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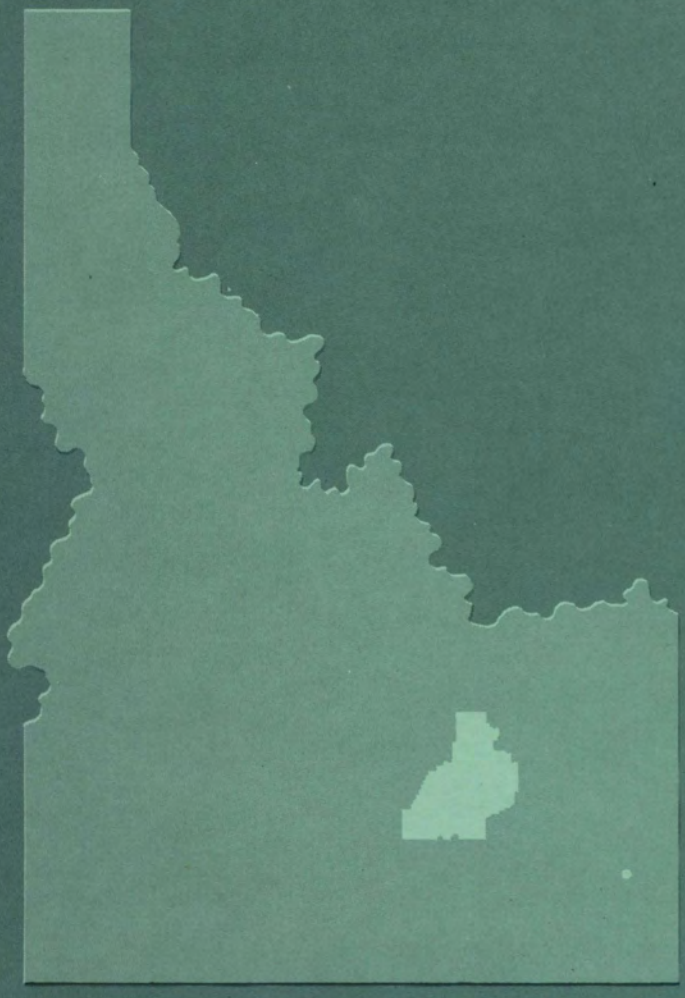
TOPIC
A FORTRAN PROGRAM FOR CALCULATING TRANSPORT OF PARTICLES IN CYLINDERS

G. E. Putnam



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A FORTRAN PROGRAM FOR CALCULATING TRANSPORT OF PARTICLES IN CYLINDERS

by

G. E. Putnam

PHILLIPS
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PREFACE

During the conceptual design of the Advanced Test Reactor (ATR), the difficulty of obtaining converged S_n solutions to some of the thermal reactor neutron transport problems with the existing codes prompted the examination of different methods of solution of the S_n difference equations. This examination led to the development of the MIST code, which employs a direct method of solution instead of an iterative one in certain phases of the solutions. It thereby attains solutions to thermal reactor problems in as little time as any other kind. The MIST code was developed by G. E. Putnam and D. M. Shapiro of Internuclear Company, Clayton, Missouri for Phillips Petroleum Company and it is reported in IDO-16856.

The MIST code was a first attempt in using a direct method to solve for the fluxes in each group and it was applicable only to slab geometries. The most commonly encountered neutron transport problems, however, occur as cylindrical geometry problems. The TOPIC code is the first attempt to use the methods proved effective in MIST to help solve the more complicated cylindrical geometry problems.

D. M. Shapiro, presently employed with Washington University Computing Facilities in St. Louis, Missouri, performed almost all the programming on the MIST code. Because many of the TOPIC routines are only slight modifications of MIST routines, special acknowledgment is due Mr. Shapiro for this significant contribution to the TOPIC program.

TOPIC

A FORTRAN PROGRAM FOR CALCULATING
TRANSPORT OF PARTICLES IN CYLINDERS

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1.0 INTRODUCTION

The so-called S_n method of solution of the Boltzmann transport equation has been used in the field of reactor design for a number of years. The method, originally developed by Bengt G. Carlson at LASL, has been programmed for almost all the high speed digital computers at various installations; and the family of S_n codes has grown steadily with a continuing major effort to speed the convergence of the iterative processes used in the programs. The Program S codes of B. H. Duane of HAPO are probably the most sophisticated S_n codes presently in existence.

Unfortunately, the customary iterative methods used in solving for the fluxes in a neutron energy group with the S_n codes make some problems practically impossible to solve. If a problem contains a region which is many mean free paths thick and which has a very low absorption to scattering cross section ratio, convergence becomes so slow that machine time requirements negate the use of the usual S_n codes.

The MIST code (IDO-16856) for slab geometries was the first S_n code to overcome these difficulties by using a feasible direct method of solution which eliminated the troublesome iterations altogether. In cylindrical geometries, however, the introduction of another angle into the Boltzmann transport equation greatly complicates the solution as compared to slab geometry cases. The direct method of solution used in the MIST code becomes immediately infeasible from the standpoint of machine time when applied to the much larger matrices encountered in the cylindrical geometry S_n problems. Consequently, a method was sought which would be partially iterative and partially direct and combine the advantages of the two methods. The TOPIC code for cylindrical geometry is the first S_n code to try such a combination of methods. The extra angular variable of cylindrical geometry is handled by Gauss quadrature, exactly as in the Program S versions. However, instead of handling the fluxes at each Gauss quadrature point iteratively, the fluxes at the first Gauss point are obtained by the direct method proved effective in the MIST code. Then given the fluxes at the first Gauss point, the other fluxes are obtained by the usual iterative process - with one very important difference which greatly speeds convergence: the values actually

solved for at the second and higher Gauss points are the differences between each flux value and the corresponding flux value of the first Gauss point.

The TOPIC code is written in Fortran II and it was originally intended only to try the effectiveness of the new method. It contains many inefficient routines because of the lack of any tape input and output buffering. However, it has been shown to be very much faster than the other S_n codes on the difficult problems; and even on the problems that run in very short times with the other S_n codes (because only a few iterations suffice) it does not require much more time.

2.0 SUMMARY

The TOPIC codes are designed to solve the one-dimensional Boltzmann equation in cylindrical geometry with up to six energy groups, two hundred and forty space points, forty regions, and anisotropic (P_1) scattering.

The boundary conditions for each group can be independently specified and the flexibility of the specifications permit:

a) Perfect mirror reflection or symmetry.

b) Isotropic reflection (Lambert surface reflection).

c) Anisotropic diffuse sources by means of either a P_1 Legendre series or a short table of point values for the angular flux.

Independent specification of isotropic fixed volume sources for each group is also allowed.

As implied, both homogeneous and inhomogeneous problems are solved, and fissions can occur in either type of problem.

A direct method of solution which was shown to be effective in the MIST code is used to solve for some of the angular flux points in each group and an inner iterative solution is used to solve for the others. The result is that most problems converge in as few outer iterations as diffusion theory solutions, and usually only one or two inner iterations need be performed.

The numerical approximation to the Boltzmann equation is a linear one with respect to variation in the space variable r and one of the angular variables. Gauss quadrature is used to handle the second (polar) angular variable. The difference equations solved are exactly identical to those solved by B. H. Duane's S codes. (See GE ANP report No. XDC-59-9-118.)

TOPIC solutions require about 50% more time than would the solution of a comparable problem with MIST. An S_4 problem with four groups of neutrons, 75 mesh points, and 3 Gauss quadrature points took 5 outer iterations to converge to a pointwise variation of less than the fraction 1×10^{-3}. The running time was 2.93 minutes on the 7090. A comparable MIST problem took 2.1 minutes.

Because of the automatic tape buffering which is available with the Fortran IV monitoring system, some of the inefficiencies of the Fortran II version of TOPIC are removed when the program is run on the 7040. Consequently, TOPIC problems require less time on the IBM 7040, even though it is a basically slower machine. (The times required are usually from 0.4 to 0.9 of the time required on the 7090.)

3.0 The Derivation of the Difference Equations of TOPIC

3.1 The Boltzmann Equation in Cylindrical Coordinates

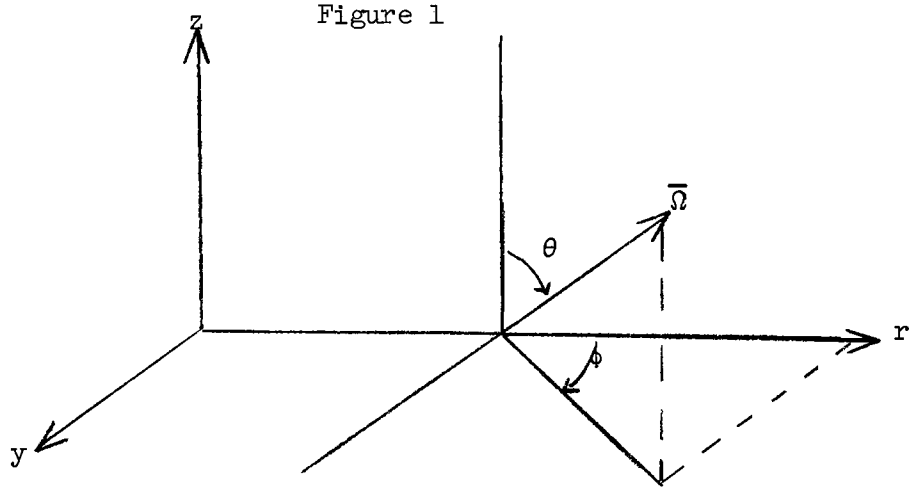
The basic equation solved approximately by the TOPIC code is the Boltzmann equation which expresses the balance or conservation of particles in a cylindrical coordinate system. It is assumed that there is no variation of the angular distribution of the flux in either the z direction (parallel to the cylinder axis) or the direction perpendicular to both the z and the radial directions. The reader is referred to A. M. Weinberg and E. P. Wigner, The Physical Theory of Neutron Chain Reactors (University of Chicago Press, 1958), pp. 275-276 for a derivation of the monoenergetic Boltzmann equation in cylindrical coordinates. This monoenergetic form of the equation is the form which is solved in each energy group in order to solve the usual reactor problems. The equation solved in each group can be written as

$$\frac{\sqrt{1-\mu^2}\cos\phi}{r}\,\frac{\partial\left[rf(r,\mu,\phi)\right]}{\partial r} - \frac{\sqrt{1-\mu^2}}{r}\,\frac{\partial\left[\sin\phi\, f(r,\mu,\phi)\right]}{\partial\phi} \qquad (3.a)$$

$$+\sum(r)f(r,\mu,\phi) = \int_0^{2\pi}\int_{-1}^{1} \sum_S(r,\mu_o)\, f(r,\mu',\phi')\, d\mu' d\phi' + S(r,\mu,\phi).$$

In Figure 1, $\overline{\Omega}$ is the unit vector in the direction μ,ϕ, where $\mu = \cos\theta$. θ is the polar between the z direction and $\overline{\Omega}$, and ϕ is the azimuthal angle between the radial direction and the projection of $\overline{\Omega}$ onto the r-y plane.

Figure 1



$f(r,\mu,\phi)d\Omega\ dV$ is the number of particles in the volume element dV at r with directions of travel in $d\Omega$ about $\bar{\Omega}$ which, per unit time cross the area $d\Omega$ (perpendicular to $\bar{\Omega}$) on the unit sphere. $f(r,\mu,\phi)\ d\Omega\ dV$ is equal to the number of particles in dV and $d\Omega$ multiplied by the speed of particles.

$\sum(r)$ is the total collision cross section (area per unit volume) at r and $\sum_s(r,\mu_o)$ is the scattering cross section (area per unit volume) for scattering of particles from $d\Omega'$ into $d\Omega$. $d\Omega'$ is the solid angle $d\mu'\ d\phi'$ about the direction $\bar{\Omega}'$ which is defined by μ' and ϕ'. μ_o is the cosine of the angle between $\bar{\Omega}'$ and $\bar{\Omega}$. Only axially symmetric scattering is assumed.

$S(r,\mu,\phi)$ is the fixed source for the particle energy group. It is always handled in Eq. 3.a as if it were a constant function insofar as the $f(r,\mu,\phi)$ of the group is concerned; however, it can be a function of $f(r,\mu,\phi)$ obtained from a previous "outer iteration," as defined later. $S(r,\mu,\phi)$ is the sum of three terms. These are:

a) $q(r)$, the fixed volume source of particles equal to the number of particles emitted isotropically per unit volume per unit solid angle per second at r.

b) $\frac{1}{\lambda}\ \frac{\chi_g}{4\pi}\ \sum_{g'=1}^{G} \nu\Sigma_{f_{g'}}(r)\ \Phi'_{g'}(r)$, the number of particles per unit volume per unit solid angle per second emitted at r. In a neutron reactor problem, χ_g is the fraction of neutrons released in the fissioning process which appear in the $g^{\underline{th}}$ group, $\nu\Sigma_{f_g}$ is the product of the neutrons produced per fission times the fission cross section (area per unit volume) for neutrons in the $g^{\underline{th}}$ group, and $\Phi'_g(r)$ is the total flux defined by the integral $\int_0^{2\pi}\int_{-1}^{1} f'_g(r,\mu,\phi)\ d\mu\ d\phi$. λ is 1 in inhomogeneous problems, but in homogeneous problems it is the eigenvalue obtained on the latest "outer iteration".

The primes on $\Phi(r)$ and $f(r,\mu,\phi)$ denote the fact that the quantities are those obtained on the previous "outer iteration".

c) $\sum_{g'=1}^{g-1}\int_0^{2\pi}\int_1^{1} \Sigma_{s_{g'\to g}}(r,\mu_o)\ f_{g'}(r,\mu',\phi')\ d\mu'\ d\phi'$, the number of particles per unit volume at r scattered from group g' into the solid

angle $d\Omega$ in group g. $\Sigma_{s_{g' \to g}}(r, \mu_o)$ is the scattering cross section for the scattering of particles from $d\Omega'$ in group g' into $d\Omega$ in group g.

When the $g^{\underline{th}}$ group is group 1, the term c) is zero.

3.2 The Boundary Conditions

Given $S(r, \mu, \phi)$, $\Sigma(r)$, and $\Sigma_s(r, \mu_o)$ at every point in a system, the definition of a particular problem is complete following a specification of suitable boundary conditions. The boundary condition at either boundary for each group can be expressed by the equation,

$$f(r_\nu, \mu, \phi) = A_\nu f(r_\nu, \mu, \pi-\phi) + f_b(r_\nu, \phi) + \sum_{\ell=0}^{1} \beta_{\nu,\ell} P_\ell(\sqrt{1-\mu^2} \cos \phi) \qquad (3.b)$$

$$+ \frac{B_\nu}{C} \int_{\Delta\phi'} \int_{-1}^{1} \sqrt{1-\mu^2} \cos \phi' \, f(r_\nu, \mu, \phi') \, d\mu \, d\phi'$$

In this equation the subscript ν denotes the right or left boundary depending on whether $\nu = 1$ or 2, respectively.

The range of μ is -1 to 1. However, when $\nu = 1$ (right boundary) the range of ϕ is $\frac{-\pi}{2}$ to $\frac{\pi}{2}$ and the range of ϕ' is $\frac{\pi}{2}$ to $\frac{3\pi}{2}$; and when $\nu = 2$ the range of ϕ is $\frac{\pi}{2}$ to $\frac{3\pi}{2}$ and the range of ϕ' is $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.

A_ν, B_ν and $\beta_{\nu,\ell}$ for $\ell = 0$ and 1 are arbitrary constants and $f_b(r_\nu, \phi)$ is an arbitrary function. $P_\ell(\sqrt{1-\mu^2} \cos\phi)$ is the ordinary Legendre polynomial of order ℓ with the cosine of the angle between the radial direction and μ, ϕ as argument.

The constant C is defined by the formula,

$$C = (-1)^{\nu} 4 \sum_{k=1}^{K} h_k \sqrt{1-\mu_k^2} = \int_{\Delta \phi'_s} \int_{1}^{-1} \sqrt{1-\mu^2} \cos \phi' \, d\mu \, d\phi',$$

where the integral over the range of μ is approximated by Gauss quadrature. Hence the μ_k are the roots of $P_{2K}(\mu)$ and the h_k are defined by

$h_k = \int_{-1}^{1} p_k(\mu) \, d\mu$, where the $p_k(\mu)$ is the polynomial of μ of degree 2K-1 which is zero at all roots of $P_{2K}(\mu)$ except at μ_k, where it is unity.

The formula for $p_k(\mu)$ is $p_k(\mu) = \dfrac{P_{2K}(\mu)}{(\mu-\mu_k)\left[\frac{dP_{2K}(\mu_k)}{d\mu}\right]}$.

Table I presents the h_k values and other pertinent Gauss quadrature data corresponding to various values of K.

The reason for the approximate calculation of C is that in the TOPIC code all integrals over the range of μ are calculated in this manner, including the integral which appears in the B_s term of Eq. 3.b. It is necessary to compute C in this manner in order to prevent a fictitious source of particles. When $B_{\mathcal{S}} = 1$, it would be found that C/π instead of 1.0 times the outward current would be reflected back, thus causing a spurious source.

The physical meanings of each of the terms on the right hand side of Eq. 3.b can be stated as follows:

a) $A_{\mathcal{S}}$ is a "mirror albedo coefficient" such that the fraction $A_{\mathcal{S}}$ of any outward flux will be reflected as if by a mirror. $A_{\mathcal{S}}$ is used, therefore, to represent a symmetry boundary condition. For $\mathcal{S} = 1$ (at the center of a cylinder), true symmetry can exist, but at the outer boundary it is unrealistic to apply a symmetry condition - especially in neutron reactor cell problems. The reason for this is that application of the symmetry condition still leaves the angular flux equal to zero for $\phi = \frac{\pi}{2}$; i.e. $f(r_{\mathcal{S}}, \mu, \frac{\pi}{2}) = 0$.

There is an option in the TOPIC code to allow a more realistic use of the symmetry condition at the outer boundary. When $A_2 \neq 0$ and a code word MFR > 0 the TOPIC code sets $f(r_2, \mu, \frac{\pi}{2})$ equal a non-zero value in a manner which will be described in detail in 3.7 where the boundary condition difference equations are derived.

b) $B_{\mathcal{S}}$ is an "isotropic albedo coefficient" such that the fraction $B_{\mathcal{S}}$ of the outward current is reflected isotropically. This type of reflection is probably the most realistic type to apply in many nuclear reactor cell problems.

c) $\sum_{\ell=0}^{I} \mathcal{S}_{\mathcal{S},\ell} P_\ell(\sqrt{1-\mu^2} \cos \phi)$ is simply a Legendre polynomial representation of a source at a boundary. The source is axially symmetric about the radius vector. It should be noted that the Gauss quadrature used in performing all integrals over the range of μ makes it impossible, in general, to predict exactly how many particles will enter a surface in unit time and unit area for a given set, $\mathcal{S}_{\mathcal{S},0}$ and $\mathcal{S}_{\mathcal{S},1}$. However, in the

TABLE I

GAUSS QUADRATURE POINT DATA

K	k	h_k	$\cos\theta_k$	$\cos^2\theta_k$	$\sin\theta_k$	$1 - \cos^2\theta_k$ or $\sin^2\theta_k$	$\frac{4}{\pi}\sum_k h_k \sin\theta_k$,
1	1	1.0	.57735027	.33333333	.81649658	.66666667	1.03959573
2	1	.65214515	.33998104	.11558711	.94043229	.88441289	1.00603591
	2	.34785485	.86113631	.74155574	.50837412	.25844425	
3	1	.46791393	.23861919	.05693912	.97111322	.94306088	1.00195292
	2	.36076157	.66120939	.43719786	.75020140	.56280214	
	3	.17132449	.93246951	.86949939	.36124869	.13050061	
4	1	.36268378	.18343464	.03364827	.98303191	.96635173	1.00086509
	2	.31370665	.52553241	.27618431	.85077358	.72381568	
	3	.22238103	.79666648	.63467748	.60441916	.36532252	
	4	.10122854	.96028986	.92215662	.27900427	.07784339	
5	1	.29552422	.14887434	.02216357	.98885612	.97783643	1.00044938
	2	.26926021	.43339539	.18783156	.90120388	.81216844	
	3	.21908636	.67940957	.46159736	.73375925	.53840264	
	4	.14945135	.86506337	.74833463	.50166260	.25166537	
	5	.06667134	.97390653	.94849393	.22694949	.05150607	

TABLE I (continued)

K	k	h_k	$\cos\theta_k$	$\cos^2\theta_k$	$\sin\theta_k$	$1 - \cos^2\theta_k$ or $\sin^2\theta_k$	$\frac{4}{\pi}\sum_k h_k \sin\theta_k$,
6	1	.24914705	.12533341	.01570846	.99211468	.98429154	1.00026610
	2	.23349254	.36783150	.13530001	.92989246	.86469999	
	3	.20316743	.58731795	.34494237	.80935630	.65505762	
	4	.16007833	.76990267	.59275012	.63816133	.40724988	
	5	.10693933	.90411726	.81742802	.42728443	.18257198	
	6	.04717534	.98156063	.96346127	.19115107	.03653873	
7	1	.21526385	.10805495	.01167587	.99414492	.98832413	1.00017276
	2	.20519846	.31911237	.10183271	.94771689	.89816730	
	3	.18553840	.51524864	.26548116	.85704075	.73451884	
	4	.15720317	.68729290	.47237153	.72638039	.52762847	
	5	.12151857	.82720132	.68426202	.56190566	.31573798	
	6	.08015808	.92843488	.86199133	.37149519	.13800867	
	7	.03511946	.98628381	.97275575	.16505831	.02724425	
8	1	.18945061	.09501251	.00902738	.99547608	.99097262	1.00011714
	2	.18260342	.28160355	.07930056	.95953084	.92069944	
	3	.16915652	.45801678	.20977937	.88894355	.79022063	
	4	.14959599	.61787624	.38177105	.78627537	.61822895	
	5	.12462897	.75540441	.57063582	.65525886	.42936418	
	6	.09515851	.86563120	.74931737	.50068216	.25068263	
	7	.06225352	.94457502	.89222197	.32829565	.10777803	
	8	.02715245	.98940093	.97891420	.14520950	.02108580	

case when all other terms on the right hand side of Eq. 3.b are zero, the current into the surface produced by a given set, $\psi_{\nu,0}$ and $\psi_{\nu,1}$ will be

$$\int_{\Delta\phi'} \int_{-1}^{1} (\psi_{\nu,0} + \psi_{\nu,1}\sqrt{1-\mu^2}\cos\phi)\sqrt{1-\mu^2}\cos\phi \, d\mu \, d\phi$$

$$= C\psi_{\nu,0} + \pi\psi_{\nu,1} \sum_{k=1}^{K} h_k(1-\mu^2{}_k) = \sum_{k=1}^{K} 4\psi_{\nu,0} h_k\sqrt{1-\mu^2} + \frac{4\pi}{3}\psi_{\nu,1}$$

As K approaches a large number, the current will approach $\pi\psi_{\nu,0} + \frac{4\pi}{3}\psi_{\nu,1}$. When K = 2, the error in the $\psi_{\nu,0}$ term is about 0.6%.

d) The term $f_b(r_\nu, \phi)$ allows another form of boundary source specification. As is apparent, the source must be constant with respect to μ. This term facilitates the use of the output from one TOPIC problem as the input to another. The values input for $f_b(r_\nu, \phi_j)$ at each ϕ mesh point are intepreted in the same manner as are all other f functions in TOPIC; that is, all integrations are performed with the assumption that the variation of f between ϕ mesh points is linear with respect to cos ϕ.

The current due to this source (provided all other terms on the right hand side of Eq 3.b are zero) is

$4 \sum_{k=1}^{K} h_k\sqrt{1-\mu^2}_k \int_{\Delta\phi} f(r_\nu, \phi)\cos\phi \, d\phi$ where $\Delta\phi_\nu$ is the interval $\{0, \frac{\pi}{2}\}$ or the interval $\{\frac{\pi}{2}, \pi\}$ depending on whether ν = 1 or 2, respectively.

3.3 The Central Assumptions in Deriving the Difference Equations

There are three assumptions which are made in order to facilitate the derivation of the difference equations from the Boltzmann equation and the boundary conditions for each group. These assumptions, if they were rigorously correct, would permit the solution of the difference equations to be the rigorous solution of Eqs. 3.a and 3.b for each group. Conversely, the solution of the difference equations which is obtained by the TOPIC code is only as good as the approximations inherent in the assumptions for any given problem.

The assumptions are:

A. All scattering functions $\Sigma_s(r, \mu_o)$ are represented by a Legendre series as follows.

$\Sigma_s(r, \mu_o) = \sum_{\ell=0}^{L} \Sigma_\ell^r P_\ell(\mu_o)$ where $L \leq 1$ and Σ_ℓ^r is a constant throughout the $r^{\underline{th}}$ "region" on the range of the radius r. A region is defined as an

interval on the range of r in which all cross section functions are constant with respect to r and in which the mesh interval on r is constant. (See B.) In the derivations, L will be equal to 2, but TOPIC is restricted to $L \leqq 1$.

B. The half-range of ϕ from o to π is partitioned into n intervals such that $J = n + 1$ points are defined on the half-range of ϕ. The range of r is also partitioned into I-1 intervals and in each region one can speak of i_r equal intervals. Hence the total number of points defined on the range of r is $1 + \sum_{r=1}^{R} i_r = I$, where R is the number of regions. Also, one can speak of $f_{ij}(\mu) = f(r_i, \mu, \phi_j)$, where $1 \leqq i \leqq I$ and $1 \leqq j \leqq J$. It will be customary to define r_1 as the inner boundary value on r and r_I as the outer boundary value. Also, the partitioning on the half-range of ϕ is such that $\phi_1 = o$, $\phi_{J/2+1} = \frac{\pi}{2}$, and $\phi_J = \pi$ in all cases. (Because of the symmetry expressed by $f(r,\mu,\phi) = f(r,\mu,-\phi)$, it is necessary to work with only the half range of ϕ.)

