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Identifying Damping Of A Subsystem By Two Inverse-Dynamics Methods

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Abstract

A strategy is presented to develop computationally efficient models for a class of structures containing nonlinearities. Those structures are ones for which the predominant nonlinearity is in the interfaces of linear subsystems. In those cases, one hopes to achieve low order models for the linear subsystems coupled with simplistic models for the interfaces. The theme of this paper is that of deducing the properties of the nonlinear interfaces by examining the properties of the full nonlinear structure in light of the known properties of the linear subsystems. Situations where such problems arise include those where the nonlinearity derive from sliding friction or stick-slip friction. Those conditions can seriously compromise system performance if not addressed adequately, occasionally leading to either sloppy control or complete loss of stability. It is the problem of identifying those nonlinear subsystems that is addressed here.

Keywords: inverse dynamics, dynamic programming, nonlinear model reduction

2. Introduction

It is often the case that one needs either to develop a control design or a process control for a mechanical system containing nonlinear subsystems. In those cases, performance and even stability can rely on having an adequate dynamic model for the overall system. Even if most of the structure is fundamentally linear, the presence of the nonlinear subsystem is enough to make the overall system nonlinear and inscrutable. A class of nonlinear dynamic systems that may lend itself to rapid calculation will be presented below. To achieve that rapid calculation, it is necessary to get a handle on at least the nature of the intrinsic nonlinearity. The work reported here is focussed on deducing the nature of nonlinear subsystem from high resolution models of the linear subsystems coupled with observations of the system as a whole. The focus of the work reported is on nonlinear damping because of its near ubiquity in mechanical systems and its impact on control of dynamic systems.^{1, 2, 3, 4}

A paradigm of a mechanical system containing nonlinear subsystems is that of machine tools. The evolution of active control strategies to suppress vibration or regenerative chatter in machine tools requires the development of high-fidelity models. Though much of the structure is linear, the response of the structure is dominated by nonlinear processes in the interface (the cutting region) between linear components. Unfortunately, because of accessibility problems, direct measurement of the mechanics of those structures is usually nearly impossible. A characteristic picture of this situation is shown in Figure 1. Another similar class of problems are those in which linear subsystems are connected by nonlinear joints. Because of issues of motion, temperature, or accessibility, measurements can only take only of linear portions of the structure in these problems. Still, in order to devise rapid computational models for the full structure, it is necessary to deduce simple models for the combined structure.

There are three general approaches to developing numerical models for these nonlinear dynamic systems:

1. create detailed numerical models from the physics of the nonlinear subsystem and couple those with models for the dynamics of the linear components;
2. Create parametric models for the whole system from observations made at accessible points. Such approaches include attractor identification in chaos maps;
3. Using the known mechanics of the linear subsystems and experimental data for the system as a whole, deduce simple models for the nonlinear subsystem.

It is the third of these which is explored here.

The purpose of this study is to explore and test techniques to extract the properties of nonlinear subsystems that are separated from measurement by linear systems.

There are three components to this task:

- deducing the kinematics on the boundaries of the nonlinear domain from the dynamics of the linear structure
- deducing the forces on the boundaries of the nonlinear domain from the dynamics of the linear structure
- deducing a simple model relating those forces and displacements.

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Nonlinear domain

Linear domains

Figure 1: Cutting machines are generally linear structures containing small regions of large nonlinearity.

Figure 2: Linear systems having jointed connections will manifest nonlinear properties even though the nonlinearity resides exclusively in the jointed interface.

To perform the first two steps, one needs to fully characterize the linear dynamic subsystem, invert it, and then perform calculations to deduce the interface displacements and forces from observations made on linear portions of the structure. The inverse dynamics issue is one of classical difficulty, often involving band-limiting, causality, or stability problems. To mitigate some of these difficulties, we explore two separate methods of inverting dynamics, and test them against a simple paradigm problem.

3. Paradigm Problem

A simple linear two mass system is coupled to a nonlinear system. Measurements of force and displacement are taken from the left hand side of the linear system. From that data and a mathematical model for the linear system, features of the nonlinear subsystem on the right are to be deduced.

Figure 3: A simple nonlinear system composed of a two-mass linear sub-system coupled to a nonlinear sub-system whose properties are to be determined indirectly by measurements of force and displacement on the left hand side of the linear subsystem.

