A MULTIAXIAL VISCOPLASTIC MODEL FOR ADVANCED Si₃N₄ CERAMICS

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ABSTRACT

A uniaxial creep/creep rupture model developed previously based on results obtained from uniaxial tensile creep tests of an advanced Si₃N₄ ceramic has been refined and upgraded in multiaxial form to facilitate general applications. Severity of asymmetric creep behavior in tension compared to that in compression observed in recent experimental results mandates this revision. The tensorial formulation follows the plasticity theories developed for soils. Material sensitivity to pressure, asymmetry between tension and compression, changes of material structure due to hardening of grain boundary phase, and deformation induced volumetric swelling attributable to cavity and void formations have all been considered. Provisions for such features in the proposed multiaxial model are shown to have improved modeling flexibility.

INTRODUCTION

A creep/creep rupture model has been developed recently [1] based on experimental results [2] obtained from uniaxial tensile creep tests of an advanced silicon nitride (Si₃N₄) ceramic, commercially known as GN-10 Si₃N₄ (engineered and marketed by AlliedSignal Ceramic Components, Torrence, California). The exploratory model was expressed in scalar form because only uniaxial tensile data were available. The uniaxial model has been limited for general applications due to the lack of generality, i.e., in the general form of multiaxiality.

In the absence of multiaxial experimental data, a uniaxial inelastic constitutive equation is usually generalized into a tensor form using the concept of the Mises effective stress (or strain) defined as the second invariant \( J_2 \) of the stress (or strain) deviators, namely,

\[
J_2 = \frac{1}{2} \sigma' \cdot \sigma' \tag{1}
\]

where

\[
\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \sigma k \delta_{ij} \tag{2}
\]

and \( \sigma_{ij} \) and \( \delta_{ij} \) are the stress tensor and Kronecker delta symbol, respectively. The same
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relationships can be written for the strain by replacing $\sigma$ with $\varepsilon$. The plastic incompressibility and symmetry in tension and compression are corollaries. This method of generalization usually simulates the actual behavior of initially isotropic metals and alloys with reasonable accuracy.

The above approach, which is not well physically based but rather philosophical in nature, is obviously inappropriate for ceramic materials even based on limited information available on compressive creep test data previously generated for other ceramic materials [3] or other types of $\text{Si}_3\text{N}_4$ ceramic materials [4]. Ceramic materials are known to be excellent materials for practical use in compression mode, owing to their superior compressive bearing strength and compressive creep resistance compared to their behavior in tension. The importance of this asymmetric behavior was recognized in analyses of bending or flexural creep tests of ceramics [5,6].

Figure 1 shows recent creep data of GN-10 $\text{Si}_3\text{N}_4$ tested at 1300°C in compression with a series of applied stresses beginning from -125 MPa [7], including a creep curve for a specimen tested in tension at 125 MPa (ruptured in 15.2 h) for comparison. The compressive stress was increased intermittently in steps of -25 MPa increment each time the specimen completed a period of 200 to 300 h at low stresses and 1000 h at high stresses. The asymmetric creep behavior in tension and compression is clearly discerned. The creep rate under a tensile stress of 125 MPa is significantly higher than that under the same stress in compression by about three orders of magnitude. Although tensile creep curves at stresses above 125 MPa are not available for comparison, more severe contrast in creep behavior under tension and compression can be contemplated. On the basis of this observation, it is imperative that the aforementioned uniaxial creep model be refined to account for the asymmetric behavior while the uniaxial model is being generalized into a multiaxial viscoplastic model.

A UNIAXIAL CREEP MODEL

A constitutive model has been proposed for describing creep behavior of GN-10 $\text{Si}_3\text{N}_4$ ceramic material in uniaxial tension. The model [1] was formulated based on microstructural features and mechanical behavior observed at macroscopic levels in conjunction with an ideal conceptual model, which was perceived as an aggregate of small $\text{Si}_3\text{N}_4$ grains bonded with an amorphous grain boundary phase material, as shown in Fig. 2. The ideal model contains a limited number of imperfections, such as internal voids and surface microcracks, preexisting randomly in the grain boundary phase. However, the microdefects are assumed to be low in number and small in size and remain obscure at the macroscopic level, except that a large dominant one may grow and lead to eventual fracture. Deceleration of creep rate and enhancement of resistance to creep due to exposure in high temperatures are viewed as the consequences of grain boundary phase hardening, which results from continuous devitrification of the residual glass phase in the grain boundary region and phase transformations. Detailed discussions of microstructure of GN-10 $\text{Si}_3\text{N}_4$ are given elsewhere [2].