The assumption is made in the TOPIC code that for $\phi_j \leqq \phi \leqq \phi_{j+1}$ and $r_i \leqq r \leqq r_{i+1}$, given any $1 \leqq j \leqq J-1$ and any $1 \leqq i \leqq I-1$, the function $f(r,\mu,\phi)$ is a bilinear function of r and cos ϕ such that one can express $f(r,\mu,\phi)$ in the form

$$f(r,\mu,\phi) = a^{ij}(\mu) + b^{ij}(\mu)\, r + c^{ij}(\mu) \cos\phi + d^{ij}(\mu)\, r \cos\phi.$$

It can be shown that a, b, c and d are defined by

$$a^{ij}(\mu) = \left[f_{i,j}(\mu) r_{i+1} v_{j+1} + f_{i,j}(\mu) r_i v_j - f_{i+1,j}(\mu) r_i v_{j+1} - f_{i,j+1}(\mu) r_{i+1} v_j \right]$$

$$b^{ij}(\mu) = \left[f_{i+1,j}(\mu) v_{j+1} - f_{i,j}(\mu) v_{j+1} - f_{i+1,j+1}(\mu) v_j + f_{i,j+1}(\mu) v_j \right],$$

$$c^{ij}(\mu) = \left[f_{i+1,j}(\mu) r_{i+1} - f_{i,j}(\mu) r_{i+1} - f_{i+1,j+1}(\mu) r_i + f_{i+1,j}(\mu) r_i \right], \text{ and}$$

$$d^{ij}(\mu) = \left[f_{i+1,j+1}(\mu) - f_{i+1,j}(\mu) - f_{i,j+1}(\mu) + f_{i,j}(\mu) \right], \text{ where}$$

$v_j = \cos\phi_j$ and $v_{j+1} = \cos\phi_{j+1}$.

C. All integrals over the range of μ are evaluated by Gaussian quadrature, i.e.,

$$\int_{-1}^{1} g(r,\mu,\phi)\, d\mu = \sum_{\substack{k=-K \\ k\neq 0}}^{K} h_k g_k(r,\phi) = 2 \sum_{k=1}^{K} h_k g_k(r,\phi) \text{ where } g_k(r,\phi) \equiv g(r,\mu_k,\phi).$$

The h_k and μ_k are as defined in 3.2 and $g(r,\mu,\phi)$ simply denotes any function to be integrated. The symmetry of all functions to be integrated about $\mu = o$ enables one to compute the integrals with only half the usual number of points.

3.4 Integration of the Scattering Cross Section Term

In Eq. 3.a there occurs the scattering integral

$$\int_0^{2\pi}\int_{-1}^{1} \Sigma_s(r,\mu_o) f(r,\mu',\phi')\, d\mu' d\phi'. \tag{3.c}$$

The $S(r,\mu,\phi)$ term also contains scattering integrals of exactly the same form; therefore, it will suffice to show the method of integrating only the integral 3.c.

It is assumed that $\Sigma_s(r,\mu_o) = \sum_{\ell=0}^{2} \mathcal{A}_\ell^r P_\ell(\mu_o)$, so that from the orthogonality of the Legendre polynomials

$$\mathcal{A}_\ell^r = \frac{2\ell+1}{2} \int_{-1}^{1} \Sigma_s(r,\mu_o)\, P_\ell(\mu_o)\, d\mu_o.$$

However, it is customary to use as basic input data the numbers

$$\Sigma_{s\ell}^r = \int_0^{2\pi}\int_{-1}^{1} \Sigma_s(r,\mu_o)\, P_\ell(\mu_o) d\mu_o d\phi = \frac{4\pi}{2\ell+1} \mathcal{A}_\ell^r; \text{ therefore,}$$

$$\Sigma_s(r,\mu_o) = \sum_{\ell=0}^{2} \frac{2\ell+1}{4\pi} \Sigma_{s\ell}^r P_\ell(\mu_o) . \tag{3.d}$$

The addition theorem for spherical harmonics states that

$$P_\ell(\mu_o) = P_\ell(\mu)P_\ell(\mu') + (1-\delta_{\ell o}) \, 2\sum_{m=1}^{\ell} \frac{(\ell-m)!}{(\ell+m)!} P_\ell^m(\mu') \cos m(\phi-\phi') P_\ell^m(\mu) \tag{3.e}$$

where μ_o is the cosine of the angle between μ,ϕ and μ',ϕ', $\delta_{\ell o}$ is the Kronecker delta, and $P_\ell^m(\mu)$ are the associated Legendre polynomials.

If one makes use of assumption C of 3.3 to carry out the integration over the range of μ' and also uses 3.d and 3.e, the integral 3.c becomes

$$\begin{aligned} &\sum_{\ell=o}^{2} \frac{2\ell+1}{4\pi} \Sigma_{s\ell}^r P_\ell(\mu) \sum_{\substack{k=-K \\ (k\neq o)}}^{K} h_k P_\ell(\mu_k) \int_0^{2\pi} f_k(r,\phi')\, d\phi' \\ &+ \sum_{\ell=1}^{2} \frac{2\ell+1}{4\pi} \Sigma_{s\ell}^r \; 2 \sum_{m=1}^{\ell} \frac{(\ell-m)!}{(\ell+m)!} P_\ell^m(\mu) \sum_{\substack{k=-K \\ (k\neq o)}}^{K} h_k P_\ell^m(\mu_k) \int_0^{2\pi} f_k(r,\phi) \cos m(\phi-\phi')\, d\phi'. \end{aligned} \tag{3.f}$$

The symmetry properties of $f(r,\mu,\phi)$ and $\cos m(\phi-\phi')$ lead to the result that

$$\int_0^{2\pi} f_k(r,\phi')d\phi' = 2\int_0^{\pi} f_k(r,\phi')d\phi' \text{ and}$$

$$\int_0^{2\pi} f_k(r,\phi')\cos m(\phi-\phi')d\phi' = 2\cos m\phi \int_0^{\pi} f_k(r,\phi')\cos m\phi' d\phi'.$$

Furthermore, since $f_k(r,\phi) = f_{-k}(r,\phi)$, $P_1(\mu_k) = -P_1(\mu_{-k})$, and $P_\ell^m(\mu_k) = (-1)P_\ell^m(\mu_{-k})$ when ℓ-m is an odd number, it follows that the terms involving $P_1(\mu_k)$ and $P_2^1(\mu_k)$ in 3.f are zero.

Assumption B of 3.3 is next used to express the integrals over the half-range of ϕ. The integrals can be expressed in the form

$$(3.g)\qquad \int_0^{\pi} f_k(r,\phi')\cos m\phi' d\phi' = \frac{1}{4}\sum_{j=1}^{J} a_j^m f_{k,j}(r),$$

where the $\frac{a_j^m}{4}$ are the coefficients which result from the assumption of linearity of $f_k(r,\phi)$ between the mesh points on ϕ.

The detailed definitions of the a_j^m are stipulated in appendix A, Formulae A.3 and A.4.

In expanded form, the cross section integral 3.c can now be written,

$$(3.h)\qquad \int_0^{2\pi}\int_{-1}^{1} \Sigma_s(r,\mu_o)\, f(r,\mu',\phi')d\mu'd\phi' =$$

$$\frac{1}{4\pi}\sum_{k'=1}^{K}\sum_{j'=1}^{J}\Big\{\Sigma_{so}^r h_{k'}a_{j'}^o + 3\Sigma_{s1}^r \cos\phi\, P_1^1(\mu)\, h_{k'}P_1^1(\mu_{k'})a_{j'}^1$$

$$+ 5\Sigma_{s2}^r P_2(\mu)h_{k'}P_2(\mu_{k'})a_{j'}^o + \frac{5}{12}\Sigma_{s2}^r \cos 2\phi\, P_2^2(\mu)h_{k'}P_2^2(\mu_{k'})a_{j'}^2\Big\}f_{k',j'}(r)$$

where $P_2(\mu) = \frac{1}{2}(3\mu^2-1)$, $P_1^1(\mu) = \sqrt{1-\mu^2}$, and $P_2^2(\mu) = 3(1-\mu^2)$.

By the definitions of the scalar flux, $\Phi(r)$, the current, $J(r)$ and the second moment fluxes $G_2^o(r)$ and $G_2^2(r)$ (see Appendix B), it follows that Eq. 3.h can be expressed,

$$(3.i)\qquad \int_0^{2\pi}\int_{-1}^{1} \Sigma_s(r,\mu_o)\, f(r,\mu',\phi')\, d\mu'd\phi' =$$

$$\frac{1}{4\pi}\Big\{\Sigma_{so}^r\Phi(r) + 3\Sigma_{s1}^r\cos\phi\, P_1^1(\mu)J(r) + 5\Sigma_{s2}^r P_2(\mu)G_2^o(r)$$

$$+ \frac{5}{12}\Sigma_{s2}^r\cos 2\phi\, P_2^2(\mu)G_2^2(r)\Big\}$$

3.5 Development of the Equations for Each Gauss Quadrature Point

From the Boltzmann equation 3.a, one can develope K equations (one for each point in the Gauss Quadrature) by simply setting μ in Eq. 3.a equal to μ_1, μ_2, $\ldots\mu_K$. That is, the equation for $f_k(r,\phi)$ is obtained from Eq. 3.a by simply setting $\mu = \mu_k$. If the result of Eq. 3.h is used, the following equation is obtained for the $k^{\underline{th}}$ Gauss point.

$$\frac{\sqrt{1-\mu_k^2}}{r}\left\{\cos\phi\,\frac{\partial\left[rf_k(r,\phi)\right]}{\partial r} - \frac{\partial\left[\sin\phi f_k(r,\phi)\right]}{\partial\phi}\right\} + \Sigma^r f_k(r,\phi) \qquad (3.j)$$

$$= S(r,\mu_k,\phi) + \frac{1}{4\pi}\sum_{k'=1}^{K}\sum_{j'=1}^{J}\Big\{\Sigma^r_{so}\,h_{k,}a^o_{j'} + 3\Sigma^r_{s1}\cos\phi\,P_1^1(\mu_k)h_{k,}P_1^1(\mu_{k,})a^1_{j'}$$

$$+ 5\Sigma^r_{s2}P_2(\mu_k)h_{k,}P_2(\mu_{k,})a^o_{j'} + \frac{5}{12}\Sigma^r_{s2}\cos 2\phi P_2^2(\mu_k)h_{k,}P_2^2(\mu_{k,})a^2_{j'}\Big\}f_{k',j'}(r)$$

One now has, for each group, K equations which happen to be coupled through the scattering integral term. It is easy to show that this coupling can lead to very slow convergence if the f_k values are solved iteratively; i.e. if one solves for f_1 in terms of assumed values of f_k for $k\geq 2$, then for f_2 in terms of the new value of f_1 and old values for f_k for $k\geq 3$, etc. To show this, postulate a system in which the flux is isotropic and constant with respect to r. (This situation would be a rare case in actuality, but it is approached, physically, when, in a group, there is present in the system a region which is a large number of mean free paths in thickness and which is characterized by a very small value for $(1-\Sigma^r_{so}/\Sigma^r)$. Also, let $S(r,\mu,\phi)$ be constant with respect to r,μ, and ϕ. $f_k(r,\phi)$ can be called simply f_k and $S(r,\mu_k\phi)$ can be called S.

In this case the first part of Eq. 3.j within braces is zero, and if one integrates Eq. 3.j with respect to ϕ one obtains

$$2\pi\,\Sigma\,f_k = 2\pi S + \frac{2\pi}{4\pi}\Sigma_{so}\sum_{k'=1}^{K}\sum_{j'=1}^{J}h_{k,}a^o_{j'}\,f_{k'}. \quad \text{But} \quad \sum_{K}^{j'=1}a^o_{j'} = 4\pi, \text{ so}$$

one can write this equation as simply $\Sigma f_k = S + \Sigma_{so}\sum_{k'=1}^{K}h_k f_{k'}$. For simplicity let $\Sigma = 1$ and denote Σ_{so}/Σ by χ. Then the set of equations for the system becomes

(3.k)
$$
\begin{aligned}
&(1-\chi h_1)f_1 - \chi h_2 f_2 - \chi h_3 f_3 \cdots\cdots - \chi h_K f_K = S \\
&-\chi h_1 f_1 + (1-\chi h_2)f_2 - \chi h_3 f_3 \cdots\cdots - \chi h_K f_K = S \\
&-\chi h_1 f_1 - \chi h_2 f_2 + (1-\chi h_3)f_3 \cdots\cdots - \chi h_K f_K = S \\
&\quad\vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad\quad \vdots \\
&-\chi h_1 f_1 - \chi h_2 f_2 - \chi h_3 f_3 \cdots\cdots + (1-\chi h_K)f_K = S.
\end{aligned}
$$

Now, because $\sum_{k=1}^{K} h_k = 1$, the solution to this set of equations is obviously just $f_1 = f_2 = \cdots f_K = \frac{S}{1-\chi}$. However, if the set is solved by the usual Gauss Seidel iterative technique, the number of iterations required to attain this solution to within, say, an error of a few per cent will approach infinity as χ approaches one. For example, take $K = 2$, then

$$f_1^i = \frac{S}{1-\chi h_1}\left\{1 + \frac{1}{\chi h_1}\sum_{n=1}^{i-1}\left[\frac{\chi^2 h_1 h_2}{(1-\chi h_1)(1-\chi h_2)}\right]^n\right\} \quad \text{and}$$

$$f_2^i = \frac{S}{(1-\chi h_1)(1-\chi h_2)}\sum_{n=0}^{i-1}\left[\frac{\chi^2 h_1 h_2}{(1-\chi h_1)(1-\chi h_2)}\right]^n \quad \text{where } i \geq 2 \text{ denotes the}$$

value obtained on the $i^{\underline{th}}$ iteration, given that the initial value of $f_2 = o$. The change in f_1 and f_2, denoted Δf_1^i and Δf_2^i, in going from the $\underline{i^{th}}$ iteration to the $i + 1^{th}$ iteration is therefore

$$\Delta f_1^i = \frac{S}{(1-\chi h_1)(\chi h_1)}\left[\frac{\chi^2 h_1 h_2}{(1-\chi h_1)(1-\chi h_2)}\right]^i \quad \text{and}$$

$$\Delta f_2^i = \frac{S}{(1-\chi h_1)(1-\chi h_2)}\left[\frac{\chi^2 h_1 h_2}{(1-\chi h_1)(1-\chi h_2)}\right]^i .$$

As χ approaches unity Δf_1^i and Δf_2^i both approach $\frac{S}{h_1 h_2}$. This is to say that since $h_1 \doteq .652$ and $h_2 \doteq .348$, $\frac{1}{h_1 h_2} \doteq 4.4$ so that the change in f_1 and f_2 per iteration is only about 4.4 times S. Hence if, say, $\chi = .9999$ and the solution is $f_1 = f_2 = 10,000$ S, it will clearly take many more than 2000 iterations for the iterative solutions to approach the true solutions.

With present day computer speeds, quickly convergent iterative solutions are the only kinds known to be feasible for this problem. Therefore, it would be clearly infeasible to solve Eqs. 3.j by iterative techniques whenever circumstances caused the equations to display the properties of Eqs. 3.k.

The trouble can be eliminated by a simple transformation of variables; namely,

$$y_1(r,\phi) = f_1(r,\phi) \text{ and}$$
$$y_k(r,\phi) = f_k(r,\phi) - f_1(r,\phi) \text{ for } 2 \leqq k \leqq K.$$

If this transformation is made in Eqs. 3.j, the equation for $y_1(r,\phi)$ becomes

$$\frac{\sqrt{1-\mu_1^2}}{r}\left\{\cos\phi\frac{\partial\left[ry_1(r,\phi)\right]}{\partial r} - \frac{\partial\left[\sin\phi\ y_1(r,\phi)\right]}{\partial\phi}\right\} + \Sigma^r\ y_1(r,\phi) \qquad (3.m)$$

$$= S(r,\mu_1,\phi) + \frac{1}{4\pi}\sum_{k'=1}^{K}\sum_{j'=1}^{J}\left\{\Sigma^r_{so}\ H^o_{k'}\ a^o_{j'} + \Sigma^r_{s1}\cos\phi\ P^1_1(\mu_1)H^1_{k'}a^1_{j'}\right.$$

$$\left. + 5\Sigma^r_{s2}P_2(\mu_1)H^{20}_{k'}a^o_{j'} + \frac{5}{12}\Sigma^r_{s2}\cos 2\phi\ \ P^2_2(\mu_1)\ H^{22}_{k'}a^2_{j'}\right\} y_{k',j'}(r);$$

and the equation for $y_k(r,\phi)$ for $k \geqq 2$ becomes

$$\frac{\sqrt{1-\mu_k^2}}{r}\left\{\cos\phi\ \frac{\partial\left[ry_k(r,\phi)\right]}{\partial r} - \frac{\partial\left[\sin\phi\ y_k(r,\phi)\right]}{\partial\phi}\right\} + \Sigma^r y_k(r,\phi) \qquad (3.n)$$

$$\frac{\sqrt{1-\mu_k^2}}{r}\left\{\cos\phi\ \frac{\partial\left[ry_1(r,\phi)\right]}{\partial r} - \frac{\partial\left[\sin\phi\ y_1(r,\phi)\right]}{\partial\phi}\right\} + \Sigma^r y_1(r,\phi)$$

$$= S(r,\mu_k,\phi) + \frac{1}{4\pi}\sum_{k'=1}^{K}\sum_{j'=1}^{J}\left\{\Sigma^r_{so}H^o_{k'}a^o_{j'} + 3\Sigma^r_{s1}\cos\phi\ P^1_1(\mu_k)H^1_{k'}a^1_{j'}\right.$$

$$\left. + 5\ \Sigma^r_{s2}P_2(\mu_k)H^{20}_{k'}a^o_{j'} + \frac{5}{12}\Sigma^r_{s2}\cos 2\phi\ P^2_2(\mu_k)H^{22}_{k'}a^2_{j'}\right\}\ y_{k',j'}(r).$$

In Eqs. 3.m and 3.n

$$H^o_1 = \sum_{k=1}^{K} h_k,\quad H^1_1 = \sum_{k=1}^{K} h_k\sqrt{1-\mu_k^2},\quad H^{20}_1 = \sum_{k=1}^{K} h_k\ P_2(\mu_k),$$

$$H^{22}_1 = \sum_{k=1}^{K} h_k P^2_2(\mu_k);\ \text{and, for } k \geqq 2,\ H^o_k = h_k,\ H^1_k = h_k\sqrt{1-\mu_k^2},$$

$$H^{20}_k = h_k P_2(\mu_k),\ \text{and } H^{22}_k = h_k P^2_2(\mu_k).$$

With the same transformation Eqs. 3.k become

$$
\begin{aligned}
&(1-\chi)y_1 - \chi h_2 y_2 - \chi h_3 y_3 \cdots\cdots - \chi h_K y_K = S \\
&(1-\chi)y_1 + (1-\chi h_2) y_2 - \chi h_3 y_3 \cdots - \chi h_K y_K = S \\
&\quad\cdot \qquad \cdot \qquad \cdot \qquad\qquad \cdot \qquad \cdot \\
&\quad\cdot \qquad \cdot \qquad \cdot \qquad\qquad \cdot \qquad \cdot \\
&\quad\cdot \qquad \cdot \qquad \cdot \qquad\qquad \cdot \qquad \cdot \\
&(1-\chi)y_1 - \chi h_2 y_2 - \chi h_3 y_3 \cdots\cdots + (1-\chi h_K) y_K = S.
\end{aligned}
$$

The solution to this set is obviously $y_1 = \frac{S}{1-\chi}$ and $y_2 = y_3 = \cdots y_K = 0$, and it is easy to demonstrate that the solution is obtained with only one Gauss Seidel iteration.

Finally, much redundant effort is saved if one uses, instead of Eq. 3.n to solve for y_k for $k \geq 2$, the difference between Eq. 3.m and and Eq. 3.n. Therefore, the equation which will be used to solve for y_k will be

$$
\begin{aligned}
(3.o)\quad & \frac{\sqrt{1-\mu_k^2}}{r}\left\{\cos\phi\,\frac{\partial\left[r y_k(r,\phi)\right]}{\partial r} - \frac{\partial\left[\sin\phi\; y_k(r,\phi)\right]}{\partial\phi}\right\} + \Sigma^r y_k(r,\phi) = \\
& \left(\frac{\sqrt{1-\mu_k^2}-\sqrt{1-\mu_1^2}}{r_K}\right)_J\left\{\cos\phi\,\frac{\partial\left[r y_1(r,\phi)\right]}{\partial r} - \frac{\partial\left[\sin\phi\; y_1(r,\phi)\right]}{\partial\phi}\right\} + \left[S(r,\mu_k,\phi) - S(r,\mu_1,\phi)\right] \\
& + \frac{1}{4\pi}\sum_{k'=1}\sum_{j'=1}\left\{3\,\Sigma^r_{s1}\cos\phi\left[P_1^1(\mu_k)-P_1^1(\mu_1)\right]H^1_{k',a^1_{j'}} \right. \\
& \left. + 5\,\Sigma^r_{s2}\left[P_2(\mu_k) - P_2(\mu_1)\right]H^{20}_{k',a^0_{j'}} + \frac{5}{12}\,\Sigma^r_{s2}\cos 2\phi\left[P_2^2(\mu_k)-P_2^2(\mu_1)\right]H^{22}_{k',a^2_j}\right\} y_{k',j'}(r).
\end{aligned}
$$

It can be noted that the isotropic part of $S(r,\mu_k,\phi)$ is subtracted by the isotropic part of $S(r,\mu_1,\phi)$ so that $S(r,\mu_k,\phi)-S(r,\mu_1,\phi)=$

$$
\begin{aligned}
& \frac{1}{4\pi}\sum_{g'=1}^{g-1}\left\{3\,\Sigma^r_{s1g'\to g}\cos\phi\left[P_1^1(\mu_k)-P_1^1(\mu_1)\right]J_{g'}(r)\right. \\
& \left. + 5\Sigma^r_{S2g'\to g}\left[P_2(\mu_k)-P_2(\mu_1)\right]G^0_{2g'}(r) + \frac{5}{12}\,\Sigma^r_{S2g'\to g}\cos 2\phi\left[P_2^2(\mu_k) - P_2^2(\mu_1)\right]G^2_{2g'}(r)\right\}.
\end{aligned}
$$

That is, the fission source, the fixed volume source, and all isotropic down-scatter source terms have been eliminated from the equations for $y_k(r,\phi)$ for $k \geq 2$.

3.6 The Difference Equations for Each Gauss Quadrature Point

It is apparent that since there are J mesh points on the half range of Φ, I mesh points on the range of r, and K Gauss quadrature points on the half range of μ, there are J.K.I flux values ($f_{k,i,j}$) for which to solve. The J.K.I equations needed to solve for all of these flux values are obtained as follows.

a) Each of the K equations for each of the Gauss quadrature points (Eqs. 3.m and 3.o) are multiplied by r and integrated with respect to r in the interval $\{r_i, r_{i+1}\}$. There are I-1 such intervals on r, so that one now has K.(I-1) equations. Assumption B of 3.3 is used to facilitate the integration.

b) In each of the equations obtained in a), Φ is set equal to π. This results in a set of final difference equations, each of which will be used to solve for the value of $f_{k,i,J}$ for $1 \leq k \leq K$ and for $1 \leq i \leq I-1$.

c) Assumption B of 3.3 is again employed to integrate each of the equations obtained in a) with respect to Φ in the interval $\{\Phi_j, \Phi_{j+1}\}$ for $1 \leq j \leq J-1$. This results in a set of final difference equations, each of which will be used to solve for the value of $f_{k,i,j}$ for any k (i.e. $1 \leq k \leq K$), but for $2 \leq i \leq I$ when $1 \leq j \leq \frac{J-1}{2}$ and for $1 \leq i \leq I-1$ when $\frac{J+1}{2} \leq j \leq J-1$.

By this means a total of (I-1).K.J difference equations for each group are obtained from the Boltzmann equation. [(I-1).K are obtained in b) and (I-1).K.(J-1) are obtained in c)]. The other K.J necessary difference equations for each group are obtained from the boundary condition equations.

The final difference equations obtained from the Boltzmann equation as outlined above are given on the following pages. In equations 3.p.1, 3.p.2, 3.p.3, and 3.p.4, the coefficients not already defined are defined in Appendix A.

