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Evaluation of Potential Crushed-Salt Constitutive Models

G. D. Callahan, M. C. Loken, L. L. Van Sambeek, R. Chen,
T. W. Pfeifle, J. D. Nieland, F. D. Hansen

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G.D. Callahan, M.C. Loken, L.L. Van Sambeek
R. Chen, T.W. Pfeifle, J.D. Nieland
RE/SPEC Inc.
Rapid City, SD 57709-0725

F.D. Hansen
Repository Isolation Systems Department
Sandia National Laboratories
Albuquerque, NM 87185-5800

ABSTRACT

Constitutive models describing the deformation of crushed salt are presented in this report. Ten constitutive models with potential to describe the phenomenological and micromechanical processes for crushed salt were selected from a literature search. Three of these ten constitutive models, termed Sjaardema-Krieg, Zeuch, and Spiers models, were adopted as candidate constitutive models. The candidate constitutive models were generalized in a consistent manner to three-dimensional states of stress and modified to include the effects of temperature, grain size, and moisture content. A database including hydrostatic consolidation and shear consolidation tests conducted on Waste Isolation Pilot Plant and southeastern New Mexico salt was used to determine material parameters for the candidate constitutive models. Nonlinear least-squares model fitting to data from the hydrostatic consolidation tests, the shear consolidation tests, and a combination of the shear and hydrostatic tests produces three sets of material parameter values for the candidate models. The change in material parameter values from test group to test group indicates the empirical nature of the models. To evaluate the predictive capability of the candidate models, each parameter value set was used to predict each of the tests in the database. Based on the fitting statistics and the ability of the models to predict the test data, the Spiers model appeared to perform slightly better than the other two candidate models. The work reported here is a first-of-its kind evaluation of constitutive models for reconsolidation of crushed salt. Questions remain to be answered. Deficiencies in models and databases are identified and recommendations for future work are made.

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1.0 INTRODUCTION

The Waste Isolation Pilot Plant (WIPP) is designed as the first full-scale, mined geological repository for the safe management, storage, and disposal of transuranic (TRU) radioactive wastes generated by the Department of Energy (DOE) defense programs. The WIPP underground facility, located in southeastern New Mexico at a depth of approximately 655 m in bedded halites, consists of a series of underground shafts, drifts, panels, and disposal rooms. After the facility meets the requirements of the Environmental Protection Agency (EPA), each disposal room will be filled with containers holding TRU wastes of various forms. Ultimately, a seal system will be emplaced to prevent water from entering the repository and to prevent gases and brines from migrating out of the repository.

The seal system will most likely consist of short-term and long-term components, be conservative by design, and contain redundancy in component function and number. Two general periods of seal performance include: (1) short-term period and (2) long-term period. The repository operation extends through waste and waste-panel seal emplacement up to the initiation of repository decommissioning (perhaps 35 to 50 years). Thereafter, the short-term period covers perhaps 100 to 200 years, when the site is still under institutional control. The long-term period extends from the end of the short-term period to 10,000 years.

Crushed salt has been proposed as the most viable material for the permanent sealing of WIPP shafts. Crushed salt's desirable characteristics include chemical compatibility and eventual mechanical similarity with the host salt formation and availability from the site excavation. Laboratory tests have shown that crushed salt achieves desirable permeability characteristics as consolidation increases the material density. In addition, voids and fractures in salt close and heal in salt in response to applied loads. It is expected that the natural process of creep closure of the shaft will lead during the short-term seal performance period to the compaction and reconsolidation of a crushed-salt seal component placed in the shaft. Thereafter, the single long-term seal component may be the reconsolidated salt. Thus, an understanding of the consolidation processes in crushed salt is fundamental to the design of a credible seal system that will provide confidence and be able to demonstrate regulatory compliance.

To gain an understanding of the crushed-salt consolidation processes, knowledge of the overall repository isolation system is required, along with the interactions of the key components. The key components include the host geology, brine, TRU waste, waste-generated gases, and seal components. Representative material, flow, and chemical models are needed that describe the behavior of these components in the isolation system to analyze, evaluate, and demonstrate compliance of conceptual designs. The objective of this report is to examine the mechanical material models appropriate for describing crushed-salt deformation and to provide background on the research and development status

of crushed salt, including model development that has been performed to date. The WIPP crushed-salt material model or constitutive model, which relates a material's stress and strain, controls the predicted deformations and stresses for the sealing system. The magnitude of the stresses and strains throughout the isolation system determine the serviceability of the seal components.

The crushed-salt constitutive model must be technically adequate to satisfy the needs of performance assessment and regulatory compliance. To help meet this objective, this report focuses strictly on the mechanical constitutive behavior of crushed salt. In particular, Chapter 2.0 includes the results of a literature survey, summarizes phenomenological and micromechanical information on consolidation processes, and recommends candidate constitutive models from those obtained from the literature. Chapter 3.0 provides the three-dimensional generalization of the recommended candidate constitutive models and reduces the general forms to specific types of laboratory tests. Chapter 4.0 summarizes the crushed-salt experimental database used to determine the material constants for the candidate constitutive models. Chapter 5.0 describes the nonlinear least-squares fitting procedure used to fit the candidate constitutive models to the experimental database. Chapter 6.0 summarizes the candidate constitutive models' ability to reproduce the laboratory test data. The report is concluded with a summary and recommendations in Chapter 7.0. Chapter 8.0 lists the cited references. Appendix A contains a listing of the input files to the BMDP nonlinear regression program AR used in the model fitting. Plots of the candidate models' prediction of the laboratory tests are included in Appendix B. Appendix C contains a listing of the unresolved references cited in the report.

2.0 LITERATURE REVIEW

2.1 Literature Survey Sources and Search Methods

The computerized literature search was the main method used to identify appropriate constitutive models for crushed salt. Other sources for relevant information on crushed-salt constitutive model research and correlated experimental data include the WIPP-related technical reports, journal papers, and conference articles prepared by Sandia National Laboratories (SNL), Albuquerque, New Mexico, and RE/SPEC, Rapid City, South Dakota. The first step in the literature search was to gather and review these articles and their references. Computerized information retrieval systems known as *DIALOG ONESEARCH*[®] and *KNOWLEDGE-INDEX*SM were used in the literature search. The subject databases searched include:

- *COMPENDEX*[®]
- *NTIS* (The National Technical Information Service).

The search criteria used to identify potential candidate constitutive models comprise key words. Table 2-1 shows the key words and search operators used.

A bibliography on crushed-salt deformation was compiled based on the literature search and is included in Appendix A. This bibliography includes technical reports, journal papers, and conference articles identified through the literature search using Table 2-1.

2.2 Potential Candidate Model Screening Criteria

The screening criteria for the crushed-salt constitutive model include the following: (1) the potential candidate models should be applicable over the ranges of in situ stress and temperature conditions encountered at WIPP; (2) the models should be expressible in a consistent form and should include both the elastic and creep consolidation components; (3) the models should consider most of the factors that affect crushed-salt densification phenomenology, such as fractional density or porosity, applied pressure (or mean stress), deviatoric stress, temperature, grain size, and moisture content; and (4) the models should be able to reproduce intact salt behavior when the density approaches intact density.

Usually, materials exhibit a predominant behavior over a particular range of conditions. Since simple constitutive models cannot represent every aspect of material behavior, most constitutive models are developed to capture the predominant material behavior for an expected range of conditions. Therefore, it is important to define the range of conditions at WIPP to determine which phenomena and deformation

Table 2-1. Summary of Key Words and Search Operators Used in the Literature Search

Results	Key Words and Search Operators
S1	Salt <i>or</i> Potash <i>or</i> Halite
S2	S1 <i>and</i> (Crushed <i>or</i> Granular <i>or</i> Granulated)
S3	S2 <i>and</i> (Deformation <i>or</i> Creep <i>or</i> Permeability)
S4	S2 <i>and</i> (Healing <i>or</i> Backfill <i>or</i> Tailing)
S5	S2 <i>and</i> (Constitutive <i>or</i> Laboratory <i>or</i> Test)
S6	S2 <i>and</i> (Pressure <i>or</i> Sintering <i>or</i> Hot Pressing <i>or</i> Voids)
S7	S2 <i>and</i> WIPP
S8	S3 <i>and</i> S4 <i>and</i> S5 <i>and</i> S6 <i>and</i> S7
S9	S8 <i>not</i> Soil

mechanisms (i.e., material behavior) the potential candidate constitutive models must capture. The consolidation of WIPP crushed salt is known to be dependent on mean stress, stress difference, temperature, particle size, and moisture content. These conditions are present at WIPP. Therefore, it is important that constitutive models for crushed salt include functional dependence on these conditions as well.

2.3 Range of Conditions At WIPP

The range of conditions for crushed-salt behavior at WIPP includes variations in impurity, moisture content, grain size, temperature, and stress. The natural rock salt at WIPP usually contains significant quantities of impurities [Wawersik and Zeuch, 1986] of up to 5 weight percent nonhalite minerals [Stein, 1985]. The crushed salt contains inter- and intracrystalline brine inclusions of 0.1 and 1.0 weight percent [Nowak and McTigue, 1987]. WIPP crushed salt is usually coarse-grained, with a maximum grain size ranging from 0.6 to 1.3 cm [Wawersik and Zeuch, 1986]. The temperature range of interest at WIPP is 20°C to 30°C. The lithostatic stress relevant to the WIPP underground repository is 6 to 15 MPa (6 MPa at the Salado/Rustler and 15 MPa at the repository horizon). Allowing for stress concentrations around the excavations, mean deviatoric stress levels should be between 0 and 45 MPa.

2.4 Crushed-Salt Deformation Phenomenology and Mechanisms

The ability of a constitutive model to represent a material's behavior depends on how well it can reproduce experimental observations (phenomenology) and how well it can model the underlying physical processes (mechanisms). Therefore, it is important to understand the phenomenology and micromechanisms of crushed-salt deformation to enable selection of candidate constitutive models.

The importance of crushed salt as a material in the repository setting has been recognized for over 15 years [Hansen, 1976; Gnirk et al., 1978; Ratigan and Wagner, 1978]. Since that time, numerous laboratory tests have been conducted to improve understanding of the phenomenology and deformation mechanisms of crushed salt.

2.4.1 Phenomenology

The mechanical behavior of crushed salt can be divided into three basic categories:

1. elastic deformation
2. inelastic deformation
3. failure.

The inelastic behavior can further be divided into time-independent (instantaneous compaction) and time-dependent (creep consolidation) deformation.

A number of parameters or characteristics are expected to affect the mechanical behavior of crushed salt. These parameters include (but are not limited to) four material variables and three independent variables typically controlled in a laboratory environment. The variables are:

1. density (or porosity)
2. grain size and grain size distribution
3. moisture content
4. impurity content (such as clay, anhydrite, etc.).

The independent (laboratory) parameters are:

1. temperature
2. stress state (i.e., confining pressure and stress difference)
3. time or imposed strain rates.

2.4.1.1 ELASTIC DEFORMATION

Unlike intact salt, the elastic behavior of crushed salt has been observed to be nonlinear by Sjaardema and Krieg [1987]. This observation is based on one test and strongly depends on parameters such as current density and confining pressure. Research on the characteristics of crushed-salt elastic behavior was found to be quite limited.

No information was found on the effect of other parameters such as grain size, moisture content, impurities, temperature, and time (rates) on crushed-salt elastic properties. Most research has been on the effect of these parameters on the inelastic deformation (more specifically, on creep consolidation) of crushed salt.

Density Effects

Holcomb and Hannum [1982] conducted an isostatic compression test on dry crushed salt from which a nonlinear dependence of bulk modulus on current density is established [Sjaardema and Krieg, 1987]. In this test, the isostatic pressure was increased to 21 MPa, interrupted by eight depressurization cycles. During these unload-reload cycles, the response was nearly elastic. The test reveals that the bulk modulus of crushed salt increases as the density increases. Using a curve-fitting procedure, Sjaardema and Krieg [1987] propose that the bulk modulus (K) and shear modulus (G) are exponential functions of the current density (see Figure 2-1):

$$\begin{aligned} K &= K_0 e^{K_1 \rho} \\ G &= G_0 e^{G_1 \rho} \end{aligned} \tag{2-1}$$

where K_0 , K_1 , G_0 , and G_1 are material constants. No direct experimentation was performed to confirm the functional relationship for the shear modulus, but its form was assumed to be the same as that for the bulk modulus. In Equation 2-1, ρ is the current density given by:

$$\rho = \frac{\rho_0}{1 + \varepsilon_v} \tag{2-2}$$

where ρ_0 is the initial or original density of the material, and ε_v is the volumetric strain corresponding to an increase in density from ρ_0 to ρ .

Figure 2-1. Bulk Modulus (K) as a Function of Density for Dry Crushed Salt (After Sjaardema and Krieg [1987]).

Pressure Dependence

Instead of expressing the bulk modulus as a function of crushed-salt density, Holcomb and Zeuch [1988] attribute the variation of bulk modulus during densification to pressure changes (i.e., pressure dependence). They propose that the bulk modulus (K) increases linearly with increasing pressure (P) (see Figure 2-2) and can be approximated as:

$$K = a + bP \quad (2-3)$$

where a and b are fitting parameters. From the same single test used by Sjaardema and Krieg [1987] for the density-dependence of the bulk modulus, Holcomb and Zeuch [1988] obtained $a = 456$ MPa and $b = 107$.

Liedtke et al. [1986] also suggest a linear dependence of bulk modulus on applied axial stress, as shown in Figure 2-3. The mean stress is unknown in the tests done by Liedtke et al. [1986] because only the axial stress (σ_a) was measured.

2.4.1.2 INELASTIC DEFORMATION

Inelastic deformation can be divided into time-independent deformation (instantaneous compaction) and time-dependent deformation (creep consolidation). Time-independent inelastic properties are usually studied experimentally by quasi-static compaction tests. The literature is sparse concerning work done specifically to describe the characteristics of time-independent densification by means of laboratory experiments. However, some information is available regarding deformation mechanisms, which is discussed in Section 2.4.2.

Time-dependent creep consolidation of crushed salt has been studied extensively by means of isostatic creep consolidation tests. Sandia researchers suggest that the volumetric strain (ϵ_v) can be written as a function of time by the following empirical equation [Holcomb and Hannum, 1982; Zeuch and Holcomb, 1991; Holcomb and Zeuch, 1988; Holcomb and Shields, 1987; etc.]:

$$\epsilon_v = a \log_{10}(t) + b \quad (2-4)$$

where a and b are fitting constants and t is time. Clearly, this equation is not valid for $t = 0$, and the fit is usually poor for the first few hundred seconds.

Others suggest that the volumetric creep strain rate as a function of time can be approximated by a straight line in the $\ln(\frac{\dot{\epsilon}_v}{\sigma_m})$ versus $\ln(t)$ plane, as shown in Figure 2-4 [Liedtke et al., 1986]:

$$\frac{\dot{\epsilon}_v}{\sigma_m} = at^{-b} \quad (2-5)$$

Figure 2-2. Elastic Bulk Modulus (K) of Compacting Salt as a Function of Applied Pressure (After Holcomb and Zeuch [1988]).

Figure 2-3. Compressive Secant Modulus of Crushed Salt as a Function of Applied Axial Stress (After Liedtke et al. [1986]).

Figure 2-4. Normalized Volumetric Creep Strain as a Function of Time (After Liedtke et al. [1986]).

where a and b are fitting constants and t is time.

The most commonly studied variables that affect the creep densification of crushed salt include moisture content, impurity content (such as bentonite content), and grain size [Brodsky and Pfeifle, 1991, 1992; Pfeifle and Brodsky, 1991; Pfeifle et al., 1987; Pfeifle and Senseny, 1985; Holcomb and Shields, 1987; Spiers et al., 1989, 1986, 1984; Kappei, 1986; Yost and Aronson, 1987; IT Corporation, 1987; Beddoes, 1984; Peach, 1991; Johnson et al., 1984; Pufahl and Yoshida, 1982]. Temperature effects have been studied by Spiers et al. [1989], Pfeifle et al. [1987], Holcomb and Hannum [1982], and Shor et al. [1981]. Only recently has the shear or deviatoric aspect of crushed-salt consolidation been investigated (e.g., Zeuch et al. [1991]; Brodsky [1994]). The effects of stress, time, and porosity have, to some degree, been explored in all of the cited studies.

Moisture Content

During the review of the effect of moisture content on the consolidation of crushed salt, water is usually assumed to be fully saturated brine. The research on the moisture content effect conducted to date does not yield consistent results and more investigation is necessary.

Holcomb and Shields [1987] showed that the addition of a small amount of water, less than approximately 2.5 percent by weight, to the crushed-salt material significantly increases the consolidation rate relative to dry crushed-salt material. Holcomb and Shields [1987] and Pfeifle and Senseny [1985] performed several tests on the wet crushed-salt material and found that adding a small amount of water (approximately 0.5 to 3.0 weight percent) accelerated the consolidation rates by approximately two orders of magnitude. Although some researchers believe there is a minimum moisture content necessary to cause increased consolidation, Holcomb and Shields [1987] concluded that there is no moisture content threshold level.

Zeuch and Holcomb [1991] and Zeuch et al. [1991] conducted a series of isostatic and triaxial compression experiments on the time-dependent compaction behavior of crushed salt under nominally dry, "damp" (0.5–3.0 weight percent added water), and brine-saturated conditions. They found: (1) damp salt can compact rapidly; (2) though effects associated with saturation apparently have a retarding effect on consolidation, rates are reduced by less than an order of magnitude when compared with unsaturated specimens; specifically, at comparable pressures, unsaturated specimens compact 2.5 to 6.0 times faster than brine-saturated samples; (3) despite saturation, high fractional densities (≥ 0.95) are attainable, even on laboratory time scales using pressures well below lithostatic at WIPP (≈ 15 MPa). Zeuch et al. [1991] indicate that the retarding effect is presumably caused by the entrapment of brine within the specimens as the connected porosity "pinches off" and pores become isolated; this results in a reduction in drainage of the specimens and an increase in pore pressure which resists further compaction.

Spiers et al. [1989] observed that the volumetric strain rate is almost independent of brine content for values ≥ 5.0 weight percent. Fordham [1988] compared the porosity versus time curves for compaction tests performed on material at 0 percent, 2.5 percent, and 5.0 percent water contents and found that the biggest difference in behavior is between 0 percent and 2.5 percent; increasing the water content from 2.5 percent to 5.0 percent has little effect (see Figure 2-5).

Yost and Aronson [1987] proposed an exponential form for the effect of water content (w) and densification rate ($\dot{D}$) based on experimental data of Holcomb (July 1986 memorandum to L.D. Tyler, "Geomechanics Support for WIPP—Monthly Letter, July 1986") on WIPP crushed salt with added water:

$$\dot{D} \propto A(w) = a_1 \exp(a_2 w) \quad (2-6)$$

where $A(\cdot)$ is a function of water content. Figure 2-6 shows the dependence of A on water content. They concluded that water greatly enhances the compaction rate of wetted crushed salt.

A rather controversial observation on the effect of water content is the nonmonotonic effect of water content on consolidation rate observed by Wang et al. [1992]. They performed one-dimensional consolidation tests for WIPP crushed-salt samples with water content from 0.12 to 4.72 percent by weight and grain size from 0.075 to 10.0 mm under low pressure (3 MPa) and high pressure (69 MPa). They observed that an optimum value of water content exists for certain pressure and temperature combinations that gives the fastest consolidation rate (see Figure 2-7). The optimum value of water content is pressure dependent (around 2.2 percent for a pressure of 3 MPa and around 3.0 percent for a pressure of 69 MPa).

No consistent conclusions among the above researchers were reached on the correlation between the consolidation (or volumetric strain) rate and moisture content for crushed salt that is wetted. Some believe that the presence of moisture makes a big difference on consolidation rate, but there is no strong correlation between the consolidation rate and the moisture content (such as Spiers et al. [1989] and Fordham [1988]). Others believe that consolidation rates are strongly dependent on water content (such as Yost and Aronson [1987] and Wang et al. [1992]).

Particle Size

Shor et al. [1981] performed a number of consolidation tests under uniaxial strain conditions. Based on these experimental results, they developed a model which included a strong dependence of consolidation rate on particle size. The consolidation rate is proportional to d^{-3} , where d is the average grain diameter. By extrapolation, crushed salt with particle sizes of 1 cm would require about 30,000 years to consolidate to the point where permeability is in the microdarcy range. If the salt particle size were reduced to 1 mm, then the required consolidation time would reduce to about 30 years.

Figure 2-5. Effect of Moisture Content on Compaction Creep Under 10 MPa (After Fordham [1988]).

Figure 2-6. The Rate Coefficient A Versus Water Content (w) (After Yost and Aronson [1987]).

Figure 2-7. The Relationship Between the Final Fractional Density and the Water Content (After Wang et al. [1992]).

A similar strong dependence on particle size has been considered in the constitutive model of Spiers and coworkers [Spiers and Brzesowsky, 1993] for grain boundary diffusional (pressure solution-controlled) densification. They found that at low stresses (<4 MPa), the volumetric strain rate is essentially proportional to d^{-3} ; whereas, at higher stresses, it becomes less grain-size sensitive, being roughly proportional to d^{-2} at 8 MPa (see Figure 2-8), where d is the average grain diameter (ranging from about 0.1 to 0.4 mm).

Whether or not there is grain-size dependence is usually believed to depend on the presence of moisture and the densification mechanisms. Wet crushed salt consolidates through both solid- and fluid-assisted mechanisms which can act simultaneously. However, wet salt is predominantly consolidated by mechanisms such as grain boundary pressure solution, which are grain-size dependent. Dry crushed salt densifies only through solid processes, such as dislocation, which are believed to be less grain-size dependent.

Information on the effect of grain-size distribution is limited. However, grain-size distribution usually has not been considered to be important in constitutive model development.

Effects of Stress State

Shear stresses are believed to enhance the compaction of granular or porous media. However, shear consolidation tests by Brodsky [1994] show that for the small stress differences used in the study, there is no systematic correlation between the magnitude of the applied shear stress and the consolidation rate. This is consistent with results obtained by Zeuch et al. [1991] and Zeuch and Holcomb [1991]. Shear stresses are included in the models evaluated in this study.

Temperature Effects

Holcomb and Zeuch [1988] found that both the time-independent compaction and time-dependent consolidation are affected slightly by temperature in that compaction increases with increasing temperature.

Spiers et al. [1989] have shown that in the temperature range of 20–90°C, volumetric strain rate depends on temperature according to the relation $\dot{\epsilon}_v \propto A \exp\left(-\frac{\Delta H}{RT}\right)$, where $\Delta H = 24.53 \text{ kJ mol}^{-1}$, R is the universal gas constant, and T is absolute temperature. Similar temperature dependence has also been applied by other researchers in crushed-salt constitutive model development; for example, Zeuch [1989; 1990], Butcher [1980], and Zhang et al. [1993].

Figure 2-8. Typical Data for Wet Salt Illustrating the Dependence of Compaction Rate ($\dot{\epsilon}_v$) on Grain Size (d) Observed at Low and High Stresses; ($\sigma_e = \sigma_m - p$, Where σ_m is Applied Hydrostatic Stress, p is Pore Pressure, and ϵ_v is Volumetric Strain) (After Spiers and Brzesowsky [1993]).

Initial Density

Greater initial densities are generally believed to lower the initial densification rate. Also, the current density can be used as the “initial density” for subsequent densification analysis; i.e., there is no density path dependence.

Impurities

The effect of impurities appears to vary, depending on the type of impurities. None of the constitutive models described in the literature explicitly include the effect of crushed-salt impurities. However, different sets of constitutive model parameters could be used for different crushed salts or crushed-salt mixtures to account for impurities.

2.4.2 Densification Mechanisms

Granular aggregate densification mechanisms have been studied extensively in material sciences, mainly in connection with the hot pressing of crystalline powders to form dense ceramics and metallic alloys (e.g., Helle et al. [1985]). From such work, it is well established that when subjected to compactional loading, most granular materials undergo an initial stage of densification dominated by the instantaneous processes of grain rearrangement and microfracture until the aggregate assemblage “locks up.” Further densification can occur by a variety of mechanisms depending on temperature and pressure conditions and whether the material is dry or wet.

2.4.2.1 DRY AGGREGATES

Following initial locking by the instantaneous (time-independent) processes, dry aggregates densify in time-dependent modes by deformation of the individual grains (or particles), coupled with intergranular sliding. Deformation of the grains can occur by the same mechanisms seen in dense polycrystalline material; i.e., glide-controlled plasticity, recovery-controlled creep, or solid-state diffusional creep processes at high homologous temperatures [Swinkels et al., 1983]. Since intergranular sliding and rearrangement cannot proceed without deformation of the grains themselves, the solid-state deformation mechanisms generally control densification rate [Helle et al., 1985].

In the case of dry, granular salt aggregates, Zeuch [1990; 1989] and Holcomb and Zeuch [1988; 1990] suggest that under most temperature and pressure conditions of engineering interest, time-independent densification occurs by cataclasis, particle rearrangement, and plastic yield; whereas, time-dependent densification occurs predominantly by creep deformation.

2.4.2.2 WET AGGREGATES

As summarized by Spiers and Brzesowsky [1993], wet salt aggregates can, in principle, densify by the same mechanisms present in dry materials. However, the presence of the liquid phase can lead to additional deformation processes such as: (1) enhanced grain-scale cataclasis because of stress corrosion cracking [Atkinson, 1989]; (2) enhanced grain boundary mobility through interfacial solution transfer effects, which promotes recrystallization and grain growth [Drury and Urai, 1990]; and (3) densification creep by a variety of stress-induced dissolution/precipitation or pressure solution processes [Rutter, 1983; German, 1985; Lehner, 1990]. In general, two types of pressure-solution densification mechanisms are possible: grain boundary diffusional pressure solution [Stocker and Ashby, 1973; Rutter, 1983; Spiers and Schutjens, 1990] and plasticity-coupled pressure solution [Bosworth, 1981; Pharr and Ashby, 1983; Rutter, 1983; Tada and Siever, 1986]. Each of these two pressure solution densification mechanisms are reviewed in detail by Spiers and Brzesowsky [1993].

Grain Boundary Diffusional Pressure Solution

Grain boundary diffusional pressure solution is possible in wet aggregates in which grain contacts contain small quantities of the solution phase either as an absorbed thin film or as a fine-scale, island-channel network. When stressed, the solid surfaces within grain contacts develop an excess chemical potential with respect to the free pore walls. The average excess potential at any point within grain contact surfaces can be approximately written (in mole specific terms) as [Spiers and Brzesowsky, 1993]:

$$\Delta\mu_{gb} = (\sigma_n - p_l) V_m \quad (2-7)$$

where σ_n is the local average normal stress transmitted across the contact, p_l is the pressure of the liquid phase in the free pore spaces, and V_m is the molar volume of the solid. This excess potential provides a driving force for dissolution of material within grain contacts and reprecipitation in the pores, thus leading to densification creep.

Plasticity-Coupled Pressure Solution

In wet aggregates in which the solid can deform by dislocation mechanisms, densification can, in principle, proceed by plastic deformation of grains, coupled with dissolution of material at the contact margins, diffusion through the adjacent pore space, and precipitation on the free pore walls. The solution transfer is driven by gradients in chemical potential associated with strain gradients developed between grain contacts and pore walls. The excess potential of the solid at grain contact margins can be written as [Spiers and Brzesowsky, 1993]:

$$\Delta\mu_c = \Delta f + p_l \cdot \Delta V_m + 2\gamma \cdot \Delta(V_m/r) \quad (2-8)$$

where f is the molar Helmholtz free energy of the solid, γ is the solid/liquid interfacial energy, r is the mean radius of interfacial curvature, and the Δ s denote differences between contact margin and pore wall sites. Under most conditions, Δf will be composed principally of differences in defect- and elastically-stored energy and can easily be shown to far outweigh the other terms so that: $\Delta\mu_c \approx \Delta f$. The rate of densification will be governed by the kinetics of solution transfer and by the constitutive behavior of the solid.

2.4.3 Summary

The preceding paragraphs provide only a brief summary of crushed-salt densification mechanisms. Many other arguments exist as to which mechanisms control crushed-salt densification, which might be important as well. Deformational processes proposed by various researchers to date include most known mechanisms. Further study of the specific crushed salt material used for shaft seals may define which of the proposed processes are most important to reconsolidation for seals applications. Complete definition of the microprocesses giving rise to reconsolidation is beyond the scope of the current work. The conclusions reached so far are widespread. As discussed previously, Zeuch and coworkers [Zeuch, 1989; 1990] emphasize dislocation mechanisms. Their work was performed on dry salt. However, to be fully relevant to the WIPP, they recognized that fluid phase-assisted diffusional transport compaction mechanisms need to be included as well. Spiers and coworkers [Spiers and Brzesowsky, 1993] believe that diffusion is an important mechanism, based on work performed on wet salt. Yost and Aronson [1987] claim that the dislocation mechanisms may play a minimum role based on the value of stress exponent ($n = 1.8$) from experimental data of Holcomb (July 1986 memorandum to L.D. Tyler, "Geomechanics Support for WIPP—Monthly Letter, July 1986"). They believe that the dislocation mechanisms require a much larger value of stress exponent ($n \approx 5.0$). They also argue that the strong dependence on water content suggests that pressure solution is not the controlling mechanisms either, because pressure solution requires only 0.2 weight percent or less water absorbed on salt grain surfaces. Additional water has little effect. They claim that the Joffé effect [Yost and Aronson, 1987], based on descriptions given in the literature, would seem to have a stronger dependence on water content and likely be the controlling mechanism for crushed-salt densification. Wang et al. [1992] tested specimens of wet crushed WIPP salt and observed recrystallization which joins the particles together and increases the grain size.

2.5 Potential Candidate Constitutive Model

Most constitutive models for crushed salt have been developed to reproduce isostatic consolidation laboratory tests and include only the volumetric strain component. According to the approaches used in model development, crushed-salt constitutive models can be divided broadly into three groups: empirical, micromechanism-based, and viscoplastic.

Ten crushed-salt constitutive models identified from the literature search are discussed in this section. These models are:

1. Sjaardema and Krieg [1987]
2. Modified Sjaardema and Krieg [Callahan and DeVries, 1990; Labreche et al., 1993; Callahan, 1993]
3. Zeuch and Holcomb [Zeuch, 1990, 1989; Holcomb and Zeuch, 1990, 1988]
4. Spiers and coworkers [Spiers and Brzesowsky, 1993; Spiers and Schutjens, 1990; Spiers et al., 1989]
5. Yost and Aronson [1987]
6. Liedtke et al. [1986]
7. Butcher [1980]
8. Korthaus [1988]
9. Zhang et al. [1993]
10. Fordham [1988]

The models of Sjaardema–Krieg, Yost and Aronson, Korthaus, Zhang et al., and Fordham are empirical. The models of Zeuch and Holcomb, Spiers and coworkers, and Butcher are micromechanism-based. The model of Liedtke et al. is based on a viscoplastic approach. Except for the Zeuch and Holcomb model, which was derived from dry crushed-salt data, all models were derived from wetted crushed-salt data. The modified Sjaardema–Krieg model and the Sjaardema–Krieg model account for the effect of both deviatoric and volumetric consolidation; all other models include only volumetric creep consolidation.

In this section, mixed sign conventions are used. For most of the models, sign conventions are kept the same as in the original papers. Specifically, tension is positive in the Sjaardema–Krieg model and the modified Sjaardema–Krieg model, and compression is positive in other models.

Qualitative screening criteria were applied to the above ten models to determine which were suitable (or could be made suitable) in describing the consolidation of WIPP crushed salt. The screening criteria were:

- The models should be applicable over the range in field conditions encountered at the WIPP and over the range in test conditions of laboratory experiments.
- The models should be expressible as a rate equation in which the densification rate is a function of the field conditions.

- The field conditions should include (but not necessarily be limited to) fractional density, mean stress, stress difference, temperature, moisture content, and grain size.
- The behavior should approach that of intact salt as the fractional density approaches one.

Among these models, three were tentatively selected as candidate models for further examination. These are **Sjaardema–Krieg**, **Zeuch and Holcomb**, and **Spiers and coworkers**. The functional forms of these three models and model parameters that can be determined and/or adjusted by further curve-fitting exercises are suggested after model review. Motivations for their selection and reasons for the exclusion of other models are given in Section 2.6.

2.5.1 Sjaardema–Krieg Model

The Sjaardema and Krieg [1987] consolidation model is a purely empirical model. All of the parameters in the model are determined through curve fitting to laboratory test data. The model consists of a nonlinear elastic part and a creep consolidation part. The nonlinear elastic model equations and parameters are based on a single test of Holcomb and Hannum [1982]. The creep consolidation portion of the model includes volumetric strain and options for deviatoric strain are recommended based on engineering judgment. The volumetric creep consolidation equation has been fitted to the isostatic consolidation test data on WIPP salt with added water (0.5 percent to 3.0 percent) performed by Holcomb and Shields [1987]. This model has also been fitted to other laboratory testing data (e.g., Pfeifle et al. [1987], Pfeifle [1990], Brodsky and Pfeifle [1991, 1992], and Van Sambeek et al. [1992]). Although the model has no deformation-mechanism basis, it describes experimental data reasonably well, and therefore, has been used in RE/SPEC's finite element program **SPECTROM-32** [Callahan, 1994] and Sandia National Laboratories' finite element program **SANCHO** [Stone et al., 1985] to simulate consolidation of crushed salt emplaced in shafts and drifts at WIPP.

2.5.1.1 NONLINEAR ELASTIC MODEL EQUATIONS

The bulk modulus (K) and shear modulus (G) are exponential functions of the current density (ρ) as given by Equation 2-1. See Figure 2-1 for the experimental data and the fitted curve for the bulk modulus (K). The four model parameters determined by Sjaardema and Krieg [1987] are given in Table 2-2. These elastic parameters were determined from a single test on dry crushed salt by Holcomb and Hannum [1982].

2.5.1.2 CREEP CONSOLIDATION MODEL EQUATIONS

As discussed in Section 2.4.1.2, Equation 2-4 has been used to describe crushed-salt consolidation experimental data. Sjaardema and Krieg [1987] suggest a different form of Equation 2-4 to eliminate the explicit references to time and give the densification rate as a function of current density:

Table 2-2. Model Parameter Values (After Sjaardema and Krieg [1987])

Parameter	Units	Value
Elastic Model Parameters		
K_0	MPa	0.01760
K_I	m ³ / kg	0.00653
G_0	MPa	0.01060
G_I	m ³ / kg	0.00653
Creep Consolidation Model Parameters		
B_0	kg/m ³ • s ⁻¹	1.3×10^8
	kg/m ³ • yr ⁻¹	4.1×10^{15}
B_I	MPa ⁻¹	0.82
A	m ³ / kg	-1.72×10^{-2}

$$\dot{\rho} = B(P) e^{A\rho} \quad (2-9)$$

where:

$\dot{\rho}$ = densification rate

ρ = current density

P = applied pressure (or mean stress, σ_m)

$B(P)$ = pressure function

A = model parameter.

Sjaardema and Krieg [1987] further assume that the pressure function takes the following form:

$$B(P) = B_0 \left[e^{-B_1 P} - 1 \right] \quad (2-10)$$

where B_0 and B_1 , are fitting parameters. Then, Equation 2-9 can be written as:

$$\dot{\rho} = B_0 \left[e^{-B_1 P} - 1 \right] e^{A\rho} \quad (2-11)$$

2.5.1.3 FUNCTIONAL FORMS AND MODEL PARAMETERS

Using the relationship between volumetric strain and fractional density given by Equation 2-2, the functional form of volumetric creep consolidation in terms of volumetric strain rate ($\dot{\epsilon}_v$) and mean stress (σ_m) can be written as:

$$\dot{\epsilon}_v^c = \frac{(1 + \epsilon_v)^2}{\rho_0} B_0 \left[1 - e^{-B_1 \sigma_m} \right] e^{\frac{A \rho_0}{1 + \epsilon_v}} \quad (2-12)$$

where:

$\epsilon_v = \epsilon_{kk}$, volumetric strain

$\sigma_m = \frac{\sigma_{kk}}{3}$, mean stress

ρ_0 = initial density (at $\epsilon_v = 0$)

B_0, B_1, A = model parameters.

There are three creep consolidation parameters for this model (Table 2-2). In determining the creep consolidation model parameters, the parameter for the pressure function, B_1 , was determined first and the density-dependent parameters, B_0 and A , were determined with B_1 fixed. In Table 2-2, B_1 was determined based on only one test performed at two different pressure levels (Test 23JL51, see Section 4.2).

Figures 2-9 and 2-10 compare the densities predicted by Equation 2-11 to those obtained for consolidation tests at confining pressures of 1.72 MPa and 3.44 MPa, respectively [Sjaardema and Krieg, 1987]. Equation 2-12 shows that the model accounts only for the effect of applied pressure and initial crushed-salt density. Other parameters (such as temperature, water content, and particle size) are not explicitly considered. Since the density exponent A is negative, the model predicts lower initial densification rates for higher initial densities. However, the densification equation allows unlimited consolidation; therefore, a cap must be introduced to eliminate further consolidation when the intact density is attained. The performance of this constitutive model has been studied in detail by Callahan [1990; 1993]. Some of the deficiencies in this model have been overcome by the modified Sjaardema-Krieg model given in Section 2.5.2.

Figure 2-9. Comparison of Measured and Predicted Volumetric Creep for Confining Pressure of 1.72 MPa (After Sjaardema and Krieg [1987]).

Figure 2-10. Comparison of Measured and Predicted Volumetric Creep for Confining Pressure of 3.44 MPa (After Sjaardema and Krieg [1987]).

2.5.1.4 SHEAR CONSOLIDATION RESPONSE MODELS

Since no shear consolidation tests had been performed on crushed salt at that time, Sjaardema-Krieg's model development for shear consolidation is based totally on judgment. Sjaardema and Krieg [1987] suggested an elastic, an elastic-plastic, and an elastic-creep model, in order of increasing complexity.

Elastic Model

The elastic model is given as:

$$s_{ij}^T = s_{ij}^0 + 2 G \dot{\epsilon}_{ij} \Delta t \quad (2-13)$$

where:

s_{ij}^0 = initial deviatoric stress

s_{ij}^T = final deviatoric stress

G = elastic shear modulus (Equation 2-1b)

Δt = length of the time step

$\dot{\epsilon}_{ij}$ = deviatoric strain rate.

Elastic-Plastic Model

The elastic-plastic model is based on a standard von Mises type of yield condition with no hardening. Initially, a trial stress s_{ij}^T is calculated assuming no plasticity during the time step from Equation 2-13. The magnitude (s^T) of the trial deviatoric stress is then calculated as:

$$s^T = \sqrt{s_{ij}^T s_{ij}^T} \quad (2-14)$$

and then compared to the yield stress (σ_y) to determine if yield occurs. The variable κ is used to define the von Mises yield surface:

$$\kappa = \sqrt{\frac{2}{3} \frac{\sigma_y}{s^T}} \quad (2-15)$$

The final deviatoric stress state $\bar{s}_{ij}^F$ is then calculated based on κ such that:

$$\bar{s}_{ij}^F = \begin{cases} \bar{s}_{ij}^T & \text{if } \kappa \geq 1 \quad \text{no plasticity correction} \\ \kappa \bar{s}_{ij}^T & \text{if } \kappa < 1 \quad \text{plasticity correction by } \kappa \end{cases} \quad (2-16)$$

Elastic-Creep Model

The elastic-creep model is based on a secondary creep model which has been used to describe the creep behavior of intact WIPP salt [Krieg, 1984]. The porous crushed-salt uniaxial sample is assumed to be composed of cylinders of salt, each of which has the intact salt secondary creep behavior separated by areas of open space. The local stress acting on the salt cylinders is then stated in terms of the average stress on the porous sample. The cross-sectional area of the porous sample is expressed in terms of the net cross-sectional area of the salt cylinder; i.e., the areal ratio is the inverse of the fractional density of the sample. The elastic-creep model can be expressed as:

$$\dot{s}_{ij} = 2G \left(\dot{\epsilon}_{ij} - A \left(\rho_f / \rho \right)^n e^{(Q/RT)} |s_{pq}|^{n-1} s_{ij} \right) \quad (2-17)$$

where:

ρ_f = final density of intact salt

ρ = current density

T = temperature

A , R , Q , and n = the parameters of the secondary creep law for intact salt.
(Note: A is different than the A in Equation 2-9).

2.5.2 Modified Sjaardema-Krieg

Callahan and DeVries [1990], Labreche et al. [1993], and Callahan [1993] modified the Sjaardema-Krieg model by incorporating shear consolidation terms and deriving a three-dimensional generalization for creep strain rates following the approach suggested by Fossum et al. [1988]. The modified Sjaardema-Krieg model has been implemented into SPECTROM-32 [Callahan, 1994] and used to model crushed-salt behavior at WIPP [Chen and Nieland, 1994a; 1994b]. This is the only three-dimensional, crushed-salt constitutive model found in the literature. As shown by Callahan [1990] through simple uniaxial and plane strain numerical experiments, the volumetric consolidation model alone does not capture the expected behavior of crushed salt. Also, if the deviatoric portion is not included,

transition from predominately volumetric behavior at low fractional densities to predominately deviatoric behavior at high fractional densities cannot be achieved. The approach used to modify the Sjaardema–Krieg model can be used to develop the three-dimensional generalization for other candidate constitutive models.

As in the original model, the total strain (ϵ_{ij}) consists of nonlinear elastic (ϵ_{ij}^e) and creep consolidation (ϵ_{ij}^c) contributions, and the total strain rate can be written as:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^c \quad (2-18)$$

2.5.2.1 ELASTIC MODEL EQUATIONS

The infinitesimal elastic strain, (ϵ_{ij}^e), is the contribution from the stress field given by Hooke's law, which, in terms of the bulk modulus (K) and shear modulus (G), is written as:

$$\epsilon_{ij}^e = \frac{\sigma_m}{3K} \delta_{ij} + \frac{s_{ij}}{2G} \quad (2-19)$$

where:

$$\sigma_m = \frac{\sigma_{kk}}{3}, \text{ mean stress}$$

$$s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}, \text{ deviatoric stress}$$

$$\delta_{ij} = \text{Kronecker delta.}$$

The bulk and shear moduli are given by Sjaardema and Krieg [1987] (see Equation 2-1).

