer than x-ray, but x-ray requires extra time to etch away layers between measurements. Generating a profile typically takes one to several days for x-rays and several days to a week or more for neutrons.

Experimental time for the mechanical methods is generally less than for diffraction methods. Layer removal takes the longest because it requires the most material removal. The mechanical methods typically require time to install strain gauges. The total time to produce results depends on the complexity of the data reduction technique, with the most complex methods taking several days or longer.

V.F. Stress Components Measured

The various methods do not all measure all of the stress coordinates. X-ray diffraction, hole drilling, and layer removal all measure both stress components in the plane of the part, y and z components in Figure 1. Neutron diffraction can measure two components in a single measurement and all three components using multiple measurements. Compliance measures the stress component normal to the slot, the y component in Figure 1.

REFERENCES

- Beghini, M. and L. Bertini, 1990, "Residual Stress Modelling by Experimental Measurements and Finite Element Analysis," *Journal of Strain Analysis for Engineering Design*, **25**(2), 103-108.
- Bueckner, H. F., 1958, "The Propagation of Cracks and the Energy of Elastic Deformation," *Transactions of the American Society of Mechanical Engineering*, **80**, 1225-1230.
- Cheng, W. and I. Finnie, 1985, "A Method for Measurement of Axisymmetric Residual Stresses in Circumferentially Welded Thin-Walled Cylinders," *Journal of Engineering Materials and Technology*, **107**, 181-185.
- Cheng, W. and I. Finnie, 1986, "Measurement of Residual Hoop Stress in Cylinders Using the Compliance Method," *Journal of Engineering Materials and Technology*, **108**, 87-92.
- Cheng, W. and I. Finnie, 1987, "A New Method of Measurement of Residual Axial Stresses Applied to a Multi-Pass Butt Welded Cylinder," *Journal of Engineering Materials and Technology*, **109**, 337-342.
- Cheng, W. and I. Finnie, 1988, " K_I Solutions for an Edge Cracked Strip," *Engineering Fracture Mechanics*, **31**, 201-207.
- Cheng, W. and I. Finnie, 1990a, "A K_{II} Stress Intensity Solution for an Edge Cracked Strip," *Engineering Fracture Mechanics*, **36**, 355-360.
- Cheng, W. and I. Finnie, 1990b, "The Crack Compliance Method for Residual Stresses Measurement," *Welding in the World*, **28**, 103-110.
- Cheng, W., I. Finnie, and Ö. Vardar, 1991a, "Measurement of Residual Stresses Near the Surface Using the Crack Compliance Method," *Journal of Engineering Materials and Technology*, **113**, 199-204.
- Cheng, W. and I. Finnie, 1991b, "An Experimental Method for Determining Residual Stresses in Welds," in *Modeling of Casting, Welding, and Advanced Solidification Processes - V*, M. Rappaz, Ed. (The Minerals, Metals, and Materials Society, Warrendale, PA, USA), 245-252.
- Cheng, W., I. Finnie, and Ö. Vardar, 1992a, "Deformation of an Edge Cracked Strip Subjected to Normal Surface Traction on the Crack Faces," *Engineering Fracture Mechanics*, **42**, 97-107.
- Cheng, W., I. Finnie, and Ö. Vardar, 1992b, "Estimation of Axisymmetric Residual Stresses in a Long Cylinder," *Journal of Engineering Materials and Technology*, **114**, 137-140.
- Cheng, W. and I. Finnie, 1992c, "Deformation of an Edge Cracked Strip Subjected to Arbitrary Shear Surface Traction on the Crack Faces," *Engineering Fracture Mechanics*, **43**, 33-39.
- Cheng, W., I. Finnie, and M. B. Prime, 1992d, "Measurement of Residual Stresses Through the Thickness of a Strip Using the Crack Compliance Method," in *Residual Stresses III - Science and Technology, Proceedings of the Third International Conference on Residual Stress*, Vol. 2, Elsevier Applied Science, Ed.: H. Fujiwara, et al., 1127-1132.