For k = 1 and $1 \leq j \leq J - 1$

$$\left[\frac{\sqrt{1-\mu_1^2}\,T_{1i,j}}{t_{1i}\,\Delta\phi_j} + \frac{\Sigma_i \pi a_j^{01}}{\Delta\phi_j}\right] y_{1,i,j} + \left[\frac{\sqrt{1-\mu_1^2}\,T_{2i,j}}{t_{1i}\,\Delta\phi_j} + \frac{\Sigma_i \pi a_j^{02}}{\Delta\phi_j}\right] y_{1,i,j+1} \qquad (3.p.1)$$

$$+ \left[\frac{\sqrt{1-\mu_1^2}\,T_{3i,j}}{t_{1i}\,\Delta\phi_j} + \frac{\Sigma_i \pi a_j^{01} t_{2i}}{\Delta\phi_j\, t_{1i}}\right] y_{1,i+1,j} + \left[\frac{\sqrt{1-\mu_1^2}\,T_{4i,j}}{t_{1i}\,\Delta\phi_j} + \frac{\Sigma_i \pi a_j^{02} t_{2i}}{\Delta\phi_j\, t_{1i}}\right] y_{1,i+1,j+1}$$

$$- \sum_{j'=1}^{J} \left(y_{1,i,j'} + \frac{t_{2i}}{t_{1i}} y_{1,i+1,j'}\right) \left\{\Sigma_{s0i} a_{j'}^{0} + \Sigma_{s1i}\sqrt{1-\mu_1^2} H_1^1 \frac{\Delta s_j}{\Delta\phi_j} a_{j'}^{1} + 5\Sigma_{s2i} P_2(\mu_1) H_1^{20} a_{j'}^{0} + \frac{5}{12}\Sigma_{s2i} P_2^2(\mu_1) H_1^{22} \frac{\Delta s c_j}{\Delta\phi_j} a_{j'}^{2}\right\} =$$

$$+ \sum_{k'+2}^{K} \sum_{j'=1}^{J} \left(y_{k',i,j'} + \frac{t_{2i}}{t_{1i}} y_{k',i+1,j'}\right) \left\{\Sigma_{s0i} H_{k'}^{0} a_{j'}^{0} + 3\Sigma_{s1i}\sqrt{1-\mu_1^2} H_{k'}^{1} \frac{\Delta s_j}{\Delta\phi_j} a_{j'}^{1} + 5\Sigma_{s2i} P_2(\mu_1) H_{k'}^{20} a_{j'}^{0}\right.$$

$$\left. \frac{5}{12}\Sigma_{s2i} P_2^2(\mu_1) H_{k'}^{22} \frac{\Delta s c_j}{\Delta\phi_j} a_{j'}^{2}\right\} + \frac{4\pi}{\Delta\phi_j}\left\{\int_{\phi_j}^{\phi_{j+1}} \left[S_{1,i}(\phi) + \frac{t_{2i}}{t_{1i}} S_{1,i+1}(\phi)\right] d\phi\right\} .$$

For k = 1 and j = J

$$\left[\frac{\sqrt{1-\mu_1^2}\,T_{1i,J}}{t_{1i}}+\Sigma_i 4\pi\right]y_{1,i,J}+\left[\frac{\sqrt{1-\mu_1^2}\,T_{3i,J}}{t_{1i}}+\frac{\Sigma_i 4\pi t_{2i}}{t_{1i}}\right]y_{1,i+1,J} \qquad (3.p.2)$$

$$-\sum_{j'=1}^{J}\left(y_{1,i,j'}+\frac{t_{2i}}{t_{1i}}y_{1,i+1,j'}\right)\left\{\Sigma_{s0i}a_{j'}^{0}-3\Sigma_{s1i}\sqrt{1-\mu_1^2}H_1^1a_{j'}^1+5\Sigma_{s2i}P_2(\mu_1)H_1^{20}a_{j'}^0+\frac{5}{12}\Sigma_{s2i}P_2^2(\mu_1)H_1^{22}a_{j'}^2\right\}=$$

$$+\sum_{k'=2}^{K}\sum_{j'=1}^{J}y_{k',i,j'}+\frac{t_{2i}}{t_{1i}}y_{k',1+1,j'}\left\{\Sigma_{s0i}H_{k'}^0a_{j'}^0-3\Sigma_{s1i}\sqrt{1-\mu_1^2}H_{k'}^1a_{j'}^1+5\,\Sigma_{s2i}P_2(\mu_1)H_{k'}^{20}a_{j'}^0\right.$$

$$\left.+\frac{5}{12}\,\Sigma_{s2i}P_2^2(\mu_1)H_{k'}^{22}a_{j'}^2\right\}+4\pi\left[S_{1,i}(\pi)+\frac{t_{2i}}{t_{1i}}\,S_{1,i+1,}(\pi)\right].$$

For k ≧ 2 and 1 ≦ j ≦ J-1

$$\left[\frac{\sqrt{1-\mu_k^2}T_{1ij}}{t_{1i}\Delta\phi_j}+\frac{\Sigma_i\pi a_{j'}^{01}}{\Delta\phi_j}\right]y_{k,i,j}+\left[\frac{\sqrt{1-\mu_k^2}T_{2i,j}}{t_{il}\Delta\phi_j}+\frac{\Sigma_i\pi a_{j'}^{02}}{\Delta\phi_j}\right]y_{k,i,j+1}+\left[\frac{\sqrt{1-\mu_k^2}T_{3i,j}}{t_{1i}\Delta\phi_j}+\frac{\Sigma_i\pi a_{j'}^{01}t_{2i}}{\Delta\phi_j t_{1i}}\right]y_{k,i+1,j} \qquad (3.p.3)$$

$$+\left[\frac{\sqrt{1-\mu_k^2}T_{4i,j}}{t_{1i}\Delta\phi_j}+\frac{\Sigma_i\pi a_{j'}^{02}T_{2i}}{\Delta\phi_j t_{1i}}\right]y_{k,i+1,j+1}=\sum_{k'=1}^{K}\sum_{j'=1}^{J}\left(y_{k',i,j'}+\frac{t_{2i}}{t_{1i}}y_{k',i+1,j'}\right)\left\{3\Sigma_{s1i}(\sqrt{1-\mu_k^2}-\sqrt{1-\mu_1^2})H_{k'}^1\frac{\Delta\mu_j}{\Delta\phi_j}a_{j'}^1\right.$$

$$+\left.5\Sigma_{s2i}\left[P_2(\mu_k)-P_2(\mu_1)\right]H_{k'}^{20}a_{j'}^0+\frac{5}{12}\,\Sigma_{s2i}\left[P_2^2(\mu_k)-P_2^2(\mu_1)\right]H_{k'}^{22}\frac{\Delta\mu_j}{\Delta\phi_j}a_{j'}^2\right\}$$

$$+\frac{4\pi}{\Delta\phi_j}\int_{\phi_j}^{\phi_{j+1}}\left\{\left[S_{k,i}(\phi)-S_{1,i}(\phi)\right]+\frac{t_{2i}}{t_{1i}}\left[S_{k,i+1}(\phi)-S_{1,i+1}(\phi)\right]\right\}d\phi$$

$$-\left(\sqrt{1-\mu_k^2}-\sqrt{1-\mu_1^2}\right)\left[\frac{T_{1i,j}}{t_{1i}\Delta\phi_j}\,y_{1,i,j}+\frac{T_{2i,j}}{t_{1i}\Delta\phi_j}y_{1,i,j+1}+\frac{T_{3i,j}}{t_{1i}\Delta\phi_j}\,y_{1,i+1,j}+\frac{T_{4i,j}}{t_{1i}\Delta\phi_j}\,y_{1,i+1,j+1}\right].$$

For $k \geq 2$ and $j = J$

$$\left[\frac{\sqrt{1-\mu_k^2}T_{1i,J}}{t_{1i}} + \Sigma_i 4\pi\right] y_{k,i,J} + \left[\frac{\sqrt{1-\mu_k^2}T_{3i,J}}{t_{1i}} + \frac{\Sigma_i 4\pi t_{2i}}{t_{1i}}\right] = \sum_{k'=1}^{K}\sum_{j'=1}^{J}\left(y_{k',i,j'} + \frac{t_{2i}}{t_{1i}} y_{k',i+1,j'}\right) \qquad (3.p.4)$$

$$\cdot \left\{-3\Sigma_{s1i}(\sqrt{1-\mu_k^2} - \sqrt{1-\mu_1^2})\, H^{1}_{k',a^{1}_{j'}} + 5\Sigma_{s2}\left[P_2(\mu_k) - P_2(\mu_1)\right] H^{20}_{k',a^{0}_{j'}} + \frac{5}{12}\Sigma_{s2i}\left[P_2^2(\mu_k) - P_2^2(\mu_1)\right] H^{22}_{k',a^{2}_{j'}}\right.$$

$$\left. + 4\pi\left\{\left[S_{k,i}(\pi) - S_{1,i,}(\pi)\right] + \frac{t_{2i}}{t_{1i}}\left[S_{k,i+1}(\pi) - S_{1,i+1}(\pi)\right]\right\}\right.$$

$$\left. - \left[\sqrt{1-\mu_k^2} - \sqrt{1-\mu_1^2}\right]\left[\frac{T_{1i,J}}{t_{1i}}\, y_{1,i,J} + \frac{T_{3i,J}}{t_{1i}}\right] y_{1,i+1,J}\right.$$

For detailed reference, the source terms in Eqs. 3.p.1, 3.p.2, 3.p.3, and 3.p.4 are:

$$\frac{4\pi}{\Delta\Phi_j}\left\{\int_{\Phi_j}^{\Phi_{j+1}}\left[S_{1,i}(\Phi)+\frac{t_{2i}}{t_{1,i}}S_{1,i+1}(\Phi)\right]d\Phi = \left[q_i+\frac{t_{2i}}{t_{1i}}q_{i+1}\right] + \frac{\chi_{gi}}{\lambda}\sum_{g'=1}^{G}\nu\Sigma_{fg'i}\left[\Phi_{g',i}+\frac{t_{2i}}{t_{1i}}\Phi_{g',i+1}\right]\right.$$

$$+\sum_{g'=1}^{g-1}\left\{\Sigma_{s0ig'\to g}\left[\Phi_{g,i}+\frac{t_{2i}}{t_{1i}}\Phi_{g',i+1}\right]+3\Sigma_{s1ig'\to g}\sqrt{1-\mu_1^2}\,\frac{\Delta\mathcal{L}_j}{\Delta\Phi_j}\left[J_{g',i}+\frac{t_{2i}}{t_{1i}}J_{g',i+1}\right]\right.$$

$$+5\Sigma_{s2ig'\to g}P_2(\mu_1)\left[G^0_{2g',i}+\frac{t_{2i}}{t_{1i}}G^0_{2g',i+1}\right]+\frac{5}{12}\Sigma_{s2ig'\to g}P_2^2(\mu_1)\,\frac{\Delta\mathcal{L}_j}{\Delta\Phi_j}\left[G^2_{2g',i}+\frac{t_{2i}}{t_{1i}}G^2_{2g',i+1}\right];$$

$$4\pi\left[S_{1,i}(\pi)+\frac{t_{2i}}{t_{1i}}S_{1,i+1}(\pi)\right]=\left[q_i+\frac{t_{2i}}{t_{1i}}q_{i+1}\right]+\frac{\chi_{gi}}{\lambda}\sum_{g'=1}^{G}\nu\Sigma_{fg'i}\left[\Phi_{g',i}+\frac{t_{2i}}{t_{1i}}\Phi_{g',i+1}\right]$$

$$+\sum_{g'=1}^{g-1}\left\{\Sigma_{s0ig'\to g}\left[\Phi_{g',i}+\frac{t_{2i}}{t_{1i}}\Phi_{g',i+1}\right]-3\Sigma_{s1ig'\to g}\sqrt{1-\mu_1^2}\left[J_{g',i}+\frac{t_{2i}}{t_{1i}}J_{g',i+1}\right]\right.$$

$$+5\Sigma_{s2ig'\to g}P_2(\mu_1)\left[G^0_{2g',i}+\frac{t_{2i}}{t_{1i}}G^0_{2g',i+1}\right]+\frac{5}{12}\Sigma_{s2ig'\to g}P_2^2(\mu_1)\left[G^2_{2g',i}+\frac{t_{2i}}{t_{1i}}G^2_{2g',i+1}\right];$$

$$\frac{4\pi}{\Delta\Phi_j}\int_{\Phi_j}^{\Phi_{j+1}}\left\{\left[S_{k,i}(\Phi)-S_{l,i}(\Phi)\right]+\frac{t_{2i}}{t_{1i}}\left[S_{k,i+1}(\Phi)-S_{l,i+1}(\Phi)\right]\right\}d\Phi=\sum_{g'=1}^{g-1}\left\{3\Sigma_{s1ig'\to g}\left(\sqrt{1-\mu_k^2}-\sqrt{1-\mu_l^2}\right)\right.$$

$$\cdot\frac{\Delta\mathcal{A}_j}{\Delta\Phi_j}\left[J_{g',i}+\frac{t_{2i}}{t_{1i}}J_{g',i+1}\right]+5\Sigma_{s2ig'\to g}\left[P_2(\mu_k)-P_2(\mu_l)\right]\left[G^0_{2g',i}+\frac{t_{2i}}{t_{1i}}G^0_{2g',i+1}\right]$$

$$\left.+\frac{5}{12}\Sigma_{s2ig'\to g}\left[P_2^2(\mu_k)-P_2^2(\mu_l)\right]\frac{\Delta\mathcal{A}_j}{\Delta\Phi_j}\left[G^2_{2g',i}+\frac{t_{2i}}{t_{1i}}G^2_{2g',i+1}\right]\right\};$$

$$4\pi\left\{\left[S_{k,i}(\pi)-S_{l,i}(\pi)\right]+\frac{t_{2i}}{t1i}\left[S_{k,i+1}(\pi)-S_{l,i+1}(\pi)\right]\right\}=\sum_{g'=1}^{g-1}\left\{-3\Sigma_{s1ig'\to g}\left(\sqrt{1-\mu_k^2}-\sqrt{1-\mu_l^2}\right)\right.$$

$$\cdot\left[J_{g',i}+\frac{t_{2i}}{t_{1i}}J_{g',i+1}\right]+5\Sigma_{s2ig'\to g}\left[P_2(\mu_k)-P_2(\mu_l)\right]\left[G^0_{2,g',i}+\frac{t_{2i}}{t_{1i}}G^0_{2g',i+1}\right]$$

$$\left.+\frac{5}{12}\Sigma_{s2ig'\to g}\left[P_2^2(\mu_k)-P_2^2(\mu_l)\right]\left[G^2_{2g',i}+\frac{t_{2i}}{t_{1i}}G^2_{2g',i+1}\right]\right\}.$$

The definitions of Φ, J, G_2^0, and G_2^2 are stated in Appendix B.

It is important for later purposes to define a unique one-to-one correspondence between each of the difference equations and a specific flux value $y_{k,i,j}$ of a group. Hence, one can speak of the point (k,i,j), the $(k,i,j)^{\underline{th}}$ equation associated with the point, and the $f_{k,i,j}$ flux value associated with the point. In a sense, it can be said the $(k,i,j)^{\underline{th}}$ equation is used to solve for $f_{k,i,j}$. In this manner, Eqs. 3.p.1, 3.p.2, 3.p.3, and 3.p.4 are associated with the specific flux values in the following way.

When $k = 1$ and $1 \leq j \leq \frac{J-1}{2}$, Eq. 3.p.1 determines the flux values $y_{1,i+1,j}$ (for $2 \leq i + 1 \leq I$).

When $k = 1$ and $\frac{J+1}{2} \leq j \leq J-1$, Eq. 3.p.1 determines the flux values $y_{1,i,j}$ (for $1 \leq i \leq I-1$).

When $k = 1$ and $j = J$, Eq. 3.p. 2 determines the flux values $y_{1,i,J}$ (for $1 \leq i \leq I - 1$).

When $2 \leq k \leq K$ and $1 \leq j \leq \frac{J-1}{2}$, Eq. 3.p.3 determines the flux values $y_{k,i+1,j}$ (for $2 \leq i + 1 \leq I$).

When $2 \leq k \leq K$ and $\frac{J+1}{2} \leq j \leq J-1$, Eq. 3.p.3 determines the flux values $y_{k,i,j}$ (for $1 \leq i \leq I-1$).

When $2 \leq k \leq K$ and $j = J$, Eq. 3.p.4 determines the flux values $y_{k,i,J}$ (for $1 \leq i \leq I-1$).

3.7 The Boundary Condition Difference Equations

For a given group and a given Gauss point the difference equations so far derived determine all of the flux values except $y_{k,1,j}$ for $1 \leq j \leq \frac{J-1}{2}$ and $y_{k,I,j}$ for $\frac{J+1}{2} \leq j \leq J$. These flux values are determined by difference equations derived from the boundary condition equations of 3.2.

The first steps in deriving the difference equations from boundary equation 3.b are to set $\mu = \mu_k$, carry out the integration of the B_ν term, and make the transformation $y_1(r_\nu, \phi) = f_1(r_\nu, \phi)$ and $y_k(r_\nu, \phi) = f_k(r_\nu, \phi) - f_1(r_\nu, \phi)$. If this is done, one obtains:

For k = 1

$$(3.r.1)\quad y_1(r_\nu, \phi) = A_\nu y_1(r_\nu, \pi-\phi) + y_b(r_\nu, \phi) + \sum_{\ell=0}^{1} \mathcal{L}_{\nu,\ell} P_\ell(\sqrt{1-\mu_1^2}\cos\phi) + \frac{B_\nu}{c} J_{\nu B}.$$

For k ≧ 2

$$(3.r.2)\quad y_k(r_\nu, \phi) = A_\nu y_k(r_\nu, \pi-\phi) + \mathcal{L}_{\nu,1}\cos\phi\,(\sqrt{1-\mu_k^2} - \sqrt{1-\mu_1^2}).$$

In these equations,

$$J_{\nu B} = \sum_{k'=1}^{K} \sum_{j'=\frac{J+1}{2}}^{J-1} H^1_{k'} \left[a^{11}_{j'} y_{k',1,j'} + a^{12}_{j'} y_{k;1,j'+1} \right] \quad \text{for } \nu = 1$$

and $$J_{\nu B} = \sum_{k'=1}^{K} \sum_{j'=1}^{\frac{J-1}{2}} H^1_{k'} \left[a^{11}_{j'} y_{k',I,j'} + a^{12}_{j'} y_{k',I,j'+1} \right] \quad \text{for } \nu = 2.$$

Next, the equations for j = J are derived from Eqs. 3.r.1 and 3.r.2 by simply setting $\phi = \pi$. Then, in order to force the boundary conditions to balance particle currents, Eqs. 3.r.1 and 3.r.2 are multiplied by $4\cos\phi$ before integrating from ϕ_j to ϕ_{j+1} to derive the equations for $f_{k,j}(r_\nu)$ for $1 \leq j \leq J-1$.

The result of these last operations are the boundary difference equations:

For $k = 1$ and $1 \leq j \leq (J-1)/2$

$$a_j^{11} y_{1,1,j} + a_j^{12} y_{1,1,j+1} - A_1 a_j^{11} y_{1,1,J-j+1} - A_1 a_j^{12} y_{1,1,J-j} \qquad (3.\mathcal{N}.1)$$

$$+ B_1 \Delta s_j \sum_{j'=\frac{J+1}{2}}^{J-1} (a_{j'}^{11} y_{1,1,j'} + a_{j'}^{12} y_{1,1,j'+1}) = a_j^{11} y_{bj} + a_j^{12} y_{bj+1}$$

$$- \frac{B_1 \Delta s_j}{H_1^1} \sum_{k'=2}^{K} \sum_{j'=\frac{J+1}{2}}^{J-1} H_{k'}^1 (a_{j'}^{11} y_{k',1,j'} + a_{j'}^{12} y_{k',1,j'+1}) + 4\mathcal{J}_{1,0} \Delta s_j$$

$$+ 2\mathcal{J}_{1,1} (\Delta \phi_j + \Delta \alpha_j) \sqrt{1-\mu_1^2}$$

For $k = 1$ and $\frac{J+1}{2} \leq j \leq J-1$

$$a_j^{11} y_{1,I,j} + a_j^{12} y_{1,I,j+1} - A_2 a_j^{11} y_{1,I,J-j+1} (1-\delta_{j,\frac{J+1}{2}}) - A_2 a_j^{12} y_{1,I,J-j}$$

$$+ A_2 a_j^{11} y_c (\delta_{j,\frac{J+1}{2}}) - B_2 \Delta s_j \sum_{j'=1}^{\frac{J-1}{2}} (a_j^{11} y_{1,I,j'} + a_{j'}^{12} y_{1,I,j'+1}) \qquad (3.\mathcal{N}.2)$$

$$= a_j^{11} y_{bj+1} + a_j^{12} y_{bj+2} + \frac{B_2 \Delta s_j}{H_1^1} \sum_{k'=2}^{K} \sum_{j'=1}^{\frac{J-1}{2}} H_{k'}^1 (a_j^{11} y_{k',I,j'} + a_{j'}^{12} y_{k'I,j'+1})$$

$$+ 4\mathcal{J}_{2,0} \Delta s_j + 2\mathcal{J}_{2,1} (\Delta \phi_j + \Delta \alpha_j) \sqrt{1-\mu_1^2}$$

For $k = 1$ and $j = J$

$$-4 y_{1,I,J} + 4 A_2 y_{1,I,1} + B_2 \sum_{j'=1}^{\frac{J-1}{2}} (a_j^{11} y_{1,I,j'} + a_{j'}^{12} y_{1,I,j'+1}) = \qquad (3.\mathcal{N}.3)$$

$$- 4 y_{bJ+1} - \frac{B_2}{H_1^1} \sum_{k'=2}^{K} \sum_{j'=1}^{\frac{J-1}{2}} H_{k'}^1 (a_{j'}^{11} y_{k',I,j'} + a_{j'}^{12} y_{k',I,j'+1})$$

$$- 4\mathcal{J}_{2,0} + 4\mathcal{J}_{2,1} \sqrt{1-\mu_1^2}$$

For $2 \leq k \leq K$ and $1 \leq j \leq \frac{J-1}{2}$

$$(3.\mathcal{A}.4) \quad a_j^{11} y_{k,1,j} + a_j^{12} y_{k,1,j+1} - A_1 a_j^{11} y_{k,1,J-j+1} - A_1 a_j^{12} y_{k,1,J-j} = 2\mathcal{L}_{1,1}(\Delta\phi_j + \Delta\mathcal{A}c_j)\left(\sqrt{1-\mu_k^2} - \sqrt{1-\mu_1^2}\right)$$

For $2 \leq k \leq K$ and $\frac{J+1}{2} \leq j \leq J-1$

$$(3.\mathcal{A}.5) \quad a_j^{11} y_{k,I,j} + a_j^{12} y_{k,I,j+1} - A_2 a_j^{11} y_{k,I,J-j+1}(1-\delta_{j,\frac{J+1}{2}}) - A_2 a_j^{12} y_{k,I,J-j} = 2\mathcal{L}_{2,1}(\Delta\phi_j + \Delta\mathcal{A}c_j)\left(\sqrt{1-\mu_k^2} - \sqrt{1-\mu_1^2}\right)$$

For $2 \leq k \leq K$ and $j = J$

$$(3.\mathcal{A}.6) \quad -y_{k,I,J} + A_2 y_{k,I,1} = \mathcal{L}_{2,1}\left(\sqrt{1-\mu_k^2} - \sqrt{1-\mu_1^2}\right) .$$

In these equations, $\delta_{j,\frac{J+1}{2}}$ is the Kronecker delta such that

$$\delta_{j,\frac{J+1}{2}} = 0 \text{ for } j \neq \frac{J+1}{2}$$

$$\delta_{j,\frac{J+1}{2}} = 1 \text{ for } j = \frac{J+1}{2} .$$

The reason for the use of the Kronecker delta is that when $j = \frac{J+1}{2}$, $y_{k,I,j} \equiv y_{k,I,J-j+1}$ $\left[\text{Note: } J-(\frac{J+1}{2}) + 1 = \frac{J+1}{2}\right]$, so that if $A_2=1$ and $B_2=0$ the equation is indeterminate for $y_{k,I,j}$. This is clearly unacceptable because it is desired that this equation be the one used to solve for $y_{k,I,\frac{J+1}{2}}$.

The situation is associated with the fact that at the outer boundary of a cylinder a symmetry condition is not a natural physical occurrence. With mirror reflection, an albedo (A_2) other than unity leads to a contradiction of the continuity of angular flux about $\phi = \frac{\pi}{2}$ which is assumed (unless the flux at $\phi = \pi/2$ is zero).

The TOPIC program handles the trouble in two ways:

a) By the use of the Kronecker delta $\delta_{j,\frac{J+1}{2}}$ in the difference equations, mirror reflection or symmetry at the outer boundary of the cylinder is normally applied as if the flux being reflected were zero at $\phi = \frac{\pi}{2}$ -. That is, the outgoing angular flux is assumed to vary from zero to whatever it is at the next j mesh point at $j = \frac{J-1}{2}$. The incoming flux

value at $\phi = \frac{\pi}{2}+$, which is the flux solved for by the code, can be non-zero, however.

b) On a special option; namely, when the input code word MFR is set $\neq 0$, y_c in Eq. 3.4.2 is set to a non-zero value. y_c is normally zero, but when MFR $\neq 0$, y_c is calculated as follows:

$$y_c = \frac{1}{\Sigma} S(r_I, 1, \frac{\pi}{2}) + \frac{\Sigma_{s0}\phi(r_I)}{4\pi}, \qquad (3.4.7)$$

where Σ, S, Σ_{s0}, and the scalar flux $\phi(r_I)$ are, as implied, evaluated at the outer boundary. This special option is equivalent to specifying that the outgoing flux is y_c at $\phi = \frac{\pi-}{2}$ instead of zero as in a), above. It is a more realistic condition to specify than is the condition of a) in reactor cell problems.

In actual application in the code, the part of y_c which depends on $y_{1,I,j}(k=1)$ remains on the left hand side in Eq. 3.4.2 and all other terms (the $S(r_I, 1, \frac{\pi}{2})$ plus the parts of $\phi(r_I)$ dependent on $y_{k,I,j}$ for $2 \leq k$) are placed on the right hand side.

The formula for y_c follows directly from Eq. 3.a of 3.1 if one integrates the cross section term, keeps only the isotropic part of the cross section term, and sets $\mu = 1$ and $\phi = \frac{\pi}{2}$. In this case $f(r, \mu, \phi) = y_c$. The option placing $y_c \neq 0$ sets the outgoing angular flux at $\phi = \frac{\pi-}{2}$ and for all μ equal to the angular flux value which would exist in the direction parallel to the z axis of the cylinder if there were not a boundary at r_I or if continuity of the flux at $\phi = \frac{\pi}{2}$ were not assumed.