2.5.2.2 CREEP CONSOLIDATION EQUATIONS

From the application of thermodynamic concepts, the three-dimensional generalization for creep strain rates is given by Fossum et al. [1988]. Following this approach, three continuum internal variables are assumed, the inelastic volumetric strain, ϵ_{eq1}^c , and two equivalent inelastic shear strains, ϵ_{eq2}^c and ϵ_{eq3}^c . The creep consolidation strain rate is then written as:

$$\dot{\epsilon}_{ij}^c = \dot{\epsilon}_{eq1}^c \frac{\partial \sigma_{eq1}^f}{\partial \sigma_{ij}} + \dot{\epsilon}_{eq2}^c \frac{\partial \sigma_{eq2}^f}{\partial \sigma_{ij}} + \dot{\epsilon}_{eq3}^c \frac{\partial \sigma_{eq3}^f}{\partial \sigma_{ij}} \quad (2-20)$$

The first term in this equation is the contribution from volumetric consolidation and the two remaining terms are the contributions from shear consolidation.

Volumetric Consolidation Equations

The volumetric strain rate $\dot{\epsilon}_{eq_1}^c$ is taken directly from Sjaardema and Krieg [1987] (see Equation 2-12):

$$\dot{\epsilon}_{eq_1}^c = \dot{\epsilon}_v^c(\sigma_m) = \frac{(1 + \epsilon_v)^2}{\rho_0} B_0 \left[1 - e^{-B_1 \sigma_m} \right] e^{\frac{A \rho_0}{1 + \epsilon_v}} \quad (2-21)$$

where:

$\epsilon_v = \epsilon_{kk}$, total volumetric strain

$\epsilon_v^c = \epsilon_{kk}^c$, volumetric creep strain

$\sigma_m = \frac{\sigma_{kk}}{3}$, mean stress

ρ_0 = initial density (at $\epsilon_v = 0$)

B_0, B_1, A = material constants.

The invariant stress measure is given by:

$$\sigma_{eq_1}^f = \sigma_m \quad (2-22)$$

Shear Consolidation Equations

The first equivalent inelastic shear strain is taken to be:

$$\dot{\epsilon}_{eq_2}^c = \beta \dot{\epsilon}_{eq_1}^c \quad (2-23)$$

and the invariant stress is assumed to be a scalar multiple of the octahedral shear stress:

$$\sigma_{eq_2}^f = \sigma_e = \sqrt{3J_2} \quad (2-24)$$

where J_2 is the second invariant of the stress deviator. The parameter β is selected such that in a uniaxial compression test, the lateral components of $\dot{\epsilon}_{ij}^c$ equal zero (ignoring the third term in Equation 2-20). This requires that $\beta = -\frac{2}{3}$. (Note: this is true only in uniaxial compression, while in uniaxial tension, β would have to be $+\frac{2}{3}$ to obtain zero lateral strain. However, creep consolidation is not permitted to generate tensile stress.)

The third component in Equation 2-20 ($\dot{\epsilon}_{eq_3}^c$) may be selected as either the Munson-Dawson model [Munson et al., 1989] or the WIPP secondary creep model [Krieg, 1984] for intact salt, depending on which model is used to represent the creep behavior of the intact salt. In both models, the effective stress is:

$$\sigma_{eq_3}^f = \frac{\sigma_e}{\frac{\rho}{\rho_f}} \quad (2-25)$$

where:

σ_e = average effective stress

ρ = density

ρ_f = fully consolidated density.

As the material approaches full consolidation, the fractional density approaches one and the Munson-Dawson or WIPP secondary creep deviatoric component becomes the same as that for intact salt. Simultaneously, the creep consolidation portion of the model (Equation 2-21) can be capped so that it diminishes as the material approaches full consolidation. Thus, a smooth transition from crushed salt to intact salt behavior can be accomplished.

The form of the crushed-salt consolidation flow potential stress obtained for the Munson-Dawson deviatoric component is the Tresca flow potential:

$$\frac{\partial \sigma_{eq_3}^f}{\partial \sigma_{ij}} = \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \quad (2-26)$$

where:

$s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$, deviatoric stress

δ_{ij} = Kronecker delta

$t_{ij} = s_{ik} s_{kj} - \frac{2}{3} J_2 \delta_{ij}$

$J_2 = \frac{1}{2} s_{ij} s_{ji}$

$\psi = \frac{1}{3} \sin^{-1} \left[\frac{-3\sqrt{3} J_3}{2J_2^{3/2}} \right] \quad \left[-\frac{\pi}{6} \leq \psi \leq \frac{\pi}{6} \right]$

$J_3 = \frac{1}{3} s_{ij} s_{ik} s_{ki}$

Three-Dimensional Inelastic Consolidation Equations

Substituting Equations 2-21 through 2-25 and Equation 2-26 into Equation 2-20 results in the three-dimensional creep consolidation constitutive equations for crushed salt. For the Tresca flow potential, the equation is:

$$\begin{aligned} \dot{\varepsilon}_{ij}^c = & \frac{(1 + \varepsilon_v)^2}{\rho_0} B_0 [1 - e^{-B_1 \sigma_n}] e^{\frac{A \rho_0}{1 + \varepsilon_v}} \left\{ \frac{\delta_{ij}}{3} - \frac{s_{ij}}{\sigma_e} \right\} + \\ & \dot{\varepsilon}_{eq}^c \left\{ \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right\} \end{aligned} \quad (2-27)$$

where $\dot{\varepsilon}_{eq}^c$ is the equivalent strain rate measure defined, for example, by the Munson-Dawson constitutive model for intact salt [Munson et al., 1989].

2.5.2.3 COMBINED CRUSHED-SALT CONSTITUTIVE EQUATIONS

The final two equations for the total strain rate, which combines the volumetric and deviatoric contributions of the elastic and creep responses, are obtained by substituting Equation 2-19 and 2-27 into Equation 2-18. For the Tresca flow potential, the equation is:

$$\begin{aligned} \dot{\varepsilon}_{ij} = & \frac{\dot{\sigma}_m}{3K} \delta_{ij} + \frac{\dot{s}_{ij}}{2G} + \frac{(1 + \varepsilon_v)^2}{\rho_0} B_0 [1 - e^{-B_1 \sigma_n}] e^{\frac{A \rho_0}{1 + \varepsilon_v}} \left\{ \frac{\delta_{ij}}{3} - \frac{s_{ij}}{\sigma_e} \right\} \\ & + \dot{\varepsilon}_{eq}^c \left\{ \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right\} \end{aligned} \quad (2-28)$$

where $\dot{\varepsilon}_{eq}^c$ is defined by either the Munson-Dawson model or the WIPP secondary creep model.

Since the creep consolidation equation allows unlimited consolidation, a cap must be introduced that eliminates further consolidation when the intact material density ρ_f is reached. Thus, when the condition

$$|\varepsilon_v| \geq \left| \frac{\rho_0}{\rho_f} - 1 \right| \quad (2-29)$$

is satisfied, no further creep consolidation occurs.

2.5.3 Zeuch's Hot-Isostatic Pressing Model

The Zeuch Hot-Isostatic Pressing (HIPing) Model is a deformation mechanism-based constitutive model. The deformation mechanisms of crushed salt during consolidation are assumed to be similar to those occurring during isostatic hot pressing. Isostatic hot pressing is an industrial fabrication process by which finely-powdered ceramic and metallic materials are converted to near fully dense states using heat and pressure. During hot-isostatic pressing, densification occurs by mechanisms such as particle rearrangement, fracture, "instantaneous" plastic deformation by dislocation movement, and time-dependent processes including: (1) climb-controlled glide of creep, (2) transport of matter from interparticle boundaries to voids by surface diffusion, and (3) transport of matter from interparticle boundaries to voids by lattice diffusion. The last two mechanisms dominate only if the grain size is much smaller than the particle size. The simple forms of the densification rates by each of the above mechanisms are given by Helle et al. [1985]. Because the creep consolidation by these mechanisms is mutually independent, the total densification rate can be obtained from the sum of the densification rates of the individual deformation mechanisms.

Because the grain and particle sizes of WIPP crushed salt are rather coarse, it can be assumed that the Nabarro-Herring creep and Coble creep can be excluded to simplify the analysis. Zeuch [1989; 1990] considers only dry crushed salt (moisture content ≤ 0.3 weight percent) and, therefore, further excludes the diffusional mechanisms, which are believed to occur only at low pressure, high temperature, or when water is present. Another mechanism excluded by Zeuch [1989; 1990] is the initial particle rearrangement and fracturing. Therefore, only two densification mechanisms are considered in the form of the isostatic hot pressing model proposed for WIPP dry crushed salt; namely, instantaneous plastic yield and creep deformation.

The densification equations for instantaneous plastic yield from isostatic hot pressing are applied directly to WIPP crushed salt. In creep equations, the dependence of the densification rate on climb-controlled dislocation glide is derived from the commonly accepted expression for climb-controlled creep of an intact crystalline solid, modified to account for the influence of pore geometry on the local contact stresses at interparticle contacts that act to close the pores [Helle et al., 1985]. The only modification to the equations of hot-isostatic pressing when applied to WIPP crushed salt is substitution of the expression for creep of WIPP intact salt [Wawersik and Zeuch, 1986] into the equations of Helle et al. [1985] in place of climb-controlled creep of an intact crystalline solid.

Zeuch's HIPing Model fits dry crushed WIPP salt experimental data reasonably well. However, if this model is to be applied to wet crushed salt, other deformation mechanisms, especially the diffusional processes, need to be included. As indicated by Zeuch and Holcomb [1991], densification of damp and brine-saturated crushed salt almost certainly occurs by fluid phase-enhanced, diffusional creep mechanisms. No detailed effort has yet been undertaken to extend the model of Zeuch [1989; 1990] and

Holcomb and Zeuch [1988; 1990] to include these processes, although Zeuch [1989; 1990] outlined how such an extension could be accomplished, assuming that densification is rate-limited by diffusional transport of matter in the fluid phase. Constitutive model development for wet crushed-salt densification controlled by diffusional processes has been given by Spiers and coworkers and is reviewed in Section 2.5.4.

The equations in Zeuch's constitutive model [Zeuch, 1989; 1990] are taken or modified from those given by Helle et al. [1985]. The equations of Helle et al. [1985] are introduced briefly in the following section.

2.5.3.1 THE ORIGINAL HOT-PRESSING MODEL OF HELLE ET AL. [1985]

During hot-isostatic pressing, grain shape, pore size and geometry, and interparticle contact area undergo dramatic changes. As in other studies, Helle et al. [1985] simplify the analysis of this complex, continuous process by dividing it into two representative geometric stages, each persisting over a specified range of density. These are referred to as Stages 1 and 2. Stage 1 is within the density range of $D_0 \leq D \leq 0.9$ (where D_0 is the initial fractional density and D is the current fractional density); pores are "cusp-shaped" and interconnected. During Stage 2, $0.9 < D \leq 1$, the pores are small, isolated spheres located along grain boundaries.

A term that is used quite uniquely in hot-isostatic pressing is "effective stress." In the literature, the effective stress (p^*) is the local stress at interparticle contacts that acts to close the pores. To avoid confusion with the effective stress terminology used in mechanics, p^* will be referred to as "local stress" in this review. At the beginning of a hot-pressing test, p^* is greater than the applied pressure σ_m , owing to the reduced surface area on which p^* acts within the porous sample. If σ_m is held constant, p^* eventually must approach σ_m as the fractional density approaches unity.

Local Stresses

By analyzing the number of contact neighbors per particle and the average area of a contact during hot-isostatic pressing, Helle et al. [1985] propose equations for calculating the local stress for Stage 1 (p_1^*). Ignoring effects of surface energy and entrapment pressure for the low temperature of interest and for dry conditions, p_1^* is given as:

$$p_1^* \approx \frac{\sigma_m(1 - D_0)}{D^2(D - D_0)} \quad (2-30)$$

where:

D_0 = initial fractional density

D = fractional density ($D_0 < D \leq 0.9$).

During Stage 2, as the fractional density approaches 1, the effects of surface energy can also be ignored, and the local stress, p_2^* , is:

$$p_2^* \approx \sigma_m \quad (2-31)$$

Instantaneous Densification by Plastic Yield (D_{yield})

When pressure is applied to a powder, it will first deform by plastic yielding. This causes the contact area to grow and the effective pressure to decrease until the yield stress, σ_y , of the material is no longer exceeded. Yielding will occur during the initial stage provided, $p_1^* \geq 3\sigma_y$, and the external pressure that will just cause yielding is [Helle et al., 1985]:

$$\sigma_m = 3 \frac{D^3 - D^2 D_0}{(1 - D_0)} \sigma_y \approx 1.3 \frac{(D^3 - D_0^3)}{(1 - D_0)} \sigma_y \quad (2-32)$$

The compaction enters final-stage sintering during plastic yielding only if the pressure is high enough to cause yielding of the spherical shell surrounding each pore, and the pressure for yielding is:

$$\sigma_m = \frac{2}{3} \sigma_y \ln \left[\frac{1}{1 - D} \right] \quad (2-33)$$

The instantaneous plastic yield fractional density, D_{yield} , can then be obtained from Equations 2-32 and 2-33 for an applied pressure, σ_m .

For Stage 1 ($D_0 \leq 0.9$);

$$D_{\text{yield}} = \left[\frac{\sigma_m (1 - D_0)}{1.3 \sigma_y} + D_0^3 \right]^{1/3} \quad (2-34)$$

For Stage 2 ($0.9 < D_0 \leq 1$);

$$D_{\text{yield}} = 1.0 - \exp \left[\frac{-1.5 \sigma_m}{\sigma_y} \right] \quad (2-35)$$

where σ_m is the applied pressure, σ_y is the temperature-dependent polycrystalline yield stress (σ_y will be discussed in detail in Section 2.5.3.3), and D_0 is the initial fractional density.

Densification Rate for Creep

The densification rate ($\dot{D}$) for climb-controlled glide of dislocations is:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{D} = 5.3 \left[\frac{D^2 D_0}{3} \right]^{1/3} \left[\frac{D - D_0}{1 - D_0} \right]^{1/2} \left[\left[\frac{AbD_v}{kT\mu^{n-1}} \right] \left[\frac{p_1^*}{3} \right]^n \right] \quad (2-36)$$

For Stage 2 ($0.9 < D \leq 1$);

$$\dot{D} = \left[\frac{3}{2} \right] \frac{D(1 - D)}{\{1 - (1 - D)^{1/n}\}^n} \left[\left[\frac{AbD_v}{kT\mu^{n-1}} \right] \left[\frac{3p_2^*}{2n} \right]^n \right] \quad (2-37)$$

where:

D_v = volume diffusion coefficient

μ = temperature-dependent shear modulus

T = absolute temperature

n = stress exponent

A = Dorn constant

b = Burgers vector

k = Boltzmann's constant.

2.5.3.2 APPLICATION OF THE MODEL TO WIPP CRUSHED SALT

Application of the isostatic hot pressing model to WIPP crushed salt performed by Zeuch [1989; 1990] for the plastic yield and creep deformation are discussed in this section.

Instantaneous Densification by Plastic Yield (D_{yield})

The equations for instantaneous densification by plastic yield in the hot-isostatic pressing model of Helle et al. [1985] are given by Equations 2-34 and 2-35. These equations are applied directly to WIPP crushed salt.

The Densification Rate for Creep Deformation ($\dot{D}$)

The dependence of the densification rate on climb-controlled dislocation glide is included in the term in square brackets that appears on the right-hand side of Equations 2-36 and 2-37. These two terms, modified to account for the influence of pore geometry on the local pressure (i.e., using local stress instead of applied pressure), are derived from the expression for climb-controlled creep of an intact crystalline solid [Frost and Ashby, 1982]:

$$\dot{\epsilon} = \left[\frac{AbD_v}{kT\mu^{n-1}} \right] \sigma^n \quad (2-38)$$

where:

$\dot{\epsilon}$ = axial strain rate

σ = axial stress.

For WIPP salt, the expression used for creep deformation is [Wawersik and Zeuch, 1986]:

$$\dot{\epsilon} = A \left[\frac{\sigma_1 - \sigma_3}{\mu} \right]^n \exp \left[\frac{-Q_c}{RT} \right] \quad (2-39)$$

where $A = 4.5 \times 10^{-14}/\text{sec}$, $\mu = 12,400 \text{ MPa}$, $n = 4.9$, and $Q_c = 50,200 \text{ J mol}^{-1}$ are material parameters [Wawersik and Zeuch, 1986]. Comparing the above two equations, the dislocation climb-controlled creep consolidation for WIPP crushed salt can be obtained by substituting:

$$\left[\frac{A}{(\mu)^n} \right] \exp \left[\frac{-Q_c}{RT} \right] \quad (2-40)$$

for the terms within the square brackets in Equations 2-36 and 2-37. This yields:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{D} = 5.3 \left[\frac{D^2 D_0}{3} \right]^{1/3} \left[\frac{D - D_0}{1 - D_0} \right]^{1/2} \left[\left[\frac{A}{(\mu)^n} \right] \exp \left[-\frac{Q_c}{RT} \right] \left[\frac{p_1^*}{3} \right]^n \right] \quad (2-41)$$

For Stage 2 ($0.9 < D \leq 1$);

$$\dot{D} = \left[\frac{3}{2} \right] \frac{D(1 - D)}{\{1 - (1 - D)^{1/n}\}^n} \left[\left[\frac{A}{\mu^n} \right] \exp \left[-\frac{Q_c}{RT} \right] \left[\frac{3p_2^*}{2n} \right]^n \right] \quad (2-42)$$

where p_1^* and p_2^* are the local stresses given by Equations 2-30 and 2-31, respectively. Note that in order to compare results of uniaxial (Equation 2-38) and axisymmetric, triaxial compressive (Equation 2-39) creep tests, the axial stress σ in Equation 2-38 is assumed to be equivalent to the stress difference, $\sigma_1 - \sigma_3$, in Equation 2-39.

2.5.3.3 DISCUSSION OF THE YIELD STRESS σ_y

To apply the constitutive equations to crushed salt, the yield stress σ_y must be evaluated. Wawersik [Holcomb and Zeuch, 1990] suggests that within the temperature range expected at WIPP, a yield stress of 7 to 8 MPa would be appropriate. Work on pure NaCl [Zeuch, 1989] indicates that a value of 10 to 15 MPa might be more appropriate. The HIPing model predictions are particularly sensitive to the value chosen for σ_y because it directly determines the predicted plastic consolidation (Equations 2-34 and 2-35) and, thus, the density at the start of the creep phase. As Equations 2-36 and 2-37 show, the creep rate depends strongly on the density, which controls the local contact stress.

The yield stress (σ_y) is weakly temperature-dependent [Helle et al., 1985]. Zeuch [1989] suggested the following exponential equation:

$$\sigma_y = c_1 \exp [c_2 T] \quad (2-43)$$

where c_1 and c_2 are fitting parameters. Holcomb and Zeuch [1990] used the following linear fit equation for pure NaCl:

$$\sigma_y(T) = \sigma_y(20^\circ\text{C}) - c_3 T \quad (2-44)$$

where $\sigma_y(20^\circ\text{C})$ is the yield stress at 20°C (20 MPa was used by Holcomb and Zeuch [1990]) and c_3 is a fitting parameter.

2.5.3.4 MODIFIED MODEL TO ACCOUNT FOR STAGE 0 DEFORMATION

Most analyses in hot-isostatic pressing acknowledge the existence of a **Stage 0**, consisting of the instantaneous processes of particle rearrangement, fracture, and elastic adhesion; but virtually all models specifically exclude these phenomena because of lack of experimental data. Holcomb and Zeuch [1990] applied their model to test data and show that (1) the yield stress (σ_y) needs to be low (about 8 MPa) to capture most of the instantaneous compaction, which causes predicted densification rates to be about two orders of magnitude lower than the test results; (2) assuming a higher yield stress (about 20 MPa) results in improved predictions of the densification rates but greatly underpredicts instantaneous compaction; and (3) within the constraints of the standard model equations, there seems to be no way to describe both the instantaneous and the creep compaction data. The differences between the HIPing model predictions and experimental data are rather systematic, which are likely a result of errors in prediction of the instantaneous compaction at low density (i.e., a result of ignoring the Stage 0 processes).

Corrections are suggested by assuming that for low pressures and early times, brittle compaction, characterized by particle rearrangement and fracturing, occurs and increases the original density D_0 by several percent. Assuming that the effect of brittle compaction is independent of the plastic compaction described by Equations 2-34 and 2-35, then, the effective original density before plastic compaction is obtained by increasing D_0 according to the value chosen for the brittle compaction. This adjusted value of D_0 is used for the original density in Equations 2-34 and 2-35. Assuming independence between brittle and plastic compaction is equivalent to assuming that the effects of brittle compaction are the same as if the sample had been compacted before the test by shaking or packing until the density had been increased by several percent over the as-poured density.

Using their laboratory test data for time-independent instantaneous compaction of granular salt as a guide and constraint, Holcomb and Zeuch [1990] suggested that a yield stress of 20 MPa with corrections for temperature given in Equation 2-44 would describe the data fairly well if it is assumed that the brittle compaction contributes 10 to 13 percent to the consolidation as the temperature increases from 20°C to 100°C. The values chosen are not uniquely determined by the data but are reasonably well constrained. That is:

$$D_0^* = (1 + c) D_0 \quad (2-45)$$

where D_0^* is the corrected initial density, D_0 is the true initial density, and c is the contribution by the Stage 0 process, which is 10 to 13 percent depending on the temperature. As an example, Figure 2-11 shows the predicted and observed creep compaction from Holcomb and Zeuch [1990, Figure 6c].

Figure 2-11. Densification as a Function of Time at Various Pressures Showing Experimental Results (Dotted Line) and Model Predictions Using an Assumed Yield Stress of 20 MPa at 20°C . Brittle Compaction was Assumed to Increase the Original Sample Density by 10 Percent at 20°C (After Holcomb and Zeuch [1990]).

2.5.3.5 FUNCTIONAL FORMS AND MODEL PARAMETERS

No model parameters are directly included in the instantaneous plastic yield fractional density, D_{yield} (see Equations 2-34 and 2-35). Only one material property is involved; namely, the yield stress σ_y . The yield stress is temperature-dependent and is given by Equation 2-43 or Equation 2-44. There are two model parameters included in each of these two equations; namely, c_1 and c_2 in Equation 2-43 and σ_y (20°C) and c_3 in Equation 2-44. However, in Equations 2-34 and 2-35, there are constant coefficients such as 1.3, 1.5, and the 1/3 power. These coefficients are the result of certain assumptions on the grain contacts and geometry changes during densification for an arrangement of particles with uniform grain size so that the maximum number of contact neighbors per particle is 12 (see Helle et al. [1985]). For irregularly packed particles, these constants can be treated as model parameters and obtained by fitting experimental data. The same principle applies to the coefficients such as 5.3 and 3/2 in Equations 2-41 and 2-42.

Treating these coefficients as model parameters to be determined by fitting to experimental data, the instantaneous densification by plastic yield (Equations 2-34 and 2-35) can be written as:

For Stage 1 ($D_0 \leq 0.9$);

$$D_{\text{yield}} = \left[\frac{\sigma_m(1 - D_0)}{a_1 \sigma_y} + D_0^{a_2} \right]^{\frac{1}{a_2}} \quad (2-46)$$

For Stage 2 ($0.9 < D_0 \leq 1$);

$$D_{\text{yield}} = 1.0 - \exp \left[\frac{-a_3 \sigma_m}{\sigma_y} \right] \quad (2-47)$$

where σ_m is the applied pressure; σ_y is the temperature-dependent yield stress; D_{yield} is the fractional density instantaneously attained upon loading; D_0 is the initial fractional density; and a_1 , a_2 , and a_3 are model parameters.

The functional forms for Equations 2-41 and 2-42 can be written as:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{D} = b_1 (D^2 D_0)^{b_2} \left[\frac{D - D_0}{1 - D_0} \right]^{b_3} \left[\left[\frac{A}{(\mu)^n} \right] \exp \left[-\frac{Q_c}{RT} \right] (b_4 p_1^*)^n \right] \quad (2-48)$$

For Stage 2 ($0.9 < D \leq 1$);

$$\dot{D} = b_5 \frac{D(1 - D)}{\{1 - (1 - D)^{1/n}\}^n} \left[\left[\frac{A}{\mu^n} \right] \exp \left[-\frac{Q_c}{RT} \right] \left[\frac{b_6 p_2^*}{n} \right]^n \right] \quad (2-49)$$

where D is the current fractional density; D_0 is the initial fractional density; p_1^* and p_2^* are local stresses defined by Equations 2-30 and 2-31, respectively; A , μ , n , and Q_c are parameters for the WIPP creep law for intact salt [Wawersik and Zeuch, 1986]; R is the universal gas constant; and b_1 through b_6 are consolidation model parameters.

From Equation 2-49, it can be seen when $D = 1$, $\dot{D} = 0$. This corresponds to the final condition when the crushed salt is consolidated to its intact salt density. However, since only volumetric strain is considered, the model does not automatically achieve the transition from consolidation creep of crushed salt to creep of intact salt. Also, neither the plastic yield density nor the densification rate is mathematically continuous at a fractional density of 0.9. Furthermore, systematic differences exist between hot-pressing model predictions and experimental data, which are likely a result of errors in prediction of the instantaneous compaction at low density. Although empirically including Stage 0 processes have improved agreement between model predictions and test data significantly, this extension is not yet adaptable to long-term predictions. Another restriction for the application of the hot-pressing model is that the model applies to dry crushed salt only (<0.3 weight percent water) because densification by other mechanisms, especially diffusional densification, is ignored.

There could be up to 14 model parameters for crushed salt in the hot-pressing model, depending on how model parameters are selected and fitted to experimental data. Table 2-3 summarizes parameter values used by Zeuch [1989; 1990] and Holcomb and Zeuch [1988; 1990].

2.5.4 Spiers and Coworkers' Model

Spiers and coworkers' model is a deformation mechanism-based constitutive model for the densification of wet granular aggregates by grain boundary diffusional "pressure solution" or solution-precipitation creep. The models are compared with the results from one-dimensional compaction experiments using brine-saturated NaCl aggregates (grain size 100 to 400 μm) at room temperature and applied stresses from 0.5 to 8.0 MPa. The tests performed at low stresses (<4 MPa) produced mechanical data and dissolution/precipitation microstructures that broadly match the models. At higher stresses (>4 MPa), however, there seems to be a change in mechanical behavior. At higher stresses, densification is believed to become increasingly dominated by a plasticity-coupled solution transfer mechanism accompanied by cataclasis and intergranular sliding. The constitutive model, therefore, applies to the low stress regime and for the densification of wet salt by grain boundary diffusional pressure solution only. Models for other mechanisms are yet to be developed.

Table 2-3. Model Parameters and Material Properties for Hot-Pressing Model (After Zeuch [1989; 1990], Holcomb and Zeuch [1988; 1990])

Parameter	Units	Parameter Value
Instantaneous Plastic Yield Parameters		
$\sigma_y(20^\circ\text{C})$	MPa	20
a_1	—	1.3
a_2	—	3
a_3	—	1.5
Intact Creep Parameters		
A	1/sec	4.5×10^{-14}
μ	MPa	12,400
n	—	4.9
Q_c	J/mol	50,200
Time-Dependent Model Parameters		
b_1	—	$5.3/(3)^{b_2}$
b_2	—	1/3
b_3	—	1/2
b_4	—	1/3
b_5	—	3/2
b_6	—	3/2
Temperature-Dependent Yield Stress		
c_1	MPa	34.22
c_2	1/K	-2.41×10^{-3}
c_3	—	0.04
Correction for Stage 0 Deformation		
c	Percent	10 to 13

2.5.4.1 GRAIN BOUNDARY DIFFUSIONAL PRESSURE SOLUTION

As described in Section 2.4.2.2, grain boundary diffusional pressure solution is possible in wet aggregates in which grain contacts contain small quantities of the solution phase either in a stable or adsorbed thin film, or in a fine-scale, island-channel network. When such an aggregate is stressed, the solid surfaces within grain contacts develop an excess chemical potential (hence, increased solubility) with respect to the free pore walls. The average excess potential at any point within grain contact surfaces is given by Equation 2-7. This excess potential provides a driving force for dissolution of material within grain contacts, diffusion through the grain boundary solution film or island-channel network, and reprecipitation in the pores, thus leading to densification creep. Under steady-state conditions, the rate of creep is controlled by whichever of the three serial processes of dissolution, diffusion, and precipitation is the slowest. The nature of the excess chemical potential is discussed in detail by Lehner [1990], Spiers and Schutjens [1990], and Shimizu [1992].

2.5.4.2 GRAIN BOUNDARY DIFFUSIONAL PRESSURE SOLUTION: SMALL STRAIN MODEL

Spiers and Brzesowsky [1993] present two forms for their model termed the small and large strain models. The large strain model differs from the small strain model by a multiplicative geometry factor. Basic assumptions for Spiers' small strain model include: (1) grain contacts remain flat during densification; (2) pore surfaces remain spherical; (3) strains are small (< 15 percent); (4) grain-scale transport involves a succession of steady states; (5) mechanisms other than diffusional pressure solution are of negligible importance; and (6) in the three basic processes of diffusional pressure solution (i.e., dissolution at the contacts, diffusion through the intergranular solution, and precipitation in the pores), dissolution and precipitation are very rapid, and therefore, grain boundary diffusion is rate limiting. Based on these assumptions, volumetric-strain rate equations can be obtained by coupling the geometric change (and, therefore, the average normal stress on individual grain contacts; the concept is similar to that of local stress in hot pressing) during consolidation with the average excess potential equation [Spiers and Brzesowsky, 1993; Lehner, 1990; Spiers and Schutjens, 1990; Shimizu, 1992] and Fick's first law expressed in terms of potential gradient [Lehner, 1990].

The geometrical changes of a representative volume (initial volume, V_0 , current volume, V) of idealized aggregate consisting of a simple cubic pack of spherical grains of diameter d is illustrated in Figure 2-12, along with the approximate equations for calculating grain contact geometry x and r [Spiers and Brzesowsky, 1993]. The average normal stress acting on the individual grain contact is given as:

$$\sigma_n \approx \bar{A}(\sigma_m - P) \frac{(1 - \varepsilon_v)^{2/3}}{\varepsilon_v} + P \quad (2-50)$$

Figure 2-12. Geometry of Model Aggregate (a) Simple Cubic Pack in Undeformed State, (b) Grain Contact Geometry After Small Volumetric Strain ϵ_v . Note that $x = d(1 - \epsilon_v)^{1/3}/2$; also $r^2 \approx d^2\epsilon_v/6$, When ϵ_v is Small (After Spiers and Brzesowsky [1993]).

where $\bar{A} = 6/\pi$, ϵ_v is volumetric strain, σ_m is mean stress (or applied pressure), and P is pore pressure. Substituting Equation 2-50 into Equation 2-7 gives the average excess chemical potential developed at grain contact surfaces as:

$$\Delta\mu_{gb} \approx \bar{A} V_m (\sigma_m - P) \frac{(1 - \epsilon_v)^{2/3}}{\epsilon_v} \quad (2-51)$$

Since the average radial distance that matter must diffuse within grain contacts is about $r/2$ (see Figure 2-12), the average potential gradient driving this diffusion can be approximated as $G = 2\langle\Delta\mu_{gb}\rangle/r$. Coupling this with Fick's first law [Lehner, 1990] and Equation 2-51 yields the average radial diffusion flux within grain contacts ($|J|$) as:

$$|J| = \left[\frac{D_{ds} C}{RT} \right] G = 2\bar{A} V_m \left[\frac{D_{ds} C}{RT} \right] \left[(\sigma_m - P) \frac{(1 - \epsilon_v)^{2/3}}{r \epsilon_v} \right] \quad (2-52)$$

where D_{ds} is the diffusivity of the dissolved solid in the grain boundary fluid, C is the average concentration ($\approx$ solubility, volume fraction), R is the gas constant, and T is absolute temperature. The volumetric flux acts through the peripheral contact area, S_p :

$$S_p = 2\pi r S \quad (2-53)$$

where S is the average thickness of the grain boundary fluid, so that the total volume of matter leaving each grain contact per second is approximately $|J| S_p$. The velocity of grain contacts relative to grain centers is $\dot{x} \approx |J| S_p / (2\pi r^2)$ and the linear strain rate in any principal direction is $\dot{\epsilon} = -\dot{x}/x$. Coupling these relations with Equations 2-52 and 2-53 provides:

$$\dot{\epsilon} = 2\bar{A} V_m \left[\frac{D_{ds} C S}{RT} \right] \left[(\sigma_m - P) \frac{(1 - \epsilon_v)^{2/3}}{r^2 x \epsilon_v} \right] \quad (2-54)$$

Using $\dot{\epsilon}_v = -\dot{V}/V = 3\dot{\epsilon}$ and x and r given in Figure 2-12, the final equation for volumetric strain rate is:

$$\dot{\epsilon}_v = A V_m \left[\frac{Z^*}{T} \right] \left[\frac{\sigma_m - P}{d^3} \right] \left[\frac{(1 - \epsilon_v)^{1/3}}{\epsilon_v^a} \right] \quad (2-55)$$

where:

ε_v = volumetric strain

σ_m = mean stress

P = pore pressure

T = absolute temperature

V_m = molar volume of the solid

$Z^* = D_{ds}CS$, is a phenomenological coefficient representing effective grain boundary diffusivity, where D_{ds} is the diffusivity of the dissolved solid in the grain boundary fluid, C is its average concentration ($\approx$ solubility, volume fraction), S is the average thickness of the grain boundary fluid)

d = average grain diameter

$A = 72\bar{A}/R$ ($\bar{A} = 6/\pi$)

R = gas constant

$a = 2$.

2.5.4.3 GRAIN BOUNDARY DIFFUSIONAL PRESSURE SOLUTION: LARGE STRAIN MODEL

When the strain is large (> 15 percent), most of the small strain approximations used (e.g., r given in Figure 2-12) remain tolerably accurate. However, changes in microstructure caused by the intersection of neighboring contacts must be taken into account; i.e., S_p in Equation 2-53 needs to be modified. At higher strains, the intersection of contacts leads to a reduction in S_p such that $S_p \rightarrow 0$ as the porosity (ϕ) approaches zero, while Equation 2-53 remains true when $\phi \approx \phi_0$ (the initial porosity). Assuming this function is continuous and monotonic in real materials, then:

$$S_p = 2\pi r S \left(\phi / \phi_0 \right)^n \quad (2-56)$$

where n is a model parameter reflecting the evolution of contact intersection. Substituting this equation for S_p in the development of the densification model, and using $\varepsilon_v = (\phi_0 - \phi)/(1 - \phi)$ (where ε_v is volumetric strain):

$$\dot{\varepsilon}_v = AV_m \frac{Z^*}{T} \left[\frac{\sigma_m - P}{d^3} \right] \left[\frac{1 - \phi_0}{1 - \phi} \right]^{1/3} \left[\frac{1 - \phi}{\phi_0 - \phi} \right]^2 \left[\frac{\phi}{\phi_0} \right]^n \quad (2-57)$$

or:

$$\log_{10} F = \log_{10} (AV_m Z^*) + n \log_{10} (\phi / \phi_0) \quad (2-58)$$

where:

$$F = \frac{\dot{\epsilon}_v T d^3}{(\sigma_m - P)} \left[\frac{1 - \phi}{1 - \phi_0} \right]^{1/3} \left[\frac{\phi_0 - \phi}{1 - \phi} \right]^2 \quad (2-59)$$

In terms of volumetric strain, Equation 2-57 can be written as:

$$\dot{\epsilon}_v = A V_m \frac{Z^*}{T} \left[\frac{\sigma_m - P}{d^3} \right] \left[\frac{(1 - \epsilon_v)^{1/3}}{\epsilon_v^a} \right] \left[\frac{\epsilon_v - \phi_0}{\phi_0(\epsilon_v - 1)} \right]^n \quad (2-60)$$

where ϕ_0 is the initial porosity and n is a model parameter reflecting the evolution of contact intersection. Other parameters are the same as defined in Equation 2-55. This equation can be applied to relatively large strains; i.e., until the porosity becomes disconnected at $\phi \approx 5$ percent.

Spiers and Brzesowsky [1993] also state that from 20 to 90°C, Z^* depends on temperature according to the relation:

$$Z^* = Z_0 \exp \left[-\frac{\Delta H}{RT} \right] \quad (2-61)$$

where Z_0 and ΔH are material parameters.

2.5.4.4 FUNCTIONAL FORMS AND MODEL PARAMETERS

According to Spiers and Brzesowsky [1993], $\bar{A}$ in Equation 2-54 would take the value $3/(\pi\sqrt{2})$ and $A \approx 6$ for a face-centered cubic packing, so that in general, $A \approx 12 \pm 6$. This means $\bar{A}$ may be treated as a model parameter to be obtained by fitting to experimental data for packings of irregular grain shape in which A cannot be accurately calculated. The volumetric strain (ϵ_v) in the denominator in Equation 2-55 is squared because of assumptions on the change of grain geometry during densification; i.e., the calculation of x and r in Figure 2-12. Therefore, similar to A , a may also be treated as a model parameter to be obtained by fitting to experimental data. The same may also be applicable to the $1/3$ power on the $(1 - \epsilon_v)$ term and the cubic power of the grain size. There are two material parameters, V_m (the molar volume of the solid) and Z^* (a phenomenological coefficient representing effective grain boundary diffusivity), which can be treated as one model parameter together with A . Therefore, the small strain model may have up to four model parameters. Using the same principles for the large strain model, the large strain model may have up to five model parameters (four of which are the same as in the small strain model, plus parameter n). The functional form for small strain (<15 percent) can be written as:

$$\dot{\varepsilon}_v = \left[\frac{a_1}{T} \right] \left[\frac{\sigma_m - P}{d^{a_2}} \right] \left[\frac{(1 - \varepsilon_v)^{a_3}}{\varepsilon_v^{a_4}} \right] \quad (2-62)$$

and for large strain (> 15 percent):

$$\dot{\varepsilon}_v = \frac{a_1}{T} \left[\frac{\sigma_m - P}{d^{a_2}} \right] \left[\frac{(1 - \varepsilon_v)^{a_3}}{\varepsilon_v^{a_4}} \right] \left[\frac{\varepsilon_v - \phi_0}{\phi_0(\varepsilon_v - 1)} \right]^n \quad (2-63)$$

where:

ε_v = volumetric strain

σ_m = mean stress

P = pore pressure

T = absolute temperature

d = average grain diameter

a_1, a_2, a_3 , and n = model parameters

ϕ_0 = emplaced porosity.

As observed by Spiers and Brzesowsky [1993], there are two regimes of densification creep: low stress regime ($\sigma_m - P \leq 4$ MPa) and high stress regime ($\sigma_m - P > 4$ MPa). They further proposed the following empirical description for crushed-salt densification creep in the low stress regime and for volumetric strains up to 12 to 15 percent (small strain) as:

$$\dot{\varepsilon}_v = \frac{K(\sigma_m - P)^m}{d^b \varepsilon_v^c} \quad (2-64)$$

where K is a material constant, $m = 0.8$ to 1.8 , $b \approx 3$, $c = 2$ to 3 . Equation 2-64 is broadly consistent with the grain boundary diffusion-controlled pressure solution model for small strains. As the volumetric strain increases, c increases while other parameters remain basically unchanged.

In the high stress regime, the experimental data show behavior equivalent to an increase in m in Equation 2-64 to values in the range of 2 to 5, b decreases to about 2 and c increases to about 5 to 6.

These are not consistent with Equation 2-60, suggesting the need for inclusion of other deformation mechanisms.

A best-fit in $\log_{10}F/\log_{10}(\phi/\phi_0)$ space shown in Figure 2-13 was made by Spiers and Brzesowsky [1993] to experimental data and produced values for the constants in Equation 2-58 of: $n = 4.15 \pm 0.25$, $Z^* = (2.95 + 0.94, -0.71) \times 10^{-19} \text{ m}^3\text{s}^{-1}$ (taking $V_m = 2.693 \times 10^{-5} \text{ m}^3 \cdot \text{mol}^{-1}$ and $A = 12$). The observed indentation, truncation, and overgrowth microstructure provide strong evidence that densification occurred by dissolution of material within grain contacts, diffusion through the grain boundary region, and precipitation in pores. Therefore, densification in the low stress regime is indeed dominated by grain boundary diffusional pressure solution. Table 2-4 summarizes values for the parameters in Equations 2-57 and 2-64 determined by Spiers and Brzesowsky [1993].

In the large strain model, volumetric strain rate (and therefore, densification rate) becomes zero when porosity becomes zero ($\phi = 0$), which corresponds to the ultimate condition when crushed salt is consolidated to the intact salt density. However, the small strain model equations (Equation 2-55 and/or Equation 2-64) allow unlimited consolidation. Since only volumetric strain is considered, the model does not automatically achieve the transition from consolidation creep to intact salt creep. Another deficiency in this model is that it is not valid at the beginning of consolidation (when $\epsilon_v = 0$ or $\phi = \phi_0$). This is quite similar to Zeuch's creep consolidation model. In Zeuch's model, it is assumed that there is an instantaneous densification mechanism. To apply the model of Spiers and coworkers, a similar time-independent densification must be considered. The model incorporates the effect of grain size and temperature (explicitly in the model equations and through material parameter Z^*) but does not consider the effect of moisture content explicitly.

2.5.5 Yost and Aronson Model

Yost and Aronson [1987] developed a model for creep densification caused by isostatic stress based on an empirical creep power law for intact salt and mass transport into or out of a spherical body. The model is highly empirical in that it simply considers the effect of porosity in the creep power law for intact salt. The model assumes a power-dependence of volumetric strain rate on the porosity of crushed salt so that when porosity becomes zero as crushed salt becomes fully consolidated, the densification rate reduces to zero. Also, the model for crushed salt uses local stress, which is the stress acting on individual salt grains in place of the applied stress in the intact salt model. This local stress is obtained by simply dividing the applied stress by a "fractional area," which is further assumed to be equal to the volume fraction, or fractional density. No geometrical changes in the grains or pores are considered during consolidation. Yost and Aronson [1987] believe that the constitutive model is valid for several deformation mechanisms, depending on the selection of stress exponent, n . For example, they believe that the power-law creep model would describe creep by pressure solution if the exponent n were approximately unity, and $n \approx 5$ would describe flow by certain dislocation mechanisms. The model

Figure 2-13. Log-Log Diagram Showing Normalized Creep Rate Data (F) Given in Equation 2-61 From Constant Stress Tests Plotted Against the Porosity Function (ϕ/ϕ_0). The Solid Line is a Least-Squares Fit. Data Obtained at $\epsilon_v < 3$ Percent ($\log_{10}(\phi/\phi_0) > -0.025$) are not Included Because of Uncertainties in Determining Strain Rate (After Spiers and Brzesowsky [1993]).

was fitted to the creep compaction data of Holcomb (July 1986 memorandum to L.D. Tyler, "Geomechanics Support for WIPP—Monthly Letter, July 1986").