The mathematical model was formulated using the state variable approach. Two internal state variables, namely, "hardening variable ($\delta$)" and "damage variable ($\omega$)" , were used to characterize the current state of the material. The model consists of three rate equations: a flow rule, Eq. 3, shown below, which describes the inelastic strain rate ($\dot{\varepsilon}$) as a function of the hardening state variable, uniaxial tensile stress ($\sigma$), and temperature ($T$); two evolution rules, given in Eqs. 4 and 5, for the two state variables.
\[
\dot{\varepsilon} = \frac{\dot{\varepsilon}_0 (54)^n \exp \left( \frac{Q_s}{RT} \right)}{\delta} \left( \frac{\sigma - \sigma_{a}}{54} - \bar{c} \right)^n
\] (3)

\[
\dot{\delta} = \frac{\dot{\delta}_0 \exp \left( \frac{Q_s}{RT} \right)}{\delta^m}
\] (4)

\[
\dot{\omega} = \frac{\omega_0 \left( \frac{\sigma}{54} \right)^n \exp \left( \frac{Q_u}{RT} \right)}{\delta (1 - \omega)}
\] (5)

where

\[
\dot{\varepsilon}_0 = e^{78.08} \text{ (T} \leq 1200^\circ\text{C)}, \quad e^{21.92} \text{ (T} > 1200^\circ\text{C});
\]

\[
\sigma_{a} = 1765.8 - 1.12T \text{ MPa (T} \leq 1250^\circ\text{C)}, \quad 516.9 - 0.3T \text{ MPa (T} > 1250^\circ\text{C});
\]

\[
\sigma_{\text{trans}} = 1848.3 - 1.12T \text{ MPa (T} \leq 1250^\circ\text{C)}, \quad 1108.1 - 0.634T \text{ MPa (T} > 1250^\circ\text{C});
\]

For \( \sigma \leq \sigma_{\text{trans}} \), \( \bar{c} = 0 \) and \( n = 1 \),

For \( \sigma > \sigma_{\text{trans}} \),

\[
c = 1 \text{ and } n = 1.32 \text{ (T} \leq 1250^\circ\text{C)}, \quad c = 1 \text{ and } n = 1.7 \text{ (T} > 1250^\circ\text{C});
\]

\[
Q_s = 957.4 \text{ kJ/mole (T} \leq 1200^\circ\text{C)}, \quad 270 \text{ kJ/mole (T} > 1200^\circ\text{C});
\]

\[
m = 1/3;
\]

\[
\dot{\delta}_0 = e^{69.5} \text{ (T} \leq 1200^\circ\text{C)}, \quad e^{6.26} \text{ (T} > 1200^\circ\text{C});
\]

\[
Q_u = 1174 \text{ kJ/mole (T} \leq 1200^\circ\text{C)}, \quad 13.26 \text{ kJ/mole (T} > 1200^\circ\text{C});
\]

\[
\omega_0 = e^{103.46}, \quad \nu = 10.47, \text{ and } Q_u = 1497 \text{ kJ/mole}.
\]

T is the absolute temperature in K, R the gas constant, \( \sigma_{a} \) the threshold stress below which creep is assumed to be negligible, and \( \sigma_{\text{trans}} \) the transition stress below which the inelastic strain rate is a linear function of the applied stress otherwise a nonlinear function. Interestingly, the order-of-magnitude breaks in creep rate and creep lifetime observed in previous tests [2] occur approximately along the line of \( \sigma_{\text{trans}} \) given above. Note that although Eq. 4 is coupled with Eqs. 3 and 5 individually, the latter two equations are mutually independent. Physically, the relationships implies that creep deformation and fracture behavior are dependent on the material state such as the hardening of grain boundary phase, but the deformation process and damage formation do not interfere with each other. Detailed discussions concerning the model formulation are given elsewhere [1].

**GENERALIZATION TO A MULTIAXIAL VISCOPLASTIC MODEL**

Examination suggests that a weak analogy may exist between a body of compacted soil (or sand) thoroughly wetted with water and a ceramic material densified with amorphous glass phase materials as depicted in Fig. 2. The differences are that ceramic grains are usually densely packed and the grain boundary glass phase is relatively thin compared to the grain diameters.
Contrastingly, the water phase between soil particles is relatively thick compared to the size of soil particles. In soil mechanics, the inelastic behavior of wet soil is usually described by the Coulomb friction model \[^{[8]}\] as

\[\tau + \sigma \tan \Phi = k,\]