4.0 The Solution of the Difference Equations

4.1 The Expression of the Difference Equations in Matrix Form

The one-to-one correspondence of each flux value $y_{k,i,j}$ to its difference equation is defined in 3.6. If, for a given group, one takes the difference equations for a given value of k and then orders the I.J equations in a particular manner, the set of equations can be represented by a matrix equation of the form,

$$A_k Y_k = S_k + \sum_{k'=1}^{K} B_{kk'} Y_{k'},$$ where Y_k is a column vector the elements of which are $y_{k,i,j}$; A_k is a matrix of coefficients which represents the coefficients in the difference equations which originated from the divergence terms and the total collision term of Eq. 3.a. That is, the coefficients of A_k originate from the difference equation expression of the left hand side of Eq. 3.a. S_k is a column vector the elements of which originate from $S(r,\mu,\phi)$ in Eq. 3.a. Each matrix, $B_{kk'}$, is composed of the coefficients in the difference equations (for a given k) which result from the scattering integral term of Eq. 3.a and which are the coefficients of the k'^{th} flux values.

The particular ordering of the difference equations for each group and for each k is as follows:

The first equation is the equation for $y_{k,1,1}$. (This equation comes from the boundary condition Eq. 3.4.1 if k = 1 and from Eq. 3.4.4 if $2 \leq k$.) The next equation is for $y_{k,1,2}$, and the next is for $y_{k,1,3}$ etc, until the equation for $y_{k,1,J}$ is included. Then the ordering proceeds with the equation for $y_{k,2,1}$, $y_{k,2,2}$, $\cdots y_{k,2,J}$, etc. Finally the last equation is the equation for $y_{k,I,J}$ (which is a boundary condition equation - either 3.4.3 or 3.4.6).

4.2 The Solution of the Matrix Equation for k = 1

The solution for all the K.I.J values of $y_{k,i,j}$ for a given group begins with the solution for the I.J values of $y_{1,i,j}$. It can be noted that S_k is a constant in so far as the solution for $y_{k,i,j}$ within a group is concerned because it is a function of the fission source resulting from the previous outer iteration, the scattering down from groups

above the group in question, and the fixed volume and boundary sources.

When k = 1 the matrix equation is written in the implicit form, $(A_1-B_{11})\, Y_1 = S_1 + \sum_{k'=2}^{K} B_{1k'}Y_{k'}$. This equation can be rewritten as

$$M\, Y_1 = Q \quad \text{where } Q = S_1 + \sum_{k'=2}^{K} B_{1k'}Y_{k'} \text{ can be considered} \qquad (4.a)$$

a constant source vector because the $Y_{k'}$ values in $\sum_{k'=2}^{K} B_{kk'}Y_{k'}$ are always those obtained on the previous outer iteration. (On the first iteration, $Y_{k'}=0$.)

The solution to the set of equations represented by the matrix Eq. 4.a is accomplished in three steps.

a) By simple Gaussian elimination, the subdiagonal elements of M are reduced to zero in the first column, then the second, the third, etc. This is most conveniently accomplished in the following manner:

Let L_1' be a unit I.J by I.J matrix except for non-zero elements in the first column, and L_1' is such that $L_1'M = M_1'$, where M_1' has zero subdiagonal elements in the first column.

Next, let L_2' be a unit I.J by I.J matrix except for non-zero elements in the second column, and let L_2' be such that $L_2'\, M_1' = M_2'$ where M_2' has zero subdiagonal elements in the first and second columns.

Similarly, define L_3', L_4', $\ldots L_n' \ldots L_{N-1}'$ such that $L_n'M_{n-1}'=M_n'$ where M_2' has zero subdiagonal elements in the first n columns. (N = I·J.)

It is an easy matter to find L_1', M_1', then L_2', M_2', etc. until finally M_{N-1}' has no non-zero elements below the diagonal.

b) At this point one can define $L \equiv (L_{N-1}'\cdot L_{N-2}\cdots L_3'\cdot L_2'\cdot L_1')$, and $U \equiv LM = M_{N-1}'$. The second step consists simply of multiplying the vector Q by L. This is accomplished by performing $L_1'\, Q = Q_1'$, $L_2'\, Q_1' = Q_2'$, etc. until $LQ = L_{N-1}'Q_{N-2}' = Q_{N-1}'$ is obtained.

c) The third step solves the matrix equation $UY_1 = LQ$ by simple backward solution. That is, $y_{1,I,J}$ can be solved for directly, $y_{1,I,J-1}$ can be solved for in terms of $y_{1,I,J}$, etc. until the first row equation is finally solved for $y_{1,1,1}$.

It can be noted that U contains no more non-zero elements than did the diagonal and upper diagonal part of M, and each L'_n contains no more non-zero elements than did the n^{th} subdiagonal column of M (except for the elements of L'_n which are known to be unity). Therefore the U and the L matrix (L in factor form) can be stored over the elements of the original M matrix, with the result that storage is conserved and the generation of L and U need be done only once per problem in the TOPIC program.

4.3 The Solution of the Matrix Equations for $2 \leqq k$

After solving for $y_{1,i,j}$ in a given group, the TOPIC code proceeds to compute the $y_{k,i,j}$ values for each of the other values of k, starting with k = 2, then k = 3, etc.

The method of solution for Y_k for $2 \leqq k$ can be represented by the matrix equation,

$$A_k Y_k^t = (S_k + B_{k1} Y_1) + \sum_{k'=1}^{k-1} B_{kk'} Y_{k'}^t + \sum_{k'=k}^{K} B_{kk'} Y_{k'}^{t-1}, \quad \text{(4.b)}$$

where the superscript t denotes the inner iteration index. This is to say that Y_2 is computed using previously obtained values for Y_2, Y_3, ...Y_K to calculate the $B_{kk'}Y_k$ products. Y_3 is then computed using the most recent value of Y_2 in the product $B_{k2}Y_2$, but the old values for Y_3, Y_4.. Y_K in the other $B_{kk'}Y_{k'}$ products. Each Y_k is computed in turn in this manner until Y_K is calculated.

At this point, there is an option which permits one to specify that this set of computations be repeated for a given number of times. The code input word "IG" specifies the number of inner iterations which will be performed. On each inner iteration, only the values Y_2, Y_3, etc., through Y_K are computed again. S_K and Y_1 remain at their original values throughout the entire inner iteration process.

In the TOPIC code, the $B_{kk'}$ for $k' \geqq 2$ are zero if there is no anisotropic scattering in a group; that is, if $\Sigma^r_{Sl} = 0$ in all regions. Therefore, if there is no anisotropic scattering, more than one inner iteration is superfluous and wasteful.

The detailed solution of Eq. 4.b for $y_{k,i,j}$ for a given k proceeds as follows:

With the right hand side of Eq. 4.b considered constant, one solves first for $y_{k,I,J}$, then for $y_{k,I,J-1}$, etc. until the value for $y_{k,I,\frac{J+1}{2}}$ is obtained. Then one skips to the equation for $y_{k,I-1,J}$ and solves for that flux value. Thereafter one solves for $y_{k,I-1,J-1}$, $y_{k,I-1,J-2}$, $\cdots y_{k,I-1,\frac{J+1}{2}}$, then $y_{k,I-2,J}$, $y_{k,I-2,J-1}$, $\cdots y_{k,I-1,\frac{J+1}{2}}$, etc., until one has solved for $y_{k,1,J}$, $y_{k,1,J-1}$, $\cdots y_{k,1,\frac{J+1}{2}}$. Next, the order of solution is $y_{k,1,\frac{J-1}{2}}$, $y_{k,1,\frac{J-3}{2}}$, $\cdots y_{k,1,1}$; $y_{k,2,\frac{J-1}{2}}$, $y_{k,2,\frac{J-3}{2}}$, $\cdots y_{k,2,1}$; etc,; until one has finally solved for $y_{k,I,\frac{J-1}{2}}$, $y_{k,I,\frac{J-3}{2}}$, $\cdots y_{k,I,1}$.

Throughout the entire solution, the most recent values of the $y_{k,i,j}$ are used in computing the new values. Therefore, the solution is by the ordinary Gauss Seidel technique except that the order of solution of the individual equations within a given matrix equation is not the usual one.

5.0 The Outer Iterations

5.1 The Calculation Procedure

There are problems which can be run on TOPIC which require only one outer iteration. The first requirement of such problems is that there be no fission in any region (no non-zero $\nu\Sigma^r_{fg}$). In addition to this, if there is only one Gauss point (K=1), or if there is no anisotropic scattering within any group (all $\Sigma_{Sl}=0$) and no mirror reflection (all $A_2 = 0$), the correct solution is obtained on the first iteration. (Note: $\Sigma_{Slg'-g}$ for $g' \to g$ can be non-zero.) The TOPIC code actually performs two outer iterations, even in these cases. However, it will be assumed in the explanations to follow that two or more outer iterations are required in order to obtain a reasonably well-converged solution.

Before starting any calculation, the TOPIC code analyzes the input data to determine whether or not the problem is

A. Homogeneous or

B. Inhomogeneous.

If there is a non-zero boundary source or fixed volume source, the problem is inhomogeneous. All others are homogeneous.

At the beginning of the first outer iteration, if and only if the problem is homogeneous, the TOPIC code obtains a fission source distribution. This is a guess for the actual fission source distribution defined as

$$F_0(r) = \frac{1}{\lambda} \sum_{g=1}^{G} \nu \, \Sigma_{fg}(r)\phi_g(r).$$

The guess is a flat distribution if no distribution (POWR1(M)) is supplied from the input data or from a previous problem.

In either case, the fission source guess $F_0(r)$ is normalized such that

$$2\pi\int_{r_1}^{r_I} F_0(r)\, r dr = \Gamma.$$

Γ is an input data word called "FAC", and if no such word is supplied Γ is automatically set equal to unity.

In inhomogeneous problems, the TOPIC code begins the first outer iteration with either $F(r) = 0$ everywhere or with an unnormalized $F(r)$ supplied from input data or from a previous problem.

The outer iteration then begins with the solution for all flux values in the first group using the methods outlined in 4.0. The code then solves for all flux values in the second group using the recently obtained values of the first group flux to compute the scattering down source from that group into the second. The code proceeds to each succeeding group, each time using the most recent flux values in the previous groups to compute the scattering down source distribution. Finally the G^{th} (last) group fluxes are computed. At this point, new values for $\sum_{g=1} \nu\Sigma_{f_g}(r)\phi_g(r) = F''_m(r)$ are computed and the code can then calculate:

$$P_n = 2\pi\int_{r_1}^{r_I} F''_m(r) r dr \qquad (5.a)$$

where n denotes the result of the n^{th} outer iteration

$$\lambda_n = P_n/\Gamma \text{ for homogeneous problems} \qquad (5.b)$$

$\lambda_n = 1$ for inhomogeneous problems

$$F'_n(r) = F''_n(r)/\lambda_n \qquad (5.c)$$

$$E_{max} = \text{maximum value of } F'_n(r)/F_{n-1}(r) \qquad (5.d)$$

$$E_{min} = \text{minimum value of } F'_n(r)/F_{n-1}(r) \qquad (5.e)$$

$$\delta_n = (\lambda_n - \lambda_{n-1})/\lambda_n \text{ for homogeneous problems} \qquad (5.f)$$

$$\delta_n = (P_n - P_{n-1})/P_n \text{ for inhomogeneous problems}$$

$$\gamma_n = (\text{EMAX} - \text{EMIN})/\text{EMAX} \qquad (5.g)$$

$$\alpha^n_{g\,max} = \text{maximum value of } \phi_{g,n}(r)/\phi_{g,n-1}(r) \text{ in each group.}$$

(5.i) $\alpha^n_{g\,min}$ = minimum value of $\phi_{g,n}(r)/\phi_{g,n-1}(r)$ in each group

(5.j) $\beta^n_g = (\alpha_{g\,max} - \alpha_{g\,min})/\alpha_{g\,max}$

On the first outer iteration, λ_{n-1} and $\phi_{g,n-1}(r)$ do not exist; therefore the TOPIC code sets

$\lambda_0 = 1$,

$\alpha^0_{g\,max} = 1$,

$\alpha^0_{g\,min} = 1$, and

$\beta^0_g = 0$.

During the first two outer iterations the TOPIC code sets $F_n(r) = F'_n(r)$; thereafter it computes

(5.k) $F_n(r) = F'_n(r)(1+\theta) - \theta F_{n-1}(r)$, where θ is an extrapolation factor which is supplied as the input data word, "THETA".

At this point, the program is ready to begin another outer iteration using the new fission source distribution $F_n(r)$, and, of course using the better flux values in each group. (As noted before, several outer iterations will usually be required to obtain reasonably converged flux values in each group if more than one Gauss quadrature point is used.)

5.2 The Convergence Criteria on the Outer Iterations

Before beginning another outer iteration with the new fission source and fluxes, the TOPIC program checks to see if the convergence criteria are satisfied. (The only exception is that at the end of the first iteration another iteration is begun, regardless of any convergence criteria.)

The convergence criteria are determined by the values of five input data words and by the setting of Sense Switch 1. Upon finding that the convergence criteria are satisfied, the TOPIC code checks the value of the output data option word "NOT" to determine whether any detailed angular flux data is desired as output. If it is, one more iteration is performed; if it is not, the code proceeds directly to perform the output edits.

The five input data words which determine the convergence criteria are:

MIK ≡ the number of Gauss quadrature points,

LCO ≡ the fission source convergence option,

EPS1 ≡ the fission source convergence specification,

EPS2 ≡ the group scalar flux convergence specification, and

ITOUT ≡ the maximum number of outer iterations specification. The exact conditions for satisfaction of the convergence criteria in the non-trivial case are best given in a table such as Table 2, given the specific condition on each of the five input data words and the setting of Sense Switch 1.

In Table 2, the normal status of each of the input data words or the Sense Switch is on the left. (For example, the normal status of Sense Switch 1 is up.) Also the most commonly occurring cases are listed first in the table.

TABLE 2

Conditions for Convergence Test Satisfaction Versus Status of Input Control Words

Status of Sense Switch No. 1		Status of EPS1 (ϵ_1)		Status of LCO		Status of EPS2 (ϵ_2)		Status of ITOUT and MIK		Conditions Which Must Exist For the Convergence Criteria to be Satisfied
UP	DOWN	$\epsilon_1>0$	$\epsilon_1=0$	LCO =0	LCO >0	$\epsilon_2>0$	$\epsilon_2=0$	ITOUT>0 & MIK>1	ITOUT=0 & MIK=1	
X		X		X		X		X		$\gamma_n \leqq \epsilon_1$ and $\beta_g \leqq \epsilon_2$ (for all g) or n ≧ ITOUT
X		X			X		X	X		$\gamma_n \leqq \epsilon_1$ or n ≧ ITOUT
X		X			X	X		X		$\delta_n \leqq \epsilon_1$ and $\beta_g \leqq \epsilon_2$ (for all g) or n ≧ ITOUT
X		X		X			X	X		$\delta_n \leqq \epsilon_1$ or n ≧ ITOUT
X			X	not relevant		not relevant		X		$\beta_g \leqq \epsilon_2$ (for all g) or n ≧ ITOUT
X		not relevant		not relevant		not relevant			X	n≧2
	X	not relevant		not relevant		not relevant		not relevant		n≧2

(See 5.1 for definitions of γ_n, δ_n, and β_g, where n is the number of outer iterations which have been completed.)

6.0 Program Summary

The TOPIC program is a package of four codes written in Fortran and assembled with the IBM Fortran Monitor System. The four programs allow different maximum numbers of intervals on the ϕ halfspace with corresponding restrictions on the maximum number of mesh points allowed on the cylinder radius r. The number of mesh points allowed in the Fortran II and Fortran IV versions TOPIC are listed below:

Code	Maximum Number of Intervals, N, on the ϕ Halfspace	Maximum Number of Mesh Points on the Cylinder Radius r	
		Fortran II	Fortran IV
TOPIC 4	4	240	185
TOPIC 6	6	140	108
TOPIC 8	8	90	70
TOPIC 10	10	62	48

Each code includes the capability of handling all lower numbers of angular intervals; for example, TOPIC 8 can handle 8, 6, 4, or 2 angular intervals. The input data and the input data formats are the same for each code with the single exception that the fixed volume source, $4\pi q(r)$, called SVM in the data description, is stored in a different location in the Fortran IV version. (See 7.0 TOPIC Input.)

Because of core storage limitations, each code is divided logically into four chains. (The chain feature of the Fortran Monitor allows parts of a program to be stored on magnetic tape and to be subsequently called off the tape into core storage when needed. The data in "common" storage remains available in the core for use by any chain as it is brought in.) The general logical function of each chain and its subroutines can be briefly outlined as follows.

Chain 1

This chain reads, checks and prints the input data; it initializes angular and geometric data; mixes cross sections; and initializes the fission source.

Subroutines

INPR	prints input data
SCHECK	initializes the fission source
MIXX	forms the cross section mixtures

Chain 2

This chain computes the coefficients of the matrix M (see Eqs. 4.a) for each group, computes the U matrix elements, computes the L matrix, and stores L and U in binary on duplicate tapes. (The L matrix is stored in factor form.)

Subroutines

SSET	calculates various data needed for the Gauss quadrature
BUNDRY	calculates the elements of M and the source vector S (see 4.2)(for each group)which depend on the boundary condition equations
MTXSET	calculates the elements of M(for each group) which depend on the basic balance equation
MTXSEL	calculates and stores on tape the intermediate coefficients which are used to solve for the Gauss point fluxes $f_{k,i,j}$ for $2 \leq k$ in each group
FACTOR	operates on the matrix M to produce the matrices L and U such that $L^{-1}U = M$

Chain 3

This chain performs the iterative solution for all the fluxes of each group, obtains the new fission source on each outer iteration, and checks for satisfaction of the convergence criteria.

Subroutines

SOURCE	multiplies the source vector for $y_{1,i,j}$ of each

	group by the L matrix
FLUX	uses the U matrix and the result from the SOURCE subroutine to backsolve and obtain $y_{1,i,j}$ for each group
HIMODE	uses $y_{1,i,j}$ obtained in the FLUX subroutine to obtain $y_{k,i,j}$ for $2 \leq k$
CONV	calculates a new fission source and checks it for satisfaction of the convergence criteria; writes the fission source on the output tape if NOT = 4

Note: This chain calls a library subroutine TIMEB which sets a given word equal to the time in hrs., mins., and secs. at the time TIMEB is called. If this subroutine is not available at an installation all reference to it in the main of Chain 3 will have to be deleted or a dummy subroutine substituted.

Chain 4

This chain performs the edits for almost all of the output. (See 8.0 Output Description.)

This chain also calls TIMEB. See the note under Chain 3 above.

The logical tape designations as explicitly used in Fortran input - output statements are:

All BCD output	Tape 6
All binary and BCD input	Tape 5
Intermediate binary	Tape 2
Intermediate binary	Tape 4
Intermediate binary	Tape 8
Intermediate binary	Tape 9
Intermediate binary	Tape 10

In addition, the chain links are stacked on logical Tape 3.

7.0 TOPIC Input

The input to each TOPIC problem or "change case" consists of three parts:

1. Alphanumeric card
2. Fixed point data
3. Floating point data

A "change case" is a problem which is the same as the one preceding it (except for a few input data changes).

7.1 The Alphanumeric Card

This card may contain any desired alphanumeric information in columns 1-72. A "1" punch in column 1 will start all on-line and off-line printouts at the top of a new page. This card must always be physically the first card in every case.

7.2 The Fixed Point Data

The card format for the fixed point data is as follows:

Card Column 1-2	Number of pieces of data on this card* (right adjusted) $1 \leq$ No. ≤ 21
Card Column 3	1- means this is the last fixed point data card. 0 (or blank) means this is not the last fixed point data card.
Card Columns 4-7	Address of first piece of data on the card (right adjusted)
Card Columns 8-10	Up to 21 pieces of integer data (each right adjusted)
11-13	
14-16	
17-19	
20-22	
.	
.	
68-70	

* "Right adjusted" means the last significant digit of a number is at the extreme right of a field.

Fixed Point Addresses	Mnemonic Name	Meaning
1	MAX	Number of space points, I (number of intervals on r plus one) See 6.0 Program for the maximum number of space points. $3 \leq$ MAX
2	JMAX	Number of regions $1 \leq$ JMAX ≤ 40
3	NGR	Number of groups $1 \leq$ NGR ≤ 6
4	NDS	Number of downscatter groups $0 \leq$ NDS $\leq$ NGR-1
5	NMIX	Number of words needed to specify mixtures $0 \leq$ NMIX ≤ 20
6	NOT	Output control word 0 - eigenvalue, scalar fluxes, fission neutron density 1 - and particle balance by regions 2 - and average angular fluxes, $\Phi(r, \phi)$ for each r_i and ϕ_j 3 - and angular fluxes for each Gauss point; $f(r, \mu, \phi)$, for each r_i, μ_k, and ϕ_j 4 - and the fission neutron source distribution for each outer iteration
7	IG	No. of inner iterations desired for the Gauss points of k index greater than 1 (Leave blank or zero if there is no anisotropic scattering within each group.)
8	ITOUT	Maximum no. of outer iterations (This word is set to 50 unless a different value is supplied as input.)
9	LCO	Fission source convergence option 0 - pointwise 1 - eigenvalue or total fission source production
10	MUTEST	Angular interval option 1 - Equal intervals on cos ϕ

Fixed Point Addresses	Mnemonic Name	Meaning
10	MUTEST (Continued)	2 - Equal intervals on ϕ 3 - Input (This word is set to 1 if not read in as input.)
11	IDP	Input data print option 0 - Print input data 1 - Do not print input data
12	LPG	Fission source guess option 0 - Automatically computed flat fission source guess for homogeneous problems; zero for inhomogeneous problems 1 - Guess is supplied by input data (See PØWR1 in 7.3) 2 - Use guess from the previous problem
13-52	*II(40) †(Region no.)	The upper index point of each region from region 1 to region JMAX, in turn; $3 \leq II(J) \leq MAX$ and $II(J) < II(J+1)$
53-92	MIR(40) (Region no.)	Material number used in each region from region 1 to region JMAX, in turn; $1 \leq MIR(J) \leq 40$
93-112	MIX(20) (Position in mixture vector)	Material numbers listed such as to specify the mixtures
113	N	The number of angular intervals on the ϕ half-space; N must be less than or equal to 4 in TOPIC 4, 8 in TOPIC 8, etc.
114	MIK	The number of Gauss quadrature points on the μ halfspace; $1 \leq MIK \leq 7$
115	MFR	Outer boundary option for mirror reflection 0 - Standard mirror reflection 1 - $f(r, \mu, \frac{\pi}{2})$ is set non zero according to formula 3.4.7 in 3.7

* The numbers in parentheses indicate the maximum number (dimensions) of input data words which can be specified.

† The notes in parentheses indicate the meaning of the subscript (or index no.) on subscripted variables.

Fixed Point Addresses	Mnemonic Name	Meaning
116-121	MCELL(6) (Group no.)	The region number, for each group from 1 through 6 in that order, of the region in which a fixed volume source of unity (particles/sec cm^3) is desired (This source is added to whatever SVM source values may be specified in a region.) (See 7.3)

7.3 Floating Point Data

The card format for the floating point data is as follows:

Card Columns 1-2	Number of pieces of data on this card (right adjusted)
Card Column 3	1 - Means this is the last floating point data card
Card Columns 4-7	Address of first piece of data on the card (right adjusted)
Card Columns 8-17 18-27 28-37 38-47 48-57 58-67	Up to six pieces of floating point data

Note: Some valid ways of writing a floating point word in the ten column field are as follows for the number 3.141510. (The format is FORTRAN E10.5)

Card Columns

1	2	3	4	5	6	7	8	9	10
+	3	1	4	1	5	1	+	0	0
3	.	1	4	1	5	1	+	0	0
3	.	1	4	1	5	E	+	0	0
3	.	1	4	1	5	1	0	+	0
3	1	.	4	1	5	1	0	-	1
.	0	3	1	4	1	5	1	+	2
(b)	3	.	1	4	1	5	1	+	0
3	1	4	1	5	1	E	+	0	0
3	.	1	4	1	5	1	0	(b)	(b)

(b) indicates a blank.

Floating Point Addresses	Mnemonic Name	Meaning
1	EPS1	The fission source convergence specification, ϵ_1
2	EPS2	The group scalar flux convergence specification, ϵ_2
3	XIN	The value of the radius at the left boundary, r_1; $0 \leq$ XIN
4	THETA	The fission source extrapolation factor, θ $\theta = 0$ (or blank) is no extrapolation; $\theta > 1$ can cause divergence
5	FAC	Total fission source normalization constant, Γ The total no. of fission neutrons produced per cm. of axial length is equal to Γ (It is set equal to 1.0 unless it is supplied as input.)
6-45	DELR(40) (Region no.)	The value of the interval spacing Δr in each region from region 1 to region JMAX, in turn

Note: On the double and triple subscripted variables which follow, the data in each block must be listed such that all the values corresponding to the first subscript are given first. Then further values are listed until all the values corresponding to both the first and second subscript are given, etc.

Floating Point Addresses	Mnemonic Name	
46-285	SIGT(6,40) (Group, Material)	Total Cross section Σ, by group no. and material no.

46-51 Groups 1-6 for material 1
52-57 Groups 1-6 for material 2
58-63 Groups 1-6 for material 3
. . . .
. . . .
280-285 Groups 1-6 for material 40

Floating Point Addresses	Mnemonic Name	Meaning
286-525	SIGS(6,40) (Group, Material)	Σ_{S0}, the zeroth moment cross section for scattering within a group

286-291 Groups 1-6 for material 1
292-297 Groups 1-6 for material 2
298-303 Groups 1-6 for material 3
. . . .
. . . .
520-525 Groups 1-6 for material 40

526-765	SIGS(6,40) (Group, Material)	Σ_{S1}, the first moment cross section for scattering within a group

526-531 Groups 1-6 for material 1
532-537 Groups 1-6 for material 2
538-543 Groups 1-6 for material 3
. . . .
. . . .
760-765 Groups 1-6 for material 40

766-1005	VUSIG(6,40) (Group, Material)	$\nu\Sigma_f$, the number of neutrons per fission times the fission cross section

766-771 Groups 1-6 for material 1
772-777 Groups 1-6 for material 2
778-783 Groups 1-6 for material 3
. . . .
. . . .
1000-1005 Groups 1-6 for material 40

1006-1011	CHI (6) (Group)	χ, the fraction of fission neutrons released in each group

Floating Point Addresses	Mnemonic Name	Meaning
1012-1611	STR(15,40) (Position in transfer vector, material)	$\Sigma_{S0g' \to g}$, zeroth moment cross section for scattering from group g' to group g

1012-1026 Positions 1-15 for material 1
1027-1041 Positions 1-15 for material 2
1042-1056 Positions 1-15 for material 3
. . . .
. . . .
1597-1611 Positions 1-15 for material 40.

