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THE SPACE-CHARGE IMPEDANCE OF RF-SHIELDING WIRES

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Abstract

We studied the electrostatic fields due to the longitudinal and transverse perturbations of a charged particle beam with a uniform distribution propagating inside an rf-shielding cage constructed from evenly-spaced conducting wires. Simple formulae are derived for estimating the space-charge impedances. Numerical examples are given for illustration.

1 Introduction

An rf-shielding cage, or an rf-cage, used in an accelerator or storage ring is a cage-like structure made of conducting wires stretched in parallel to the direction of the circulating charged particle beam.^[1] The conducting wires on the cage are arranged to surround the beam to create an electromagnetically shielded environment for the beam. This kind or the similar kinds of devices together with ceramic beam pipes have been implemented^[1] and planned^[2-4], or is being planned^[5] in some high-intensity rapid cycling proton synchrotrons.

There are two reasons for choosing the rf-cage instead of solid beam pipe. The first reason is to avoid excess eddy current that may be induced on the beam pipe by the fast-changing magnetic field. The second reason is that, for reducing the coupling impedance, it is easier to vary the cross-section of an rf-cage than varying the cross section of a solid pipe. In the long wavelength region, an appropriately designed rf-cage can provide electromagnetic shielding near that of a solid beam pipe.

Although an rf-cage has been built and implemented in an existing proton synchrotron,^[1] a serious study of the electromagnetic field of a charged particle beam propagating in an rf-cage has never been documented until the relative recent.^[6] In Ref. 6, a rigorous formalism was established to investigate the electrostatic field of a charged particle beam with a uniform distribution inside an rf-shielding cage constructed from evenly-spaced conducting wires. However, mistakes were later found in that work. The purpose of this present work is to revisit the old problem considered in Ref. 6. We will present the correct solution of the electrostatic fields of a perturbed beam traversing inside of an rf-shielding cage. Simple formulae will be derived for both the longitudinal and transverse coupling impedances in the long wavelength regime. Numerical examples will be given to illustrate the effect of rf-shielding.

2 The Electrostatic Potential of a Perturbed Beam

The system considered here is shown in Fig. 1. A beam having a circular cross-section of radius r_b and a uniform charge distribution is propagating inside of an rf-cage composed of N conducting wires extended in the direction parallel to the beam. For simplicity, we shall limit our discussion to the geometry in which wires are evenly distributed over a circle and the rf-cage is positioned concentric with the beam. The radius of the rf-cage, measured from the center of the cage to the centers of wires, is r_c . We assume that wires are electrically grounded and all wires have the same circular cross-section of radius ρ_w . The discussions in the following will be restricted to the regime of $\rho_w \ll r_c$ and $N \gg 1$.

Fig. 1. Cross-sectional view of a beam inside an rf-cage and beam pipe. r_c and r_b are the radii of the rf-cage and the beam, respectively. Δ is the angle subtended by two adjacent wires, and ρ_w is the radius of a wire.

We choose a cylindrical coordinate system (r, θ, z) such that the z -axis coincides with the central axis of the beam. We shall call this coordinate system the “beam coordinate system” or the “global coordinate system”. In order to make it convenient to describe the electric field near an individual wire, we shall also use another cylindrical coordinate system (ρ, ψ, z) in which the z -axis coincides with the central axis of a wire as shown in Fig. 2. This “local coordinate system” will also be referred to as the “wire coordinate system” in the following.

Fig. 2. The local and the global coordinates adopted in this study. The origins of the local and the global coordinates are located at the center of beam and the center of a wire, respectively.

2.1 The Potential due to a Longitudinal Perturbation

Here, we shall concentrate on the electrostatic potential due to the longitudinal charge-density perturbation that varies in the z -direction according to e^{ikz} , where k is the wave-number of the perturbation. The Poisson equation we want to solve is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = \begin{cases} 0, & \text{for } r > r_b, \\ -(\sigma/\epsilon_o)e^{ikz}, & \text{for } r \leq r_b, \end{cases} \quad (1)$$

where σ is the volume charge density associated with the perturbation, ϵ_o is the permittivity of free space, and r_b is the radius of the beam. We are interested in the solution of Eq. (1) in the region where $kr_c \ll 1$.