The equations for this linear system are:

$$m_1 \ddot{u}_1 = f_1 + k(u_2 - u_1) + c(\dot{u}_2 - \dot{u}_1) \quad (1)$$

and

$$m_2 \ddot{u}_2 = f_2 - k(u_2 - u_1) - c(\dot{u}_2 - \dot{u}_1) \quad (2)$$

It is the displacement u_2 , its derivatives, and force f_2 which must be deduced from u_1 and f_1

Before selecting a method for deducing u_2 and f_2 for difficult problems such as nonlinear damping, we start out with the simplest possible problem, where the "unknown" system is a linear spring. We then apply the more promising method to addressing nonlinear damping.

In exploring methods of performing the inverse dynamics, we perform the following steps:

1. Specify a mechanism (possibly nonlinear) for the right hand side.
2. Calculate the dynamics of the full system in response to specified forces f_1 applied to the left hand side, saving only the histories of f_1 and u_1 .
3. Forgetting how the right hand side was specified, invert the dynamics and using the above histories of f_1 and u_1 to calculate the histories of u_2 and f_2 .

Because of the difficulties inherent to the inverse dynamics problem, two distinct approaches that offer some hope of mitigating those difficulties were selected and tested: a Fourier method and an optimization method.

4. Force Reconstruction by Fourier Analysis

Fourier analysis is appealing because, as an integral method, it offers the potential of being more forgiving and because it preserves causality. Further, this approach also has the advantage of building on the art and science of modal testing and analysis⁵.

Fourier transforming Equations 1 and 2 and solving for the transforms of u_2 and f_2 ,

$$U_2 = \frac{U_1(-\omega^2 m_1 + k + i\omega c) - F_1}{k + i\omega c} \quad (3)$$

and

$$F_2 = (k + i\omega c - \omega^2)U_2 - (k + i\omega c)U_1 \quad (4)$$

where initial conditions are assumed to be homogeneous; $U_1 = F(u_1)$; $U_2 = F(u_2)$; $F_1 = F(f_1)$; $F_2 = F(f_2)$; and $F()$ indicates Fourier transform of its argument. In the numerical experiments presented below, Fourier integrals are approximated by discrete fast-Fourier transforms over finite intervals.

Immediately, we see that at high frequencies, U_2 is of order ω and that F_2 is of order ω^3 . Unless test functions U_1 are used that decay faster than $\omega^{1/3}$, the Fourier transforms U_2 and F_2 will not decay and will not be invertible back to time space. Unfortunately, our problem is such that we can specify F_1 , but cannot specify U_1 . Further, in some problems, such as stick-slip, one might want to specify force histories $f_1(t)$ that are designed to excite that phenomenon but whose Fourier transforms will probably not decay quickly with frequency. The significance of this restriction is explored below.

The parameters for the two-mass system were selected according to the following table:

Table 1: Parameters of Two-Mass System

Parameter	m_1	m_2	k	c
Value	1.0	1.0	1.0	0.01

Numerical experiments were performed with both impulsive and oscillatory loads f_1 . The two figures shown below are associated with a driving force $f_1(t) = 1 - \cos(t)$, a force history that one might use to explore stick-slip friction. In these experiments fourth-order Runge-Kutta "forward" calculations are performed to predict the response of the whole system to the prescribed force. In what follows, the linear system on the left is referred to as the "known" system and the (possibly nonlinear) system on the right as the "unknown" system. The prescribed force f_1 and the resulting displacement u_1 of the "known" system are used along with the linear model to estimate the responses of the "unknown" system.

For the first test case, the "unknown" system is a simple linear spring and the initial results are not very encouraging. The following three figures (Figure 4 for computed Fourier transform of displacement, Figure 5 for the computed Fourier transform of force, and Figure 6 force and displacement in the time domain) demonstrate the potential of this method. We see that the estimates of the Fourier transform of the displacement of the "unknown" system are reasonably good for small frequencies, but diverge systematically at larger frequencies. Since the Fourier transforms of the forces are derived from those of the displacement in Eq 4, it is not surprising that the estimates for the Fourier transforms for the force f_2 are similarly systematically off. These curves are mapped back to the time domain in Figure 6. Here we see that errors at high frequency in the Fourier transforms of the force and displacement result in oscillations in the estimated displacement about the true curve and errors in the force that overwhelm the true force. This problem is that discussed above: the transfer functions in Equations 3 and 4 increase very strongly with ω and the test functions U_1 and F_1 do not decrease quickly enough with ω to cause U_2 and F_2 to decrease with ω - a necessary condition for meaningful transformation back to the time domain.