Basic assumptions in the Yost and Aronson model include [Yost and Aronson, 1987]: (1) the material is homogeneous throughout the compaction process (so that $\rho = \rho(t)$, density is a function of time only) and (2) salt is neither created nor destroyed (e.g., through chemical reaction) and the rate of change of salt mass inside a spherical volume is caused by flow into (or out of) the volume through its surface, then:

$$\frac{\partial \rho(t)}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} [\rho(t) V(r,t) r^2] \quad (2-65)$$

where:

$\rho(t)$ = salt density

$V(r,t)$ = local salt velocity

r = radius of the reference sphere.

The model assumes that salt creep is governed by the following power law:

$$\dot{\epsilon} = A \sigma_e^n \quad (2-66)$$

where stress difference is the effective stress (σ_e), and A and n are model parameters. Since isostatic stress applied to the sphere cannot compact the crushed salt any denser than the intact density, ρ_f , the assumed constitutive relation for strain rate, is modified to be:

$$\dot{\epsilon} = A \sigma_e^n \phi^m \quad (2-67)$$

where ϕ is porosity ($\phi = 1 - D$, where D is fractional density), and m is an empirical model parameter. When Equation 2-67 is used for volumetric strain of crushed salt [Yost and Aronson, 1987], σ_e is taken as the stress acting on individual salt grains or the "local stress," which approaches the applied mean stress, σ_m , as ρ approaches ρ_f . This local stress can be approximated by:

$$\sigma_e = \frac{\rho_f}{\rho} \sigma_m \quad (2-68)$$

Substituting Equation 2-68 into Equation 2-67 provides the creep power law for crushed salt as:

$$\dot{\epsilon} = A \phi^m \left[\frac{\rho_f}{\rho} \right]^n \sigma_m^n \quad (2-69)$$

Table 2-4. Parameter Values in the Model of Spiers and Brzesowsky [1993]

Regimes	Parameters	Units	Values
Parameters in Equation 2-60			
$(\sigma_m - p) < 4 \text{ MPa}$	n	—	4.15 ± 0.25
	$Z^{*(a)}$	$\text{m}^3 \text{ s}^{-1}$	$(2.95 + 0.94, -0.71) \times 10^{-19}$
	V_m	$\text{m}^3 \cdot \text{mol}^{-1}$	2.693×10^{-5}
	A	—	12
Parameters in Equation 2-64			
$(\sigma_m - p) < 4 \text{ MPa}$	K	—	—
	m	—	0.8 to 1.8
	b	—	3
	c	—	$2 - 3^{(a)}$
$(\sigma_m - p) > 4 \text{ MPa}$	K	—	—
	m	—	2 to 5
	b	—	2
	c	—	5 to 6

(a) This value is to be increased towards higher volumetric strain (finite strain).

The local salt velocity is defined as:

$$V(r, t) = \frac{\partial U(r, t)}{\partial r} \quad (2-70)$$

and the volumetric strain rate is:

$$\dot{\epsilon} = \frac{\partial^2 U(r, t)}{\partial r \partial t} = \frac{\partial V(r, t)}{\partial r} \quad (2-71)$$

Integrating Equation 2-71 and considering Equation 2-69 yield:

$$V(r, t) = rA \left[\frac{\rho_f}{\rho} \right]^n \phi^m \sigma_m^n \quad (2-72)$$

Substituting Equation 2-72 into 2-65 yields the densification rate for crushed salt as:

$$\dot{D} = 3AD^{1-n} [1 - D]^m \sigma_m^n \quad (2-73)$$

where:

D = fractional density

σ_m = mean stress

A , m , and n = model parameters.

There are three empirical model parameters in Equation 2-73. Yost and Aronson [1987] determined these parameters by fitting the model to experimental data of Holcomb (July 1986 memorandum to L.D. Tyler, "Geomechanics Support to WIPP—Monthly Letter, July 1986"). They assumed that parameter A is a function of water content w (see Figure 2-6):

$$A = a_1 \exp(a_2 w) \quad (2-74)$$

where w is water content, and a_1 and a_2 are fitting parameters. Parameter values determined by Yost and Aronson [1987] are presented in Table 2-5.

Table 2-5. Parameters Values for the Model of Yost and Aronson [1987]

Parameters	Units	Parameter Value
n	—	1.75 ± 0.87
m	—	5.88 ± 1.12
a_1	MPa ⁻ⁿ	11.56×10^{-6}
a_2	(%) ⁻¹	1.69

The model predicts a zero densification rate when crushed salt becomes intact salt. However, the assumptions about local stress and the ultimate volumetric strain are quite simplified. Also, the first part of this model; i.e., mass transfer into or out of a spherical body (Equation 2-65 and, therefore, Equations 2-71 and 2-72) does not seem necessary. Equation 2-69 already provides a complete description of the crushed-salt stress-strain relation. An equation in the form of Equation 2-73 can easily be obtained by considering the relationship between density and volumetric strain:

$$\varepsilon_v = 1 - \frac{D_0}{D} \quad (2-75)$$

where D is fractional density and D_0 is the initial fractional density. Differentiating Equation 2-75 and substituting in Equation 2-69 yields:

$$\dot{D} = \frac{A}{D_0} D^{2-n} (1 - D)^m \sigma_m^n \quad (2-76)$$

which is different from the final equation of Yost and Aronson's model in that Equation 2-76 depends on the initial fractional density; whereas, Equation 2-69 does not, which makes the credibility of this model doubtful.

2.5.6 Liedtke Model

Liedtke et al. [1986] present a viscoplastic model in which the viscoplastic potential depends on material constants, stress invariants, and the accumulated equivalent total and deviatoric viscoplastic strains. The basic model equations include both the deviatoric and volumetric consolidation components. However, the actual material-specific model is for volumetric consolidation only. Two types of experiments have been used to determine model constants: (1) triaxial compression tests carried out to determine the ultimate load bearing capacity and (2) creep consolidation tests. The model has been implemented into the updated Lagrangian Hencky formulation available in the computer program **ADINA** (Automatic Dynamic Incremental Nonlinear Analysis) [Bathe et al., 1986].

2.5.6.1 BASIC CONSIDERATIONS AND EQUATIONS

Liedtke followed the basic approach of associative viscoplasticity; i.e., starting from a flow rule and deriving the constitutive model by assuming a material-specific plastic potential function. The major effort in the constitutive model development included selecting the plastic potential function, which is identical to the yield function for associative flow. The yield function was selected based on several criteria. These criteria required that the yield function be:

1. Convex in the stress space.
2. Pressure-sensitive. In the $\sqrt{J_2} - I_1$ plane, it should intersect the negative I_1 axis, where I_1 is the first invariant of stress tensor and J_2 is the second invariant of deviatoric stress tensor.
3. A continuous function to simplify its numerical implementation and, therefore, increase the reliability of the results it provides.
4. Free of any *a priori* assumptions on the equivalence of stress states. Therefore, for an isotropic material model, the potential function should be of the form:

4. Free of any *a priori* assumptions on the equivalence of stress states. Therefore, for an isotropic material model, the potential function should be of the form:

$$F = F(I_1, I_2, I_3) \quad (2-77)$$

where I_1 , I_2 , and I_3 are the three stress invariants, defined as:

$$\begin{aligned} I_1 &= \sigma_{ij} \delta_{ij} \\ I_2 &= \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} + \begin{vmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33} \end{vmatrix} + \begin{vmatrix} \sigma_{33} & \sigma_{31} \\ \sigma_{13} & \sigma_{11} \end{vmatrix} \\ I_3 &= \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix} \end{aligned}$$

5. Simplest function available to satisfy the above requirements.

The following yield function for instantaneous plasticity of geological materials was selected for the crushed-salt constitutive model development:

$$F = J_2 + \alpha_1 I_1^2 - \beta_1 I_1 I_3^{1/3} + \gamma_1 I_1 - k_1^2 = 0 \quad (2-78)$$

where:

$$J_2 = \frac{1}{2} s_{ij} s_{ij}$$

and α_1 , γ_1 , and k_1 are model parameters, which are assumed to be related to the ultimate state of the material, and β_1 is a hardening function defined as [Desai and Faruque, 1984]:

$$\beta_1 = 3 \alpha_1 \left[1 - \frac{\beta_a}{i + (\bar{\epsilon}^{vp})^{\eta_1} (1 - \beta_b (r_D))^{\eta_2}} \right] \quad (2-79)$$

where $\bar{\epsilon}^{vp} = \int (d\bar{\epsilon}_{ij}^{vp} d\bar{\epsilon}_{ij}^{vp})^{1/2}$ is the equivalent viscoplastic strain; r_D is the ratio of the equivalent deviatoric viscoplastic strain ($\bar{\epsilon}_D^{vp}$) to the viscoplastic strain ($\bar{\epsilon}^{vp}$); and i , β_a , β_b , η_1 , and η_2 are additional material parameters. They further proposed that the above yield function is to be used with the following conditions: $\alpha_1 > 0$, $\gamma_1 > 0$, and $-\infty < \beta_1 \leq 3\alpha_1$, where $\beta_1 = 3\alpha_1$ corresponds to an ultimate state for the material.

2.5.6.2 CONSTITUTIVE EQUATIONS

With the yield function selected, the functional form of the constitutive model can, then, be obtained by substituting the yield function into the following associated flow rule:

$$\dot{\epsilon}_{ij}^{vp} = \gamma F \frac{\partial F}{\partial \sigma_{ij}} \quad (2-80)$$

where F is the static yield function defined by Equation 2-78 and γ is the viscosity constant of the material (fluidity parameter). Although Equations 2-78 and 2-79 include both volumetric strain and the deviatoric strain components, Liedtke et al. [1986] considered only the volumetric strain under isostatic pressure.

For oedometer test conditions (assumed $J_2 = 0$), the viscoplastic potential function (Equation 2-78) reduces to:

$$F = 3(3\alpha_1 - \beta_1) \sigma_m^2 - 3\gamma_1 \sigma_m - k_1^2 \quad (2-81)$$

For the hardening function (Equation 2-79), the nonlinear elastic behavior is assumed to start at the beginning of loading; therefore, $i = 0$. Neglecting the dependence of the hardening function β_1 on r_D , then $\beta_b = 0$, and $\eta_2 = 0$. The hardening function can then be written with $\bar{\epsilon}^{vp} = \epsilon_v/\sqrt{3}$ as:

$$\beta_1 = 3\alpha_1 \left[1 - \frac{\beta_a}{\left(\epsilon_v/\sqrt{3}\right)^{\eta_1}} \right] \quad (2-82)$$

Substituting Equation 2-82 into 2-81, differentiating Equation 2-81 with respect to σ_m , and substituting the result into Equation 2-80, one obtains the volumetric strain rate as:

$$\dot{\epsilon}_v = 3\gamma \left[9\alpha_1 \sigma_m^2 \frac{\beta_a}{\left(\epsilon_v/\sqrt{3}\right)^{\eta_1}} - \left(3\gamma_1 \sigma_m + k_1^2\right) \right] \left[18\alpha_1 \sigma_m \frac{\beta_a}{\left(\epsilon_v/\sqrt{3}\right)^{\eta_1}} - 3\gamma_1 \right] \quad (2-83)$$

where:

ϵ_v = volumetric strain

σ_m = applied pressure

γ = the viscosity constant (fluidity parameter)

$\alpha_1, \gamma_1, k_1, \eta_1$, and β_a = model parameters.

2.5.6.3 MODEL PARAMETERS

Parameters depending on the ultimate state of the material include α_1 , γ_1 , and κ_1 . These material constants can be determined from triaxial compression tests. At least three triaxial tests with different pressures are required. A linear system of equations based on Equation 2-83 for the tests can be solved to obtain α_1 , γ_1 , and κ_1 .

Parameters that define the creep characteristics of the material include γ , β_a , and η_1 . To determine these parameters, Liedtke assumed the following empirical relation:

$$\dot{\epsilon}_v = b_1 t^{-b_2} \quad (2-84)$$

where b_1 and b_2 are the fitting parameters. Using b_1 , Equation 2-84, and the ultimate conditions, the rest of the model parameters can be determined. Table 2-6 presents model parameters determined by Liedtke et al. [1986]. Model predictions using ADINA, a finite element code, are compared with oedometer test results in Figure 2-14 [Liedtke and Bleich, 1985].

Table 2-6. Model Parameter Values for the Constitutive Model of Liedtke et al. [1986]

Parameters	Units	Parameter Values
Parameters Depending on the Ultimate State of Material		
α_1	—	0.1446
γ_1	MPa	4.6286
κ_1	MPa	1.8943
Parameters Depending on Creep Characteristics		
η_1	—	9.3359
β_a	—	0.3816×10^{-8}
γ	$d^{-1} \text{ MPa}^{-3}$	0.1345×10^{-7}
Elastic Parameters		
$E^{(a)}$	MPa	51
ν	—	0.45

(a) Corresponding to data in Figure 2-3.

Figure 2-14. Comparison of Experimental Data With ADINA Results (After Liedtke et al. [1986]).

2.5.7 Spherical Void Model of Butcher

Butcher [1980] conducted one of the earliest efforts on crushed-salt constitutive model development. His study addresses the creep consolidation of crushed salt in a manner identical to his earlier shock mitigation studies [Butcher et al., 1974]. The basic assumptions of this model include: (1) voids in crushed salt become approximately spherical in shape after a short transient period; (2) the compaction is caused by the symmetric collapse of a hollow sphere of homogeneous isotropic incompressible elastic-plastic material; (3) a time-dependent pressure is applied to the outer boundary of a spherical void while the inner boundary remains stress free (i.e., neglecting entrapped pressure or pore pressure); (4) the porous aggregate is imagined to be a three-dimensional array of these spheres, with the outer boundaries overlapping so that the only void volume is that of the spherical voids, and the voids are not interacting so that the pressure supported by the spheres is assumed to be the pressure supported by the porous aggregate. The constitutive model for crushed salt is obtained by combining the compaction strain of spherical voids with the following steady-state creep law for intact salt:

$$\dot{\epsilon} = B \exp \left[-\frac{Q}{RT} \right] \left[\frac{\sigma_e}{\mu} \right]^n \quad (2-85)$$

where:

$\dot{\epsilon}$ = creep strain rate

σ_e = effective stress $\left(\sqrt{3 J_2} \right)$

T = absolute temperature

μ = shear modulus

B , Q/R , and n = steady-state creep parameters for intact salt.

Using Equation 2-85 and the theory of a spherical void model, Butcher [1980] developed the following compaction pressure relation:

$$\sigma_m = \frac{2\mu}{3n} \left[\frac{2A \exp \left[\frac{Q}{RT} \right] [-\dot{\phi}]}{3[1 - \phi]} \right]^n \left[\frac{1 - \phi^n}{\phi^n} \right] \quad (2-86)$$

where ϕ is porosity and $A = B^{-1}$. Using the relationship between porosity and volumetric strain:

$$\varepsilon_v = \frac{\phi_0 - \phi}{1 - \phi} \quad (2-87)$$

Equation 2-85 can be written as:

$$\dot{\varepsilon}_v = \frac{3B}{2} \exp \left[-\frac{Q}{RT} \right] \frac{\phi(\phi_0 - 1)}{(\phi - 1)} \left[\frac{3n}{2\mu} \frac{\sigma_m}{1 - \phi^n} \right]^{1/n} \quad (2-88)$$

where:

$\dot{\varepsilon}_v$ = volumetric strain rate

ϕ = porosity

ϕ_0 = initial porosity.

Model parameters are presented in Table 2-7 [Butcher, 1980]. According to this model, an infinite stress would be required to completely eliminate all porosity. Butcher concluded that this is a *natural* consequence of any spherical void model.

Table 2-7. Parameter Values for the Spherical Void Model of Butcher [1980]

Parameters	Units	Parameter Values
B	s ⁻¹	6.47 × 10 ¹³
Q	kJ/mol	0.42 (10 kcal/mol)
n	—	0.2
μ	GPa	9.62
R	J/mol·K	8.317 (1.987 cal/mol·K)

2.5.8 Korthaus Empirical Model

Korthaus [1988] developed a simple empirical model for strongly consolidated material. The model was incorporated into a finite element code known as **KOLA**. A newer version of this constitutive model that describes the isostatic, temperature-dependent creep consolidation of the backfill crushed-salt material is included in a finite element program known as **MAUS** [Albers, 1984] and has been used for the thermal and thermomechanical analyses of drifts in a repository in rock salt in Germany [Pudewills et al., 1994]. However, very little information is available in the literature regarding the newer version of the model.

2.5.8.1 CONSTITUTIVE MODEL EQUATIONS

Korthaus proposes that the consolidation of granulated salt is governed by a material law consisting of three components: (1) prompt (instantaneous) consolidation, (2) transient creep consolidation, and (3) the spherical void model proposed by Butcher [1980].

Prompt Consolidation: The prompt consolidation equation is used to describe approximately the consolidation obtained in laboratory tests with short loadings (e.g., 100 seconds):

$$\varepsilon_{vp} = c_1 \cdot \log \left(\sigma_m / c_2 + 1 \right) \quad (2-89)$$

where:

ε_{vp} = prompt volumetric strain

σ_m = applied pressure

c_1 and c_2 = model parameters.

Transient Creep Consolidation: The transient creep consolidation equation is used to describe the $t^{-\alpha}$ time dependence ($\alpha \approx 1$) at constant pressure as observed in laboratory tests for time spans up to about 100 days:

$$\dot{\varepsilon}_{vt} = c_3 \cdot \sigma_m^{c_4} \frac{(\phi_0 - \varepsilon_v)^{c_5}}{(\varepsilon_v - \varepsilon_{vp}(\sigma_m) + c_6)^{c_7}} \quad (2-90)$$

where:

ε_{vt} = transient creep volumetric strain

ε_v = total volumetric strain

ϕ_0 = initial porosity

$c_1, \dots, c_7$ = model parameters.

Korthaus [1988] realized that the extrapolation of Equation 2-90 or similar relations to long time is arbitrary as long as it has not been confirmed by experiments. In particular, the case of α close to 1, the consolidation rate decreases very rapidly according to Equation 2-90, which does not seem realistic, at least not for higher pressure (and/or temperatures). To overcome this problem, Korthaus [1988] suggested using a “mechanical model assumption for the already strongly consolidated material” and selected the spherical void model of Butcher [1980]. Then, the total volumetric strain rate is obtained by the summation of transient consolidation and spherical void consolidation. The model parameters were

determined from the experimental data of Holcomb and Hannum [1982] and Korthaus [1988]. Different sets of model parameters are obtained for different tests. These parameters are presented in Table 2-8.

Table 2-8. Model Parameters in the Consolidation Model of Korthaus [1988]

Parameters	Units	Parameter Values	
Prompt Consolidation (Holcomb data)			
c_1	—	0.0615	
c_2	MPa	0.47	
Transient Creep Consolidation			
		Model I (Holcomb data)	Model II (Liedtke data)
c_3	$\text{d}^{-1} \text{MPa}^{-c_4}$	10^{-13}	1.46×10^{-6}
c_4	—	2.5	1
c_5	—	10	4
c_6	—	0.02	0.0125
c_7	—	10	4

2.5.8.2 THE NEWER VERSION OF THE CONSTITUTIVE MODEL EQUATIONS

An alternative form of the constitutive model described by Equation 2-90 used in Germany is given by Equation 2-91.

$$\dot{\epsilon}_v = c_1 (\phi_0 - \epsilon_v)^6 \left(\sigma_m - c_3 \epsilon_v^{c_4} \right)^2 \quad (2-91)$$

where c_1 and c_3 are temperature-dependent constants defined as:

$$\begin{aligned} c_1(T) &= c_1 \exp \left(c_2 \left(1/T_{\text{ref}} - 1/T \right) \right) \\ c_3(T) &= c_3 \left(c_5 + c_6 (T - T_0) + c_7 (T - T_0)^2 \right) \end{aligned} \quad (2-92)$$

where:

ε_v = volumetric strain
 ϕ_0 = initial porosity
 σ_m = applied pressure
 T = temperature
 T_0, T_{ref} = reference temperature
 $c_1, \dots, c_7$ = model parameters.

Model parameter values are presented in Table 2-9 (from Pudewills and Korthaus [1993]).

Table 2-9. Parameter Values Used in the Constitutive Model of Pudewills and Korthaus [1993]

Parameters	Units	Parameter Values
Thermo Elastic Properties		
Modulus of Elasticity (E)	MPa	24,000
Poisson's Ratio (ν)	—	0.25
Coefficient of Thermal Expansion	K ⁻¹	4.2×10^{-5}
Creep Compaction Parameters		
T_0	K	273
T_{ref}	K	392
c_1	d ⁻¹	170
c_2	K	4,000
c_3	—	160
c_4	—	1.8
c_5	—	2.6
c_6	K ⁻¹	-0.021
c_7	K ⁻²	5×10^{-5}

2.5.9 Empirical Model of Zhang

Zhang et al. [1993] describe a model that is supposedly based on “theoretical and phenomenal considerations and experimental results.” However, the basis for the model is not clear. Laboratory tests were performed to verify the constitutive model and to evaluate model parameters. The experimental data of Holcomb and Zeuch [1988] were also used to determine model parameters. As most other constitutive models for crushed salt reviewed in this study, Zhang’s model describes volumetric consolidation only and does not include an elastic compaction component. Zhang et al. [1993] believe that the constitutive model can describe both the instantaneous compaction and time-dependent creep compaction behavior of crushed salt for stresses up to 100 MPa, temperatures to 200°C, and fractional densities to 0.98. Mathematically, Zhang’s model is:

$$\dot{\epsilon}_v = A \exp \left\{ -\frac{Q}{RT} \right\} \left\{ \frac{\sigma}{\sigma_o} \right\}^n \left[\ln \left(\frac{D(1 - D_0)}{D_0(1 - D)} \right) \right]^{-m} \quad (2-93)$$

where:

$\dot{\epsilon}_v$ = volumetric strain rate

σ = isostatic stress

σ_o = reference stress

T = absolute temperature

Q = activation energy for creep

R = universal gas constant

D = fractional density

D_0 = initial fractional density

A , n , and m = model parameters.

As Equation 2-93 indicates, the constitutive model consists of two parts. The first part is:

$$A \exp \left\{ -\frac{Q}{RT} \right\} \left\{ \frac{\sigma}{\sigma_o} \right\}^n \quad (2-94)$$

which is a typical power creep law for intact salt. The second part is:

$$\left[\ln \left[\frac{D(1 - D_0)}{D_0(1 - D)} \right] \right]^{-m} \quad \text{or} \quad \left[\ln \left[\frac{e_0}{e} \right] \right]^{-m} \quad (2-95)$$

where:

e = void ratio

e_0 = initial void ratio

which is a simple density correction for crushed salt with respect to intact salt. As the crushed salt is compacted, the fractional density (D) increases. Crushed salt can eventually be compacted to intact salt density. However, from the equations given by Zhang et al. [1993], the model does not seem to be valid at the ultimate condition; i.e., when $D = 1$. Table 2-10 presents model parameters used by Zhang et al. [1993].

Table 2-10. Parameter Values Used by Zhang et al. [1993]

Parameters	Units	Parameter Values			
		Wet $w=1\%$	Wet $w=1\%$	Dry $w=0\%$	Dry $w=0\%$
		$d=0-2 \text{ mm}$	$d=0-1 \text{ mm}$	$d=0-16 \text{ mm}$	$d=0-64 \text{ mm}$
A	min^{-1}	7.37×10^{-5}	1.32×10^{-4}	8.34×10^{-4}	6.63×10^{-5}
n	—	7.1	8.3	6.4	6.8
m	—	10.9	12.9	9.6	8.3

NOTE: d = grain diameter.

w = water content.

2.5.10 Empirical Model of Fordham

Fordham [1988] presents an empirical compaction creep model for DPPC (Dennison Potacan Potash Co.) halite backfill. The model includes cataclasis, pressure solution, and particulate sliding, and is based on the following assumptions: (1) compaction creep occurs in two creep rate regimes, initially a high rate regime dominated by cataclasis, followed by a low rate regime dominated by either pressure solution or particulate sliding, depending on the stress level; (2) in each regime, the $\log \dot{\epsilon}_v / \log \phi$ curves are linear; i.e. the experimental results of $\log \dot{\epsilon}_v / \log \phi$ curves can be approximated as bilinear, where $\dot{\epsilon}_v$

is volumetric strain rate and ϕ is porosity; (3) the rate of decrease in strain rates for both mechanisms is a function of stress; (4) the point at which high rate processes cease and low rate processes begin ($\dot{\epsilon}_c$, characteristic volumetric strain rate and ϕ_c , the characteristic porosity) is stress- and initial porosity-dependent. As Fordham [1988] noticed, at high stress (> 10 MPa), cataclastic effects may reduce the overall grain size of the backfill, increase the rate of pressure solution creep, and make model predictions incorrect. In addition, most $\log \dot{\epsilon}_v / \log \phi$ curves can hardly be described as bilinear.

The model is based on the assumption that under all stresses and initial porosities, a certain amount of cataclasis (the high rate regime) will occur. Once the porosity reaches the characteristic value, ϕ_c , pressure solution/particulate sliding creep (the low rate regime) begins to dominate. The model equations are:

$$\begin{aligned} \dot{\epsilon}_v &= \frac{\dot{\epsilon}_c}{\phi_c^{l_1}} \phi^{l_1} & \text{when } \phi > \phi_c \\ \dot{\epsilon}_v &= \frac{\dot{\epsilon}_c}{\phi_c^{l_2}} \phi^{l_2} & \text{when } \phi \leq \phi_c \end{aligned} \quad (2-96)$$

where (based on curve-fitting from test data):

$$\begin{aligned} \phi_c &= \phi_0 - \left(0.0487 \times \sigma_m^{0.581} \right) \\ \dot{\epsilon}_c &= 0.0131 \times \phi_0^{10.633} \\ l_1 &= 105 \times \sigma_m^{-0.725} \\ l_2 &= 18.6 \times \sigma_m^{-0.463} \end{aligned}$$

where l_1 and l_2 are the stress-dependent slopes of the $\log \dot{\epsilon}_v / \log \phi$ curves for the two creep rate regimes. $\dot{\epsilon}_c$ is the characteristic strain rate and ϕ_c is the characteristic porosity. $\dot{\epsilon}_c$ and ϕ_c separate the two creep rate regimes in the $\log \dot{\epsilon}_v / \log \phi$ plot, ϕ_0 is the initial porosity, and σ_m is the applied pressure. All experimental work consisted of uniaxial strain (oedometer) tests.

Fordham [1988] implemented the model in a **BASIC** program to predict constant strain-rate compaction creep test data. A number of comparisons between model predictions and test data are included in the porosity/time plots and porosity/stress plots (see Fordham [1988]).

2.6 Literature Review Summary and Discussion

The models for mechanical behavior of crushed salt can be divided according to whether they are based on elastic deformation or inelastic deformation. Sjaardema and Krieg's nonlinear elastic model is the only elastic model found in the crushed-salt literature. Therefore, their model will be used in further crushed-salt constitutive model evaluations. Research on cataclasis of crushed salt is limited. The only effort to establish failure criteria for crushed salt is that of Fordham [1988], which is empirical and material-specific. Failure of a shaft seal component, such as crushed salt, is not likely because the state of stress is almost entirely compressional with only minimal shear stress expected. Therefore, failure of crushed salt is not reviewed or discussed in this report.

Crushed-salt research has primarily concentrated on inelastic deformation, specifically volumetric creep consolidation. Ten densification constitutive models for crushed salt were identified through the literature survey. These represent contemporaneous, state-of-the-art research efforts on constitutive model development for crushed salt. Based on the approaches used in model development, these constitutive models can be grouped into three broad categories: empirical, micromechanism-based, and viscoplastic.

Empirical constitutive models for crushed salt are usually based on curve-fitting exercises to experimental data and do not necessarily or explicitly model specific deformation mechanisms. Since empirical constitutive models are developed based on experimental data, they usually describe test results well. Generally, empirical models are simple in mathematical form and easy to understand. However, as observed by many researchers, these models are usually material-specific and apply only to specific conditions of stress, temperature, moisture content, and grain size. Because of these limitations, the extrapolation of empirical models to in situ conditions and long time spans is a concern. Nevertheless, these models may resemble certain deformation mechanisms. Therefore, if empirical models are applied with the knowledge of specific densification mechanisms, the utility of these models should not be overlooked.

A representative empirical constitutive model for crushed salt is the Sjaardema-Krieg model, which has been used extensively in describing time-dependent consolidation experimental data. The model is developed specifically for WIPP crushed salt and has been implemented into numerical analysis codes and used to perform analyses of WIPP crushed-salt backfill problems by both Sandia National Laboratories [Stone et al., 1985] and RE/SPEC [Callahan, 1990]. Because of its prior use, this model is included as one of the candidate constitutive models for further model examination.

From a deformation mechanism point of view, the inelastic densification of crushed salt includes an instantaneous (time-independent) process dominated by grain rearrangement and microfracturing. Further densification then occurs by a variety of mechanisms depending on conditions and whether the crushed salt is dry or wet. The time-dependent densification of dry crushed salt is believed to be controlled predominantly by creep; whereas, wet crushed salt densification is controlled predominantly by grain

boundary diffusional pressure solution and plasticity-coupled pressure solution. Apparently, an ideal deformation mechanism-based constitutive model should consider at least the following mechanisms: (1) instantaneous processes, (2) creep, (3) grain boundary diffusional pressure solution, and (4) plasticity-coupled pressure solution. None of the models identified from the literature search account for all of these mechanisms. Also, no effort has been made to develop compaction models for plasticity-coupled pressure solution of crushed salt. Unlike empirical models, densification mechanism-based constitutive models usually have fewer model parameters but more material parameters (i.e., parameters with a physical basis). Almost all of these models are based on certain simplifying assumptions about grain-size distribution and the perplexing evolution in grain geometry during densification. Therefore, the mechanistic models do not usually fit laboratory experimental data as well as the empirical models. However, in reality, some material parameters and model coefficients can be treated as arbitrary model parameters because of deviations from the assumed systematic packing of grains.

The model of Zeuch and Holcomb [Zeuch, 1990, 1989; Holcomb and Zeuch, 1990, 1988] and that of Spiers and coworkers [Spiers and Brzesowsky, 1993; Spiers and Schutjens, 1990] are good examples of densification mechanism-based constitutive models. Since they emphasize different aspects of crushed-salt densification mechanisms, both models are selected as candidates for further examination. Both of these models have significant physical and mechanical bases and can fit experimental data reasonably well. The model of Zeuch utilizes the well-developed, isostatic hot-pressing theories developed in the ceramics and metal forming industry. Currently, Zeuch's model considers only the dislocation-controlled densification mechanism. Therefore, it applies only to dry crushed salt. However, the model equations for other mechanisms are available in the hot-pressing theory and can be applied readily to wet crushed salt. In contrast to Zeuch's model, the model of Spiers and coworkers accounts for the grain boundary pressure solution-controlled deformation mechanism and applies to wet crushed salt. Because creep consolidation by different mechanisms are mutually independent, the total densification rate can be obtained by the sum of the densification rates for the deformation mechanisms considered.

Other models are excluded from further examination for various reasons. The model of Yost and Aronson [1987] is based on power law creep for intact salt. It considers the effect of porosity by simply assuming a power-dependence of volumetric strain on the current porosity of crushed salt so that when porosity becomes zero as crushed salt consolidates over time, the densification rate becomes zero. The basic idea is similar to that of the "local stress" in hot pressing and other densification mechanism-based theories. However, the Yost and Aronson model is still empirical in nature because change in grain geometry during densification and specific deformation mechanisms are not considered. The model appears to be a much simplified version of the dislocation-controlled densification model (i.e., Zeuch's hot-pressing model). For similar reasons, the model of Zhang et al. [1993] is excluded from further examination. In the Zhang model, the volumetric strain equation is mathematically invalid when crushed salt is completely consolidated; i.e., when the fractional density becomes one. Application of the empirical transient creep consolidation model of Korthaus [1988] is very limited. The Korthaus model

applies only within 100 days of crushed-salt consolidation and uses the spherical void model for the longer term consolidation. A very important assumption in the spherical void model of Butcher [1980] is that the voids are spherical and interconnected. As indicated by Butcher [1980], the initial compaction response needs to be obtained experimentally up to the time when the voids have developed sufficient spheroidal form. This assumption makes the application of the spherical void model difficult because the voids in crushed salt may never approach being spherical, so it is almost impossible to determine when the voids become sufficiently spherical. In addition, the mechanism of densification by void collapse might not be significant at all compared to other mechanisms, such as grain boundary pressure solution and dislocation motion. Void collapse might occur only at the very early stage of crushed-salt consolidation and is most likely time-independent. According to Butcher's model, an infinite stress would be required to completely eliminate all porosity. In Fordham's empirical model, the assumption of the bilinearity of the $\log \dot{\epsilon}_v / \log \phi$ curves (where $\dot{\epsilon}_v$ is volumetric strain rate and ϕ is porosity) does not seem to be applicable to WIPP crushed salt. Liedtke et al. [1986] exercised a viscoplastic approach in developing the constitutive model for crushed salt. A major effort in the viscoplastic model development includes selecting the viscoplastic potential function and hardening function. Liedtke et al. simplified a yield function for instantaneous plasticity of geological material and used it for crushed salt. Since the selection of this yield function is based neither on experimental data nor on crushed-salt densification mechanisms, the model has not been demonstrated for crushed salt.

Most of the existing constitutive models for crushed salt are volumetric creep consolidation models. Sjaardema and Krieg [1987] recognized the need for a deviatoric component in the model, but because of the general lack of experimental data, they could only recommend several options based on judgment. The three-dimensional generalization of the Sjaardema-Krieg model; i.e., the modified Sjaardema-Krieg model proposed by Callahan [1993], is the only three-dimensional constitutive model for crushed salt. This approach can be used to generalize other candidate models to three-dimensional states of stress.

Several factors have been identified that affect the creep consolidation of crushed salt, including: (1) density (or porosity), (2) stress state (confining pressure and stress difference), (3) time or strain rate, (4) temperature, (5) average grain size and grain-size distribution, (6) moisture content, and (7) impurities. All of the existing constitutive models can be expressed as volumetric strain rate in terms of density (or accumulated volumetric strain) and applied pressure. Among the constitutive models reviewed, four models include temperature dependency explicitly in the model equations. Only the model of Spiers include a grain size effect. This might be because of the general belief that grain size effect is only important for pressure solution-controlled deformation mechanisms. No existing models have included moisture or impurity content explicitly. However, most constitutive models have been developed specifically for either dry crushed salt (e.g., Zeuch) or wet crushed salt (e.g., Spiers). Yost and Aronson [1987] incorporated the effect of moisture content by assuming that one of the model parameters is a function of water content. Basically, a similar approach can be used for the other constitutive models to account for the effect of moisture content.

2.7 Conclusions

The densification of crushed salt can be divided into elastic densification and inelastic densification. The inelastic densification can be further divided into time-independent (instantaneous) densification and time-dependent densification. A number of parameters have been found to affect the mechanical behavior of crushed salt, including density, grain size and grain-size distribution, moisture content, impurities, temperature, stress state, and time. Studies on these effects can be summarized as follows:

1. The elastic properties (bulk modulus and shear modulus) of crushed salt depend on crushed-salt density or applied pressure. The effects of other parameters on crushed-salt elastic properties are not clear.
2. The presence of water significantly increases the crushed-salt consolidation rate relative to a dry crushed-salt material, and there is no threshold observed for the moisture content effects. No consistent correlations were found between the consolidation rate and water content for a crushed-salt material that is wetted. Some researchers believe that even though some moisture makes a big difference on consolidation rate, there is no strong correlation between the consolidation rate and moisture content. Others believe consolidation rate increases with increasing moisture content.
3. A strong dependence of densification rate on grain size is observed for diffusion-controlled mechanisms (for wet crushed salt); the larger the grain size, the lower the densification rate. However, there is no apparent grain-size dependence observed for creep-controlled densification. The effect of grain-size distribution has not been determined.
4. Most research reveals a weak temperature-dependence of the consolidation rate; the higher the temperature, the higher the consolidation rate. However, higher temperatures seem to reduce the rate for pressure solution-controlled densification.
5. Laboratory tests generally show that the higher the initial density, the lower the initial densification rate. Also, the higher the current density, the lower the current densification rate — all other conditions being the same.
6. Shear stresses are believed to enhance the compaction of granular material. However, no systematic connection or correlation between the magnitude of the applied shear stress and the consolidation rate has been observed experimentally (only small stress differences have been used so far).

From a deformation mechanism point of view, the inelastic densification of crushed salt includes (but may not be limited to):

1. Instantaneous or time-independent processes dominated by grain rearrangement and microfracturing.
2. Grain boundary pressure solution (diffusion) which controls the rate of creep.
3. Plastic-coupled pressure solution.
4. Creep.

Ten crushed-salt constitutive models were identified through a computerized literature survey. Model equations are presented and evaluated against model screening criteria and the phenomenology and deformation mechanisms of crushed-salt densification. Three of the ten constitutive models are recommended for further evaluation by fitting them to experimental data: (1) Sjaardema and Krieg [Sjaardema and Krieg, 1987], (2) Zeuch and Holcomb [Zeuch, 1989, 1990; Holcomb and Zeuch, 1988, 1990], and (3) Spiers and coworkers [Spiers and Brzesowsky, 1993; Spiers and Schutjens, 1990].

The state-of-the-art research on constitutive model development for crushed salt may be summarized as follows:

1. There is only one elastic constitutive model described in the literature: the nonlinear elastic model proposed by Sjaardema and Krieg [1987].
2. The existing constitutive models for creep consolidation can be grouped into three categories: empirical, deformation mechanism-based, and viscoplastic.
3. The empirical models are usually simple in mathematical form and can fit experimental data well, but do not explicitly model any specific deformation mechanisms. However, the empirical model equations may resemble certain deformation mechanisms. Therefore, the utility of these models should not be overlooked, provided they are applied with the knowledge of densification mechanisms.
4. Deformation mechanism-based models are usually mathematically complicated and account for only certain specific densification mechanisms. However, researchers generally agree that densification by various mechanisms are mutually independent and act in parallel and therefore, densification rates can be added.
5. Most existing constitutive models are densification rate equations with density (or volumetric strain), applied pressure (usually isostatic stress), temperature, and other material state or geometric parameters as independent variables. The model of Spiers is the only model that explicitly includes grain size effect. Water content is only included in the model of Yost and Aronson. None of the existing models explicitly consider the effect of grain-size distribution or impurity content.
6. With the exception of the modified Sjaardema-Krieg model, the existing constitutive models consider only volumetric creep consolidation, although the necessity of including deviatoric

components is generally realized. The three-dimensional generalization of the Sjaardema–Krieg model [Callahan, 1990; 1993] is the only model that considers deviatoric components.

3.0 CANDIDATE CONSTITUTIVE MODELS

In the previous chapter, the phenomenology and mechanisms of crushed-salt deformation were summarized. Ten crushed-salt constitutive models were identified, and three of those models (Sjaardema-Krieg, Zeuch, and Spiers models) were recommended for further evaluation. This chapter addresses development of the general form for the crushed-salt constitutive model, development of the three-dimensional generalization for the candidate constitutive models, and development of the recommended constitutive model forms necessary to fit the laboratory data and evaluate the material model parameters. Recommended modifications to the models to include the phenomenological aspects of crushed-salt deformation deemed necessary are included since none of the identified models describe all of the potentially important aspects of crushed-salt deformation.

3.1 GENERAL FORM FOR THE CRUSHED-SALT CONSTITUTIVE MODEL

The total strain rate for the crushed-salt constitutive model is assumed to consist of three components. The components include nonlinear elastic ($\dot{\epsilon}_{ij}^e$), creep consolidation ($\dot{\epsilon}_{ij}^c$), and creep ($\dot{\epsilon}_{ij}^i$) contributions, and the total strain rate is written as:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^c + \dot{\epsilon}_{ij}^i \quad (3-1)$$

When the strains become finite, the expressions for strain rate should be interpreted as the rate of deformation. Both the nonlinear elastic and creep consolidation portions of the model describe the material behavior in bulk (volumetric) and in shear (deviatoric). However, the creep portion of the model only describes deviatoric material behavior and is, in fact, the creep deformation model used for intact salt. Each of these contributions to the total strain rate for crushed salt is discussed separately.

3.2 Nonlinear Elastic Model

For the nonlinear elastic model, the functional forms for the elastic constants given by Sjaardema and Krieg [1987] are adopted. They propose bulk and shear moduli as exponential functions of the current density, ρ . Tensile stresses and extensile strains are assumed to be positive. Functional forms of the elastic constants are written in terms of the total engineering volumetric strain, ϵ_v , using the relation:

$$\rho = \frac{\rho_0}{1 + \varepsilon_v} \quad (3-2)$$

where ρ_0 is the initial or original density of the material. The bulk modulus (K) and shear modulus (G) are given by:

$$K = K_0 e^{\frac{K_1 \rho_0}{1 + \varepsilon_v}} \quad (3-3)$$

$$G = G_0 e^{\frac{G_1 \rho_0}{1 + \varepsilon_v}}$$

where K_0 , K_1 , G_0 , and G_1 are material constants.