In the absence of any boundary, we assume $\Phi = 0$ at infinity. The solution of Eq. (1) in this case is

$$\Phi = \phi_b = \begin{cases} b_{\parallel} K_0(kr) e^{ikz} , & \text{for } r > r_b, \\ \frac{\sigma}{\epsilon_o k^2} \left[1 - kr_b K_1(kr_b) I_0(kr) \right] e^{ikz} , & \text{for } r \leq r_b, \end{cases} \quad (2)$$

where

$$b_{\parallel} = \left(\frac{\sigma r_b}{k \epsilon_o} \right) I_1(kr_b) ; \quad (3)$$

$I_n(x)$ and $K_n(x)$ are the n th order modified Bessel functions of the first kind and the second kind, respectively.

Moving into the discussion of solving Eq. (1) in the presence of wires, we consider the solution in the region of $r \geq r_c$ first. In the presence of wires, the beam field will induce electric charges on the surfaces of wires. These induced charges also have an electric potential associated with them. We then assume that each wire induces a field which can be expanded into a Fourier series of $e^{in\psi}$ in the local coordinate as

$$\phi_w = \sum_{n=-\infty}^{\infty} C_n K_n(k\rho) e^{in\psi} e^{ikz} , \quad (4)$$

where C_n is an unknown quantity to be solved. Applying the addition theorem of Bessel functions,^[7] ϕ_w can also be expressed in the global coordinate variables as

$$\phi_w = \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_n I_{n+l}(kr_c) K_l(kr) e^{il\theta} e^{ikz} . \quad (5)$$

Note that Eq. (4) is virtually the multipole expansion of the induced field in the wire coordinate system.

Since all wires are electrically identical and are evenly distributed, one can study the field around wires by considering the electric potential around any individual wire. Thus, we call the wire under consideration the 0th wire and number all others by their relative locations with respect to the 0th wire. If S_w is the sum of the induced potentials from all other wires nearby the 0th one, we have

$$S_w = \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_l \left[\sum_{\mu=1}^{N-1} e^{i(n+l)(\pi-\mu\Delta)/2} \right] K_{l-n}(kd_{\mu}) I_n(k\rho) e^{in\psi} e^{ikz} , \quad (6)$$

where $\Delta = 2\pi/N$ is the angular separation between two adjacent wires; d_{μ} is the distance between the centers of the 0th and the μ th wires.

The total electric potential around the 0th wire is

$$\Phi = \phi_b + \phi_w + S_w . \quad (7)$$

On the surface of each wire, the potential due to the induced charge should cancel the potential due to the beam and the potential contributed by all other wires, i.e., $\Phi = 0$ at $\rho = \rho_w$. Thus, substituting Eqs. (2), (4) and (6) into Eq. (7), we derive that on the surface of the 0th wire,

$$\sum_{l=-\infty}^{\infty} C_l \left[\sum_{\mu=1}^{N-1} e^{i(n+l)(\pi-\mu\Delta)/2} \right] K_{l-n}(kd_\mu) + \frac{C_n}{h_n(k\rho_w)} + b_{\parallel} K_n(kr_c) = 0, \quad (8)$$

where

$$h_n(x) = I_n(x)/K_n(x). \quad (9)$$

The closed-form solution of Eq. (8) appears to be inaccessible. However, in the regime of $k\rho_w \ll kr_c \ll 1$, it is possible to find a solution expanded in the power series of $h_n(k\rho_w)$. We define a quantity η_n such that

$$C_n = -b_{\parallel} \eta_n / [N I_n(kr_c)]. \quad (10)$$

Then, if the multipole coupling is neglected, the lowest order solution of η_n is

$$\eta_n \approx N h_n(k\rho_w) I_n(kr_c) K_n(kr_c) D_n, \quad (11)$$

where

$$D_n = \left[1 + (-1)^n h_n(k\rho_w) \sum_{\mu=1}^{N-1} e^{-in\mu\Delta} K_0(kd_\mu) \right]^{-1}. \quad (12)$$