The relative position of each cross section in the transfer vector is indicated below.

Position	$\Sigma_{S0g' \to g}$
1	$\Sigma_{S01 \to 2}$
2	$\Sigma_{S02 \to 3}$
3	$\Sigma_{S03 \to 4}$
4	$\Sigma_{S04 \to 5}$
5	$\Sigma_{S05 \to 6}$
6	$\Sigma_{S01 \to 3}$
7	$\Sigma_{S02 \to 4}$
8	$\Sigma_{S03 \to 5}$
9	$\Sigma_{S04 \to 6}$
10	$\Sigma_{S01 \to 4}$
11	$\Sigma_{S02 \to 5}$
12	$\Sigma_{S03 \to 6}$
13	$\Sigma_{S01 \to 5}$
14	$\Sigma_{S02 \to 6}$
15	$\Sigma_{S01 \to 6}$

Floating Point Addresses	Mneomic Name	Meaning
1612-2211	STRJ(15,40) (Position in transfer vector, Material)	$\Sigma_{Slg' \to g}$, the first moment cross section for scattering from group g' to g The positions of the cross sections in the transfer vector are exactly as given for $\Sigma_{SOg'-g}$ above.

1612-1626 Positions 1-15 for material 1
1627-1641 Positions 1-15 for material 2
1642-1656 Positions 1-15 for material 3
. . . .
. . . .
. . . .
2197-2211 Positions 1-15 for material 40

2212-2223	ALPHA(2,6) (Boundary index, Group)	A_ν, the mirror reflection (or symmetry) coefficients or albedos, by boundary index and group no.

2212-2213 Left boundary value and right boundary value for group 1
2214-2215 Left boundary value and right boundary value for group 2
2216-2217 Left boundary value and right boundary value for group 3
. . . . . .
. . . . . .
2222-2223 Left boundary value and right boundary value for group 6

2224-2235	BETA(2,6) (Boundary, index, Group)	B_ν, the isotropic reflection coefficients or albedos

2224-2225 Left value and right value for group 1
2226-2227 Left value and right value for group 2
2228-2229 Left value and right value for group 3
. . . . .
. . . . .
2234-2235 Left value and right value for group 6

2236-2259	GAMMA(2,2,6) (L, Boundary index, Group)	$\mathcal{S}_{s,L}$, the Legendre polynomial coefficients for a diffuse boundary source; by polynomial order, boundary index, and group no.

2236-2237 L = 0 value and L = 1 value for left boundary, group 1
2238-2239 L = 0 value and L = 1 value for right boundary, group 1
2240-2241 L = 0 value and L = 1 value for left boundary, group 2
2242-2243 L = 0 value and L = 1 value for right boundary, group 2
. . . . . .
etc. etc.

Floating Point Address	Mnemonic Name	Meaning
2260-2331	DELTA(12,6) (Position in source vector, Group)	$f_b(r_s, \phi_j)$, the fixed boundary source fluxes; by position in source vector and by group no.

The order of the input data is given below for an S_4 problem (four angular intervals on the ϕ halfspace).

Address	DELTA Value	Boundary	ϕ Value	Group No.
2260	DELTA(1,1)	Right	$\phi_J = \phi_5 = \pi$	1
2261	DELTA(2,1)	Right	ϕ_4	1
2262	DELTA(3,1)	Right	$\phi_3 = \pi/2$	1
2263	DELTA(4,1)	Left	$\phi_3 = \pi/2$	1
2264	DELTA(5,1)	Left	ϕ_2	1
2265	DELTA(6,1)	Left	$\phi_1 = 0$	1
2272	DELTA(1,2)	Right	$\phi_J = \phi_5 = \pi$	2
2273	DELTA(2,2)	Right	ϕ_4	2
2274	DELTA(3,2)	Right	ϕ_3	2
2275	DELTA(4,2)	Left	ϕ_3	2
2276	DELTA(5,2)	Left	ϕ_2	2
2277	DELTA(6,2)	Left	$\phi_1 = 0$	2
.	.	.	.	.
.	.	.	.	.
etc.	etc.	etc.	etc.	etc.

2332-2351	CONC(20) (Position in weighting vector)	A list of atomic densities, volume fractions, or weighting factors to be used in conjunction with the MIX(20) mixture vector to form mixtures of cross sections (See explanation in 7.4.2 under MIX.)

Floating Point Addresses	Mnemonic Name	Meaning
2352-2363	EMU(12) (Position in angular interval specification vector)	Values for optional specification of the cos ϕ_j points directly, given in reverse j order (i.e. from j=J to 1)

The following example is for an S_6 problem (six angular intervals on the ϕ halfspace)

Address	EMU	cos ϕ Value	j index
2352	EMU(1)	cos ϕ_J = cos ϕ_7=-1.0	7
2353	EMU(2)	cos ϕ_6 = -.8	6
2354	EMU(3)	cos ϕ_5 = -.4	5
2355	EMU(4)	cos ϕ_4 = 0.0	4
2356	EMU(5)	cos ϕ_3 = +.4	3
2357	EMU(6)	cos ϕ_2 = +.8	2
2358	EMU(7)	cos ϕ_1 = +1.0	1

*2364-2642	POWR1(279) (Source point index)	$F_0(r) = \frac{1}{\lambda} \sum_{g=1}^{G} \nu\Sigma_f(r)\phi_g(r)$, the fission density source guess; by source point; $3 \leq \ell \leq$ MAX + JMAX-1 (There are two source points at each interface between regions. The source point index is ℓ = i+m-1, where i is the mesh index on r and m is the no. of region interfaces which have been crossed.)

* In the Fortran IV version the addresses are 2364-2587 and $3 \leq \ell \leq 224$ instead of 279.

Floating Point Addresses	Mnemonic Name	Meaning
†2643-4316	SVM(279,6) (Source, point index, Group)	$4\pi q(r)$, the fixed volume source; by source point index and by group no. [The dimensions on SVM (or $4\pi q(r)$)are $\frac{\text{particles emitted}}{\text{unit volume}\cdot\text{unit time}}$.] (See POWR1, above, for the definition of the source point index, ℓ.)

2643-2921 Source points 1 through MAX + JMAX-1 for group 1
2922-3200 Source points 1 through MAX + JMAX-1 for group 2
3201-3479 Source points 1 through MAX + JMAX-1 for group 3
. . . . . .
. . . . . .
4038-4316 Source points 1 through MAX + JMAX-1 for group 6

7.4 Explanation of Input Data

7.4.1 Card Formats and Addressing

The fixed or floating point data may be entered in any order and on as many cards as desired, so long as all the fixed point data cards precede all the floating point cards. Zero values need not be read in except for ITOUT, which the program sets to 50 unless a value is read in. A given piece of data can be entered more than once, in which case the last value read in is used. If more than one data word is entered on a card, the words following the first are stored in consecutive addresses. Blanks are considered zeros; hence, care must be taken to assure that the specified number of pieces of data in cols. 1-2 is the desired quantity; otherwise, zeros may be stored in locations where they are not wanted.

In order to run change cases, just those data words which are different from the preceding case need be entered. Caution should be exercised when running change cases because unwanted data from the preceding cases may be carried over. Every problem or case must have at least 3 cards; i.e., one Alphanumeric card, one fixed point data card, and one floating point data card.

† All addresses and dimensions shown are those of the Fortran II version for the 7090. In the Fortran IV version for the 7040, the addresses are 2588-3931 and the dimensions of SVM are (224,6). Therefore group 1 values occupy 2588-2811, group 2 occupies 2812-3035, etc.

7.4.2 Explanation of Fixed Point Data

MAX — This is the total number of mesh points on the range of r. In the text, 3.6, this number is referred to as I. It is equal to the number of intervals on r plus one. If a value of MAX read in is too large, an error indication is printed off-line.

JMAX — This is the total number of regions. If $1 \leq JMAX \leq 40$ is not satisfied an error indication is printed. Different regions can have different materials in them or different intervals spacings on r.

NGR — This is the total number of groups, G; and if $1 \leq NGR \leq 6$ is not satisfied an error indication is printed.

NDS — The number of downscatter groups can be defined as one plus the maximum number of groups which can be skipped in scattering down from any group. It ranges from zero (no downscatter) to a maximum of 5 in TOPIC (when group 1 can scatter to group 6). If $0 \leq NDS \leq 5$ is not satisfied, an error indication is printed.

NMIX — There is a limited amount of cross section mixing allowed in TOPIC. NMIX specifies the number of data words in the MIX vector needed to define the mixtures. If NMIX is zero or blank, no mixtures are formed; and if $0 \leq NMIX \leq 20$ is not satisfied an error indication is printed. Also, if NMIX is not zero it must be two or greater or an error indication is printed.

NOT — This word controls the amount of output data from each problem. The higher the value of NOT, the more output data will be given in accordance with the explanation under NOT in 7.2.

IG — After solving for $y_{1,i,j}$ (k=1) in a given group, TOPIC always solves for $y_{k,i,j}$ for all $k \geq 2$. If $IG \geq 2$, the code repeats the solution for $y_{k,i,j}$ for all $k \geq 2$, using the same values of $y_{1,i,j}$, until IG inner iterations have been performed. If all Σ_{Sl} values are zero; i.e., if there is no anisotropic scattering within any group, then any $y_{k,i,j}$ for $k \geq 2$ depends only on the $y_{1,i,j}$ values and inner iterations are unnecessary. Therefore, IG should be left blank, zero or 1, in this case, to avoid wasteful calculation.

ITOUT — This word specifies the maximum number of outer iterations which will be performed, with the exceptions that at least two iterations are always performed and when NOT ≥ 2 then ITOUT + 1 outer iterations (as a maximum) are done to store extra data needed for the output edits.

LCO — LCO has meaning only if there are fissions. If there are fissions, then an LCO = 0 requires that $\gamma_n \leq \epsilon_1$, or pointwise convergence for satisfaction of the convergence criteria. (See formula 5.g.) When LCO ≥ 1, $\delta_n \leq \epsilon_1$, or total fission source convergence for satisfaction of the convergence criteria.

MUTEST — The TOPIC code allows three options with regard to the specification of the intervals on the ϕ halfspace. If MUTEST = 1, the code automatically chooses the intervals such that $(\cos \phi_{j+1} - \cos \phi_j)$ = a constant for $1 \leq j \leq J-1$. If MUTEST = 2, the code automatically chooses the intervals such that $\phi_{j+1} - \phi_j$ = a constant for $1 \leq j \leq J-1$. If MUTEST = 3, the code will compute the interval spacing from the input values given for EMU in the floating point data.

In all three cases, the code makes sure that $\phi_1 = 0$, $\phi_{J+1/2} = \frac{\pi}{2}$, and $\phi_J = \pi$. In addition, it checks to see if $\cos \phi_{j+1} < \cos \phi_j$ and if $\cos \phi_j = -\cos \phi_{J-j+1}$. If these conditions are not met, an error indication is printed.

IDP — If this word is given a value greater than zero, no off line printing of the input data will occur.

LPG — This word has no meaning if there are no fissioning regions in a problem. The following table outlines how the various values supplied for LPG will affect the choice of the initial fission source spectrum.

LPG Value	Meaning in Homogeneous Problem	Meaning in Inhomogeneous Problem
0	*A flat fission source guess in each fissioning region is generated by the code.	The initial fission source guess is set equal to zero everywhere.

* Any fission source guess distribution is first normalized such that the total fission neutron production is FAC, Γ, before it is used in a homogeneous problem.

LPG (Continued)	1	*The fission source guess distribution is taken from the POWR1 input data.	The fission source guess distribution is taken from the POWR1 input data.
	2	*The fission source guess distribution is taken from the previous problem.	The fission source guess distribution is taken from the previous problem.

II — This word specifies the value of the i index on r at the upper (right) boundary of each region. Each region must have at least one interval therefore $II(1) \geq 2$. Also, an error indication is printed if $II(J) \leq II(I)$ for $J > I$ or if $II(JMAX) \neq MAX$.

MIR — These data words specify the material number of the material which is to be used in each region. Each region must have a material number specified. If a region is void, a material with zero cross sections should be used. The maximum material number is 40. (The material placement and numbering is arbitrary and many regions can have the same material number specified.)

MIX — These data words specify the cross section mixtures to be formed. Each word of the MIX list can be regarded as an element of the mixture vector. Each element of the mixture vector is a material number which corresponds either to a material for which cross sections have been read in or to a material to be formed by mixing other materials. There is a one to one correspondence, in order, of the words in the mixture vector MIX to the words in the weighting vector CONC. If the corresponding word in CONC is zero, the material named in MIX is formed by mixing all the materials named by the succeeding words until a material is reached which corresponds to another zero in CONC.

The example to follow demonstrates the proper usage of MIX, CONC, and NMIX.

* Any fission source guess distribution is first normalized such that the total fission neutron production is FAC, Γ, before it is used in a homogeneous problem.

Example

Cross sections are given in the input data for materials 2, 3, 10, 15, 20, and 5.

It is desired that material 7 be such that for any cross section of material 7, $\Sigma^7 = \Sigma^2 w_7^2 + \Sigma^3 w_7^3$, where $\Sigma^i \equiv$ cross section for material i and w_j^i is the proper wieghting factor for material i to use in forming material j. Similarly, let it be desired that for material 6, $\Sigma^6 = \Sigma^{10} w_6^{10} + \Sigma^{15} w_6^{15} + \Sigma^{20} w_6^{20}$, and for material 40, $\Sigma^{40} = \Sigma^7 w_{40}^7 + \Sigma^5 w_{40}^5$.

Then the proper mixture vector and weighting vector elements are as follows:

Element Number	1	2	3	4	5	6	7	8	9	10
MIX Word	7	2	3	6	15	10	20	40	5	7
CONC Word	0	w_7^2	w_7^3	0	w_6^{15}	w_6^{10}	w_6^{20}	0	w_{40}^5	w_{40}^7

There are 10 elements in the mixture vector, so NMIX = 10.

A material formed earlier in the mixture vector can be used to form another material, as with material 7 in the above example. It can be used to form material 40 because it was specified earlier in the mixture vector.

The maximum length of the mixture vector is 20; therefore, NMIX $\leq$ 20. Also, no mixed material can have a material number greater than 40. Error indications are printed if these criteria are not met, if two consecutive words in the CONC vector are zero, if NMIX = 1, if CONC(1) $\neq$ 0, or if CONC(NMIX) = 0.

N — This word specifies the number of intervals on the angle Φ in the halfspace $\{0, \pi\}$. N must always be even and it must be no higher than indicated on the table under 6.0 Program for each version of TOPIC. N is always set to 4 if no value is read in. Error indications are printed if N is too high.

MIK — This word specifies the number of Gauss quadrature points, K, which will be used in TOPIC to perform all integrations on the θ halfspace $\left\{0, \frac{\pi}{2}\right\}$. One indication of how the probable error

in the integration is reduced as K increases is given in the table below. If one computes the current due to an isotropic flux of unity in, say, the hemisphere, $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$, $0 \leq \theta \leq \pi$, the value obtained is $\pi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{\pi} (1) \sin^2 \theta \cos \phi \, d\theta \, d\phi$. However, an isotropic flux of unity using Gauss quadrature will result in a current of C, where $C = \sum_{k=1}^{K} h_k \sqrt{1-\mu_k}$ as in 3.2.

MIK (K)	Value of C	% Error= $100\left[(C/\pi)-1\right]$
1	3.2659647	3.960
2	3.1605551	0.6036
3	3.1477279	0.1953
4	3.1443104	0.08651
5	3.1430044	0.04494
6	3.1424286	0.02661
7	3.1421354	0.01728

MFR

The word MFR (in location or address 115) specifies whether normal mirror reflection is desired at the outer boundary of a problem or whether the flux at $\phi=\frac{\pi}{2}$ should be altered such that a cell problem is more realistically handled. MFR should be made greater than zero if the latter is the case. The option has meaning only for those groups in which the mirror reflection coefficient A is non-zero at the outer boundary. (See 3.7 and formula 3.N.7 for a detailed description of the result of applying the option MFR> 0.)

MCELL

The six words in the MCELL vector specify a region number for each of the six groups. In the specified region, a fixed volume source, 4π q(r), equal to 1.0 is added to whatever fixed volume source is specified by the SVM data. For example if the third word in MCELL (location 118) is equal to 5, then the TØPIC program will place a uniform fixed source of one particle per second per cm^3 at every source point in Region 5 in Group 3.

Only one region can be specified in each group and if MCELL is zero (or blank) for a group no fixed source is added and the fixed volume sources for the group are specified by the

original SVM data.

The fixed sources added by the MCELL specifications become part of SVM and are listed as such in the input data edits. Therefore, care should be taken when running change cases, for if an MCELL value is left non-zero from one problem to the next then another set of unit fixed volume sources will be added to all previously specified fixed volume sources in the regions named.

7.4.3 Explanation of Floating Point Data

EPS1 This is ϵ_1, the fission source convergence specification. It has no meaning if there are no fissions, and in Table 2, 5.2 ϵ_1 is interpreted as being equal to zero in this case. In any problem, if ϵ_1 is zero or left blank, the convergence of a fission source is evaluated and printed out as if ϵ_1 were greater than zero, but the convergence criteria normally depending on ϵ_1 need not be satisfied and control of the convergence process automatically passes to ϵ_2. The convergence criterion dependent on ϵ_1 is on the pointwise fission source distribution or on the total fission source according to whether LCO is zero or non zero, respectively.

EPS2 This is ϵ_2, the group scalar flux convergence specification. The convergence criteria are such that if $\epsilon_2 > 0$, the pointwise flux distribution in each group must be converged to the degree specified by the value of ϵ_2. (See Table 2, 5.2 and the definitions in 5.1.) The convergence criterion on ϵ_2 must be satisfied unless $\epsilon_1 > 0$ and $\epsilon_2 = 0$, in which case only the fission source convergence specification is in control.

XIN This word specifies the value of the cylinder radius at the left or (inner) boundary. It need not be zero, but it cannot be negative.

THETA This is the fission source density extrapolation factor θ. (See 5.k in 5.1.) $\theta = 0$ (or blank) corresponds to no extrapolation. A $\theta > 0$ linearly extrapolates the value of the fission source distribution at each point a distance $\theta(F'_n - F_{n-1})$ beyond the unextrapolated value F'_n. (F_{n-1} is the value at a

point which was used on the previous outer iteration.) θ should be between zero and one to avoid possible divergence when $\theta > 1$.

FAC — This is the constant, Γ, referred to in 5.1. The total fission neutron source is always set equal to this value in homogeneous problems. FAC has no meaning in either an inhomogeneous problem or one without fissions. If no value for FAC is read in, it is set equal to 1.0, automatically.

DELR — The values read in the DELR list are the interval widths Δr which are used in each region. Each Δr must be greater than zero or an error indication will be printed.

Choosing a Δr too large in a region can be one cause of negative flux values. Although not much experience exists, there is some indication that a Δr choice is a "safe" one if, in any group and region, the following inequality is satisfied.

$$\Delta r \cdot \Sigma \leq \frac{1+\Sigma_{S0}/\Sigma}{4(N-1)}$$, where Σ is the total cross section, Σ_{S0} is the isotropic cross section for scattering within the group, and N is the number of intervals on the Φ halfspace. This criterion can be greatly relaxed in many instances in which it is known beforehand that most of the particles at any point in the region are produced by sources or scattering down from higher groups rather than direct transport from neighboring points.

SIGT — This is the total neutron collision cross section, Σ, entered by group no. and material no.. Either macroscopic or microscopic cross sections may be read in, of course; but since only macroscopic values are used by the program, any microscopic cross sections should be multiplied by the proper atomic densities (by use of the cross section mixing option).

Cross sections for a given material can be read in without being used in any region of a given problem. This permits one to read in all the cross sections for a series of problems in one block with the first problem. Then one need only refer to the proper material numbers in the succeeding change cases.

SIGS — The cross sections, Σ_{S0}, for isotropic (zeroth moment) scattering within all groups for all materials are given in the SIGS list. These values are only those for non-degrading (no scattering down) scattering.

SIGS1 — This cross section list gives the first moment scattering cross sections, Σ_{S1}, for scattering within a group for all groups and materials.

VUSIG — This list is the number of neutrons per fission times the fission cross section, $\nu\Sigma_f$, for each group and material.

CHI — This is the χ list which defines the fraction of fission neutrons released in each group.

STR — This list of $\Sigma_{S0g'\to g}$ values gives, for each material, the cross sections for isotropic (zeroth moment) transfer of particles from each group to the others.

Note: These zeroth moment cross sections govern the actual quantities of particles which are transferred by scattering from one group to any other. The $\Sigma_{S1g'\to g}$ values given in the STRJ list alter angular distributions but they do not alter quantities transferred.

STRJ — This list of $\Sigma_{S1g'\to g}$ values gives, for each material, the first moment scattering cross section for scatterings from each group to the others.

ALPHA — These are the mirror reflection (or symmetry) boundary condition coefficients, $A_{\mathcal{S}}$. If, for a given group at a given boundary, the coefficient is $A_{\mathcal{S}}$, the fraction $A_{\mathcal{S}}$ of all outbound particles are reflected back as if by a mirror.

BETA — These are the isotropic reflection boundary condition coefficients, $B_{\mathcal{S}}$. If, for a given group at a given boundary, the coefficient is $B_{\mathcal{S}}$, the fraction $B_{\mathcal{S}}$ of all outbound particles is reflected back in an isotropic (constant) angular distribution.

Note: If $A_{\mathcal{S}} + B_{\mathcal{S}} > 1.0$ at a boundary, there will be multiplication of particles at the boundary.

GAMMA — These are the boundary source coefficients $\mathcal{S}_0$ and $\mathcal{S}_1$ which are given for each boundary for each group. See the explanation under c) in 3.2.

DELTA — These are the fixed boundary source fluxes $f_b(r_{\mathcal{B}}, \Phi_j)$ which are given for the relevant Φ_j points at each boundary for each group. For either a right or a left boundary (in a given group) there must be (J+1)/2 values given. Hence, for an S_4 case, 3 values of f_b are required at either boundary for a group.

CONC — These are the concentrations or weighting factors corresponding to the materials listed in the MIX data. (See MIX under 7.4.2.)

EMU — These data are for the optional specification of the points Φ_j on the Φ halfspace. If equal intervals on cos Φ or Φ is not desired, one sets MUTEST = 3 and enters the desired values for each cos Φ_j in the EMU list. The value cos Φ_J is given first, then cos Φ_{J-1}, etc. until cos Φ_1 is given. No matter what is read in for cos Φ_J, cos $\Phi_{\frac{J+1}{2}}$, and cos Φ_1, TOPIC automatically sets these values to -1, 0, and 1, respectively. In addition, if cos $\Phi_j \neq$ cos Φ_{J-j+1} or if cos $\Phi_{j+1} \not>$ cos Φ_j an error indication is printed.

POWR1 — If there is fissioning in a problem, these input data can be read in as the optional fission source distribution guess, provided LPG has been set equal to 1. The data are entered as $F_{0,\ell} = \frac{1}{\lambda} \sum_{g=1}^{G} \nu\Sigma_{fg\ell}\phi_{ji}$ where ℓ = i+m-1, i is the usual r point index and m is the number of region interfaces which have been crossed. A source guess obtained either from POWR1 input data or from the previous problem is accepted by TOPIC regardless of what composes the $F_{0,\ell}$ list. However, in homogeneous problems it is necessary that at least one point in at least one fissioning region have a non-zero value of $F_{0,\ell}$. Otherwise, trivial (zero) values will result for all fluxes and sources.

SVM — These values are the fixed volume sources read in by source point index, ℓ = i+m-1 (see POWR1, above), for each group. The units on this source should be particles per unit volume per unit time. If the SVM value for point ℓ and group g is $Q_{\ell g}$; then, since these sources are assumed isotropic,

$q_\ell = \frac{Q_{\ell g}}{4\pi}$, where q_ℓ is the q referred to in a) of 3.1. In a given group, there must be one value of Q (SVM) given at each mesh point i not on a region interface and two values at region interfaces (as with POWR1).

7.5 Explanation of Input Error Indicators

When an error is detected in the input data of a problem by the TOPIC program an error indication is printed off-line and the problem and all succeeding change cases are skipped (i.e., the job is terminated).

In the list to follow, the underlined words are the error indication statements which are printed off-line. The explanation of the error then follows.

1. Error, Address Format For Fixed Point Data
   a) Number of data words on card (cols. 1-2) is zero, negative, or greater than twenty-one (21).
   b) Fixed point address (cols. 4-7) is zero, negative, or greater than 130.
2. Error, Address Format For Floating Point Data
   a) Number of data words on card (cols. 1-2) is zero, negative, or greater than 6.
   b) Floating point address (cols. 4-7) is zero, negative, or greater than 4316 (3826 for the 7040 version).
3. Error, Number of Points Too Large
   The number of mesh points I = MAX, on the range of r has been set too large for whatever version of TOPIC is being run.
4. Error, Number of Points Too Small
   I = MAX has been set too small, i.e., less than three.
5. Error, Number of Regions Too Large
   JMAX has been set greater than forty.
6. Error, Number of Regions Is Zero
   JMAX has been set ≤ 0.
7. Error, Number of Groups Too Large
   G = NGR has been set greater than six.

8. Error, Number of Groups Is Zero

G = NGR has been set $\leq$ 0.

9. Error, Number of Downscatter Groups Too Large

NDS has been set greater than NGR-1.

10. Error, Zero or Negative Material Number

A material number, MIR, for some region has been set $\leq$ 0 or left out.