At any time, the current values of Young's modulus, E , and Poisson's ratio, ν , are computed from the current values of bulk and shear modulus using the relations:

$$E = \frac{9KG}{3K + G} \quad (3-4)$$

$$\nu = \frac{3K - 2G}{2(3K + G)}$$

The rate of deformation is computed using the relation (e.g., Prager [1961]):

$$\varepsilon_{ij}^e = C_{ijkl} \dot{\sigma}_{kl} \quad (3-5)$$

where C_{ijkl} is the matrix of elastic constants and $\dot{\sigma}_{kl}$ is a co-rotational stress rate. For an isotropic body, there are two independent elastic constants and Equation 3-5 can be written as:

$$\varepsilon_{ij}^e = \frac{1}{E} \left[(1 + \nu) \dot{\sigma}_{ij} - \nu \dot{\sigma}_{kk} \delta_{ij} \right] \quad (3-6)$$

In the strictest sense, Equation 3-6 does not apply to the nonlinear elastic model since the coefficients are variable depending on the material density. However, in the neighborhood of a stress-free state, the general hypoelastic expression given in Equation 3-5 can be approximated by Equation 3-6. Since the expressions for the variable elastic moduli were developed for the expected repository stress, deformation, and load path conditions, Equation 3-6 is an acceptable approximation. Equation 3-6 may also be written in terms of the bulk modulus (K) and shear modulus (G) for the material as:

$$\epsilon_{ij}^e = \frac{\bar{\sigma}_m}{3K} \delta_{ij} + \frac{\bar{s}_{ij}}{2G} \quad (3-7)$$

where:

$$\begin{aligned} \sigma_m &= \frac{\sigma_{kk}}{3}, \text{ mean stress} \\ s_{ij} &= \sigma_{ij} - \sigma_m \delta_{ij}, \text{ deviatoric stress} \\ \delta_{ij} &= \text{Kronecker delta.} \end{aligned} \quad (3-8)$$

Equation 3-6 may be inverted to give the elastic constitutive equations for stress in terms of strain:

$$\bar{\sigma}_{ij} = \frac{E}{(1 + \nu)(1 - 2\nu)} \left[(1 - 2\nu) \bar{\epsilon}_{ij}^e + \nu \bar{\epsilon}_{kk}^e \delta_{ij} \right] \quad (3-9)$$

3.3 Creep Consolidation Model

The creep consolidation model is of primary concern in this study, and thus, this section is divided into subsections that address important issues regarding the creep consolidation portion of the crushed-salt compaction model. Typically, equations proposed to describe the behavior of a particular material are written in one-dimensional form or as scalar relationships. To be useful in numerical analyses and applicable to a variety of laboratory experiments with different load paths, any constitutive model must be generalized to include three-dimensional states of stress. The first subsection presents the generalization of the candidate constitutive model forms. The next subsection collapses or reduces the generalized form of the creep consolidation equation to specific laboratory test conditions, which yields simplified forms that can be used in fitting the candidate models to the experimental database. Since many of the candidate models are written in terms of variables such as densification and fractional densification rates, which are readily related to engineering volumetric strain rates, the third subsection presents relationships between these quantities as well as their relationship to true or logarithmic strain rates. The fourth subsection reviews considerations for moisture, particle size, and temperature effects on the deformation of crushed salt. Since no single model includes all of these state variables, a functional form is proposed to include their influence in the models. The fifth subsection presents the candidate constitutive model forms as modified to capture the phenomena associated with the crushed-salt creep consolidation component of the constitutive model.

3.3.1 Creep Consolidation Model Generalization

To develop the creep consolidation constitutive equation, general considerations are first observed, and then specific functional forms are selected for evaluation using available laboratory data. From the application of thermodynamic concepts, the three-dimensional generalization of kinetic equations is given by Fossum et al. [1988]. Following this approach, the generalized average kinetic equation for the creep consolidation inelastic flow is given as:

$$\dot{\varepsilon}_{ij}^c = \dot{\varepsilon}_{eq}^c \left(\sigma_{eq}^f \right) \frac{\partial \sigma_{eq}}{\partial \sigma_{ij}} \quad (3-10)$$

where $\dot{\varepsilon}_{ij}^c$ is the inelastic strain rate tensor and σ_{eq}^f and $\dot{\varepsilon}_{eq}^c$ are the power-conjugate equivalent stress measure and equivalent inelastic strain rate measure for creep consolidation, respectively; and the equivalent inelastic strain rate measure is written as a function of the equivalent stress measure. For use in the flow potential, another equivalent stress measure, σ_{eq} , is used to provide a nonassociative formulation that provides flexibility in governing the magnitude of the volumetric behavior.

Motivation for selection of the equivalent stress measures comes from laboratory experiments. Tests on crushed-salt specimens exhibit a strong dependence on the pressure (mean stress) applied to the specimens. Shear consolidation experiments also exhibit differing behavior depending on the magnitude of applied stress difference. Thus, the appropriate stress measure should include both mean and deviatoric stress dependencies. Laboratory tests on intact salt specimens show little dependence on moderate mean stress levels (> 2 MPa), and typically, the deformation of intact salt is described as a volume-preserving process. Therefore, one would expect the mean stress influence to decrease as the crushed salt approaches full consolidation. With these considerations, the equivalent stress measures are given by:

$$\begin{aligned} \sigma_{eq}^f &= \eta \left[1 - e^{-\eta_2 \sigma_m} \right] - \frac{|\sigma_1 - \sigma_3|^{m_1}}{\sigma_{r1}^{m_1 - 1}} \\ \sigma_{eq} &= \kappa \left[1 - e^{-\kappa_2 \sigma_m} \right] - \frac{|\sigma_1 - \sigma_3|^{m_2}}{\sigma_{r2}^{m_2 - 1}} \end{aligned} \quad (3-11)$$

where:

$$\eta = \eta_0 \left[1 - \frac{\rho}{\rho_f} \right]^{\eta_1}$$

$$\kappa = \kappa_0 \left[1 - \frac{\rho}{\rho_f} \right]^{\kappa_1} \quad (3-12)$$

ρ_f = final or intact density

σ_1, σ_3 = principal stresses

σ_{r1}, σ_{r2} = reference stresses [1 MPa]

$\eta_0, \eta_1, \eta_2,$
 $\kappa_0, \kappa_1, \kappa_2, m_1, m_2$ = material parameters.

The equivalent stress measures are defined to be negative quantities for compressive states of stress that drive the consolidation processes, which requires that η , η_2 , κ , and κ_2 be positive. As the crushed salt approaches full consolidation, η and κ approach zero, and the mean stress dependencies diminish. Twice the maximum shear stress or Tresca equivalent stress is used in the equivalent stress measures to describe the shear behavior and is given by:

$$|\sigma_1 - \sigma_3| = 2 \cos \psi \sqrt{J_2} \quad (3-13)$$

with the second and third invariants of the deviator stress defined by:

$$J_2 = \frac{1}{2} s_{ij} s_{ji} \quad (3-14)$$

$$J_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki}$$

where the Lode angle (ψ), which is a convenient alternative invariant to J_3 , is given by:

$$\psi = \frac{1}{3} \sin^{-1} \left[\frac{-3\sqrt{3} J_3}{2J_2^{3/2}} \right], \quad \left[-\frac{\pi}{6} \leq \psi \leq \frac{\pi}{6} \right] \quad (3-15)$$

The partial derivative given in Equation 3-10 may be determined using the chain rule as:

$$\frac{\partial \sigma_{eq}}{\partial \sigma_{ij}} = \frac{\partial \sigma_{eq}}{\partial \sigma_m} \frac{\partial \sigma_m}{\partial \sigma_{ij}} + \frac{\partial \sigma_{eq}}{\partial J_2} \frac{\partial J_2}{\partial \sigma_{ij}} + \frac{\partial \sigma_{eq}}{\partial \psi} \frac{\partial \psi}{\partial J_3} \frac{\partial J_3}{\partial \sigma_{ij}} \quad (3-16)$$

The derivatives of the invariants in Equation 3-16 are the same regardless of the invariant stress and strain measures selected. These derivatives are:

$$\begin{aligned} \frac{\partial \sigma_m}{\partial \sigma_{ij}} &= \frac{\delta_{ij}}{3} \\ \frac{\partial J_2}{\partial \sigma_{ij}} &= s_{ij} \\ \frac{\partial \psi}{\partial J_3} \frac{\partial J_3}{\partial \sigma_{ij}} &= - \frac{\sqrt{3}}{2J_2^{3/2} \cos 3\psi} t_{ij} \end{aligned} \quad (3-17)$$

where:

$$t_{ij} = s_{ip}s_{pj} - \frac{2}{3}J_2\delta_{ij}$$

Equation 3-16, defining the flow potential, also requires the partial derivatives of the equivalent stress measure (Equation 3-11) with respect to stress. Performing the required differentiation yields:

$$\begin{aligned} \frac{\partial \sigma_{eq}}{\partial \sigma_m} &= \kappa \kappa_2 e^{-\kappa_2 \sigma_m} \\ \frac{\partial \sigma_{eq}}{\partial J_2} &= \frac{-m_2}{\sigma_{r_2}^{m_2-1}} \left(2 \cos \psi \sqrt{J_2} \right)^{m_2-1} \left[\frac{\cos 2\psi}{\sqrt{J_2} \cos 3\psi} \right] \\ \frac{\partial \sigma_{eq}}{\partial \psi} &= \frac{m_2}{\sigma_{r_2}^{m_2-1}} \left(2 \cos \psi \sqrt{J_2} \right)^{m_2-1} \left[2 \sin \psi \sqrt{J_2} \right] \end{aligned} \quad (3-18)$$

Substituting Equations 3-16, 3-17, and 3-18 into Equation 3-10 provides the tensorial strain rate components for the creep consolidation portion of the crushed-salt model, which is given by:

$$\dot{\epsilon}_{ij}^c = \dot{\epsilon}_{eq}^c \left\{ \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \delta_{ij} - \frac{m_2}{\sigma_{r_2}^{m_2-1}} \left(2 \cos \psi \sqrt{J_2} \right)^{m_2-1} \right. \\ \left. \left[\left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right] \right\} \quad (3-19)$$

Equation 3-19 is seen to be undefined as the Lode angle approaches $\pm\pi/6$. In other words, the flow potential forms corners at $\psi = \pm\pi/6$, and the direction of straining is not unique. To eliminate this problem computationally, the flow potentials on either side of the corner are averaged, which produces an indeterminate form that is examined in the limit as $\psi \rightarrow \pm\pi/6$. Performing this limiting operation, Equation 3-19 becomes:

$$\lim_{\psi \rightarrow \pm \frac{\pi}{6}} \dot{\epsilon}_{ij}^c = \dot{\epsilon}_{eq}^c \left\{ \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \delta_{ij} - \frac{m_2 \left(\sqrt{3} J_2 \right)^{m_2-1}}{\sigma_{r_2}^{m_2-1}} \left[\frac{s_{ij}}{\sqrt{3} J_2} \mp \frac{t_{ij}}{2 J_2} \right] \right\} \quad (3-20)$$

The flow potential contains four material parameters (κ_0 , κ_1 , κ_2 , and m_2) that need to be determined from laboratory experiments. To complete the crushed-salt creep consolidation description, the kinetic equation for the crushed-salt model needs to be defined. In general, this kinetic equation could be a form of any of the specific crushed-salt models under consideration (e.g., Sjaardema-Krieg model). Discussion of these specific forms is included in Section 3.3.5 following presentation of other general considerations for the crushed-salt consolidation model.

3.3.2 Evaluation of Laboratory Test Equation Forms

To compare the crushed-salt constitutive equations to laboratory experiments, one needs to reduce the equations to the applicable test conditions. In particular, tests of interest are the hydrostatic compression, triaxial compression, and extension tests. To derive these forms, one simply needs to substitute the applicable test conditions into the constitutive equations and reduce them to their simplest forms. The hydrostatic compression test conditions can be easily obtained from the triaxial compression test conditions by letting $\Delta\sigma$ equal zero and are not presented separately. The $\{zz\}$ direction is taken to be the axial direction.

Reduction to Triaxial Compression Creep Test ($\psi = \pi/6$)

The triaxial compression creep test is prescribed by controlling a constant stress state defined by $\sigma_{zz} < \sigma_{xx} = \sigma_{yy}$. Under these stress conditions, the stress measures included in Equation 3-20 reduce to:

$$\begin{aligned}
 \sqrt{3J_2} &= \Delta\sigma = \sigma_1 - \sigma_3 \\
 J_2 &= \frac{\Delta\sigma^2}{3} \\
 s_{zz} &= \sigma_{zz} - \sigma_m = -\frac{2}{3}\Delta\sigma \\
 s_{xx} &= s_{yy} = \sigma_{xx} - \sigma_m = \frac{\Delta\sigma}{3} \\
 t_{zz} &= s_{zz}^2 + s_{zx}^2 + s_{zy}^2 - \frac{2}{3}J_2 = \frac{2\Delta\sigma^2}{9} \\
 t_{xx} &= t_{yy} = s_{xx}^2 + s_{xy}^2 + s_{xz}^2 - \frac{2}{3}J_2 = -\frac{\Delta\sigma^2}{9} \\
 \left[\frac{s_{zz}}{\sqrt{3J_2}} - \frac{t_{zz}}{2J_2} \right] &= -1 \\
 \left[\frac{s_{xx}}{\sqrt{3J_2}} - \frac{t_{xx}}{2J_2} \right] &= \frac{1}{2}
 \end{aligned} \tag{3-21}$$

Therefore, the axial (a), lateral (l), and volumetric (v) strain components for the creep consolidation portion of the model may be determined from Equations 3-20 and 3-21 as:

$$\begin{aligned}
\Delta \varepsilon_a^c &= \left[\frac{m_2 (\Delta \sigma)^{m_2-1}}{\sigma_{r_2}^{m_2-1}} + \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \right] \dot{\varepsilon}_{eq}^c \Delta t \\
\Delta \varepsilon_l^c &= \left[-\frac{m_2 (\Delta \sigma)^{m_2-1}}{2 \sigma_{r_2}^{m_2-1}} + \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \right] \dot{\varepsilon}_{eq}^c \Delta t \\
\Delta \varepsilon_v^c &= \kappa \kappa_2 e^{-\kappa_2 \sigma_m} \dot{\varepsilon}_{eq}^c \Delta t
\end{aligned} \tag{3-22}$$

where $\dot{\varepsilon}_{ij}^c \approx \frac{\Delta \varepsilon_{ij}^c}{\Delta t}$ and Δt represents a time increment.

Reduction to Triaxial Extension Test ($\psi = -\pi/6$)

The triaxial extension creep test is prescribed by controlling a constant stress state defined by $\sigma_{zz} > \sigma_{xx} = \sigma_{yy}$. Under these stress conditions, the stress measures in Equation 3-20 reduce to:

$$\begin{aligned}
\sqrt{3J_2} &= \Delta \sigma = \sigma_1 - \sigma_3 \\
J_2 &= \frac{\Delta \sigma^2}{3} \\
s_{zz} &= \sigma_{zz} - \sigma_m = \frac{2}{3} \Delta \sigma \\
s_{xx} &= s_{yy} = \sigma_{xx} - \sigma_m = -\frac{\Delta \sigma}{3} \\
t_{zz} &= s_{zz}^2 + s_{xx}^2 + s_{yy}^2 - \frac{2}{3} J_2 = \frac{2\Delta \sigma^2}{9} \\
t_{xx} &= t_{yy} = s_{xx}^2 + s_{xy}^2 + s_{xz}^2 - \frac{2}{3} J_2 = -\frac{\Delta \sigma^2}{9}
\end{aligned} \tag{3-23}$$

$$\begin{aligned}
\left[\frac{s_{zz}}{\sqrt{3J_2}} + \frac{t_{zz}}{2J_2} \right] &= 1 \\
\left[\frac{s_{xx}}{\sqrt{3J_2}} + \frac{t_{xx}}{2J_2} \right] &= -\frac{1}{2}
\end{aligned}$$

Therefore, the axial (a), lateral (l), and volumetric (v) strain components for the creep consolidation portion of the model may be determined from Equations 3-20 and 3-23 as:

$$\begin{aligned}\Delta \varepsilon_a^c &= \left[-\frac{m_2 (\Delta \sigma)^{m_2-1}}{\sigma_{r_2}^{m_2-1}} + \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \right] \dot{\varepsilon}_{eq}^c \Delta t \\ \Delta \varepsilon_l^c &= \left[\frac{m_2 (\Delta \sigma)^{m_2-1}}{2 \sigma_{r_2}^{m_2-1}} + \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \right] \dot{\varepsilon}_{eq}^c \Delta t \\ \Delta \varepsilon_v^c &= \kappa \kappa_2 e^{-\kappa_2 \sigma_m} \dot{\varepsilon}_{eq}^c \Delta t\end{aligned}\tag{3-24}$$

3.3.3 Finite Strain Considerations

Emplaced crushed salt in the shafts and rooms at WIPP will undoubtedly undergo significant volumetric strain as the crushed salt goes to full consolidation. Since the crushed salt will experience finite strains, care must be taken to express the constitutive models appropriately. Typically, laboratory test data are expressed in terms of true strains. During the model fitting exercise, care must be taken to ensure that the response models correctly incorporate the finite strain effects. Material models for the consolidation of crushed salt may be written in terms of variables such as density (ρ), fractional density (D), and volumetric strain. The purpose of this section is to define the relationships between these variables and to develop the relationships between the true (ε_v) and engineering (ε_v) volumetric strain measures. First, consider the following basic density relationships:

$$\begin{aligned}\rho_0 &= \frac{m}{V_0} \\ \rho &= \frac{m}{V} \\ V &= V_0 + \Delta V \\ \rho &= \frac{\rho_0}{1 + \frac{\Delta V}{V_0}} = \frac{\rho_0}{1 + \varepsilon_v} \\ D &= \frac{\rho}{\rho_f}\end{aligned}\tag{3-25}$$

where:

m = mass [kg]

V_0 = original volume [m³]

V = current volume [m³]

ρ_0 = original density $\left[\frac{\text{kg}}{\text{m}^3} \right]$

ρ_f = final or intact density $\left[\frac{\text{kg}}{\text{m}^3} \right]$

ϵ_v = engineering volumetric strain

D = fractional density.

Therefore, by substituting and performing the required differentiation, the relationships between the rate of change of fractional density, density, and engineering volumetric strain may be written as:

$$\dot{D} = \frac{\dot{\rho}}{\rho_f}$$

$$\dot{D} = -\frac{\rho^2 \dot{\epsilon}_v}{\rho_0 \rho_f} = -\frac{\rho D \dot{\epsilon}_v}{\rho_0}$$

$$\dot{\rho} = \rho_f \dot{D}$$

$$\dot{\rho} = -\frac{\rho^2 \dot{\epsilon}_v}{\rho_0} \quad (3-26)$$

$$\dot{\epsilon}_v = -\frac{\rho_0 \dot{\rho}}{\rho^2}$$

$$\dot{\epsilon}_v = -\frac{\rho_0 \rho_f \dot{D}}{\rho^2} = -\frac{\rho_0 \dot{D}}{\rho D}$$

To determine similar relationships for true or logarithmic strain, consider the determinant of the deformation gradient (F), which is given as (e.g., Malvern [1969]):

$$\det F = |F| = \frac{V}{V_0} = \frac{\rho_0}{\rho} \quad (3-27)$$

with the true or logarithmic volumetric strain (e_v) given by:

$$e_v = \ln(|F|) = \ln \left[\frac{V}{V_0} \right] = \ln \left[\frac{\rho_0}{\rho} \right] \quad (3-28)$$

which may also be written as:

$$e_v = \ln \left[1 + \frac{\Delta V}{V_0} \right] = \ln(1 + \varepsilon_v) \quad (3-29)$$

Rearranging Equation 3-29 gives the engineering volumetric strain in terms of the true volumetric strain as:

$$\varepsilon_v = e^{e_v} - 1 \quad (3-30)$$

In terms of the true volumetric strain, the density may be expressed as:

$$\rho = \rho_0 e^{-e_v} \quad (3-31)$$

Then, the relationships between fractional densification, densification, and true volumetric strain rates given in Equation 3-26 may be written as:

$$\begin{aligned} \dot{D} &= -\frac{\rho}{\rho_f} \dot{e}_v \\ \dot{\rho} &= -\rho \dot{e}_v \\ \dot{e}_v &= -\frac{\dot{\rho}}{\rho} \\ \dot{e}_v &= -\frac{\rho_f \dot{D}}{\rho} = -\frac{\dot{D}}{D} \\ \dot{\varepsilon}_v &= \frac{\rho_0 \dot{e}_v}{\rho} = e^{e_v} \dot{e}_v \end{aligned} \quad (3-32)$$

The rates given in Equations 3-26 and 3-32 are used to express the kinetic equations in terms of variables consistent with the constitutive model routines in **SPECTROM-32** [Callahan, 1994].

3.3.4 Moisture, Particle Size, and Temperature Effects

No constitutive model form obtained from the literature includes the effects of moisture, particle size, and temperature on the consolidation rate of crushed salt. Some of the models include one or two effects, but definitive research has not yet been completed to produce general agreement on the effects of all three variables. However, general agreement does exist that these variables influence the consolidation rate of crushed salt. Therefore, an attempt is made here (at least in an empirical sense) to include the influence of these variables in the constitutive models.

Moisture

As discussed in Chapter 2.0, the addition of a small amount of moisture significantly increases the consolidation rate of crushed salt. In addition, there appears to be a quantity of moisture above which further increases in the consolidation rate are not noticeable. However, the amount of moisture producing the largest consolidation rate may vary depending on temperature and pressure. When a significant amount of moisture is available, a retardation in the consolidation rate could occur if the interconnected porosity disappears and the moisture is trapped leading to the generation of pore pressures. However, this functional form is meant only to represent the increased consolidation rate created by the addition of small amounts of moisture and not to represent the retardation caused by large amounts of trapped moisture. To incorporate some of these ideas into a functional relationship for moisture (M), consider the following:

$$M = 1 + a_1(1 - e^{-a_2 w}) \quad (3-33)$$

where:

w = percent moisture by weight

a_1, a_2 = material parameters.

Equation 3-33 has a value of one when the moisture content is zero and asymptotically approaches a value of $a_1 + 1$ as the moisture content increases without bound. The rate at which M approaches the asymptote is governed by parameter a_2 . Equation 3-33 serves as a direct multiplier on the equivalent consolidation strain rate to capture the effect of moisture content.

Particle Size

Particle size or grain-size dependencies have been considered by several researchers as discussed in Chapter 2.0. Typically, they have found that the consolidation rate of crushed salt is inversely proportional to the cube of the average grain-size diameter. Grain size is believed to be more important

when moisture is present because densification through pressure solutioning is operative. To incorporate grain-size dependence, the form used by Shor et al. [1981] and Spiers and Brzesowsky [1993] is used, and the grain-size effect (G) is given by:

$$G = \frac{L}{d^p} \quad (3-34)$$

where:

d = average grain diameter [m]

p = material parameter

L = dimensional normalizing parameter [m^p].

Equation 3-34 also serves as a multiplier on the equivalent consolidation strain.

Temperature

Crushed-salt consolidation rates have been observed to increase with increasing temperature. This most likely occurs because of the thermally activated dislocation motion within the crushed-salt particles. Most researchers have adopted the same temperature dependence forms for crushed salt and intact salt. The same form is used here for inclusion of the temperature dependence (θ) into the constitutive models for crushed salt, which is given by:

$$\theta = \exp \left[-\frac{Q_c}{RT} \right] \quad (3-35)$$

where:

R = universal gas constant $\left[\frac{\text{J}}{\text{mol-K}} \right]$

T = absolute temperature [K]

Q_c = material parameter $\left[\frac{\text{J}}{\text{mol}} \right]$.

Combined Moisture, Particle Size, and Temperature Effects

To introduce the combined effects of moisture, particle size, and temperature into the candidate constitutive models for crushed salt, the functional forms provided in Equations 3-33, 3-34, and 3-35 are combined in a multiplicative manner to produce the combined-effects parameter (ξ) given by:

$$\xi = \frac{L}{d^p} \left[1 + a_1 (1 - e^{-a_2 w}) \right] e^{-\frac{Q_c}{RT}} \quad (3-36)$$

3.3.5 Equivalent Inelastic Strain-Rate Forms

Three different equivalent inelastic strain-rate forms are recommended in Chapter 2.0 to describe the creep consolidation portion of the crushed-salt material model. These forms include the following:

- Sjaardema and Krieg Empirical Model [Sjaardema and Krieg, 1987]
- Zeuch's Hot-Isostatic Pressing Model [Zeuch, 1990]
- Spiers Pressure Solution Model [Spiers and Brzesowsky, 1993]

Each of these functional forms is described separately. In all instances, compaction/ consolidation strain and compressive stress are assumed to be negative. Use of this sign convention results in sign changes from the original referenced models.

Modified Sjaardema-Krieg Model

The engineering volumetric strain rate $\dot{\epsilon}_v^c$ can be derived from the densification rate described empirically by Sjaardema and Krieg [1987] based on hydrostatic laboratory test data on crushed salt as:

$$\dot{\epsilon}_v^c = \frac{(1 + \epsilon_v)^2}{\rho_0} C_0 \left[1 - e^{-B_1 \sigma_m} \right] e^{\frac{A \rho_0}{1 + \epsilon_v}} \quad (3-37)$$

where:

$\epsilon_v = \epsilon_{kk}$, total engineering volumetric strain

$\epsilon_v^c = \epsilon_{kk}^c$, engineering volumetric creep consolidation strain

C_0, B_1, A = material constants.

The functional form given in Equation 3-37 is used as a potential kinetic equation by modifying the stress dependence term and is written as:

$$\dot{\epsilon}_{eq}^c = \frac{(1 + \epsilon_v)^2}{\rho_0} C_0 \sigma_{eq}^f \exp \left[\frac{A \rho_0}{1 + \epsilon_v} \right] \quad (3-38)$$

where σ_{eq}^f is given by Equation 3-11. The effects of moisture, particle size, and temperature are added to the model by multiplying the right-hand side of Equation 3-38 by Equation 3-36, which gives:

$$\dot{\epsilon}_{eq}^c = \frac{B_0 (1 + \epsilon_v)^2}{\rho_0 d^p} \sigma_{eq}^f \exp \left[\frac{A \rho_0}{1 + \epsilon_v} \right] \left[1 + a_1 (1 - e^{-a_2 w}) \right] e^{-\frac{Q_c}{RT}} \quad (3-39)$$

Notice in Equation 3-39 that material constant L appearing in Equation 3-36 has been combined with parameter C_0 since the two constants are multiplicative. Thus, B_0 is defined as:

$$B_0 = C_0 L = \text{material constant} \left[\frac{\text{kg} \cdot \text{m}^p}{\text{m}^3 \cdot \text{MPa} \cdot \text{s}} \right]$$

Following the above development and using Equations 3-26 and 3-32, Equation 3-39 may be converted to true strain, which is:

$$\epsilon_{eq}^c = \frac{B_0 e^{\epsilon_v}}{\rho_0 d^p} \sigma_{eq}^f \exp [A \rho_0 e^{-\epsilon_v}] \left[1 + a_1 (1 - e^{-a_2 w}) \right] e^{-\frac{Q_c}{RT}} \quad (3-40)$$

The modified Sjaardema-Krieg model has ten material constants – η_0 , η_1 , η_2 , m_1 , a_1 , a_2 , p , Q_c , B_0 , and A .

Modified Zeuch Hot-Isostatic Pressing Model

Review of the equations describing Zeuch's two-stage model reveals inclusion of redundant material constants. Thus, before restating Zeuch's model, the following material constants are defined:

$$\begin{aligned} b_9 &= b_1 \left[\frac{A}{\mu^n} \right] b_4^n \\ b_{10} &= b_5 \left[\frac{A}{\mu^n} \right] \left[\frac{b_6}{n} \right]^n \end{aligned} \quad (3-41)$$

where μ is the shear modulus and the other variables on the right-hand side of Equation 3-41 are constants in Zeuch's model. With these definitions, Zeuch's two-stage model (in terms of the fractional density, D) may be restated as:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{D} = -b_9 D_0^{b_2} D^{2(b_2-n)} \left[\frac{D - D_0}{1 - D_0} \right]^{b_3-n} \exp \left[-\frac{Q_c}{RT} \right] \text{sgn}(\sigma_m) |\sigma_m|^n \quad (3-42)$$

For Stage 2 ($0.9 < D \leq 1$);

$$\dot{D} = -b_{10} \frac{D(1 - D)}{\{1 - (1 - D)^{1/n}\}^n} \exp \left[-\frac{Q_c}{RT} \right] \text{sgn}(\sigma_m) |\sigma_m|^n \quad (3-43)$$

where the signum of σ_m (i.e., $\text{sgn}(\sigma_m) = \sigma_m / |\sigma_m|$) represents the sign of the mean stress with the convention that $\text{sgn}(0)=0$. The engineering volumetric strain rate for Zeuch's model can be written (using Equation 3-26) as:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{\epsilon}_v^c = \frac{\rho_0 b_9}{\rho} D_0^{b_2} D^{2(b_2-n)-1} \left[\frac{D - D_0}{1 - D_0} \right]^{b_3-n} \exp \left[-\frac{Q_c}{RT} \right] \text{sgn}(\sigma_m) |\sigma_m|^n \quad (3-44)$$

For Stage 2 ($0.9 < D \leq 1$);

$$\dot{\epsilon}_v^c = \frac{\rho_0 b_{10}}{\rho} \frac{(1 - D)}{\{1 - (1 - D)^{1/n}\}^n} \exp \left[-\frac{Q_c}{RT} \right] \text{sgn}(\sigma_m) |\sigma_m|^n \quad (3-45)$$

The functional forms given in Equations 3-44 and 3-45 are taken to define the potential kinetic equations by modifying the stress-dependent term (using the equivalent stress measure defined in Equation 3-11). The effects of moisture, particle size, and temperature are included by multiplying the right-hand sides of Equations 3-44 and 3-45 by Equation 3-36. With these modifications, the Zeuch model becomes:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{\epsilon}_{eq}^c = \frac{\rho_0 b_7}{\rho d^p} D_0^{b_2} D^{2(b_2-n)-1} \left[\frac{D - D_0}{1 - D_0} \right]^{b_3-n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \sigma_{eq}^f \quad (3-46)$$

For Stage 2 ($0.9 < D \leq 1$);

$$\dot{\epsilon}_{eq}^c = \frac{\rho_0 b_8}{\rho d^p} \frac{(1 - D)}{\{1 - (1 - D)^{1/n}\}^n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \sigma_{eq}^f \quad (3-47)$$

where $D = D_0 / (1 + \varepsilon_v)$. Notice that material constant L appearing in Equation 3-36 has been combined with the parameters b_9 and b_{10} in Equations 3-46 and 3-47. Thus, these parameters are redefined as:

$$b_7 = b_9 L = \text{material constant} \left[\frac{\text{m}^p}{\text{MPa} \cdot \text{s}} \right]$$

$$b_8 = b_{10} L = \text{material constant} \left[\frac{\text{m}^p}{\text{MPa} \cdot \text{s}} \right]$$

Following the above development and using Equations 3-26, 3-31, and 3-32, Equations 3-46 and 3-47 may be converted to true strain, which is:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{\varepsilon}_{eq}^c = \frac{b_7}{d^p} D_0^{b_2} D^{2(b_2-n)-1} \left[\frac{D - D_0}{1 - D_0} \right]^{b_3-n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \sigma_{eq}^f \quad (3-48)$$

For Stage 2 ($0.9 < D \leq 1$);

$$\dot{\varepsilon}_{eq}^c = \frac{b_8}{d^p} \frac{(1 - D)}{\{1 - (1 - D)^{1/n}\}^n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \sigma_{eq}^f \quad (3-49)$$

where $D = D_0 / e^{\varepsilon_v}$. The modified Zeuch model has 13 material constants – η_0 , η_1 , η_2 , m_1 , a_1 , a_2 , p , Q_c , b_2 , b_3 , b_7 , b_8 , and n .

Modified Spiers Pressure Solution Model

Spiers grain boundary diffusional pressure solution model for wet crushed salt is given as:

$$\dot{\varepsilon}_v^c = A V_m \frac{\exp \left[-\frac{\Delta H}{RT} \right]}{T} \left[\frac{\sigma_m - P}{d^{r_2}} \right] \left[\frac{(1 + \varepsilon_v)^{r_3}}{|\varepsilon_v|^{r_4}} \right] \Gamma \quad (3-50)$$

where A , V_m , ΔH , r_2 , r_3 , and r_4 are material constants; T is absolute temperature; R is the universal gas constant; and P is the pore pressure. The model consists of two functional forms — one for small strain and one for large strain, which are invoked depending on the prescribed value for Γ given as:

where ϕ_0 is the initial porosity and n is a material parameter. Spiers and Brzesowsky [1993] developed the function Γ to account for increasing surface contact (increasing area and decreasing stress) as the

$$\Gamma = \begin{cases} 1 & \text{small strain } (\varepsilon_v > -15\%) \\ \left[\frac{\varepsilon_v + \phi_0}{\phi_0(\varepsilon_v + 1)} \right]^n & \text{large strain } (\varepsilon_v < -15\%) \end{cases} \quad (3-51)$$

strains become large. This geometrically interpreted variable serves to decrease the magnitude of the driving force.

The functional form given in Equation 3-50 is modified by changing the stress-dependent term (using the equivalent stress measure defined in Equation 3-11) and adding the effects of moisture, particle size, and temperature by multiplying the right-hand side of Equation 3-50 by the function given in Equation 3-36. In addition, the pore pressure dependency (parameter P in Equation 3-50) has been dropped from the model. Pore pressure could be an important consideration at high fractional densities with sufficient moisture available. However, at this time, insufficient test data are available to include pore pressure as a state variable. With these modifications, the modified Spiers model used to define the kinetic equation becomes:

$$\dot{\varepsilon}_{eq}^c = \frac{r_1}{d^p} \left[1 + a_1 (1 - e^{-a_2 w}) \right] e^{-\frac{Q_c}{RT}} \left[\frac{(1 + \varepsilon_v)^{r_3}}{|\varepsilon_v|^{r_4}} \right] \Gamma \sigma_{eq}^f \quad (3-52)$$

Notice that material constant L appearing in Equation 3-36 and material constants A and V_m appearing in Equation 3-50 have been combined into one parameter, r_1 , in Equation 3-52 defined as:

$$r_1 = AV_m L = \text{material constant} \left[\frac{\text{m}^p}{\text{MPa} \cdot \text{s}} \right]$$

In addition, the temperature dependency in Equation 3-36 was used in place of the function proposed by Spiers. Following the above development, Equation 3-52 may be converted to true strain, which is:

$$\dot{\varepsilon}_{eq}^c = \frac{r_1 e^{-\varepsilon_v}}{d^p} \left[1 + a_1 (1 - e^{-a_2 w}) \right] e^{-\frac{Q_c}{RT}} \left[\frac{e^{r_3 \varepsilon_v}}{|e^{\varepsilon_v} - 1|^{r_4}} \right] \Gamma \sigma_{eq}^f \quad (3-53)$$

where Γ becomes:

$$\Gamma = \begin{cases} 1 & \text{small strain } (e^{e_v} - 1 > -15\%) \\ \left[\frac{e^{e_v} + \phi_0 - 1}{\phi_0 e^{e_v}} \right]^n & \text{large strain } (e^{e_v} - 1 < -15\%) \end{cases} \quad (3-54)$$

A problem is evident in Spiers' model at time zero. Both Equations 3-52 and 3-53 can be indeterminate since when $\sigma_{eq}^f = 0$, the initial volumetric strain is zero. To eliminate this problem during computations, some initial value must be assumed for the volumetric strain. The modified Spiers model has 12 material constants - η_0 , η_1 , η_2 , m_1 , a_1 , a_2 , p , Q_c , r_1 , r_3 , r_4 , and n .

3.4 Creep Model

To develop the creep portion of the crushed-salt constitutive equation given in Equation 3-1, the approach used for the creep consolidation portion of the model is applied. Following that approach, the generalized average kinetic equation for the creep inelastic flow is given as:

$$\dot{\epsilon}_{ij}^i = \dot{\epsilon}_{eq}^i \left(\sigma_{eq}^g \right) \frac{\partial \sigma_{eq}^g}{\partial \sigma_{ij}} \quad (3-55)$$

where $\dot{\epsilon}_{ij}^i$ is the inelastic strain rate tensor and σ_{eq}^g , $\dot{\epsilon}_{eq}^i$ are the power-conjugate equivalent stress measure and equivalent inelastic strain rate measure for the creep, respectively. Creep is assumed to be a volume-preserving process over moderate ranges of mean stress, and thus, the power-conjugate equivalent stress and the flow potential equivalent stress measures are assumed to be the same (associative flow). The equivalent stress measure is the maximum shear or Tresca equivalent stress measure given by:

$$\sigma_{eq}^g = |\sigma_1 - \sigma_3| = 2 \cos \psi \sqrt{J_2} \quad (3-56)$$

When the creep model is used to describe deformation in crushed salt, the stress measure used in the kinetic equation is modified to account for the porous nature of the material. This modification stems from envisioning that the porous crushed salt is composed of cylinders of salt, each of which exhibits the creep behavior of intact salt separated by areas of open space as suggested by Sjaardema and Krieg [1987]. The local stress acting on the salt cylinders is stated in terms of the average stress acting on the porous crushed salt. The cross-sectional area of the porous sample is expressed in terms of the net cross-sectional area of the salt cylinders. This implied areal ratio is the inverse of the fractional density. Amplification of the effective stress by the fractional density is analogous to implementation of damage (ω) into constitutive models. Typically, damage will appear in the denominator as $1 - \omega$ with a stress measure in the numerator. As damage accumulates, ω increases ($\omega \rightarrow 1$), magnifying the influence of

the stress measure. The consolidation process is basically the reverse of damage; whereby, the fractional density divisor serves as a *consolidation* parameter reducing the influence of the stress measure as the crushed salt consolidates. Therefore, in this model, the effective stress used in the multimechanism deformation (M-D) model [Munson, 1979; Munson and Dawson, 1979; Munson et al., 1989] equivalent inelastic strain-rate measure is expressed as:

$$\sigma_{eq}^g = \frac{2 \rho_f \cos \psi \sqrt{J_2}}{\rho} \quad (3-57)$$

The modified equivalent stress measure given in Equation 3-57 increases the magnitude of creep but does not affect the flow potential direction since the stress tensor is also amplified by the porous nature of the material, and the density amplification factor cancels in the stress derivatives.

Therefore, to define completely the inelastic tensorial strain-rate components defined in Equation 3-55, the partial derivative of the equivalent stress measure with respect to stress given by Equation 3-16 and the equivalent inelastic strain-rate measure are required. The M-D model is used for the kinetic equation and is defined later. For the partial derivative of the effective stress measure, the Lode angle (Equation 3-15) is again taken as a convenient alternative to the third invariant of the stress deviator. The derivatives of the invariants are given in Equation 3-17. The derivatives of the equivalent stress measure (Equation 3-56) with respect to the invariants are given by:

$$\begin{aligned} \frac{\partial \sigma_{eq}^g}{\partial \sigma_m} &= 0 \\ \frac{\partial \sigma_{eq}^g}{\partial J_2} &= \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{1}{\sqrt{J_2}} \\ \frac{\partial \sigma_{eq}^g}{\partial \psi} &= -2 \sin \psi \sqrt{J_2} \end{aligned} \quad (3-58)$$

Substituting Equations 3-16, 3-17, and 3-58 into Equation 3-55 gives the generalization for the M-D creep model used to describe the creep portion of the crushed-salt model:

$$\dot{\epsilon}_{ij}^i = \dot{\epsilon}_{eq}^i \left\{ \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right\} \quad (3-59)$$

Equation 3-59 is seen to be undefined as the Lode angle approaches $\pm\pi/6$. In other words, the flow potential forms corners at $\psi = \pm\pi/6$ and the direction of straining is not unique. To eliminate this

problem computationally, the flow potentials on either side of the corner are averaged, which produces an indeterminant form that is evaluated in the limit as $\psi \rightarrow \pm\pi/6$. Performing this limiting operation, Equation 3-59 becomes:

$$\lim_{\psi \rightarrow \pm \frac{\pi}{6}} \dot{\epsilon}_{ij}^i = \dot{\epsilon}_{eq}^i \left\{ \frac{s_{ij}}{\sqrt{3J_2}} \mp \frac{t_{ij}}{2J_2} \right\} \quad (3-60)$$

3.4.1 Evaluation of Laboratory Test Equation Forms

Reduction of the creep equations to triaxial compression and triaxial extension test forms follows the process completed for the creep consolidation equations. Using Equations 3-21 and 3-60, the axial, lateral, and volumetric strain components for the creep deformation portion of the model for triaxial compression conditions are:

$$\begin{aligned} \Delta \epsilon_a^i &= -\dot{\epsilon}_{eq}^i \Delta t \\ \Delta \epsilon_l^i &= \frac{\dot{\epsilon}_{eq}^i \Delta t}{2} \\ \Delta \epsilon_v^i &= 0 \end{aligned} \quad (3-61)$$

Using Equations 3-23 and 3-60, the axial, lateral, and volumetric strain components for the creep deformation portion of the model for triaxial extension conditions are:

$$\begin{aligned} \Delta \epsilon_a^i &= \dot{\epsilon}_{eq}^i \Delta t \\ \Delta \epsilon_l^i &= -\frac{\dot{\epsilon}_{eq}^i \Delta t}{2} \\ \Delta \epsilon_v^i &= 0 \end{aligned} \quad (3-62)$$

3.4.2 Equivalent Inelastic Strain-Rate Form

The equivalent inelastic strain-rate measure for the creep portion of the crushed-salt model is given by the multimechanism deformation (M-D) material model [Munson et al., 1989]. The model is written as:

$$\dot{\epsilon}_{eq}^i = F \dot{\epsilon}_s \quad (3-63)$$

where $\dot{\epsilon}_{eq}^i$ is the kinetic equation and $\dot{\epsilon}_s$ is the steady-state strain rate. The transient function F consists of three branches — a work-hardening branch, an equilibrium branch, and a recovery branch and is written in that order as:

$$F = \begin{cases} \exp \left[\Delta \left(1 - \frac{\zeta}{\epsilon_t^f} \right)^2 \right] & \zeta < \epsilon_t^f \\ 1 & \zeta = \epsilon_t^f \\ \exp \left[-\delta \left(1 - \frac{\zeta}{\epsilon_t^f} \right)^2 \right] & \zeta > \epsilon_t^f \end{cases} \quad (3-64)$$

Δ and δ are the work-hardening and recovery parameters, respectively, and ϵ_t^f is the transient strain-rate limit. The internal variable ζ is governed by the evolution equation:

$$\dot{\zeta} = (F - 1) \dot{\epsilon}_s \quad (3-65)$$

and the transient strain limit is given by:

$$\epsilon_t^f = K_0 e^{cT} \left(\frac{\sigma_e^g}{\mu} \right)^m \quad (3-66)$$

where:

T = temperature

μ = shear modulus

K_0 , c , and m = material parameters.