- Cheng, W. and I. Finnie, 1993a, "A Comparison of the Strains due to Edge Cracks and Cuts of Finite Width in a Semi-Infinite Elastic Plate," *Journal of Engineering Materials and Technology*, **115**, 220-226.
- Cheng, W. and I. Finnie, 1993b, "Measurement of Residual Stress Distributions Near the Toe of an Attachment Welded on a Plate Using the Crack Compliance Method," *Engineering Fracture Mechanics*, **46**, 79-91.
- Cheng, W., I. Finnie, M. Gremaud, and M. B. Prime, 1994a, "Measurement of Near Surface Residual Stresses Using Electrical Discharge Wire Machining," *Journal of Engineering Materials and Technology*, **116**, 1-7.
- Cheng, W., I. Finnie, M. Gremaud, A. Rosselet, and R. D. Streit, 1994b, "The Compliance Method for Measurement of Near Surface Residual Stresses - Application and Validation for Surface Treatment by Laser and Shot-Peening," *Journal of Engineering Materials and Technology*, **116**, 556-560.
- Cheng, W. and I. Finnie, 1994c, "An Overview of the Crack Compliance Method for Residual Stress Measurement," in *Proceedings of the 4th International Conference on Residual Stress*, Baltimore, Maryland, 449-458.
- Fett, T., 1987, "Bestimmung von Eigenspannungen mittels bruchmechanischer Beziehungen (Determination of Residual Stresses by Use of Fracture Mechanical Relations)," *Materialprüfung*, **29**, 92-92.
- Fett, T., 1996a, "Determination of Residual Stresses in Components Using the Fracture Mechanics Weight Function," *Engineering Fracture Mechanics*, **55**(4), 571-576.
- Fett, T. and G. Thun, 1996b, "Residual Stresses in PVC-Cylinders Determined with the Weight Function Method," *Engineering Fracture Mechanics*, **55**(5), 859-863.
- Finnie, S., W. Cheng, I. Finnie, J-M. Drezet, and M. Gremaud, 1996, "The Computation and Measurement of Residual Stress in Laser Deposited Layers," in *Proceedings of the 4th European Conference on Residual Stress*, Cluny-en-Bourgogne, France, June 1996.
- Galatolo, R. and A. Laciotti, 1997, "Fatigue Crack Propagation in Residual Stress Fields of Welded Plates," *International Journal of Fatigue*, **19**(1), 43-49.
- Gremaud, M., W. Cheng, M. Prime, and I. Finnie, 1992, "Measurement of Residual Stresses in Laser Treated Layers Using the Crack Compliance Method," in *Proceedings of International Conference on Laser Advanced Materials Processing*, Eds.: A. Matasunawa and S. Katayama, 713-718.
- Gremaud, M., W. Cheng, I. Finnie, and M. B. Prime, 1994, "The Compliance Method for Measurement of Near Surface Residual Stresses - Analytical Background," *Journal of Engineering Materials and Technology*, **116**, 550-555.
- Hermann, R., 1994, "Fatigue Crack Growth in Ductile Materials Under Cyclic Compressive Loading," *Fatigue and Fracture of Engineering Materials*, **17**(1), 93-103.
- Hermann, R., 1995, "Crack Growth and Residual Stress in Al-Li Metal Matrix Composites Under Far-Field Cyclic Compression," *Journal of Materials Science*, **30**, 3782-3790.
- Joerms, M. W., 1987, "Calculation of Residual Stresses in Railroad Rails and Wheels from Sawcut Displacement," in *Residual Stress in Design, Process and Materials Selection, Proceedings of ASM's Conference on Residual Stress in Design, Process and Materials Selection*, Cincinnati, Ohio, USA, 27-29 April 1987, W. B. Young, Ed., 205-209.