11. Error, Material Number Greater Than 40

A material number, MIR, for some region has been set greater than forty.

12. Error, Upper Region Boundary Point Non-Increasing Or II(1) Is Less Than Or Equal To 1

Either II(J) is $\leq$ II(J-1) for some $1 \leq J \leq$ JMAX or II(1) $\leq$ 1; that is, some region has less than one mesh interval in it.

13. Error, Zero Delta R

The interval width, Δr = DELR, has been set $\leq$ 0 for some region.

14. Mixing Vector Must Have At Least Two Elements

NMIX has been set equal to 1. It must be at least two if it is not zero.

15. Inconsistent Concentration Vector

One or more of the following erroneous specifications have been made:

a) CONC(1) $\neq$ 0,

b) Either CONC(NMIX) = 0 or NMIX is too large, or

c) Two successive CONC values are zero.

16. Material Number Greater Than 40 In Mixture Vector

Some number in the MIX list is greater than forty.

17. Error, Inconsistent Angular Input

One or more of the following has occurred:

a) N, the number of intervals on the Φ halfspace is too large for the version of TOPIC being run.

b) N is odd.

c) $\cos \Phi_j \neq \cos \Phi_{J-j+1}$ for some $2 \leq j \leq (J-1)/2$

d) $\cos \Phi_{j+1} \leq \cos \Phi_j$ for some $2 \leq j \leq (J-1)/2$

18. Error, No Fission Cross Sections or Sources

There are no non-zero values for $\nu\Sigma_f$ in any group for any region and there are no fixed sources, either volume sources or boundary sources; therefore, no non-trivial solution exists. The MIR list may be wrong; i.e., a fissioning material may have been left off the list.

19. Error, No Non-Zero Chi

There are fissions (i.e., some non-zero $\nu\Sigma_f$ in some group) in some region but χ, the fission spectrum, is zero for all groups.

8.0 Output Description

The amount of output data normally obtained off-line from a given TOPIC problem is controlled by the two fixed point input words NOT and IDP.

If sense switch 3 is down or if the inner iteration control word IG is negative, a large amount of unlabeled data is obtained which is useful for checkout or debugging purposes. The use of these debugging options will require a problem to consume much more machine time than is normally required.

The first line of the output data always consists of whatever was on the Alphanumeric card of the problem; the next line tells which version of TOPIC is being run (i.e., whether it is TOPIC 4, TOPIC 10, etc.).

8.1 The Input Data Edit

If IDP is less than 1, the next data listed consists of an edit of the problem input data. The input data listed are:

a) the fixed point data consisting of IG, MAX, JMAX, NGR, NDS, MUTEST, N, LPG, LCO, NOT, NMIX, IDP, ITOUT, and MIK
b) the floating point data consisting of EPS1, EPS2, XIN, THETA, and FAC
c) the basic region data consisting of (for each region) the region no., the material used in the region, the upper r mesh point value of the region, the Δr of the region, and the upper value of r for the region (i.e., J, MIR(J), II(J), DELR(J), and the value of r_i for i = II(J))
d) the angular interval data consisting of the cos ϕ_j value for each value of the index j on the ϕ halfspace
e) the cross section mixture data (if any) consisting of the values of CONC and MIX corresponding to each numbered position in the mixture vector
f) the fixed volume source data (if any) consisting of the value of SVM at every source point for each group (There are MAX + JMAX-1 source points listed for each group.)
g) the boundary condition specification data for each group [The data for each group and for each boundary, right and left, includes the values for ALPHA (Aα), BETA (Bα), the GAMMA'α ($\delta_{\alpha\ell}$ for ℓ = 0 and 1), and the DELTA'α ($f_b(r_\alpha, \phi_j)$ at j=1 through J and with two values at j =(J+1)/2).]
h) the cross section data for each region consisting, first, of the values of Σ, Σ_{S0}, Σ_{S1}, and $\nu\Sigma_f$ for each group and, second, of the values of the transfer or scattering down cross sections (The latter data are listed in the order, $\Sigma_{S0g\to g+1}$ for g=1-5, $\Sigma_{S1g\to g+1}$ for g=1-5, $\Sigma_{S0g\to g+2}$ for g=1-4, $\Sigma_{S1g\to g+2}$ for g=1-4, etc. until finally $\Sigma_{S1g\to G}$ is listed for g=1.)
i) finally, the fission spectrum data consisting of the value of CHI (χ) for each group

8.2 The Outer Iteration Monitor

At the end of each outer iteration, a set of data is printed out which summarizes the rate and degree of convergence of various quantities. These data are in two sections:

a) If and only if NOT ≥ 4 and there are fissions, the fission source distribution, $F_n'(r)$, is printed. $F_n'(r)$ is the fission source distribution normalized such that $\int_R F_n'(r)\, 2\pi r dr = \Gamma$ (FAC) if the problem is homogeneous and unnormalized in an inhomogeneous problem. It is the source computed from the fluxes computed on the $n^{\underline{th}}$ outer iteration; and, except for extrapolation, it is the source which will be used to begin the $n+1^{\underline{th}}$ iteration. As stated, $F_n'(r)$ is not the extrapolated source $F_n(r)$. (See 5.k in 5.1 and THETA in 7.4.3.)

For each iteration, MAX + JMAX-1 values are printed, one for each mesh point on r, except at region interfaces where two are given.

b) Various data showing the state of the convergence process are always printed out at the end of each outer iteration.

The first line of data gives the fission source convergence summary. It consists of the iteration number (n), the eigenvalue λ_n in homogeneous problems or the value of P_n in inhomogeneous problems, E_{max}(EMAX), E_{min} (EMIN), the value of γ_n if LCO = 0 or the value of δ_n if LCO > 0, and the value of the input word ϵ_1 (EPS1). (See 5.1 for the definitions of these terms.)

The next G+1 (NGR+1) lines of data give the scalar flux convergence summary for each group. The iteration number n is printed first. This is followed by G lines which give, for each group, the following data: the group number g, the maximum value of $\alpha^n_{g\ max}$ (called PHIMAX), the point i at which this maximum occurs, the minimum value of $\alpha^n_{g\ min}$ (called PHIMIN), the point i at which this minimum occurs, the value of β_g (called CALC. EPS2), and the input word ϵ_2 (EPS2).

In addition there can be in some versions of TOPIC a line listing various times. These times which are printed out are intended simply for use by one familiar with the TOPIC program and FORTRAN to determine the amount of time the machine spends in various parts of the outer iterative cycle.

8.3 The Problem Solution Output

The first line printed as output for the converged problem simply states the final outer iteration number n and either λ_n or P_n, depending on whether the problem is homogneous or inhomogeneous. If there are no fissions, this line is replaced one by which states "No Fission Problem - No Eigenvalue".

Next, MAX + JMAX-1 (the number of source points) lines are printed. On each line there is listed the mesh point number i on r, the value of r_i, the material number MIR at the point, the fission source distribution $F_n(r_i)$ (right and left hand values are given at each region interface for MIR and $F_n(r_i)$), and the value of the scalar flux $\Phi(r_i)$ for each of the groups.

The data described above are always obtained. In addition, if NOT≧2, the next data printed for each group are: For each mesh point r; there is printed the value of i, the value of r_i, the current $J(r_i)$, the left hemispherical current $\vec{J}(r_i)$, the right hemispherical current $\overleftarrow{J}(r_i)$, and the values of the average angular flux $\Phi(r_i, \phi_j)$ for j=1 through J. (See Appendix B for the definitions of these quantities.)

If NOT ≧ 1, the next data listed are the balance characteristics for each group. These data for each group consists of JMAX + 1 lines of data - one for each region and one for the total of all the regions. For each region there is listed the region number, the integrated flux of the region $(\bar{\Phi}V)_J$, the integrated fixed volume source $(\bar{Q}V)_J$, the total absorptions $(\bar{\Phi}V\Sigma_a)_J$, the total fissions divided by $\lambda_n\left[(\bar{\Phi}V\nu\Sigma_f)_J/\lambda_n\right]$, the net leakage (L_J), the average flux $(\bar{\Phi})_J$, and the average fission source $(\bar{\Phi}\nu\Sigma_f/\lambda_n)_J$. (See Appendix B for the definitions of these quantities.) These data are followed by a line of data giving the sum over all regions for each of the above quantities except for the average quantities. The average flux and average fission source values printed on the "Total" line for each group are, respectively,

$$\sum_J(\bar{\Phi}V)_J/\sum_J(V)_J \text{ and}$$

$$\sum_{J'}(\bar{\Phi}V\nu\Sigma_f)_{J'}/\lambda_n \Big/ \sum_{J'}(V)_{J'},$$ where the sum over J' denotes that only regions with fissions are included in the sum.

After all these data for each group are listed, a line is printed which totals the quantities $(\bar{\Phi}V)_J$, $(\bar{Q}V)_J$, etc. for all groups as well as all regions. In this case, the average flux and average fission source values printed are simply the sums of the values printed on the "Total" line for each group.

If NOT ≥ 3 the last data output are the detailed angular flux values $f_{k,i,j}$ for $1 \leq k \leq K$, $1 \leq i \leq I$, and $1 \leq j \leq J$ for each group. For each group and Gauss point k there are listed a total of I = MAX lines, and on each line are the following data: the mesh point index i, the value r_i, the values of $f_{k,i,j}$ for j = 1 through J.

Appendix A

Definitions of the Difference Equation Coefficients

Define for all $1 \leqq i < I$ and all $1 \leqq j < J$:

$$\Delta r_i = r_{i+1} - r_i ,$$

$$\Delta \phi_j = \phi_{j+1} - \phi_j ,$$

$$\Delta s_j = \sin \phi_{j+1} - \sin \phi_j ,$$

$$\Delta c_j = \cos \phi_{j+1} - \cos \phi_j , \text{ and}$$

$$\Delta sc_j = \cos \phi_{j+1} \sin \phi_{j+1} - \cos \phi_j \sin \phi_j .$$

Then under the assumption B of 3.3 it follows that

A.1 $$\int_{r_i}^{r_{i+1}} f(r,\phi)dr = \frac{\Delta r_i}{2} f_i(\phi) + \frac{\Delta r_i}{2} f_{i+1}(\phi),$$ where the u variable has been omitted for the sake of simplification.

A.2 $$\int_{r_i}^{r_{i+1}} rf(r,\phi)dr = t_{1i} f_i(\phi) + t_{2i} f_{i+1}(\phi),$$ where

$$t_{1i} = \frac{\Delta r_i}{2}\left(\frac{r_{i+1} + 2r_i}{3}\right) \text{ and } t_{2i} = \frac{\Delta r_i}{2}\left(\frac{2r_{i+1} + r_i}{3}\right).$$

A.3 $$\int_{\phi_j}^{\phi_{j+1}} d\phi \cos m\phi f(r,\phi) = \frac{a_j^{m1}}{4} f_j(r) + \frac{a_j^{m2}}{4} f_{j+1}(r),$$ where $1 \leqq j < J$ and

$$a_j^{01} = \frac{4(\cos\phi_{j+1}\Delta\phi_j - \Delta s_j)}{\Delta c_j} , \quad a_j^{02} = \frac{4(\Delta s_j - \cos\phi_j \Delta\phi_j)}{\Delta c_j} ,$$

$$a_j^{11} = \frac{2}{\Delta c_j}\left[\cos\phi_{j+1}\Delta s_j - \sin\phi_j \Delta c_j - \Delta\phi_j\right],$$

$$a_j^{12} = \frac{2}{\Delta c_j}\left[\sin\phi_{j+1}\Delta c_j - \cos\phi_j \Delta s_j + \Delta\phi_j\right],$$

$$a_j^{21} = \frac{4}{3\Delta c_j}\left[-3\sin\phi_j \cos\phi_j \cos\phi_{j+1} + 2\sin\phi_j \cos^2\phi_j + \sin\phi_{j+1}\cos^2\phi_{j+1} - \Delta s_j\right], \text{ and}$$

$$a_j^{22} = \frac{4}{3\Delta c_j}\left[-3\sin\phi_{j+1}\cos\phi_{j+1}\cos\phi_j + 2\sin\phi_{j+1}\cos^2\phi_{j+1} + \sin\phi_j \cos^2\phi_j + \Delta s_j\right].$$

A.4 $$\int_0^{\pi} \cos m\phi \; f(r,\phi)d\phi = \frac{1}{4} \sum_{j=1}^{J} a_j^m f_j(r), \text{ where}$$

$$a_1^m = a_1^{m1}, \; a_j^m = a_j^{m1} + a_{j-1}^{m2} \text{ for } 1 < j < J, \text{ and } a_J^m = a_{J-1}^{m2} .$$

A.5 $$\int_{r_i}^{r_{i+1}} \left\{ \cos\phi \frac{\partial [rf(r,\phi)]}{\partial r} - \frac{\partial [\sin\phi f(r,\phi)]}{\partial \phi} \right\} dr \Bigg|_{\text{at } \phi = \pi}^{\text{evaluated}}$$

$$= \frac{T_{1i,J}}{4\pi} f_{i,J} + \frac{T_{3i,J}}{4\pi} f_{i+1,J}, \text{ where}$$

$$T_{1i,J} = 2\pi(r_{i+1}+r_i) \text{ and } T_{3i,J} = -2\pi(r_{i+1}+r_i).$$

A.6 $$\int_{r_i}^{r_{i+1}} \int_{\phi_j}^{\phi_{j+1}} \left\{ \cos\phi \frac{\partial [rf(r,\phi)]}{\partial r} - \frac{\partial [\sin\phi f(r,\phi)]}{\partial \phi} \right\} d\phi dr$$

$$= \frac{1}{4\pi} \left\{ T_{1i,j} f_{i,j} + T_{2i,j} f_{i,j+1} + T_{3i,j} f_{i+1,j} + T_{4i,j} f_{i+1,j+1} \right\}$$

for $1 \leq j < J$, where

$$T_{1i,j} = -\pi a_j^{11} r_i + 2\pi \sin\phi_j \Delta r_i ,$$
$$T_{2i,j} = -\pi a_j^{12} r_i - 2\pi \sin\phi_{j+1} \Delta r_i ,$$
$$T_{3i,j} = \pi a_j^{11} r_{i+1} + 2\pi \sin\phi_j \Delta r_i , \text{ and}$$
$$T_{4i,j} = \pi a_j^{12} r_{i+1} - 2\pi \sin\phi_{j+1} \Delta r_i ; \text{ for } 1 \leq j < J.$$

A.7 $$\int_{\phi_j}^{\phi_{j+1}} \cos\phi \, d\phi = \Delta\mathscr{S}_j \quad \text{for } 1 \leq j < J.$$

A.8 $$\int_{\phi_j}^{\phi_{j+1}} \cos 2\phi d\phi = \Delta\mathscr{C}_j \text{ for } 1 \leq j < J.$$

A.9 $$\int_{\phi_j}^{\phi_{j+1}} \cos^2\phi d\phi = \frac{\Delta\phi_j + \Delta\mathscr{C}_j}{2} .$$

A.10 $C = (-1)^{\mathscr{S}} 4H_1^1$, where $\mathscr{S}$=1 or 2 and

$$H_1^1 = \sum_{k=1}^{K} h_k \sqrt{1-\mu_k^2} .$$

Appendix B

Definition of Some Basic Physical Quantities

The following definitions apply regardless of the group index.

B.1 Scalar flux (or simply "flux"), Φ

$$\Phi(r) = \int_0^{2\pi} \int_{-1}^{1} f(r,\mu,\phi)\,d\mu\,d\phi = \sum_{k=1}^{K} \sum_{j=1}^{J} h_k\, a_j^0\, f_{k,j}(r).$$

B.2 Current, J

$$J(r) = \int_0^{2\pi} \int_{-1}^{1} \sqrt{1-\mu^2}\, \cos\phi\, f(r,\mu,\phi)\, d\mu\,d\phi$$

$$\doteq \sum_{k=1}^{K} \sum_{j=1}^{J} h_k \sqrt{1-\mu_k^2}\; a_j^1\, f_{k,j}(r).$$

(This represents the net number of particles per unit time crossing a unit area of the cylinder surface perpendicular to r.)

B.3 Left Hemispherical Current, $\overleftarrow{J}$, and Right Hemispherical Current, $\overrightarrow{J}$

$$\overleftarrow{J}(r) = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_{-1}^{1} \sqrt{1-\mu^2}\, \cos\phi\, f(r,\mu,\phi)\, d\mu\,d\phi, \text{ and}$$

$$\overrightarrow{J}(r) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{-1}^{1} \sqrt{1-\mu^2}\, \cos\phi\, f(r,\mu,\phi)\, d\mu\,d\phi.$$

(Note that $J(r) = \overleftarrow{J}(r) + \overrightarrow{J}(r)$.)

$$\overleftarrow{J}(r) \doteq \sum_{k=1}^{K} \sum_{j=\frac{J+1}{2}}^{J-1} h_k \sqrt{1-\mu_k} \left[a_j^{11}\, f_{k,j}(r) + a_j^{12}\, f_{k,j+1}(r) \right]$$

$$\overrightarrow{J}(r) \doteq \sum_{k=1}^{J} \sum_{j=1}^{\frac{J-1}{2}} h_k \sqrt{1-\mu_k} \left[a_j^{11}\, f_{k,j}(r) + a_j^{12}\, f_{k,j+1}(r) \right].$$

B.4 Second Moment Fluxes, $G_2^0(r)$ and $G_2^2(r)$

$$G_2^0(r) = \int_0^{2\pi}\int_{-1}^{1} P_2(\mu)\, f(r,\mu,\phi)\, d\mu d\phi = \sum_{k=1}^{K}\sum_{j=1}^{J} h_k \frac{1}{2}(3\mu_k^2-1)a_j^0 f_{k,j}(r).$$

$$G_2^2(r) = \int_0^{2\pi}\int_{-1}^{1} P_2^2(\mu)\cos 2\phi\, f(r,\mu,\phi)d\mu d\phi$$

$$= \sum_{k=1}^{K}\sum_{j=1}^{J} h_k\, 3(1-\mu_k^2)\, a_j^2\, f_{k,j}(r).$$

B.5 Integrated Flux (for a Region)

Let the lower bound of a region be r_{i_ℓ} and the upper bound be r_{i_μ}, then

$$(\bar{\Phi}V)_J = \int_{r_{i_\ell}}^{r_{i_\mu}} \phi(r)dV = \int_{r_{i_\ell}}^{r_{i_\mu}} \phi(r)\, 2\pi r dr = 2\pi \sum_{i=i_\ell}^{i_\mu-1} (t_{1i}\phi_i + t_{2i}\phi_{i+1}),$$

where $V = \int_{r_{i_\ell}}^{r_{i_\mu}} 2\pi r dr.$

B.6 Total Absorptions (for a Region)

Define, for the J^{th} region,

$$\Sigma_a^J = \Sigma^J - \Sigma_{SO}^J - \sum_{g=g'+1}^{G} \Sigma_{SOg'\to g}^J, \text{ where } \Sigma_{SOg'\to g}^J = \int_0^{2\pi}\int_{-1}^{1} \Sigma_{Sg'\to g}(r,\mu_0)d\mu_0 d\phi.$$

in the r^{th} region. Then the total absorption of the region is

$$(\bar{\Phi}\, V\Sigma_a)_J = (\bar{\Phi}\, V)_J\, \Sigma_a^J.$$

B.7 Net Leakage from a Region

$$L_J = J_{i_\mu} - J_{i_\ell}, \text{ for the } J^{\text{th}} \text{ region.}$$

B.8 Average Angular Flux

$$\Phi(r,\phi) = \frac{\int_{-1}^{1} f(r,\mu,\phi)d\mu}{\int_{-1}^{1} d\mu} = \sum_{k=1}^{K} h_k f_k(r,\phi).$$

(This is called simply "Angular Flux" on the output data sheets.)

B.9 Integrated Fixed Volume Source (for a Region)

$$(\overline{Q}\ V)_J = \int_{r_{i\ell}}^{r_{i\mu}} 2\pi r Q(r)dr, \text{ where } Q(r) = 4\pi q(r).$$

B.10 Integrated (or Total) Fissions (for a Region)

$$(\overline{\Phi}\ V\ \nu\Sigma_f)_J = \nu\Sigma_f^J \int_{r_{i\ell}}^{r_{i\mu}} 2\pi r\Phi(r)dr.$$

B.11 Average Flux (for a Region)

$$(\overline{\Phi})_J = (\overline{\Phi V})_J / V_J = \int_{r_{i\ell}}^{r_{i\mu}} 2\pi r\Phi(r)dr \Big/ \int_{r_{i\ell}}^{r_{i\mu}} 2\pi r dr.$$

B.12 Average Fission Source (for a Region)

$$(\overline{\Phi}\nu\Sigma_f/\lambda_n)_J = \int_{r_{i\ell}}^{r_{i\mu}} 2\pi r\nu\Sigma_f\phi(r)dr \Big/ \lambda_n \int_{r_{i\ell}}^{r_{i\mu}} 2\pi r dr$$ where λ_n is the eigenvalue obtained on the last outer iteration.

D.1 Input Data Listing

```
1SAMPLE PROBLEM FOR TOPIC 4
16 1 40 4 2 1 7 4 1 2 50 2 0 0 1 1 3 33 40
4 53 2 7 7 7
7 93 2 1 4 7 3 6 8
21 1 3 4 2
6 1+1. 0000-04+1. 0000-04 +.5 0000+00+1. 0000+00+.7 5000+00
3 7+1. 2500+00+1. 0000+0 1+1. 0000+00
2 46+.0 6000+00+.0 8000+00
2 58+.5 0000-02+.1 0000-01
2 64+.0 9C00+00+.1 2000+00
2 76+.0 2500+00+.0 5000+00
2 82+.0 2000+00+.0 4000+00
2 286+.3 2000-01+.2 0000-01
2 298+.0 3000-01+.9 9990-02
2 304+.0 4800+00+.3 0000-01
2 316+.0 1500+00+.4 9995-01
2 328+.0 1200+00+.3 9996-01
2 544+.2 0000-01+.1 0000-01
2 568+.0 0400+00+.0 0120+00
2 766+.0 7200+00+.2 2400+00
2 784+.1 0800+00+.3 3600+00
1 1006+1. 0000+00
1 1012+4. 0000-04
1 1042+.0 0200+00
1 1057+6. 0000-04
1 1087+.0 1000+00
1 1117+.8 0000-02
1 1657+1. 2000-04
1 1717+5. 0000-02
3 2212+1. 0000+00 +1. 0000+00
6 2332 +.2 0000-04+1. 3300+00 +.1 0000-01+.2 0000+01
1 2338+4. 0000-03
```

D.2 Output Data Listing

```
  SAMPLE PROBLEM FOR TOPIC 4

PROGRAM TOPIC 4
 INPUT DATA **************
    INNER ITERATION INDEX *  1
    NUMBER OF POINTS ****** 40
    NUMBER OF REGIONS *****  4
    NUMBER OF GROUPS ******  2
    DOWNSCATTER GROUPS ****  1
    ANGULAR APPROXIMATION *  2
    ANGULAR INTERVALS *****  4
    POWER GUESS OPTION ****  0
    CONVERGENCE OPTION ****  0
    OUTPUT OPTION *********  4
    ELEMS. IN MIX. VECT. **  7
    INPUT PRINT OPTION ****  0
    ITERATION MAXIMUM ***** 25
    NO. OF GAUSS POINTS****  2

    EPSILON1***************10.00000E-05
    EPSILON2***************10.00000E-05
    INITIAL RADIUS ********-0.
    EXTRAPOLATION FACTOR ** 5.00000E-01
    NORMALIZATION FACTOR ** 1.00000E 00


REGION DATA

      REGION        REGION         MAXIMUM         DELTA        OUTER
       NO.         MATERIAL     POINT INDEX          R          RADIUS
        1             2             11        7.50000E-01 7.50000E 00
        2             7             13        1.25000E 00 1.00000E 01
        3             7             33        1.00000E 01 2.10000E 02
        4             7             40        1.00000E 00 2.17000E 02


 ANGULAR DATA
     J= 5*-1.00000E 00
     J= 4*-7.07107E-01
     J= 3* 0.
     J= 2* 7.07107E-01
     J= 1* 1.00000E 00

  MIXTURE DATA
      1     2   -0.
      2     1    2.00000E-05
      3     4    1.33000E 00
      4     7   -0.
      5     3   10.00000E-03
      6     6    2.00000E 00
      7     8    4.00000E-03
```

BOUNDARY CONDITION SPECIFICATION

GROUP 1 *****	**LEFT**	**RIGHT**
ALPHA ******	0.10000E 01	-0.
BETA ******	0.	0.
GAMMA ******		
L=0 ******	0.	0.
L=1 ******	0.	0.
DELTA ******		
J=(1)(5)*	0.	0.
J=(2)(4)*	0.	0.
J=(3)(3)*	0.	0.

GROUP 2 *****	**LEFT**	**RIGHT**
ALPHA ******	0.10000E 01	0.
BETA ******	0.	0.
GAMMA ******		
L=0 ******	0.	0.
L=1 ******	0.	0.
DELTA ******		
J=(1)(5)*	0.	0.
J=(2)(4)*	0.	0.
J=(3)(3)*	0.	0.

CROSS SECTION DATA

REGION 1	SIGMA TOTAL	SIGMA S ZERO	SIGMA S ONE	NU-SIGMA FISSION
GROUP 1	1.19701E-01	6.38406E-02	2.66000E-02	1.43641E-01
GROUP 2	1.59602E-01	3.99004E-02	1.33000E-02	4.46884E-01

TRANSFER MATRIX

DOWN**** 1	7.98008E-04	0.	0.	0.	0.
DOWN**** 1	1.59600E-04	0.	0.	0.	0.

REGION 2	SIGMA TOTAL	SIGMA S ZERO	SIGMA S ONE	NU-SIGMA FISSION
GROUP 1	5.01300E-02	3.00780E-02	1.60000E-05	0.
GROUP 2	1.00260E-01	1.00250E-01	4.80000E-06	0.