(6)

where \(\sigma\) and \(\tau\) are the normal and shear stresses, respectively, as shown in Fig. 3; \(k\) the cohesive strength, and \(\Phi\) the friction angle. Equation (6) indicates that the shear stress required to yield plastic deformation along the slip plane is governed by the magnitude and direction of the normal stress being applied on the slip plane. The tensile normal stress (\(\sigma\) is positive in Fig. 3a) acting on the slip plane facilitates slip, and the compressive normal stress (\(\sigma\) is shown in Fig. 3b) impedes slip. It is postulated that the asymmetric creep behavior in tension and compression is attributed to the difference in resistance to shear. The cohesive strength \(k\) in Eq. (6) represents the intrinsic shear resistance of the material in the absence of the normal stress. If \(k = 0\), the critical shear stress required to initiate slip is equal to \(\sigma \tan \Phi\). Because this situation is analogous to the general friction model, \(\Phi\) is therefore called the friction angle. When \(k\) is not zero, Eq. (6) implies that the critical shear stress is the sum of \(\sigma \tan \Phi\) and \(k\).

Equation (6) may be generalized into a tensor form by replacing \(\tau\) with \(q = (3\eta)^{[9]}\) and \(\sigma\) with \(-p = (1/3)\sigma_{\text{in}}\), resulting in the Drucker-Prager yield criterion for soils \[^{[9]}\] as

\[q - p \tan \Phi = k\]

(7)

where \(k\) is an appropriate yield strength in tension, compression, and shear whichever is applicable.

To aid in generalizing Eq. 3 into a tensor form, Eq. 7 is not a convenient form for an obvious reason that it is not a rate-dependent expression. If \(\sigma_{\text{in}}\) in Eq. 3 is viewed as the static yield stress and \((\sigma - \sigma_{\text{in}})\) as the overstress (or viscous stress) \[^{[10]}\] defined in overstress viscoplasticity theories, it is not difficult to recognize that Eq. 3 is a viscoplastic flow equation which indicates that the inelastic strain rate is driven by \((\sigma - \sigma_{\text{in}})\). Therefore, the inelastic strain rate is assumed to be derived from a flow potential \(\Omega\) as

\[\dot{\varepsilon}_y = \frac{\partial \Omega}{\partial \sigma_y}\]

(8)

With reference to the form of Eq. 3, an appropriate form of \(\Omega\) may be written in a power law form of \((\delta - \sigma_{\text{in}})\) as

\[\Omega = \dot{\varepsilon}_0 \exp \left( \frac{Q_0}{RT} \right) \left( \frac{\delta}{54} \right)^{n+1} \left( \frac{\delta - \sigma_{\text{in}}}{54} \right)^{n+1},\]

(9)

where
Substituting Eq. 9 into Eq. 8 yields

\[ \dot{\varepsilon}_y = \frac{\alpha_0}{\delta} \left( \frac{g - \sigma_{sh}}{54} - c \right)^n \frac{\partial g}{\partial \sigma_y} \]  \hspace{1cm} (11)

or

\[ \dot{\varepsilon}_y = \frac{\alpha_0}{\delta} \left( \frac{g - \sigma_{sh}}{54} - c \right)^n \frac{\sqrt{3} - \sigma_y' + \frac{1}{2} \delta_y \tan \Phi}{1 + \frac{1}{3} \tan \Phi} \]  \hspace{1cm} (12)

in which \( \alpha_0 = \dot{\varepsilon}_d(54)^n \exp(-Q/RT) \). Equation 12 together with Eq. 4 form a multiaxial viscoplastic constitutive model for describing the inelastic deformation of Si₃N₄ ceramics. Note that when \( g = \sigma_{sh} \), Eq. 10 reduces to Eq. 7, and Eq. 11 in turn yields \( \dot{\varepsilon}_y = 0 \).

It can be demonstrated that \( g \) reduces to \( \sigma \) and Eq. 12 to Eq. 3 in uniaxial tension (e.g., all \( \sigma_i = 0 \) except \( \sigma_{11} = \Sigma \)). Under uniaxial compression loading (all \( \sigma_i = 0 \) except \( \sigma_{11} = -\Sigma \)), Eq. 12 reduces to

\[ \dot{\varepsilon}_{	ext{com}} = \frac{\alpha_0}{\delta} \left( \frac{\Sigma \theta - \sigma_{sh}}{54} - c \right)^n (-\theta) \]  \hspace{1cm} (13)

where \( \theta = (1-\sqrt{\tan \Phi})/(1+\sqrt{\tan \Phi}) \). With the aid of Eq. 13, the ratio of creep rate in compression \( (\dot{\varepsilon}_{	ext{com}}) \) to that in tension \( (\dot{\varepsilon}_{	ext{un}}) \), under the same stress amplitude of \( \sigma \) can be written as

\[ \Gamma = \frac{\dot{\varepsilon}_{	ext{com}}}{\dot{\varepsilon}_{	ext{un}}} = \frac{-\theta (\Sigma \theta - \sigma_{sh} - 54c)^n}{(\Sigma - \sigma_{sh} - 54c)^n} \]  \hspace{1cm} (14)

Assuming that \( (\sigma - \sigma_{sh})/\sigma < \theta \) to ensure that the applied stress is sufficiently high enough to induce creep. Equation 14 illustrates that the model predicts asymmetric creep behavior in tension and compression.