- Kang, K. J., J. H. Song, and Y. Y. Earmme, 1989, "A Method for the Measurement of Residual Stresses Using a Fracture Mechanics Approach," *Journal of Strain Analysis*, **24**, 23-30.
- Kang, K. J. and S. Y. Seol, 1996, "Measurement of Residual Stresses in a Circular Ring Using the Successive Cracking Method," *Journal of Engineering Materials and Technology*, **118**, 217-223.
- Kelsey, R. A., 1956, "Measuring Non-Uniform Residual Stresses by the Hole Drilling Method," in *Proceedings of the Society for Experimental Stress Analysis*, **14**(1), 181-194.
- Lai, M. O., J. T. Oh, and A. Y. C. Nee, 1993, "Fatigue Properties of Holes with Residual Stresses," *Engineering Fracture Mechanics*, **45**(5), 551-557.
- Lai, M. O. and Y. H. Siew, 1995, "Fatigue Properties of Cold Worked Holes," *Journal of Materials Processing Technology*, **48**, 533-540.
- Leggatt, R. H., D. J. Smith, S. D. Smith, and F. Faure, 1996, "Development and Experimental Validation of the Deep Hole Method for Residual Stress Measurement," *Journal of Strain Analysis*, **31**(3), 177-186.
- Lu, J., Editor, 1996, *Handbook of Measurement of Residual Stress* (The Fairmont Press, Inc., Lilburn, Georgia, USA).
- Nisitani, H., 1978, "Solutions of Notch Problems by Body Force Method," in *Stress Analysis of Notch Problems*, G. C. Sih, Ed., Vol. 5 of *Mechanics of Fracture* (Nordhoff International Publishing), 1-68.
- Oh, J. T., M. O. Lai and A. Y. C. Nee, 1993, "Stress Analysis of a Ballised Hole," *Journal of Materials Processing Technology*, **37**, 137-147.
- Orkisz, J. and A. Skrzat, 1996, "Reconstruction of Residual Stresses in Railroad Vehicle Wheels Based on Enhanced Saw Cut Measurements, Formulation and Benchmark Tests," *Wear*, **191**, 188-198.
- Perl, M. and R. Aroné, 1994, "An Axisymmetric Stress Release Method for Measuring the Autofrettage Level in Thick-Walled - Parts I & II," *Journal of Pressure Vessel Technology*, **116**, 384-395.
- Popelar, C. H., T Barber, and J. Groom, 1982, "A Method for Determining Residual Stresses in Pipes," *Journal of Pressure Vessel Technology*, **104**, 223-228.
- Prime, M. B. and I. Finnie, 1996, "Surface Strains Due to Face Loading of a Slot in a Layered Half-Space," *Journal of Engineering Materials and Technology*, **118**, 410-418.
- Read, D. T., 1989, "Measurement of Applied J -Integral Produced by Residual Stress," *Engineering Fracture Mechanics*, **32**(1), 147-153.
- Reid, C. N., 1988a, "A Method of Mapping Residual Stress in a Compact Tension Specimen," *Scripta Metallurgica*, **22**(4), 451-456.
- Reid, C. N., J. Moffatt, and R. Hermann, 1988b, "Fatigue Under Compressive Loading and Residual Stress," *Scripta Metallurgica*, **22**, 1743-48.
- Ritchie, D. and R. H. Leggatt, May 1987, "The Measurement of the Distribution of Residual Stress Through the Thickness of a Welded Joint," *Strain*, 61-70.
- Saxena, A. and S. J. Hudak, Jr., 1978, "Review and Extension of Compliance Information for Common Crack Growth Specimens," *International Journal of Fracture*, **14**(5), 453-467.

Schajer, G. S., 1981, "Application of Finite Element Calculations to Residual Stress Measurements," *Journal of Engineering Materials and Technology*, **103**, 157-163.

Schindler, H. J., W. Cheng, and I. Finnie, 1994, "Measurement of the Residual Stress Distribution in a Disk or a Solid Cylinder Using the Crack Compliance Method," in *Proceedings of the 4th International Conference on Residual Stress*, Baltimore, Maryland, 1266-1274.

Schindler, H. J., 1995, "Determination of Residual Stress Distributions from Measured Stress Intensity Factors," *International Journal of Fracture*, **74**(2), R23-R30.

Schwaighofer, J., 1964, "Determination of Residual Stresses on the Surface of Structural Parts," *Experimental Mechanics*, **4**(2), 54-56.

Soete, W. and R. VanCrombrugge, 1954, "Determination of Residual Stresses Below the Surface," in *Residual Stresses in Metals and Metal Construction*, W. R. Osgood, Ed. (Reinhold Publishers, New York, NY, USA), **8**(1), 17-28.

Vaidyanathan, S. and I. Finnie, 1971, "Determination of Residual Stresses from Stress Intensity Factor Measurements," *Journal of Basic Engineering*, **93**, 242-246.