In order to make U_2 and F_2 invertible to the time domain, their values at high frequency must be suppressed. This is done by applying the filter shown in Figure 7 to them before transforming to the time domain. When the filtered transforms are mapped back to the time domain, the results are more reasonable. The calculated forces and displacement are shown in Figure

Figure 4: Numerically computed real and imaginary parts of the Fourier transform of the displacement u_2 of the "unknown" system. The solid curves are the exact solution and the crosses are computed from Equation 3. The lower curves are the data for the upper curves expanded at lower frequencies.

8. One sees that the inverted Fourier transforms approximate the true solutions reasonably well, except at very short times. A plot of force versus displacement and force versus velocity is presented in Figure 9. This plot does show the linear force-displacement of the spring. The force-velocity plot shows that force and velocity stay out of phase.

Note that the filter used above is a low-pass filter, so information associated with rapid changes is lost. Those band-limiting errors are manifest by the difference between true and calculated values for force and displacement at short times.

The conclusions to be drawn about this method are:

- This method can be made to reproduce the forces and displacements on the linear structure due to the presence of the "unknown" system, for this case.
- Either narrowly selected forcing functions must be used or a low-pass filter must be designed to assure correct inversion to the time domain.
- The repeated oscillations that one would like to impose on the system to explore stick-slip processes are expected to have Fourier transforms that decay slowly with frequency, so it would be necessary to use a low-pass filter.
- Because much of the important stick-slip information will be high frequency, a low-pass filter would be a serious impediment to capturing that phenomena.

It would be desirable to find a method for reconstructing the forces and displacement due to the interface between known and unknown systems more suitable to capture stick-slip-like phenomena.

5. Force Reconstruction by Dynamic Programming

In the second method presented here, the inverse dynamics problem is formulated as a discrete-time optimal control problem. In this formulation, the forces at the linear system boundary are treated as control variables. The optimal control problem is

Figure 5: Numerically computed real and imaginary parts of the Fourier transform of the force f_2 of the "unknown" system. The solid curves are the exact solution and the crosses are computed from Equation 4. The lower curves are the data for the upper curves expanded at lower frequencies.

solved using a efficient dynamic programming algorithm [6, 7]. This algorithm has the attractive feature that the number of mathematical operations required grows only linearly with the number of discrete times. In the problem at hand, the algorithm was configured to find the unknown function f_2 that minimizes the functional:

$$R(f_2) = \int_0^T [(u_1(t) - \hat{u}_1(t))^2 + \alpha(f_2(t))^2] dt \quad (5)$$

where u_1 is the observed displacement on the left-hand-side, $\hat{u}_1$ is that which would result from the application of force f_2 to the linear subsystem, and α is an adjustable parameter (set equal to 10^{-7} in these calculations). Note that this approach is computationally attractive: the core algorithm is very fast and there is no transformation to and from frequency space. Two other features of the formulation are:

- Minimizing $R(f_2)$ involved finding an f_2 that generates a $\hat{u}_1$ that closely approximates the observed u_1 , but without wild excursions in f_2 itself. This second constraints behaves like a low-pass filter.
- because f_2 at each time is determined to maximize the agreement of u_1 and $\hat{u}_1$ at all times, we can expect this process to be non-causal.

Figure 6: The displacement and force on the "unknown" system in the time domain. The solid curves are the exact solution and the crosses are those obtained by numerical inversion of the curves on the previous two figures.

For the test case of a linear spring as the "unknown" system, the dynamic programming method works very well. Figure 10 shows the true and computed force and displacement for the case of a linear spring and force reconstruction of the unknown quantities by dynamic programming. The agreement between true and computed force is better than was the case with the Fourier technique, though there is still some error at the beginning and end of the time interval. The anomalies occurring at early-time and are artifacts of both band-limiting and non-causality. Errors occurring at late time are due to noncausality. The force displacement curves are shown in Figure 11, reproducing the properties of the linear spring. This particular plot was constructed without the first 10% of the data points, thereby removing the systematic error found at those times.