The work-hardening parameter is defined as a function of stress:

$$\Delta = \alpha + \beta \log \left(\frac{\sigma_e^g}{\mu} \right) \quad (3-67)$$

where α and β are material parameters. Because of insufficient data, the recovery parameter (δ) is taken to be a constant. However, functional forms similar to Equation 3-67 are sometimes used for recovery.

The steady-state strain rate is the sum of the three individual strain-rate mechanisms acting in parallel:

$$\dot{\epsilon}_s = \sum_{i=1}^3 \dot{\epsilon}_{s_i} \quad (3-68)$$

The three contributing mechanisms — dislocation climb, an undefined mechanism, and slip are written respectively as:

$$\dot{\epsilon}_{s_1} = A_1 \left[\frac{\sigma_e^g}{\mu} \right]^{n_1} \exp \left[-\frac{Q_1}{RT} \right] \quad (3-69)$$

$$\dot{\epsilon}_{s_2} = A_2 \left[\frac{\sigma_e^g}{\mu} \right]^{n_2} \exp \left[-\frac{Q_2}{RT} \right] \quad (3-70)$$

$$\dot{\epsilon}_{s_3} = \left(B_1 e^{-Q_1/RT} + B_2 e^{-Q_2/RT} \right) \sinh \left[q \left[\frac{\sigma_e^g - \sigma_0}{\mu} \right] \right] H(\sigma_e^g - \sigma_0) \quad (3-71)$$

where:

μ = shear modulus [MPa]

q = activation volume

$A_1, A_2, B_1, B_2, n_1, n_2, Q_1, Q_2$

$\sigma_0, K_0, m, \alpha, \beta, \delta$, and Δ = experimental constants

R = universal gas constant $\left[\frac{J}{\text{mol} \cdot \text{K}} \right]$

$H(.)$ = Heaviside step function.

Finally, substitution of Equation 3-63, along with associated Equations 3-64 through 3-71, into Equation 3-59 gives the generalization of the M-D model using the Tresca (maximum shear) type of flow potential for the creep deformation portion of the crushed salt constitutive model.

3.5 Summary of Crushed-Salt Model Forms

Equations describing three candidate crushed-salt constitutive models are summarized in this section — modified Sjaardema-Krieg model, modified Zeuch model, and modified Spiers model.

3.5.1 Modified Sjaardema-Krieg Model

The final equation for the total strain rate in the modified Sjaardema-Krieg constitutive model for crushed salt is obtained by substituting Equations 3-7, 3-19, 3-39, 3-59, and 3-63 into Equation 3-1:

$$\begin{aligned} \dot{\epsilon}_{ij} = & \frac{\bar{\sigma}_m}{3K} \delta_{ij} + \frac{\bar{s}_{ij}}{2G} + \left[1 + a_1 (1 - e^{-a_2 w}) \right] \times \\ & \frac{B_0 (1 + \epsilon_v)^2}{\rho_0 d^P} \exp \left[\frac{A \rho_0}{(1 + \epsilon_v)} \right] \left[\eta [1 - e^{-\eta_2 \sigma_m}] - |\sigma_1 - \sigma_3|^{m_1} \right] e^{-\frac{Q_c}{RT}} \times \\ & \left\{ \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \delta_{ij} - \frac{m_2 (2 \cos \psi \sqrt{J_2})^{m_2-1}}{\sigma_{r_2}^{m_2-1}} \left[\left[\frac{\cos 2 \psi}{\cos 3 \psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3 \psi} \right] t_{ij} \right] \right\} + \quad (3-72) \\ & F \dot{\epsilon}_s \left\{ \left[\frac{\cos 2 \psi}{\cos 3 \psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3 \psi} \right] t_{ij} \right\} \end{aligned}$$

Obviously, as the material approaches full consolidation, the fractional density approaches 1, and the M-D model creep component becomes the same as that for intact salt. Simultaneously, the creep consolidation portion of the model diminishes as the material approaches full consolidation. Therefore, the described model provides a smooth transition from crushed salt to intact salt behavior provided that the creep consolidation portion of the model is insignificant compared to the creep portion of the model near full consolidation.

Equation 3-72 may be used to derive the total engineering volumetric strain-rate ($\dot{\epsilon}_v$) expression for the model. To accomplish this, it is assumed that a reasonable representation of the volumetric strain rate can be obtained from the trace of the strain-rate tensor and that increments in time are reasonably small so that volumetric strain increments are well represented. Performing this operation yields:

$$\dot{\epsilon}_v = \frac{\dot{\sigma}_m}{K} + \left[\kappa \kappa_2 e^{-\kappa_2 \sigma_m} \right] \frac{B_0 (1 + \dot{\epsilon}_v)^2}{\rho_0 d^p} \exp \left[\frac{A \rho_0}{1 + \epsilon_v} \right] \times$$

$$\left[1 + a_1 (1 - e^{-a_2 w}) \right] \left[\eta [1 - e^{-\eta_2 \sigma_m}] - |\sigma_1 - \sigma_3|^{m_1} \right] e^{-\frac{Q_c}{RT}} \quad (3-73)$$

Equation 3-73 may be rewritten in terms of true strain using Equation 3-29 as:

$$\dot{\epsilon}_v = \frac{\dot{\sigma}_m}{K} + \left[\kappa \kappa_2 e^{-\kappa_2 \sigma_m} \right] \frac{B_0 e^{\epsilon_v}}{\rho_0 d^p} \exp \left[A \rho_0 e^{-\epsilon_v} \right] \times$$

$$\left[1 + a_1 (1 - e^{-a_2 w}) \right] \left[\eta [1 - e^{-\eta_2 \sigma_m}] - |\sigma_1 - \sigma_3|^{m_1} \right] e^{-\frac{Q_c}{RT}} \quad (3-74)$$

3.5.2 Modified Zeuch Model

Similar to the modified Sjaardema-Krieg model, the final equation for the total strain rate in the modified Zeuch two-stage constitutive model for crushed salt is obtained by substituting Equations 3-7, 3-19, 3-46, 3-47, 3-59, and 3-63 into Equation 3-1:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{\epsilon}_{ij} = \frac{\dot{\sigma}_m}{3K} \delta_{ij} + \frac{\dot{s}_{ij}}{2G} +$$

$$\frac{\rho_o b_7}{\rho d^p} D_0^{b_2} D^{2(b_2-n)-1} \left[\frac{D - D_0}{1 - D_0} \right]^{b_3-n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \sigma_{eq}^f \times$$

$$\left\{ \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \delta_{ij} - \frac{m_2 (2 \cos \psi \sqrt{J_2})^{m_2-1}}{\sigma_{r_2}^{m_2-1}} \left[\left[\frac{\cos 2 \psi}{\cos 3 \psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3 \psi} \right] t_{ij} \right] \right\} + \quad (3-75)$$

$$F \dot{\epsilon}_s \left\{ \left[\frac{\cos 2 \psi}{\cos 3 \psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3 \psi} \right] t_{ij} \right\}$$

For Stage 2 ($0.9 < D \leq 1.0$);

$$\begin{aligned} \dot{\epsilon}_{ij} = & \frac{\dot{\sigma}_m}{3K} \delta_{ij} + \frac{\dot{s}_{ij}}{2G} + \\ & \frac{\rho_0 b_8}{\rho d^p} \frac{(1-D)}{\{1-(1-D)^{1/n}\}^n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \sigma_{eq}^f \times \\ & \left\{ \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \delta_{ij} - \frac{m_2 (2 \cos \psi \sqrt{J_2})^{m_2-1}}{\sigma_{r_2}^{m_2-1}} \left[\left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right] \right\} + \quad (3-76) \\ & F \dot{\epsilon}_s \left\{ \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right\} \end{aligned}$$

Obviously, as the material approaches full consolidation, the fractional density approaches 1, and again, the M-D model creep component becomes the same as that for intact salt. Simultaneously, the creep consolidation portion of the model diminishes as the material approaches full consolidation and becomes zero when $D = 1$. Therefore, the described model provides a smooth transition from crushed salt to intact salt behavior.

Similar to the modified Sjaardema-Krieg model, the modified Zeuch model given in Equations 3-75 and 3-76 may be used to derive the engineering volumetric strain as:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\begin{aligned} \dot{\epsilon}_v = & \frac{\dot{\sigma}_m}{K} + \left[\kappa \kappa_2 e^{-\kappa_2 \sigma_m} \right] \left[\eta [1 - e^{-\eta_2 \sigma_m}] - |\sigma_1 - \sigma_3|^{m_1} \right] \times \\ & \frac{\rho_0 b_7}{\rho d^p} D_0^{b_2} D^{2(b_2-n)-1} \left[\frac{D - D_0}{1} \right]^{b_3-n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \\ & - D_0 \end{aligned} \quad (3-77)$$

For Stage 2 ($0.9 < D \leq 1.0$);

$$\begin{aligned} \dot{\epsilon}_v = & \frac{\dot{\sigma}_m}{K} + \left[\kappa \kappa_2 e^{-\kappa_2 \sigma_m} \right] \left[\eta \left[1 - e^{-\eta_2 \sigma_m} \right] - |\sigma_1 - \sigma_3|^{m_1} \right] \times \\ & \frac{\rho_0 b_8}{\rho d^p} \frac{(1 - D)}{\left\{ 1 - (1 - D)^{1/n} \right\}^n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \end{aligned} \quad (3-78)$$

Equations 3-77 and 3-78 may be rewritten in terms of true strain using Equations 3-31 and 3-32 as:

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\begin{aligned} \dot{\epsilon}_v = & \frac{\dot{\sigma}_m}{K} + \left[\kappa \kappa_2 e^{-\kappa_2 \sigma_m} \right] \left[\eta \left[1 - e^{-\eta_2 \sigma_m} \right] - |\sigma_1 - \sigma_3|^{m_1} \right] \times \\ & \frac{b_7}{d^p} D_0^{b_2} D^{2(b_2-n)-1} \left[\frac{D - D_0}{1 - D_0} \right]^{b_3-n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \end{aligned} \quad (3-79)$$

For Stage 2 ($0.9 < D \leq 1.0$);

$$\begin{aligned} \dot{\epsilon}_v = & \frac{\dot{\sigma}_m}{K} + \left[\kappa \kappa_2 e^{-\kappa_2 \sigma_m} \right] \left[\eta \left[1 - e^{-\eta_2 \sigma_m} \right] - |\sigma_1 - \sigma_3|^{m_1} \right] \times \\ & \frac{b_8}{d^p} \frac{(1 - D)}{\left\{ 1 - (1 - D)^{1/n} \right\}^n} \left[1 + a_1 (1 - e^{-a_2 w}) \right] \exp \left[-\frac{Q_c}{RT} \right] \end{aligned} \quad (3-80)$$

3.5.3 Modified Spiers Model

The final equation for the total strain rate described by the modified Spiers constitutive model for crushed salt is obtained by substituting Equations 3-7, 3-19, 3-52, 3-59, and 3-63 into Equation 3-1:

$$\begin{aligned} \dot{\epsilon}_{ij} = & \frac{\bar{\sigma}_m}{3K} \delta_{ij} + \frac{\bar{s}_{ij}}{2G} + \frac{r_1}{d^p} \left[1 + a_1 (1 - e^{-a_2 w}) \right] e^{-\frac{Q_c}{RT}} \left[\frac{(1 + \epsilon_v)^{r_2}}{|\epsilon_v|^{r_3}} \right] \Gamma \sigma_{eq}^f \times \\ & \left\{ \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \delta_{ij} - \frac{m_2 (2 \cos \psi \sqrt{J_2})^{m_2-1}}{\sigma_{r_2}^{m_2-1}} \left[\left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right] \right\} + \\ & F \dot{\epsilon}_s \left\{ \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right\} \end{aligned} \quad (3-81)$$

Similar to the other models, as the material approaches full consolidation, the fractional density approaches 1, and the M-D model creep component becomes the same as that for intact salt. Simultaneously, the creep consolidation portion of the model diminishes as the material approaches full consolidation. This decrease is implemented through the geometric Γ function, which becomes zero at full consolidation. Therefore, the described model provides a smooth transition from crushed salt to intact salt behavior.

Similar to the other models, the modified Spiers model given in Equation 3-81 may be used to derive the engineering volumetric strain as:

$$\begin{aligned} \dot{\epsilon}_v = & \frac{\bar{\sigma}_m}{K} + \left[\kappa \kappa_2 e^{-\kappa_2 \sigma_m} \right] \left[\eta [1 - e^{-\eta_2 \sigma_m}] - |\sigma_1 - \sigma_3|^{m_1} \right] \times \\ & \frac{r_1}{d^p} \left[1 + a_1 (1 - e^{-a_2 w}) \right] e^{-\frac{Q_c}{RT}} \left[\frac{(1 + \epsilon_v)^{r_3}}{|\epsilon_v|^{r_4}} \right] \Gamma \end{aligned} \quad (3-82)$$

Equation 3-82 may be rewritten in terms of true strain using Equations 3-31 and 3-32 as:

$$\begin{aligned} \dot{\epsilon}_v = & \frac{\bar{\sigma}_m}{K} + \left[\kappa \kappa_2 e^{-\kappa_2 \sigma_m} \right] \left[\eta [1 - e^{-\eta_2 \sigma_m}] - |\sigma_1 - \sigma_3|^{m_1} \right] \times \\ & \frac{r_1 e^{-\epsilon_v}}{d^p} \left[1 + a_1 (1 - e^{-a_2 w}) \right] e^{-\frac{Q_c}{RT}} \left[\frac{e^{r_3 \epsilon_v}}{|e^{\epsilon_v} - 1|^{r_4}} \right] \Gamma \end{aligned} \quad (3-83)$$

4.0 CRUSHED-SALT EXPERIMENTAL DATABASE SUMMARY

The development of a technically sound constitutive model for crushed-salt consolidation requires an extensive database from which model parameters can be estimated through model fitting. Constitutive models relate kinematical quantities such as displacement or strain to statical quantities such as stress. Therefore, the database must contain data acquired from experiments in which both kinematical and statical quantities are known (accurately controlled) or measured. A detailed discussion of the technical approach for identifying and accumulating crushed-salt data from WIPP and southeastern New Mexico for use in the development of crushed-salt constitutive models is given by Pfeifle [1995]. The purpose of this chapter is to summarize the experimental database compiled by Pfeifle, which was used to determine the parameter values for the candidate constitutive models.

4.1 Identification and Accumulation of Crushed-Salt data

Not all of the WIPP/southeastern New Mexico crushed-salt data identified in the literature search described by Pfeifle [1995] are appropriate for the development of a crushed-salt constitutive model. For example, some types of the tests did not provide measurements of the required kinematic and static quantities. Therefore, a data assessment was performed to identify appropriate data for WIPP crushed-salt model development. The data assessment was performed by establishing acceptance criteria against which the test data could be compared. The assessment criteria were as follows:

- Crushed-salt source: WIPP or other southeastern New Mexico sites.
- Bentonite content: 0 percent (by weight).
- Temperature: 293K to 373K.
- Grain size: < 10 mm.
- Specimen size: > 80 mm (in diameter).
- Moisture content: 0 percent (by weight) to saturated.
- Hydrostatic stress: 0 to 20 MPa.
- Stress difference: 0 to 5 MPa.
- Kinematic and static quantities: all known or measured.

When these criteria were applied to the data acquired from the tests on crushed salt and crushed-salt mixtures, the data from only five studies [Holcomb and Hannum, 1982; Pfeifle and Senseny, 1985; Holcomb and Shields, 1987; Zeuch et al., 1991; and Brodsky, 1994] were accepted for use in the constitutive model development. These five studies represent 81 tests, which include the following:

- 17 hydrostatic compaction tests.
- 53 hydrostatic consolidation tests.
- 11 triaxial compression shear consolidation tests.

These specific types of tests are described in the following paragraphs.

Hydrostatic Compaction Tests. In this test, a right-circular, cylindrically-shaped specimen is jacketed in a flexible protective sleeve and then placed in a standard triaxial compression testing machine or pressure vessel. Hydrostatic stress (σ_{hyd}) or mean stress (σ_m) is applied to the specimen by increasing the three principal stresses (σ_1 , σ_2 , and σ_3) at equivalent and uniform rates ($\dot{\sigma}_1 = \dot{\sigma}_2 = \dot{\sigma}_3 = \dot{\sigma}_{hyd}$) until a specified stress level is achieved. During the test, both the principal stresses and either the density, ρ , or the volumetric strain, ϵ_v , are measured. Also, in some cases, the loading is interrupted at various levels of stress or deformation to perform unload-reload cycles. The initial loading as well as the unloading and reloading are performed rapidly to minimize time-dependent deformations, and therefore, to isolate both time-independent inelastic deformations and elastic deformations. Since both the kinematic and static quantities are known (either controlled or measured), the test results are useful in the development of constitutive models.

Hydrostatic Consolidation Tests. A jacketed right-circular cylindrical specimen is also used in this test and is placed in a standard triaxial compression testing machine or pressure vessel. The test is performed by first hydrostatically loading (as described for the hydrostatic compaction test) the specimen to a specified hydrostatic stress level, and then maintaining the hydrostatic stress at the specified level for a prescribed period of time. The period of time while the hydrostatic stress is maintained constant is defined as the consolidation period or stage. During the hydrostatic loading, the density increases from the initial specimen density to the density at the initiation of the consolidation stage (ρ_o to ρ_i) or the volumetric strain increases from zero to the value corresponding to the initiation of consolidation stage ($\epsilon_v = \epsilon_{vo} = 0$ to $\epsilon_v = \epsilon_{vi}$). During the consolidation stage, the density increases from ρ_i to ρ_f (or $\epsilon_v = \epsilon_{vi}$ to $\epsilon_v = \epsilon_{vf}$). In general, only ρ_o and ρ_i (ϵ_{vo} and ϵ_{vi}) are measured during the hydrostatic loading; however, ρ and ϵ_v are measured continuously during the consolidation stage. The kinematical and statical quantities of deformation and stress are known (measured or controlled) during these tests, and therefore, can be used for constitutive model evaluations.

Triaxial Compression Shear Consolidation Tests. As with both the hydrostatic compaction and consolidation tests, the shear consolidation test is performed on a jacketed right-circular cylindrical specimen and makes use of a standard triaxial testing machine. The test is initiated by first loading the specimen hydrostatically to a specified hydrostatic stress level and then applying a stress difference by increasing the axial stress (σ_3) to a specified level above the hydrostatic stress. The specified hydrostatic stress level may be either the mean stress corresponding to the shear consolidation stage (i.e., $\sigma_m =$

$(\sigma_1 + \sigma_2 + \sigma_3)/3$) or the confining pressure specified for the shear consolidation stage. The consolidation period in this test is defined as the time when the specimen is subjected to the shear stress. As in the hydrostatic consolidation test, the density increases from ρ_o to ρ_i (or $\varepsilon_v = \varepsilon_{vo} = 0$ to $\varepsilon_v = \varepsilon_{vi}$) during the hydrostatic loading stage and from ρ_i to ρ_f or (from $\varepsilon_v = \varepsilon_{vi}$ to $\varepsilon_v = \varepsilon_{vf}$) during the shear consolidation stage. Again, only ρ_o and ρ_i (ε_{vo} and ε_{vi}) are measured during the hydrostatic loading; however, ρ , ε_v , and the axial strain, ε_3 , are measured continuously during the consolidation stage. The kinematical and statical quantities of deformation and stress are known (measured or controlled) during these tests, and therefore, can be used for constitutive model evaluations.

Although data from a large number of tests are available (i.e., 81), some limitations to the database exist. These limitations are summarized as follows:

1. All data are generated from stress-path controlled tests (i.e., the stresses rather than the strains were controlled during the tests).
2. Insufficient testing has been conducted for elastic properties (only one hydrostatic compaction test with unload/reload cycles has been completed, and no tests for either Young's modulus or shear modulus have been conducted).
3. All shear consolidation tests were performed at the same nominal temperature (i.e., 293 K).
4. All shear consolidation tests were performed at the same nominal moisture content (i.e., 2.5 wt%).

Because only stress-path controlled tests comprise the data base, we are unable to evaluate the model against strain controlled deformation that might simulate shaft closure more accurately. Limited elastic properties measurements mean that calculations of stresses within the consolidating salt mass will contain uncertainty. Constant temperature test data at 293 K mean that no temperature dependence can be determined, but this is of minor concern to the model because the salt column in the shaft will be nearly isothermal. Moisture content has been shown to be a very important parameter, so having a preponderance of the data at a constant moisture content limits refinement of this parameter.

4.2 Crushed-Salt Database For Model Evaluations

The crushed-salt data identified in the previous section provide the basis for the WIPP/southeastern New Mexico crushed-salt database used for constitutive model evaluations. The data were examined for completeness and then reformatted in a manner consistent with that required for the model fitting effort. The reformatting procedure was verified by plotting the data from each test and comparing the plots with similar plots presented in the original reference material. This section summarizes the results of this examination.

4.2.1 Examination and Summary of the Crushed-Salt Database

The data acquired from Sandia National Laboratories and the RE/SPEC data archive were examined for completeness by comparing data file names with the filenames identified for use in the constitutive

model fitting. From this examination, several data files (individual test results) were determined to be missing from the acquired data packages. Several attempts were made to locate the missing data with only moderate success. As a result, the following tests were removed from further consideration because the data could not be found or were not available:

<u>Reference</u>	<u>Missing Data File</u>
Holcomb and Hannum, 1982	09JUN82
	09JUN82(M)
	10JUN82
	10JUN82(M)
	11JUN82
	11JUN82(M)
	14JUN82
	14JUN82(M)
	15JUN82(M)
	16JUN82
Zeuch et al., 1991	08MAR89-2
	18AP89-2

The data files included under the first reference were to have contained hydrostatic compaction (HComp) results, while those listed under the second reference were to have contained hydrostatic consolidation (HCons) results. In addition to these data files, one shear consolidation (SCons) data file (08MY88 from Zeuch et al. [1991]) contained incomplete information (axial strain data were missing) and was also removed from further consideration. Therefore, ten hydrostatic compaction tests, two hydrostatic consolidation tests, and one shear consolidation test were removed from the identified crushed-salt data discussed in Section 4.1.

With the deletion of the identified tests, the WIPP/southeastern New Mexico crushed-salt database for constitutive model evaluations contains the data from 70 tests as described in Tables 4-1 through 4-5. The number of tests is 70 because a staged shear consolidation test that was originally a single test file was separated into different test files for each stage of the test (see Table 4-5, Tests 12OC891, 12OC892, and 12OC893), which added two test files. These 70 tests comprise results from:

Hydrostatic Compaction Tests	7 Tests
Hydrostatic Consolidation Tests	51 Tests
Shear Consolidation Tests	<u>12 Tests</u>
Total	70 Tests

Table 4-1. Summary of Test Conditions for Data Files in RSIALL1.CSV

Test I.D.	Test Type	Seq. No.	T (K)	σ_1 (MPa)	σ_3 (MPa)	Mean Grain Size (mm)	Added Water (%)	Test Duration (days)	ρ_o (kg/m ³)	ρ_i (kg/m ³)	ρ_f (kg/m ³)
CS1	1	1	298	-1.72	-1.72	2.30	0	4.00	1,430	1,490	1,548
CS2	1	2	298	-1.72	-1.72	1.36	0	4.30	1,480	1,570	1,720
CS3	1	3	298	-1.72	-1.72	0.77	0	4.03	1,410	1,480	1,552
CS4	1	4	298	-1.72	-1.72	0.42	0	4.02	1,290	1,400	1,485
CS5	1	5	298	-1.72	-1.72	2.30	4.44	3.80	1,360	1,440	1,697
CS6	1	6	298	-1.72	-1.72	1.36	4.28	4.01	1,440	1,500	1,775
CS7	1	7	298	-1.72	-1.72	0.77	4.57	4.34	1,420	1,470	1,572
CS8	1	8	298	-1.72	-1.72	0.42	4.61	3.99	1,400	1,430	1,822
CS9	1	9	298	-1.72	-1.72	2.30	0	37.01	1,480	1,530	1,759
CS10	1	10	298	-3.45	-3.45	2.30	0	34.21	1,470	1,570	1,648

Table 4-2. Summary of Test Conditions for Data Files in RSIALL2.CSV

Test I.D.	Test Type	Seq. No.	T (K)	σ_1 (MPa)	σ_3 (MPa)	Mean Grain Size (mm)	Added Water (%)	Test Duration (days)	ρ_o (kg/m ³)	ρ_i (kg/m ³)	ρ_f (kg/m ³)
HC1A	1	11	298	-1.72	-1.72	1.56	Sat	28.77	1,462	1,539	1,921
HC2A	1	12	298	-1.72	-1.72	1.56	Sat	98.66	1,388	1,574	2,026
HC3A	1	13	298	-3.45	-3.45	1.56	Sat	49.94	1,448	1,550	2,042
HC4A	1	14	298	-3.45	-3.45	1.56	Sat	110.76	1,409	1,610	2,065
HC5A	1	15	298	-6.90	-6.90	1.56	Sat	53.00	1,496	1,741	2,124
HC6A	1	16	298	-6.90	-6.90	1.56	Sat	77.94	1,395	1,721	2,125
SC1B	2	1	298	-3.45	-4.14	1.56	2.34	62.94	1,449	1,574	2,009
SC2A	2	2	298	-3.45	-4.83	1.56	2.25	61.92	1,479	1,608	2,014
SC3A	2	3	298	-3.45	-5.52	1.56	2.21	61.73	1,448	1,575	2,005
SC4A	2	4	298	-6.90	-7.59	1.56	2.27	60.85	1,428	1,607	2,007
SC5A	2	5	298	-6.90	-8.97	1.56	2.52	61.42	1,522	1,764	2,135
SC6A	2	6	298	-6.90	-10.34	1.56	2.19	67.91	1,375	1,701	2,053
SC7A	2	7	298	-5.17	-6.55	1.56	2.33	65.97	1,455	1,665	2,091
SC8A	2	8	298	-5.17	-7.93	1.56	2.29	60.77	1,423	1,611	2,000
SC9B	2	9	298	-5.17	-9.31	1.56	2.33	60.94	1,415	1,683	2,080

Table 4-3. Summary of Test Conditions for Data Files in SNLALLD.CSV

Test I.D.	Test Type	Seq. No.	T (K)	σ_1 (MPa)	σ_3 (MPa)	Mean Grain Size (mm)	Added Water (%)	Test Duration (days)	ρ_o (kg/m ³)	ρ_i (kg/m ³)	ρ_f (kg/m ³)
27JUL82	1	1	294	-1.72	-1.72	2.5	0	2.65	1,273	1,390	1,440
04MAY82	1	2	313	-1.72	-1.72	2.5	0	2.77	1,396	1,563	1,621
30APR82	1	3	333	-1.72	-1.72	2.5	0	2.99	1,448	1,605	1,677
07MAY82	1	4	353	-1.72	-1.72	2.5	0	2.87	1,240	1,415	1,492
12MAY82	1	5	373	-1.72	-1.72	2.5	0	4.81	1,371	1,621	1,738
05FEB82	1	6	294	-3.44	-3.44	2.5	0	3.40	1,263	1,432	1,466
09APR82	1	7	313	-3.44	-3.44	2.5	0	3.02	1,330	1,495	1,561
26MAR82	1	8	333	-3.44	-3.44	2.5	0	3.89	1,242	1,393	1,451
02APR82	1	9	353	-3.44	-3.44	2.5	0	2.72	1,249	1,428	1,501
13APR82	1	10	373	-3.44	-3.44	2.5	0	2.79	1,240	1,424	1,522
29JAN82	1	11	294	-6.70	-6.70	2.5	0	3.20	1,279	1,506	1,557
26FEB82	1	12	294	-10.1	-10.1	2.5	0	3.06	1,256	1,538	1,601

Table 4-3. Summary of Test Conditions for Data Files in SNLALLD.CSV (Continued)

Test I.D.	Test Type	Seq. No.	T (K)	σ_1 (MPa)	σ_3 (MPa)	Mean Grain Size (mm)	Added Water (%)	Test Duration (days)	ρ_o (kg/m ³)	ρ_i (kg/m ³)	ρ_f (kg/m ³)
08JUN82 ^(a)	3	1	294	(b)	(b)	2.5	0	3,001	1,290	(c)	1,658
08JUN82 ^(d)	3	2	294	(b)	(b)	2.5	0	2,718	1,280	(c)	1,653
23FEB82	3	3	313	(b)	(b)	2.5	0	6,359	1,300	(c)	1,677
25FEB82	3	4	333	(b)	(b)	2.5	0	6,607	1,280	(c)	1,716
16MAR82	3	5	353	(b)	(b)	2.5	0	6,728	1,310	(c)	1,788
25MAR82	3	6	373	(b)	(b)	2.5	0	7,350	1,220	(c)	1,644
24MAY82 ^(a)	3	7	294	(b)	(b)	2.5	0	Unknown	1,390	(c)	1,825

(a) Number of data points for Tests 08Jun82 and 24May82 are 94 and 497, respectively.

(b) Hydrostatic compaction from 0 to -20 MPa.

(c) No consolidation stage performed.

(d) WIPP Crushed Salt — all other tests on Mississippi Chemical Co. crushed salt.

Table 4-4. Summary of Test Conditions for Data Files in SNLALLW1.CSV

Test I.D.	Test Type	Seq. No.	T (K)	σ_1 (MPa)	σ_3 (MPa)	Mean Grain Size (mm)	Added Water (%)	Test Duration (days)	ρ_o (kg/m ³)	ρ_i (kg/m ³)	ρ_f (kg/m ³)
27JU61	1	1	293	-0.69	-0.69	1.75	2.5	31.76	1,440	1,493	1,732
23JL51	1	2	293	-1.72	-1.72	1.75	0.5	19.70	1,498	1,593	1,827
14NV51	1	3	293	-1.72	-1.72	1.75	1.5	24.08	1,477	1,571	1,860
25FE61	1	4	293	-1.72	-1.72	1.75	2.0	36.64	1,455	1,548	1,853
10MY51	1	5	293	-1.72	-1.72	1.75	2.4	28.94	1,412	1,601	1,890
20AU51	1	6	293	-1.72	-1.72	1.75	3.0	20.03	1,348	1,516	1,821
16JL51	1	7	293	-3.44	-3.44	1.75	0.5	20.58	1,391	1,608	1,920
18JU51	1	8	293	-3.44	-3.44	1.75	1.0	21.63	1,455	1,668	1,930
30OC51	1	9	293	-3.44	-3.44	1.75	1.5	13.78	1,477	1,648	1,930
16JA61	1	10	293	-3.44	-3.44	1.75	2.0	27.94	1,489	1,640	1,950
19DC44	1	11	293	-3.44	-3.44	1.75	2.4	52.12	1,434	1,566	1,899
13AU51	1	12	293	-3.44	-3.44	1.75	3.0	21.71	1,370	1,586	1,899
24OC611 ^(a)	1	13	293	-0.69	-0.69	1.75	2.4	11.70	1,470	1,496	1,676
24OC612 ^(a)	1	14	293	-0.34	-0.34	1.75	2.4	12.33	1,669	1,675	1,695
24OC613 ^(a)	1	15	293	-1.72	-1.72	1.75	2.4	15.39	1,691	1,709	1,801
24OC614 ^(a)	1	16	293	-3.44	-3.44	1.75	2.4	10.23	1,798	1,820	1,891
24OC615 ^(a)	1	17	293	-6.90	-6.90	1.75	2.4	48.23	1,883	1,907	2,018

(a) Number of data points for Tests 24OC611, 24OC612, 24OC613, 24OC614, and 24OC615 are 81, 30, 81, 72, and 80, respectively.

Table 4-5. Summary of Test Conditions for Data Files in SNLALLW2.CSV

Test I.D.	Test Type	Seq. No.	T (K)	σ_1 (MPa)	σ_3 (MPa)	Mean Grain Size (mm)	Added Water (%)	Test Duration (days)	ρ_o (kg/m ³)	ρ_i (kg/m ³)	ρ_f (kg/m ³)
24JL71	1	18	293	-1.72	-1.72	2	Sat	185.60	1,348	1,528	1,976
09JU881	1	19	293	-3.45	-3.45	2	Sat	128.60	1,370	1,491	1,941
09JU882	1	20	293	-6.90	-6.90	2	Sat	55.79	1,941	1,941	2,033
08MR89	1	21	293	-3.45	-3.45	2	Sat	258.28	1,348	1,492	1,973
20SE89	1	22	293	-6.90	-6.90	2	Sat	116.38	1,284	1,532	2,045
18AP89	1	23	293	-6.90	-6.90	2	Sat	193.99	1,284	1,532	1,893
12OC891	2	1	293	-3.26	-3.70	2	2.4	33.75	1,284	1,516	1,805
12OC892	2	2	293	-3.25	-3.97	2	2.4	17.50	1,804	1,806	1,836
12OC893	2	3	293	-3.24	-4.57	2	2.4	51.86	1,836	1,837	1,888

The data files for each contain 100 data points per test (except where noted) and are concatenated to form five separate major data files defined as RSIALL1.CSV, RSIALL2.CSV, SNLALLD.CSV, SNLALLW1.CSV, and SNLALLW2.CSV. The test data within each of these major data files are summarized in Tables 4-1 through 4-5. The *Test Type* code in the tables is defined as: 1 = hydrostatic consolidation (HCons), 2 = shear consolidation (SCons), and 3 = hydrostatic compaction (HComp). The maximum and minimum principal are given under σ_1 and σ_3 , respectively. The initial dry density, the density at the beginning of the consolidation stage, and the final density at the end of the test are given under ρ_0 , ρ_i , and ρ_f .

4.2.2 Specific Tests Excluded From the Model Fitting

An unwritten assumption used in model fitting is that the data are *good* data. During the preliminary model fitting exercises, several of the tests were determined to be *outliers*. Oftentimes, the conditions that make specific tests nontypical cannot be reconstructed. However, outliers must be discovered because their retention can dramatically affect the model fits. In this case, many of the tests excluded from the database for the model fitting were later stages from multistage tests. In these instances, the initial conditions for subsequent stages of a test are typically ill-defined. As shown in Chapter 3.0, all of the candidate constitutive models depend strongly on the initial density or volumetric strain. Eight tests were excluded from the model fitting database. Seven of these tests were excluded because they were subsequent stages of a particular test. One of the tests was excluded because it only contained 81 data points. All eight of these excluded tests were apparent outliers during the preliminary model fitting conducted. The specific tests in the crushed-salt database excluded from the model fitting exercise and reasons for their exclusion include:

Test I.D.	Description	Reason for Exclusion
24OC611	Table 4-4	Only contained 81 data points
24OC612	Table 4-4	Stage 2 of Test 24OC611
24OC613	Table 4-4	Stage 3 of Test 24OC611
24OC614	Table 4-4	Stage 4 of Test 24OC611
24OC615	Table 4-4	Stage 5 of Test 24OC611
09JU882	Table 4-5	Stage 2 of Test 09JU881
12OC892	Table 4-5	Stage 2 of Test 12OC891
12OC893	Table 4-5	Stage 3 of Test 12OC892

The final database used for fitting the constitutive models consisted of 55 tests, including 45 hydrostatic consolidation tests and 10 shear consolidation tests. None of the creep consolidation constitutive models were fitted to the hydrostatic compaction (i.e., quasi-static) tests in this study.

4.2.3 Presentation and Examination of the Crushed-Salt Database

Tables 4-6 and 4-7 present the hydrostatic consolidation and shear consolidation tests, respectively. The tables are constructed so that the number of tests conducted for a particular test variable (i.e., mean stress, added water, temperature, and mean grain size) is readily discernable. From Table 4-6, a bias in the hydrostatic consolidation tests is seen toward mean grain sizes of 1.75 and 2.5 mm and mean stress levels of -1.72 and -3.44 MPa. Table 4-7 shows that insufficient shear consolidation testing has been performed to determine the effects of added water, temperature, and grain size. An additional table is not presented for the hydrostatic compaction tests because Table 4-3 is sufficiently detailed to describe these tests.

The experimental data for the hydrostatic consolidation tests are shown in Figures 4-1 through 4-3, which plot the true inelastic volumetric strain as a function of time and represent the data included in the database. The tests are categorized by mean stress level (-1.72, -3.44 and -3.45, and -6.7 and -6.9 MPa) and plotted together. Each group of tests is plotted twice — once to indicate similar mean grain sizes and once to indicate similar water contents. For example, Figure 4-1 includes the -1.72 MPa mean stress tests. The top graph in Figure 4-1 plots tests with mean grain sizes greater than or equal to 2.0 mm as a solid line and tests with mean grain sizes less than 2.0 mm as a dotted line. The bottom graph in Figure 4-1 plots information identical to the top graph with tests saturated with brine plotted as a dot-dash line, tests with water additions nominally 4.5 percent by dry weight plotted as a dotted line, tests with water additions between 0.5 and 3.0 percent by dry weight plotted as a dashed line, and test with no added water plotted as a solid line. Figure 4-2 is plotted in the same manner for the -3.44 and -3.45 MPa mean stress level tests, and Figure 4-3 plots the -6.7 and -6.9 MPa mean stress level tests. All of the figures included in this section exclude those tests determined to be outliers.

The shear consolidation experimental data are shown in Figures 4-4 through 4-6, which plot the true inelastic volumetric, axial, and lateral strains, respectively, as a function of time. Figures 4-4 through 4-6 segregate the data by stress difference level ($\Delta\sigma = \sigma_1 - \sigma_3$) in the upper graphs and by mean stress level ($\sigma_m = (2\sigma_1 + \sigma_3)/3$) in the lower graphs. Recall that σ_1 is the axial stress (maximum compressive principal stress) and σ_3 is the lateral stress (minimum compressive principal stress).

The hydrostatic compaction test data are shown in Figures 4-7 and 4-8. Figure 4-7 illustrates mean stress as a function of time (load-time plot) in the upper graph and shows true inelastic volumetric strain as a function of time in the lower graph. Figure 4-8 gives the stress-strain curves for the tests. Except for the 24MAY82 experiment which had an initial density of 1,390 kg/m³, the range in initial densities for these specimens was from 1,220 kg/m³ to 1,310 kg/m³ (see Table 4-3). The hydrostatic compaction tests are segregated by loading rate. The slower loading rate tests accumulate more deformation which appears to indicate a rate dependency in the crushed salt. However, Figure 4-7 shows that little volumetric strain accumulates during the period in the tests where the load was held nearly constant.

Table 4-6. Summary of the Hydrostatic Consolidation Tests

Mean Stress (MPa)	Water Added (% Wt)	Temp. (K)	Mean Grain Size (mm)								Mean Stress Group Totals	
			2.5	2.3	2.0	1.75	1.56	1.36	0.77	0.42		Total
-0.34	2.4	293				1 ^(a)					1	1
-0.69	2.4	293				1					1	
	2.5	293				1 ^(a)					1	2
-1.72	0	294	1								1	
	0	298		2				1	1	1	5	21
	0	313	1								1	
	0	333	1								1	
	0	353	1								1	
	0	373	1								1	
	0.5	293				1					1	
	1.5	293				1					1	
	2.0	293				1					1	
	2.4	293				2 ^(a)					2	
	3.0	293				1					1	
	4.44	298		1							1	
	4.28	298						1			1	
	4.57	298							1		1	
	4.61	298								1	1	
	sat	293			1						1	
	sat	298					2				2	
-3.44	0	294	1								1	12
	0	313	1								1	
	0	333	1								1	
	0	353	1								1	
	0	373	1								1	
	0.5	293				1					1	
	1.0	293				1					1	
	1.5	293				1					1	
	2.0	293				1					1	
	2.4	293				2 ^(a)					2	
	3.0	293				1					1	
-3.45	0	298		1							1	
	sat	293			2						2	3
	sat	298					2				2	
-6.7	0	294									1	1
			1									
-6.9	2.4	293				1 ^(a)					1	6
	sat	293			3 ^(a)						3	
	sat	298					2				2	
-10.1	0	294	1								1	1
	Total		12	4	6	17	6	2	2	2	51	51

(a) One test determined to be an outlier and excluded from model fitting.

Table 4-7. Summary of the Shear Consolidation Tests

Stress Difference (MPa)	Mean Stress (MPa)	Water Added (% Wt)	Temp. (K)	Mean Grain Size (mm)			Stress Difference Group Totals
				2.0	1.56	Total	
0.44	-3.407	2.40	293	1		1	1
0.69	-3.68	2.34	298		1	1	
	-7.13	2.27	298		1	1	2
0.72	-3.49	2.40	293	1 ^(a)		1	1
1.33	-3.68	2.40	293	1 ^(a)		1	1
1.38	-3.91	2.25	298		1	1	
	-5.63	2.33	298		1	1	2
2.07	-4.14	2.21	298		1	1	
	-7.59	2.52	298		1	1	2
2.76	-6.09	2.29	298		1	1	1
3.44	-8.05	2.19	298		1	1	
4.14	-6.55	2.33	298		1	1	1
Total				3	9	12	12

(a) Test determined to be an outlier and excluded from model fitting.

Figure 4-1. Comparison of Hydrostatic Consolidation Test Results Conducted at a Mean Stress of -1.72 MPa.

Figure 4-2. Comparison of Hydrostatic Consolidation Test Results Conducted at a Mean Stress Level of -3.44 and -3.45 MPa.

Figure 4-3. Comparison of Hydrostatic Consolidation Test Results Conducted at a Mean Stress Level of -6.7 and -6.9 MPa.

Figure 4-4. Comparison of Shear Consolidation Tests True Inelastic Volumetric Strain Results.

Figure 4-5. Comparison of Shear Consolidation Tests True Inelastic Axial Strain Results.

Figure 4-6. Comparison of Shear Consolidation Tests True Inelastic Lateral Strain Results.

Figure 4-7. Comparison of the Loading Histories and True Volumetric Strain Results for the Hydrostatic Compaction Tests.