The lowest order solution of η_n that includes the multipole coupling is

$$\eta_n \approx N h_n(k\rho_w) I_n(kr_c) D_n \left\{ K_n(kr_c) - \sum_{l=-\infty}^{\infty} (1 - \delta_{ln}) h_l(k\rho_w) \times K_l(kr_c) \sum_{\mu=1}^{N-1} e^{i(n+l)(\pi-\mu\Delta)/2} K_{n-l}(kd_\mu) \right\}. \quad (13)$$

The contribution of the potential from all wires now can be expressed as

$$\phi_w + S_w = b_{\parallel} \sum_{p=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{\eta_n}{I_n(kr_c)} I_{n+pN}(kr_c) K_{pN}(kr) e^{ipN\theta} e^{ikz}. \quad (14)$$

In arriving at Eq. (14), use has been made of the relation (holds good only when $\Delta = 2\pi/N$)

$$\sum_{j=0}^{N-1} e^{-ijl\Delta} = N \delta_{l,pN} = \begin{cases} N, & \text{for } l = pN, p = 0, \pm 1, \pm 2, \dots, \\ 0, & \text{for } l \neq pN. \end{cases} \quad (15)$$

Substituting the potentials in Eqs. (2) and (14) into Eq. (7) yields the following total potential in the region of $r \geq r_c$

$$\Phi_{\parallel} = b_{\parallel} K_0(kr) e^{ikz} - b_{\parallel} \sum_{p=-\infty}^{\infty} B_p K_{pN}(kr) e^{ipN\theta} e^{ikz} , \quad (16)$$

where

$$B_p = (1 - \delta_{N0}) \sum_{n=-\infty}^{\infty} \frac{\eta_n}{I_n(kr_c)} I_{n+pN}(kr_c) , \quad (17)$$

Next, we consider the solution in the region of $r \leq r_c$. In this region,

$$\phi_w + S_w = b_{\parallel} \sum_{p=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{\eta_n}{I_n(kr_c)} K_{n+pN}(kr_c) I_{pN}(kr) e^{ipN\theta} e^{ikz} . \quad (18)$$

From this, we can derive

$$\Phi_{\parallel} = b_{\parallel} K_0(kr) e^{ikz} - b_{\parallel} \sum_{p=-\infty}^{\infty} \hat{B}_p I_{pN}(kr) e^{ipN\theta} e^{ikz} , \quad (19)$$

for $r_c \geq r \geq r_b$, and

$$\Phi_{\parallel} = \frac{\sigma}{\epsilon_0 k^2} \left[1 - kr_b K_1(kr_b) I_0(kr) \right] e^{ikz} - b_{\parallel} \sum_{p=-\infty}^{\infty} \hat{B}_p I_{pN}(kr) e^{ipN\theta} e^{ikz} , \quad (20)$$

for $r \leq r_b$, where

$$\hat{B}_p = (1 - \delta_{N0}) \sum_{n=-\infty}^{\infty} \frac{\eta_n}{I_n(kr_c)} K_{n+pN}(kr_c) . \quad (21)$$

2.2 The Potential due to a Transverse Perturbation

We now consider the electrostatic potential due to a sinusoidal transverse perturbation in a beam propagating inside an rf-cage. The model of the perturbation to be studied here is a shell with surface charge density varying according to $e^{ikz} \cos \theta$. Here we assume the beam coordinate is oriented in such a way that the maximal charge density is at the angle of $\theta = 0$. The Poisson equation is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = -\frac{\sigma \bar{d}}{2\epsilon_0} \delta(r - r_b) (e^{i\theta} + e^{-i\theta}) e^{ikz} , \quad (22)$$

where $\bar{d}$ is the averaged displacement of the beam.