With the encouragement of the success of the dynamic programming method applied to the problem of a linear spring, we go on to examining damping. The first of these explorations is the case of linear damping. Figure 12 shows computed force f_2 versus time and computed velocity u_2 versus time as well as computed force versus computed velocity. Both the force and velocity vary sinusoidally with time and the force varies linearly with velocity, reproducing the properties of a linear damper. Again, anomalies occur at early and late time. The linearity of force with velocity is as expected for a linear damper.

This brings us to the case of sliding friction, where

$$f_2 = -F_{\mu_k} \text{sign}(\dot{u}_2) \quad (6)$$

Figure 7: The filter used to remove the erroneous high frequency information from the estimates of the Fourier transforms of the force f_2 and displacement u_2 .

and F_{μ_k} is the sliding friction, equal to the product of weight times the coefficient of kinematic friction. In our calculations, we have set $F_{\mu_k} = 0.1$. The predictions for this case of sliding friction are shown in Figure 13. In this figure one sees plots of computed force f_2 and computed velocity $\dot{u}_2$ as functions of time. Also shown is a plot of computed force versus computed velocity. The force versus time plot shows force jumping back and forth between positive and negative values, as one expects with sliding friction. The plot of force versus velocity does appear to be trying to reproduce the properties of Equation 6.

The most difficult problem is addressed last. This is that of stick-slip friction where

$$f_2 = \begin{cases} -F_{spring} & \text{for } |\dot{u}_2| < \delta \text{ and } |F_{spring}| < F_{\mu_s} \\ -F_{\mu_k} \text{sign}(F_{spring}) & \text{for } |\dot{u}_2| < \delta \text{ and } F_{\mu_s} \leq |F_{spring}| \\ -F_{\mu_k} \text{sign}(\dot{u}_2) & \text{for } \delta < |\dot{u}_2| \end{cases} \quad (7)$$

Above, $F_{spring} = -k(u_2 - u_1) - c(\dot{u}_2 - \dot{u}_1)$, F_{μ_k} is sliding friction, F_{μ_s} is static friction, and δ defines a range of small velocities at which static friction prevails. In our calculations, we have set $F_{\mu_s} = 1.0$. Plots of computed force versus time and computed velocity versus time as well as computed force versus computed velocity are shown in Figure 14. The force

Figure 9: The force-displacement and the force velocity curves for a linear spring. Values computed using Fourier methods are shown with crosses (+). These curves are achieved by applying a sharp filter to the Fourier curves presented above. The near linear plot of force displacement reproduces the stiffness of the spring.

8. References

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Figure 10: Computed displacement and force for the case of a linear spring and force reconstruction of the unknown quantities by dynamic programming. In the displacement plot, the reconstructed displacement is represented by crosses(+) and the "true" displacement is a continuous curve.

Figure 11: The computed force versus computed displacement of the unknown system. The linear behavior reproduces the properties of the linear spring. This plot was constructed from the last 90% of the data to avoid systematic error associated with early time.

Figure 12: Plots of computed velocity versus time, computed force versus time, and computed force versus computed velocity, all for the case of linear damping. In the velocity plot, the reconstructed velocity is represented by crosses(+) and the "true" velocity is a continuous curve. The third plot was constructed deleting the first 10% of the data points to remove corresponding artifacts. The very linear nature of force versus velocity reproduces the properties of the linear damper.

Figure 13: Computed velocity $\dot{u}_2$ and computed force f_2 as functions of time for the case of sliding friction. Also shown is a plot of computed force versus computed velocity. The force versus time plot shows force jumping back and forth between positive and negative values, as one expects with sliding friction. Because of anomalous values in force are seen at early times, only the last 90% of the time steps were used in the last of these plots. The exact force/velocity curve is drawn in light gray.

Figure 14: Computed velocity $\dot{u}_2$ and computed force f_2 as functions of time for the case of stick-slip friction. Also shown is a plot of computed force versus computed velocity. The force versus time plot shows force jumping back and forth between positive and negative values of kinematic friction, as well as additional, higher values near zero velocity - signifying stick. The force versus velocity plot shows the expected discontinuities near zero velocity. The exact force/velocity curve for stick-slip is drawn in light gray.