Figure 4-8. Mean Stress-True Volumetric Strain Relations for the Hydrostatic Compaction Tests.

Figures 4-1 through 4-8 present the experimental test results as they appear in the database with no modifications. While Figures 4-1 through 4-6 provide a good illustration of the actual test data, they tend to confound the effects of other variables in the tests because of the different initial densities for each of the tests. Therefore, Figures 4-9 through 4-14 replot the information given in Figures 4-1 through 4-6 with the data **translated** so that each test starts at the same initial density. The data translation was accomplished by selecting the largest initial density from the group of tests being plotted and translating the time and strain data pairs for the tests so that all of the starting data points (0,0) correspond to the same initial density. The base test (i.e., the test with the density that the other tests were translated to match) was selected so that as many tests as possible were included in the graphical presentation. Obviously, some of the tests never reached the magnitude of the base test density and, thus, do not appear on the plots. For the hydrostatic consolidation test plots (Figure 4-9 through 4-11), the initial densities for the -1.72, -3.44 and -3.45, and -6.9 MPa mean stress level tests are 1,621, 1,668, and 1,741 kg/m³, respectively (i.e., Tests 12MAY82, 18JU51, and HC5A form the bases, respectively). For the shear consolidation plots showing true inelastic volumetric, axial, and lateral strains (Figure 4-12 through 4-14), the initial density was 1,764 kg/m³ (i.e., Test SC5A is the basis). Ignoring history effects, these figures give a fair representation of the specimen-to-specimen variability, which appears to be fairly great if one discounts mean grain size and moisture effects. No attempt was made to adjust the initial conditions on the hydrostatic compaction tests.

By comparing the grain size/added water figures with each other (the upper and lower plots in Figures 4-9 through 4-11), one can see the effects of mean grain size and added moisture on the deformation of crushed salt. In all cases, an increase in deformation is seen with smaller mean grain sizes and added moisture. Anomalous behavior appears on each of the added moisture plots (the lower plot in Figures 4-9 through 4-11). In Figure 4-9, one of the saturated tests results appears to be too low; in Figure 4-10, two of the saturated tests exhibit the least deformation; and in Figure 4-11, two of the saturated tests exhibit about one-third of the deformation of the other two tests. In all cases, the seemingly low deformation level, saturated tests were conducted by Sandia National Laboratories. The test assembly used by Sandia National Laboratories differs slightly from RE/SPEC's test assembly in that the porous felt metal was placed between the beveled face plates rather than against the specimen. Thus, the Sandia National Laboratories' specimens were not as well-drained as those saturated tests conducted at RE/SPEC. This assembly difference was also noticed and discussed for permeability measurements on crushed salt [Brodsky et al., 1995]. The anomalous, saturated, hydrostatic consolidation tests are included in the results of the current model fitting effort. The influence of these tests on the parameter values is believed to be small because of the large size of the overall database and the fact that qualitatively the anomalous tests were similar. However, these tests should be removed from consideration in future model fitting studies.

Figures 4-12 through 4-14 illustrate the true inelastic volumetric, axial, and lateral strains as a function of time for the shear consolidation tests, respectively, which are translated to the same initial

Figure 4-9. Comparison of Translated Hydrostatic Consolidation Test Results Conducted at a Mean Stress of -1.72 MPa.

Figure 4-10. Comparison of Translated Hydrostatic Consolidation Test Results Conducted at a Mean Stress of -3.44 MPa.

Figure 4-11. Comparison of Translated Hydrostatic Consolidation Test Results Conducted at a Mean Stress Level of -6.7 and -6.9 MPa.

Figure 4-12. Comparison of Translated Shear Consolidation Tests True Inelastic Volumetric Strain Results.

Figure 4-13. Comparison of Translated Shear Consolidation Tests True Inelastic Axial Strain Results.

Figure 4-14. Comparison of Translated Shear Consolidation Tests True Inelastic Lateral Strain Results.

density. The upper and lower graphs in each figure present the same data with the upper graph showing the effect of stress difference level and the lower graph showing the effect of mean stress level. The true inelastic volumetric strain and axial strain figures show, in general, that the magnitude of the strain increases with both increasing stress difference and mean stress. However, this increase in strain magnitude is not evident for the true lateral strains illustrated in Figure 4-14. The reason for this type of lateral strain behavior can be explained, at least in part, by the fact that the deviatoric and mean stress driven portions of the deformation are competing processes for the lateral strain. The deviatoric stress (i.e., stress generated by a larger compressive axial stress) creates a positive or outward movement in the lateral direction of the test specimens; whereas, the mean stress creates a negative or inward movement in the lateral direction of the test specimens. Therefore, the magnitude of the lateral strains in the tests depends on the relative magnitudes of the deviatoric and mean stress driving forces generated in the experiment.

5.0 APPROACH TO MODEL FITTING

5.1 Introduction

A fundamental component of the constitutive model screening process involves the evaluation of the ability of each of the models to reproduce laboratory-measured responses. Three of the ten candidate models identified in the literature review (Chapter 2.0) were judged to be potentially able to reproduce the laboratory-measured data and were mathematically fit to the database comprised of hydrostatic consolidation and shear consolidation tests. The three constitutive models that were fit to the laboratory database are:

- Sjaardema-Krieg
- Zeuch
- Spiers.

The constitutive models were fit to the laboratory data by determining the model parameters such that the weighted square of the difference between the measured and calculated response was minimized. Application of the weighted least-squares fitting criterion to the constitutive models being considered results in a requirement to solve simultaneous nonlinear equations. These complex nonlinear equations were solved with the **BMDP** statistical software package [Frane et al., 1982].

Two different types of laboratory test responses were used in the least-squares fitting: hydrostatic consolidation tests and shear consolidation tests. Three distinct weighted least-squares fits were performed for each of the constitutive models. The first fit was made to the hydrostatic consolidation test database only. The second fit was made to the shear consolidation test database only, and the final fit was made to a database containing both the hydrostatic and the shear consolidation tests. At the onset, pertinent statements can be made relevant to the databases. As developed in Chapter 3.0, each of the creep consolidation constitutive models contains a volumetric part and a deviatoric part. No information can be obtained about the deviatoric portion of the models when the hydrostatic consolidation database is used because the deviatoric stress (stress difference) is zero in these experiments. Therefore, no information can be obtained for parameters m_1 and m_2 from the hydrostatic consolidation tests. As shown in Table 4-7, the temperature, moisture content, and particle size variations in the shear consolidation experimental database are insufficient to obtain information regarding these variables; therefore, no information can be obtained for parameters a_1 , a_2 , p , and Q_c/R from the shear consolidation tests. In the remaining subsections of this chapter, the model fitting procedure, the statistical measures used to evaluate goodness of fit, and the use of the commercial statistical computer software, **BMDP**, are discussed.

5.2 Modeling Procedure

The object function is the mathematical function that is to be minimized in the least-squares sense. For example, when performing a least-squares fit of some data to a straight line, the object function is the difference between the measured response (the data) and the predicted response from the equation for the straight line. The equation for the straight line when performing linear regression is called the response model.

The object function is evaluated at discrete points. These points are the points where data have been taken or are available. Some of the measurements may be more important than others and should hold more importance or “weight” than the other points. Mathematically, the relative importance of the level of fit to each data point is accommodated through a weighting function.

Application of least squares results in an estimation of the parameters of the response model. In the example cited above, application of least squares will result in the estimation of the two parameters necessary to specify a straight line. Application of least squares involves an implicit assumption that the response of the system will be normally distributed about some mean for fixed values of the (hopefully) independent parameters. For complex models, the independence of the parameters is not always obvious before fitting. For example, in the case of the straight-line model considered above, the measured output at fixed independent parameter conditions would be uniformly distributed about a mean. If the system response is not uniformly distributed, but in fact, “skewed” about some mean response, the parameter estimators from the least-squares application may not be appropriate.

A selected response model will almost never reproduce all of the measured data points. The difference between the measured data points and the predicted model response is termed the residual. When a weighting function is used in the model fitting, the weighted residual is also of interest. The weighted residual is the usual residual squared multiplied by the weighting function value for the specific observation or data point. Three different object functions are minimized in the fitting of the constitutive models to the hydrostatic consolidation and shear consolidation tests. The first object function involves the hydrostatic tests only, the second involves the shear consolidation tests only, and the third involves both the hydrostatic and shear consolidation tests.

The object functions in this section are developed under the following assumptions and reflect the relative importance associated to individual data points:

1. The data at the beginning of each test are equal in importance in comparison to the data at the end of each test. Within a given test (either hydrostatic or shear consolidation), there are an equal number of measurement (data) points per test; i.e., 100. However, these data points are not uniformly distributed with respect to time; the data are more dense early in time when the strain rates are highest. Thus, the weight function that was used in this model fitting reflects the time step

size relative to each data point in each test, such that higher weights are assigned to data points at the end of the test than are assigned to early-time data points.

2. Tests of longer duration are assumed to be more important than tests of shorter duration. As the crushed salt consolidates with time, the fractional density asymptotically approaches one. It is important to characterize this asymptotic behavior; i.e., as the crushed salt becomes intact salt, the predicted response of the models should be continuous.
3. Residuals were normalized. They were represented as percent deviations from the measured data. Thus, a 10 percent deviation from a very small strain results in the same error in the object function as a 10 percent deviation from a very large strain.

5.2.1 Response Function

In the hydrostatic consolidation tests, there is only one independent measurement; namely, the volumetric strain (ϵ_v^m). The axial (ϵ_a^m) and lateral (ϵ_l^m) strains are assumed to be equal and related to the volumetric strain as:

$$\begin{aligned}\epsilon_v^m &= \epsilon_a^m + 2\epsilon_l^m \\ &= 3\epsilon_a^m\end{aligned}\tag{5-1}$$

When fitting to the hydrostatic consolidation tests only, the response function is the predicted axial strain (ϵ_a^p).

In the shear consolidation tests, there are two independent measurements; namely, the volumetric strain (ϵ_v^m) and the axial strain (ϵ_a^m). When fitting to the shear consolidation tests, two response functions were required; namely, the predicted axial strain (ϵ_a^p) and the predicted lateral strain (ϵ_l^p), where:

$$\epsilon_l^p = (\epsilon_v^p - \epsilon_a^p)/2\tag{5-2}$$

Each of the three candidate models is given in terms of rate equations that express the axial ($\dot{\epsilon}_a^p$) and lateral ($\dot{\epsilon}_l^p$) strain rates as functions of mean stress (σ_m), stress difference ($\Delta\sigma$), initial fractional density (ρ_0), current volumetric strain (ϵ_v), mean particle size (d), water content (w), and temperature (T), of the form:

$$\begin{aligned}\dot{\epsilon}_a^p &= f_1(\rho_0, \epsilon_v) f_a(\sigma_m, \Delta\sigma) \sigma_{eq}^f(\sigma_m, \Delta\sigma) \xi(d, w, T) \\ \dot{\epsilon}_l^p &= f_1(\rho_0, \epsilon_v) f_l(\sigma_m, \Delta\sigma) \sigma_{eq}^f(\sigma_m, \Delta\sigma) \xi(d, w, T)\end{aligned}\tag{5-3}$$

where f_1 describes the functional dependence on ρ_0 and ε_v unique to each of the candidate models, and:

$$\varepsilon_v = \text{current volumetric strain} = \varepsilon_a + 2\varepsilon_l$$

$$\varepsilon_a = \text{current axial strain}$$

$$\varepsilon_l = \text{current lateral strain}$$

$$f_a(\sigma_m, \Delta\sigma) = \frac{m_2(\Delta\sigma)^{m_2-1}}{\sigma_{r_2}^{m_2-1}} + \kappa\kappa_2 \exp(-\kappa_2\sigma_m)/3$$

$$f_l(\sigma_m, \Delta\sigma) = -\frac{m_2(\Delta\sigma)^{m_2-1}}{2\sigma_{r_2}^{m_2-1}} + \kappa\kappa_2 \exp(-\kappa_2\sigma_m)/3$$

$$\sigma_{eq}^f(\sigma_m, \Delta\sigma) = \eta(1 - \exp(-\eta_2\sigma_m)) - \Delta\sigma^{m_1}$$

$$\xi(d, w, T) = \left\{ 1 + a_1[1 - \exp(-a_2w)] \right\} \exp(-Q_c/RT)/d^p$$

$$\eta = \eta_0(1 - D)^{\eta_1}$$

$$\kappa = \kappa_0(1 - D)^{\kappa_1}$$

$$D = \text{current fractional density} = \rho/\rho_f$$

$$\rho = \text{current density} = \rho_0/(1 + \varepsilon_v)$$

$$\rho_0 = \text{emplaced density}$$

$$\rho_f = \text{intact density} = 2,160 \text{ kg/m}^3$$

$$\sigma_m = \text{mean stress}$$

$$= \sigma_1 + \sigma_2 + \sigma_3$$

$$\Delta\sigma = |\sigma_1 - \sigma_3|$$

$$\sigma_1, \sigma_2, \sigma_3 = \text{principal stresses}$$

$$\eta_0, \eta_1, \eta_2, \kappa_0, \kappa_1, \kappa_2, m_1, m_2, a_1, a_2, Q_c/R, \text{ and } p = \text{material parameters.}$$

Modified Sjaardema-Krieg Model

The functional dependence of f_1 on initial density (ρ_0) and current volumetric strain (ε_v) for the modified Sjaardema-Krieg model is given by:

$$f_1(\rho_0, \varepsilon_v) = B_0(1 + \varepsilon_v)^2 \exp(A\rho)/\rho_0 \quad (5-4)$$

where B_0 and A are additional material parameters. Thus, the modified Sjaardema-Krieg model has a total of 14 total material parameters.

Modified Zeuch Model

The functional dependence of f_1 on ρ_0 and ε_v according to the modified Zeuch model is given by:

For Stage 1 ($D_0 < D < 0.9$);

$$f_1(\rho_0, \varepsilon_v) = b_7 D_0^{b_2} D^{2(b_2-n)-1} \left[(D - D_0)/(1 - D_0) \right]^{b_3-n} \quad (5-5)$$

For Stage 2 ($0.9 < D < 1.0$);

$$f_1(\rho_0, \varepsilon_v) = b_8 (1 - D) / \left\{ 1 - (1 - D)^{1/n} \right\}^n \quad (5-6)$$

where:

$$D_0 = \text{initial fractional density} = \rho_0/\rho_f$$

and b_2 , b_3 , b_7 , b_8 , and n are additional material parameters. Thus, the modified Zeuch model has a total of 17 material parameters.

Modified Spiers Model

The functional dependence of f_1 on ρ_0 and ε_v according to the modified Spiers model is given by:

$$f_1(\rho_0, \varepsilon_v) = r_1(1 + \varepsilon_v)^{r_3} / |\varepsilon_v|^{r_4} \Gamma \quad (5-7)$$

where:

$$\Gamma = \begin{cases} 1 & \varepsilon_v < 0.15 \\ \left[\frac{\varepsilon_v + \phi_0}{\phi_0(\varepsilon_v + 1)} \right]^n & \varepsilon_v > 0.15 \end{cases} \quad (5-8)$$

ϕ = initial porosity

and r_1 , r_3 , r_4 , and n are additional material parameters. Thus, the modified Spiers model has a total of 16 material parameters.

5.2.2 Object Function

The object function is the sum-of-squared errors (SS_E):

$$SS_E = \sum_{j=1}^J \int_0^{t_j^* / \bar{t}} \left\{ \left[1 - \frac{\varepsilon_a^p(\tau)}{\varepsilon_a^m(\tau)} \right]^2 + \left[1 - \frac{\varepsilon_l^p(\tau)}{\varepsilon_l^m(\tau)} \right]^2 \right\} d\tau \quad (5-9)$$

$$= \sum_{j=1}^J \sum_{i=1}^I \left\{ \left[1 - \frac{\varepsilon_a^p(i,j)}{\varepsilon_a^m(i,j)} \right]^2 + \left[1 - \frac{\varepsilon_l^p(i,j)}{\varepsilon_l^m(i,j)} \right]^2 \right\} \frac{\Delta t(i,j)}{\bar{t}}$$

where:

J = number of tests

I = number of data points per test

t_j^* = total time of the j^{th} test

$\varepsilon_a^p(t)$ = predicted axial strain at time t

$\varepsilon_a^m(t)$ = measured axial strain at time t

$\varepsilon_l^p(t)$ = predicted lateral strain at time t

$\varepsilon_l^m(t)$ = measured lateral strain at time t

Δt = time step size

$\bar{t}$ = normalizing time = 10^6 s.

In Equation 5-9, the weighted residual for each test is integrated over the total time domain of that test. The specific weighting associated with this fit is equal to the reciprocal of the measured axial strain squared $(\epsilon_a^m)^2$, the reciprocal of the measured lateral strain squared $(\epsilon_l^m)^2$, and time increment $(\Delta t(i,j))$. Thus, the SS_E term has several desirable characteristics. If the response models predict zero axial strain and zero lateral strain over the entire duration of the test, the term SS_E will have a value of two times the number of million seconds in the test for that test. If the response model is capable of reproducing the measured axial and lateral strains over the duration of the test, the term SS_E will have a value of zero for that test. Each test contributes to the total SS_E . Therefore, if the response model predicts zero axial and lateral strains for all of the hydrostatic consolidation tests, the maximum value of SS_E will be equal to $2 \sum_{j=1}^J t_j^* / \bar{t} = 2(152.3) = 304.6$, where J is the number of tests included in the fit, and t_j^* is the total time of the j th hydrostatic test. Similarly, if the response model is a perfect predictor for each of the tests, the term SS_E will sum to zero. In the case of the shear consolidation tests, the maximum value of SS_E is $2(51.7) = 103.4$.

5.3 Statistical Measures

One key aspect of the model-fitting effort is deciding if one model is better than another. In this section, the statistical measures for evaluating and comparing the candidate constitutive models are discussed.

5.3.1 Weighted Residual

In all types of regression, the residual squared times a weighting function is the function that is minimized. The residual is defined as the difference between the observed and predicted functional values. Thus, if one uses a common database (observed values) and the same weighting function, the weighted residual can be used to evaluate various response models. Obviously, the lower the weighted residual, the better the fit.

5.3.2 Coefficient of Multiple Determination

A measure of the adequacy of regression model that is widely recognized in statistics is the coefficient of multiple determination, R_p^2 , defined as:

$$R_p^2 = 1 - \frac{SS_E}{SS_t} \quad (5-10)$$

where:

$$SS_t = \sum_{j=1}^J \left(\varepsilon_j^m - \bar{\varepsilon}^m \right)^2 w_j$$

J = total number of test data points

$\bar{\varepsilon}^m$ = mean value of all measured strains

$$= \sum_{j=1}^J \varepsilon_j^m / J.$$

R_p^2 increases and approaches one as SS_E decreases. Thus, models with large values of R_p^2 are preferred. Clearly, use of this measure requires judgment on the part of the analyst.

5.3.3 Mean-Squared Error

The residual mean-squared error (MS_E) can be expressed mathematically as:

$$MS_E = \frac{SS_E}{\sum_{j=1}^J w_j - p} \quad (5-11)$$

where p is the number of material parameters for a particular model.

In general, one attempts to minimize the residual mean square. Because the sum-of-squared error (SS_E) decreases as the number of parameters (p) increases, the mean-squared error (MS_E) initially decreases, then stabilizes, and eventually may increase. The eventual increase in MS_E occurs when the reduction in SS_E from adding a parameter to the model is not sufficient to compensate for the loss of one degree of freedom in the denominator of Equation 5-11. This measure can be used to judge whether an additional parameter added to the model is useful in that it provides a substantial increase in the goodness of fit.

5.3.4 Parameter Multicollinearity

When fitting a mathematical relation to a set of laboratory-measured data, it is desirable to have enough information in the measured database to ensure that each of the parameters can be uniquely determined. Two possibilities can occur that will not allow for a unique determination of each parameter in a mathematical relation. Firstly, the mathematical relation may inherently include parameters that are dependent upon one another. For example, in fitting the candidate models to the hydrostatic consolidation tests ($\Delta\sigma = 0$), the rate equation defining the predicted axial strain ($\dot{\varepsilon}_a$) reduces to:

$$\begin{aligned}\dot{\epsilon}_a &= f_1(\rho_0, \epsilon_v) \kappa \kappa_2 \exp(-\kappa_2 \sigma_m) / 3 \eta [1 - \exp(-\eta_2 \sigma_m)] \xi(d, w, T) \\ &= \kappa_0 \kappa_2 \eta_0 (1 - D)^{\eta_1 + \kappa_1} f_1(\rho_0, \epsilon_v) \exp(-\kappa_2 \sigma_m) / 3 [1 - \exp(-\eta_2 \sigma_m)] \xi(d, w, T)\end{aligned}\quad (5-12)$$

where κ_0 , κ_1 , κ_2 , η_0 , η_1 and η_2 are material parameters.

Clearly, the parameters κ_0 , κ_2 and η_0 are not independent in this model, and likewise for parameters η_1 and κ_1 . In addition, the parameters m_1 and m_2 that describe the dependence on $\Delta\sigma$ in Equation 3-11 are nonfunctional in Equation 5-12.

Secondly, there may not be sufficient data in the database to uniquely determine the magnitude of certain model parameters. For example, there is no variation in grain size (d), moisture content (w), or temperature (T) in the shear consolidation tests ($\Delta\sigma \neq 0$). Thus, in fitting the candidate models to these tests, the parameters p , a_1 , a_2 , and Q_c/R in the term:

$$\xi(d, w, T) = \left\{ 1 + a_1 [1 - \exp(-a_2 w)] \right\} \exp(-Q_c/RT) / d^p \quad (5-13)$$

cannot be uniquely determined.

When either of the two aforementioned conditions exist, a condition of multicollinearity is said to exist [Montgomery and Peck, 1982]. An examination of the parameter covariance matrix will determine the degree to which the material parameters are correlated to each other. The higher the parameter covariance, the higher the parameter collinearity. When a multicollinearity situation exists, one must (1) expand the database of measurements or (2) modify the functional form of the model.

The elements of the correlation matrix (C_{ij}) can be expressed as:

$$C_{ij} = \frac{q_{ij}}{\sqrt{q_{ii} q_{jj}}} \quad (5-14)$$

where:

$$\begin{aligned}q_{ij} &= [z_{ip} z_{pj}]^{-1} \\ z_{ij} &= \frac{\partial \epsilon^m(x_j, \bar{p})}{\partial p_i} \\ p_i &= i^{\text{th}} \text{ parameter} \\ x_j &= j^{\text{th}} \text{ model variable.}\end{aligned}$$

A scalar quantity, termed the parameter correlation measure (pcm), was used in this study as a global measure of the parameter correlations associated with each model's fit to each database. This measure is defined as:

$$\text{pcm} = \frac{\frac{1}{2} \left[\sum_{i=1}^p \sum_{j=1}^p C_{ij}^2 - p \right]}{\sum_{i=1}^{p-1} i} = \frac{\sum_{i=1}^p \sum_{j=i}^p C_{ij}^2 - p}{p^2 - p} \quad (5-15)$$

where p is the number of parameters in the model and $0 \leq \text{pcm} \leq 1$. The numerator in Equation 5-15 is the sum of the squares of the lower triangular components of the parameter correlation matrix (Equation 5-14), and the denominator simply normalizes the result by the number of components in the sum. Obviously, the lower the parameter correlations, the lower the magnitude of the parameter correlation measure.

5.3.5 Parameter Variation Amongst Fits

The candidate models were fit to three databases: (1) the hydrostatic consolidation tests only, (2) the shear consolidation tests only, and (3) the combination of (1) and (2). If a specific model is truly a constitutive law in representing the consolidation of crushed salt, the material parameters in each of the different fits should be the same or nearly the same. To evaluate the parameter variation in each of the models, comparisons of the parameter values determined in the fits to the hydrostatic and shear consolidation tests were made to the parameter values determined in the fit to the combined database. The ratio of the number of parameters that changed by more than one order of magnitude to the total number of model parameters was calculated for each of the fits to the hydrostatic and shear tests.

5.3.6 Predictive Capability

The aptness of each of the candidate models (predictive capability) can be demonstrated using the parameter values determined by fitting to the individual test databases. The material parameters determined in the fit to the shear consolidation tests were used to predict each of the hydrostatic consolidation tests. A quantitative measure of the predictive capability is simply the sum-of-squared error (Equation 5-9). This measure was calculated using the BioMedical Data Processing (BMDP) Program AR (Section 5.4) by fixing the parameter values at those determined in the fit to the shear consolidation tests and fitting (predicting) the hydrostatic consolidation tests.

5.4 BMDP Statistical Software

The purpose of this section is to describe in detail the input to the BioMedical Data Processing (BMDP) nonlinear regression program AR used in the model fitting. BMDP is a statistical software package developed at the University of California at Berkeley supported by grants from the Biotechnology Resources Branch of the National Institutes of Health and the National Science Foundation.

The BMDP Program AR is a derivative-free, nonlinear regression program that estimates the parameters of a nonlinear function by least squares using a pseudo Gauss-Newton algorithm. The rate equations defining the regression function are specified in the input file. Response model derivatives are evaluated numerically. In addition to calculating the weighted sum-of-squared error (SSE_E), the program AR calculates the coefficient of multiple determination (R_p^2), the residual mean-squared error (MS_E), and the parameter correlation matrix. The various program features and supporting documentation of AR are provided in various BMDP technical reports, including Frane et al. [1982].

The input files required to execute an AR analysis are described in detail by Dixon et al. [1985] and in the BMDP Users' Digest [Hill, 1984]. Basically, the input files can be thought of as a series of paragraphs which are further subdivided into sentences using key words. Each paragraph is activated by a backslash followed immediately by a key word. Each of these sentences are activated by key words and ended with periods. All comment cards are noted using the (#) symbol. The BMDP input files for each of the three models fit to each of the three databases are given in Appendix A.

These input files are composed of 12 paragraphs. The first paragraph (/INPUT) contains the title card, the database file name, the format to be used in reading the data, and the number of variables in the database. The second paragraph (/VARIABLES) names each of the 18 variables contained in the database file. The third paragraph (/TRANS) identifies the added variables that are used in the fit. MS and DS are the mean stress and stress difference, respectively. D0 and DI are the emplaced and initial fractional density, respectively. The variable USE identifies which test database is to be used in the fit, and the variable WT is the weight function. The fourth paragraph (/REGRESS) identifies the dependent variable (DEPEND), the number of parameters in the model, which variable is the weight function (WEIGHT), which variable number is the integration variable (ITIME) in the rate equations, the number of differential equations (NEQN), the number of iterations (ITER), the number of interval halving for each iteration (HALVINGS), and the maximum number of integration steps (MAXC). The fifth paragraph (/PARAMETER) identifies the parameter names and the initial estimates, as well as the minimum and maximum values for each of the parameters. The sixth paragraph (/DIFIN) identifies the initial values for each of the differential equations. The seventh paragraph (/DIFEQ) specifies the differential equation (DZ1). The eighth paragraph (/FUN) identifies the dependent variable function (F). In the case of the fit to the hydrostatic consolidation tests only, the dependent variable is axial strain (EAT). In the case of the fits to the shear consolidation test database and to the combined test database

the dependent variable is the function (ρ) defined below. The ninth through eleventh paragraphs (/SAVE, /PRINT, and /PLOT) specify output options for AR. The last paragraph (/END) identifies the end of the input file.

To find a single "measurement quantity" (ρ) as an object function to the combined test database, the bracketed quantity in Equation 5-9 is rewritten as:

$$\left[(1 - \alpha)^2 + (1 - \beta)^2 \right] a = (1 - \rho)^2 \quad (5-16)$$

where:

$$\alpha = \varepsilon_a^p / \varepsilon_a^m$$

$$\beta = \varepsilon_t^p / \varepsilon_t^p$$

$$a = \Delta t / \bar{t}$$

Solving for the desired-single object function:

$$\rho = 1 - \left\{ \left[(1 - \alpha)^2 + (1 - \beta)^2 \right] a \right\}^{1/2} \quad (5-17)$$

As defined in the /TRANSFORM paragraph, the calculated quantity ρ is fit to the dependent variable ONE.

6.0 RESULTS

In this section of the report, the results of the model fits for each of the three candidate constitutive laws are presented. The response models presented in Section 5.2.1 of this report have each been fit to (1) the hydrostatic consolidation test database, (2) the shear consolidation test database, and (3) a combined database of both types of tests. The results of the fits are discussed in terms of the model parameters and the statistical measures of the fits for each of the constitutive laws.

Each of the models contains 12 common parameters; i.e., η_0 , η_1 , η_2 , κ_0 , κ_1 , κ_2 , m_1 , m_2 , a_1 , a_2 , Q_c/R , and p in Equation 5-3. The modified Sjaardema-Krieg model contains an additional two parameters (B_0 and A in Equation 5-4). The modified Zeuch model contains an additional five parameters (b_2 , b_3 , b_7 , b_8 , and n in Equations 5-5 and 5-6). The modified Spiers model contains an additional four parameters (r_1 , r_3 , r_4 , and n in Equations 5-7 and 5-8). All of the parameter values were determined in the fit to the combined database. Consequently, this fit was performed first for all three models. In fitting to the hydrostatic consolidation database, the stress difference ($\Delta\sigma$) is zero. Thus, in performing the fit to the hydrostatic consolidation test database, the two parameters (i.e., m_1 and m_2) that define the $\Delta\sigma$ dependence are nonfunctional. In addition (as pointed out in Section 5.3.4 and demonstrated in Equation 5-12), not all of the parameters are independent in the hydrostatic model. Thus, three parameters (i.e., κ_1 , η_0 , and η_1) were fixed at the values determined in the fit to the combined database. The shear consolidation tests were performed at nearly constant water content (w), constant temperature (T), and constant grain size (d). Thus, in performing the fit to the shear consolidation test database, the four parameters (i.e., a_1 , a_2 , Q_c/R , and p) that define the dependence on w , T , and d were fixed at the values determined in the fit to the combined database.

6.1 Parameter Values

This section presents the sets of parameters values that were obtained for the fits to the hydrostatic consolidation test database, the shear consolidation test database, and the combined database, respectively. The parameter values for the modified Sjaardema-Krieg model, the modified Zeuch model, and the modified Spiers model are given in Tables 6-1 through 6-3, respectively. For comparative purposes, the parameter values used by the original researchers of these models are given in the rightmost column in each table.

For all three models, the parameters η_0 and κ_0 are leading coefficients, the parameters κ_2 and η_2 define the functional dependence on mean stress, the parameters m_1 and m_2 define the functional dependence on stress difference, the parameters η_1 and κ_1 define the functional dependence on density, the parameters a_1 and a_2 define the functional dependence on moisture content, the parameter p defines the functional dependence on mean particle size, and the parameter Q_c/R defines the functional

dependence on temperature. For the modified Sjaardema-Krieg model, the parameter B_0 is an additional leading coefficient and the parameter A defines additional functional dependence on density. For the modified Zeuch model, the parameters b_7 and b_8 are additional leading coefficient parameters for fractional densities less than 90 percent and greater than 90 percent, respectively. The parameters b_3 and n describe additional functional dependence on density, and the parameter b_2 describes the functional dependence on the initial density. In the Spiers model, the parameter r_1 is an additional leading coefficient, and the parameters r_3 , r_4 , and n describe additional functional dependence on accumulated volumetric strain (or equivalently, the current density).

Table 6-1. Crushed-Salt Constitutive Parameters for Modified Sjaardema-Krieg Model

Parameter	Units	Test Database			Previous Values
		Hydrostatic	Shear	Combined	
η_0	MPa	(a)	$1.45(10^1)$	$4.70(10^5)$	
η_1	—	(a)	$3.56(10^0)$	$2.46(10^0)$	
η_2	1/MPa	$2.63(10^{-4})$	$9.04(10^{-1})$	$7.29(10^{-5})$	
κ_0	MPa	(a)	$1.91(10^2)$	$9.67(10^2)$	
κ_1	—	$-7.89(10^{-15})$	$9.67(10^{-2})$	$-2.42(10^{-13})$	
κ_2	1/MPa	$2.02(10^{-3})$	$7.12(10^{-2})$	$1.43(10^{-2})$	
m_1	—	—	$4.67(10^{-1})$	$7.05(10^{-1})$	
m_2	—	—	$1.93(10^0)$	$1.85(10^0)$	
a_1	—	$1.36(10^0)$	(b)	$1.70(10^1)$	
a_2	—	$5.74(10^1)$	(b)	$4.75(10^1)$	
Q_c/R	K	$3.06(10^3)$	(b)	$4.01(10^3)$	
p	—	$5.40(10^{-1})$	(b)	$5.64(10^{-1})$	
B_0	$\text{kg}\cdot\text{mm}^3/\text{m}^3\cdot\text{MPa}\cdot\text{s}$	$6.21(10^4)$	$5.46(10^7)$	$2.04(10^6)$	—
A	m^3/kg	$-8.04(10^{-3})$	$-8.70(10^{-3})$	$-8.70(10^{-3})$	$-1.72(10^{-2})$

(a) Parameters fixed in fit to hydrostatic database at values determined in fit to combined database.

(b) Parameters fixed in fit to shear database at values determined in fit to combined database.

6.2 Statistical Measures

Six statistical measures were used to evaluate the sets of parameter values reported above. These measures include: (1) the sum-of-squared error, SS_E ; (2) the coefficient of multiple determination, R_p^2 ; (3) the mean-squared error, MS_E ; (4) parameter multicollinearity; (5) parameter variation amongst fits, and (6) predictive capability. Each of these measures are discussed in the following subsections.

Table 6-2. Crushed-Salt Constitutive Parameters for Modified Zeuch Model

Parameter	Units	Test Database			Previous Values
		Hydrostatic	Shear	Combined	
η_0	MPa	(a)	6.46(10 ⁰)	2.03(10 ¹)	
η_1	—	(a)	0.00	6.97(10 ⁻¹⁵)	
η_2	1/MPa	1.77(10 ⁻²)	3.10(10 ⁻²)	6.66(10 ⁻³)	
κ_0	MPa	(a)	1.34(10 ²)	1.64(10 ²)	
κ_1	—	1.68(10 ⁻³⁰)	-2.78(10 ⁻¹⁵)	4.93(10 ⁻³²)	
κ_2	1/MPa	3.18(10 ⁻²)	8.65(10 ⁻²)	7.37(10 ⁻²)	
m_1	—	—	1.09(10 ⁰)	1.46(10 ⁰)	
m_2	—	—	2.04(10 ⁰)	2.02(10 ⁰)	
a_1	—	1.99(10 ¹)	(b)	2.01(10 ¹)	
a_2	—	9.98(10 ¹)	(b)	9.66(10 ¹)	
Q_c/R	K	9.16(10 ⁻¹⁷)	(b)	9.26(10 ⁻¹⁷)	
p	—	4.11(10 ⁻¹)	(b)	3.96(10 ⁻¹)	
b_2	—	8.90(10 ⁻²⁶)	1.07(10 ⁻²⁷)	6.51(10 ⁻²⁸)	1/3
b_3	—	2.69(10 ⁻¹⁹)	6.70(10 ⁻²¹)	1.05(10 ⁻²⁰)	1/2
b_7	mm ^p /MPa · s	2.18(10 ⁻¹¹)	1.42(10 ⁻¹¹)	1.43(10 ⁻¹¹)	—
b_8	mm ^p /MPa · s	7.07(10 ⁻¹³)	3.78(10 ⁻¹¹)	1.09(10 ⁻¹¹)	—
n	—	2.95(10 ⁰)	2.73(10 ⁰)	3.22(10 ⁰)	4.9

(a) Parameters fixed in fit to hydrostatic database at values determined in fit to combined database.

(b) Parameters fixed in fit to shear database at values determined in fit to combined database.

6.2.1 Sum-of-Squared Error

The sum-of-squared error (SS_E) is the object function that was minimized in each of the fits, as calculated using Equation 5-9. The sum-of-squared error is summarized in Table 6-4 for each of the three model fits to each of the three databases. This statistic is consistently the lowest for the modified Spiers model and consistently the highest for the modified Sjaardema-Krieg model. This indicates that the Spiers model fits the test data somewhat better than the other two models. However, the percentage variation in the sum-of-squared error amongst the three models is not large.

6.2.2 Coefficient of Multiple Determination

The coefficient of multiple determination (R_p^2) was calculated using Equation 5-10 for each of the fits. This statistical measure is summarized in Table 6-5. This measure is basically a normalization of the sum-of-squared error and can be used to determine which data the models fit better. This statistic

Table 6-3. Crushed-Salt Constitutive Parameters for Modified Spiers Model

Parameter	Units	Test Database			Previous Values
		Hydrostatic	Shear	Combined	
η_0	MPa	(a)	2.01(10 ⁰)	5.81(10 ⁰)	
η_1	—	(a)	2.13(10 ⁻²⁴)	9.03(10 ⁻²²)	
η_2	1/MPa	1.79(10 ⁻¹)	4.61(10 ⁻¹)	1.42(10 ⁻¹)	
κ_0	MPa	(a)	2.59(10 ²)	3.52(10 ²)	
κ_1	—	2.30(10 ⁻¹⁷)	-1.89(10 ⁻¹)	1.14(10 ⁻¹⁷)	
κ_2	1/MPa	5.70(10 ⁻²)	3.86(10 ⁻²)	4.25(10 ⁻²)	
m_1	—	—	4.44(10 ⁻²⁷)	1.60(10 ⁻²⁸)	
m_2	—	—	1.97(10 ⁰)	2.03(10 ⁰)	
a_1	—	8.84(10 ¹)	(b)	7.11(10 ¹)	
a_2	—	5.30(10 ⁻¹)	(b)	6.26(10 ⁻¹)	
Q_c	K	1.04(10 ⁻¹⁹)	(b)	1.80(10 ²)	
R	—	9.36(10 ²)	(b)	3.22(10 ⁻⁵)	
P					
r_1	mm ^P /MPa · s	8.22(10 ⁻¹⁰)	2.57(10 ⁻⁹)	6.06(10 ⁻¹¹)	—
r_3	—	1.97(10 ¹)	1.03(10 ¹)	1.58(10 ¹)	1/3
r_4	—	1.99(10 ⁰)	4.17(10 ⁻²⁶)	2.16(10 ⁰)	2
n	—	1.59(10 ⁻²)	3.55(10 ⁰)	5.22(10 ⁻¹)	4.15

(a) Parameters fixed in fit to hydrostatic database at values determined in fit to combined database.

(b) Parameters fixed in fit to shear database at values determined in fit to combined database.

Table 6-4. Sum-of-Squared Error, SS_E

Model	Test Database		
	Hydrostatic	Shear	Combined
Sjaardema-Krieg	3.62	0.52	8.20
Zeuch	2.89	0.62	6.84
Spiers	2.77	0.31	6.27

Table 6-5. Coefficient of Multiple Determination, R_p^2

Model	Test Database		
	Hydrostatic	Shear	Combined
Sjaardema-Krieg	0.903	0.993	0.942
Zeuch	0.917	0.991	0.945
Spiers	0.934	0.996	0.956

is consistently the highest for the modified Spiers model and consistently the lowest for the modified Sjaardema-Krieg model. In addition, this statistic indicates that the fits are consistently the best (highest R_p^2) to the shear consolidation tests than to either the combined database or the hydrostatic tests.

6.2.3 Mean-Squared Error

The mean-squared error (MS_E) was calculated using Equation 5-11 for each of the fits. This statistical measure is summarized in Table 6-6. This measure is used to determine the effectiveness of increasing the number of parameters in a model to reduce the sum-of-squared error. In these model fits, the Sjaardema-Krieg model has the fewest parameters (14), and the Zeuch model has the most (17). This statistic is consistently the lowest for the modified Spiers model and consistently the highest for the modified Sjaardema-Krieg model. For all of the these models, the mean-squared error is lowest for the fits to the shear tests and highest for the fits to the hydrostatic consolidation tests.

Table 6-6. Mean-Squared Error, MS_E

Model	Test Database		
	Hydrostatic	Shear	Combined
Sjaardema-Krieg	$1.86(10^{-4})$	$0.65(10^{-4})$	$1.63(10^{-4})$
Zeuch	$1.59(10^{-4})$	$0.85(10^{-4})$	$1.54(10^{-4})$
Spiers	$1.28(10^{-4})$	$0.44(10^{-5})$	$1.23(10^{-4})$

6.2.4 Parameter Multicollinearity

The parameter correlation matrices for each of the nine fits are given in Tables 6-7 through 6-15. High parameter correlations (>0.90) are indicative of possible parameter multicollinearity. High parameter correlations indicate deficiencies in the functional form of the response model or (2) deficiencies in the test database. Parameter correlations of exactly zero indicates either that (1) the parameter fits perfectly to the database or (2) the magnitude of the parameter is such that it renders that part of the functional form of the response model nonfunctional.

The Sjaardema-Krieg (S-K) parameter correlation matrices are shown in Tables 6-5 to 6-9. In the S-K model fit to the hydrostatic consolidation tests (Table 6-7), the following three sets of parameters are exactly correlated to each other ($B_0-\eta_2$, $B_0-\kappa_2$, and $\eta_2-\kappa_2$). The parameter correlations for the S-K model fit to the shear consolidation tests are given in Table 6-8. The parameters A and B_0 are exactly correlated. In addition, the correlations are high (>0.90) between the following sets of parameters (η_0-B_0 and η_0-A). The parameter correlation matrix for the S-K fit to the combined database is shown in Table 6-9. The following two sets of parameters are exactly correlated to each other in this fit ($\kappa_0-\kappa_2$ and $\eta_2-\eta_2$). In addition, the correlation between parameters A and η_1 is high.

The Zeuch model parameter correlation matrices are shown in Tables 6-10 to 6-12. In the Zeuch model fit to the hydrostatic consolidation tests (Table 6-10), there are no high parameter correlations. The parameter correlations for the Zeuch model fit to the shear consolidation tests are given in Table 6-11. The parameter correlations are high between the following three sets of parameters (b_7-b_8 , $\kappa_0-\kappa_1$, and $\eta_0-\eta_1$). The parameter correlation matrix for the Zeuch model fit to the combined database is shown in Table 6-12. With three exceptions (b_7-b_8 , $\kappa_0-\kappa_2$, and $\eta_0-\eta_2$), the correlation coefficients are not high.