Because the system under consideration is symmetric with respect to θ , we shall solve the equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = -\frac{\sigma \bar{d}}{2\epsilon_0} \delta(r - r_b) e^{i\theta} e^{ikz}, \quad (23)$$

first. The solution of Eq. (22) then can be obtained from the solution of Eq. (23) by the substitution of $e^{in\theta}/2 \rightarrow \cos \theta$ appropriately. In the absence of any boundary, Eq. (23) has the solution

$$\Phi = \phi_b = \begin{cases} b_{\perp} K_1(kr) e^{i\theta} e^{ikz}, & \text{for } r > r_b, \\ b_{\perp} [K_1(kr_b)/I_1(kr_b)] I_1(kr) e^{i\theta} e^{ikz}, & \text{for } r \leq r_b, \end{cases} \quad (24)$$

where

$$b_{\perp} = \left(\frac{\sigma \bar{d} r_b}{2\epsilon_0} \right) I_1(kr_b). \quad (25)$$

For the solution including the rf-shielding wires, we again consider the region of $r \geq r_c$ first. Different from the case of longitudinal perturbation, the axial symmetry does not exist here and the angular positions of wires need to be taken into account when describing the potential around each wire. Take the wire located at the angle of $m\Delta$, the m th wire, as an example. When applying the addition theorem of Bessel functions, one has to make a change of variable from θ to $\theta - m\Delta$ for translating the description of the potential from the global coordinate variables to the local coordinate variables around the m th wire. Therefore, the multipole expansion coefficients of the field induced on each wire depend on the wire's location.

The analysis will follow the same path as in Section 2.1. Thus, we assume the m th wire induces a potential which can be described in terms of the wire coordinate system variables as

$$\phi_w = \sum_{n=-\infty}^{\infty} P_{m,n} K_n(k\rho) e^{in\psi} e^{ikz}, \quad (26)$$

where $P_{m,n}$ is the unknown we want to solve. The contribution from all other $N - 1$ wires, S'_w is

$$S'_w = \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{j=0}^{N-1} (1 - \delta_{j,m}) P_{j,l} e^{i(n+l)[\pi - (j-m)\Delta]/2} \\ \times K_{l-n}(kd_{j,m}) I_n(k\rho) e^{in\psi} e^{ikz}, \quad (27)$$

where $d_{j,m}$ is the distance between the centers of the m th and the j th wires. Using some algebra, one can prove that $P_{m,n} = e^{im\Delta} P_n$, where $P_n = P_{0,n}$ is the multipole expansion coefficient for the 0th wire. Applying this relation, the requirement of the total potential be zero on the surfaces of all wires can be reduced to a single equation similar to Eq. (8):

$$\sum_{l=-\infty}^{\infty} P_l \left[\sum_{\mu=1}^{N-1} e^{i(n+l)(\pi-\mu\Delta)/2} \right] K_{n-l}(kd_\mu) + \frac{P_n}{h_n(k\rho_w)} + b_\perp K_{n+1}(kr_c) = 0 \quad (28)$$

Eq. (28) can be solved in the same procedure as solving Eq. (8). Thus, defining η'_n according to

$$P_n = -b_\perp \eta'_n / \left[N I_{n+1}(kr_c) \right] \quad , \quad (29)$$

then the lowest order solution of η'_n without the multipole coupling is

$$\eta'_n \approx N h_n(k\rho_w) I_{n+1}(kr_c) K_{n+1}(kr_c) D'_n \quad . \quad (30)$$

where

$$D'_n = \left[1 + (-1)^n h_n(k\rho_w) \sum_{\mu=1}^{N-1} e^{-i(n-1)\mu\Delta} K_0(kd_\mu) \right]^{-1} \quad . \quad (31)$$

Including the multipole coupling, the lowest order solution of η'_n is

$$\begin{aligned} \eta'_n \approx N h_n(k\rho_w) I_{n+1}(kr_c) D'_n & \left\{ K_{n+1}(kr_c) - \sum_{l=-\infty}^{\infty} (1 - \delta_{ln}) h_l(k\rho_w) \right. \\ & \times K_{l+1}(kr_c) \sum_{\mu=1}^{N-1} e^{i(n+l)(\pi-\mu\Delta)/2} e^{in\Delta} K_{n-l}(kd_\mu) \left. \right\} \quad . \quad (32) \end{aligned}$$