TRANSFER MATRIX

DOWN**** 1	2.00520E-02	0.	0.	0.	0.
DOWN**** 1	2.00000E-04	0.	0.	0.	0.

REGION 3	SIGMA TOTAL	SIGMA S ZERO	SIGMA S ONE	NU-SIGMA FISSION
GROUP 1	5.01300E-02	3.00780E-02	1.60000E-05	0.
GROUP 2	1.00260E-01	1.00250E-01	4.80000E-06	0.

TRANSFER MATRIX

DOWN**** 1	2.00520E-02	0.	0.	0.	0.
DOWN**** 1	2.00000E-04	0.	0.	0.	0.

REGION 4	SIGMA TOTAL	SIGMA S ZERO	SIGMA S ONE	NU-SIGMA FISSION
GROUP 1	5.01300E-02	3.00780E-02	1.60000E-05	0.
GROUP 2	1.00260E-01	1.00250E-01	4.80000E-06	0.

TRANSFER MATRIX

DOWN**** 1	2.00520E-02	0.	0.	0.	0.
DOWN**** 1	2.00000E-04	0.	0.	0.	0.

```
     FISSION SPECTRUM
     GROUP****  1  0.10000E 01
     GROUP****  2  0.

   END OF INPUT PRINT
ITERATION BEGUN AT TIME = 014622


  FISSION DENSITY-ITERATION   1
 0.55772E-02  0.55209E-02  0.55267E-02  0.55323E-02  0.55418E-02  0.55573E-02  0.55817E-02  0.56187E-02
 0.56731E-02  0.57476E-02  0.59905E-02  0.            0.            0.            0.            0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE     EMAX          EMIN          CAL.-EPS.     INP.-EPS.
  1    0.18149E 01  0.10586E 01  0.97562E 00  0.78385E-01  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   1
      GROUP  PHIMAX      POINT   PHIMIN      POINT   CALC. EPS2      INPUT EPS2
         1  0.10000E 01     1 0.10000E 01      1    0.              1.00000E-04
         2  0.10000E 01     1 0.10000E 01      1    0.              1.00000E-04
TIME1=014628 TIME2=014629 TIME3=014630 TIME4=014630 TIME5=014631 TIME6=014632 TIME7=014633 TIME8=014633 TIME9=000000

  FISSION DENSITY-ITERATION   2
 0.54078E-02  0.54114E-02  0.54201E-02  0.54356E-02  0.54591E-02  0.54929E-02  0.55408E-02  0.56075E-02
 0.56990E-02  0.58233E-02  0.61414E-02  0.            0.            0.            0.            0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE     EMAX          EMIN          CAL.-EPS.     INP.-EPS.
  2    0.19718E 01  0.10252E 01  0.96963E 00  0.54192E-01  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   2
      GROUP  PHIMAX      POINT   PHIMIN      POINT   CALC. EPS2      INPUT EPS2
         1  0.10491E 01   10-0.62234E 00     32    0.15932E 01     1.00000E-04
         2  0.11769E 01   15 0.10000E 01      1    0.15031E-00     1.00000E-04
TIME1=014640 TIME2=014641 TIME3=014642 TIME4=014642 TIME5=014643 TIME6=014644 TIME7=014645 TIME8=014646 TIME9=000000


  FISSION DENSITY-ITERATION   3
 0.53676E-02  0.53685E-02  0.53806E-02  0.54007E-02  0.54303E-02  0.54720E-02  0.55294E-02  0.56072E-02
 0.57113E-02  0.58494E-02  0.61807E-02  0.            0.            0.            0.            0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE     EMAX          EMIN          CAL.-EPS.     INP.-EPS.
  3    0.19932E 01  0.10064E 01  0.99164E 00  0.14674E-01  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   3
```

```
      GROUP  PHIMAX      POINT   PHIMIN      POINT   CALC. EPS2      INPUT EPS2
         1  C.17794E 01   31-0.11102E 01     32    0.16239E 01     1.00000E-04
         2  C.10297E 01   15 0.10000E 01      1    0.28861E-01     1.00000E-04
TIME1=014652 TIME2=014653 TIME3=014654 TIME4=014655 TIME5=014655 TIME6=014656 TIME7=014657 TIME8=014658 TIME9=000000


  FISSION DENSITY-ITERATION   4
 0.53442E-02  0.53506E-02  0.53641E-02  0.53864E-02  0.54190E-02  0.54644E-02  0.55259E-02  0.56082E-02
 0.57170E-02  0.58594E-02  0.61929E-02  0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE      EMAX          EMIN        CAL.-EPS.      INP.-EPS.
  4    0.19967E 01  0.10008E 01  0.99878E 00  0.20025E-02  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   4
      GROUP  PHIMAX      POINT   PHIMIN      POINT   CALC. EPS2      INPUT EPS2
         1  C.13046E 01   36 0.99528E 00      1    0.23708E-00     1.00000E-04
         2  C.10065E 01   15 0.10000E 01      1    0.64597E-02     1.00000E-04
TIME1=014704 TIME2=014705 TIME3=014706 TIME4=014707 TIME5=014708 TIME6=014709 TIME7=014709 TIME8=014710 TIME9=000000


  FISSION DENSITY-ITERATION   5
 0.53442E-02  0.53506E-02  0.53641E-02  0.53864E-02  0.54190E-02  0.54644E-02  0.55259E-02  0.56082E-02
 0.57169E-02  0.58592E-02  0.61935E-02  0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE      EMAX          EMIN        CAL.-EPS.      INP.-EPS.
  5    0.19976E 01  0.10007E 01  0.99960E 00  0.11108E-02  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   5
      GROUP  PHIMAX      POINT   PHIMIN      POINT   CALC. EPS2      INPUT EPS2
         1  C.10101E 01   36 0.99850E 00     30    0.11480E-01     1.00000E-04
         2  C.10014E 01   16 0.10000E 01      1    0.13579E-02     1.00000E-04
TIME1=014717 TIME2=014718 TIME3=014718 TIME4=014719 TIME5=014720 TIME6=014721 TIME7=014722 TIME8=014722 TIME9=000000


  FISSION DENSITY-ITERATION   6
 0.53434E-02  0.53498E-02  0.53634E-02  0.53857E-02  0.54185E-02  0.54640E-02  0.55257E-02  0.56082E-02
 0.57171E-02  0.58598E-02  0.61941E-02  0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE      EMAX          EMIN        CAL.-EPS.      INP.-EPS.
  6    0.19977E 01  0.10001E 01  0.99975E 00  0.30754E-03  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   6
      GROUP  PHIMAX      POINT   PHIMIN      POINT   CALC. EPS2      INPUT EPS2
         1  C.10007E 01   36 0.99978E 00      1    0.91671E-03     1.00000E-04
```

```
        2  0.10003E 01   16 0.10000E 01      1    0.32559E-03    1.00000E-04
TIME1=014729 TIME2=014730 TIME3=014731 TIME4=014731 TIME5=014732 TIME6=014733 TIME7=014734 TIME8=014734 TIME9=000000


  FISSION DENSITY-ITERATION   7
 0.53434E-02  0.53498E-02  0.53634E-02  0.53857E-02  0.54185E-02  0.54640E-02  0.55257E-02  0.56082E-02
 0.57171E-02  0.58597E-02  0.61941E-02  0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE     EMAX          EMIN         CAL.-EPS.     INP.-EPS.
  7    0.19978E 01  0.10001E 01  0.99998E 00  0.14529E-03  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   7
      GROUP  PHIMAX     POINT   PHIMIN     POINT   CALC. EPS2     INPUT EPS2
        1  0.10000E 01    6 0.99996E 00     30    0.61347E-04    1.00000E-04
        2  0.10001E 01   16 0.10000E 01      1    0.67751E-04    1.00000E-04
TIME1=014741 TIME2=014742 TIME3=014743 TIME4=014743 TIME5=014744 TIME6=014745 TIME7=014746 TIME8=014747 TIME9=000000


  FISSION DENSITY-ITERATION   8
 0.53434E-02  0.53498E-02  0.53634E-02  0.53857E-02  0.54185E-02  0.54640E-02  0.55257E-02  0.56082E-02
 0.57171E-02  0.58597E-02  0.61941E-02  0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE     EMAX          EMIN         CAL.-EPS.     INP.-EPS.
  8    0.19978E 01  0.10000E 01  0.99995E 00  0.63001E-04  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   8
      GROUP  PHIMAX     POINT   PHIMIN     POINT   CALC. EPS2     INPUT EPS2
        1  0.10000E 01   36 0.99999E 00      1    0.26099E-04    1.00000E-04
        2  0.10000E 01   16 0.10000E 01      1    0.17374E-04    1.00000E-04
TIME1=014753 TIME2=014754 TIME3=014755 TIME4=014756 TIME5=014756 TIME6=014757 TIME7=014758 TIME8=014759 TIME9=000000


  FISSION DENSITY-ITERATION   9
 0.53434E-02  0.53498E-02  0.53634E-02  0.53857E-02  0.54185E-02  0.54640E-02  0.55257E-02  0.56082E-02
 0.57171E-02  0.58597E-02  0.61941E-02  0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.           0.           0.
 0.           0.           0.

ITER.  EIGENVALUE     EMAX          EMIN         CAL.-EPS.     INP.-EPS.
  9    0.19978E 01  0.10000E 01  1.00000E 00  0.17659E-05  1.00000E-04

  FLUX CONVERGENCE SUMMARY,ITERATION   9
      GROUP  PHIMAX     POINT   PHIMIN     POINT   CALC. EPS2     INPUT EPS2
        1  0.10000E 01    8 1.00000E 00     38    0.35954E-05    1.00000E-04
        2  0.10000E 01   17 0.10000E 01      1    0.35912E-05    1.00000E-04
TIME1=014805 TIME2=014806 TIME3=014807 TIME4=014808 TIME5=014808 TIME6=014809 TIME7=014810 TIME8=014811 TIME9=000000
```