The Spiers model parameter correlation matrices are shown in Tables 6-13 to 6-15. The correlation between parameters κ_2 and η_2 is exact in the Spiers model fit to the hydrostatic tests (Table 6-13). The correlations among the remaining parameters are low. The Spiers model parameter correlations in the fit to the shear tests are given in Table 6-14. The parameters r_1 and η_0 are exactly correlated, and the correlations between the following two sets of parameters are high ($r_1-\eta_0$ and $\eta_0-\eta_2$). The parameter correlation matrix for the Spiers model fit to the combined database is shown in Table 6-15. With two exceptions ($Q_c/R-r_1$ and $\kappa_0-\kappa_2$), the correlation coefficients are not high.

The parameter correlation measure (pcm) was calculated using Equation 5-15 for each model's fit to each of the three databases. A summary of this calculation is given in Table 6-16. As can be seen in this table, these results indicate that the parameters are most correlated to each other in the S-K model and least correlated to each other in the Zeuch model. Also these results indicate that the parameters are more highly correlated to each other in the fits to the combined test database than to either of the two individual test databases.

Table 6-7. Correlation Matrix for Modified Sjaardema-Krieg Model Fit to Hydrostatic Consolidation Test Database

	B_0	A	κ_1	κ_2	η_2	a_1	a_2	Q_c/R	p
B_0	1.00								
A	-0.03	1.00							
κ_1	0	0	1.00						
κ_2	1.00	-0.03	0	1.00					
η_2	-1.00	0.03	0	-1.00	1.00				
a_1	0.20	-0.53	0	0.20	-0.20	1.00			
a_2	0	0	0	0	0	0	1.00		
Q_c/R	0.09	0.09	0	0.09	-0.09	0.38	0	1.00	
p	-0.10	0.22	0	-0.10	0.10	-0.34	0	-0.09	1.00

Table 6-8. Correlation Matrix for Modified Sjaardema-Krieg Model Fit to Combined Database

	B_0	A	κ_0	κ_1	κ_2	m_2	η_0	η_1	η_2	m_1
B_0	1.00									
A	-1.00	1.00								
κ_0	0.09	-0.08	1.00							
κ_1	0.01	0.01	0.01	1.00						
κ_2	-0.04	0.05	-0.65	0.74	1.00					
m_2	0.02	-0.03	-0.43	-0.12	0.31	1.00				
η_0	-0.94	0.94	-0.03	-0.15	-0.11	-0.06	1.00			
η_1	-0.66	0.68	0.02	-0.13	-0.12	-0.02	0.77	1.00		
η_2	0.45	-0.42	0.07	0.05	0	0.03	-0.38	0.29	1.00	
m_1	0.33	-0.35	0.26	0.07	-0.17	-0.52	-0.23	-0.41	-0.21	1.00

Table 6-9. Correlation Matrix for Modified Sjaardema-Krieg Model Fit to Shear Consolidation Test Database

	B_0	A	κ_0	κ_1	κ_2	m_2	η_0	η_1	η_2	m_1	a_1	a_2	Q_c/R	p
B_0	1.00													
A	-0.76	1.00												
κ_0	0.11	-0.12	1.00											
κ_1	0	0	0	1.00										
κ_2	-0.11	0.12	-1.00	0	1.00									
m_2	-0.05	0.05	-0.49	0	0.54	1.00								
η_0	0.11	-0.13	-0.58	0	0.58	0.32	1.00							
η_1	-0.73	0.94	-0.16	0	0.16	0.07	-0.21	1.00						
η_2	-0.11	0.14	0.58	0	-0.58	-0.32	-1.00	0.21	1.00					
m_1	0.24	-0.32	0.20	0	-0.22	-0.60	-0.07	-0.32	0.07	1.00				
a_1	0.33	-0.15	0.03	0	-0.03	-0.02	-0.11	0.02	0.11	0.06	1.00			
a_2	0	0	0	0	0	0	0	0	0	0	0	1.00		
Q_c/R	0.60	0.07	0.01	0	-0.01	0	-0.02	0.05	0.02	0.01	0.42	0	1.00	
p	-0.09	0.08	-0.09	0	0.09	0.06	0.13	-0.01	-0.13	-0.01	-0.34	0	-0.11	1.00

Table 6-10. Correlation Matrix for Modified Zeuch Model Fit to Hydrostatic Consolidation
Test Database

	B_2	B_3	B_7	B_8	N	κ_1	κ_2	η_2	a_1	a_2	Q_c/R	p
B_2	1.00											
B_3	0	1.00										
B_7	0	0	1.00									
B_8	0	0	-0.05	1.00								
N	0	0	-0.32	-0.12	1.00							
κ_1	0	0	0	0	0	1.00						
κ_2	0	0	-0.24	-0.36	0.29	0	1.00					
η_2	0	0	-0.52	0.37	-0.25	0	-0.64	1.00				
a_1	0	0	-0.25	-0.09	0.60	0	0.16	-0.23	1.00			
a_2	0	0	0	0	0	0	0	0	0	1.00		
Q_c/R	0	0	0	0	0	0	0	0	0	0	1.00	
p	0	0	0.21	0.13	-0.18	0	0.09	-0.05	-0.26	0	0	1.00

Table 6-11. Correlation Matrix for Modified Zeuch Model Fit to Shear Consolidation Test Database

	B_2	B_3	B_7	B_8	N	κ_0	κ_1	κ_2	m_2	η_0	η_1	η_2	m_1
B_2	1.00												
B_3	0	1.00											
B_7	0	0	1.00										
B_8	0	0	0.96	1.00									
N	0	0	-0.80	-0.82	1.00								
κ_0	0	0	0.03	0	0.09	1.00							
κ_1	0	0	0	0	0	0	1.00						
κ_2	0	0	0	0.02	-0.08	-0.96	0	1.00					
m_2	0	0	0.07	0.03	0.02	-0.37	0	0.59	1.00				
η_0	0	0	-0.47	-0.38	0.06	-0.09	0	0.02	-0.20	1.00			
η_1	0	0	0.59	0.76	-0.57	-0.10	0	0.07	-0.07	-0.22	1.00		
η_2	0	0	0.52	0.51	-0.21	0.03	0	0.01	0.13	-0.94	0.50	1.00	
m_1	0	0	-0.76	-0.75	0.34	-0.01	0	-0.10	-0.41	0.63	-0.45	-0.61	1.00

Table 6-12. Correlation Matrix for Modified Zeuch Model Fit to Combined Test Database

	B_2	B_3	B_7	B_8	N	κ_0	κ_1	κ_2	m_2	η_0	η_1	η_2	m_1	a_1	a_2	Q_c/R	p
B_2	1.00																
B_3	0	1.00															
B_7	0	0	1.00														
B_8	0	0	0.95	1.00													
N	0	0	-0.73	-0.75	1.00												
κ_0	0	0	0.13	0.20	-0.18	1.00											
κ_1	0	0	0	0	0	0	1.00										
κ_2	0	0	-0.14	-0.19	0.18	-0.96	0	1.00									
m_2	0	0	-0.07	-0.07	0.08	-0.25	0	0.50	1.00								
η_0	0	0	-0.06	-0.06	0.03	-0.12	0	0.11	0.03	1.00							
η_1	0	0	-0.06	-0.07	0.08	-0.02	0	0.02	0.01	0	1.00						
η_2	0	0	0.03	0.05	-0.04	0.10	0	-0.10	-0.05	-0.99	0	1.00					
m_1	0	0	-0.27	-0.35	0.04	-0.07	0	-0.05	-0.48	0.01	0.01	0.05	1.00				
a_1	0	0	-0.83	-0.83	0.57	-0.09	0	0.08	0.01	0.07	0.06	-0.07	0.08	1.00			
a_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00		
Q_c/R	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	
p	0	0	0.40	0.42	-0.16	-0.04	0	0.06	0.08	-0.04	0	0.06	-0.19	-0.19	0	0	1.00

Table 6-13. Correlation Matrix for Modified Spiers Model Fit to Hydrostatic Consolidation Test Database

	R_1	R_3	R_4	N	κ_1	κ_2	η_2	a_1	a_2	Q_c/R	p
R_1	1.00										
R_3	0.26	1.00									
R_4	-0.46	-0.88	1.00								
N	0.26	-0.25	-0.17	1.00							
κ_1	0	0	0	0	1.00						
κ_2	-0.57	-0.26	0.23	0.24	0	1.00					
η_2	0.56	0.28	-0.23	-0.24	0	-1.00	1.00				
a_1	-0.35	-0.55	0.73	-0.13	0	0.26	-0.25	1.00			
a_2	0.11	-0.15	-0.17	0.56	0	0.01	-0.02	-0.34	1.00		
Q_c/R	0.56	-0.32	0.09	0.49	0	0.27	-0.28	0.15	0.07	1.00	
p	0	0	0	0	0	0	0	0	0	0	1.00

Table 6-14. Correlation Matrix for Modified Spiers Model Fit to Shear Consolidation Test Database

	R_1	R_3	R_4	N	κ_0	κ_1	κ_2	m_2	η_0	η_1	η_2	m_1
R_1	1.00											
R_3	0.52	1.00										
R_4	0	0	1.00									
N	-0.39	-0.74	0	1.00								
κ_0	-0.24	-0.02	0	0.13	1.00							
κ_1	-0.22	-0.04	0	-0.27	-0.30	1.00						
κ_2	0.08	0.01	0	-0.25	-0.88	0.71	1.00					
m_2	0.30	0.25	0	-0.25	-0.41	0.02	0.38	1.00				
η_0	-1.00	-0.46	0	0.36	0.26	0.22	-0.09	-0.29	1.00			
η_1	0	0	0	0	0	0	0	0	0	1.00		
η_2	-0.95	0.47	0	-0.26	-0.26	-0.21	0.09	0.21	-0.96	0	1.00	
m_1	0	0	0	0	0	0	0	0	0	0	0	1.00

Table 6-15. Correlation Matrix for Modified Spiers Model Fit to Combined Test Database

	R_1	R_3	R_4	N	κ_0	κ_1	κ_2	m_2	η_0	η_1	η_2	m_1	a_1	a_2	Q_c/R	p
R_1	1.00															
R_3	-0.03	1.00														
R_4	-0.05	-0.91	1.00													
N	0.28	-0.26	-0.08	1.00												
κ_0	0.03	-0.04	0	0.11	1.00											
κ_1	0	0	0	0	0	1.00										
κ_2	-0.03	0.05	0.01	-0.14	-0.99	0	1.00									
m_2	-0.08	0.06	0.02	-0.23	-0.40	0	0.52	1.00								
η_0	0.18	-0.06	-0.02	0.05	-0.53	0	0.50	0.12	1.00							
η_1	0	0	0	0	0	0	0	0	0	1.00						
η_2	-0.09	0.23	-0.12	-0.05	0.62	0	-0.63	-0.38	-0.62	0	1.00					
m_1	0	0	0	0	0	0	0	0	0	0	0	1.00				
a_1	0.25	-0.55	0.71	-0.09	-0.01	0	0.01	0.03	-0.03	0	0.02	0	1.00			
a_2	0.15	-0.17	-0.03	0.40	-0.02	0	0	-0.11	0.40	0	-0.22	0	-0.11	1.00		
Q_c/R	0.93	-0.36	0.32	0.23	0	0	-0.01	-0.07	0.23	0	-0.15	0	0.51	0.18	1.00	
p	-0.65	0.12	-0.21	0.09	0.01	0	-0.02	-0.02	-0.06	0	0	0	-0.49	-0.25	-0.71	1.00

Table 6-16. Parameter Correlation Measure

Test	Model		
	S-K	Zeuch	Spiers
Hydrostatic	0.105	0.030	0.103
Shear	0.158	0.122	0.105
Combined	0.088	0.052	0.072

6.2.5 Parameter Variation Among Fits

A quantitative measure of the parameter variation among fits was defined in Section 5.3.5. This measure simply indicates the percentage of parameters that changed by more than one order of magnitude in relation to the values determined in the fit to the combined database. This measure is summarized in Table 6-17 for each of the model fits to the hydrostatic and shear tests. These results indicate that the variation in parameters is least in the S-K model for the fit to the hydrostatic tests and in the Zeuch model for the fit to the shear tests. Similarly, the parameter variation is greatest in the Zeuch and Spiers models in the fits to the hydrostatic and shear tests, respectively.

Table 6-17. Effect of Parameter Variation

Test	Model		
	S-K	Zeuch	Spiers
Hydrostatic	2/9 = 0.222	4/12 = 0.333	3/11 = 0.273
Shear	4/10 = 0.400	2/13 = 0.154	5/12 = 0.417

6.2.6 Predictive Capability

The predictive capability of each of the models can be seen in how well the models predict the data of one type (e.g., hydrostatic consolidation) using the parameters that were determined from the fit to the data of the second type (e.g., shear consolidation). For example, in Appendix B, Figure B-1 shows the three model fits to Hydrostatic Consolidation Test CS1. The third plot in this figure represents the fit to the consolidation test and illustrates how well each of the models fit to the data. The second plot in this figure was generated using the model parameters that were fit to the shear consolidation database, and hence shows the predictive capability of each of the models.

The sum-of-squared error (SS_E), calculated using Equation 5-9 in predicting the 45 hydrostatic consolidation tests using the parameters that were fit to the shear consolidation tests, are summarized in Table 6-18. The Zeuch model generates the minimum SS_E , and the S-K model gives the greatest SS_E .

Table 6-18. Sum-of-Squared Error in Predicting Hydrostatic Tests Using Parameters Determined From Fit to Shear Consolidation Tests

Model	SS_E
Sjaardema-Krieg	14.01
Zeuch	7.13
Spiers	8.24

7.0 SUMMARY AND CONCLUSIONS

A major long-term component of the shaft seal system at the WIPP site is crushed salt emplaced in the shaft within the Salado Formation. This seal gradually becomes effective as the salt is consolidated over time by shaft closure resulting from creep of the surrounding intact salt. An understanding of creep consolidation of crushed salt is important to WIPP seals design and analyses. The work reported here represents a very thorough study of existing constitutive models for crushed salt reconsolidation, and the understanding of the creep consolidation of WIPP crushed salt has been greatly improved as a result of this study.

Credible prediction of crushed salt consolidation is an important factor in design calculations performed in support of the WIPP seals project in two specific areas. First, the crushed salt consolidates with time as the intact salt surrounding the shaft creeps. This consolidation process resists movement of the surrounding intact salt or provides a "backstress," which promotes healing of the DRZ in the salt immediately surrounding the shaft. This healing greatly reduces the DRZ's flow characteristics and helps eliminate this potential pathway. Second, the consolidation of the crushed salt reduces the permeability and porosity of the crushed salt itself, greatly enhancing the crushed salt as an effective seal material.

Before this study was initiated, the only model used in characterizing creep consolidation of crushed salt was that proposed by Sjaardema and Krieg. The elastic component of this model is empirically based on the results of one laboratory experiment. The original Sjaardema-Krieg (S-K) creep consolidation model included only the effects of mean stress and density in its functional form. Other potentially important parameters, such as deviatoric stress, temperature, moisture content, and particle size, were not included. Parameter values defining the dependence on mean stress and density were based on curve fitting to 12 hydrostatic consolidation tests. Since the original fit of S-K, a significant number of shear and hydrostatic consolidation experiments have been conducted. The additional test data were used in the current study.

7.1 Summary

This report examines the mechanical behavior of crushed salt, which is to be used as the long-term seal component for the shafts at the WIPP site. A comprehensive literature search was first performed to determine potential candidate constitutive models that might be appropriate for the expected temperature, stress, and moisture conditions expected at the WIPP site. Ten potential crushed-salt constitutive models were identified in the literature search with widely varying underlying assumptions. Three of these models were selected for further evaluation. The three models are those attributed to Sjaardema-Krieg, Zeuch, and Spiers. These models were then generalized to three-dimensional forms and modified where deemed necessary. A database comprised of hydrostatic and shear consolidation tests

was created and used to determine the material parameters of the candidate constitutive models. Nine model fits in all were performed; each of the three models were fit to three databases. The three databases were comprised of hydrostatic consolidation tests only, shear consolidation tests only, and a combination of both test types. The parameters were determined using the commercially available nonlinear regression program AR from the statistical package **BMDP**. Six statistical measures were used to evaluate each of the models.

The original S-K model is adequate for engineering approximations and scoping calculations for many of the WIPP related sealing activities. With the modified S-K model, more sophistication was added to characterize consolidation of crushed salt under varying conditions of deviatoric stress, temperature, moisture content, and particle size. The addition of small amounts of moisture is known to increase the consolidation rate of crushed salt. The database collected as a part of this study includes tests performed at different temperatures, moisture contents, and particle sizes. Thus, an objective of this study was to include these parameters in the crushed salt models.

The new generalization to three-dimensional states of stress has improved the modified S-K model dramatically. The modified model can capture shear consolidation behavior; whereas, the earlier version of the S-K model could not. In addition, the literature survey performed as a part of this study discovered two additional models that are based on deformation mechanisms. Zeuch's model (dislocation mechanisms) is based on work performed on dry, crushed salt. Spiers' model (diffusional transport mechanisms) is based on work performed on wet, crushed salt. Based on the model fitting performed in this study, both of these models are overall superior to the modified S-K model in capturing the consolidation of crushed salt over a variety of stress, temperature, moisture, and particle size conditions.

As an illustration, consider the model predictions for shear consolidation Test SC5A in Figure 7-1. The original S-K model overpredicts the axial strain by more than 50 percent and predicts zero lateral strain. The modified S-K model fits the experimental data much better. As can be seen in this figure, the modified Spiers model fits the experimental data even better than the improved S-K model.

A rank ordering of the models based on the six statistical measures is given in Table 7-1. For the first three measures (sum-of-squared error, coefficient of multiple determination, and residual-mean-squared error), a rank ordering of the three models is consistently (1) Spiers, (2) Zeuch, and (3) Sjaardema-Krieg. All of the models have some parameters that are highly correlated (correlation coefficient greater than 0.90). The rank ordering of the three models to parameter multicollinearity is based on the number of pairs of parameters for each model that have a high correlation coefficient; i.e., Spiers = 1, Zeuch = 2, and Sjaardema-Krieg = 3. Parameter variation amongst the three fits is used as another index criterion. The rank ordering assigned to this measure is based on the number of parameters for each model that vary by more than 50 percent amongst the fits. In this case, Zeuch = 1, Sjaardema-Krieg = 2, and Spiers = 3. A rank ordering of the models based on their predictive

Figure 7-1. Model Predictions of Shear Consolidation Test SC5A.

capability (using the parameters determined from the fit to the shear consolidation database to predict the hydrostatic consolidation tests) would be Spiers = 1, Zeuch = 2, and Sjaardema-Krieg = 3. Taken collectively, these six statistical measures indicate that the modified Spiers model is better than either the modified Zeuch model or the modified Sjaardema-Krieg model in defining the constitutive behavior of WIPP crushed salt. This may be fortuitous, since the modified Spiers model is based on grain boundary pressure solution (which requires moisture), and many of the tests in the database were tested under dry conditions.

Table 7-1. Rank Ordering of Statistical Measures

Statistical Measure	Model		
	Sjaardema-Krieg	Zeuch	Spiers
SS_E	3	2	1
R_p^2	3	2	1
MS_E	3	2	1
Parameter Multicollinearity	3	2	1
Parameter Variation Amongst Fits	2	1	3
Predictive Capability	3	1	2
TOTAL	17	11	8

1 = Highest ranking.

2 = Average ranking.

3 = Lowest ranking.

7.2 Conclusions

As a result of the model development effort for consolidation of crushed salt, several areas have been identified where improvements were made to the model and where further improvements could be made to the model. The new constitutive models are characterized by the following enhancements:

1. deformation mechanism based models (two of the models)
2. model parameter estimates based on a significantly larger database with two types of tests

3. models based on an improved three-dimensional generalization that captures shear consolidation behavior
4. functional relationship developed for comparing effects of moisture, temperature, and grain size.

Areas identified where improvements could be made to the crushed salt consolidation constitutive models to help reduce the uncertainty in the deformation of the crushed salt, and thus the uncertainty in the temporal and spatial variations in the calculated density of a seal component, include:

1. The modified Zeuch and Spiers models are poorly defined in terms of the initial conditions. The initial densification rate of the modified Zeuch model is zero (when $D = D_0$), and the initial densification rate of the modified Spiers model is infinite (since ε_v is initially zero). To perform the model fitting, adjustments had to be made to both models. In the case of the modified Zeuch model, a nonzero initial densification rate was generated by using the fractional density at the beginning of the consolidation portion of the test (D_i) instead of the initial fractional density (D_0). In this way, the term $(D - D_0)^{b_3 - n}$ is nonzero in Equation 3-77 at the beginning of the test ($D = D_i$) since $D_i > D_0$. In the case of the modified Spiers model, the total volumetric strains were used rather than the creep-consolidation volumetric strains. In this way, $\varepsilon_v > 0$ in the denominator of Equation 3-82 at the beginning of each test because an elastic volumetric strain accumulates with initial load application. The initial strain rates predicted by both of these models are very sensitive to the selection of the starting conditions. Initial conditions for these models should be modified to avoid these problems.
2. The models are fit to tests that include only one load path; i.e., constant-stress creep tests. The models should be fit to tests that include other load paths, such as constant strain rate or quasi-static tests. Tests of these types would evaluate contributions from both the elastic and creep-consolidation portions of the constitutive model. With additional test types, the constitutive quality of the models could be more fully examined. In addition, the predictive capability of the models could be better evaluated.
3. The database to which the models are fit is insufficient to exercise all parts of the models. For example, all of the shear consolidation tests were biased toward similar moisture content, grain size, and temperature conditions. In instances where the database is insufficient to describe a portion of a model (analogous to defining a line with one point), those parts of the model should be forced to be inoperative rather than included as independent parameters that appear only as redundant multipliers.

4. Two of the models (i.e., Zeuch and Spiers) are bifunctional in nature. The modified Spiers model contains different functional forms for small strain (<0.15) and for large strain (>0.15). The modified Zeuch model describes different processes at densities below and above 0.90. For both cases, the densification rates are not continuous at these break points. Thus, the creep-consolidation strains (the integrated densification rates) exhibit kinks at these points. Furthermore, in terms of application of these models to the WIPP seals program, crushed salt is proposed to be emplaced at a fractional density of 0.90 [Ahrens and Hansen, 1995]. Thus, neither model is likely to exercise both parts of its functional form in practice. It is recommended that either (1) the functional forms be made continuous at their break points or (2) the fits be performed at WIPP conditions (i.e., fractional densities of greater than 0.90 and/or volumetric strains of less than 0.15).
5. The modified Zeuch model (dislocation mechanisms) is based on work performed on dry crushed salt. The modified Spiers model (diffusional transport mechanisms) is based on work performed on wet crushed salt. A comprehensive model should be developed that includes contributions from both types of densification mechanisms. In essence, the modified Zeuch model and the modified Spiers model should be combined or, alternatively, the diffusional process could be added to the modified Zeuch model as outlined by Zeuch [1990].

The following modifications are suggested for each of the candidate models based on the parameter values determined in the model fitting.

Modified Sjaardema-Krieg

No obvious model reductions are indicated by the parameter values given in Table 6-1.

Modified Zeuch

In Equations 3-11 and 3-12, the terms

$$\eta_0 \left[1 - \frac{\rho}{\rho_f} \right]^{\eta_1}$$

and

$$\kappa_0 \left[1 - \frac{\rho}{\rho_f} \right]^{\kappa_1}$$

are inoperative and can be included in the leading coefficients b_7 and b_8 . The terms η_1 and κ_1 are very small, as given in Table 6-2. Also, the terms b_2 and b_3 are very, very small compared to n and can be ignored.

Spiers

Since the term η_1 is very small (Table 6-3), in Equation 3-10 the factor

$$\eta_0 \left[1 - \frac{\rho}{\rho_f} \right]^{\eta_1}$$

is reduced to η_0 . Similarly, the magnitude of the term κ_1 in Equation 3-11 renders the factor

$$\kappa_0 \left[1 - \frac{\rho}{\rho_f} \right]^{\kappa_1}$$

equal to κ_0 except in the shear consolidation database case. However, since κ_1 is very small in the combined database, which includes the shear tests, it is probably not needed. The parameter m_1 is very small (Table 6-3) and is not needed.

Incorporating the above observations obtained from parameter values and excluding the particle size and moisture content functions, the equivalent inelastic strain-rate forms for the three constitutive models may be written as:

Modified Sjaardema-Krieg Model

$$\begin{aligned} \dot{\epsilon}_{ij} = & \frac{\dot{\sigma}_m}{3K} \delta_{ij} + \frac{\dot{s}_{ij}}{2G} + \frac{B_0 (1 + \epsilon_v)^2}{\rho_0} \\ & \exp \left[\frac{A \rho_0}{(1 + \epsilon_v)} \right] \left[\eta [1 - e^{-\eta_2 \sigma_m}] - \frac{|\sigma_1 - \sigma_3|^{m_1}}{\sigma_{r1}^{m_1 - 1}} \right] \times \\ & \left\{ \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \delta_{ij} - \frac{m_2 (2 \cos \psi \sqrt{J_2})^{m_2 - 1}}{\sigma_{r2}^{m_2 - 1}} \left[\left[\frac{\cos 2 \psi}{\cos 3 \psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3 \psi} \right] t_{ij} \right] \right\} + \\ & \dot{\epsilon}_s \left\{ \left[\frac{\cos 2 \psi}{\cos 3 \psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3 \psi} \right] t_{ij} \right\} \end{aligned} \quad (7-1)$$

Modified Zeuch Model

For Stage 1 ($D_0 \leq D \leq 0.9$);

$$\dot{\varepsilon}_{ij} = \frac{\bar{\sigma}_m}{3K} \delta_{ij} + \frac{\bar{s}_{ij}}{2G} + \frac{\rho_0 b_7}{\rho} D^{-2n} \left[\frac{D - D_0}{1 - D_0} \right]^{-n} \sigma_{eq}^f \times \Lambda \quad (7-2)$$

where:

$$\Lambda = \left\{ \frac{\kappa \kappa_2 e^{-\kappa_2 \sigma_m}}{3} \delta_{ij} - \frac{m_2 \left(2 \cos \psi \sqrt{J_2} \right)^{m_2-1}}{\sigma_{r_2}^{m_2-1}} \left\{ \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right\} \right\} + \quad (7-3)$$

$$F \dot{\varepsilon}_s \left\{ \left[\frac{\cos 2\psi}{\cos 3\psi} \right] \frac{s_{ij}}{\sqrt{J_2}} + \left[\frac{\sqrt{3} \sin \psi}{J_2 \cos 3\psi} \right] t_{ij} \right\}$$

For Stage 2 ($0.9 < D \leq 1.0$);

$$\dot{\varepsilon}_{ij} = \frac{\bar{\sigma}_m}{3K} \delta_{ij} + \frac{\bar{s}_{ij}}{2G} + \frac{\rho_0 b_8}{\rho} \frac{(1 - D)}{\{1 - (1 - D)^{1/n}\}^n} \sigma_{eq}^f \times \Lambda \quad (7-4)$$

where:

$$\sigma_{eq}^f = \left[1 - e^{-\eta_2 \sigma_m} \right] - \frac{|\sigma_1 - \sigma_3|^{m_1}}{\sigma_{r_1}^{m_1-1}} \quad (7-5)$$

Modified Spiers Model

$$\dot{\varepsilon}_{ij} = \frac{\bar{\sigma}_m}{3K} \delta_{ij} + \frac{\bar{s}_{ij}}{2G} + r_1 \left[\frac{(1 + \varepsilon_v)^{r_2}}{|\varepsilon_v|^{r_3}} \right] \Gamma \sigma_{eq}^f \times \Lambda \quad (7-6)$$

where Γ is defined in Equations 3-51 and 3-54, and:

$$\sigma_{eq}^f = \left[1 - e^{-\eta_2 \sigma_m} \right] - |\sigma_1 - \sigma_3| \quad (7-7)$$

6. The final recommendation is that the new functional forms given in the above equations for each of the three candidate models should be fit to the existing database.

Obviously, our understanding of the behavior of crushed salt is not complete as a result of this study. But in addition to enhancing our knowledge of the behavior of crushed salt, this study has isolated and identified areas for future work to improve our understanding of the consolidation of WIPP crushed salt. This study has identified areas of inadequacies in the current laboratory test database. Future laboratory testing of WIPP crushed salt should be used in conjunction with the current database to fill these gaps. As pointed out earlier, the models should be simplified and/or combined to reflect the nature of the laboratory test database. The models of Zeuch and Spiers could be combined to represent the mechanisms of each model. Field (in situ) experiments may be useful to validate the models on tests of a larger scale than laboratory tests. The new models should be incorporated into computer codes and used in design calculations to predict the structural response of the crushed salt emplaced in the shafts. These results can be compared to results calculated previously using the original S-K model.

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APPENDIX A

INPUT FILES TO BMDP
NONLINEAR REGRESSION PROGRAM AR

APPENDIX A

INPUT FILES TO BMDP NONLINEAR REGRESSION PROGRAM AR

This appendix contains the nine input files used in the fitting of the three models (Sjaardema-Krieg, Zeuch, and Spiers) to each of the three databases (hydrostatic, shear, and combined). These files are input to the nonlinear regression program AR described in Section 5.4. The files all have the suffix .INP, and the prefix of each file corresponds to the table below (Table A-1).

Table A-1. Input File Prefixes Used in Model Fitting

		Model		
		Sjaardema-Krieg	Zeuch	Spiers
Test Database	Hydrostatic	S-K1.	ZEUCH1.	SPIERS1.
	Shear	S-K2.	ZEUCH2.	SPIERS2.
	Combined	S-K3.	ZEUCH3.	SPIERS3.

The input files for the Sjaardema-Krieg model fits to the hydrostatic, shear, and combined test databases are given in Figures A-1, A-2, and A-3, respectively. Similarly, Figures A-4 through A-6 are the input files for the Zeuch model fits to the hydrostatic, shear, and combined databases. The Spiers model fit input files are given in Figures A-7 through A-9, respectively, for the hydrostatic, shear, and combined fits.


```

# File = j:\325\cs\s-k1.inp
#
# Fit to hydrostatic consolidation tests only (icase=1).
#
# =====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
# 1. mean stress (SM)
# 2. stress difference (DS)
# 3. emplaced density (RHO0)
# 4. initial density for differential equation (RHOI)
# 5. grain size (DD)
# 6. absolute temperature (T)
# 7. moisture content (W)
#
# =====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
  TITLE = 'SJAARDEMA-KRIEG MODEL FIT TO 45 HYDROSTATIC CONSOLIDATION TESTS'.
  FILE = 'J:\325\CS\FIT4\BMDP\RSISNL.CSV'. # Data file name.
  FORMAT = '(2I3,3F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
  VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
  NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS
  MS = (2.0*LS+AS)/3.0. # mean stress, as=axial stress, ls=conf. pres.
  DS = ABS(AS - LS). # stress difference
  D0 = RHO0/2160. # rho0=initial density, 2160=intact density
  DI = RHOI/2160. # fractional density at the beginning of creep
  ONE = 1.0.
  USE = ICASE EQ 1.
  WT = (DT/EAT**2)/1.0E+6.
#

```

Figure A-1. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to Hydrostatic Consolidation Test Database Only (Page 1 of 4).

```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = EAT.          # Dependent variables
  PARAMETERS=15.         # Number of regression parameters
  WEIGHT = WT.           # WEIGHT VARIABLE
  ITIME = 3.             # INTEGRATION VARIABLE NUMBER
  NEQN = 1.              # Number of differential equations
  ITER = 20.             # NUMBER OF ITERATIONS
  HALVINGS = 8.          # NUMBER OF INTERVAL HALVINGS
  MAXC = 100000.         # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=
        B0,      A,      KAPPA0,
        KAPPA1,  KAPPA2,  M2,
        ETA0,    ETA1,    ETA2,
        M1,      A1,      A2,
        QCR,     P,       JUNK.
  INIT =
        6.206E+04, -8.035E-03, 9.667E+02,
        -7.886E-15, 2.016E-03, 1.850E+00,
        4.698E+05, 2.464E+00, 2.625E-04,
        7.047E-01, 1.360E+01, 5.743E+01,
        3.057E+03, 5.401E-01, 0.
  DELTA =
        6.206E+02, 8.035E-05, 9.667E+00,
        7.886E-17, 2.016E-05, 1.850E-02,
        4.698E+03, 2.464E-02, 2.625E-06,
        7.047E-03, 1.360E-01, 5.743E-01,
        3.057E+01, 5.401E-03.
  MINIMUM =
        5.206E+04, -9.035E-03, 8.667E+02,
        -8.886E-15, 1.016E-03, 0.850E+00,
        3.698E+05, 1.464E+00, 1.625E-04,
        6.047E-01, 0.360E+01, 4.743E+01,
        2.057E+03, 4.401E-01.

```

Figure A-1. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to Hydrostatic Consolidation Test Database Only (Page 2 of 4).

```

MAXIMUM =          7.206E+04,   -7.035E-03,   1.067E+03,
                 -6.886E-15,   3.016E-03,   2.850E+00,
                 5.698E+05,   3.464E+00,   3.625E-04,
                 8.047E-01,   2.360E+01,   6.743E+01,
                 4.057E+03,   6.401E-01.
#   fixed =          B0,          A,          KAPPA0,
#               KAPPA1,          KAPPA2,          M2,
#               ETA0,          ETA1,          ETA2,
#               M1,          A1,          A2,
#               QCR,          P.
      FIXED = KAPPA0,M2,ETA0,ETA1,M1.
#
# ---- INITIAL VALUES FOR DIFFERENTIAL EQUATIONS ----
#
/DIFIN
      Z1=EAT.
#
# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
      IF (TIME NE 0.0) THEN (
        VOL=3*Z1.
        FD = D0/EXP(VOL).
        IF (FD LE 1.0) THEN (
          KAPPA=KAPPA0*(1-D0/EXP(VOL))**KAPPA1.
          ETA=ETA0*(1-D0/EXP(VOL))**ETA1.
          T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
          SEQF=ETA*(1-EXP(-ETA2*MS)).
          FA=T1/3.
          G=1/DD**P.
          M=1+A1*(1-EXP(-A2*W)).
          THETA=EXP(-QCR/TEMP).
          SK=B0*EXP(VOL)/RHO0*EXP(A*RHO0*EXP(-VOL)).
          DZ1=SK*FA*SEQF*M*G*THETA.
          IF (FD GT 1.0) THEN DZ1=0.).
      IF (TIME EQ 0.0) THEN DZ1=0.
#
# ---- SPECIFY THE DEPENDENT VARIABLE FUNCTION
#

```

Figure A-1. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to Hydrostatic Consolidation Test Database Only (Page 3 of 4).

```

/FUN
  F=Z1.
  IF(TIME EQ 0.) THEN (
    NEW = 0.
    Z1=EAT.
    F=Z1).
#
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----
#
/SAVE
  FILE = 'J:\325\CS\FIT4\BMDP\S-K1.SAV'.
  NEW.
  KEEP = TIME,EAT.
  FORMAT = '(5E12.3)'.
/PRINT
  FORMAT = E.
/PLOT
  RESIDUAL.
/END

```

Figure A-1. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to Hydrostatic Consolidation Test Database Only (Page 4 of 4).

```

# File = j:\325\cs\fit4\bmdp\s-k2.inp
#
# Fit to shear consolidation tests only (icase=2).
#
# =====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
#   1. mean stress                (SM)
#   2. stress difference          (DS)
#   3. emplaced density          (RHO0)
#   4. initial density for differential equation (RHOI)
#   5. grain size                (DD)
#   6. absolute temperature      (T)
#   7. moisture content          (W)
#
# =====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
  TITLE = 'SPIERS MODEL FIT TO 10 SHEAR CONSOLIDATION TESTS'.
  FILE = 'J:\325\CS\FIT4\BMDP\RSISNL.CSV'.    # Data file name.
  FORMAT = '(2I3,3F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
  VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
  NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS
  MS = (2.0*LS+AS)/3.0.    # mean stress, as=axial stress, ls=conf. pres.
  DS = ABS(AS - LS).       # stress difference
  ELT = (EVT - EAT)/2.     # lateral strain, evt=vol. strain, eat=ax.strain
  D0 = RHO0/2160.          # rho0=initial density, 2160=intact density
  DI = RHOI/2160.          # fractional density at the beginning of creep
  ONE = 1.0.
  USE = ICASE EQ 2.
  WT = 1.0.
#

```

Figure A-2. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to Shear Consolidation Test Database Only (Page 1 of 4).

```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = ONE.          # Dependent variables
  PARAMETERS=15.         # Number of regression parameters
  WEIGHT = WT.           # WEIGHT VARIABLE
  ITIME = 3.             # INTEGRATION VARIABLE NUMBER
  NEQN = 2.              # Number of differential equations
  ITER = 10.             # NUMBER OF ITERATIONS
  HALVINGS = 10.         # NUMBER OF INTERVAL HALVINGS
  MAXC = 300000.         # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=
        B0,      A,      KAPPA0,
        KAPPA1,  KAPPA2,  M2,
        ETA0,    ETA1,    ETA2,
        M1,      A1,      A2,
        QCR,     P,       JUNK
  INIT =
        5.457E+07, -1.043E-02, 1.907E+02,
        9.670E-02, 7.116E-02, 1.933E+00,
        1.446E+01, 3.561E+00, 9.037E-01,
        4.670E-01, 1.697E+01, 4.746E+01,
        4.005E+03, 5.639E-01.
  MINIMUM =
        4.457E+07, -2.043E-02, 0.907E+02,
        8.670E-02, 6.116E-02, 0.933E+00,
        0.446E+01, 2.561E+00, 8.037E-01,
        3.670E-01, 0.697E+01, 3.746E+01,
        3.005E+03, 4.639E-01.
  MAXIMUM =
        6.457E+07, -0.043E-02, 2.907E+02,
        1.067E-01, 8.116E-02, 2.933E+00,
        2.446E+01, 4.561E+00, 1.004E+00,
        5.670E-01, 2.697E+01, 5.746E+01,
        5.005E+03, 6.639E-01.

```

Figure A-2. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to Shear Consolidation Test Database Only (Page 2 of 4).

```

DELTA =          5.457E+05,    1.043E-04,    1.907E+00,
                9.670E-04,    7.116E-04,    1.933E-02,
                1.446E-01,    3.561E-02,    9.037E-03,
                4.670E-03,    1.697E-01,    4.746E-01,
                4.005E+01,    5.639E-03.
#  FIXED=          B0,          A,          KAPPA0,
#                KAPPA1,        KAPPA2,        M2,
#                ETA0,          ETA1,          ETA2,
#                M1,          A1,          A2,
#                QCR,          P.
      FIXED = A1,A2,P,QCR.
#
# ---- INITIAL VALUES FOR DIFFERENTIAL EQUATIONS ----
#
/DIFIN
      Z1=EAT.
      Z2=ELT.
#
# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
      IF (TIME NE 0.0) THEN (
        VOL=Z1+2*Z2.
        FD = D0/EXP(VOL).
        IF (FD LE 1.0) THEN (
          KAPPA=KAPPA0*(1-D0/EXP(VOL))**KAPPA1.
          ETA=ETA0*(1-D0/EXP(VOL))**ETA1.
          T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
          T2=M2*DS**(M2-1).
          SEQF=ETA*(1-EXP(-ETA2*MS))-DS**M1.
          FA=(T1/3+T2).
          FL=(T1/3-0.5*T2).
          G=1/DD**P.
          M=1+A1*(1-EXP(-A2*W)).
          THETA=EXP(-QCR/TEMP).
          SK=B0*EXP(VOL)/RHO0*EXP(A*RHO0*EXP(-VOL)).
          DZ1=SK*FA*SEQF*M*G*THETA.
          DZ2=SK*FL*SEQF*M*G*THETA.).
        IF (FD GT 1.0) THEN (
          DZ1=0.
          DZ2=0.).)
      IF (TIME EQ 0.0) THEN (
        DZ1=0.
        DZ2=0.).
#

```

Figure A-2. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to Shear Consolidation Test Database Only (Page 3 of 4).

```
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----  
#  
/SAVE  
    FILE = 'J:\325\CS\FIT4\BMDP\S-K2.SAV'.  
    NEW.  
    KEEP = TIME,EAT.  
    FORMAT = '(5E12.3)'.  
/PRINT  
    FORMAT = E.  
/PLOT  
    RESIDUAL.  
/END
```

Figure A-2. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to Shear Consolidation Test Database Only (Page 4 of 4).


```

# File = j:\325\cs\fit4\bmdp\s-k3.inp
#
# Fit to the combined hydrostatic consolidation tests (icase=1) and shear
# consolidation tests (icase=2)
#
# =====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
#   1. mean stress                (SM)
#   2. stress difference          (DS)
#   3. emplaced density          (RHO0)
#   4. initial density for differential equation (RHOI)
#   5. grain size                (DD)
#   6. absolute temperature      (T)
#   7. moisture content          (W)
#
# =====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
TITLE = 'SJAARDEMA-KRIEG MODEL FIT TO RSISNL.CSV'. # 55 RSI and SNL tests.
FILE = 'J:\325\CS\FIT4\BMDP\RSISNL.CSV'.          # Data file name.
FORMAT = '(2I3,3F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS
MS = (2.0*LS+AS)/3.0. # mean stress, as=axial stress, ls=conf. pres.
DS = ABS(AS - LS).    # stress difference
ELT = (EVT - EAT)/2.  # lateral strain, evt=vol. strain, eat=ax.strain
D0 = RHO0/2160.        # rho0=initial density, 2160=intact density
DI = RHOI/2160.        # fractional density at the beginning of creep
ONE = 1.0.
USE = ICASE EQ 1 OR ICASE EQ 2.
WT = 1.0.
#

```

Figure A-3. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to the Combined Test Database Only (Page 1 of 4).

```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = ONE.           # Dependent variables
  PARAMETERS=15.          # Number of regression parameters
  WEIGHT = WT.            # WEIGHT VARIABLE
  ITIME = 3.              # INTEGRATION VARIABLE NUMBER
  NEQN = 2.               # Number of differential equations
  ITER = 10.              # NUMBER OF ITERATIONS
  HALVINGS = 8.           # NUMBER OF INTERVAL HALVINGS
  MAXC = 300000.          # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=
        B0,      A,      KAPPA0,
        KAPPA1,  KAPPA2,  M2,
        ETA0,    ETA1,    ETA2,
        M1,      A1,      A2,
        QCR,     P,       JUNK.
  INIT =
        2.043E+06, -8.703E-03, 9.667E+02,
        -2.417E-13, 1.432E-02, 1.850E+00,
        4.698E+05, 2.464E+00, 7.285E-05,
        7.047E-01, 1.697E+01, 4.746E+01,
        4.005E+03, 5.639E-01, 0.
  MINIMUM =
        1.043E+06, -9.703E-03, 8.667E+02,
        -3.417E-13, 0.432E-02, 0.850E+00,
        3.698E+05, 1.464E+00, 6.285E-05,
        6.047E-01, 0.697E+01, 3.746E+01,
        3.005E+03, 4.639E-01.
  MAXIMUM =
        3.043E+06, -7.703E-03, 1.067E+03,
        -1.417E-13, 2.432E-02, 2.850E+00,
        5.698E+05, 3.464E+00, 8.285E-05,
        8.047E-01, 2.697E+01, 5.746E+01,
        5.005E+03, 6.639E-01.