Using these results, we can obtain the following solution of Eq. (22):

$$\Phi_\perp = 2b_\perp K_1(kr) \cos \theta e^{ikz} - 2b_\perp \sum_{p=-\infty}^{\infty} B'_p I_{pN+1}(kr) \cos \theta e^{ipN\theta} e^{ikz} \quad , \quad (33)$$

for $r \geq r_c$,

$$\Phi_\perp = 2b_\perp K_1(kr) \cos \theta e^{ikz} - 2b_\perp \sum_{p=-\infty}^{\infty} \hat{B}'_p I_{pN+1}(kr) \cos \theta e^{ipN\theta} e^{ikz} \quad , \quad (34)$$

for $r_c \leq r \leq r_b$, and

$$\Phi_\perp = 2b_\perp \left[\frac{K_1(kr_b)}{I_1(kr_b)} \right] I_1(kr) \cos \theta e^{ikz} - 2b_\perp \sum_{p=-\infty}^{\infty} \hat{B}'_p I_{pN+1}(kr) \cos \theta e^{ipN\theta} e^{ikz} \quad , \quad (35)$$

for $r \leq r_b$, where

$$B'_p = (1 - \delta_{N0}) \sum_{n=-\infty}^{\infty} \frac{\eta'_n}{I_{n+1}(kr_c)} I_{n+pN+1}(kr_c) \quad . \quad (36)$$

and

$$\hat{B}'_p = (1 - \delta_{N0}) \sum_{n=-\infty}^{\infty} \frac{\eta'_n}{I_{n+1}(kr_c)} K_{n+pN+1}(kr_c) . \quad (37)$$

3 The Space-Charge Impedance

Before proceeding to the derivation of space-charge impedance, it is worthwhile to discuss the limits of our solutions when $N \rightarrow \infty$ and $\rho_w \rightarrow 0$. At these limits, the summation over wires can be replaced by an integration to show that the values of both η_n and η'_n approach one. Then from Eqs. (19), (20), (34), and (35), one obtains the perturbed potentials for a beam inside a conducting pipe without wires. Thus, for an rf-cage made of infinite number of thin wires, the shielding effects is the same as that of a conducting beam pipe.

We now consider the space-charge impedance in the long wavelength region, i.e. the region where $k\rho_w \ll kr_c \ll 1$. We shall concentrate our discussions to the field at the center of the beam, i.e. at $r = 0$. From Eqs. (20) and (35), we can derive the perturbed electric field at the center of the beam:

$$E_z = -\frac{\partial \Phi_{\parallel}}{\partial z} \approx \frac{-i\sigma}{\epsilon_o k} \left\{ 1 - kr_b K_1(kr_b) - \frac{\eta_0 kr_b I_1(kr_b) K_0(kr_c)}{I_0(kr_c)} \right\} e^{ikz} , \quad (38)$$

and

$$E_y = -\frac{\partial \Phi_{\perp}}{\partial r} \Big|_{\theta=0} \approx \frac{-\sigma k \bar{dr}_b}{2\epsilon_o} \left\{ K_1(kr_b) - \frac{\eta'_0 I_1(kr_b) K_1(kr_c)}{I_1(kr_c)} \right\} e^{ikz} . \quad (39)$$

where we have retained only the most significant terms (the $n = 0$ or the monopole terms) in the summations, and we have left off the couplings among the multipole fields since they are negligibly small in the parameter range of our interest. Then, applying the small argument expansions of Bessel functions, we derive from Eqs. (11) and (30) that for $N \gg 1$

$$\eta_0 \approx \frac{N \ln(kr_c)}{N \ln(kr_c) - \ln(r_c/\rho_w)} \approx \frac{N \ln(kr_c)}{N \ln(kr_c) - \ln[N/(\pi f_w)]} , \quad (40)$$

and

$$\eta'_0 \approx \frac{N}{N + 2 \ln(r_c/\rho_w)} \approx \frac{N}{N + 2 \ln[N/(\pi f_w)]} . \quad (41)$$

where the "wire filling factor" f_w is defined as the ratio between the angle subtended by a wire in the beam coordinate system θ_w and Δ , i.e.