As mentioned in Ref. [8], one important characteristic of Eq. 12 is that the volumetric strain rate

\[ \dot{\varepsilon}_{sh} = \frac{\alpha_0}{\delta} \left( \frac{g - \sigma_{sh}}{54} - c \right)^n \frac{\tan \Phi}{1 + (1/3) \tan \Phi} \]  \hspace{1cm} (15)
derived from Eq. 12, is always a positive, nonzero, constant value regardless of whether the material is in tension or in compression. However, this paradoxical observation does not contradict reported experimental test results [4]. Ceramic materials show swelling under either tension or compression, resulting from cavity or void formation at grain boundaries when the materials were crept for a long period of time.

DISCUSSION

Predictability of Eq. 13 was examined using the parametric values given in the section following Eqs. 3 to 5. To simulate the compressive creep curve for -125 MPa depicted in Fig. 1, an appropriate value of $\theta$ must be calculated from $\tan \phi$ which can not be measured experimentally at this time. Therefore, $\theta$ must be calculated from Eq. 14 with a known value of $\Gamma$. The dilemma is that the characteristic features of the tensile creep curve and those of compressive one are contrastingly different, indicating a fast creep rate and a short life under tension whereas slow creep rate and potentially a long creep life are portrayed under compression. Besides, the shortness of the tensile creep curve indicates that the test was interrupted before completing transient creep. For the purpose of illustration, $\dot{\epsilon}_{\text{tensile}}$ at $t=100$ h was calculated using Eq. 1 and $\dot{\epsilon}_{\text{compressive}}$ at $t=100$ h was measured directly from the compressive creep curve (Fig. 4), yielding a strain rate ratio $\Gamma=0.0187$. This in turn yields $\theta=0.8143$ upon substituting the $\Gamma$ value in Eq. 14, and a value of $\tan \phi=0.307$.

Simulated compressive creep curve calculated from Eqs. 4 and 13 is plotted in Fig. 4 for comparison with measured creep curve. Agreement between the simulation and experimental data is respectable in consideration of the fact that the model is exploratory in nature. The calculated creep curve could have been plotted to fit closer to the end portion of the experimental curve by choosing a different value for $\Gamma$, but this type of ploy is meaningless and has no value to model development. A major deficiency of the calculated curve is the failure to simulate the initial portion of primary creep because of the characteristic differences in geometric features of tensile and compressive creep curves as discussed in the preceding section. It should be noted that discrepancies in primary creep stage of simulation and measured creep curve are not uncommon even for creep in tension. However, the exploratory model in the present form is far from complete and needs further refinement. For example, features capable of predicting the effects of grain boundary hardening on creep behavior under compressive loading and the volume shrinkage under hydraulic pressure are desirable.

Since the emphasis of this paper is placed on generalization of the deformation model, no mention has been or will be made as of how to generalize the damage growth equation in a tensor form. The approach to upgrade the damage equation must be physically based rather than philosophical in nature. A simple concept may be utilized by replacing tensile stress $\sigma$ with the maximum positive principal stress. In this case, only tensile stress is considered to promote the growth of damage. Modeling approaches may be borrowed from a recent work [11] which discussed the development of a continuum damage model for elastic solids.

CONCLUSION

An exploratory multiaxial viscoplastic model was studied using the Coulomb friction plasticity model previously developed for soil mechanics use. The multiaxial model is capable of simulating asymmetric creep behavior in tension compared to that in compression. The model includes
material sensitivity to pressure, changes of material structure due to hardening of grain boundary phase, and deformation induced volumetric swelling attributable to cavity and void formations. Qualitative agreement between a simulated compressive creep curve and experimentally obtained creep data are respectable in view of the model which is exploratory in nature.

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REFERENCES

Fig. 1 - Creep curves of GN-10 Si$_3$N$_4$ tested in tension (ruptured in 15.2 h) and in compression at 1300°C. The compressive stress was increased intermittently in steps of -25 MPa each time the specimen completed a period of time when steady-state creep was discerned.

Fig. 2 - A conceptual ceramic model.
Fig. 3 - An ideal soil model (a) in tension and (b) in compression, showing the normal stress (σ) and shear stress (τ) acting on a slip plane.

Fig. 4 - Comparison of simulated and measured compressive creep curves for a GN-10 Si₃N₄ specimen tested at 1300°C with an applied stress of 125 MPa.
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