```

Figure A-3. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to the Combined Test Database Only (Page 2 of 4).

```

DELTA =          2.043E+04,    8.703E-05,    9.667E+00,
                2.417E-15,    1.432E-04,    1.850E-02,
                4.698E+03,    2.464E-02,    7.285E-07,
                7.047E-03,    1.697E-01,    4.746E-01,
                4.005E+01,    5.639E-03,    0.
#  FIXED=          B0,          A,          KAPPA0,
#                KAPPA1,        KAPPA2,        M2,
#                ETA0,          ETA1,          ETA2,
#                M1,          A1,          A2,
#                QCR,          P.
#
# ---- INITIAL VALUES FOR DIFFERENTIAL EQUATIONS ----
#
/DIFIN
  Z1=EAT.
  Z2=ELT.
#
# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
  IF (TIME NE 0.0) THEN (
    VOL=Z1+2*Z2.
    FD = D0/EXP(VOL).
    IF (FD LE 1.0) THEN (
      KAPPA=KAPPA0*(1-D0/EXP(VOL))**KAPPA1.
      ETA=ETA0*(1-D0/EXP(VOL))**ETA1.
      T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
      IF (ICASE EQ 2) THEN (
        T2=M2*DS**(M2-1).
        SEQF=ETA*(1-EXP(-ETA2*MS))-DS**M1.).
      IF (ICASE EQ 1) THEN (
        T2=0.
        SEQF=ETA*(1-EXP(-ETA2*MS)).).
      FA=(T1/3+T2).
      FL=(T1/3-0.5*T2).
      G=1/DD**P.
      M=1+A1*(1-EXP(-A2*W)).
      THETA=EXP(-QCR/TEMP).
      SK=B0*EXP(VOL)/RHO0*EXP(A*RHO0*EXP(-VOL)).
      DZ1=SK*FA*SEQF*M*G*THETA.
      DZ2=SK*FL*SEQF*M*G*THETA.).

```

Figure A-3. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to the Combined Test Database Only (Page 3 of 4).

```

      IF (FD GT 1.0) THEN (
        DZ1=0.
        DZ2=0.).)
      IF (TIME EQ 0.0) THEN (
        DZ1=0.
        DZ2=0.).
#
# ---- SPECIFY THE DEPENDENT VARIABLE FUNCTION
#
/FUN
  ALPHA = Z1/EAT.
  BETA = Z2/ELT.
  WT1 = DT/1.0E+6.
  F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).
  IF(TIME EQ 0.) THEN (
    NEW = 0.
    Z1=EAT.
    Z2=ELT.
    ALPHA = Z1/EAT.
    BETA = Z2/ELT.
    WT1 = DT/1.0E+6.
    F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).
#
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----
#
/SAVE
  FILE = 'J:\325\CS\FIT4\BMDP\S-K3.SAV'.
  NEW.
  KEEP = TIME,EAT,ELT.
  FORMAT = '(5E12.3)'.
/PRINT
  FORMAT = E.
/PLOT
  RESIDUAL.
/END

```

Figure A-3. Input File to BMDP Nonlinear Regression Program AR for Sjaardema-Krieg Model Fit to the Combined Test Database Only (Page 4 of 4).

```

# File = r:\bmdp\zeuch\zeuch1.inp
#
# Fit to hydrostatic consolidation tests only (icase=1).
#
# =====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
# 1. mean stress (SM)
# 2. stress difference (DS)
# 3. emplaced density (RHO0)
# 4. initial density for differential equation (RHOI)
# 5. grain size (DD)
# 6. absolute temperature (T)
# 7. moisture content (W)
#
# =====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
  TITLE = 'ZEUCH MODEL FIT TO 45 HYDROSTATIC CONSOLIDATION TESTS'.
  FILE = 'R:\BMDP\RSISNL.CSV'. # Data file name
  FORMAT = '(2I3,3F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
  VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
  NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS # DEFINE MEAN STRESS (SM), Fraction density AND WEIGHT FUNCTION (WT)
  MS = (2.0*LS+AS)/3.0. # mean stress, as=axial stress, ls=conf. pres.
  DS = ABS(AS - LS). # stress difference
  D0 = RHO0/2160. # rho0=initial density, 2160=intact density
  DI = RHOI/2160. # fractional density at the beginning of creep
  ONE = 1.0.
  USE = ICASE EQ 1.
  WT = (DT/EAT**2)/1.0E+6.
#

```

Figure A-4. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to Hydrostatic Consolidation Test Database Only (Page 1 of 4).

```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = EAT.           # Dependent variables
  PARAMETERS=17.          # Number of regression parameters
  WEIGHT = WT.            # WEIGHT VARIABLE
  ITIME = 3.              # INTEGRATION VARIABLE NUMBER
  NEQN = 2.               # Number of differential equations
  ITER = 100.             # NUMBER OF ITERATIONS
  HALVINGS = 10.          # NUMBER OF INTERVAL HALVINGS
  MAXC = 300000.          # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=      B2,      B3,      B7,
              B8,      N,      KAPPA0,
              KAPPA1,  KAPPA2,  M2,
              ETA0,    ETA1,    ETA2,
              M1,
              A1,      A2,      P,      QCR.
INITIAL = 6.316763E-28, 1.051233E-20, 1.401666E-11,
          1.049219E-11, 3.219941E+00, 1.636756E+02,
          4.850311E-32, 7.271277E-02, 2.021284E+00,
          2.029039E+01, 6.966930E-15, 6.914775E-03,
          1.459375E+00,
          2.017534E+01, 9.674910E+01, 3.980428E-01, 9.257368E-17.
MINIMUM = 0.0,      0.0,      0.0,
          0.0,      0.0,      0.0,
          0.0,      0.0,      0.0,
          0.0,
          0.0,      -10.0,      0.0,      -10.0.

```

Figure A-4. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to Hydrostatic Consolidation Test Database Only (Page 2 of 4).

```

MAXIMUM =    100.0,    100.0,    100.0,
            500.0,    100.0,    500.0,
            100.0,    500.0,    100.0,
            100.0,    100.0,    100.0,
            100.0,
            100.0,    1000.0,    100.0,    10000.
FIXED = KAPPA0,M2,ETA0,ETA1,M1.
#
# ---- INITIAL VALUES FOR DIFERENTIAL EQUATIONS ----
#
/DIFIN
    Z1=EAT.
#
# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
    IF (TIME NE 0.0) THEN (
        VOL=3*Z1.
        DEN=D0/EXP(VOL).
        KAPPA=KAPPA0*(1-D0/EXP(VOL))**KAPPA1.
        ETA=ETA0*(1-D0/EXP(VOL))**ETA1.
        T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
        SEQF=ETA*(1-EXP(-ETA2*MS)).
        FA=T1/3.
        G=1/DD**P.
        M=1+A1*(1-EXP(-A2*W)).
        THETA=EXP(-QCR/TEMP).
        IF (DEN LE 1.0) THEN (
            IF (DEN LE 0.9) THEN (
                ZEUCH=B7*(D0**B2)*DEN**(2*(B2-N)-1)*((DEN-D0)/(1-D0))**(B3-N).
            IF (DEN GT 0.9) THEN (
                ZEUCH=B8*(1-DEN)/((1-(1-DEN)**(1/N))**N).).
            IF (DEN GT 1.0) THEN ZEUCH=0.0.
            DZ1=ZEUCH*FA*SEQF*M*G*THETA.).
        IF (TIME EQ 0) THEN DZ1=0.
    )
#
# ---- SPECIFY THE DEPENDENT VARIABLE FUNCTION
#

```

Figure A-4. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to Hydrostatic Consolidation Test Database Only (Page 3 of 4).

```
/FUN
  F=Z1.
  IF(TIME EQ 0.) THEN (
    NEW = 0.
    Z1=EAT.
    F=Z1).
#
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----
#
/SAVE
  FILE = 'R:\BMDP\ZEUCH\ZEUCH1.SAV'.
  NEW.
  KEEP = TIME,EAT.
  FORMAT = '(5E12.3)'.
/PRINT
  FORMAT = E.
/PLOT
  RESIDUAL.
/END
```

Figure A-4. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to Hydrostatic Consolidation Test Database Only (Page 4 of 4).


```

# File = r:\bmdp\zeuch\zeuch2.inp
#
# Fit to shear consolidation tests only (icase=2)
#
# =====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
#   1. mean stress                (SM)
#   2. stress difference          (DS)
#   3. emplaced density          (RHO0)
#   4. initial density for differential equation (RHOI)
#   5. grain size                (DD)
#   6. absolute temperature      (T)
#   7. moisture content          (W)
#
# =====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
  TITLE = 'ZEUCH MODEL FIT TO RSISNL.CSV'.      # 55 RE/SPEC and SNL tests.
  FILE = 'R:\BMDP\RSISNL.CSV'.                 # Data file name
  FORMAT = '(2I3,3F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
  VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
  NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS
  MS = (2.0*LS+AS)/3.0.    # mean stress, as=axial stress, ls=conf. pres.
  DS = ABS(AS - LS).       # stress difference
  ELT = (EVT - EAT)/2.     # lateral strain, evt=vol. strain, eat=ax.strain
  D0 = RHO0/2160.          # rho0=initial density, 2160=intact density
  DI = RHOI/2160.          # fractional density at the beginning of creep
  ONE = 1.0.
  USE = ICASE EQ 2.
  WT = 1.0.
#

```

Figure A-5. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to Shear Consolidation Test Database Only (Page 1 of 4).

```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = ONE.          # Dependent variables
  PARAMETERS=17.         # Number of regression parameters
  WEIGHT = WT.           # WEIGHT VARIABLE
  ITIME = 3.             # INTEGRATION VARIABLE NUMBER
  NEQN = 2.              # Number of differential equations
  ITER = 100.            # NUMBER OF ITERATIONS
  HALVINGS = 10.         # NUMBER OF INTERVAL HALVINGS
  MAXC = 300000.         # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=      B2,      B3,      B7,
              B8,      N,      KAPPA0,
              KAPPA1,  KAPPA2,  M2,
              ETA0,    ETA1,    ETA2,
              M1,
              A1,      A2,      P,      QCR.
INITIAL = 1.065699E-27, 6.699538E-21, 1.418777E-11,
          3.781116E-11, 2.732519E+00, 1.342958E+02,
          1.536979E-35, 8.645950E-02, 2.043231E+00,
          6.457341E+00, 5.958929E-15, 3.105514E-02,
          1.085479E+00,
          2.014929E+01, 9.657048E+01, 3.960221E-01, 9.256731E-17.
MINIMUM = 0.0,      0.0,      0.0,
          0.0,      0.0,      0.0,
          0.0,      0.0,      0.0,
          0.0,
          0.0,      -10.0,      0.0,      -10.0.

```

Figure A-5. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to Shear Consolidation Test Database Only (Page 2 of 4).

```

MAXIMUM =      100.0,      100.0,      100.0,
              500.0,      100.0,      500.0,
              100.0,      500.0,      100.0,
              100.0,      100.0,      100.0,
              100.0,
              100.0,      1000.0,      100.0,      10000.
FIXED = A1,A2,P,QCR.
#
# ---- INITIAL VALUES FOR DIFFERENTIAL EQUATIONS ----
#
/DIFIN
  Z1=EAT.
  Z2=ELT.
#
# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
  IF (TIME NE 0.0) THEN (
    VOL=Z1+2*Z2.
    DEN=D0/EXP(VOL).
    KAPPA=KAPPA0*(1-D0/EXP(VOL))**KAPPA1.
    ETA=ETA0*(1-D0/EXP(VOL))**ETA1.
    T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
    T2=M2*DS**(M2-1).
    SEQF=ETA*(1-EXP(-ETA2*MS))-DS**M1.).
    FA=(T1/3+T2).
    FL=(T1/3-0.5*T2).
    G=1/DD**P.
    M=1+A1*(1-EXP(-A2*W)).
    THETA=EXP(-QCR/TEMP).
    IF (DEN LE 1.0) THEN (
      IF (DEN LE 0.9) THEN (
        ZEUCH=B7*(D0**B2)*DEN**(2*(B2-N)-1)*((DEN-D0)/(1-D0))**(B3-N).).
      IF (DEN GT 0.9) THEN (
        ZEUCH=B8*(1-DEN)/((1-(1-DEN)**(1/N))**N).).
      IF (DEN GT 1.0) THEN ZEUCH=0.0.
      DZ1=ZEUCH*FA*SEQF*M*G*THETA.
      DZ2=ZEUCH*FL*SEQF*M*G*THETA.).
    IF (TIME EQ 0) THEN (
      DZ1=0.
      DZ2=0.).
#

```

Figure A-5. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to Shear Consolidation Test Database Only (Page 3 of 4).

```

# ---- SPECIFY THE DEPENDENT VARIABLE FUNCTION
#
/FUN
  ALPHA = Z1/EAT.
  BETA = Z2/ELT.
  WT1 = DT/1.0E+6.
  F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).
  IF(TIME EQ 0.) THEN (
    NEW = 0.
    Z1=EAT.
    Z2=ELT.
    ALPHA = Z1/EAT.
    BETA = Z2/ELT.
    WT1 = DT/1.0E+6.
    F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).).
#
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----
#
/SAVE
  FILE = 'R:\BMDP\ZEUCH\ZEUCH2.SAV'.
  NEW.
  KEEP = TIME,EAT,ELT.
  FORMAT = '(5E12.3)'.
/PRINT
  FORMAT = E.
/PLOT
  RESIDUAL.
/END

```

Figure A-5. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to Shear Consolidation Test Database Only (Page 4 of 4).

```

# File = r:\bmdp\zeuch\zeuch3.inp
#
# Fit to the combined hydrostatic consolidation tests (icase=1) and shear
# consolidation tests (icase=2)
#
# =====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
# 1. mean stress (SM)
# 2. stress difference (DS)
# 3. emplaced dnesity (RHO0)
# 4. initial density for differential equation (RHOI)
# 5. grain size (DD)
# 6. absolute temperature (T)
# 7. moisture content (W)
#
# =====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
  TITLE = 'ZEUCH MODEL FIT TO RSISNL.CSV'. # 55 RE/SPEC and SNL tests.
  FILE = 'R:\BMDP\RSISNL.CSV'. # Data file name
  FORMAT = '(I3,F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
  VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
  NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS # DEFINE MEAN STRESS (SM), Fraction density AND WEIGHT FUNCTION (WT)
  MS = (2.0*LS+AS)/3.0. # mean stress, as=axial stress, ls=conf. pres.
  DS = ABS(AS - LS). # stress difference
  ELT = (EVT - EAT)/2. # lateral strain, evt=vol. strain, eat=ax.strain
  D0 = RHO0/2160. # rho0=initial density, 2160=intact density
  DI = RHOI/2160. # fractional density at the beginning of creep
  ONE = 1.0.
  USE = ICASE EQ 1 OR ICASE EQ 2.
  WT = 1.0.
#

```

Figure A-6. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to the Combined Test Database Only (Page 1 of 4).

```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = ONE.           # Dependent variables
  PARAMETERS=17.          # Number of regression parameters
  WEIGHT = WT.            # WEIGHT VARIABLE
  ITIME = 3.              # INTEGRATION VARIABLE NUMBER
  NEQN = 2.               # Number of differential equations
  ITER = 100.             # NUMBER OF ITERATIONS
  HALVINGS = 10.          # NUMBER OF INTERVAL HALVINGS
  MAXC = 300000.          # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=    B2,      B3,      B7,
            B8,      N,      KAPPA0,
            KAPPA1,  KAPPA2,  M2,
            ETA0,    ETA1,    ETA2,
            M1,
            A1,      A2,      P,      QCR.
INITIAL = 6.506557E-28, 1.053548E-20, 1.429409E-11,
          1.086919E-11, 3.222074E+00, 1.636756E+02,
          4.930381E-32, 7.369839E-02, 2.021284E+00,
          2.029039E+01, 7.021788E-15, 6.655437E-03,
          1.459375E+00,
          2.014929E+01, 9.657048E+01, 3.960221E-01, 9.256731E-17.
MINIMUM = 0.0,      0.0,      0.0,
          0.0,      0.0,      0.0,
          0.0,      0.0,      0.0,
          0.0,
          0.0,      -10.0,      0.0,      -10.0.

```

Figure A-6. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to the Combined Test Database Only (Page 2 of 4).

```

MAXIMUM =    100.0,    100.0,    100.0,
             500.0,    100.0,    500.0,
             100.0,    500.0,    100.0,
             100.0,    100.0,    100.0,
             100.0,
             100.0,    1000.0,    100.0,    10000.

#
# ---- INITIAL VALUES FOR DIFERENTIAL EQUATIONS ----
#
/DIFIN
  Z1=EAT.
  Z2=ELT.
#
# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
  IF (TIME NE 0.0) THEN (
    VOL=Z1+2*Z2.
    DEN=D0/EXP(VOL).
    KAPPA=KAPPA0*(1-D0/EXP(VOL))*KAPPA1.
    ETA=ETA0*(1-D0/EXP(VOL))*ETA1.
    T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
    IF (ICASE EQ 2) THEN (
      T2=M2*DS**(M2-1).
      SEQF=ETA*(1-EXP(-ETA2*MS))-DS**M1.).
    IF (ICASE EQ 1) THEN (
      T2=0.
      SEQF=ETA*(1-EXP(-ETA2*MS)).).
    FA=(T1/3+T2).
    FL=(T1/3-0.5*T2).
    G=1/DD**P.
    M=1+A1*(1-EXP(-A2*W)).
    THETA=EXP(-QCR/TEMP).
    IF (DEN LE 1.0) THEN (
      IF (DEN LE 0.9) THEN (
        ZEUCH=B7*(D0**B2)*DEN**(2*(B2-N)-1)*((DEN-D0)/(1-D0))**(B3-N).).
      IF (DEN GT 0.9) THEN (
        ZEUCH=B8*(1-DEN)/((1-(1-DEN)**(1/N))**N).).
      IF (DEN GT 1.0) THEN ZEUCH=0.0.
      DZ1=ZEUCH*FA*SEQF*M*G*THETA.
      DZ2=ZEUCH*FL*SEQF*M*G*THETA.).).
    IF (TIME EQ 0) THEN (
      DZ1=0.
      DZ2=0.).
#

```

Figure A-6. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to the Combined Test Database Only (Page 3 of 4).

```

# ---- SPECIFY THE DEPENDENT VARIABLE FUNCTION
#
/FUN
  ALPHA = Z1/EAT.
  BETA = Z2/ELT.
  WT1 = DT/1.0E+6.
  F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).
  IF(TIME EQ 0.) THEN (
    NEW = 0.
    Z1=EAT.
    Z2=ELT.
    ALPHA = Z1/EAT.
    BETA = Z2/ELT.
    WT1 = DT/1.0E+6.
    F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).
  )
#
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----
#
/SAVE
  FILE = 'R:\BMDP\ZEUCH\ZEUCH3.SAV'.
  NEW.
  KEEP = TIME,EAT,ELT.
  FORMAT = '(5E12.3)'.
/PRINT
  FORMAT = E.
/PLOT
  RESIDUAL.
/END

```

Figure A-6. Input File to BMDP Nonlinear Regression Program AR for Zeuch Model Fit to the Combined Test Database Only (Page 4 of 4).


```

# File = r:\bmdp\spiers\spiers1.inp
#
# Fit to hydrostatic consolidation tests only (icase=1).
#
#=====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
# 1. mean stress (SM)
# 2. stress difference (DS)
# 3. emplaced density (RHO0)
# 4. initial density for differential equation (RHOI)
# 5. grain size (DD)
# 6. absolute temperature (T)
# 7. moisture content (W)
#
#=====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
TITLE = 'SPIERS MODEL FIT TO 45 HYDROSTATIC CONSOLIDATION TESTS'.
FILE = 'R:\BMDP\RSISNL.CSV'. # Data file name.
FORMAT = '(2I3,3F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS
MS = (2.0*LS+AS)/3.0. # mean stress, as=axial stress, ls=conf. pres.
DS = ABS(AS - LS). # stress difference
D0 = RHO0/2160. # rho0=initial density, 2160=intact density
DI = RHOI/2160. # fractional density at the beginning of creep
ONE = 1.0.
USE = ICASE EQ 1.
WT = (DT/EAT**2)/1.0E+6.
#

```

Figure A-7. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to Hydrostatic Consolidation Test Database Only (Page 1 of 3).

```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = EAT.          # Dependent variables
  PARAMETERS=16.         # Number of regression parameters
  WEIGHT = WT.           # WEIGHT VARIABLE
  ITIME = 3.             # INTEGRATION VARIABLE NUMBER
  NEQN = 2.              # Number of differential equations
  ITER = 100.            # NUMBER OF ITERATIONS
  HALVINGS = 10.         # NUMBER OF INTERVAL HALVINGS
  MAXC = 300000.         # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=  R1,      R3,      R4,
          N,      KAPPA0,   KAPPA1,
          KAPPA2,   M2,      ETA0,
          ETA1,     ETA2,    M1,
          A1,      A2,      P,      QCR.
  INIT = 3.930969E-10, 1.967375E+01, 2.005564E+00,
        2.911066E-02, 3.523686E+02, 5.260176E-17,
        4.671744E-02, 2.026037E+00, 5.806067E+00,
        9.032922E-22, 2.004772E-01, 1.596766E-28,
        8.069893E+01, 5.318675E-01, 5.706072E-19, 6.884243E+02.
  MINIMUM = 0.0, 0, 0, -1, 0, -1, 0, 0,
            0, -1, 0, -1, 0, 0, 0, 0.
  MAXIMUM =50.0, 50, 50, 50, 500, 50, 50, 50,
            50, 50, 50, 50, 500, 50, 50, 1000.
  FIXED = KAPPA0,M2,ETA0,ETA1,M1.
#
# ---- INITIAL VALUES FOR DIFFERENTIAL EQUATIONS ----
#
/DIFIN
  Z1=EAT.
#

```

Figure A-7. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to Hydrostatic Consolidation Test Database Only (Page 2 of 3).

```

# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
  IF (TIME NE 0.0) THEN (
    VOL=3*Z1.
    KAPPA=KAPPA0*(1-D0/EXP(VOL))**KAPPA1.
    ETA=ETA0*(1-D0/EXP(VOL))**ETA1.
    T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
    SEQF=ETA*(1-EXP(-ETA2*MS)).
    FA=T1/3.
    G=1/DD**P.
    M=1+A1*(1-EXP(-A2*W)).
    THETA=EXP(-QCR/TEMP).
    CR=ABS(EXP(VOL)-1).
    IF(CR LE 0.15) THEN GAMMA=1.0.
    IF(CR GT 0.15) THEN GAMMA=(ABS((D0-EXP(VOL))/((1-D0)*EXP(VOL))))**N.
    X3=EXP((R3-1)*VOL)/(ABS(1-EXP(VOL)))**R4.
    SPIERS=R1*X3*GAMMA.
    DZ1=SPIERS*FA*SEQF*M*G*THETA.).
  IF (TIME EQ 0.0) THEN DZ1=0.
#
# ---- SPECIFY THE DEPENDENT VARIABLE FUNCTION
#
/FUN
  F=Z1.
  IF(TIME EQ 0.) THEN (
    NEW = 0.
    Z1=EAT.
    F=Z1).
#
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----
#
/SAVE
  FILE = 'R:\BMDP\SPIERS\SPIERS1.SAV'.
  NEW.
  KEEP = TIME,EAT.
  FORMAT = '(5E12.3)'.
/PRINT
  FORMAT = E.
/PLOT
  RESIDUAL.
/END

```

Figure A-7. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to Hydrostatic Consolidation Test Database Only (Page 3 of 3).

```

# File = r:\bmdp\spiers\spiers2.inp
#
# Fit to shear consolidation tests only (icase=2).
#
# =====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
# 1. mean stress (SM)
# 2. stress difference (DS)
# 3. emplaced density (RHO0)
# 4. initial density for differential equation (RHOI)
# 5. grain size (DD)
# 6. absolute temperature (T)
# 7. moisture content (W)
#
# =====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
  TITLE = 'SPIERS MODEL FIT TO 10 SHEAR CONSOLIDATION TESTS'.
  FILE = 'R:\BMDP\RSISNL.CSV'. # Data file name.
  FORMAT = '(2I3,3F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
  VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
  NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS
  MS = (2.0*LS+AS)/3.0. # mean stress, as=axial stress, ls=conf. pres.
  DS = ABS(AS - LS). # stress difference
  ELT = (EVT - EAT)/2. # lateral strain, evt=vol. strain, eat=ax.strain
  D0 = RHO0/2160. # rho0=initial density, 2160=intact density
  DI = RHOI/2160. # fractional density at the beginning of creep
  ONE = 1.0.
  USE = ICASE EQ 2.
  WT = 1.0.
#

```

Figure A-8. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to Shear Consolidation Test Database Only (Page 1 of 4).

```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = ONE.           # Dependent variables
  PARAMETERS=16.          # Number of regression parameters
  WEIGHT = WT.            # WEIGHT VARIABLE
  ITIME = 3.              # INTEGRATION VARIABLE NUMBER
  NEQN = 2.               # Number of differential equations
  ITER = 100.             # NUMBER OF ITERATIONS
  HALVINGS = 10.          # NUMBER OF INTERVAL HALVINGS
  MAXC = 300000.          # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=  R1,      R3,      R4,
          N,      KAPPA0,   KAPPA1,
          KAPPA2,   M2,      ETA0,
          ETA1,     ETA2,    M1,
          A1,      A2,      P,      QCR.
  INIT = 2.636508E-09, 1.029438E+01, 3.796323E-26,
        3.542610E+00, 2.592413E+02, -1.901144E-01,
        3.855113E-02, 1.974461E+00, 1.965395E+00,
        2.428668E-24, 4.607594E-01, 4.372353E-27,
        7.110572E+01, 6.260268E-01, 3.218921E-05, 1.803877E+02.
  MINIMUM = 0.0, 0, 0, 0, 0, -10, 0, 0,
            0, 0, 0, -10, 0, 0, 0, 0.
  MAXIMUM =50.0, 50, 50, 50, 500, 50, 50, 50,
            50, 50, 50, 50, 500, 50, 50, 500.
  FIXED = A1,A2,P,QCR.
#
# ---- INITIAL VALUES FOR DIFFERENTIAL EQUATIONS ----
#
/DIFIN
  Z1=EAT.
  Z2=ELT.
#

```

Figure A-8. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to Shear Consolidation Test Database Only (Page 2 of 4).

```

# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
  IF (TIME NE 0.0) THEN (
    VOL=Z1+2*Z2.
    KAPPA=KAPPA0*(1-D0/EXP(VOL))**KAPPA1.
    ETA=ETA0*(1-D0/EXP(VOL))**ETA1.
    T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
    T2=M2*DS**(M2-1).
    SEQF=ETA*(1-EXP(-ETA2*MS))-DS**M1.
    FA=(T1/3+T2).
    FL=(T1/3-0.5*T2).
    G=1/DD**P.
    M=1+A1*(1-EXP(-A2*W)).
    THETA=EXP(-QCR/TEMP).
    CR=ABS(EXP(VOL)-1).
    IF(CR LE 0.15) THEN GAMMA=1.0.
    IF(CR GT 0.15) THEN GAMMA=(ABS((D0-EXP(VOL))/((1-D0)*EXP(VOL))))**N.
    X3=EXP((R3-1)*VOL)/(ABS(1-EXP(VOL)))**R4.
    SPIERS=R1*X3*GAMMA.
    DZ1=SPIERS*FA*SEQF*M*G*THETA.
    DZ2=SPIERS*FL*SEQF*M*G*THETA.).
  IF (TIME EQ 0.0) THEN (
    DZ1=0.
    DZ2=0.).
#
# ---- SPECIFY THE DEPENDENT VARIABLE FUNCTION
#
/FUN
  ALPHA = Z1/EAT.
  BETA = Z2/ELT.
  WT1 = DT/1.0E+6.
  F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).
  IF(TIME EQ 0) THEN (
    NEW = 0.
    Z1=EAT.
    Z2=ELT.
    ALPHA = Z1/EAT.
    BETA = Z2/ELT.
    WT1 = DT/1.0E+6.
    F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1)).
#

```

Figure A-8. Input File to **BMDP** Nonlinear Regression Program AR for Spiers Model Fit to Shear Consolidation Test Database Only (Page 3 of 4).

```
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----  
#  
/SAVE  
    FILE = 'R:\BMDP\SPIERS\SPIERS2.SAV'.  
    NEW.  
    KEEP = TIME,EAT,ELT.  
    FORMAT = '(5E12.3)'.  
/PRINT  
    FORMAT = E.  
/PLOT  
    RESIDUAL.  
/END
```

Figure A-8. Input File to **BMDP** Nonlinear Regression Program AR for Spiers Model Fit to Shear Consolidation Test Database Only (Page 4 of 4).

```

# File = f:\bmdp\spiers\spiers3.inp
#
# Fit to the combined hydrostatic consolidation tests (icase=1) and shear
# consolidation tests (icase=2)
#
# =====
#
# This code uses the BMDP routine AR to fit the differential
# equations that describe the axial and lateral strain rates
# as functions of the following test conditions:
#   1. mean stress                (SM)
#   2. stress difference          (DS)
#   3. emplaced density          (RHO0)
#   4. initial density for differential equation (RHOI)
#   5. grain size                (DD)
#   6. absolute temperature      (T)
#   7. moisture content          (W)
#
# =====
#
# ---- INPUT PARAGRAPH ----
#
/INPUT
  TITLE = 'SPIERS MODEL FIT TO RSISNL.CSV'.      # 55 RE/SPEC and SNL tests.
  FILE = 'F:\BMDP\RSISNL.CSV'.                  # Data file name.
  FORMAT = '(2I3,3F10.0,F6.1,2F9.4,4F9.5,F9.3,F9.6,2F9.3,F5.2,F7.2)'.
  VARIABLES = 18.
#
# ---- VARIABLES PARAGRAPH ----
#
/VARIABLE
  NAMES=ICASE,ITEST,TIME,DT,TF,TEMP,AS,LS,EVT,EVC,EAT,EAC,RHO,D,RHO0,RHOI,DD,W.
#
# ---- TRANSFORMATION PARAGRAPH ----
#
/TRANS
  MS = (2.0*LS+AS)/3.0.    # mean stress, as=axial stress, ls=conf. pres.
  DS = ABS(AS - LS).       # stress difference
  ELT = (EVT - EAT)/2.     # lateral strain, evt=vol. strain, eat=ax.strain
  D0 = RHO0/2160.          # rho0=initial density, 2160=intact density
  DI = RHOI/2160.          # fractional density at the beginning of creep
  ONE = 1.0.
  USE = ICASE EQ 1 OR ICASE EQ 2.
  WT = 1.0.
#

```

Figure A-9. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to the Combined Test Database (Page 1 of 4).


```

# ---- REGRESS PARAGRAPH ----
#
/REGRESS
  DEPEND = ONE.          # Dependent variables
  PARAMETERS=16.         # Number of regression parameters
  WEIGHT = WT.           # WEIGHT VARIABLE
  ITIME = 3.             # INTEGRATION VARIABLE NUMBER
  NEQN = 2.              # Number of differential equations
  ITER = 100.            # NUMBER OF ITERATIONS
  HALVINGS = 8.          # NUMBER OF INTERVAL HALVINGS
  MAXC = 300000.         # MAXIMUM NUMBER OF TIMES 'DIFEQ' IS USED
#
# ---- PARAMETERS PARAGRAPH ----
# Specify the names, initial estimates, and the range of the sixteen
# regression parameters, where R1 is the over all leading parameter; R3, R4,
# and N are strain dependent parameters; KAPPA0, KAPPA1, KAPPA2, M2 determine
# the equivalent stress measure for the flow-rule; ETA0, ETA1, ETA2, and M1
# determine the equivalent stress measure in strain rate measure; A1 and A2
# are water-content dependent parameters; P defines grain size dependence;
# and QCR defines temperature effect.
#
/PARAMETER
  NAMES=  R1,      R3,      R4,
          N,      KAPPA0,   KAPPA1,
          KAPPA2,   M2,     ETA0,
          ETA1,    ETA2,    M1,
          A1,      A2,      P,      QCR.
  INIT = 6.059439E-11, 1.583143E+01, 2.161328E+00,
        5.220879E-01, 3.523686E+02, 1.141932E-17,
        4.245049E-02, 2.026037E+00, 5.806067E+00,
        9.032922E-22, 1.422396E-01, 1.596766E-28,
        7.110572E+01, 6.260268E-01, 3.219821E-05, 1.803877E+02.
  MINIMUM = 0.0, 0, 0, 0, 0, -1, 0, 0,
            0, -1, 0, -1, 0, 0, 0, 0.
  MAXIMUM = 50.0, 50, 50, 50, 500, 50, 50, 50,
            50, 50, 50, 50, 500, 50, 50, 500.
#  FIXED = KAPPA0,M2,ETA0,ETA1,M1,QCR.
#
# ---- INITIAL VALUES FOR DIFFERENTIAL EQUATIONS ----
#
/DIFIN
  Z1=EAT.
  Z2=ELT.
#

```

Figure A-9. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to the Combined Test Database (Page 2 of 4).

```

# ---- SPECIFY THE DIFFERENTIAL EQUATION ----
#
/DIFEQ
  IF (TIME NE 0.0) THEN (
    VOL=Z1+2*Z2.
    KAPPA=KAPPA0*(1-D0/EXP(VOL))**KAPPA1.
    ETA=ETA0*(1-D0/EXP(VOL))**ETA1.
    T1=KAPPA*KAPPA2*EXP(-KAPPA2*MS).
    IF (ICASE EQ 2) THEN (
      T2=M2*DS**(M2-1).
      SEQF=ETA*(1-EXP(-ETA2*MS))-DS**M1.).
    IF (ICASE EQ 1) THEN (
      T2=0.
      SEQF=ETA*(1-EXP(-ETA2*MS)).).
    FA=(T1/3+T2).
    FL=(T1/3-0.5*T2).
    G=1/DD**P.
    M=1+A1*(1-EXP(-A2*W)).
    THETA=EXP(-QCR/TEMP).
    CR=ABS(EXP(VOL)-1).
    IF(CR LE 0.15) THEN GAMMA=1.0.
    IF(CR GT 0.15) THEN GAMMA=(ABS((D0-EXP(VOL))/((1-D0)*EXP(VOL))))**N.
    X3=EXP((R3-1)*VOL)/(ABS(1-EXP(VOL)))**R4.
    SPIERS=R1*X3*GAMMA.
    DZ1=SPIERS*FA*SEQF*M*G*THETA.
    DZ2=SPIERS*FL*SEQF*M*G*THETA.).
  IF (TIME EQ 0.0) THEN (
    DZ1=0.
    DZ2=0.).
#
# ---- SPECIFY THE DEPENDENT VARIABLE FUNCTION
#
/FUN
  ALPHA = Z1/EAT.
  BETA = Z2/ELT.
  WT1 = DT/1.0E+6.
  F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).
  IF(TIME EQ 0.) THEN (
    NEW = 0.
    Z1=EAT.
    Z2=ELT.
    ALPHA = Z1/EAT.
    BETA = Z2/ELT.
    WT1 = DT/1.0E+6.
    F=1.0 - SQRT(((1.0-ALPHA)**2 + (1.0-BETA)**2)*WT1).).

```

Figure A-9. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to the Combined Test Database (Page 3 of 4).

```
#
# ---- SPECIFY VARIOUS PRINTING AND PLOTTING OPTIONS ----
#
/SAVE
    FILE = 'F:\BMDP\SPIERS\SPIERS3.SAV'.
    NEW.
    KEEP = TIME,EAT,ELT.
    FORMAT = '(5E12.3)'.
/PRINT
    FORMAT = E.
/PLOT
    RESIDUAL.
/END
```

Figure A-9. Input File to BMDP Nonlinear Regression Program AR for Spiers Model Fit to the Combined Test Database (Page 4 of 4).

APPENDIX B

CONSTITUTIVE MODEL FITS TO THE HYDROSTATIC AND SHEAR CONSOLIDATION TESTS

APPENDIX B

CONSTITUTIVE MODEL FITS TO THE HYDROSTATIC AND SHEAR CONSOLIDATION TESTS

This appendix contains 55 figures. Each figure represents the results of a laboratory test; either a hydrostatic consolidation test or a shear consolidation test. Each figure shows the model fits for each of the three constitutive laws. The three plots contained in each figure represent the three sets of parameter values determined for each model.

Figure B-1. Hydrostatic Consolidation Test CS1.

Figure B-2. Hydrostatic Consolidation Test CS2.

Figure B-3. Hydrostatic Consolidation Test CS3.

Figure B-4. Hydrostatic Consolidation Test CS4.

Figure B-5. Hydrostatic Consolidation Test CS5.

B-8

Figure B-6. Hydrostatic Consolidation Test CS6.

Figure B-7. Hydrostatic Consolidation Test CS7.

Figure B-8. Hydrostatic Consolidation Test CS8.

B-11

— Test Data Spiers Predicted ---- Zeuch Predicted -.-.- S-K Predicted

Figure B-9. Hydrostatic Consolidation Test CS9.

Figure B-10. Hydrostatic Consolidation Test CS10.

Figure B-11. Hydrostatic Consolidation Test HC1A.

Figure B-12. Hydrostatic Consolidation Test HC2A.

B-15

Figure B-13. Hydrostatic Consolidation Test HC3A.

Figure B-14. Hydrostatic Consolidation Test HC4A.

Figure B-15. Hydrostatic Consolidation Test HC5A.

Figure B-16. Hydrostatic Consolidation Test HC6A.

Figure B-17. Shear Consolidation Test SC1B.

Figure B-18. Shear Consolidation Test SC2A.

Figure B-19. Shear Consolidation Test SC3A.

Figure B-20. Shear Consolidation Test SC4A.

Figure B-21. Shear Consolidation Test SC5A.

Figure B-22. Shear Consolidation Test SC6A.

Figure B-23. Shear Consolidation Test SC7A.

Figure B-24. Shear Consolidation Test SC8A.

Figure B-25. Shear Consolidation Test SC9B.

Figure B-26. Hydrostatic Consolidation Test 27JUL82.

Figure B-27. Hydrostatic Consolidation Test 04MAY82.

Figure B-28. Hydrostatic Consolidation Test 30APR82.

Figure B-29. Hydrostatic Consolidation Test 07MAY82.

Figure B-30. Hydrostatic Consolidation Test 12MAY82.

Figure B-31. Hydrostatic Consolidation Test 05FEB82.

Figure B-32. Hydrostatic Consolidation Test 09APR82.

Figure B-33. Hydrostatic Consolidation Test 26MAR82.

Figure B-34. Hydrostatic Consolidation Test 02APR82.

Figure B-35. Hydrostatic Consolidation Test 13APR82.

Figure B-36. Hydrostatic Consolidation Test 29JAN82.

Figure B-37. Hydrostatic Consolidation Test 26FEB82.

Figure B-38. Hydrostatic Consolidation Test 27JU61.

Figure B-39. Hydrostatic Consolidation Test 23JL51.

B-42

Figure B-40. Hydrostatic Consolidation Test 14NV51.

B-43

Figure B-41. Hydrostatic Consolidation Test 25FE61.

Figure B-42. Hydrostatic Consolidation Test 10MY51.

Figure B-43. Hydrostatic Consolidation Test 20AU51.

B-46

Figure B-44. Hydrostatic Consolidation Test 16JL51.

Figure B-45. Hydrostatic Consolidation Test 18JU51.

Figure B-46. Hydrostatic Consolidation Test 300C51.

Figure B-47. Hydrostatic Consolidation Test 16JA61.

Figure B-48. Hydrostatic Consolidation Test 19DC44.

Figure B-49. Hydrostatic Consolidation Test 13AU51.

Figure B-50. Hydrostatic Consolidation Test 24JL71.

Figure B-51. Hydrostatic Consolidation Test 09JU881.

Figure B-52. Hydrostatic Consolidation Test 08MR89.

Figure B-53. Hydrostatic Consolidation Test 20SE89.

Figure B-54. Hydrostatic Consolidation Test 18AP89.

Figure B-55. Shear Consolidation Test 12OC891.

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WIPP Panel**

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Oxyrase, Incorporated
7327 Oak Ridge Highway
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Board of Radioactive Waste Management
GF456
2101 Constitution Ave.
Washington, DC 20418

Rodney C. Ewing
Department of Geology
University of New Mexico
Albuquerque, NM 87131

Charles Fairhurst
Department of Civil and Mineral Engineering
University of Minnesota
500 Pillsbury Dr. SE
Minneapolis, MN 55455-0220

B. John Garrick
PLG Incorporated
4590 MacArthur Blvd., Suite 400
Newport Beach, CA 92660-2027

Leonard F. Konikow
US Geological Survey
431 National Center
Reston, VA 22092

Carl A. Anderson, Director
Board of Radioactive Waste Management
National Research Council
HA 456
2101 Constitution Ave. NW
Washington, DC 20418

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ICF Kaiser Engineers
1800 Harrison St., 7th Floor
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Savannah River Ecology Lab
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Aiken, SC 29802

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Stanford, CA 94305-2115

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Pennsylvania State University
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FRANCE

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Centre d'Etudes Nucleaires de la Vallee Rhone
CEN/VALRHO
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