$$f_w = \frac{\theta_w}{\Delta} \approx \frac{N \rho_w}{\pi r_c} . \quad (42)$$

Making use of the approximations in Eqs. (40) and (41), we obtain from Eqs. (38) and (39) that

$$E_z \approx \frac{-i\sigma k r_b^2}{2\epsilon_o} \left[1 + 2 \ln\left(\frac{r_c}{r_b}\right) + W_{\parallel} \right] e^{ikz} , \quad (43)$$

and

$$E_y \approx \frac{-\sigma \bar{d} r_b^2}{2\epsilon_o} \left(\frac{1}{r_b^2} - \frac{1}{r_c^2} + \frac{W_{\perp}}{r_c^2} \right) e^{izk} , \quad (44)$$

where

$$W_{\parallel} = -2(1 - \eta_0) \approx \frac{2 \ln(kr_c) \ln[N/(\pi f_w)]}{\ln[N/(\pi f_w)] - N \ln(kr_c)} , \quad (45)$$

and

$$W_{\perp} = 1 - \eta'_0 \approx \frac{2 \ln[N/(\pi f_w)]}{N + 2 \ln[N/(\pi f_w)]} . \quad (46)$$

Note that in Eqs. (43) and (44), when $W_{\parallel} = 0$ and $W_{\perp} = 0$, one obtains the electric field inside a solid beam pipe. In a crude way, the quantities $W_{\parallel}$ and $W_{\perp}$ can be linked to the electric flux leaks out from the rf-cage; the quantities η_n and η'_n can be linked to the electric flux contained by the rf-cage. Thus, the approximations in Eqs. (43) and (44) allow one to estimate how the usual impedances of a solid beam pipe are modified when using an rf-cage for shielding. For example, in a hypothetical case of the rf-cage occupies the whole ring, the longitudinal and the transverse space-charge impedances are

$$Z_{\parallel} = \frac{i n Z_o}{2\beta\gamma^2} \left[1 + 2 \ln\left(\frac{r_c}{r_b}\right) + W_{\parallel} \right] , \quad (47)$$

and

$$Z_{\perp} = \frac{i R Z_o}{\beta^2 \gamma^2} \left(\frac{1}{r_b^2} - \frac{1}{r_c^2} + \frac{W_{\perp}}{r_c^2} \right) , \quad (48)$$

respectively, where

$$W_{\parallel} \approx \frac{2 \ln(nr_c/R) \ln[N/(\pi f_w)]}{\ln[N/(\pi f_w)] - N \ln(nr_c/R)} , \quad (49)$$

$W_{\perp}$ is the same as in Eq. (46), n is the azimuthal harmonic of the perturbation around the ring, $Z_o = 377\Omega$, β is equal to the averaged particle speed divided by the speed of light, $\gamma = (1 - \beta^2)^{-1/2}$, and R is the machine's effective radius.

Numerical examples of $W_{\parallel}$ and $W_{\perp}$ are given in Figs. 3 and 4 for $kr_c = 0.1$ (or $nr_c/R = 0.1$). These results indicate that in the long wavelength regime, the diameter of wires is not an important parameter when large number of wires are used for shielding. Also note that because of the approximations we have taken in deriving the results given in Eqs. (43) and (44), the values of $W_{\parallel}$ and $W_{\perp}$ will not be zero when $f_w = 1$.

Fig. 3. $W_{\parallel}$ as a function of the total number of wires N for (a) $f_w = 0.1$, (b) $f_w = 0.3$, (c) $f_w = 0.55$, and (d) $f_w = 0.9$. Where $W_{\parallel}$ is calculated using Eq. (45) for $kr_c = 0.1$.

Fig. 4. $W_{\perp}$ as a function of the total number of wires N for (a) $f_w = 0.1$, (b) $f_w = 0.3$, (c) $f_w = 0.6$, and (d) $f_w = 0.9$. Where $W_{\perp}$ is calculated using Eq. (46) for $kr_c = 0.1$.

4 Conclusions

For a charged particle beam propagating inside of an rf-shielding cage made of evenly-spaced conducting wires, the electrostatic fields due to sinusoidal longitudinal and transverse perturbations have been solved analytically for the case that both the cage and wires have circular cross sections. Only the dipole mode has been treated for the transverse perturbation. We have assumed that the beam has a uniform charge distribution and the unperturbed system is azimuthally symmetric. Using the calculated fields, we have derived simple formulae for the coupling impedances in the long wavelength region. Numerical examples were given to show the shielding effects of the rf-cage.

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