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  ****OUTPUT DATA****
FINAL ITERATION=  9          FINAL MULTIPLICATION FACTOR=  1.99778E 00
```

POINT NUMBER	RADIUS	MATERIAL	FISSION DENSITY	GROUP 1 FLUX	GROUP 2 FLUX	GROUP 3 FLUX	GROUP 4 FLUX	GROUP 5 FLUX	GROUP 6 FLUX
1	-0.	2	5.3434E-03	4.8154E-02	8.4091E-03				
2	7.5000E-01	2	5.3498E-03	4.8005E-02	8.4859E-03				
3	1.5000E 00	2	5.3634E-03	4.7656E-02	8.6587E-03				
4	2.2500E 00	2	5.3857E-03	4.7065E-02	8.9484E-03				
5	3.0000E 00	2	5.4185E-03	4.6204E-02	9.3716E-03				
6	3.7500E 00	2	5.4640E-03	4.5036E-02	9.9505E-03				
7	4.5000E 00	2	5.5257E-03	4.3515E-02	1.0715E-02				
8	5.2500E 00	2	5.6082E-03	4.1575E-02	1.1708E-02				
9	6.0000E 00	2	5.7171E-03	3.9087E-02	1.2994E-02				
10	6.7500E 00	2	5.8597E-03	3.5563E-02	1.4765E-02				
11	7.5000E 00	2	6.1941E-03	2.7902E-02	1.8722E-02				
11	7.5000E 00	7	0.	2.7902E-02	1.8722E-02				
12	8.7500E 00	7	0.	2.1295E-02	2.2446E-02				
13	1.0000E 01	7	0.	1.7311E-02	2.4553E-02				
13	1.0000E 01	7	0.	1.7311E-02	2.4553E-02				
14	2.0000E 01	7	0.	5.1716E-03	2.9338E-02				
15	3.0000E 01	7	0.	2.4510E-03	2.7781E-02				
16	4.0000E 01	7	0.	1.1825E-03	2.5460E-02				
17	5.0000E 01	7	0.	6.2700E-04	2.2965E-02				
18	6.0000E 01	7	0.	3.4153E-04	2.0595E-02				
19	7.0000E 01	7	0.	1.9102E-04	1.8404E-02				
20	8.0000E 01	7	0.	1.0859E-04	1.6406E-02				
21	9.0000E 01	7	0.	6.2521E-05	1.4587E-02				
22	1.0000E 02	7	0.	3.6342E-05	1.2929E-02				
23	1.1000E 02	7	0.	2.1286E-05	1.1412E-02				
24	1.2000E 02	7	0.	1.2545E-05	1.0019E-02				
25	1.3000E 02	7	0.	7.4309E-06	8.7339E-03				
26	1.4000E 02	7	0.	4.4206E-06	7.5423E-03				
27	1.5000E 02	7	0.	2.6392E-06	6.4328E-03				
28	1.6000E 02	7	0.	1.5804E-06	5.3954E-03				
29	1.7000E 02	7	0.	9.4832E-07	4.4215E-03				
30	1.8000E 02	7	0.	5.6948E-07	3.5044E-03				
31	1.9000E 02	7	0.	3.4107E-07	2.6368E-03				
32	2.0000E 02	7	0.	2.0203E-07	1.8154E-03				
33	2.1000E 02	7	0.	1.1367E-07	1.0187E-03				
33	2.1000E 02	7	0.	1.1367E-07	1.0187E-03				
34	2.1100E 02	7	0.	1.0665E-07	9.3886E-04				
35	2.1200E 02	7	0.	9.9783E-08	8.5812E-04				
36	2.1300E 02	7	0.	9.3021E-08	7.7591E-04				
37	2.1400E 02	7	0.	8.6299E-08	6.9132E-04				
38	2.1500E 02	7	0.	7.9534E-08	6.0276E-04				
39	2.1600E 02	7	0.	7.2616E-08	5.0743E-04				
40	2.1700E 02	7	0.	6.5394E-08	3.9993E-04				

AUXILIARY OUTPUT FOR GROUP 1

PT.	RADIUS	NET CURRENT	HEMISPHERE-CURRENT-LEFT	HEMISPHERE-CURRENT-RIGHT	ANGULAR FLUXES FROM MU=1. TO -1.			
1	-0.	-2.045554E-10	-1.163928E-02	1.163928E-02	3.832236E-03	3.831977E-03	3.831694E-03	3.831977E-03
					3.832236E-03			
2	7.5000C0E-01	9.987349E-04	-1.110387E-02	1.210260E-02	4.016474E-03	3.959365E-03	3.820805E-03	3.680963E-03
					3.622652E-03			
3	1.5000C0E 00	2.010573E-03	-1.051300E-02	1.252357E-02	4.186183E-03	4.074092E-03	3.794820E-03	3.511405E-03
					3.393756E-03			
4	2.2500C0E 00	3.047128E-03	-9.852472E-03	1.289960E-02	4.341445E-03	4.175252E-03	3.749119E-03	3.317218E-03
					3.141477E-03			
5	3.0000C0E 00	4.122168E-03	-9.110133E-03	1.323230E-02	4.483373E-03	4.263968E-03	3.679568E-03	3.092372E-03
					2.862823E-03			
6	3.7500C0E 00	5.251789E-03	-8.271684E-03	1.352347E-02	4.613198E-03	4.341471E-03	3.581188E-03	2.829216E-03
					2.554564E-03			
7	4.5000C0E 00	6.455349E-03	-7.319559E-03	1.377491E-02	4.731997E-03	4.409367E-03	3.447771E-03	2.517772E-03
					2.213034E-03			
8	5.2500C0E 00	7.756711E-03	-6.231343E-03	1.398805E-02	4.840628E-03	4.469955E-03	3.269816E-03	2.144927E-03
					1.834019E-03			
9	6.0000C0E 00	9.186528E-03	-4.974013E-03	1.416054E-02	4.938779E-03	4.528452E-03	3.018268E-03	1.693636E-03
					1.412811E-03			
10	6.7500C0E 00	1.079141E-02	-3.474072E-03	1.426549E-02	5.016848E-03	4.611320E-03	2.532484E-03	1.143297E-03
					9.452557E-04			
11	7.5000C0E 00	1.274824E-02	-1.403973E-03	1.415222E-02	4.978214E-03	4.966533E-03	6.299480E-04	4.712545E-04
					4.298117E-04			
12	8.7500C0E 00	1.035651E-02	-1.194213E-03	1.155073E-02	5.383255E-03	2.461139E-03	5.326115E-04	4.006117E-04
					3.649476E-04			
13	1.0000C0E 01	8.609397E-03	-1.042078E-03	9.651476E-03	4.954530E-03	1.477600E-03	4.743826E-04	3.486645E-04
					3.165615E-04			
14	2.0000C0E 01	2.715555E-03	-3.911356E-04	3.106691E-03	1.805458E-03	1.134343E-04	1.754827E-04	1.302025E-04
					1.180816E-04			
15	3.0000C0E 01	1.188659E-03	-1.917148E-04	1.380374E-03	6.580079E-04	1.958243E-04	8.790954E-05	6.366993E-05
					5.737881E-05			
16	4.0000C0E 01	5.780374E-04	-9.801004E-05	6.760474E-04	3.322736E-04	7.755651E-05	4.487076E-05	3.249977E-05
					2.925208E-05			
17	5.0000C0E 01	3.010097E-04	-5.317573E-05	3.541854E-04	1.685246E-04	4.558131E-05	2.447811E-05	1.761510E-05
					1.582118E-05			
18	6.0000C0E 01	1.626239E-04	-2.955011E-05	1.921740E-04	9.051824E-05	2.516292E-05	1.363745E-05	9.781720E-06
					8.773773E-06			
19	7.0000C0E 01	9.017148E-05	-1.675187E-05	1.069234E-04	4.981733E-05	1.444048E-05	7.751704E-06	5.542176E-06
					4.965233E-06			
20	8.0000C0E 01	5.091003E-05	-9.624305E-06	6.053433E-05	2.798868E-05	8.348732E-06	4.462962E-06	3.182727E-06
					2.848693E-06			
21	9.0000C0E 01	2.913594E-05	-5.586814E-06	3.472275E-05	1.595254E-05	4.877471E-06	2.595472E-06	1.846901E-06
					1.651735E-06			
22	1.0000C0E 02	1.684971E-05	-3.269181E-06	2.011889E-05	9.194832E-06	2.869738E-06	1.521167E-06	1.080425E-06
					9.655922E-07			
23	1.1000C0E 02	9.825666E-06	-1.925352E-06	1.175102E-05	5.346745E-06	1.698313E-06	8.971129E-07	6.361563E-07
					5.682032E-07			
24	1.2000C0E 02	5.768510E-06	-1.139908E-06	6.908418E-06	3.131406E-06	1.009827E-06	5.317901E-07	3.765653E-07
					3.361615E-07			
25	1.3000C0E 02	3.405631E-06	-6.778346E-07	4.083466E-06	1.844814E-06	6.028234E-07	3.165853E-07	2.238884E-07
					1.997642E-07			
26	1.4000C0E 02	2.020169E-06	-4.045121E-07	2.424681E-06	1.092237E-06	3.610642E-07	1.891517E-07	1.336008E-07
					1.191386E-07			
27	1.5000C0E 02	1.203250E-06	-2.420652E-07	1.445315E-06	6.493962E-07	2.168791E-07	1.133583E-07	7.995432E-08
					7.124482E-08			

28	1.600000E 02	7.193159E-07	-1.450755E-07	8.643914E-07	3.874985E-07	1.305872E-07	6.810574E-08	4.793726E-08
					4.265817E-08			
29	1.700000E 02	4.315555E-07	-8.686393E-08	5.184195E-07	2.319387E-07	7.878361E-08	4.098896E-08	2.873423E-08
					2.549876E-08			
30	1.800000E 02	2.599836E-07	-5.164649E-08	3.116301E-07	1.391889E-07	4.758806E-08	2.468471E-08	1.712938E-08
					1.510628E-08			
31	1.900000E 02	1.576112E-07	-2.998734E-08	1.875986E-07	8.368867E-08	2.874637E-08	1.481265E-08	1.000183E-08
					8.696389E-09			
32	2.000000E 02	9.675634E-08	-1.614813E-08	1.129045E-07	5.037851E-08	1.725770E-08	8.873572E-09	5.418516E-09
					4.566181E-09			
33	2.100000E 02	6.132057E-08	-6.271487E-09	6.759206E-08	3.014982E-08	1.042653E-08	4.612798E-09	2.073512E-09
					1.647058E-09			
34	2.110000E 02	5.882634E-08	-5.400663E-09	6.422700E-08	2.863312E-08	9.956614E-09	4.187401E-09	1.774411E-09
					1.397303E-09			
35	2.120000E 02	5.648411E-08	-4.532032E-09	6.101615E-08	2.717600E-08	9.530611E-09	3.727507E-09	1.476841E-09
					1.152256E-09			
36	2.130000E 02	5.429047E-08	-3.660508E-09	5.795098E-08	2.577096E-08	9.155066E-09	3.215863E-09	1.180205E-09
					9.117573E-10			
37	2.140000E 02	5.224317E-08	-2.780028E-09	5.502320E-08	2.440903E-08	8.839832E-09	2.628563E-09	8.840938E-10
					6.758265E-10			
38	2.150000E 02	5.034146E-08	-1.883159E-09	5.222462E-08	2.307906E-08	8.599538E-09	1.932049E-09	5.883849E-10
					4.447314E-10			
39	2.160000E 02	4.858651E-08	-9.605291E-10	4.954704E-08	2.176654E-08	8.455813E-09	1.078662E-09	2.933773E-10
					2.190868E-10			
40	2.170000E 02	4.698216E-08	0.	4.698216E-08	2.045200E-08	8.440597E-09	0.	-0.
					0.			

AUXILIARY OUTPUT FOR GROUP 2

PT.	RADIUS	NET CURRENT	HEMISPHERE-CURRENT-LEFT	HEMISPHERE-CURRENT-RIGHT	ANGULAR FLUXES FROM MU=1. TO -1.			
1	-0.	2.573367E-12	-2.271817E-03	2.271817E-03	6.689762E-04	6.691917E-04	6.694187E-04	6.691917E-04
					6.689762E-04			
2	7.5000C0E-01	-3.653846E-04	-2.474051E-03	2.109666E-03	6.054188E-04	6.255534E-04	6.747262E-04	7.249383E-04
					7.460603E-04			
3	1.5000C0E 00	-7.390670E-04	-2.705675E-03	1.966608E-03	5.494055E-04	5.876136E-04	6.861585E-04	7.897127E-04
					8.337860E-04			
4	2.2500C0E 00	-1.128447E-03	-2.975009E-03	1.846562E-03	5.007661E-04	5.557351E-04	7.057472E-04	8.667109E-04
					9.348122E-04			
5	3.0000C0E 00	-1.542271E-03	-3.289687E-03	1.747416E-03	4.588772E-04	5.292883E-04	7.354298E-04	9.592719E-04
					1.051394E-03			
6	3.7500C0E 00	-1.990718E-03	-3.659047E-03	1.668328E-03	4.231429E-04	5.077244E-04	7.777848E-04	1.071821E-03
					1.186094E-03			
7	4.5000C0E 00	-2.485954E-03	-4.094840E-03	1.608886E-03	3.931701E-04	4.904351E-04	8.362453E-04	1.210361E-03
					1.341902E-03			
8	5.2500C0E 00	-3.042884E-03	-4.612089E-03	1.569206E-03	3.688048E-04	4.766711E-04	9.155750E-04	1.383101E-03
					1.522322E-03			
9	6.0000C0E 00	-3.680639E-03	-5.231412E-03	1.550772E-03	3.504558E-04	4.647592E-04	1.026327E-03	1.601311E-03
					1.731440E-03			
10	6.7500C0E 00	-4.428889E-03	-5.991863E-03	1.562974E-03	3.418417E-04	4.453785E-04	1.221016E-03	1.880267E-03
					1.973806E-03			
11	7.5000C0E 00	-5.398944E-03	-7.080212E-03	1.681268E-03	4.081623E-04	2.604201E-04	2.156230E-03	2.236670E-03
					2.251561E-03			
12	8.7500C0E 00	-4.057354E-03	-7.226932E-03	3.169578E-03	3.426032E-04	1.499824E-03	2.242109E-03	2.287494E-03
					2.293753E-03			
13	1.0000C0E 01	-3.097908E-03	-7.306751E-03	4.208843E-03	6.548684E-04	1.940232E-03	2.287069E-03	2.316412E-03
					2.316669E-03			
14	2.0000C0E 01	3.814398E-05	-7.234629E-03	7.272773E-03	2.173350E-03	2.487022E-03	2.374136E-03	2.304982E-03
					2.278098E-03			
15	3.0000C0E 01	6.447581E-04	-6.649294E-03	7.294052E-03	2.332440E-03	2.301492E-03	2.210098E-03	2.121278E-03
					2.088486E-03			
16	4.0000C0E 01	7.946938E-04	-6.010171E-03	6.804865E-03	2.183701E-03	2.136775E-03	2.011608E-03	1.918614E-03
					1.884874E-03			
17	5.0000C0E 01	7.949940E-04	-5.391239E-03	6.186233E-03	1.996322E-03	1.927794E-03	1.811163E-03	1.721676E-03
					1.689196E-03			
18	6.0000C0E 01	7.487138E-04	-4.819319E-03	5.568033E-03	1.797370E-03	1.733720E-03	1.623356E-03	1.539482E-03
					1.508949E-03			
19	7.0000C0E 01	6.891623E-04	-4.297432E-03	4.986594E-03	1.610231E-03	1.551410E-03	1.450724E-03	1.373154E-03
					1.344765E-03			
20	8.0000C0E 01	6.293731E-04	-3.823272E-03	4.452645E-03	1.437570E-03	1.385272E-03	1.293507E-03	1.222030E-03
					1.195720E-03			
21	9.0000C0E 01	5.740945E-04	-3.391911E-03	3.966006E-03	1.280368E-03	1.233810E-03	1.150418E-03	1.084567E-03
					1.060183E-03			
22	1.0000C0E 02	5.247487E-04	-2.998209E-03	3.522958E-03	1.137351E-03	1.095935E-03	1.019933E-03	9.591349E-04
					9.364905E-04			
23	1.1000C0E 02	4.813729E-04	-2.637297E-03	3.118670E-03	1.007038E-03	9.699888E-04	9.004885E-04	8.441823E-04
					8.230939E-04			
24	1.2000C0E 02	4.434682E-04	-2.304887E-03	2.748356E-03	8.878401E-04	8.545081E-04	7.906744E-04	7.383395E-04
					7.186364E-04			
25	1.3000C0E 02	4.103713E-04	-1.997279E-03	2.407650E-03	7.783245E-04	7.481407E-04	6.892455E-04	6.404233E-04
					6.219528E-04			
26	1.4000C0E 02	3.814152E-04	-1.711326E-03	2.092741E-03	6.772299E-04	6.497207E-04	5.951427E-04	5.494265E-04
					5.320545E-04			
27	1.5000C0E 02	3.559932E-04	-1.444362E-03	1.800355E-03	5.834721E-04	5.582535E-04	5.074454E-04	4.644967E-04
					4.481063E-04			

28	1.6000C0E 02	3.335784E-04	-1.194131E-03	1.527709E-03	4.961402E-04	4.728752E-04	4.254142E-04	3.849074E-04	3.693997E-04
29	1.7000C0E 02	3.137244E-04	-9.587104E-04	1.272435E-03	4.144369E-04	3.928964E-04	3.483212E-04	3.100491E-04	2.953425E-04
30	1.8000C0E 02	2.960570E-04	-7.364752E-04	1.037532E-03	3.377510E-04	3.176119E-04	2.757993E-04	2.393581E-04	2.254211E-04
31	1.9000C0E 02	2.802644E-04	-5.259651E-04	8.062295E-04	2.653913E-04	2.467035E-04	2.068819E-04	1.723585E-04	1.592832E-04
32	2.0000C0E 02	2.660869E-04	-3.259845E-04	5.920714E-04	1.972177E-04	1.790575E-04	1.425097E-04	1.083249E-04	9.653027E-05
33	2.1000C0E 02	2.533086E-04	-1.339936E-04	3.873022E-04	1.315749E-04	1.154342E-04	7.802257E-05	4.491297E-05	3.727179E-05
34	2.1100C0E 02	2.521006E-04	-1.150970E-04	3.671976E-04	1.250050E-04	1.094496E-04	7.108169E-05	3.849573E-05	3.156796E-05
35	2.1200C0E 02	2.509045E-04	-9.620029E-05	3.471048E-04	1.183387E-04	1.036725E-04	6.370673E-05	3.204012E-05	2.592997E-05
36	2.1300C0E 02	2.497203E-04	-7.727478E-05	3.269951E-04	1.115041E-04	9.822068E-05	5.563144E-05	2.554943E-05	2.037675E-05
37	2.1400C0E 02	2.485478E-04	-5.827892E-05	3.069268E-04	1.043799E-04	9.329865E-05	4.640699E-05	1.903829E-05	1.493846E-05
38	2.1500C0E 02	2.473870E-04	-3.914911E-05	2.868361E-04	9.675381E-05	8.927501E-05	3.524583E-05	1.254324E-05	9.664251E-06
39	2.1600C0E 02	2.462377E-04	-1.978225E-05	2.668199E-04	8.823583E-05	8.684169E-05	2.070264E-05	6.143123E-06	4.636717E-06
40	2.1700C0E 02	2.450998E-04	0.	2.450998E-04	7.807617E-05	8.735340E-05	0.	-0.	0.

BALANCE CHARACTERISTICS GROUP 1

	INTEGRATED FLUX	INTEGRATED FIXED SOURCE	TOTAL ABSORPTIONS	TOTAL FISSIONS/K	NET LEAKAGE	AVERAGE FLUX	AVERAGE FISSION SOURCE
REGION 1	7.147316E 00	0.	3.935494E-01	5.138968E-01	6.007469E-01	4.044553E-02	2.908061E-03
REGION 2	2.982371E 00	0.	6.943874E-10	0.	-5.980249E-02	2.169870E-02	0.
REGION 3	2.697305E 01	0.	6.280151E-09	0.	-5.408635E-01	1.951315E-04	0.
REGION 4	8.404493E-04	0.	1.956823E-13	0.	-1.685270E-05	8.950261E-08	0.
TOTAL***	3.710357E 01	0.	3.935494E-01	5.138968E-01	6.406061E-05	2.508109E-04	2.908061E-03

BALANCE CHARACTERISTICS GROUP 2

	INTEGRATED FLUX	INTEGRATED FIXED SOURCE	TOTAL ABSORPTIONS	TOTAL FISSIONS/K	NET LEAKAGE	AVERAGE FLUX	AVERAGE FISSION SOURCE
REGION 1	2.173102E 00	0.	2.601229E-01	4.861033E-01	-2.544192E-01	1.229724E-02	2.750782E-03
REGION 2	3.048616E 00	0.	3.056731E-05	0.	5.977197E-02	2.218068E-02	0.
REGION 3	1.195168E 03	0.	1.198349E-02	0.	5.288802E-01	8.646220E-03	0.
REGION 4	6.804003E 00	0.	6.822114E-05	0.	-5.130470E-05	7.245839E-04	0.
TOTAL***	1.207193E 03	0.	2.722052E-01	4.861033E-01	3.341816E-01	8.160325E-03	2.750782E-03

TOTAL BALANCE

TOTAL***	1.244297E 03	0.	6.657546E-01	1.000000E 00	3.342456E-01	8.411136E-03	5.658643E-03

ANGULAR FLUXES FOR GROUP 1, GAUSS POINT 1

PT.	RADIUS	ANGULAR DATA FROM MU=1. TO MU=-1.				
1	-0.	0.34131E-02	0.34131E-02	0.34131E-02	0.34131E-02	0.34131E-02
2	0.7500CE 00	0.36119E-02	0.35508E-02	0.34031E-02	0.32554E-02	0.31941E-02
3	0.1500CE 01	0.37985E-02	0.36759E-02	0.33771E-02	0.30810E-02	0.29603E-02
4	0.2250CE 01	0.39727E-02	0.37875E-02	0.33305E-02	0.28853E-02	0.27094E-02
5	0.3000CE 01	0.41353E-02	0.38862E-02	0.32596E-02	0.26636E-02	0.24401E-02
6	0.3750CE 01	0.42870E-02	0.39729E-02	0.31601E-02	0.24107E-02	0.21510E-02
7	0.4500CE 01	0.44288E-02	0.40487E-02	0.30270E-02	0.21201E-02	0.18410E-02
8	0.5250CE 01	0.45615E-02	0.41152E-02	0.28523E-02	0.17842E-02	0.15088E-02
9	0.6000CE 01	0.46845E-02	0.41776E-02	0.26036E-02	0.13937E-02	0.11531E-02
10	0.6750CE 01	0.47893E-02	0.42602E-02	0.21136E-02	0.93869E-03	0.77387E-03
11	0.7500CE 01	0.48241E-02	0.44995E-02	0.55473E-03	0.41095E-03	0.37347E-03
12	0.8750CE 01	0.52102E-02	0.23115E-02	0.47339E-03	0.35124E-03	0.31855E-03
13	0.1000CE 02	0.48661E-02	0.14145E-02	0.42462E-03	0.30680E-03	0.27714E-03
14	0.2000CE 02	0.20289E-02	0.12093E-03	0.16197E-03	0.11696E-03	0.10521E-03
15	0.3000CE 02	0.81633E-03	0.18426E-03	0.82076E-04	0.57550E-04	0.51368E-04
16	0.4000CE 02	0.41539E-03	0.83301E-04	0.42336E-04	0.29560E-04	0.26319E-04
17	0.5000CE 02	0.21699E-03	0.46423E-04	0.23238E-04	0.16075E-04	0.14270E-04
18	0.6000CE 02	0.11744E-03	0.26447E-04	0.13010E-04	0.89499E-05	0.79292E-05
19	0.7000CE 02	0.65110E-04	0.15220E-04	0.74226E-05	0.50806E-05	0.44936E-05
20	0.8000CE 02	0.36723E-04	0.88663E-05	0.42862E-05	0.29221E-05	0.25809E-05
21	0.9000CE 02	0.20986E-04	0.52064E-05	0.24987E-05	0.16977E-05	0.14978E-05
22	0.1000CE 03	0.12117E-04	0.30769E-05	0.14674E-05	0.99415E-06	0.87619E-06
23	0.1100CE 03	0.70543E-05	0.18278E-05	0.86685E-06	0.58584E-06	0.51589E-06
24	0.1200CE 03	0.41349E-05	0.10903E-05	0.51459E-06	0.34703E-06	0.30536E-06
25	0.1300CE 03	0.24375E-05	0.65268E-06	0.30673E-06	0.20645E-06	0.18153E-06
26	0.1400CE 03	0.14438E-05	0.39188E-06	0.18347E-06	0.12325E-06	0.10828E-06
27	0.1500CE 03	0.85874E-06	0.23590E-06	0.11006E-06	0.73781E-07	0.64749E-07
28	0.1600CE 03	0.51256E-06	0.14231E-06	0.66180E-07	0.44231E-07	0.38741E-07
29	0.1700CE 03	0.30687E-06	0.86008E-07	0.39863E-07	0.26485E-07	0.23107E-07
30	0.1800CE 03	0.18421E-06	0.52040E-07	0.24021E-07	0.15733E-07	0.13613E-07
31	0.1900CE 03	0.11080E-06	0.31488E-07	0.14430E-07	0.90927E-08	0.77360E-08
32	0.2000CE 03	0.66744E-07	0.18966E-07	0.86290E-08	0.47946E-08	0.39507E-08
33	0.2100CE 03	0.40022E-07	0.11591E-07	0.42903E-08	0.17327E-08	0.13535E-08
34	0.2110CE 03	0.38026E-07	0.11089E-07	0.38452E-08	0.14695E-08	0.11398E-08
35	0.2120CE 03	0.36114E-07	0.10630E-07	0.33682E-08	0.12113E-08	0.93253E-09
36	0.2130CE 03	0.34278E-07	0.10220E-07	0.28476E-08	0.95800E-09	0.73175E-09
37	0.2140CE 03	0.32512E-07	0.98622E-08	0.22695E-08	0.70961E-09	0.53759E-09
38	0.2150CE 03	0.30807E-07	0.95623E-08	0.16166E-08	0.46655E-09	0.35042E-09
39	0.2160CE 03	0.29154E-07	0.93272E-08	0.86828E-09	0.22959E-09	0.17089E-09
40	0.2170CE 03	0.27545E-07	0.91654E-08	0.	0.	0.

ANGULAR FLUXES FOR GROUP 1, GAUSS POINT 2

PT.	RADIUS	ANGULAR DATA FROM MU=1. TO MU=-1.				
1	-0.	0.46180E-02	0.46173E-02	0.46165E-02	0.46173E-02	0.46180E-02
2	0.7500CE 00	0.47750E-02	0.47254E-02	0.46038E-02	0.44789E-02	0.44261E-02
3	0.1500CE 01	0.49130E-02	0.48207E-02	0.45779E-02	0.43182E-02	0.42063E-02
4	0.2250CE 01	0.50327E-02	0.49022E-02	0.45339E-02	0.41270E-02	0.39514E-02
5	0.3000CE 01	0.51360E-02	0.49722E-02	0.44670E-02	0.38963E-02	0.36554E-02
6	0.3750CE 01	0.52247E-02	0.50325E-02	0.43707E-02	0.36139E-02	0.33111E-02
7	0.4500CE 01	0.53004E-02	0.50856E-02	0.42365E-02	0.32633E-02	0.29105E-02
8	0.5250CE 01	0.53640E-02	0.51350E-02	0.40526E-02	0.28212E-02	0.24438E-02
9	0.6000CE 01	0.54156E-02	0.51862E-02	0.37957E-02	0.22560E-02	0.18997E-02
10	0.6750CE 01	0.54434E-02	0.52697E-02	0.33178E-02	0.15269E-02	0.12666E-02
11	0.7500CE 01	0.52672E-02	0.58421E-02	0.77096E-03	0.58432E-03	0.53544E-03
12	0.8750CE 01	0.57077E-02	0.27416E-02	0.64364E-03	0.49317E-03	0.45194E-03
13	0.1000CE 02	0.51204E-02	0.15959E-02	0.56768E-03	0.42714E-03	0.39048E-03
14	0.2000CE 02	0.13865E-02	0.99388E-04	0.20082E-03	0.15503E-03	0.14221E-03
15	0.3000CE 02	0.36120E-03	0.21750E-03	0.98846E-04	0.75144E-04	0.68647E-04
16	0.4000CE 02	0.17644E-03	0.66787E-04	0.49624E-04	0.38011E-04	0.34750E-04
17	0.5000CE 02	0.77663E-04	0.44004E-04	0.26803E-04	0.20503E-04	0.18729E-04
18	0.6000CE 02	0.40054E-04	0.22756E-04	0.14813E-04	0.11341E-04	0.10357E-04
19	0.7000CE 02	0.21147E-04	0.12979E-04	0.83687E-05	0.64075E-05	0.58495E-05
20	0.8000CE 02	0.11614E-04	0.73783E-05	0.47944E-05	0.36714E-05	0.33507E-05
21	0.9000CE 02	0.65158E-05	0.42608E-05	0.27770E-05	0.21266E-05	0.19404E-05
22	0.1000CE 03	0.37165E-05	0.24813E-05	0.16220E-05	0.12422E-05	0.11332E-05
23	0.1100CE 03	0.21455E-05	0.14556E-05	0.95385E-06	0.73048E-06	0.66627E-06
24	0.1200CE 03	0.12500E-05	0.85894E-06	0.56403E-06	0.43194E-06	0.39391E-06
25	0.1300CE 03	0.73363E-06	0.50935E-06	0.33505E-06	0.25658E-06	0.23396E-06
26	0.1400CE 03	0.43311E-06	0.30329E-06	0.19981E-06	0.15300E-06	0.13949E-06
27	0.1500CE 03	0.25693E-06	0.18122E-06	0.11954E-06	0.91528E-07	0.83424E-07
28	0.1600CE 03	0.15304E-06	0.10860E-06	0.71716E-07	0.54885E-07	0.50002E-07
29	0.1700CE 03	0.91452E-07	0.65240E-07	0.43100E-07	0.32950E-07	0.29983E-07
30	0.1800CE 03	0.54789E-07	0.39241E-07	0.25930E-07	0.19747E-07	0.17905E-07
31	0.1900CE 03	0.32854E-07	0.23606E-07	0.15530E-07	0.11706E-07	0.10497E-07
32	0.2000CE 03	0.19697E-07	0.14054E-07	0.93322E-08	0.65882E-08	0.57200E-08
33	0.2100CE 03	0.11642E-07	0.82426E-08	0.52174E-08	0.27125E-08	0.21975E-08
34	0.2110CE 03	0.11024E-07	0.78343E-08	0.48289E-08	0.23460E-08	0.18801E-08
35	0.2120CE 03	0.10420E-07	0.74686E-08	0.44011E-08	0.19746E-08	0.15642E-08
36	0.2130CE 03	0.98217E-08	0.71582E-08	0.39062E-08	0.15968E-08	0.12492E-08
37	0.2140CE 03	0.92179E-08	0.69231E-08	0.33018E-08	0.12112E-08	0.93499E-09
38	0.2150CE 03	0.85911E-08	0.67946E-08	0.25235E-08	0.81679E-09	0.62154E-09
39	0.2160CE 03	0.79159E-08	0.68221E-08	0.14731E-08	0.41296E-09	0.30945E-09
40	0.2170CE 03	0.71538E-08	0.70817E-08	0.	-0.	0.

ANGULAR FLUXES FOR GROUP 2, GAUSS POINT 1

PT.	RADIUS	ANGULAR DATA FROM MU=1. TO MU=-1.				
1	-0.	0.80838E-03	0.80852E-03	0.80867E-03	0.80852E-03	0.80838E-03
2	0.7500CE 00	0.73503E-03	0.75801E-03	0.81391E-03	0.87049E-03	0.89413E-03
3	0.1500CE 01	0.66892E-03	0.71363E-03	0.82610E-03	0.94118E-03	0.98919E-03
4	0.2250CE 01	0.61006E-03	0.67574E-03	0.84735E-03	0.10234E-02	0.10954E-02
5	0.3000CE 01	0.55795E-03	0.64385E-03	0.87960E-03	0.11198E-02	0.12141E-02
6	0.3750CE 01	0.51206E-03	0.61749E-03	0.92523E-03	0.12338E-02	0.13464E-02
7	0.4500CE 01	0.47199E-03	0.59622E-03	0.98724E-03	0.13695E-02	0.14939E-02
8	0.5250CE 01	0.43750E-03	0.57948E-03	0.10697E-02	0.15320E-02	0.16577E-02
9	0.6000CE 01	0.40892E-03	0.56565E-03	0.11838E-02	0.17277E-02	0.18394E-02
10	0.6750CE 01	0.39118E-03	0.54170E-03	0.14022E-02	0.19642E-02	0.20401E-02
11	0.7500CE 01	0.41984E-03	0.42073E-03	0.21788E-02	0.22476E-02	0.22585E-02
12	0.8750CE 01	0.33473E-03	0.15015E-02	0.22525E-02	0.22890E-02	0.22916E-02
13	0.1000CE 02	0.58833E-03	0.19238E-02	0.22908E-02	0.23118E-02	0.23084E-02
14	0.2000CE 02	0.20471E-02	0.24841E-02	0.23607E-02	0.22803E-02	0.22493E-02
15	0.3000CE 02	0.23242E-02	0.23109E-02	0.21980E-02	0.20955E-02	0.20581E-02
16	0.4000CE 02	0.22050E-02	0.21464E-02	0.20016E-02	0.18943E-02	0.18558E-02
17	0.5000CE 02	0.20239E-02	0.19427E-02	0.18032E-02	0.16996E-02	0.16625E-02
18	0.6000CE 02	0.18270E-02	0.17477E-02	0.16169E-02	0.15197E-02	0.14846E-02
19	0.7000CE 02	0.16383E-02	0.15657E-02	0.14455E-02	0.13553E-02	0.13227E-02
20	0.8000CE 02	0.14637E-02	0.13987E-02	0.12893E-02	0.12060E-02	0.11757E-02
21	0.9000CE 02	0.13042E-02	0.12466E-02	0.11469E-02	0.10700E-02	0.10419E-02
22	0.1000CE 03	0.11591E-02	0.11079E-02	0.10170E-02	0.94589E-03	0.91969E-03
23	0.1100CE 03	0.10269E-02	0.98113E-03	0.89799E-03	0.83206E-03	0.80761E-03
24	0.1200CE 03	0.90598E-03	0.86491E-03	0.78853E-03	0.72718E-03	0.70429E-03
25	0.1300CE 03	0.79499E-03	0.75788E-03	0.68740E-03	0.63009E-03	0.60861E-03
26	0.1400CE 03	0.69261E-03	0.65885E-03	0.59353E-03	0.53982E-03	0.51959E-03
27	0.1500CE 03	0.59773E-03	0.56683E-03	0.50603E-03	0.45553E-03	0.43641E-03
28	0.1600CE 03	0.50941E-03	0.48096E-03	0.42416E-03	0.37649E-03	0.35839E-03
29	0.1700CE 03	0.42684E-03	0.40054E-03	0.34720E-03	0.30213E-03	0.28495E-03
30	0.1800CE 03	0.34940E-03	0.32484E-03	0.27481E-03	0.23187E-03	0.21559E-03
31	0.1900CE 03	0.27635E-03	0.25359E-03	0.20596E-03	0.16521E-03	0.15001E-03
32	0.2000CE 03	0.20766E-03	0.18543E-03	0.14199E-03	0.10119E-03	0.88118E-04
33	0.2100CE 03	0.14118E-03	0.12234E-03	0.76311E-04	0.39405E-04	0.31939E-04
34	0.2110CE 03	0.13448E-03	0.11656E-03	0.69003E-04	0.33375E-04	0.26766E-04
35	0.2120CE 03	0.12765E-03	0.11107E-03	0.61151E-04	0.27406E-04	0.21729E-04
36	0.2130CE 03	0.12063E-03	0.10597E-03	0.52495E-04	0.21521E-04	0.16855E-04
37	0.2140CE 03	0.11332E-03	0.10142E-03	0.42661E-04	0.15758E-04	0.12180E-04
38	0.2150CE 03	0.10559E-03	0.97658E-04	0.31129E-04	0.10175E-04	0.77539E-05
39	0.2160CE 03	0.97250E-04	0.94953E-04	0.17197E-04	0.48693E-05	0.36534E-05
40	0.2170CE 03	0.88057E-04	0.93596E-04	0.	0.	0.

ANGULAR FLUXES FOR GROUP 2, GAUSS POINT 2

PT.	RADIUS	ANGULAR DATA FROM MU=1. TO MU=-1.				
1	-0.	0.40762E-03	0.40798E-03	0.40836E-03	0.40798E-03	0.40762E-03
2	0.7500CE 00	0.36242E-03	0.37722E-03	0.41379E-03	0.45207E-03	0.46846E-03
3	0.1500CE 01	0.32535E-03	0.35136E-03	0.42380E-03	0.50575E-03	0.54244E-03
4	0.2250CE 01	0.29587E-03	0.33075E-03	0.44027E-03	0.57297E-03	0.63372E-03
5	0.3000CE 01	0.27314E-03	0.31452E-03	0.46514E-03	0.65825E-03	0.74641E-03
6	0.3750CE 01	0.25645E-03	0.30193E-03	0.50135E-03	0.76812E-03	0.88547E-03
7	0.4500CE 01	0.24540E-03	0.29212E-03	0.55316E-03	0.91206E-03	0.10570E-02
8	0.5250CE 01	0.24002E-03	0.28393E-03	0.62670E-03	0.11040E-02	0.12684E-02
9	0.6000CE 01	0.24085E-03	0.27562E-03	0.73113E-03	0.13644E-02	0.15290E-02
10	0.6750CE 01	0.24935E-03	0.26480E-03	0.88140E-03	0.17228E-02	0.18495E-02
11	0.7500CE 01	0.38627E-03	-0.40131E-04	0.21139E-02	0.22161E-02	0.22386E-02
12	0.8750CE 01	0.35735E-03	0.14967E-02	0.22226E-02	0.22847E-02	0.22978E-02
13	0.1000CE 02	0.77961E-03	0.19710E-02	0.22800E-02	0.23251E-02	0.23321E-02
14	0.2000CE 02	0.24101E-02	0.24924E-02	0.23992E-02	0.23513E-02	0.23320E-02
15	0.3000CE 02	0.23478E-02	0.22839E-02	0.22328E-02	0.21696E-02	0.21455E-02
16	0.4000CE 02	0.21437E-02	0.21188E-02	0.20305E-02	0.19642E-02	0.19393E-02
17	0.5000CE 02	0.19446E-02	0.18999E-02	0.18262E-02	0.17630E-02	0.17392E-02
18	0.6000CE 02	0.17418E-02	0.17075E-02	0.16354E-02	0.15767E-02	0.15545E-02
19	0.7000CE 02	0.15576E-02	0.15246E-02	0.14605E-02	0.14066E-02	0.13861E-02
20	0.8000CE 02	0.13886E-02	0.13600E-02	0.13015E-02	0.12521E-02	0.12333E-02
21	0.9000CE 02	0.12357E-02	0.12099E-02	0.11570E-02	0.11118E-02	0.10945E-02
22	0.1000CE 03	0.10966E-02	0.10736E-02	0.10255E-02	0.98396E-03	0.96799E-03
23	0.1100CE 03	0.96987E-03	0.94910E-03	0.90518E-03	0.86691E-03	0.85213E-03
24	0.1200CE 03	0.85383E-03	0.83500E-03	0.79469E-03	0.75927E-03	0.74553E-03
25	0.1300CE 03	0.74708E-03	0.72989E-03	0.69271E-03	0.65979E-03	0.64697E-03
26	0.1400CE 03	0.64840E-03	0.63261E-03	0.59817E-03	0.56743E-03	0.55543E-03
27	0.1500CE 03	0.55675E-03	0.54217E-03	0.51010E-03	0.48132E-03	0.47003E-03
28	0.1600CE 03	0.47126E-03	0.45772E-03	0.42777E-03	0.40068E-03	0.39004E-03
29	0.1700CE 03	0.39118E-03	0.37857E-03	0.35042E-03	0.32490E-03	0.31483E-03
30	0.1800CE 03	0.31592E-03	0.30406E-03	0.27764E-03	0.25340E-03	0.24385E-03
31	0.1900CE 03	0.24484E-03	0.23379E-03	0.20862E-03	0.18577E-03	0.17667E-03
32	0.2000CE 03	0.17764E-03	0.16711E-03	0.14348E-03	0.12169E-03	0.11230E-03
33	0.2100CE 03	0.11356E-03	0.10249E-03	0.81231E-04	0.55239E-04	0.47269E-04
34	0.2110CE 03	0.10725E-03	0.96110E-04	0.74979E-04	0.48095E-04	0.40571E-04
35	0.2120CE 03	0.10089E-03	0.89807E-04	0.68498E-04	0.40729E-04	0.33806E-04
36	0.2130CE 03	0.94398E-04	0.83700E-04	0.61512E-04	0.33102E-04	0.26979E-04
37	0.2140CE 03	0.87613E-04	0.78071E-04	0.53429E-04	0.25189E-04	0.20110E-04
38	0.2150CE 03	0.80181E-04	0.73560E-04	0.42964E-04	0.16982E-04	0.13246E-04
39	0.2160CE 03	0.71336E-04	0.71634E-04	0.27275E-04	0.85312E-05	0.64801E-05
40	0.2170CE 03	0.59365E-04	0.75649E-04	0.	-0.	0.

OUTPUT EDIT BEGUN AT TIME=014812, EDIT FINISHED AT TIME=014818

PHILLIPS
PETROLEUM
COMPANY

ATOMIC ENERGY DIVISION